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Effect of Supply Reliability in a Retail Setting with Joint Marketing and Inventory Decisions

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This paper studies the impact of supply reliability on a retail firm's performance under joint marketing and inventory decisions. The firm sells a product in a single selling season and can exert marketing effort to influence consumer demand. We develop a modeling framework to quantify the value of improving supply reliability and investigate how this value depends on different model parameters. Our results provide useful insights into how firms should make investment decisions on adopting new technologies to improve supply reliability. First, we establish a necessary and sufficient condition under which the maximum unit cost a firm is willing to pay to improve supply reliability increases in product price. We further show that this condition would hold in most practical situations. Thus, with some caveats, our result supports the intuition that a firm is willing to pay more to improve supply reliability for products with a higher price. Next, we show that for two products with the same price, a firm is willing to pay more to improve supply reliability for the product with a higher product cost. This implies that it is not necessarily true that emerging technologies for improving supply reliability should be first adopted for products with the highest unit contribution margin. Finally, we show that a product with a lower marketing cost function always benefits more from improved supply reliability than a product with a higher marketing cost function. This finding suggests that the priority of adopting new technologies should be given to situations where the firm can effectively induce greater demand through promotional effort.

Key words: supply uncertainty; inventory; marketing and operations interface; information technology; stochastic orderings
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1. Introduction

Because the success of a firm depends upon matching supply with demand, both supply and demand uncertainties can present a major obstacle to achieving this goal. Demand uncertainty is inherent in almost all practical business environments and has been studied extensively in the inventory management literature. Although supply uncertainty has received less research attention, it would also have a significant impact on a firm's bottom-line performance. Take the retail industry as an example. It is critical for any retailer to move the right quantity of products to the right location at the right time to turn potential consumer demand into sales revenue. This requires a reliable supply process that usually consists of many intermediate steps, starting from producing the products at the supplier to shelving the products in the retailer's store. Understanding the impact of supply uncertainty and the value of improving supply reliability is therefore among one of the major keys to achieving the final goal of matching supply and demand.

Recent studies have shown that a variety of supply chain glitches (e.g., unexpected supply disruption, incorrect shipment quantity, transportation delays) can cause severe stockout situations (Hendricks and Singhal 2005). Furthermore, a careful selection of suppliers and proper management of supplier relationship can be important in reducing uncertainty in the supply channel. For example, in situations where there is scarce supply of products or capacity, a supplier's allocation decision will affect his ability of meeting an individual retailer's order, which in turn contributes to the retailer's uncertain supply. Understanding the adverse impact of unreliable supply on profitability helps a retailer select a more reliable supplier or provide proper incentives to the supplier in negotiating for an adequate allocation of scarce products.

Interestingly, recent evidence demonstrates that it is also the last few feet from the backroom to the store shelves that determines whether the goods are available to meet consumer demand. According to Raman et al. (2001), about 16% of product items at a leading retailer could not be found by customers, not because the store is out of stock as is typically assumed, but because the items were misplaced in the backroom. See also Ton and Raman (2004) and DeHoratius and Raman (2008) for more empirical studies along this line. Moreover, Fisher et al. (2006) find that inventory availability can significantly affect overall customer satisfaction and firms' sales. Thus, any retailer that pursues operational excellence needs to reduce execution errors to improve supply reliability.

Reliable supply is especially important for products with a relatively short selling season such as fashion and high-tech products with short life cycles and promotional products. For fashion goods, misplaced stock or late shipments that show up after the selling season are of little value due to the perishable nature of the goods. For promotional products, missing the promotional time window means that it will be harder to push these products to consumers. Firms realize the importance of improving supply reliability for these products. For instance, Gillette has adopted radio frequency identification (RFID) technology to manage its promotional display program, and is able to monitor the product flows from distribution centers to back rooms of retail stores to sales floors. This has greatly improved the supply reliability for Gillette's products and yielded excellent results. Subirana et al. (2006) reported that sales at stores that used RFID were, on average, 19% higher than those stores that did not use it. Other firms using new inventory tracking systems to improve promotion effectiveness include Proctor & Gamble and Walgreens (Liard and Shah 2007).

In this research, we analyze the effects of supply uncertainty for a retailer facing both marketing and inventory decisions. Here, we use supply uncertainty as a collective term to refer to various factors that may contribute to a less reliable supply, including production yield and quality problems, insufficient capacity allocation due to scarce supply, stochastic shipping lead time, shrinkage and theft, store execution errors, etc. Any combination of these factors limits the ability of the retailer to put an appropriate amount of stock on store shelves when demand arrives. The retailer faces uncertain demand in a single selling season and needs to decide on the order quantity before the selling season starts. The single-period model is intended to capture the key features of the perishable goods and promotional products mentioned above. In addition, the retailer may exert marketing effort such as advertising, promotional displays, and consumer incentives (free gifts, direct mail service, etc.) to induce greater consumer demand (Krishnan et al. 2004). The objective of our research is to address two fundamental questions in such a retail setting. First, what is the impact of supply reliability on the retailer's optimal decisions and the corresponding profit? Second, what is the value of improving supply reliability and how does this value depend on the underlying product characteristics?

To address the above two research questions, we introduce a so-called supply reliability factor, which is defined as the random fraction of the order quantity that is available to satisfy demand. We show that a firm will increase his optimal marketing effort to induce a higher demand level as the supply reliability factor becomes larger or less variable. Furthermore, the expected profit of the firm will also increase. However, the optimal order quantity does not necessarily decrease as supply reliability improves. In addition, we demonstrate that with a higher marketing cost function, a firm will induce lower demand and order fewer products, resulting in a lower expected profit.

Despite the apparent benefits of improving supply reliability, many firms are concerned with the high cost of adopting emerging information technologies to achieve this objective. How should firms determine how much to invest in these information technologies? Our research attempts to provide some managerial insights on firms' investment decisions by analyzing the trade-off between the costs and benefits of improving supply reliability. Specifically, we analyze the maximum unit cost that a retailer is willing to pay to implement the technology to improve supply reliability. We define this maximum willingness to pay as the *threshold unit cost*, i.e., the retailer will not choose the technology if the additional unit cost for adopting this technology is higher than this threshold value.

We highlight the following results regarding the value of improving supply reliability. First, we analyze the impact of product's economic parameters (e.g., procurement cost and retail price) on the threshold value for adopting a technology. We find that, all else being equal, the maximum unit cost a firm is willing to pay to improve supply reliability may either increase or decrease in product price. We then establish a necessary and sufficient condition under which this maximum unit cost is actually increasing in product price. We further show that the condition would hold in most practical situations, and thus the intuition that a firm is willing to pay more for products with a higher price is reasonably robust. Second, we show that for two products with the same price, a firm is willing to pay more to improve supply reliability for the product with a higher unit product cost. This result suggests that it is not necessarily true that expensive emerging technologies for improving supply reliability should be first adopted for products with the highest unit contribution margin. Finally, we investigate how the firm's marketing cost function affects the value of improving supply reliability. We find that for any given unit technology cost, the absolute value of improving supply reliability is always higher when the firm's marketing cost function is lower. This means that improving supply reliability will be more rewarding for products for which sales effort is more effective in attracting consumer demand.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents our model and characterizes the firm's optimal decisions. In §4 we analyze the impact of supply reliability and marketing cost function on the optimal decisions and expected profit. Section 5 quantifies the value of improving supply reliability and studies how different product characteristics can affect this value. The paper concludes with §6. All proofs are given in the appendix.

2. Literature Review

This paper is related to the literature on optimal inventory decisions under supply uncertainty due to random production yield. Yano and Lee (1995) provide a comprehensive review of this literature. Our model differs from this literature in that we consider effort-dependent demand for the firm, and we also evaluate the value of improving supply reliability.

Recently, Li and Zheng (2006) studied the optimal pricing and inventory decisions of a firm under both random yield and stochastic demand, which is closely related to our work. However, our paper differs from theirs in several important aspects. First, Li and Zheng (2006) consider a periodic-review model with price-dependent demand. In contrast, we utilize a single-period model with marketing effort-dependent demand. Second, when modeling demand uncertainty, Li and Zheng (2006) use an additive demand function, which implies that demand variance remains the same as price decreases or the demand level increases. We use a multiplicative demand function where demand variance increases as the demand level increases. Third, Li and Zheng (2006) focus on characterizing the structure of the optimal inventory policy for the firm, whereas the focus of our paper is to understand the value of improving supply reliability under various product characteristics. Therefore, the analytical results and managerial insights are quite different.

There is a growing literature focusing on how emerging information technologies can help improve operational efficiency. Apte et al. (2006) address some general issues regarding RFID selection and adoption. Zipkin (2006) discusses how RFID can improve transaction processes as well as some security and privacy issues associated with this technology. Kok and Shang (2006) analyze an inventory system with accumulating inaccuracy in inventory records and study the value of RFID in improving inventory tracking in such an inventory system. Heese (2007) studies a two-level supply chain involving a manufacturer (as a Stackelberg leader) and a retailer with inventory inaccuracy and random demand, and derives the threshold value for the tag cost for adopting RFID.

Gaukler et al. (2008) quantify the value of obtaining the supplier's order progress information using RFID for a retailer with random lead time and stochastic demand. Gaukler et al. (2007) study the value of RFID in a supply chain setting and address the incentive compatibility issue in which the supply chain members have conflicting interests in adopting RFID. Lee and Özer (2007) provide a review of current research on modeling the value of RFID. Although our paper does not explicitly model the details of how RFID can improve the underlying supply reliability, our paper contributes to this literature by providing a general framework for modeling and analyzing the impact of improving supply reliability under joint marketing and stocking decisions.

Finally, there are many papers in the literature involving joint marketing (e.g., advertising, promotions, and sales force incentives) and operations (e.g., manufacturing, procurement, and inventory planning) decisions. Recently, the *Journal of Operations Management* and *Management Science* published special issues focusing on marketing and operations interfaces and their coordination; see Malhotra and Sharma (2002) and Ho and Tang (2004) and the references therein. Our paper considers joint marketing and inventory decisions under supply uncertainty, which is different from the majority of studies in this literature.

3. Model Formulation

A retailer sells a product in a single selling season. Due to a long supply lead time, the firm needs to decide on the order quantity before the selling season starts. However, the available stock to meet demand can be less than this order quantity due to various supply uncertainty factors. The firm can invest in demand-enhancing effort to induce customer demand. We assume that the firm needs to make the marketing decision before the actual available stock is realized, because demand-enhancing effort such as promotional campaigns is usually planned well before the start of the selling season.

There is a fixed retail price p for the product, but the retailer can exert sales effort to attract some target demand level D. The fixed price assumption may reflect the retailer's limited pricing power when facing intense competition in the market. This assumption is also reasonable in situations where the retail firm has little control over product price, especially in the electronics and fashion industries, where manufacturers dictate the retail price.[1] Let $A(D)$ denote the marketing cost for attracting demand level D. We assume that $A(D)$ is a strictly increasing, convex function satisfying $A(0) = 0$ and $A(\infty) = \infty$. That is, as the target demand level increases, it becomes increasingly more expensive to acquire additional demand.

Given a target demand level D, the firm's actual demand is stochastic and is equal to ωD, where ω is a positive random variable independent of D. Under this multiplicative demand function, demand variance increases in the target demand level D. Let G and g denote the distribution and density functions of ω, respectively. Assume that G is twice differentiable. Without loss of generality, we assume $E(\omega) = 1$ such that the expected realized demand is equal to the target demand level. This demand model has been widely used in the operations and marketing literatures; see, for example, Taylor (2002) and Rao (1990).

Let Q be the firm's order quantity before the selling season. However, the actual stock available to meet demand during the selling season is uncertain because the supply process is not perfectly reliable. Specifically, the available stock to meet demand is given by ϵQ, where ϵ is a positive random variable.[2] We refer to ϵ as the supply reliability factor. Let F and f denote the distribution and density functions of ϵ. Assume that F is twice differentiable. Also, we assume that the support of ϵ is a subinterval within $[0, 1]$, i.e., the actual available stock to meet demand is always less than Q due to supply uncertainty, which is most realistic. However, our analysis also applies to the general case where ϵ can be greater than one.

The retailer obtains the products at a unit cost c, $0 < c < p$. There is a unit shortage cost m for any

[1] Many manufacturers control the retail price by using resale price maintenance contracts. For example, although the resale prices of different retailers might differ slightly, Apple restricts its retailers to some minimum price for selling popular goods such as the iPod. Some manufacturers such as Nike and Gucci even fix the resale price of their products for all retailers.

[2] An alternative way to model the effect of supply reliability is an additive model in which the available stock to meet demand is equal to $Q + \epsilon$. The additive model, however, implies that the magnitude of shrinkage and the variance of the available stock is independent of the actual inventory amount Q, which is less realistic.

unmet demand and a unit salvage value v for any leftover inventory at the end of the selling season. The retailer's procurement cost depends on the assumption of whether the retailer is responsible for the unavailable inventory $(1-\epsilon)Q$. If the supply uncertainty is due to misplacement or damage at the retail store, then the retailer would need to pay for the total order quantity and the procurement cost is equal to cQ. However, if the supply uncertainty is caused by quality or capacity allocation issues, then the retailer would only pay for the available inventory ϵQ and the procurement cost is equal to $c\epsilon Q$. We assume that the retailer's procurement cost is equal to cQ throughout the paper. That is, the supply reliability is mostly the retailer's responsibility and the retailer has to pay for the entire order.[3]

Similarly, the leftover inventory at the end of the selling season depends on the assumption of whether the unavailable inventory $(1-\epsilon)Q$ would show up after the selling season. For supply uncertainty due to late delivery or misplaced inventory that can be recovered at the end of the selling season, the leftover inventory includes this unavailable quantity $(1-\epsilon)Q$. However, for supply uncertainty due to theft or shrinkage, we do not need to include this quantity. In our model, we consider the unavailable inventory as leftover inventory. The analysis and results for the other scenario are similar and therefore omitted.

Let $\pi(D, Q)$ be the expected profit of the firm, given marketing decision D and inventory decision Q. Also, let $x^+ = \max(0, x)$ denote the positive part of any real number x. Then, the firm's optimization problem can be formulated as

$$\begin{aligned}\max_{D,Q} \pi(D, Q) &= \{pE[\min(\omega D, \epsilon Q)] - m(\omega D - \epsilon Q)^+ + v(\epsilon Q - \omega D)^+ \\ &\quad - cQ + v(1-\epsilon)Q - A(D)\}. \quad (1)\end{aligned}$$

[3] A more general model is to assume that the supply reliability factor ϵ can be decomposed into two parts: $\epsilon = \epsilon_r \epsilon_s$, where ϵ_s denotes the uncertainty due to supplier's production problems and ϵ_r refers to the retailer's execution errors. The retailer's procurement cost is then equal to $c\epsilon_s Q$. This more general model can be analyzed in a similar way. See the e-companion available on the *Manufacturing & Serivce Operations Management* website (http://msom.pubs.informs.org/ecompanion.html) for details.

We next analyze the expected profit function $\pi(D, Q)$ and characterize the optimal solution (D^*, Q^*) for the firm's problem in (1). Let $\mu = E(\epsilon)$ denote the mean value of the supply reliability factor. Alternatively, we can interpret μ as the probability that an ordered unit will be available to meet demand.

Proposition 1. (i) *The expected profit function $\pi(D, Q)$ is jointly concave in D and Q.*

(ii) *If $\mu \le (c-v)/(p+m-v)$, then $D^* = Q^* = 0$ with $\pi^* = \pi(D^*, Q^*) = 0$. Otherwise, the optimal solution (D^*, Q^*) can be characterized by the optimality conditions*

$$A'(D^*) = (p+m-v)\int_0^\infty \left[1 - F\left(\frac{D^*}{Q^*}y\right)\right] y g(y)\, dy - m, \quad (2)$$

$$\int_0^\infty \left[\int_0^{(D^*/Q^*)y} x f(x)\, dx\right] g(y)\, dy = \frac{c-v}{p+m-v}, \quad (3)$$

and the optimal expected profit is given by

$$\pi^* = \pi(D^*, Q^*) = D^* A'(D^*) - A(D^*). \quad (4)$$

Proposition 1 characterizes the optimal solutions for the retailer. In particular, we show that when supply reliability is too low ($\mu \le (c-v)/(p+m-v)$), it is not profitable for the firm to attract any demand or stock any product. Hence, $\mu = (c-v)/(p+m-v)$ represents the minimum mean supply reliability for the firm to market and sell the product. We can explain this result as follows. Because μ represents the probability that an ordered unit is available to meet demand, it takes an average of $1/\mu$ units of order quantity to meet one demand. Also, we can interpret $(p+m-v)$ as the unit contribution revenue per unit sold and $(c-v)$ as the effective unit product cost after taking into account the unit shortage cost and salvage value. Thus, if the unit contribution revenue $(p+m-v)$ is less than or equal to $(c-v)/\mu$, then it is not profitable for the firm to attract any demand. To avoid this trivial case, we shall assume that $\mu > (c-v)/(p+m-v)$ in all our subsequent analysis, so Equations (2) and (3) give the first-order conditions for finding the optimal decisions. To simplify our discussion, from now on we also assume that both the shortage cost and salvage value equal to zero (i.e., $m = 0$ and $v = 0$) except in §5.4, which discusses the impact of the shortage cost and salvage value on the value of improving supply reliability. This assumption does not change the qualitative insights but can improve the exposition of this paper.

4. Impact of Supply Reliability on Firm Performance

One main objective of this research is to understand the impact of supply reliability on the retailer's optimal decisions and profit. For this purpose, we need to apply the concept of stochastic dominance. We first introduce the notion of first-order stochastic dominance. Let X_1 and X_2 be two random variables with distribution functions F_1 and F_2, respectively.

Definition 1. X_1 stochastically dominates X_2 in the first order, denoted by $X_1 \geq_1 X_2$, if $F_1(x) \leq F_2(x)$ for all x.

For two supply processes with reliability factors ϵ_1 and ϵ_2, we say process one is more reliable (or has a higher supply reliability) if $\epsilon_1 \geq_1 \epsilon_2$. That is, for any fixed order quantity, a more reliable supply process will yield a stochastically larger quantity available to meet demand in the selling season. It is clear that if $\epsilon_1 \geq_1 \epsilon_2$, then $E[\epsilon_1] \geq E[\epsilon_2]$.

Although the first-order stochastic dominance is a natural concept for comparing supply processes, it does not provide much information about the variability of the supply processes. To capture the impact of the variability, we next introduce the notion of second-order stochastic dominance.

Definition 2. X_1 stochastically dominates X_2 in the second order, denoted by $X_1 \geq_2 X_2$, if $\int_0^x (F_1(y) - F_2(y))\,dy \leq 0$ for all x .

Note that $X_1 \geq_2 X_2$ is equivalent to $E(g(X_1)) \geq E(g(X_2))$ for all increasing concave function g. Also, when $E(X_1) = E(X_2)$, $X_1 \geq_2 X_2$ implies that $\sigma(X_1) \leq \sigma(X_2)$. This allows us to interpret $X_1 \geq_2 X_2$ as X_1 being *less variable* than X_2.[4] Suppose there are two supply reliability factors ϵ_i, $i = 1, 2$. Let (D_i^*, Q_i^*) denote the respective optimal marketing and stocking decisions, and π_i^* denote the corresponding expected profit.

Proposition 2. *Suppose $\epsilon_1 \geq_2 \epsilon_2$. Then, $D_1^* \geq D_2^*$ and $\pi_1^* \geq \pi_2^*$.*

Proposition 2 shows that a firm with a less variable supply reliability will increase its target demand level, resulting in a higher expected profit. Therefore, it is beneficial for a firm to reduce the variability of supply reliability, even if the mean supply reliability remains the same. The above proposition extends some of the existing results in the literature to a more general setting. For example, Gupta and Cooper (2005) show that a convex-order dominant yield rate leads to a higher expected profit for a firm when demand is exogenously given. Li and Zheng (2006) compare two extremes (with and without variability) and prove that eliminating the underlying yield variability always results in a higher expected profit.

It is well known that first-order stochastic dominance implies second-order stochastic dominance, i.e., $X_1 \geq_1 X_2$ implies that $X_1 \geq_2 X_2$. The next result follows directly from Proposition 2:

Corollary 1. *Suppose $\epsilon_1 \geq_1 \epsilon_2$. Then, $D_1^* \geq D_2^*$ and $\pi_1^* \geq \pi_2^*$.*

Corollary 1 shows that as the supply reliability increases (stochastically), the firm will increase the target demand level, resulting in a higher expected profit. Therefore, it is always beneficial for a firm to increase its supply reliability, as long as the associated cost in improving the reliability does not exceed the increase in expected profit. We shall examine the trade-off between the cost of increasing supply reliability and its associated benefit in the next section.

Corollary 1 and Proposition 2 show that as the supply reliability increases or becomes less variable, both the target demand and expected profit increase. However, the corresponding impact on the optimal order quantity Q^* is more complex. To explain, observe that the optimal order quantity Q^* is equal to the target demand level D^* multiplied by an inflation factor due to the uncertain supply. Let $\delta = D^*/Q^*$. Then, $1/\delta$ is the inflation factor because $Q^* = (1/\delta)D^*$. The improvement of supply reliability has two effects on the stocking decision. On one hand, it increases the target demand D^*. On the other hand, it might decrease the inflation factor $1/\delta$ because the firm does not need to inflate the stocking quantity as much with an improved supply reliability. Therefore, the combined effect of these two factors is unclear.

To further illustrate how improving supply reliability would affect the optimal stocking quantity, let us consider the special case where $A(D) = aD^b$ $(b > 1)$,

[4] We refer readers to Song (1994) for another (stronger) definition for comparing the relative variability of two random variables, as well as Shaked and Shanthikumar (1994) and Muller and Stoyan (2002) for additional concepts and properties on stochastic orderings.

$\omega = 1$ with probability one, and the supply reliability factor ϵ follows a uniform distribution with mean μ and standard deviation σ.[5] Using (2) and (3) in Proposition 1, it is straightforward to show that the optimal marketing and stocking decisions are given by

$$D^* = \left[\frac{p(\mu + \sqrt{3}\sigma - \delta)}{2\sqrt{3}\sigma ab}\right]^{1/(b-1)} \tag{5}$$

$$Q^* = \frac{1}{\delta}\left[\frac{p(\mu + \sqrt{3}\sigma - \delta)}{2\sqrt{3}\sigma ab}\right]^{1/(b-1)} \tag{6}$$

with optimal profit $\pi^* = (b-1)A(D^*)$, where $\delta = \sqrt{(\mu - \sqrt{3}\sigma)^2 + 4\sqrt{3}\sigma c/p}$. Keep σ constant. Then, it can be easily shown that D^* is increasing in μ while the inflation factor $1/\delta$ is decreasing in μ. Because $Q^* = D^*/\delta$, these two opposite forces affect the optimal stocking quantity Q^* as μ increases. Furthermore, it can be shown that Q^* is increasing (decreasing) in μ if

$$\frac{[\delta - (\mu - \sqrt{3}\sigma)]\delta}{(\mu + \sqrt{3}\sigma - \delta)(\mu - \sqrt{3}\sigma)} \underset{(\le)}{\ge} (b-1).$$

Thus, the optimal stocking quantity is likely to be increasing in μ for small values of b and is likely to be decreasing in μ for large values of b. In other words, as supply reliability increases, a firm tends to increase its stocking quantity if the cost to attract demand is relatively low. Essentially, increasing supply reliability has a larger impact on the optimal target demand level (D^*) than that of the corresponding inflation factor $1/\delta$ due to the low demand acquisition cost (small values of b). On the other hand, when demand acquisition cost is high (large values of b), increasing supply reliability has a smaller impact on the optimal target demand level than that of the corresponding inflation factor, resulting in a decrease in the optimal stocking quantity. This example illustrates the fact that the optimal stocking quantity does not necessarily increase or decrease as supply reliability increases. Rather, it depends on the specific values of the model parameters.

[5] For a uniform distribution with range parameters $[\alpha, \beta]$, its mean and standard deviation are given by $\mu = (\alpha+\beta)/2$ and $\sigma = (\beta-\alpha)/2\sqrt{3}$, respectively. Equivalently, we have $\alpha = \mu - \sqrt{3}\sigma$ and $\beta = \mu + \sqrt{3}\sigma$.

Finally, we can illustrate how the marketing cost function affects the firm's optimal decisions and expected profit under unreliable supply. Specifically, it is straightforward to show that as the marketing cost increases, the firm will target a lower demand level and order fewer products, resulting in a lower expected profit. This is an intuitive result and the details are presented in the e-companion (online).

5. Value of Improving Supply Reliability

Most firms understand the potential benefits of increasing supply reliability, but at the same time they are concerned with the investment costs needed to achieve this objective. For instance, recently there has been much interest in using RFID to track product flows in a supply chain. Clearly, such a technology can greatly increase supply reliability and help retailers put the desirable amount of stock on store shelves to meet consumer demand. However, many firms are wary about the associated cost of RFID tags, which may still be too high to justify the adoption of the technology at the item level, especially for small-ticket, low-margin products.[6] Thus, it is important to quantify the value of improving supply reliability so that decision makers can evaluate the trade-off between the implementation costs and the associated benefits. Our model can serve as a useful framework to gain some insights into this important question.

Consider a situation where the retailer can invest to improve its supply reliability factor from ϵ to $\tilde{\epsilon}$. Let $F(f)$ and $\tilde{F}(\tilde{f})$ be their distribution (density) functions, respectively. In general, the associated investment costs may or may not depend on the order quantity. For example, the retailer can invest in a better security or monitoring system in the store to reduce theft or shrinkage. In this case, the associated cost is a fixed cost independent of the order quantity. Alternatively, the retailer can put RFID tags on the products to track and improve supply reliability. In some situations, the retailer could source from

[6] Wal-Mart was among the earliest retailers to embrace this technology by mandating that its top 100 suppliers apply RFID tags to all shipments by 2005. However, many suppliers failed to meet this deadline, and Wal-Mart has quietly dropped the RFID requirement in 2007 (see McWilliams 2007).

a more reliable supplier who might charge a higher unit cost for the product. For these two situations, the associated cost depends on the order quantity. Thus, we assume that the investment costs generally consist of a fixed cost that is independent of the order quantity (e.g., expenditures on security systems) and a variable cost that is proportional to the order quantity (e.g., costs of RFID tags). Because the addition of a fixed cost will not affect the firm's marketing and stocking decisions, its impact on the firm's investment decision is quite straightforward. So, unless otherwise mentioned, we assume that the fixed cost is zero and focus mainly on the variable cost.

Let t represent the unit technology cost for improving supply reliability and define $\tilde{c} = c + t$ as the new unit product cost. Let (D^*, Q^*, π^*) denote the optimal solution for the original system and $(\widetilde{D}, \widetilde{Q}, \widetilde{\pi})$ denote the corresponding optimal solution after improving supply reliability. Then, $\Delta\pi = \widetilde{\pi} - \pi^*$ represents the value (benefit) of adopting the technology to increase supply reliability. Next, we analyze how this value, and therefore the firm's investment decision, depends on t.

Clearly, the resulting optimal expected profit $\widetilde{\pi}$ is decreasing in t. Now define $\bar{t}$ to be the unit cost such that $\Delta\pi = 0$. In other words, a technology is valuable (beneficial) to the firm (i.e., $\Delta\pi > 0$) if and only if the actual technology cost is smaller than $\bar{t}$. Call this the threshold unit cost. We may derive an expression for $\bar{t}$ as follows. Recall that $\delta = D^*/Q^*$ satisfies

$$\int_0^\infty \left[\int_0^{\delta y} x f(x)\, dx\right] g(y)\, dy = \frac{c}{p}, \tag{7}$$

and define $\hat{\delta}$ by

$$\int_0^\infty \widetilde{F}(\hat{\delta} y) y g(y)\, dy = \int_0^\infty F(\delta y) y g(y)\, dy. \tag{8}$$

We can show that the threshold unit cost $\bar{t}$ is given by

$$\bar{t} = p\left\{\int_0^\infty \left[\int_0^{\hat{\delta} y} x \tilde{f}(x)\, dx\right] g(y)\, dy\right\} - c. \tag{9}$$

To see this, suppose $t = \bar{t}$. It follows from (7) and (9) that $\delta = D^*/Q^*$ and $\hat{\delta} = \widetilde{D}/\widetilde{Q}$ satisfy the first-order condition (3) for the respective systems. Using (8) and the first-order condition (2) for the respective systems, we have

$$\begin{aligned} A'(D^*) &= p\int_0^\infty [1 - F(\delta y)] y g(y)\, dy \\ &= p\int_0^\infty [1 - \widetilde{F}(\hat{\delta} y)] y g(y)\, dy = A'(\widetilde{D}). \end{aligned}$$

Thus, $D^* = \widetilde{D}$, which implies that $\pi^* = \widetilde{\pi}$ using (4). As $\widetilde{\pi}$ is decreasing in t, we have $\Delta\pi = \widetilde{\pi} - \pi^* \geq 0$ if and only if $t \leq \bar{t}$.

Hence, the threshold value $\bar{t}$ in (9) gives the maximum unit cost that a firm should be willing to pay for improving its supply reliability factor from ϵ to $\tilde{\epsilon}$. Thus, one may simply compare $\bar{t}$ and the actual cost t to decide whether to adopt the technology. Observe that this threshold value $\bar{t}$ does not depend on the marketing cost function $A(D)$. It only depends on the unit product cost and price, demand distribution, and the corresponding supply reliability distributions.

Suppose there are two different technologies that can improve supply reliability from ϵ to $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$, respectively. Let $\bar{t}_1$ and $\bar{t}_2$ denote their corresponding threshold unit costs.

Proposition 3. *Suppose* $\tilde{\epsilon}_1 \geq_2 \tilde{\epsilon}_2$. *Then,* $\bar{t}_1 \geq \bar{t}_2$.

Proposition 3 shows that the threshold unit cost is higher for the technology that provides a less variable supply reliability. Because $\tilde{\epsilon}_1 \geq_1 \tilde{\epsilon}_2$ implies that $\tilde{\epsilon}_1 \geq_2 \tilde{\epsilon}_2$, Proposition 3 implies that $\bar{t}_1 \geq \bar{t}_2$ if $\tilde{\epsilon}_1 \geq_1 \tilde{\epsilon}_2$, i.e., the threshold unit cost is higher for the technology that gives a (stochastically) higher supply reliability. Thus, we also know that a firm would be willing to pay more for a technology that provides a higher supply reliability.

For the rest of this section, we conduct sensitivity analysis to understand how various product characteristics affect the threshold value for adopting reliability-improving technologies.

5.1. Product Price

We first analyze how the product price affects the threshold unit cost $\bar{t}$ given in (9). All else being equal, a higher price corresponds to a higher unit contribution margin $(p - c)$. It might seem intuitive that a firm should be willing to spend more to increase supply reliability for products with a higher price. However, this intuition is not necessarily true as shown in the following example. Consider two products with parameters $c_1 = c_2 = 1$, $p_1 = 2$, $p_2 = 3$, and $A(D) = D^2/100$. Demand is deterministic, i.e., $\omega = 1$ with probability one. Both the original supply reliability distribution ϵ and improved supply reliability distribution $\tilde{\epsilon}$ follow uniform distributions with $\epsilon \sim U[0.5, 0.7]$ and $\tilde{\epsilon} \sim U[0.6, 1]$. Then, in this example, we have $\bar{t}_1 = 0.32 > \bar{t}_2 = 0.29$.

Why would the threshold value $\bar{t}$ decrease in product price p? To explain, we make the following two observations. First, our numerical experiments suggest that the increase in the expected profits with improved supply reliability is always higher for a high-price product when $t = 0$. Second, we observe that an increase in the unit cost c hurts the high-price product more. Specifically, we can show that $d\pi^*/dc = -Q^*$, where π^* is the optimal expected profit, and the optimal order quantity Q^* can be shown to be increasing in p. Thus, the expected profit for the high-price product decreases faster as the unit cost c increases due to the fact that the optimal order quantity is larger for the high-price (or high unit contribution margin) product.

Clearly, the driving forces underlying the above two observations act against each other: Although increasing supply reliability benefits the high-price product more (at $t = 0$), the additional unit cost t also hurts the high-price product more as t increases. Hence, it is possible for some large value of t that the benefit of increasing supply reliability for a low-price product is larger than that for a high-price product, resulting in a higher threshold value $\bar{t}$. (Figure 1 in the e-companion illustrates these two countervailing driving forces.) Next, we provide a condition under which the trade-off between the associated cost and benefit could indeed be more favorable for the low-price product.

Proposition 4. *$d\bar{t}/dp \le 0$ if and only if*

$$\frac{\int_0^\infty [\int_0^{\delta y} F(x)\,dx] g(y)\,dy}{\delta} \le \frac{\int_0^\infty [\int_0^{\hat{\delta} y} \tilde{F}(x)\,dx] g(y)\,dy}{\hat{\delta}}, \qquad (10)$$

where δ and $\hat{\delta}$ are given by (7) *and* (8).

The above proposition shows that the surprising result (i.e., a low-price product has a higher threshold value) can occur if and only if condition (10) holds. To gain more insight, we conducted a set of numerical experiments using a wide range of parameters and distributions. We observe that this surprising result can only occur when there is a sufficiently large increase in the variance of the reliability distribution. (See §B of the e-companion for a more detailed analysis and discussion.) In most practical settings, one would expect that a new information technology for tracking product flows should not significantly increase the variability of the supply uncertainty. Therefore, our result shows that the intuition that, all else being equal, new expensive technologies should first be adopted for products with a higher price is reasonably robust.

Finally, we point out that Proposition 4 remains valid when demand is exogenously given, and is not driven by the sale-effort component of the model. Also, the product price is fixed in our model and the firm can only affect demand through marketing efforts. How the above results would change when price is an endogenous decision variable remains an open question and is a potential topic for future research.

5.2. Product Cost

We next illustrate how the product cost affects the threshold value $\bar{t}$. Suppose the firm can (stochastically) increase supply reliability from ϵ to $\tilde{\epsilon}$ for two products with cost–price pairs (c_1, p_1) and (c_2, p_2), respectively. Let $\bar{t}_1$ and $\bar{t}_2$ denote the corresponding threshold unit costs. We can establish the following result.

Proposition 5. *Suppose $\tilde{\epsilon} \ge_1 \epsilon$. If $c_1/p_1 \le c_2/p_2$, then $\bar{t}_1/p_1 \le \bar{t}_2/p_2$.*

Proposition 5 shows that a firm is willing to pay a higher proportion of the price to increase supply reliability for products with a higher ratio of c/p. By setting $p_1 = p_2$, it follows immediately from Proposition 5 that $d\bar{t}/dc \ge 0$, i.e., the product with a higher unit cost also has a higher threshold value $\bar{t}$ for any fixed product price. In other words, for two products with the same price, a firm would be willing to pay a higher unit cost to improve supply reliability for the high-cost product than that for the low-cost product. The intuition behind this result can be explained using the following observations. First, increasing supply reliability reduces the required order quantity to meet any given target demand level, which provides a larger savings for the product with a higher unit cost. Second, the order quantity is higher for the product with a lower unit cost due to a higher contribution margin, and so an increase in the unit cost c for improving supply reliability also hurts the product with a lower unit cost more. These two driving forces act in the same direction: Increasing supply reliability benefits the high-cost product more, although the associated additional unit cost t also hurts the low-cost product more. Consequently, the product with a higher unit cost always has a higher threshold value $\bar{t}$.

An immediate implication of Proposition 5 is that a firm may be willing to pay more to improve supply reliability for products with a lower unit contribution margin. In fact, we have shown that for two products with the same price, the threshold cost for technology adoption is higher for the product with a lower margin. Hence, our finding does not support the common industry belief that expensive emerging technologies such as RFID should always be first considered for products with the highest profit margin; for example, see Deltech (2005) and Grocery Manufacturers Association (2006). We need to take cautions when making technology adoption decisions based on the profitability of products.

5.3. Marketing Cost Function

We proceed to investigate how the marketing cost affects the value of improving supply reliability. Is it more valuable to invest in technologies such as RFID for products with a high marketing cost function or low marketing cost function? One may view the marketing cost function as a proxy for the degree of difficulty in attracting demand, e.g., whether consumers are sensitive to product advertisement. Thus, the answer to this question may help us understand what products should receive a higher priority for improving supply reliability from the marketing perspective.

Consider two products that differ only in their marketing cost functions, $A_i(D)$ $(i = 1, 2)$. Assume that both $A_i(D)$ are strictly increasing and convex, and $A_1'(D) \leq A_2'(D)$ for all $D \geq 0$. Suppose the supply reliability can be increased from ϵ to $\tilde{\epsilon}$ for the two products at a unit cost t, $0 < t < \bar{t}$. Note from (7)–(9) that the threshold value $\bar{t}$ is independent of the marketing cost function. Therefore, the two products have the same threshold unit cost $\bar{t}$ although they have different marketing cost functions. For product i, let π_i^* be the optimal profit in the original system with ϵ and $\tilde{\pi}_i$ be the optimal profit in the improved system with $\tilde{\epsilon}$. We can establish the following result.

Proposition 6. *Suppose $A_1'(D) \leq A_2'(D)$ for all $D \geq 0$. Then, $\tilde{\pi}_1 - \pi_1^* \geq \tilde{\pi}_2 - \pi_2^*$.*

Proposition 6 shows that the marketing cost function has an unambiguous impact on the value of increasing supply reliability. In particular, as long as it is beneficial to improve supply reliability $(t < \bar{t})$, the benefit derived from improved supply reliability is always higher for the product with a lower marketing cost function. Thus, a reliable supply is more valuable when the retailer's promotional efforts are more effective in attracting demand.

As noted above, the threshold value $\bar{t}$ is independent of the marketing cost function. This is true when there is only a variable cost for adopting the technology. Suppose there is also a fixed cost associated with the technology adoption. It then follows from Proposition 6 that the threshold value for the product with a lower marketing cost will be higher than that for the product with a higher marketing cost. That is, under the presence of a fixed technology cost, the retailer should be willing to pay a higher threshold unit cost to increase supply reliability for the product with a lower marketing cost.

5.4. Shortage Cost and Salvage Value

Finally, we add the shortage cost (m) and the salvage value (v) into the model and discuss their impact on technology adoption decisions. It is clear from (3) that the unit shortage cost m has the same effect as the unit price p. In particular, the threshold value $\bar{t}$ may either increase or decrease in m depending on whether condition (10) holds or not. Thus, we can obtain similar insights as those in §5.1 regarding the impact of the shortage cost on the threshold value for technology adoption.

We next illustrate the effect of the unit salvage value v on the threshold value $\bar{t}$. Consider the situation where the unit salvage value v is changed from v_1 to v_2, and let $\bar{t}_1$ and $\bar{t}_2$ denote the respective threshold values defined in (9). Suppose $v_1 \geq v_2$. Then, $(c - v_1)/(p - v_1) \leq (c - v_2)/(p - v_2)$. In view of (3) and Proposition 5, it follows directly that $\bar{t}_1/(p-v_1) \le \bar{t}_2/(p-v_2)$, which implies $\bar{t}_1 \le \bar{t}_2((p-v_1)/(p-v_2)) \le \bar{t}_2$. Thus, we obtain the insight that the threshold value $\bar{t}$ increases as the unit salvage value v decreases. In other words, all else being equal, the firm is willing to pay a higher unit cost to increase supply reliability for products with lower salvage values.[7]

6. Conclusion

This paper studies the impact of supply reliability on a firm's performance under joint marketing and inventory decisions. The firm sells a product in a single selling season and needs to make a stocking quantity decision before the selling season starts. In addition, the firm may exert marketing effort to influence consumer demand. We show that as the supply reliability becomes (stochastically) higher or less variable, the firm will target a higher demand level and earn a higher profit. However, the optimal stocking quantity does not necessarily increase or decrease as supply reliability improves.

There is much interest today in adopting emerging technologies to improve supply reliability, so we apply our model to quantify the value of improving supply reliability. We analyze how the product characteristics affect the value of increasing supply reliability. First, we establish a necessary and sufficient condition under which the maximum unit cost a firm is willing to pay to improve supply reliability would increase in product price. We find that this condition, although not always true, would hold in most practical situations. Then we show that for two products with the same price, a firm is willing to pay more to improve supply reliability for the product with a higher product cost. Thus, our result suggests that it is not necessarily true that emerging technologies for improving supply reliability should be first adopted for products with the highest unit contribution margin. Finally, we find that the value of increasing supply reliability is always higher for products with a lower marketing cost function. That is, it is more rewarding to implement the new technologies when it is less costly to induce greater consumer demand using various marketing tools. These results provide useful insights into how firms should make investment decisions on adopting new technologies to improve product supply reliability.

Several research directions deserve further attention. First, the retail price is fixed in this paper. The marketing cost function can partially capture the effect of a price-setting retailer, i.e., reducing price to attract more demand is similar to a price promotion in which the lost revenue can be incorporated in the marketing cost function $A(D)$. However, it would be interesting to extend our model to include situations where price is also a decision variable. Second, our model considers a monopolist seller, and a potential research topic is to introduce competition among multiple firms facing supply uncertainties. Third, our single-period model considers only execution errors that result in inventory loss for the entire selling season. Some stores do count inventory frequently and recover misplaced items during the selling season. In this situation, a multiperiod model is needed to better capture the impact of such errors. Finally, multisourcing is a common strategy used by firms to reduce supply risks. It would be interesting to study how a retail firm can use multisourcing to improve supply reliability under joint marketing and inventory decisions.

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Appendix

Proof of Proposition 1. (i) Using the transformations $(\epsilon Q-\omega D)^+ = \{\epsilon Q - \min(\epsilon Q, \omega D)\}$ and $(\omega D-\epsilon Q)^+ = \{\omega D - \min(\epsilon Q, \omega D)\}$, we can express the retailer's expected profit function as

$$\pi(D, Q) = E\{(p+m-v)[\min(\omega D, \epsilon Q)] - m\omega D - (c-v)Q - A(D)\},$$

which can be rewritten as

$$\pi(D, Q) = (p+m-v)\left\{\int_0^\infty \left[\int_0^{(D/Q)y} xQf(x)\,dx\right]g(y)\,dy + \int_0^\infty \left[\int_{(D/Q)y}^1 yDf(x)\,dx\right]g(y)\,dy\right\} - mD - (c-v)Q - A(D). \tag{11}$$

[7] The assumption that the unavailable inventory $(1-\epsilon)Q$ can be recovered and salvaged at the end of the selling season is critical for this result. It may not hold if the supply uncertainty is due to theft and shrinkage for which the $(1-\epsilon)Q$ portion cannot be recovered.

By differentiating $\pi(D, Q)$ with respect to D and Q, it can be shown that the Hessian matrix of the profit function is negative semidefinite (see §A of the e-companion for details). Hence, the expected profit function $\pi(D, Q)$ is jointly concave in D and Q.

(ii) It follows from part (i) that any interior optimal solution must satisfy the first-order conditions $\partial\pi(D, Q)/\partial D = 0$ and $\partial\pi(D, Q)/\partial Q = 0$, which are equivalent to (2) and (3). We can rewrite (3) as

$$\mu - \int_0^\infty \left[\int_{(D/Q)y}^1 x f(x)\,dx\right] g(y)\,dy = \frac{c-v}{p+m-v}.$$

If $\mu < (c-v)/(p+m-v)$, there exists no solution to (3). For any fixed $D > 0$, it is clear that $\pi(D, Q) \to -\infty$ as $Q \to \infty$. Therefore, $Q^* = 0$ for any fixed $D > 0$. Because $A(D) > 0$ for $D > 0$, $\pi(D, 0) = -mD - A(D) < 0$. Because $\pi(0, 0) = 0$, $D^* = Q^* = 0$ if $\mu < (c-v)/(p+m-v)$.

Observe that the function $H(\delta) = \int_0^\infty [\int_{\delta y}^1 x f(x)\,dx] g(y)\,dy$ is continuous and strictly decreasing in δ with $H(0) = \mu$. Furthermore, it is easy to show that $H(\infty) = 0$. Therefore, for $\mu > (c-v)/(p+m-v)$, there must exist some $\delta^* > 0$ such that $H(\delta^*) = \mu - (c-v)/(p+m-v) > 0$. Because $A'(0) = 0$ and A' is strictly increasing, there exists some unique $D^* > 0$ such that D^* satisfies (2). Thus, the solution $(D^*, D^*/\delta^*)$ satisfies (2) and (3), and is thus optimal. Finally, substituting (2) and (3) into (11) and after simplification, we can show that $\pi^* = \pi^*(D^*, Q^*) = D^* A'(D^*) - A(D^*)$.

Proof of Proposition 2. Let F_1 and F_2 denote the distribution functions of ϵ_1 and ϵ_2, respectively. We first show that

$$\int_0^\infty [F_2(\delta_2 y) - F_1(\delta_1 y)] y g(y)\,dy \ge 0, \tag{12}$$

where δ_1 and δ_2 are given by

$$\int_0^\infty \left[\int_0^{\delta_1 y} x f_1(x)\,dx\right] g(y)\,dy = \int_0^\infty \left[\int_0^{\delta_2 y} x f_2(x)\,dx\right] g(y)\,dy = \frac{c}{p}. \tag{13}$$

Suppose $\delta_2 \le \delta_1$. Using integration by parts, we have

$$\int_0^\infty \left[\int_0^{\delta_2 y} x f_2(x)\,dx\right] g(y)\,dy = \int_0^\infty \left[F_2(\delta_2 y)\delta_2 y - \int_0^{\delta_2 y} F_2(x)\,dx\right] g(y)\,dy$$

and

$$\int_0^\infty \left[\int_0^{\delta_1 y} x f_1(x)\,dx\right] g(y)\,dy = \int_0^\infty \left[F_1(\delta_1 y)\delta_2 y - \int_0^{\delta_2 y} F_1(x)\,dx + F_1(\delta_1 y)(\delta_1 y - \delta_2 y) - \int_{\delta_2 y}^{\delta_1 y} F_1(x)\,dx\right] g(y)\,dy.$$

It then follows from (13) that

$$\int_0^\infty [F_2(\delta_2 y) - F_1(\delta_1 y)]\delta_2 y g(y)\,dy = \int_0^\infty \left[\int_0^{\delta_2 y} [F_2(x) - F_1(x)]\,dx\right] g(y)\,dy + \int_0^\infty \left[F_1(\delta_1 y)(\delta_1 y - \delta_2 y) - \int_{\delta_2 y}^{\delta_1 y} F_1(x)\,dx\right] g(y)\,dy. \tag{14}$$

Observe that the first term on the right side of (14) is nonnegative because $\epsilon_1 \ge_2 \epsilon_2$, and the second term is nonnegative because F_1 is an increasing function. Thus, (12) holds.

Now Suppose $\delta_2 > \delta_1$. In this case, we can write

$$\int_0^\infty \left[\int_0^{\delta_2 y} x f_2(x)\,dx\right] g(y)\,dy = \int_0^\infty \left[F_2(\delta_2 y)\delta_1 y + F_2(\delta_2 y)(\delta_2 y - \delta_1 y) - \int_0^{\delta_2 y} F_2(x)\,dx\right] g(y)\,dy,$$

$$\int_0^\infty \left[\int_0^{\delta_1 y} x f_1(x)\,dx\right] g(y)\,dy = \int_0^\infty \left[F_1(\delta_1 y)\delta_1 y - \int_0^{\delta_2 y} F_1(x)\,dx + \int_{\delta_1 y}^{\delta_2 y} F_1(x)\,dx\right] g(y)\,dy.$$

Using (13), we have

$$\int_0^\infty [F_2(\delta_2 y) - F_1(\delta_1 y)]\delta_1 y g(y)\,dy = \int_0^\infty \left[\int_0^{\delta_2 y} [F_2(x) - F_1(x)]\,dx\right] g(y)\,dy + \int_0^\infty \left[\int_{\delta_1 y}^{\delta_2 y} F_1(x)\,dx - F_2(\delta_2 y)(\delta_2 y - \delta_1 y)\right] g(y)\,dy. \tag{15}$$

Suppose that (12) does not hold, i.e.,

$$\int_0^\infty [F_2(\delta_2 y) - F_1(\delta_1 y)] y g(y)\,dy < 0,$$

Observe that the first term on the right side of (15) is nonnegative because $\epsilon_1 \ge_2 \epsilon_2$. Additionally, because

$$\int_{\delta_1 y}^{\delta_2 y} F_1(x)\,dx > F_1(\delta_1 y)(\delta_2 y - \delta_1 y),$$

we have

$$\int_0^\infty \left[\int_{\delta_1 y}^{\delta_2 y} F_1(x)\,dx - F_2(\delta_2 y)(\delta_2 y - \delta_1 y)\right] g(y)\,dy > (\delta_2 - \delta_1)\int_0^\infty [F_1(\delta_1 y) - F_2(\delta_2 y)] y g(y)\,dy > 0.$$

Thus, the right side of (15) is positive. On the other hand, the left side of (15) is negative, leading to a contradiction. Therefore, (12) must also hold in this case.

From Proposition 1, we have

$$A'(D_1^*) = p\int_0^\infty [1 - F_1(\delta_1 y)] y g(y)\,dy,$$

$$A'(D_2^*) = p\int_0^\infty [1 - F_2(\delta_2 y)] y g(y)\,dy.$$

It then follows from (12) that $A'(D_2^*) \geq A'(D_1^*)$, which implies that $D_2^* \geq D_1^*$ as $A'(D)$ is increasing in D.

From (4), we have $d\pi^*/dD^* = D^* A''(D^*) > 0$, which implies that π^* is increasing in D^*. Because $D_2^* \geq D_1^*$, we have $\pi_2^* \geq \pi_1^*$.

Proof of Proposition 3. Let $\tilde{F}_1$ and $\tilde{F}_2$ denote the distribution functions of $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$, respectively. Using (9) and (3), we have

$$\bar{t}_1 = p\int_0^\infty \left[\int_0^{\hat{\delta}_{t_1} y} x\, d\tilde{F}_1(x)\right] g(y)\, dy - c,$$

$$\bar{t}_2 = p\int_0^\infty \left[\int_0^{\hat{\delta}_{t_2} y} x\, d\tilde{F}_2(x)\right] g(y)\, dy - c.$$

Therefore,

$$\begin{aligned}\bar{t}_1 - \bar{t}_2 &= p\int_0^\infty \left[\int_0^{\hat{\delta}_{t_1} y} x\, d\tilde{F}_1(x) - \int_0^{\hat{\delta}_{t_2} y} x\, d\tilde{F}_2(x)\right] g(y)\, dy \\ &= p\int_0^\infty \left\{\left[\hat{\delta}_{t_1} y \tilde{F}_1(\hat{\delta}_{t_1} y) - \int_0^{\hat{\delta}_{t_1} y} \tilde{F}_1(x)\, dx\right]\right. \\ &\qquad \left. - \left[\hat{\delta}_{t_2} y F_2(\hat{\delta}_{t_2} y) - \int_0^{\hat{\delta}_{t_2} y} \tilde{F}_2(x)\, dx\right]\right\} g(y)\, dy, \end{aligned} \tag{16}$$

where the second equality follows from integration by parts. From the definitions of δ_{t_1} and δ_{t_2} and using (8), we have

$$\int_0^\infty \tilde{F}_1(\hat{\delta}_{t_1} y) y g(y) gy = \int_0^\infty \tilde{F}_2(\hat{\delta}_{t_2} y) y g(y) gy. \tag{17}$$

If $\hat{\delta}_{t_1} \geq \hat{\delta}_{t_2}$, we can rewrite (16) as

$$\begin{aligned}\bar{t}_1 - \bar{t}_2 &= p\int_0^\infty \left\{[\hat{\delta}_{t_2} y + (\hat{\delta}_{t_1} y - \hat{\delta}_{t_2} y)]\tilde{F}_1(\hat{\delta}_{t_1} y)\right. \\ &\qquad \left. - \int_0^{\hat{\delta}_{t_2} y} \tilde{F}_1(x)\, dx - \int_{\hat{\delta}_{t_2} y}^{\hat{\delta}_{t_1} y} \tilde{F}_1(x)\, dx\right\} g(y)\, dy \\ &\quad - p\int_0^\infty \left[\hat{\delta}_{t_2} y \tilde{F}_2(\hat{\delta}_{t_2} y) - \int_0^{\hat{\delta}_{t_2} y} \tilde{F}_2(x)\, dx\right] g(y)\, dy.\end{aligned}$$

Using (17) and after simplification, we have

$$\begin{aligned}\bar{t}_1 - \bar{t}_2 &= p\int_0^\infty \left\{\int_0^{\hat{\delta}_{t_2} y} [\tilde{F}_2(x) - \tilde{F}_1(x)]\, dx\right. \\ &\qquad \left. + \int_{\hat{\delta}_{t_1} y}^{\hat{\delta}_{t_2} y} [\tilde{F}_1(\hat{\delta}_{t_1} y) - \tilde{F}_1(x)]\, dx\right\} g(y)\, dy.\end{aligned}$$

The first term on the right side of above expression is nonnegative because $\tilde{\epsilon}_1 \geq_2 \tilde{\epsilon}_2$, and the second term is nonnegative because $\tilde{F}_1$ is an increasing function. Thus, $\bar{t}_1 - \bar{t}_2 \geq 0$.

If $\hat{\delta}_{t_1} \leq \hat{\delta}_{t_2}$, we can rewrite (16) as

$$\begin{aligned}\bar{t}_1 - \bar{t}_2 &= p\int_0^\infty \left\{[\hat{\delta}_{t_2} y - (\hat{\delta}_{t_2} y - \hat{\delta}_{t_1} y)]\tilde{F}_1(\hat{\delta}_{t_1} y)\right. \\ &\qquad \left. - \int_0^{\hat{\delta}_{t_2} y} \tilde{F}_1(x)\, dx + \int_{\hat{\delta}_{t_2} y}^{\hat{\delta}_{t_1} y} \tilde{F}_1(x)\, dx\right\} g(y)\, dy \\ &\quad - p\int_0^\infty \left\{\hat{\delta}_{t_2} y \tilde{F}_2(\hat{\delta}_{t_2} y) - \int_0^{\hat{\delta}_{t_2} y} \tilde{F}_2(x)\, dx\right\} g(y)\, dy.\end{aligned}$$

Again, using (17) and after simplification, we have

$$\begin{aligned}\bar{t}_1 - \bar{t}_2 &= p\int_0^\infty \left\{\int_0^{\hat{\delta}_{t_2} y} [\tilde{F}_2(x) - \tilde{F}_1(x)]\, dx\right. \\ &\qquad \left. + \int_{\hat{\delta}_{t_1} y}^{\hat{\delta}_{t_2} y} [\tilde{F}_1(x) - \tilde{F}_1(\hat{\delta}_{t_1} y)]\, dx\right\} g(y)\, dy.\end{aligned}$$

The first term on the right side of the above expression is nonnegative because $\tilde{\epsilon}_1 \geq_2 \tilde{\epsilon}_2$, and the second term is also nonnegative because $\tilde{F}_1$ is an increasing function. Thus, we also have $\bar{t}_1 - \bar{t}_2 \geq 0$ in this case.

Proof of Proposition 4. Taking derivative of $\bar{t}$ with respective to p, we obtain

$$\frac{d\bar{t}}{dp} = \frac{\partial \bar{t}}{\partial p} + \frac{\partial \bar{t}}{\partial \hat{\delta}}\frac{d\hat{\delta}}{dp} = \frac{\partial \bar{t}}{\partial p} + \frac{\partial \bar{t}}{\partial \hat{\delta}}\frac{d\hat{\delta}}{d\delta}\frac{d\delta}{dp}. \tag{18}$$

From (7) and (8), respectively, we have

$$\frac{d\delta}{dp} = -\frac{c}{p^2 \int_0^\infty \delta y^2 f(\delta y) g(y)\, dy} \tag{19}$$

$$\frac{d\hat{\delta}}{d\delta} = \frac{\int_0^\infty y^2 f(\delta y) g(y)\, dy}{\int_0^\infty y^2 \tilde{f}(\hat{\delta} y) g(y)\, dy}. \tag{20}$$

Substituting (19) and (20) in (18), we have

$$\frac{d\bar{t}}{dp} = \int_0^\infty \left[\int_0^{\hat{\delta} y} x\tilde{f}(x)\, dx\right] g(y)\, dy - \frac{\hat{\delta}}{\delta}\int_0^\infty \left[\int_0^{\delta y} x f(x)\, dx\right] g(y)\, dy.$$

Using integration by parts and in view of (8), it is easy to see that $d\bar{t}/dp \leq 0$ if and only if condition (10) holds.

Proof of Proposition 5. This proof is lengthy and therefore presented in §A of the e-companion.

Proof of Proposition 6. We use the notation $\pi_i^*(t)$ and $\tilde{\pi}_i(t)$ to show the dependence of the optimal expected profits on the unit cost increase t under supply reliability distributions F and $\tilde{F}$, respectively. Under this notation, we need to prove that $\tilde{\pi}_1(t) - \pi_1^*(0) > \tilde{\pi}_2(t) - \pi_2^*(0)$. By the definition of $\bar{t}$, we have $\tilde{\pi}_1(\bar{t}) - \pi_1^*(0) = \tilde{\pi}_2(\bar{t}) - \pi_2^*(0) = 0$, which implies that $\tilde{\pi}_1(\bar{t}) - \tilde{\pi}_2(\bar{t}) = \pi_1^*(0) - \pi_2^*(0)$.

Applying the envelope theorem to the profit function $\pi^*(c) = pE[\min(\omega D^*, \epsilon Q^*)] - cQ^* - A(D^*)$, we obtain $d\pi^*(c)/dc = -Q^*$. Therefore, $d(\pi_1^*(c) - \pi_2^*(c))/dc = Q_2^* - Q_1^*$. It can be shown (see Proposition B1 in the e-companion) that $Q_1^* \geq Q_2^*$, which implies that $d(\pi_1^*(c) - \pi_2^*(c))/dc < 0$. Thus, for $t < \bar{t}$, $\tilde{\pi}_1(t) - \tilde{\pi}_2(t) \geq \tilde{\pi}_1(\bar{t}) - \tilde{\pi}_2(\bar{t}) = \pi_1^*(0) - \pi_2^*(0)$, which leads to $\tilde{\pi}_1(t) - \pi_1^*(0) \geq \tilde{\pi}_2(t) - \pi_2^*(0)$ for $t < \bar{t}$.

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