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# Competition, Cooperation, and Information Sharing in a Two-Echelon Assembly System

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This paper studies a two-echelon assembly system and explores several important issues in managing decentralized supply chains. First, we investigate the behavior of the assembly system under decentralized control. It is shown that the Nash equilibrium of the competitive assembly system exists but is never system optimal. Next, we examine the role of information in the assembly system. We consider horizontal information sharing on the inventory status between the suppliers and demonstrate when information is valuable from the perspectives of the system and the individual player. We find that information sharing mitigates the system's competition penalty in some cases, but there are also instances in which information sharing can hurt a decentralized system. Although the manufacturer always prefers information sharing, it is sometimes in the interest of the suppliers to refuse to share information. Finally, we propose a demand-independent coordination scheme for managing decentralized supply chains with either a serial or an assembly structure. The coordination scheme has practical value because very often the firm's head (or the owner of the supply chain) does not have the accurate demand information that the local managers have. Alternatively, the demand varies over time, and the coordination contracts have to be modified. Under the transfer payment contract, the system optimal policy is the unique Nash equilibrium in the decentralized supply chain. Discussion on the relationship between our coordination scheme and existing methods in the literature is also provided.

*Key words*: assembly system; decentralization; information sharing; supply chain coordination
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## 1. Introduction

The past decade has witnessed drastic changes in the manufacturing sector due to an increasingly competitive business environment. Manufacturing companies have begun to outsource most production activities to their suppliers to focus on the business of core competencies. For example, in the automotive industry, major parts and vehicle systems are processed by suppliers. It is projected that by 2010, suppliers in the automotive industry will provide over 70% of vehicle costs (De Koker 2001). Similar changes are observed in the electronics and telecommunications industries. High-tech giants such as Dell, Lucent, Cisco, Hewlett-Packard, and Ericsson outsource a major portion of their output. According to Lehman Brothers, the value of outsourced electronic goods will reach $280 billion in about three years (Selbert 2002).

Evidence shows that a main theme of future manufacturing will be disintegration. Or, in short, "do what you are best at" rather than "do it all." Traditional original equipment manufacturers (OEMs) are converting themselves into pure assemblers that draw components from a reducing base of suppliers. Such a strategy benefits the OEMs in several ways. First, they can devote their full attention and resources to essential activities including product design and development, assembly, marketing, and selling. Second, cost efficiency can be achieved because suppliers are probably more competitive in production and logistics. Third, the reduced assembly time enables OEMs to promptly respond to rapidly changing customer demand. Despite the advantages, however, disintegration creates information and incentive problems too, which may cause huge supply chain inefficiency. Information problems arise when members in a decentralized supply chain do not have sufficient information to implement system optimal policies. More importantly, supply chain members often have conflicting objectives, which make efficient solutions for the whole system nonexecutable.

Recently, there has been a rapidly expanding literature written on decentralized supply chains. Most studies consider serial supply chains in which a single component is needed to produce a final product. Although these studies provide important managerial insights into how to improve supply chain efficiency, they are not comprehensive enough to embody further interactions in real manufacturing systems. Usually an assembler obtains multiple components from a number of independent suppliers. Coordination with only a partial set of the suppliers is insufficient because a shortage at any supplier will bring the assembly line to a halt. In the automotive industry, even a brief shutdown of the assembly line costs millions of dollars. Consequently, some auto assemblers charge parts suppliers thousands of dollars for each minute of lost production due to late delivery (ARC Advisory Group 2002).

This paper considers a two-echelon assembly system consisting of a manufacturer and two suppliers. The manufacturer produces a final product using components provided by the suppliers. Each supplier replenishes from an ample source with a positive lead time. There is also a lead time between the suppliers and the manufacturer. This lead time represents the transportation time and the time to transform the components into the final product. When the transportation time is zero, the model resembles a vendor managed inventory (VMI) program, in which the suppliers manage the inventory of the components at the manufacturer's site. Demand occurs only at the manufacturer and is i.i.d. across periods. Unsatisfied demand is backlogged to the next period. The manufacturer and suppliers are independent firms whose objective is to minimize their own expected costs. The model also applies to situations in which the suppliers and manufacturer belong to the same firm, but are managed as independent cost centers.

One aim of this paper is to investigate the competitive behavior of such an assembly system. We study inventory games played by members of the assembly system. Given specified holding and backorder costs, the players choose their base-stock levels independently. By comparing the equilibrium outcomes to the system optimal solution, we will know under what conditions the decentralized system is inefficient and, therefore, when a coordination effort is justified. We are also interested in the role of information in the decentralized assembly system. Information sharing has long been identified as improving supply chain efficiency. Most existing papers consider vertical information sharing—that is, the upstream supplier receives information on the downstream retailer's sales, forecasts, and inventory level through IT platforms like EDI. However for the assembly system, we focus on horizontal information sharing—that is, the suppliers exchange information on their inventory status. We know horizontal information sharing is necessary to implement the centralized optimal policy. However, it is unclear whether such information is valuable in the decentralized assembly system. Will information sharing help coordinate suppliers' inventories? What is the impact on the individual players and the whole system? The answers to these questions will be explored.

We analyze the inventory games under two information structures. In the first, the suppliers' inventory status is transparent. That is, besides its own inventory status, each supplier can see the inventory status of the other supplier. Under this information structure, the supplier with a shorter lead time can use a replenishment plan contingent on the other supplier's inventory status. We show that although multiple equilibria exist, the cost outcome is unique. In the second information structure, the suppliers know only their own inventory status. Under this information structure, the contingent policy is no longer possible and there exists a unique equilibrium. We find that under both information structures, the equilibrium outcomes are never system optimal, so decentralized decision making leads to supply chain inefficiency. A numerical example shows that information sharing helps reduce inefficiency in many cases, but it may hurt the system as well. There are cases in which information sharing makes the decentralized system less efficient. Although the manufacturer always prefers information sharing, suppliers may find it best not to share information.

Supply chain members can be better off through coordination because the equilibrium outcomes in the inventory games are never Pareto optimal. Several coordination schemes have already been proposed in the literature. A common feature of these schemes is that they require the demand distribution as an input (the only exception is Porteus 2000). This may cause a problem in practice because any change in the demand distribution necessitates modification of the coordination contract. When the supply chain is owned by a single firm, the firm's head may not have the accurate demand information that the local inventory managers do. Or, very likely, the firm's head cannot react swiftly to market changes. To solve this problem, we propose a new coordination scheme independent of demand distribution. Because an assembly system can be converted to an equivalent serial system, we illustrate the scheme using a serial system. Linear transfer payments among supply chain members are specified in such a way that their incentives are aligned. We also show that under our coordination scheme, the system optimal policy is the unique Nash equilibrium in the decentralized system.

The rest of the paper is organized as follows: §2 is a literature review. Section 3 describes the model and characterizes the optimal inventory policy for the assembly system. The inventory games are analyzed in §4. Comparisons among equilibrium outcomes and the optimal solution are made in §5. Section 6 discusses the demand-independent coordination scheme. The paper concludes with §7.

## 2. Literature Review

Some important studies have reported on the optimal inventory policy for centralized multiechelon supply chains: Clark and Scarf (1960) and Federgruen and Zipkin (1984), for serial systems; Rosling (1989), for assembly systems; and Roundy (1985), for distribution systems. Chen and Zheng (1994) provide lower bounds for all three systems and prove the optimal policies of the first two. Schmidt and Nahmias (1985) investigate an assembly system with two suppliers and one assembler under a finite horizon. These studies provide methods for evaluating the costs in multiechelon supply chains, and characterize the optimal policies as well.

This paper fits into the growing literature on competitive multiechelon inventory management. Cachon and Zipkin (1999) consider a two-stage serial system under periodic review, and Cachon (2001) studies a distribution system with one supplier and multiple retailers under continuous review. Parker and Kapuscinski (2003) further consider a decentralized serial supply chain with capacity limits at each stage. Our paper complements the above by investigating the competitive behavior of an assembly system. In Cachon and Zipkin (1999), only vertical competition exists between the supplier and retailer. They show that unique equilibrium exists for the games using either echelon or local inventory tracking methods. However for the assembly system, there is one manufacturer and two (or more) suppliers, so there is both vertical (suppliers versus manufacturer) and horizontal (supplier versus supplier) competition. Multiple equilibria may exist in the assembly game. This is mainly because in an assembly system, the supplier with the shorter lead time would adopt a contingent policy. However, despite the multiple equilibria produced by the contingent inventory policy, the equilibria can be characterized and the equilibrium cost outcome is shown to be unique.

There is a growing interest in studying decentralized assembly systems. Gerchak and Wang (2004) consider a two-echelon assembly system in which suppliers decide on the capacities to build before demand is observed. This system shows that the suppliers always understock and contracts are proposed to coordinate the system. Wang and Gerchak (2003) investigate two different capacity games in an assembly system. In one game the assembler acts as the leader, while in the other the suppliers lead by setting wholesale prices. Both papers use a single-period model, which makes the analysis different from ours. In contrast, we formulate a periodic review inventory model to explore the strategic interaction among supply chain members in a dynamic setting. This enables us to introduce positive lead times for the suppliers and the assembler, as well as the information-sharing issue. Bernstein and DeCroix (2004) study a three-level assembly system in which a manufacturer procures from the first-tier subassemblers, who in turn replenish from the second-tier suppliers. Their paper differs from ours in that it focuses on a price and capacity setting game and provides insights for structural decisions like how to group components to form a subassembly. Feng and Zhang (2005) study the impact of modular assembly approach on supply chain efficiency. Bernstein and DeCroix (2006) consider a similar two-stage assembly system under periodic review, but their model allows the manufacturer to hold unmatched components. In our model, the manufacturer only holds finished products, and the suppliers are responsible for any unmatched components. Nagarajan and Bassok (2003) develop a bargaining framework for an assembly system. The purpose of that paper is to shed light on the impact of negotiation processes on system performance and profit allocation.

Information sharing is becoming an important research arena in supply chain management. Chen (2003) reviews the literature in this category. Notable studies include Chen (1998), Gavirneni et al. (1999), Cachon and Fisher (2000), and Lee et al. (2000). In these papers, information on demand or downstream inventory status is passed to the upper portion of the supply chain. Vertical information sharing in the presence of horizontal competition is studied by Li (2002). In his paper, multiple retailers decide whether to share their private demand signals with the manufacturer. Their information sharing affects the competitive outcome of the distribution system. We consider another type of information sharing in which the suppliers in the same echelon exchange information on their inventory status. We are interested in knowing whether information is helpful in coordinating suppliers in a decentralized assembly setting. Bernstein and DeCroix (2006) also study the impact of information sharing between suppliers in an assembly system, but, again, their model is different from ours because the manufacturer can hold unmatched components.

Some papers show how to ensure fast delivery from suppliers. Cachon and Zhang (2003) consider demand allocation policies for a buyer to promote competition among suppliers in multisourcing. Cachon and Zhang (2006) investigate a buyer's procurement strategies under the presence of information asymmetry, given that sole sourcing will be used by the buyer. In these two papers, the suppliers provide a substitutable product or service, and the lead times the buyer gets depend on capacities built by the suppliers. In this paper, the suppliers provide complementary components and hold inventory to satisfy the manufacturer's orders. Nevertheless, the spirit is the same: from the manufacturer's perspective, higher inventory levels at the suppliers mean more responsive delivery. Thus, the manufacturer has incentive to coordinate the lead times from both suppliers.

This paper is also closely related to the line of research on supply chain coordination. Reviews can be found in Cachon (1998, 2003). Most existing coordination schemes require the demand information as an input. We propose a new scheme to coordinate decentralized multiechelon inventory systems. One contribution of our paper is that the coordination scheme is independent of the demand distribution. Detailed discussion on the relationship between our coordination contract and the existing schemes is given in §6.2.

## 3. Model Settings

We consider the inventory policy for a two-echelon assembly system under decentralized control. The assembly system consists of a manufacturer and two suppliers. The analysis can, however, be extended to more than two suppliers. The manufacturer assembles a final product using the components provided by the suppliers. Assume that the unit has been properly scaled so that each final product requires one unit of each component. The suppliers replenish from outside sources with positive lead times. The outside sources are assumed to have ample stock. There is also a lead time between the suppliers and manufacturer, which could include the shipping and assembly time. For ease of exposition, we assume that the shipping time is zero and consider the VMI program. We also assume that the manufacturer has sufficient capacity and the assembly time does not depend on the lot size. The system is under periodic review with an infinite horizon.

Let $L_i$ be the lead time for stage $i$, $i = 1, 2, 3$. Define $s(i)$ as the set containing $i$ and all the successors of stage $i$. Define the *total lead time* for stage $i$ by $M_i = \sum_{j\in s(i)} L_j$. Assume that the stages are indexed in such a way that $M_i$ increases in $i$. In short, denote Stage 1 by S1, Stage 2 by S2, etc. Note that for the assembly system, S1 is the manufacturer while the others are suppliers.

Demand occurs only at S1. Unsatisfied demand is backlogged. There is no setup cost for order placing and processing. Stage $i$ incurs an echelon inventory holding cost $h_i$ per unit of stock per period. We follow the standard assumption that $h_3 > 0$ and $h_1, h_2 \geq 0$. This implies that a lower stage incurs higher holding cost, which is commonly observed in practice. Therefore, in the assembly system, S1's installation holding cost is $h = h_1 + h_2 + h_3$, S2 and S3's installation holding costs are $h_2$ and $h_3$, respectively. The suppliers are responsible for both the inventory on hand and inventory in assembly. The manufacturer only incurs holding cost for the finished products.

In a centralized system, it is usually assumed that backorder costs are incurred only at the most downstream stage. In reality, all supply chain members dislike backorders. The backorder cost is a proxy for various consequences that result from the backorders. First, it represents financing cost—that is, the customers pay on delivery of the product; second, it provides an approximation of the loss of goodwill; third, it estimates the impact of the lost sales on profit. For each backorder, stage $i$ incurs a backorder cost $p_i$. Let $p = p_1 + p_2 + p_3$ and $\alpha_i = p_i/p$. Thus, it would be equivalent to say that $p$ is the system backorder cost and all stages share such a cost. The underlying assumption is that the sum of the backorder costs for individual stages equals the backorder penalty for the centralized system. Such a method is used in Cachon and Zipkin (1999) and Caldentey and Wein (2003). The $\alpha_i$'s can be viewed as the relative extent to which stage $i$ dislikes backorders. Note that $0 < \alpha_i < 1$ and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

The timing of events in each period is as follows: (1) shipments arrive at each stage; (2) orders are submitted and shipments immediately released (as for the manufacturer's orders, only matched components can enter assembly); (3) customer demand occurs and unsatisfied demand is backlogged; and (4) holding cost and backorder costs are charged.

The decentralized assembly system can be formulated as a (stochastic) dynamic game. In each period, the players make their own procurement decisions, which may depend on the current state and history of the assembly system. Detailed definitions of dynamic games can be found in Fudenberg and Tirole (1991) and Heyman and Sobel (1984). In this paper, we follow the literature (e.g., Cachon and Zipkin 1999, Bernstein and Federgruen 2004) to assume that all players adopt stationary base-stock policies, i.e., we focus on a subset of the strategy space. Under this assumption, the players actually face a static game in which they simultaneously choose their base-stock levels. The stationary policies are of particular interest for several reasons. First, in most settings, it seems unlikely that firms would use complex, dynamic procedures to manage their inventory. In fact, simple base-stock policies have been widely accepted in practice for ease of use. Second, if all firms $j \neq i$ use a stationary base-stock policy in the dynamic game, then it is straightforward to show that it is optimal for firm $i$ to use a stationary base-stock policy. Therefore, any Nash equilibrium in the static game is also a Nash equilibrium in the dynamic game. Third, the stationary policy assumption facilitates the comparison between the centralized and decentralized systems because the centralized optimal solution involves stationary base-stock policies.

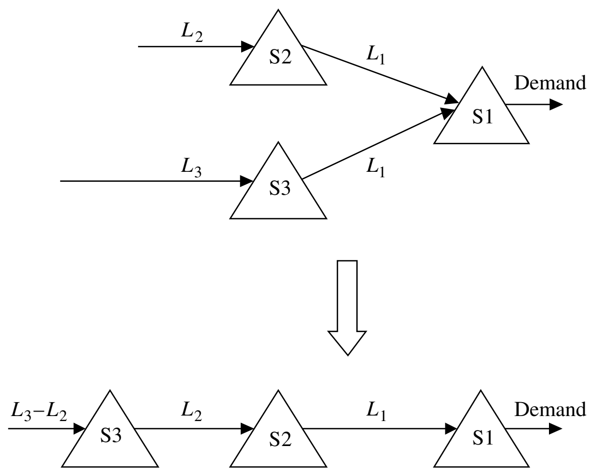
We need some notations to describe the state of the system. All the following notations are for stage $i$ in period $t$. Let $IT_{it}$ be the *in-transit inventory* to stage $i$; let $IL_{it}$ be the *echelon inventory level* (all inventory at stage $i$ and lower in the system minus the backorders at S1); let $\overline{IL}_{it}$ be the *local inventory level* (inventory at stage $i$ minus the backorders at stage $i$); let $IP_{it}$ be the *echelon inventory position*, $IP_{it} = IL_{it} + IT_{it}$; and let $\overline{IP}_{it}$ be the *local inventory position*, $\overline{IP}_{it} = \overline{IL}_{it} + IT_{it}$. Throughout the paper, we use overbars to distinguish between echelon inventory and local inventory. Denote stage $i$'s echelon base-stock level by $s_i$ and local base-stock level by $\bar{s}_i$.

Demand is random and i.i.d. in each period. Use $D^\tau$ to denote the demand over $\tau$ periods, so $D^1$ is the one-period demand and $D^{L_i}$ is the lead time demand for stage $i$. For future use, define $\mu^\tau = ED^\tau$ as the mean demand over $\tau$ periods. Let $\Phi^\tau$ and $\phi^\tau$ be the cumulative distribution function and density function, respectively, of demand over $\tau$ periods. Assume $\Phi^\tau(x)$ is differentiable for all positive integers $\tau$. In other words, the demand has continuous density. Furthermore, assume $\Phi^1(0) = 0$, so only positive demand occurs in each period.

For concision, define the following mathematical notations: $(x)^+ = \max(x, 0)$, $(x)^- = \max(-x, 0)$, and $x \wedge y = \min(x, y)$, $x \vee y = \max(x, y)$. Let $E$ be the expectation operation, and let a prime denote the derivative of a function of a single variable.

Here, for future comparison, we present the optimal inventory policy for a centralized assembly system. Rosling (1989) and Chen and Zheng (1994) demonstrate that the assembly system can be viewed as a serial with modified lead times: $l_i = M_i - M_{i-1}$ for all $i$,

**Figure 1** **An Illustration of the Assembly System and Its Equivalent Serial System**




where $l_i$ are the equivalent lead times for stage $i$ in the serial system. Indexing by the total lead time implies $L_3 \geq L_2$, that is, S3's lead time is at least as long as S2's. Hence, we have $l_1 = L_1$, $l_2 = L_2$, and $l_3 = L_3 - L_2$. See Figure 1 for an illustration.

In the serial system, the installation holding costs for S1, S2, and S3 are $h$, $h_2 + h_3$, and $h_3$, respectively. According to Chen and Zheng (1994), the expected total cost per period for the serial system can be written as

$$C^s = E[h_1 IL_{1t} + h_2 IL_{2t} + h_3 IL_{3t} + (p+h)B_t], \tag{1}$$

where $B_t$ is the backorder level at S1. The expected total cost per period for the original assembly system can be similarly written as

$$C^a = E[h_1 IL_{1t} + h_2 IL_{2t} + h_3 IL_{3t} + (p+h)B_t]. \tag{2}$$

Superscripts $s$ and $a$ are used to represent serial and assembly systems, respectively. More details of the above two cost functions are given in §6. Note that $C^s$ and $C^a$ are exactly the same except the term $IL_{3t}$. In the serial system, $IL_{3t}$ includes the in-transit inventory from S3 to S2, which actually does not exist in the assembly system. Thus, the serial system overestimates the total cost for the assembly system by a constant $h_3 \mu^{L_2}$ on average. However, this constant has no impact on optimization. Next, we characterize the optimal inventory policy for the serial system.

In the optimal policy, each stage adopts an echelon base-stock level. Denote the base-stock levels by $s^o = (s_1^o, s_2^o, s_3^o)$. Based on Chen and Zheng (1994) and Cachon and Zipkin (1999), we derive the following first-order conditions for $s_i^o$:

The first-order condition for $s_1^o$ is

$$\Phi^{l_1+1}(s_1^o) = \frac{h+p-h_1}{h+p}. \tag{3}$$

The first-order condition for $s_2^o$ is

$$-(p+h-h_1-h_2) + (p+h-h_1)\Phi^{l_2}(s_2^o - s_1^o) + (p+h)\int_{s_2^o - s_1^o}^{\infty} \phi^{l_2}(x)\Phi^{l_1+1}(s_2^o - x)\,dx = 0. \tag{4}$$

S3's optimal echelon base-stock level, $s_3^o$ is given by

$$\begin{aligned} 0 = {} & -(p+h-h_1-h_2-h_3) + (p+h-h_1-h_2)\Phi^{l_3}(s_3^o - s_2^o) \\ & + (p+h-h_1)\int_{s_3^o - s_2^o}^{\infty} \phi^{l_3}(x)\Phi^{l_2}(s_3^o - x - s_1^o)\,dx \\ & + (p+h)\int_{s_3^o - s_2^o}^{\infty} \phi^{l_3}(x) \\ & \cdot \int_{s_3^o - x - s_1^o}^{\infty} \phi^{l_2}(z)\Phi^{l_1+1}(s_3^o - x - z)\,dz\,dx. \end{aligned} \tag{5}$$

The above first-order conditions can be used to compute the optimal inventory policy for the serial system. Once we have the optimal base-stock levels $s^o$, the optimal policy for the assembly system is as follows: S1 uses the base-stock level $s_1^o$, and S3 uses the base-stock level $s_3^o$; S2 uses the base-stock level $s_2^o$ contingent on S3's inventory status. In particular, S2 orders up to $\min(s_2^o, s_3^o - D^{l_3})$, where $s_3^o - D^{l_3}$ is S3's echelon inventory position when S2 orders. This result has been proven in Theorem 2 in Chen and Zheng (1994). As in the serial system, the optimal echelon inventory policy can be implemented with local base-stock policies. The local base-stock levels for different stages are $\bar{s}_1^o = s_1^o$, $\bar{s}_2^o = s_2^o - s_1^o$, and $\bar{s}_3^o = s_3^o - s_1^o$, respectively. Again, S2 must use the contingent local base-stock policy. Note that for the assembly system, the optimal policy cannot be executed with only local inventory information.

## 4. Inventory Games in the Assembly System

Now we study the noncooperative inventory game played by the independent members in the assembly system. In this game, the players simultaneously select their *local* base-stock level $\bar{s}_i$. Assume that the strategy spaces are the same for all players: $\sigma = [0, M]$, where $M$ is a sufficiently large number that never constrains the players. Once the players have chosen their strategies, they are fixed and the system runs for a sufficiently long time.

Throughout the paper, we assume that all players are risk neutral and all model parameters are common knowledge. However, there are two different information structures. With information sharing, S2 (the supplier with the shorter lead time) can observe the inventory status at S3 (the supplier with the longer lead time) and thus is able to use a contingent base-stock policy. But without information sharing, both suppliers only use the simple base-stock policy. Details will be discussed later. The players know what game they are playing (the information structure).

Let $\bar{s}_1$, $\bar{s}_2$, and $\bar{s}_3$ be the local base-stock levels chosen by S1, S2, and S3, respectively. Then the echelon base-stock levels $s_1 = \bar{s}_1$, $s_2 = \bar{s}_1 + \bar{s}_2$, and $s_3 = \bar{s}_1 + \bar{s}_3$ are also determined. See Axsäter and Rosling (1993) for more discussions on echelon and local-stock policies. For S1, because the local inventory and echelon inventory are the same ($s_1 = \bar{s}_1$), we use $s_1$ for both situations in the rest of the paper. Let $\bar{s}_{-i}$ denote the strategies taken by all players except $i$, a standard game theory notation. For instance, $\bar{s}_{-1} = (\bar{s}_2, \bar{s}_3)$. Let $\overline{H}_i(s_1, \bar{s}_2, \bar{s}_3)$ be the cost function for player $i$ (in each period). Then the best reply mapping for player $i$ is

$$\bar{r}_i(\bar{s}_{-i}) = \{\bar{s}_i \in \sigma \mid \bar{s}_i \text{ minimizes } \overline{H}_i(\bar{s}_i, \bar{s}_{-i})\}, \quad i = 1, 2, 3.$$

Note that we use the same notations $\overline{H}_i$ and $\bar{r}_i$ for different games (i.e., different information structures). Using the above notations, the inventory game can be represented by ($\sigma_i$, $\overline{H}_i$, $i = 1, 2, 3$) with $\sigma_i = [0, M]$ for all $i$. A Nash equilibrium is a set of strategies under which no player has a unilateral incentive to deviate. Therefore, if a unique Nash equilibrium exists, it can be used to predict the *rational* behavior of the players. Note that there could be both pure-strategy and mixed-strategy Nash equilibria. In this paper, we are only concerned with the pure-strategy Nash equilibrium. Mixed strategies make little practical sense in operations. Also, no mixed-strategy Nash equilibrium exists if there is a unique pure-strategy Nash equilibrium. Suppose $(s_1^*, \bar{s}_2^*, \bar{s}_3^*)$ is a Nash equilibrium. By definition, there is

$$s_1^* \in \bar{r}_1(\bar{s}_2^*, \bar{s}_3^*), \qquad \bar{s}_2^* \in \bar{r}_2(s_1^*, \bar{s}_3^*), \qquad \bar{s}_3^* \in \bar{r}_3(s_1^*, \bar{s}_2^*).$$

As preparation for game analysis, we present the cost function for S1 first. In each period, S1 incurs a holding cost $h$ for the inventory on hand and $\alpha_1 p$ for the backorders. Define

$$\widehat{G}_1(x) = h(x)^+ + \alpha_1 p(x)^-.$$

Let $y$ be the inventory position in period $t$. Then the cost incurred at S1 in period $t + L_1$ is

$$\begin{aligned} G_1(y) &= E[\widehat{G}_1(y - D^{L_1+1})] \\ &= h(y - \mu^{L_1+1}) + (h + \alpha_1 p)\int_y^\infty (x - y)\phi^{L_1+1}(x)\,dx. \end{aligned}$$

It can be readily shown that $G_1(y)$ is strictly convex. Using function $G_1$, S1's cost function $\overline{H}_1(s_1, \bar{s}_2, \bar{s}_3)$ can be written as

$$\overline{H}_1(s_1, \bar{s}_2, \bar{s}_3) = E\big[G_1(s_1 \wedge (s_2 - D^{L_2}) \wedge (s_3 - D^{L_3}))\big], \quad (6)$$

where $s_2 = s_1 + \bar{s}_2$, and $s_3 = s_1 + \bar{s}_3$ are the corresponding echelon base-stock levels.

S1's cost function is independent of the information structure. However, in the following sections, we will see that S2 and S3's cost functions depend on the information structure.

### 4.1. Inventory Game with Information Sharing

The stages are indexed in such a way that $L_2 \le L_3$. In this section, we focus on $L_2 < L_3$. Later we will see that $L_2 = L_3$ can be analyzed as a special case without information sharing. To analyze the inventory game, we need to derive the cost functions for the players. Suppose S2 and S3 have chosen $\bar{s}_2$ and $\bar{s}_3$ as their base-stock levels. Since $L_2 < L_3$, when S2 makes its ordering decision in period $t - L_2$, it has already observed the demand $D^{l_3}$ that occurred during the time interval $[t - L_3, t - L_2)$ (from period $t - L_3$ to period $t - L_2 - 1$). So S2 knows that in period $t$, the number of pairs of components that can enter for assembly will be constrained by $\bar{s}_3 - D^{l_3}$. Therefore, in period $t - L_2$, S2 should bring its own inventory position to $\bar{s}_2 \wedge (\bar{s}_3 - D^{l_3})$ instead of $\bar{s}_2$. That is, while S3 uses a true base-stock policy, S2 uses an ordering policy contingent on S3's inventory position.

Define $\widehat{G}(x) = p(x)^-$. Let $y$ be the inventory position at S1 in period $t$. With base-stock levels $(s_1, \bar{s}_2, \bar{s}_3)$, we know

$$y = s_1 \wedge (\bar{s}_2 + s_1 - D^{L_2}) \wedge (\bar{s}_3 + s_1 - D^{L_2} - D^{l_3}).$$

The system's backorder cost in period $t + L_1$ will be

$$G(y) = E[\widehat{G}(y - D^{L_1+1})],$$

where $y - D^{L_1+1}$ is S1's inventory level at the end of period $t + L_1$. As a result, stage $i$'s backorder cost in period $t + L_1$ will be $\alpha_i G(y)$. It can be shown that $G' < 0$ and $G'' > 0$.

The suppliers incur holding cost for the inventory on hand and in assembly. By Little's law, the average inventory in assembly is $\mu^{L_1}$, the mean demand in $L_1$ periods. We have shown that in period $t - L_2$, S2's inventory position will be $\bar{s}_2 \wedge (\bar{s}_3 - D^{l_3})$. Thus, S2's inventory on hand in period $t$ is

$$[(\bar{s}_2 - D^{L_2}) \wedge (\bar{s}_3 - D^{l_3} - D^{L_2})]^+.$$

The analysis for S3 is a bit more involved. The inventory on hand at S3 may also include the components held up due to S2's failure to match with S3. For example, if S2 and S3 have inventory 5 and 10, respectively, and there is a demand for 10, then only 5 pairs of the matched components can enter for assembly, leaving S3 with 5 still in hand. Thus, S3's inventory on hand in period $t$ is

$$(\bar{s}_3 - D^{l_3}) - \bar{s}_2 \wedge (\bar{s}_3 - D^{l_3}) \wedge D^{L_2} = [(\bar{s}_3 - \bar{s}_2 - D^{l_3}) \vee (\bar{s}_3 - D^{l_3} - D^{L_2})]^+.$$

Now we can write out the cost functions for S2 and S3:

$$\begin{aligned} &\bar{H}_2(s_1, \bar{s}_2, \bar{s}_3) \\ &= h_2\mu^{L_1} + h_2 E[(\bar{s}_2 - D^{L_2}) \wedge (\bar{s}_3 - D^{L_2} - D^{l_3})]^+ \\ &\quad + \alpha_2 E[G(s_1 \wedge (\bar{s}_2 + s_1 - D^{L_2}) \wedge (\bar{s}_3 + s_1 - D^{L_2} - D^{l_3}))] \qquad (7) \\ &\bar{H}_3(s_1, \bar{s}_2, \bar{s}_3) \\ &= h_3\mu^{L_1} + h_3 E[(\bar{s}_3 - D^{l_3} - D^{L_2}) \vee (\bar{s}_3 - \bar{s}_2 - D^{l_3})]^+ \\ &\quad + \alpha_3 E[G(s_1 \wedge (\bar{s}_2 + s_1 - D^{L_2}) \wedge (\bar{s}_3 + s_1 - D^{L_2} - D^{l_3}))]. \qquad (8) \end{aligned}$$

The cost functions have the following properties.

Lemma 1. *$\bar{H}_1$ is strictly convex in $s_1$, $\bar{H}_3$ is strictly convex in $\bar{s}_3$, and $\bar{H}_2$ is quasiconvex in $\bar{s}_2$.*

Proof. Since

$$\begin{aligned} &\bar{H}_1(s_1, \bar{s}_2, \bar{s}_3) \\ &\quad = \int_{\bar{s}_3 - \bar{s}_2}^{\infty} \phi^{l_3}(x) \int_{\bar{s}_3 - x}^{\infty} \phi^{L_2}(y) G_1(s_1 + \bar{s}_3 - y - x)\,dy\,dx \\ &\qquad + \int_{\bar{s}_3 - \bar{s}_2}^{\infty} \phi^{l_3}(x) \int_0^{\bar{s}_3 - x} \phi^{L_2}(y) G_1(s_1)\,dy\,dx \\ &\qquad + \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G_1(s_1 + \bar{s}_2 - y)\,dy\,dx \\ &\qquad + \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x) \int_0^{\bar{s}_2} \phi^{L_2}(y) G_1(s_1)\,dy\,dx \end{aligned}$$

and $G_1(\cdot)$ is strictly convex, $\bar{H}_1(s_1, \bar{s}_2, \bar{s}_3)$ is strictly convex in $s_1$.

We can show that $\bar{H}_3(s_1, \bar{s}_2, \bar{s}_3)$ is strictly convex in $\bar{s}_3$ by checking the second-order derivatives.

Taking derivative of $\bar{H}_2(s_1, \bar{s}_2, \bar{s}_3)$ gives

$$\begin{aligned} &\frac{\partial \bar{H}_2(s_1, \bar{s}_2, \bar{s}_3)}{\partial \bar{s}_2} \\ &\quad = h_2 \Phi^{l_3}(\bar{s}_3 - \bar{s}_2) \Phi^{L_2}(\bar{s}_2) \\ &\qquad + \alpha_2 \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G'(s_1 + \bar{s}_2 - y)\,dy\,dx \\ &\quad = \left( \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x)\,dx \right) \\ &\qquad \cdot \left( h_2 \Phi^{L_2}(\bar{s}_2) + \alpha_2 \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G'(s_1 + \bar{s}_2 - y)\,dy \right). \end{aligned}$$

The first part $\int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x)\,dx$, is positive for $\bar{s}_2 < \bar{s}_3$ and zero for $\bar{s}_2 \geq \bar{s}_3$. Let

$$f(\bar{s}_2) = h_2 \Phi^{L_2}(\bar{s}_2) + \alpha_2 \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G'(s_1 + \bar{s}_2 - y)\,dy,$$

then

$$\begin{aligned} f'(\bar{s}_2) &= h_2 \phi^{L_2}(\bar{s}_2) + \alpha_2 \int_{\bar{s}_2}^{\infty} \phi^{L_2}(y) G''(s_1 + \bar{s}_2 - y)\,dy \\ &\quad - \alpha_2 \phi(\bar{s}_2) G'(s_1). \end{aligned}$$

Since $G' < 0$ and $G'' > 0$, we know $f'(\bar{s}_2) > 0$ for $\bar{s}_2 \geq 0$. So $f$ is strictly increasing. Note that

$$f(0) = \alpha_2 \int_0^{\infty} \phi^{L_2}(y) G'(s_1 - y)\,dy < 0.$$

Consider the equation $f(\bar{s}_2) = 0$ for $\bar{s}_2 \in (0, \bar{s}_3)$. Since $f(0) < 0$ and $f' > 0$, the equation has at most one solution in the range $(0, \bar{s}_3)$. If the equation has no solution, then $(\partial \bar{H}_2(s_1, \bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) < 0$ for $\bar{s}_2 < \bar{s}_3$ and $(\partial \bar{H}_2(s_1, \bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) = 0$ for $\bar{s}_2 \geq \bar{s}_3$. This implies that $\bar{H}_2(s_1, \bar{s}_2, \bar{s}_3)$ is quasi-convex in $\bar{s}_2$. If the equation has one solution, denoted by $x^* \in (0, \bar{s}_3)$, then $(\partial \bar{H}_2(s_1, \bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) < 0$ for $\bar{s}_2 < x^*$, $(\partial \bar{H}_2(s_1, \bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) > 0$ for $x^* < \bar{s}_2 < \bar{s}_3$, while $(\partial \bar{H}_2(s_1, \bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) = 0$ for $\bar{s}_2 \geq \bar{s}_3$. This also implies that $\bar{H}_2(s_1, \bar{s}_2, \bar{s}_3)$ is quasi-convex in $\bar{s}_2$. □

The following lemma follows immediately from Lemma 1 and Theorem 1.2 in Fudenberg and Tirole (1991).

Lemma 2. *There exists a Nash equilibrium in the inventory game with information sharing.*

We have proven the existence of the equilibrium. In the rest of this subsection, we further show that although multiple equilibria may exist, they possess the same cost outcome. That is, there is a unique equilibrium cost outcome. Define $\bar{r}_2(s_1) = \bar{r}_2(s_1, \infty)$ as the best reply function for S2 assuming that S3 always has ample stock. Recall that $s_2 = s_1 + \bar{s}_2$, and $s_3 = s_1 + \bar{s}_3$ are the corresponding echelon base-stock levels. For the suppliers, once the manufacturer's decision $s_1$ has been made, choosing local inventory levels is actually equivalent to choosing echelon inventory levels. Define $r_2(s_1) = s_1 + \bar{r}_2(s_1)$ as the echelon best reply of S2 assuming that S3 has ample stock. Define $r_3(s_1, s_2)$ as the echelon best reply of S3 given the echelon stock levels chosen by S1 and S2. Lemmas 3 to 6 demonstrate some important properties of these best reply functions.

Lemma 3. *In the inventory game with information sharing,* S2*'s best reply to the game is*

$$\bar{r}_2(s_1, \bar{s}_3) = \begin{cases} \bar{r}_2(s_1) & \text{if } \bar{s}_3 > \bar{r}_2(s_1) \\ [\bar{s}_3, M] & \text{if } \bar{s}_3 \leq \bar{r}_2(s_1). \end{cases}$$

Proof. Note that

$$\bar{H}_2(s_1, \bar{s}_2, \infty) = h_2\mu^{L_1} + h_2 E(\bar{s}_2 - D^{L_2})^+ + \alpha_2 E[G(s_1 \wedge (\bar{s}_2 + s_1 - D^{L_2}))],$$

which is strictly convex in $\bar{s}_2$ and minimized by $\bar{r}_2(s_1)$. In other words, S2 should order up to $\bar{r}_2(s_1)$ if S3 has ample stock to match. Now consider $\bar{H}_2(s_1, \bar{s}_2, \bar{s}_3)$. Recall that S2 can use a contingent policy. Since $\bar{r}_2(s_1)$ is the minimizer of $\bar{H}_2(s_1, \bar{s}_2, \infty)$, S2 should order up to $\bar{r}_2(s_1)$ in the game unless S3 does not have sufficient inventory to match. If $\bar{s}_3 > \bar{r}_2(s_1)$, then $\bar{r}_2(s_1)$ is the strictly dominant strategy for S2. If $\bar{s}_3 \leq \bar{r}_2(s_1)$, then $\bar{s}_3 - D^{l_3} < \bar{r}_2(s_1)$. That is, S3 never has sufficient stock to match S2's best order-up-to level. S2's cost function $\bar{H}_2(s_1, \bar{s}_2, \bar{s}_3)$ remains the same for any $\bar{s}_2 \geq \bar{s}_3$ due to the contingent policy. □

Lemma 4. *In the inventory game with information sharing,*

$$-1 < (d\bar{r}_1(\bar{s}_2 + t, \bar{s}_3 + t))/(dt) < 0,$$

$$-1 < (\partial \bar{r}_1(\bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) < 0,$$

*and*

$$-1 < (\partial \bar{r}_1(\bar{s}_2, \bar{s}_3))/(\partial \bar{s}_3) < 0.$$

Proof. We find that

$$\begin{aligned} \frac{\partial^2 \bar{H}_1}{\partial s_1^2} &= \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x) \Big[ \Phi^{L_2}(\bar{s}_2 + t - x) G_1''(s_1) \\ &\qquad + \int_{\bar{s}_3 + t - x}^{\infty} \phi^{L_2}(y) G_1''(s_1 + \bar{s}_3 + t - y - x)\, dy \Big] dx \\ &\quad + \Phi^{l_3}(\bar{s}_3 - \bar{s}_2) \Big[ \Phi^{L_2}(\bar{s}_2 + t) G_1''(s_1) \\ &\qquad + \int_{\bar{s}_2 + t}^{\infty} \phi^{L_2}(y) G_1''(s_1 + \bar{s}_2 + t - y)\, dy \Big], \\ \frac{\partial^2 \bar{H}_1}{\partial s_1 \partial t} &= \int_0^{\bar{s}_3 - \bar{s}_2} \phi^{l_3}(x) \int_{\bar{s}_3 + t - x}^{\infty} \phi^{L_2}(y) \\ &\quad \cdot G_1''(s_1 + \bar{s}_3 + t - y - x)\, dy\, dx \\ &\quad + \Phi^{l_3}(\bar{s}_3 - \bar{s}_2) \int_{\bar{s}_2 + t}^{\infty} \phi^{L_2}(y) G_1''(s_1 + \bar{s}_2 + t - y)\, dy. \end{aligned}$$

We know $(\partial^2 \bar{H}_1)/(\partial s_1^2) > 0$ and $(\partial^2 \bar{H}_1)/(\partial s_1 \partial t) > 0$ because $G_1'' > 0$. Observe also that $(\partial^2 \bar{H}_1)/(\partial s_1^2) > (\partial^2 \bar{H}_1)/(\partial s_1 \partial t)$. Thus,

$$-1 < \frac{d\bar{r}_1(\bar{s}_2 + t, \bar{s}_3 + t)}{dt} = -\frac{\partial^2 \bar{H}_1}{\partial s_1^2} \bigg/ \frac{\partial^2 \bar{H}_1}{\partial s_1 \partial t} > 0.$$

Similarly, we can show that $-1 < (\partial \bar{r}_1(\bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2) < 0$ and $-1 < (\partial \bar{r}_1(\bar{s}_2, \bar{s}_3))/(\partial \bar{s}_3) < 0$. □

Lemma 5. *In the inventory game with information sharing,* $0 < r_2'(s_1) < 1$ *and* $-1 < \bar{r}_2'(s_1) < 0$.

Proof. Lemma 2 in Cachon and Zipkin (1999) shows $0 < r_2'(s_1) < 1$. Since $r_2(s_1) = s_1 + \bar{r}_2(s_1)$ by definition, the second result follows. □

Lemma 6. *In the inventory game with information sharing, the echelon best reply of* S3, $r_3(s_1, s_2)$, *satisfies* $0 < (\partial r_3(s_1, s_2))/(\partial s_1) < 1$ *and* $0 \le (\partial r_3(s_1, s_2))/(\partial s_2) < 1$.

Proof. S3's cost function expressed in the echelon inventory notation is $\overline{H}_3(s_1, s_2 - s_1, s_3 - s_1)$. Taking derivatives gives

$$\begin{aligned}\frac{\partial^2 \overline{H}_3}{\partial s_3^2} &= h_3 \int_0^{s_2-s_1} \phi^{L_2}(x)\phi^{l_3}(s_3 - s_1 - x)\,dx\\ &\quad + h_3 \int_{s_2-s_1}^{\infty} \phi^{L_2}(x)\phi^{l_3}(s_3 - s_2)\,dx\\ &\quad + \alpha_3 \int_{s_3-s_2}^{\infty} \phi^{l_3}(x) \int_{s_3-s_1-x}^{\infty} \phi^{L_2}(y)\\ &\quad \cdot G''(s_3 - y - x)\,dy\,dx\\ &\quad - \alpha_3 \int_{s_3-s_2}^{\infty} \phi^{l_3}(x)\phi^{L_2}(s_3 - s_1 - x)G'(s_1)\,dx\\ &\quad - \alpha_3 \phi^{l_3}(s_3 - s_2) \int_{s_2-s_1}^{\infty} \phi^{L_2}(y)G'(s_2 - y)\,dy,\\ \frac{\partial^2 \overline{H}_3}{\partial s_1 \partial s_3} &= -h_3 \int_0^{s_2-s_1} \phi^{L_2}(x)\phi^{l_3}(s_3 - s_1 - x)\,dx\\ &\quad + \alpha_3 \int_{s_3-s_2}^{\infty} \phi^{l_3}(x)\phi^{L_2}(s_3 - s_1 - x)G'(s_1)\,dx,\\ \frac{\partial^2 \overline{H}_3}{\partial s_2 \partial s_3} &= -h_3 \int_{s_2-s_1}^{\infty} \phi^{L_2}(x)\phi^{l_3}(s_3 - s_2)\,dx\\ &\quad + \alpha_3 \phi^{l_3}(s_3 - s_2) \int_{s_2-s_1}^{\infty} \phi^{L_2}(y)G'(s_2 - y)\,dy.\end{aligned}$$

Note that $G' < 0$ and $G'' > 0$. So $\partial^2 \overline{H}_3/\partial s_3^2 > 0$ and $\partial^2 \overline{H}_3/\partial s_1 \partial s_3 < 0$. But $\partial^2 \overline{H}_3/\partial s_2 \partial s_3 < 0$ when $s_3 > s_2$ and $\partial^2 \overline{H}_3/\partial s_2 \partial s_3 = 0$ when $s_3 \le s_2$. Since

$$\frac{\partial r_3(s_1, s_2)}{\partial s_1} = -\frac{\partial^2 \overline{H}_3}{\partial s_1 \partial s_3} \bigg/ \frac{\partial^2 \overline{H}_3}{\partial s_3^2},$$
$$\frac{\partial r_3(s_1, s_2)}{\partial s_2} = -\frac{\partial^2 \overline{H}_3}{\partial s_2 \partial s_3} \bigg/ \frac{\partial^2 \overline{H}_3}{\partial s_3^2},$$

the desired result follows. □

Now we are in a position to prove that despite the potential multiple equilibria, the cost outcome in equilibrium is unique.

Lemma 7. *Suppose* $(s_1, \bar{s}_2, \bar{s}_3)$ *is a Nash equilibrium of the inventory game with information sharing, then* $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ *is also a Nash equilibrium with the same cost outcome.*

Proof. If $\bar{s}_2 = \bar{r}_2(s_1)$, then the proof is done. If $\bar{s}_2 \ne \bar{r}_2(s_1)$, then according to Lemma 3, there must be $\bar{s}_3 \le \bar{r}_2(s_1)$ and $\bar{s}_3 \le \bar{s}_2$. We claim that $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ is a Nash equilibrium. The argument is as follows. S1's payoff function in (6) remains the same because both $\bar{s}_2$ and $\bar{r}_2(s_1)$ are greater than $\bar{s}_3$. Thus, $s_1$ is still the best reply given $\bar{s}_{-1} = (\bar{r}_2(s_1), \bar{s}_3)$. Given $\bar{s}_{-2} = (s_1, \bar{s}_3)$, S2's cost is the same for all $\bar{s}_2 \ge \bar{s}_3$ (recall the contingent policy). We know $\bar{r}_2(s_1) \ge \bar{s}_3$. So $\bar{r}_2(s_1)$ and $\bar{s}_2$ yields the same cost for S2. However, then $\bar{r}_2(s_1)$ is a best reply to $\bar{s}_{-2} = (s_1, \bar{s}_3)$ because $\bar{s}_2$ is a best reply. Last, we argue that $\bar{s}_3$ is still the best reply for S3 given $\bar{s}_{-3} = (\bar{s}_1, \bar{r}_2(s_1))$. S3's cost in (8) is minimized at $\bar{r}_3(s_1, \bar{s}_2) = \bar{s}_3 \le \bar{s}_2$, so $\bar{r}_3(s_1, \bar{s}_2)$ does not change in $\bar{s}_2$ as long as $\bar{s}_2 \ge \bar{s}_3$. However, $\bar{r}_2(s_1) \ge \bar{s}_3$, hence, we have $\bar{r}_3(s_1, \bar{r}_2(s_1)) = \bar{r}_3(s_1, \bar{s}_2) = \bar{s}_3$. That is, $\bar{s}_3$ is still the best reply for S3 given $\bar{s}_{-3} = (s_1, \bar{r}_2(s_1))$. Therefore, we have shown that $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ is also an equilibrium. Following the same argument, we know that $(s_1, \bar{s}_2, \bar{s}_3)$ and $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ have the same cost outcome. □

Lemma 8. *Suppose* $(s_1, \bar{r}_2(\bar{s}_1), \bar{s}_3)$ *and* $(s_1', \bar{r}_2(s_1'), \bar{s}_3')$ *are two Nash equilibria of the inventory game with information sharing, then they are identical.*

Proof. If $\bar{r}_2(s_1) = \bar{r}_2(s_1')$, then it implies that $\bar{s}_1 = s_1'$ (recall that $\bar{r}_2$ is strictly decreasing by Lemma 5), and thus $\bar{s}_3 = \bar{s}_3'$, and the proof is done. Next, we show that $\bar{r}_2(s_1) > \bar{r}_2(\bar{s}_1')$ cannot hold.

Suppose $\bar{r}_2(s_1) > \bar{r}_2(\bar{s}_1')$, then $s_1 < s_1'$ by Lemma 5. Define $\Delta = (\bar{r}_2(s_1) - \bar{r}_2(s_1')) \vee (\bar{s}_3 - \bar{s}_3')$, the larger one of the two differences in S2 and S3's order-up-to levels. By Lemma 4, $s_1' - s_1 < \Delta$.

If $\Delta = \bar{r}_2(s_1) - \bar{r}_2(s_1')$, then $s_1' - s_1 < \Delta$ leads to $s_1 + \bar{r}_2(s_1) > s_1' + \bar{r}_2(s_1')$, or $s_2 > s_2'$. However $s_2 > s_2'$ implies $s_1 > s_1'$ by Lemma 5, a contradiction.

If $\Delta = \bar{s}_3 - \bar{s}_3'$, then $s_1' - s_1 < \Delta$ leads to $s_1 + \bar{s}_3 > s_1' + \bar{s}_3'$, or $s_3 > s_3'$. However this implies $s_1 > s_1'$, since otherwise $s_1 < s_1'$ implies $s_2 < s_2'$ (by Lemma 5) and further $s_3 < s_3'$ by Lemma 6. Again, a contradiction.

Therefore, $\bar{r}_2(s_1) > \bar{r}_2(s_1')$ cannot be true. Similarly, we can show that $\bar{r}_2(s_1) < \bar{r}_2(s_1')$ cannot be true, either. □

Theorem 1. *For any two Nash equilibria* $(s_1, \bar{s}_2, \bar{s}_3)$ *and* $(s_1', \bar{s}_2', \bar{s}_3')$ *of the inventory game with information sharing, there is* $s_1 = s_1'$ *and* $\bar{s}_3 = \bar{s}_3'$. *In addition, they give the same cost outcome.*

Proof. By Lemma 7, $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ and $(s_1', \bar{r}_2(s_1'), \bar{s}_3')$ are also Nash equilibrium. By Lemma 8, these two equilibria are identical. Furthermore, $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ and $(s_1, \bar{s}_2, \bar{s}_3)$ give the same cost outcome, and so do $(s_1', \bar{r}_2(s_1'), \bar{s}_3')$ and $(s_1', \bar{s}_2', \bar{s}_3')$. Thus, $(s_1, \bar{s}_2, \bar{s}_3)$ and $(s_1', \bar{s}_2', \bar{s}_3')$ give the same costs as well. □

Theorem 1 asserts that S1 and S3's strategies are the same across all Nash equilibria. In fact, S2's strategy can differ across Nash equilibria just because of the contingent stock policy. Nevertheless, the cost outcome is the same in all Nash equilibria. From this point on, we view $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ as the unique Nash equilibrium in the game.

### 4.2. Inventory Game Without Information Sharing

If the suppliers do not exchange information on their inventory status, the contingent inventory policies are no longer possible. Note that when $L_2 = L_3$, the contingent policy cannot be implemented either. Thus, the analysis for no information sharing applies to $L_2 \le L_3$ (we can simply set $l_3 = 0$ when $L_2 = L_3$). The cost functions for S1 and S3 remain the same as in (6) and (8). For S2, the inventory on hand in period $t$ (without contingent inventory policy) now becomes

$$[\bar{s}_2 - (\bar{s}_3 - D^{l_3})^+ \wedge D^{L_2}]^+ = [(\bar{s}_2 - (\bar{s}_3 - D^{l_3})^+) \vee (\bar{s}_2 - D^{L_2})]^+,$$

and the cost function becomes

$$\begin{aligned}\overline{H}_2(s_1, \bar{s}_2, \bar{s}_3) &= h_2\mu^{L_1} + h_2E[(\bar{s}_2 - (\bar{s}_3 - D^{l_3})^+) \vee (\bar{s}_2 - D^{L_2})]^+ \\ &\quad + \alpha_2E[G(s_1 \wedge (s_1 + \bar{s}_2 - D^{L_2}) \wedge (s_1 + \bar{s}_3 - D^{L_2} - D^{l_3}))].\end{aligned} \tag{9}$$

By checking the second-order derivatives, it is straightforward to prove the convexity of the cost functions.

Lemma 9. *In the inventory game without information sharing, the cost function $\overline{H}_i$ is strictly convex in $\bar{s}_i$ for all $i$.*

The following theorem shows that there exists a unique Nash equilibrium in the inventory game without information sharing. Therefore, there is also a unique cost outcome in the inventory game without information sharing.

Theorem 2. *In the inventory game without information sharing, there exists a unique Nash equilibrium.*

Proof. The existence of a Nash equilibrium is from Lemma 9 and Theorem 1.2 in Fudenberg and Tirole (1991). Uniqueness can be established if the best reply mapping $\bar{s}_i = \bar{r}_i(\bar{s}_{-i})$, $i = 1, 2, 3$ is a contraction mapping. The Hessian matrix of the best reply mapping is

$$\mathbf{H} = \begin{vmatrix} \dfrac{\partial^2 \overline{H}_1}{\partial \bar{s}_1^2} & \dfrac{\partial^2 \overline{H}_1}{\partial \bar{s}_1 \partial \bar{s}_2} & \dfrac{\partial^2 \overline{H}_1}{\partial \bar{s}_1 \partial \bar{s}_3} \\ \dfrac{\partial^2 \overline{H}_2}{\partial \bar{s}_2 \partial \bar{s}_1} & \dfrac{\partial^2 \overline{H}_2}{\partial \bar{s}_2^2} & \dfrac{\partial^2 \overline{H}_2}{\partial \bar{s}_2 \partial \bar{s}_3} \\ \dfrac{\partial^2 \overline{H}_3}{\partial \bar{s}_3 \partial \bar{s}_1} & \dfrac{\partial^2 \overline{H}_3}{\partial \bar{s}_3 \partial \bar{s}_2} & \dfrac{\partial^2 \overline{H}_3}{\partial \bar{s}_3^2} \end{vmatrix}.$$

It suffices to show that $\mathbf{H}$ satisfies the *diagonal dominance* property (see Horn and Johnson 1996, Cachon and Netessine 2004):

$$\sum_{j \ne i} \left| \frac{\partial^2 \overline{H}_i}{\partial \bar{s}_i \partial \bar{s}_j} \right| < \left| \frac{\partial^2 \overline{H}_i}{\partial \bar{s}_i^2} \right|, \quad i = 1, 2, 3.$$

This can be shown by checking all the second-order derivatives. □

## 5. Comparing the Equilibria and the Optimal Solution

In this section, we first compare the Nash equilibria in the inventory games (with and without information sharing) to the system optimal policy, and then compare the Nash equilibria under different information structures. Numerical studies are also carried out to generate more insights from the comparisons.

### 5.1. Nash Equilibria vs. the System Optimal Policy

Lemma 10. *Let $(s_1, \bar{r}_2(s_1), \bar{s}_3)$ be the Nash equilibrium in the game with information sharing, then $\bar{r}_2(s_1) + s_1 < s_2^o$.*

Proof. Note that $\bar{r}_2(s_1) + s_1 = r_2(s_1)$ and choosing $\bar{r}_2(s_1)$ is equivalent to choosing $r_2(s_1)$. Recall that $r_2(s_1)$ is the minimizer of $\overline{H}_2(s_1, s_2 - s_1, \infty)$. Theorem 15 in Cachon and Zipkin (1999) shows that $r_2(s_1) < s_2^o$, regardless of $s_1$. □

Theorem 3. *The Nash equilibrium in the inventory game with information sharing is never system optimal. Either $s_2 < s_2^o$ or $s_3 < s_3^o$ or both hold.*

Proof. In any Nash equilibrium $(s_1, \bar{s}_2, \bar{s}_3)$, if $\bar{s}_2 < \bar{s}_3$, then $\bar{s}_2 = \bar{r}_2(s_1) = r_2(s_1) - s_1$ by Lemma 3. But from Lemma 10, $s_2 = r_2(s_1) < s_2^o$.

If $\bar{s}_2 \ge \bar{s}_3$, then by Lemma 3, $\bar{s}_3 \le r_2(s_1)$ and, thus, $s_3 = \bar{s}_3 + s_1 \le \bar{r}_2(s_1) + s_1 < s_2^o \le s_3^o$. In this case, the system stock is bounded by $s_3$ ($s_2$ can be arbitrarily large, but that does not alter the cost because of the contingent policy).

So the equilibria are never system optimal. $s_2 < s_2^o$ and/or $s_3 < s_3^o$ imply that the system understocks compared to the system optimal policy. □

Theorem 4. *The Nash equilibrium in the inventory game without information sharing is never system optimal. The echelon stock level at* S2 *is always less than that in the system optimal policy, i.e.,* $s_2 < s_2^o$.

Proof. Let $(s_1, \bar{s}_2, \bar{s}_3)$ be the unique Nash equilibrium, where $\bar{s}_2 = \bar{r}_2(s_1, \bar{s}_3)$ is the best reply function in the game without information sharing. Consider the cost function for S2 in (9). It can be readily shown that $(\partial \overline{H}_2(s_1, \bar{s}_2, \infty))/(\partial \bar{s}_2) < (\partial \overline{H}_2(s_1, \bar{s}_2, \bar{s}_3))/(\partial \bar{s}_2)$. So $\bar{r}_2(s_1, \bar{s}_3) < \bar{r}_2(s_1)$ and, thus, $s_2 = r_2(s_1, \bar{s}_3) < r_2(s_1)$, which in turn is less than $s_2^o$ regardless of $s_1$ by Lemma 10. □

Theorems 3 and 4 show that under both information structures, the competitive outcome is never system optimal. More importantly, in equilibrium, there is either $s_2 < s_2^o$ or $s_3 < s_3^o$. Because only the matched components can enter assembly, understocking at either supplier implies understocking for the whole system.

### 5.2. Nash Equilibria Under Different Information Structures

Let $(s_1^w, \bar{s}_2^w, \bar{s}_3^w)$ be the Nash equilibrium with information sharing and $(s_1^{wo}, \bar{s}_2^{wo}, \bar{s}_3^{wo})$ be the Nash equilibrium without information sharing. Next, we compare these two equilibria.

Lemma 11. *In the inventory game without information sharing,* S2*'s best reply function satisfies* $-1 < (\partial \bar{r}_2(\bar{s}_1, \bar{s}_3))/(\partial \bar{s}_1) < 0$ *and* $0 < (\partial \bar{r}_2(\bar{s}_1, \bar{s}_3))/(\partial \bar{s}_3) < 1$; S3*'s best reply function satisfies* $-1 < (\partial \bar{r}_3(\bar{s}_1, \bar{s}_2))/(\partial \bar{s}_1) < 0$ *and* $0 < (\partial \bar{r}_3(\bar{s}_1, \bar{s}_2))/(\partial \bar{s}_2) < 1$.

Proof. Similar to the proof of Lemma 4. □

Theorem 5. *Both suppliers' inventory levels in the game with information sharing are higher than those in the game without information sharing, i.e.,* $\bar{s}_2^w > \bar{s}_2^{wo}$ *and* $\bar{s}_3^w > \bar{s}_3^{wo}$. *However the opposite holds for the manufacturer's inventory levels, i.e.,* $\bar{s}_1^w < \bar{s}_1^{wo}$.

Proof. Imagine the process of searching for $(s_1^{wo}, \bar{s}_2^{wo}, \bar{s}_3^{wo})$, the Nash equilibrium without information sharing. Suppose we start from

$$(s_1^{(0)}, \bar{s}_2^{(0)}, \bar{s}_3^{(0)}) = (s_1^w, \bar{s}_2^w, \bar{s}_3^w),$$

the equilibrium with information sharing. We have shown that the best reply functions form a contraction mapping (Theorem 4), so finally the process will converge to $(\bar{s}_1^{wo}, \bar{s}_2^{wo}, \bar{s}_3^{wo})$ by iteration. In the $k$th iteration, we set

$$\begin{aligned}(s_1^{(k)}, \bar{s}_2^{(k)}, \bar{s}_3^{(k)}) \\ = \big(\bar{r}_1(\bar{s}_2^{(k-1)}, \bar{s}_3^{(k-1)}), \bar{r}_2(s_1^{(k-1)}, \bar{s}_3^{(k-1)}), \bar{r}_3(s_1^{(k-1)}, \bar{s}_2^{(k-1)})\big).\end{aligned}$$

Note that here $\bar{r}_1$, $\bar{r}_2$, and $\bar{r}_3$ are the best reply functions in the inventory game without information sharing. Consider the first iteration $k = 1$. There will be $s_1^{(1)} = s_1^w$ and $s_3^{(1)} = s_3^w$ since S1 and S3's cost functions are the same under both information structures. However $\bar{s}_2^{(1)} = \bar{r}_2(s_1^w, \bar{s}_3^w) < \bar{s}_2^w$ since $\partial \overline{H}_2^{wo}/\partial \bar{s}_2 > \partial \overline{H}_2^w/\partial \bar{s}_2$ ($\overline{H}_2^w$ and $\overline{H}_2^{wo}$ are given in Equations (7) and (9), respectively). In the second iteration, there will be $s_1^{(2)} > s_1^{(1)} = s_1^w$ by Lemma 4 (since S2's order-up-to level has decreased in the previous iteration), $\bar{s}_2^{(2)} = \bar{s}_2^{(1)}$ (since S1 and S3's order-up-to levels have not been changed in the previous iteration), and $s_3^{(2)} < s_3^{(1)} = s_3^w$ by Lemma 11 (again, since S2's order-up-to level has decreased in the previous iteration).

The iteration process continues until the unique equilibrium $(s_1^{wo}, \bar{s}_2^{wo}, \bar{s}_3^{wo})$ is reached. As we can see, in each iteration, the suppliers' order-up-to levels decrease (or at least never increase) by Lemma 11, and the manufacturer's order-up-to level increases (or at least never decreases) by Lemma 4. Therefore, there must be $\bar{s}_1^w < \bar{s}_1^{wo}$, $\bar{s}_2^w > \bar{s}_2^{wo}$, and $\bar{s}_3^w > \bar{s}_3^{wo}$. □

From the above theorem, we know that the suppliers will hold less inventory without information sharing. This is an intuitive result because the components at different suppliers are complementary. It is pointless to hold inventory for one component when the other is not available. So without information sharing, it is riskier for suppliers (especially S2) to maintain a high level of inventory. As a result, the manufacturer has to stock more to compensate for understocking by the suppliers.

**Table 1 Competition Penalty Under Different Allocations of Backorder Costs**

| $\alpha_1$ | Minimum | 25th percentile | 50th percentile | 75th percentile | Maximum |
|---|---|---|---|---|---|
| With information sharing | | | | | |
| 0.01 | 123 | 139 | 156 | 168 | 214 |
| 0.20 | 21 | 22 | 24 | 25 | 28 |
| 0.40 | 7 | 7 | 8 | 8 | 9 |
| 0.60 | 3 | 3 | 4 | 5 | 7 |
| 0.80 | 1 | 3 | 5 | 7 | 12 |
| 0.99 | 1 | 4 | 7 | 10 | 18 |
| Without information sharing | | | | | |
| 0.01 | 126 | 142 | 159 | 171 | 215 |
| 0.20 | 24 | 26 | 26 | 27 | 30 |
| 0.40 | 10 | 11 | 11 | 12 | 13 |
| 0.60 | 3 | 4 | 5 | 6 | 11 |
| 0.80 | 1 | 3 | 5 | 7 | 12 |
| 0.99 | 1 | 4 | 7 | 10 | 18 |

### 5.3. Numerical Studies

To obtain more insight from the comparisons, we conduct two sets of numerical experiments. The purpose of the first one is to investigate how large the difference is between the costs for decentralized and centralized systems. In particular, we want to find out how the allocation of backorder costs between the lower and upper echelons affect that difference. Single-period demand is normal with a mean of 20 and a variance of 25. Fix $p=1$, $L_1=1$, and $L_2=2$. $L_3$ is selected from $\{3,5,7\}$. Holding costs $h_i$ $(i=1,2,3)$ take values in $\{0.05, 0.10\}$. Assume the two suppliers have equal shares of backorder costs, i.e., $\alpha_2=\alpha_3=\frac{1}{2}(1-\alpha_1)$. $\alpha_1$ take values in the set $\{0.01, 0.2, 0.4, 0.6, 0.8, 0.99\}$. We compute the Nash equilibrium for each parameter combination and compare with the system optimal solution. The results are displayed in Table 1. The competition penalty is defined to be the percentage increase in the cost in the Nash equilibrium over the system optimal cost.

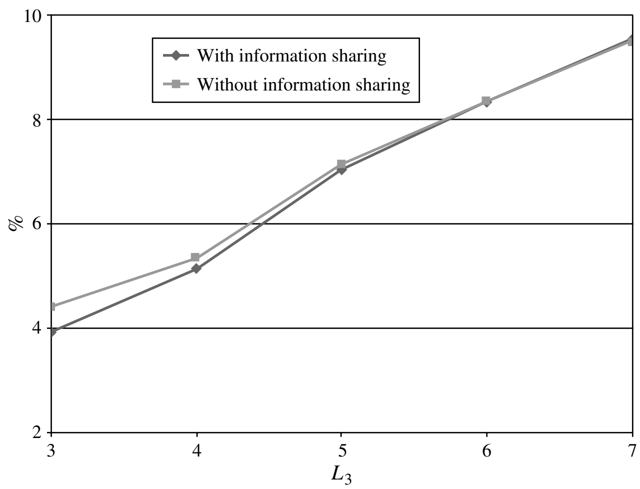
Table 1 shows that unbalanced allocations of backorder costs may cause huge cost increases in the decentralized system. When $\alpha_1=0.01$, the supply chain total cost is at least doubled for all cases compared to the optimal solution. However, if the manufacturer bears a larger portion of the backorder cost, e.g., $\alpha_1=0.6$ or higher, the competition penalty drops dramatically to less than 10%. This is because the manufacturer has the higher holding cost $(h=h_1+h_2+h_3)$ and tends to carry insufficient inventory from the system's perspective. Hence, a larger backorder cost forces the manufacturer to carry more inventory, which benefits the entire supply chain. However note that too large a backorder cost for the manufacturer (e.g., $\alpha_1=0.99$) drives up the competition penalty again. Therefore, in summary, the competition penalty increases when the manufacturer's backorder cost is not appropriately matched with his holding cost. The pattern holds for both information structures.

In the second set of experiments, we try to answer questions like: How does the difference in suppliers' lead times affect the competition penalty, and how does the competition penalty differ in different information structures? Fix $p=1$, $\alpha_1=0.8$, $\alpha_1=\alpha_2=0.1$, $L_1=1$, and $L_2=2$. Let $L_3$ take values in $\{3,4,5,6,7\}$ and $h_i$ $(i=1,2,3)$ take values in $\{0.01, 0.03, 0.05\}$. The demand is the same as in the previous example, that is, $D^1 \sim N(20,25)$. Thus, for each value of $L_3$, there are 27 parameter combinations.

Figure 2 shows the average competition penalty for different information structures. The horizontal axis is S3's lead time, $L_3$. When $L_3$ increases, the average competition penalty increases for both structures. This implies that unbalanced supply lead times tend to aggravate the competition penalty. In other words, one way to reduce the competition penalty is to shorten S3's lead time (S3 is the supplier with the longer lead time by definition). This also suggests that supply chain coordination is more important in situations in which suppliers have very different lead times.

Define the value of information sharing as the percentage reduction in the competition penalty by

**Figure 2** **The Impact of the Lead Time $L_3$ on the Competition Penalty**




sharing information. Table 2 presents the values of information sharing for different $L_3$. The negative numbers indicate that information sharing may lead to a higher competition penalty than without information sharing. Such outcomes tend to occur when both $h_2$ and $h_3$ are large. One possible explanation is as follows. Without information sharing, the manufacturer is forced to hold more inventory while the suppliers hold less. Shifting inventory from suppliers to manufacturer reduces the suppliers' holding costs while helping satisfy customer demand, so this might lower the system's total cost. This is an interesting example showing that more information is not necessarily beneficial to a decentralized operational system. Having said that, sharing information sometimes can reduce the competition penalty by almost a third in certain cases. Therefore, from the system's perspective, the value of information sharing can be either positive or negative. As to individual members in the assembly system, we find in this numerical example that the manufacturer always prefers information sharing.

**Table 2** **The Value of Information Sharing for Different $L_3$**

| $L_3$ | Minimum | 25th percentile | 50th percentile | 75th percentile | Maximum |
|---|---|---|---|---|---|
| 3 | −11 | −2 | 5 | 9 | 32 |
| 4 | −6 | −3 | 1 | 3 | 21 |
| 5 | −7 | −3 | 0 | 1 | 13 |
| 6 | −6 | −3 | −1 | 1 | 11 |
| 7 | −8 | −3 | −2 | 1 | 8 |

This is because the suppliers will carry more inventory under information sharing, which is desirable to the manufacturer. However, the suppliers may or may not prefer information sharing. That is, the suppliers' costs could be higher or lower with information sharing than without information sharing. The finding is true for both S2 and S3. In general, when suppliers have large holding costs, they tend to refuse to share information.

## 6. A Demand-Independent Coordination Contract

We have shown that in the decentralized assembly system, different stages act in their own interest and never choose the system optimal policy. Hence, all stages can benefit from coordination. Several coordination methods have been proposed in the literature, but primarily for serial systems. In this section, we propose a new coordination scheme and discuss its relationship with existing methods. The new coordination scheme works for supply chains with either serial or assembly structure, and it does not require demand distribution as an input. In addition, we demonstrate that under the new scheme, the system optimal policy will be the unique Nash equilibrium for the decentralized supply chain.

### 6.1. The Coordination Contract

For ease of exposition, the coordination contract is illustrated using a serial system first. Modification of the assembly system will be discussed later. Below, we introduce the coordination contract in two steps. In the first step, an incentive-compatible cost allocation scheme is presented. A cost allocation scheme is incentive compatible if each stage will choose the system optimal policy. Then in the second step, we show that the cost allocation scheme can be implemented via a linear transfer payment contract.

Use the notations defined earlier in this paper. Let $s^o = (s_1^o, \ldots, s_n^o)$ be the system optimal policy for an $n$-stage serial system. According to Chen and Zheng (1994), the system's total cost per period can be written as

$$C^s = \sum_{i=1}^{n} h_i IL_{it} + (p+h)B_t,$$

where $IL_{it}$ is the echelon inventory level at stage $i$, and $B_t$ is the backorder level at S1. S1's optimal base-stock level $s_1^o$ is the minimizer of the following cost function

$$G_1(y) = E\big[h_1(y - D^{L_1+1}) + (p+h)(y - D^{L_1+1})^-\big],$$

where $(y - D^{L_1+1})$ is the echelon inventory level at S1, and $(y - D^{L_1+1})^-$ is the backorder level at S1. Suppose that in the decentralized system, S1 incurs a cost rate of $h_1$ for the echelon inventory and $(p+h)$ for the backorders at S1 in each period. Then, if S2 always makes immediate delivery, S1's cost will be

$$\begin{aligned} J_1(y) &= G_1(y) \\ &= E\big[h_1(y - D^{L_1+1}) + (p+h)(y - D^{L_1+1})^-\big]. \quad (10) \end{aligned}$$

Thus, S1 will choose base-stock level $s_1^o$, assuming that S2 always makes immediate delivery. Actually, since $G_1(y)$ is convex, $s_1^o$ is also optimal for S1 even if S2 cannot bring S1's inventory position to $s_1^o$.

Next, we consider S2's decision. In the system optimal solution, $s_2^o$ should minimize

$$G_2(y) = E\big[h_2(y - D^{L_2+1}) + G_1^2(y - D^{L_2})\big],$$

where

$$\begin{aligned} G_1^2(x) &= G_1(x \wedge s_1^o) - G_1(s_1^o) \\ &= E\big[h_1(x \wedge s_1^o - D^{L_1+1}) + (p+h)(x \wedge s_1^o - D^{L_1+1})^-\big] \\ &\quad - G_1(s_1^o) \end{aligned}$$

is the additional cost incurred at S1 due to S2's supply constraint. Notice that $G_1(s_1^o)$ is a constant and has no impact on optimization. Suppose that in the decentralized system, S2 incurs a cost rate of $h_2$ for the echelon inventory at S2, $h_1$ for the echelon inventory S1, and $(p+h)$ for the backorders at S1. Then, if S3 always makes immediate full delivery, S2's cost will be

$$\begin{aligned} J_2(y) &= E\big[h_2(y - D^{L_2+1})\big] + E\big[h_1((y - D^{L_2}) \wedge s_1^o - D^{L_1+1}) \\ &\qquad + (p+h)((y - D^{L_2}) \wedge s_1^o - D^{L_1+1})^-\big] \\ &= G_2(y) + G_1(s_1^o). \quad (11) \end{aligned}$$

Thus, S2 will choose $s_2^o$, which is defined to minimize $G_2(y)$. Since $J_2(y)$ is convex, we know that even with S3's supply constraint, $s_2^o$ will be chosen by S2.

To summarize the analysis so far, we want S1 to incur a cost based on S1's inventory and backorder levels; we want S2 to incur a cost based on S2 and S1's echelon inventory levels and the backorder level at S1. Such a scheme can be generalized as follows: stage $i$ incurs costs based on all downstream stage $j$'s ($j = 1, 2, \ldots, i$) echelon inventory levels and S1's backorder level. With notations, we want the cost incurred at stage $i$ to be

$$J_i = \sum_{j=1}^{i} h_j IL_{jt} + (p+h)B_t.$$

Note that $J_n$ ($n$ is the most upstream stage) is just the total costs incurred in the supply chain. This is interesting because our scheme asserts that the most upstream stage should be responsible for the whole supply chain. In addition, any linear transformation of the above cost functions can achieve coordination: If stage $i$ incurs a cost $\widehat{C}_i^s = \beta_i J_i + \gamma_i$ ($\beta_i > 0$), then the cost scheme is still incentive compatible. $\beta_i$ can be viewed as a scaling factor, while $\gamma_i$ a constant transfer payment.

How do we implement the above incentive-compatible cost scheme? When the supply chain is owned by a single firm, the firm head can directly charge each stage according to the above cost scheme. The firm head is then responsible for any taxes or subsidies needed afterward. However, when the supply chain members are independent firms, it is less appealing to assume that there exists a third party who will take care of the cost allocation and any taxes or subsidies. A more realistic solution is to use a transfer payment contract under which the system's total cost is preserved. Below, we describe how to derive the contract for the three-stage serial system depicted in Figure 1.

The cost functions for different stages in the decentralized serial system are

$$\begin{aligned} C_1^s &= hIL_{1t} + (\alpha_1 p + h)B_t, \\ C_2^s &= (h_2 + h_3)(IL_{2t} - IL_{1t}) + \alpha_2 pB_t, \\ C_3^s &= h_3(IL_{3t} - IL_{2t}) + \alpha_3 pB_t, \end{aligned}$$

where $IL_{it} - IL_{i-1,t}$ ($i = 2, 3$) is the inventory on hand at stage $i$ plus the inventory in transit to stage $i-1$. Summing up the above cost functions gives the total cost function $C^s$ in Equation (1). Suppose that S1 plays the role of a coordinator. The coordinator is imaginary and only used to facilitate the derivation of the coordination contract. First, let S1 pay S2 and S3 $C_2^s$

and $C_3^s$, respectively. That is, let S1 bear all the costs in the system. Second, let S1 charge S2 $\eta_2 J_2$ and S3 $\eta_3 J_3$ ($\eta_2, \eta_3 > 0$, and $\eta_2 + \eta_3 < 1$). Then the cost functions for S2 and S3 become $\eta_2 J_2$ and S3 $\eta_3 J_3$, respectively, and S1 faces a cost of

$$\begin{aligned}\widehat{C}_1^s &= C^s - \eta_2 J_2 - \eta_3 J_3 \\ &= (1-\eta_2-\eta_3)J_1 + h_2(1-\eta_2-\eta_3)IL_{2t} \\ &\quad + h_3(1-\eta_3)IL_{3t}.\end{aligned}$$

Note that $IL_{2t}$ and $IL_{3t}$ are determined by S2 and S3's order-up-to levels ($IL_{it} = s_i - D^{L_i+1}$), so the last two terms in the cost function are independent of S1's order-up-to level $s_1$. Therefore, S1 will choose $s_1^o$ that minimizes $(1-\eta_2-\eta_3)J_1$. Given that $s_1^o$ is chosen by S1, S2 and S3 will in turn choose $s_2^o$ and $s_3^o$ that minimize $\eta_2 J_2$ and $\eta_3 J_3$, respectively. Combining the above two steps, we derive the net transfer payments from S1 to S2 and S3, respectively:

$$\begin{aligned}T_{12}^s &= -(\eta_2 h_1 + h_2 + h_3)IL_{1t} + [(1-\eta_2)h_2 + h_3]IL_{2t} \\ &\quad + [(\alpha_2 - \eta_2)p - \eta_2 h]B_t, \\ T_{13}^s &= -\eta_3 h_1 IL_{1t} - (\eta_3 h_2 + h_3)IL_{2t} + (1-\eta_3)h_3 IL_{3t} \\ &\quad + [(\alpha_3 - \eta_3)p - \eta_3 h]B_t.\end{aligned}$$

Consider the terms in $T_{12}^s$. The first two terms assert that S2 subsidizes S1 for S1's inventory level while getting compensated for her own inventory level. The sign of the coefficient of the third term depends on the magnitude of $\alpha_2$ and $\eta_2$. The interpretation of $T_{13}^s$ is similar. To coordinate the serial system, we can apply the above transfer payment contract and ask each stage to choose its own echelon order-up-to level independently. The result is stated by the following theorem.

Theorem 6. *Under the transfer payment contract* $\{T_{12}^s, T_{13}^s\}$, *all stages will choose the system optimal policy* $(s_1^o, \ldots, s_n^o)$ *independently, which is the unique Nash equilibrium in the serial system.*

Proof. Note that $J_i$ ($i = 1, 2, \ldots, n$) is convex in $s_i$, so the Nash equilibrium can be characterized by the first-order conditions. By construction, we know the first-order conditions for the Nash equilibrium are the same as those for the system optimal policy. Because the system optimal policy $(s_1^o, \ldots, s_n^o)$ is unique, there is a unique Nash equilibrium for the decentralized serial system. □

The same idea can be applied to assembly systems with minor modification. Consider the assembly system depicted in Figure 1. First, to coordinate the system, we need to assume that S2 can observe S3's inventory status in such a way that a contingent ordering policy is feasible. Second, each stage is asked to choose its own echelon order-up-to level. The cost functions for different stages are

$$\begin{aligned}C_1^a &= hIL_{1t} + (\alpha_1 p + h)B_t, \\ C_2^a &= h_2(IL_{2t} - IL_{1t}) + \alpha_2 pB_t, \\ C_3^a &= h_3(IL_{3t} - IL_{1t}) + \alpha_3 pB_t.\end{aligned}$$

Again, summing up the above cost functions gives the total costs in Equation (2). Unlike the serial system, S3 incurs holding cost for $(IL_{3t} - IL_{1t})$ rather than $(IL_{2t} - IL_{1t})$, since S3's immediate successor is S1 in the assembly system. Consider the following transfer payment contract

$$\begin{aligned}T_{12}^a &= -(\eta_2 h_1 + h_2)IL_{1t} + (1-\eta_2)h_2 IL_{2t} \\ &\quad + [(\alpha_2 - \eta_2)p - \eta_2 h]B_t, \\ T_{13}^a &= -(\eta_3 h_1 + h_3)IL_{1t} - \eta_3 h_2 IL_{2t} + (1-\eta_3)h_3 IL_{3t} \\ &\quad + [(\alpha_3 - \eta_3)p - \eta_3 h]B_t.\end{aligned}$$

Under this contract, the firms' new cost functions are as follows:

$$\begin{aligned}\widehat{C}_1^a &= (1-\eta_2-\eta_3)J_1 + h_2(1-\eta_2-\eta_3)IL_{2t} \\ &\quad + h_3(1-\eta_3)IL_{3t}, \\ \widehat{C}_2^a &= \eta_2 J_2, \\ \widehat{C}_3^a &= \eta_3 J_3.\end{aligned}$$

Given that a contingent policy is used by S2, the $J_i$'s in the assembly system are the same as the $J_i$'s in the equivalent serial system except that in the serial system, $IL_{3t}$ includes the in-transit inventory from S3 to S2, which does not exist in the assembly system (the outside source is responsible for that inventory). However the average inventory in-transit from S3 to S2 in the serial system is a constant $\mu^{L_2}$. So the difference does not affect the firm's decisions. Therefore, with cost functions $\{\widehat{C}_1^a, \widehat{C}_2^a, \widehat{C}_3^a\}$, the firms will choose the optimal inventory policies for the equivalent serial system, which is also optimal for the assembly system. Thus, we have the following theorem. The proof is similar and omitted.

THEOREM 7. *Under the transfer payment contract* $\{T_{12}^a, T_{13}^a\}$, *all stages will choose the system optimal policy* $(s_1^o, \ldots, s_n^o)$ *independently, which is the unique Nash equilibrium in the assembly system.*

### 6.2. Relationship with Existing Schemes

Our coordination scheme has several interesting properties. First, it is independent of the demand distribution. Supply chain owners or firms' heads usually do not have the accurate information that local managers do, so the coordination contract provides an appealing decentralization option. If the supply chain members are independent firms, the transfer payment contract is still attractive because no revision of the contract terms is needed when demand parameters shift over time. Second, the most upstream stage is actually evaluated by the performance of the whole supply chain. This provides incentive to correct any wrongdoings by its downstream stage. For example, the manager of the most upstream stage may refuse to make full delivery if the downstream stage orders too much. Third, the computational burden increases when moving from S1 to the most upstream stage. S1 faces a simple newsvendor problem, while the most upstream stage needs to solve the system optimal policy for the whole system. This suggests that the firm head should place more experienced managers at upstream stages. Besides the above properties, it appears that our idea can improve some existing methods and more importantly, serve as a general framework unifying some existing coordination methods.

Chen (1999) proposes a decentralization scheme in which a firm's head manages a serial supply chain in which each stage is an independent cost center. His scheme hinges on the concept of *accounting inventory*, which is defined as the inventory record under the hypothetical scenario in which all orders are filled immediately. The firm's head is assumed to bear all costs in the supply chain. Then, each stage is charged holding and backorder costs as in the newsvendor problem. The cost parameters are especially designed to make stage $i$ choose $s_i^o$. Watson and Zheng (2005) extend Chen's idea and propose a demand-focused decentralization scheme. They demonstrate that by using the *echelon accounting inventory*, each stage's performance is independent of the other stages' decisions. They further offer a demand-independent heuristic measurement scheme that is shown to be close to incentive-compatible via numerical tests. Our coordination scheme improves upon the line of research opened by Chen in two ways. First, because newsvendor cost functions are convex, it is not necessary to use the accounting inventory. Even if the orders are not always instantly delivered, each stage will not deviate from $s_i^o$. Second, Chen observes that decentralization may be justified by the lack of demand information for the firm's head. Using numerical examples, he finds that asking local managers to choose their own inventory policies is more effective than dictating the system with inaccurate information. Inspired by Chen's work, we show that a demand-independent decentralization scheme exists and is quite simple.

A couple of papers study coordination when supply chain members are independent firms. Cachon and Zipkin (1999) consider a two-stage supply chain consisting of one supplier and one retailer. They demonstrate that an array of linear transfer payment contracts can coordinate the supply chain. The supplier pays (charges) the retailer proportionally based on the retailer's on-hand inventory and the backorders at both the supplier and retailer. The transfer payments are constructed in such a way that the system optimal policy $(s_1^o, s_2^o)$ is a solution to the first-order condition for each firm. This guarantees that the system optimal policy is a Nash equilibrium of the inventory game. However, it is unclear whether the inventory game has a unique Nash equilibrium under the contract. Bernstein and DeCroix (2006) extend this contract to coordinate an assembly system, but, again, the uniqueness of equilibrium is not formally addressed. In contrast, we have demonstrated that under our coordination contract, the system optimal policy is the unique Nash equilibrium in the decentralized supply chain. Lee and Whang (1999) interpret Clark and Scarf's cost decomposition idea as a transfer payment contract that coordinates the supply chain. That is, the supplier compensates the retailer for any loss incurred due to the supplier's failure to make instant delivery. In their contract (for a two-stage supply chain), the retailer is actually charged $J_1$ (or $G_1(y)$, whose minimizer is $s_1^o$) while the supplier's cost is just $J_2 - G_1(s_1^o)$. Therefore, Lee and Whang's contract fits into the framework $\alpha_i J_i + \beta_i$

($\alpha_i > 0$). Their contract involves a nonlinear transfer payment. Both coordination schemes in Cachon and Zipkin (1999) and Lee and Whang (1999) require the demand distribution as an input.

Porteus (2000) operationalizes Lee and Whang's scheme to administer a decentralized supply chain. It is shown that with the aid of the responsibility tokens, the shortage reimbursement in Lee and Whang (1999) can be based on actual rather than expected consequences. Responsibility tokens also resemble the accounting inventory method in Chen (1999) because they essentially make immediate delivery feasible. Cachon (1998) shows that using the responsibility tokens, the retailer in a two-stage supply chain actually bears a cost $G_1(y)$ (recall $J_1 = G_1$), which is minimized by $s_1^o$. Then the supplier will choose $s_2^o$ because its cost equals all the remaining costs in the system (i.e., $G_2(y) = J_2 - G_1(s_1^o)$). So the scheme using responsibility tokens in essence takes the form $\alpha_i J_i + \beta_i$, which is indeed consistent with Lee and Whang (1999)'s scheme. Notice that by using the responsibility tokens, the firm's head can also avoid the requirement of demand information. This is the only existing decentralization scheme we are aware of that is demand-independent.

## 7. Conclusion

We study a two-echelon assembly system in which the suppliers and manufacturer competitively choose inventory policies to minimize their own expected costs. Inventory games under two information structures are analyzed. With information sharing, multiple equilibria may exist, but the cost outcome is unique. Without information sharing, the game has a unique equilibrium. It is shown that the Nash equilibrium is never system optimal under both information structures. In particular, the suppliers always understock in the Nash equilibrium. Therefore, competition among supply chain members causes efficiency loss. The competition penalty, defined as the percentage increase in the total cost of the Nash equilibrium over the system optimal cost, can be quite large when the manufacturer bears too little or too much of the backorder cost. On average, we find that the competition penalty increases in the difference between the suppliers' lead times.

In assembly systems, sharing information horizontally between the suppliers plays a critical role. With information sharing, the supplier with the shorter lead time can adopt a contingent ordering policy, which is necessary in the centralized optimal solution. However, the value of information sharing in a decentralized assembly system is found to be ambiguous. Numerical experiments show that in some cases, sharing information between the suppliers can mitigate the competition penalty. However, it may also hurt the supply chain if supply chain members behave noncooperatively. We prove that in the Nash equilibrium without information sharing, the suppliers always hold less inventory compared to the Nash equilibrium with information sharing. The opposite is true for the manufacturer. Therefore, while the manufacturer always prefers information sharing, the suppliers may refuse to share the information.

A new coordination contract is proposed in this paper. The coordination contract is independent of the demand distribution and applies to supply chains with either a serial or an assembly structure. In this contract, linear transfer payments based on the echelon inventory levels and the backorder level at the manufacturer are used. Under such a contract, the supply chain members will choose the system optimal policy as the unique Nash equilibrium.

As mentioned in the model settings, this paper has focused on a subset of the strategy space, i.e., stationary base-stock policies. We have shown that a unique equilibrium (or unique equilibrium cost outcome) exists with stationary base-stock policies. However, additional equilibria may arise if we include more sophisticated policies. They may exhibit different structures and lead to different results. For instance, the supply chain members can use a strategy that is contingent on the history of the game. In this case, one may wonder if an equilibrium exists that even coordinates the supply chain (e.g., all the players follow the supply chain optimal policy, and if one deviates, then the Nash equilibrium of the static game is played as punishment). Therefore, a complete analysis of the dynamic inventory game is interesting and merits a separate study. In addition, few studies have been reported on supply chain contracting within a dynamic setting. Due to repeated interactions, issues such as long-term relationships and reputation may arise. This also has great potential for future research.

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