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Household Debt Overhang and Unemployment

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ABSTRACT

We use a labor-search model to explain why the worst employment slumps often follow expansions of household debt. We find that households protected by limited liability suffer from a household-debt-overhang problem that leads them to require high wages to work. Firms respond by posting high wages but few vacancies. This *vacancy posting effect* implies that high household debt leads to high unemployment. Even though households borrow from banks via bilaterally optimal contracts, the equilibrium level of household debt is inefficiently high due to a *household-debt externality*. We analyze the role that a financial regulator can play in mitigating this externality.

Personal bankruptcy is pervasive in the United States—about one in 10 Americans will declare bankruptcy in his lifetime.[1]

Under the U.S. bankruptcy code, households are protected by limited liability. That is, they can discharge their debt and still keep a substantial amount of their assets. Such limited-liability protection distorts the incentives of indebted households, just as it distorts the incentives of indebted firms in corporate finance. In this paper, we investigate how this distortion can affect the labor market. In particular, we ask the following questions.

How does limited-liability debt distort household labor supply, and how does this affect aggregate employment in equilibrium? Further, do households take on too much limited-liability debt, and should a regulator intervene to mitigate the resulting distortions?

Model preview. To address these questions, we develop a two-date general equilibrium model of household borrowing and the labor market. At the first date, households borrow from banks. At the second date, firms post vacancies, and households and firms are randomly matched in a decentralized labor market à la Diamond–Mortensen–Pissarides. Once matched, firms and households negotiate wages bilaterally. Households then work or do not. If households work, firms produce output and pay wages.

Households use these wages to repay banks. If households do not work, firms do not produce output and do not pay wages. In this case, households cannot repay banks, so they default.

Results preview. Our first main result is that limited-liability debt on households' balance sheets leads to a *debt overhang problem* that makes indebted households reluctant to work. They act like indebted firms in corporate finance, whose equityholders are reluctant to pay the cost of new investments because they must use their cash flows to make repayments to existing creditors. Indebted households in our model are reluctant to bear the cost of working because they must use their wages to make repayments to the banks they borrowed from. Hence, firms must pay high wages to induce households to work.

Our second main result is that high levels of household debt lead firms to post relatively few vacancies, which leads, in turn, to low employment. This is a result of the household debt overhang problem. Because firms must pay indebted workers high wages, they cannot afford to hire as many of them, and thus they post fewer vacancies. This *vacancy posting effect* implies that high household debt leads to high unemployment.

Our third main result is that households take on excessive debt in equilibrium, even though they borrow from banks via bilaterally optimal contracts. This is due to a *household debt externality* that works through the vacancy posting effect. Specifically, when a household takes debt onto its balance sheet, this decreases the likelihood that households are employed, as implied by the vacancy posting effect. Since unemployed households are likely to default on their debt, this increases the default rate on all loans, including other banks' loans to other households. In other words, when households take on debt, they do not take into account the negative effect that their borrowing has on other agents in the economy through the labor market. Thus, there is scope for a financial regulator to intervene to mitigate this externality.

Our fourth main result is that banks' beliefs about future employment are self-fulfilling, which generates multiple equilibria. If banks believe that the rate of employment will be low, so household default risk is high, banks require high face values of debt to offset this risk. Households thus have high debt, so employment is indeed low due to the vacancy posting effect. In contrast, if banks believe that employment will be high, so household default risk is low, banks require low face values of debt, and employment is indeed high. Thus, there is another reason for regulatory intervention: to prevent the economy from ending up in the "bad" equilibrium with high debt and low employment.

We also show that households optimally finance themselves with debt contracts. This is because they want to minimize repayments to banks when they have the most opportunity to get rents, that is, when they have high wages. Within the class of repayment schedules that are (weakly) increasing in wages, the repayment schedule that minimizes repayments when wages are high is the one that increases most slowly with wages, that is debt. Further, we argue in an extension that repayments must indeed be increasing, since otherwise, they can be manipulated by households and firms to reduce repayments to banks (see Section IV.D).

We also explore the following three extensions, which generate further results and empirical content. (i) We include aggregate productivity shocks and discuss how household debt may contribute to sticky wages. (ii) We include household collateral and discuss how low collateral values, for example, low house prices, may exacerbate the vacancy posting effect. (iii) We include default penalties and discuss how they may attenuate the vacancy posting effect.

Policy. Our model is stylized, but may still cast light on two contemporary policy questions: Should household debt be limited, and should the personal bankruptcy code be more forgiving? Our model suggests that limiting household debt ex ante may be a good thing. In our model, caps on household debt can prevent the economy from ending up in the "bad" equilibrium with high debt and low employment. In contrast, making the bankruptcy code more debtor-friendly and limiting the liability of households ex post could be a bad thing. In our model, decreasing default penalties tightens households' limited liability constraints, which can exacerbate the vacancy posting effect (see Section III.C).

Empirical content. Our prediction that limited-liability household debt leads to a decrease in labor supply finds support in a number of recent empirical papers. Bernstein (2018) shows that instrumented negative home equity causes a decline in labor supply of between 2% and 6%; a simple linear aggregation from partial equilibrium estimates suggests this could account for over 20% of the decline in employment between 2008 and 2010—almost two million fewer U.S. jobs. Further, Herkenhoff (2012) finds a spike in the employment rate of households when their debt expires, suggesting that when households discharge their debt, the household debt overhang distortion is mitigated, which increases the employment rate. Our results are also in line with Dobbie and Goldsmith-Pinkham (2015), who find that limited recourse for mortgage debt—that is household limited liability—leads to a decrease in the employment rate.[2]

Our model captures the following stylized facts at the macroeconomic level: (i) high household leverage causes severe employment slumps (Mian and Sufi (2010), Mian and Sufi (2014b)), (ii) wages are rigid, especially downwardly[3]

(Bewley (1999), Daly and Hobijn (2015)), and (iii) negative shocks to household collateral values (house prices) contribute to labor market slumps (Mian and Sufi (2014b)).

After we present the baseline analysis in Section II, we discuss the empirical evidence in support of each of the key assumptions underlying our main mechanism (Section II.E).

Related literature. A number of papers explore how the household credit market interacts with the labor market via the aggregate demand channel (see, e.g., Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), Mian and Sufi (2011, 2014b), Midrigan and Philippon (2016), and Mishkin (1977, 1978)). Our paper is complementary to this work in that we explore how the household credit market interacts with the labor market via distortions in labor supply. In particular, Mulligan (2009, 2010) studies the costs and benefits of employment-contingent mortgage write-downs, focusing on the trade-off between preventing foreclosures and distorting labor supply. In these and other existing models of household debt overhang, the debt overhang operates on the extensive margin, insofar as indebted households are reluctant to apply for jobs at prevailing wages. In our model, in contrast, the debt overhang operates on the intensive margin, insofar as households require high wages to exert effort. This leads to lower employment because fewer firms post vacancies in anticipation of a high wage bill (not because households do not enter the labor market).

A few other papers incorporate household debt into search models of the labor market. Like us, Kehoe, Midrigan, and Pastorino (2016) identify a new channel through which household borrowing can lead to a reduction in firm vacancy posting. In their model, hiring a worker is a long-horizon investment, because workers are more valuable the longer they are on the job. Tightening credit increases the effective discount rate firms apply to these investments and, thus, decreases the value of posting vacancies. Herkenhoff (2013) analyzes how household borrowing constraints distort labor market outcomes, with a focus on credit card debt. He shows that if households can borrow on their credit cards while unemployed, they will hold out for high-wage jobs. This is because access to credit allows them to smooth consumption, making unemployment less costly. Thus, the distortion in Herkenhoff's model results from households taking on *more* debt when they are unemployed. In contrast, the distortion in our model results from households *discharging* debt when they are unemployed, an important phenomenon, for example, during the Great Recession when mortgage delinquency exceeded 10% (Mian, Sufi, and Trebbi (2015)). Finally, Bethune, Rocheteau, and Rupert (2015) connect households' role as workers in the labor market to their role as consumers in the goods market. In their model, tighter credit reduces job creation through its effect on aggregate demand. In our model, in contrast, tightening credit reduces job creation through its effect on labor supply.

In Section III.C, we show that an increase in household debt induces the same distortion as an increase in unemployment insurance, namely, it amplifies the vacancy posting effect. Acemoglu and Shimer (1999) emphasize that unemployment insurance can distort labor market search, leading to decreased employment. We show that, given household limited liability, household debt induces a similar distortion. However, the effect we characterize is likely to be even more severe than that induced by unemployment insurance, given the size of transfers to defaulting households (see Section II.E). Additionally, the household debt externality suggests that by levering up too much, households are effectively "overinsuring" employment risk.

Layout. In Section I, we present the model. In Section II, we present our main results. In Section III, we analyze extensions, and in Section IV, we show that our results are robust to relaxing a number of simplifying assumptions. We conclude in Section V. The Appendix contains all of the proofs.

I. Model

This section describes the model. There are two dates—Date 0 and Date 1—and three types of players—households, banks, and firms. Banks lend to households at Date 0 and firms employ households at Date 1. Thus, households are "borrowers" at Date 0 and "workers" at Date 1.

A. Players: Preferences and Actions

A.1. Households

There is a unit continuum of penniless households. Each has linear utility over consumption at Date 1 and requires the fixed amount B of liquidity at Date 0. That is, it maximizes its expected Date 1 payoff subject to the constraint that it meets its liquidity need at Date 0. This liquidity need creates a reason for households to borrow at Date 0; it can represent the need to smooth consumption or to make a fixed investment. At Date 1, each household may be matched with a firm, in which case it must work to generate output. Working entails a cost *c*, which implies that firms have to compensate households to work, and hence gives rise to the individual rationality constraint that determines wages.

A.2. Firms

There is a large continuum of competitive, profit-maximizing firms. At Date 1, each firm can pay the cost *k* to post a vacancy and attract a household/worker. If a firm is matched with a household, it produces output *y* if the household works. Otherwise, it produces nothing. We assume that $y > c + k$, so the benefits *y* of production are greater than the costs *c* and *k* of working and posting vacancies.

A.3. Banks

There is a large continuum of deep-pocketed, profit-maximizing banks. Banks lend to households/borrowers at Date 0 and discount the future at rate zero.

B. Labor Market

We model the labor market using a one-shot random search model. The number of households is fixed (with unit mass) and the number of firms is determined by endogenous entry. We denote the ratio of searching households to firms posting vacancies, which we refer to as the "queue length" *q*. This ratio determines the probability that households and firms are matched: The higher the *q* the harder it is for a household to be matched with a firm and the easier it is for a firm to be matched with a household. Specifically, each household is matched with a firm with probability $\alpha(q)$, and each firm is matched with a household with probability $q\alpha(q)$, where $\alpha'(q) < 0$ and $(q\alpha(q))' > 0$.[4]

In Assumption 1, we put more structure on α to make it easy to solve the model, but in Section IV.B we show that our qualitative results are not sensitive to this specification.

C. Contracts

C.1. Labor Contracts

After a firm and a household are matched, they negotiate a labor contract, which constitutes the wage *w* that the firm pays the household when output equals *y* (the wage is zero when output equals zero, since firms cannot pay more than they have). The contract is determined in order to split the surplus between the firm and the household, which we model via a simple random-proposer bargaining protocol: With probability one-half, the firm makes a take-it-or-leave-it offer to the household, and with probability one-half, the household makes a take-it-or-leave-it offer to the firm.[5]

C.2. Financial Contracts

Each household borrows *B* from a bank at Date 0. In exchange, the household makes the repayment $R(w)$ when it receives wage *w*, where *R* is a function that is determined optimally, that is, it maximizes the household's expected utility subject to the constraints that banks break even and households are protected by limited liability, $R(w) \leq w$. For now, we also assume that *R* is (weakly) increasing. This assumption is common in the literature (see, e.g., Brennan and Kraus (1987), Harris and Raviv (1989), Nachman and Noe (1994)), but it precludes some contracts that are optimal in some contexts (see, e.g., Innes (1990)). In Section IV.B, we extend the model to show that our results are robust to relaxing this assumption.

Note that banks and households are all "small" (they are indexed by continua), so they take the employment probability $\alpha(q)$ as given when they negotiate these lending contracts.

D. Timing

The sequence of moves is as follows. At Date 0, each household negotiates a lending contract *R* with a bank. At Date 1, firms post vacancies and they are randomly matched with households according to the matching technology described above. Next, firms and households negotiate wages, and households work or do not. Finally, output is realized and contracts are settled.

E. Equilibrium Definition

We look for the subgame-perfect equilibria of the game described above. This constitutes (i) the lending contract *R*, (ii) the labor contract *w* given *R*, that is, the wages w_h when the household proposes and w_f when the firm proposes, (iii) the households' decisions to work or not given *w* and *R*, and (iv) firms' entry decisions, which determine the queue length *q* such that (i) to (iv) are chosen optimally given players' beliefs, and these beliefs are consistent.

See Lemma 1 for an expression of the equilibrium contracts as the solution to an optimization program.

F. Assumptions

We make several assumptions on parameters. These assumptions allow us to solve the model in closed form, but they are not essential for our qualitative results (in fact, we do not use them until Section II.B).

Assumption 1. The matching probability α takes the functional form

$$\alpha(q) = \frac{a}{\sqrt{q}}, \tag{1}$$

where *a* is a positive constant.[6]

The next assumption guarantees that the matching probabilities are between zero and one in equilibrium.

Assumption 2.

$$a^2\left(y + c + \sqrt{(y-c)^2 - \frac{8kB}{a^2}}\right) < 4k < y + c - \sqrt{(y-c)^2 - \frac{8kB}{a^2}}. \tag{2}$$

This assumption is satisfied as long as firms are sufficiently productive (*y* is large) and the labor market is sufficiently frictional (*a* is small). However, we need it only to ensure that the labor market matching probabilities α and $q\alpha$ are well defined and interior, $\alpha, q\alpha \in (0,1)$, given Assumption 1 (see the Appendix for details). Moreover, our qualitative results do not depend on these assumptions, as we show formally in Section IV.B.

Finally, we assume that a household's Date 0 liquidity need is not too large.

Assumption 3.

$$B < \frac{a^2(y-c)^2}{8k}. \tag{3}$$

This ensures that the equilibrium face value of household debt exists (see equation 17).

II. Results

We now present the main analysis of our model. We solve for the optimal labor and lending contracts as well as the equilibrium entry of firms. We show that (i) the optimal contract is debt, but (ii) there is a household debt overhang problem that leads indebted households to require high wages. In equilibrium, this leads to (iii) the vacancy posting effect, whereby high levels of household debt lead to low employment. However, (iv) households do not take into account the effect of their debt on aggregate employment, that is, there is a household debt externality. Finally, we show that (v) there are multiple self-fulfilling equilibrium outcomes.

A. Optimal Contracts

We first solve for the optimal labor contract between a firm and a household and the optimal lending contract between a household and a bank. Recall that a labor contract is determined by the random-proposer bargaining protocol—firms and households each make take-it-or-leave-it offers with probability one-half. When the firm proposes, it maximizes its payoff subject to the constraint that the household is willing to work at cost *c*, that is, it proposes the wage w_f to solve

$$\text{maximize} \quad y - w \tag{4}$$

subject to the household's individual rationality constraint

$$w - R(w) - c \geq 0. \tag{5}$$

Note that the household's repayment *R* appears only on the left-hand side of its individual rationality constraint. This is because $R(0) = 0$ due to limited liability. When the household proposes, it maximizes its payoff subject to the constraint that the firm is willing to participate and pay the wage. That is, it proposes the wage w_h to solve

$$\text{maximize} \quad w - R(w) - c \tag{6}$$

subject to the firm's individual rationality constraint

$$y - w \geq 0. \tag{7}$$

The optimal lending contract is determined taking as given that the labor contracts w_f and w_h solve the problems above. At Date 0, a household makes a bank a take-it-or-leave-it offer to determine the repayment $R(w)$. The household and the bank anticipate that the household will be employed with probability α. Thus, it will get wage w_f with probability $\alpha/2$, wage w_h with probability $\alpha/2$, and wage zero with probability $1-\alpha$ (when it is unemployed). We can now set up this contracting problem as an optimization program. Recall that since all players are small, they do not take into account the effect of their actions on aggregate employment. Thus, optimal contracts are determined taking the employment rate α as given.

Lemma 1. Given an employment rate α, an optimal lending contract *R* solves the following program to maximize the household's expected payoff,

$$\text{maximize} \quad \alpha\left(\frac{1}{2}(w_f - R(w_f) - c\right) + \frac{1}{2}\left(w_h - R(w_h) - c\right)\Big), \tag{8}$$

subject to the constraints:

- the wage w_f maximizes the firm's payoff subject to the constraint that it is individually rational for the household to work, given the repayment *R*,

$$w_f \in \arg\max \left\{ y - w \,\middle|\, w - R(w) - c \geq 0 \right\}, \tag{9}$$

- the wage w_h maximizes a household's payoff subject to the constraint that it is individually rational for the firm to participate, given the repayment *R*,

$$w_h \in \arg\max \left\{ w - R(w) - c \,|\, y - w \geq 0 \right\}, \tag{10}$$

- banks break even, given the employment rate α,

$$\alpha \mathbb{E}\left[R(w)\right] \geq B, \tag{11}$$

- households have limited liability, $R(w) \leq w$, and
- *R* is weakly increasing.

Our first main result is that the optimal lending contract can be implemented with defaultable debt. We denote the face value of a representative household's debt by *F*.

Proposition 1. Given an employment rate α, defaultable debt with face value $F := B/\alpha$ is an optimal lending contract. That is, $R(w) = \min\{B/\alpha, w\}$ is a solution to the program in Lemma 1.

To see why debt is optimal, think of an arbitrary contract in which the household's debt repayment is weakly increasing in its wage. Now, observe that the household always gets a zero net payoff (the wage minus the debt repayment minus the cost of effort) when the firm proposes the wage, since in this case, the firm pushes the household to its participation constraint. Thus, the household chooses the lending contract *R* to minimize its repayment when it proposes the wage, since this is the only opportunity for the household to capture a rent. Given that the wage is relatively high when the household proposes, the household chooses the lending contract to minimize the repayment $R(w)$ when the wage *w* is high. To make the bank break even in expectation, the household must then increase the repayment $R(w)$ when the wage *w* is low (which occurs when the firm proposes). Since *R* must be monotonic, the contract that minimizes the repayment for high wages and maximizes the repayment for low wages is the flat contract, which is debt.

Now, given the face value of debt *F*, the firm proposes the wage w_f to make the household's individual rationality constraint bind, and the household proposes the wage w_h to make the firm's individual rationality constraint bind.

Proposition 2. The equilibrium wages are $w_f = F + c$ and $w_h = y$. Thus, the average wage is

$$\overline{w} := \mathbb{E}\left[w\right] = \frac{y + F + c}{2}. \tag{12}$$

Note that the expected wage is increasing in the face value of debt *F*. This is because the more indebted the household is, the more of its wage goes to the bank, and as a result, the more the firm has to compensate it for working. This finding, that wages are increasing in household debt, is the key to the vacancy posting effect, which we turn to next.

B. Firm Vacancy Posting

We next solve for the queue length *q* and employment rate $\alpha(q)$, which are determined by firms' willingness to post vacancies. Recall that firms are matched with households with probability $q\alpha(q)$. If they are matched, they get $y - \overline{w}$ on average, so their expected payoff from posting vacancies is $q\alpha\left(q\right)\left(y - \overline{w}\right)$. Since they must pay the cost *k* to post vacancies, firms post vacancies whenever

$$q\alpha\left(q\right)\left(y - \overline{w}\right) \geq k. \tag{13}$$

We can now solve for *q* by substituting in for $\overline{w}$ from Proposition 2 and observing that the inequality must bind in equilibrium since firms compete away all the rent from posting vacancies.

Proposition 3. Given the face value of debt *F*, the queue length and employment rate are

$$q = \left(\frac{2k}{a(y - F - c)}\right)^2 \tag{14}$$

and

$$\alpha\left(q\right) = \frac{a^2\left(y - F - c\right)}{2k}, \tag{15}$$

as long as $\alpha(q)$ is between zero and one.

This proposition leads us immediately to the vacancy posting effect, whereby firms post fewer vacancies when the level of household debt is high, leading to low employment.

Corollary 1. The employment rate α is decreasing in the level of household debt *F*.

This vacancy posting effect works through the effect of household debt on wages. Recall that increasing the level of household debt *F* increases the average wage $\overline{w}$ (by Corollary 2). Thus, the higher is *F*, the higher is a firm's wage bill and the lower is its profit. As a result, fewer firms can afford to enter and post vacancies.

C. Household Debt and Unemployment in Equilibrium

Above we characterize the face value of household debt *F* as a function of the employment rate α (Proposition 1), and we characterize the employment rate α as a function of the face value of household debt *F* (Proposition 3). We now solve for the equilibrium of the model by finding the face value of debt that makes these findings consistent with each other. In other words, the face value of debt is determined as a fixed point: $F(\alpha(F)) = F$. Specifically, the household offers the bank the face value *F* so that the bank breaks even, that is

$$\alpha F = B, \tag{16}$$

where α is determined in equilibrium as a function of *F*. Substituting in for α from Proposition 3, we have that

$$\frac{a^2}{2k}\left(y - F - c\right)F = B. \tag{17}$$

This is a quadratic equation in *F* and has two solution. That is, the model has two equilibria, which correspond to different levels of household debt and different employment rates.[7]

Proposition 4. Define

$$d := \frac{2Bk}{a^2}. \tag{18}$$

There are two equilibria: an equilibrium with a low face value of debt

$$F_- = \frac{y - c - \sqrt{(y-c)^2 - 4d}}{2} \tag{19}$$

and a high employment rate

$$\alpha_- = \frac{a^2}{2k}\left(y - F_- - c\right), \tag{20}$$

and an equilibrium with a high face value of debt

$$F_+ = \frac{y - c + \sqrt{(y-c)^2 - 4d}}{2} \tag{21}$$

and a low employment rate

$$\alpha_{+} = \frac{a^2}{2k}\left(y - F_{+} - c\right). \tag{22}$$

There are multiple equilibria because banks' beliefs about future employment are self-fulfilling. When banks believe that the rate of employment will be high, making household default unlikely, banks demand low face values of debt and employment is indeed high. Similarly, when banks believe that the rate of employment will be low, making household default likely, banks demand high face values of debt and unemployment is indeed high.

D. The Constrained-Efficient Outcome

We define the constrained-efficient outcome as the queue length q and the employment rate $\alpha(q)$ that maximize total surplus given the search friction; this outcome maximizes the total output minus the total costs of working and vacancy posting. Recall that there is a unit of households. Thus, α is the number of firm-household matches and $1/q$ is the number of firms that pay k to enter. Therefore, the constrained-efficient outcome must maximize the output αy minus the costs of working αc and the costs of posting vacancies k/q, that is, must solve

$$\text{maximize } \alpha(q)(y - c) - \frac{k}{q}. \tag{23}$$

Lemma 2. The constrained-efficient queue length and employment rate are given by

$$q_{\mathrm{CE}} = \left(\frac{2k}{a(y - c)}\right)^2 \tag{24}$$

and

$$\alpha_{\mathrm{CE}} = \frac{a^2(y - c)}{2k}. \tag{25}$$

We now examine whether the equilibrium outcome in Proposition 4 is constrained efficient. The next proposition indicates that the answer is no.

Proposition 5. Employment is too low even in the high-employment equilibrium: The employment rate in the high-employment equilibrium in Proposition 4 is lower than the employment rate in the constrained-efficient outcome in Lemma 2. That is,

$$\alpha_{-} < \alpha_{\mathrm{CE}}. \tag{26}$$

The equilibrium outcome is not constrained efficient due to a *household debt externality* that works as follows. When banks lend to households, they take the employment rate α as given. However, bank lending decreases the employment rate via the vacancy posting effect (Corollary 1). This increases the default rate on all loans—including other banks' loans to other households—since unemployed households default on their debts. In other words, when banks lend to households, they do not take into account the negative effect that their lending has on other banks and households through labor market externalities.

Given this externality, there is scope for a regulator to intervene in labor and credit markets to improve efficiency.

Proposition 6. A regulator can implement the constrained-efficient outcome by regulating wages and household debt. If the regulator sets

$$w_{\mathrm{CE}} = \frac{y+c}{2}, \tag{27}$$

then the constrained-efficient outcome is achieved as long as household debt is not too high. Specifically, the regulator must set

$$F \leq F_{\mathrm{CE}} = \frac{y-c}{2}. \tag{28}$$

The intuition for this result is as follows. In equilibrium, the employment rate $\alpha(q)$ is determined by firms' entry condition: Firms continue to post vacancies as long as the cost of posting is less than their expected profit from posting given the wage *w*, so

$$k = q\alpha(q)(y-w). \tag{29}$$

We find that $q = q_{\mathrm{CE}}$ exactly when $w = w_{\mathrm{CE}}$. In other words, setting w_{CE} implements the constrained-efficient outcome. However, it must be individually rational for the household to work. That is, it must be true that

$$w_{\mathrm{CE}} - F \geq c. \tag{30}$$

This individual rationality constraint gives the upper bound on *F* in the proposition above. It implies that a regulator may not be able to implement the constrained-efficient outcome by intervening in the labor market alone, even though the household-debt externality works through wages. Indeed, a regulator may need to cap and/or write down household debt to stimulate the labor market.

Corollary 2. Household debt is too high in equilibrium in the following two senses:

(i) The level of household debt in the high-debt equilibrium in Proposition 4 is higher than the upper bound on the level of household debt in the constrained-efficient outcome in Proposition 6, that is, $F_{+} > F_{\mathrm{CE}}$. Thus, the regulator cannot implement the constrained-efficient outcome even if it can intervene in the labor market and set wages.

(ii) If wages are determined bilaterally by firms and households given household debt *F* as in Proposition 2, then decreasing *F* brings the economy closer to the constrained-efficient outcome (it increases the objective function in equation (23)).

To the extent that capping household debt occurs via regulations imposed on the banks that lend to households, this proposition implies that the central bank, in its regulatory role, can affect employment through prudential bank regulations. This provides the central bank with a new way to target employment as an alternative to monetary policy.

E. Discussion of Assumptions

In this subsection, we discuss the empirical support for the microeconomic ingredients that drive our main results.

The mechanism behind the vacancy posting effect relies on four ingredients: (i) households default when they are unemployed, (ii) households are protected by limited liability, (iii) households take their limited liability protection into account, and (iv) firms internalize this household preference distortion when posting vacancies. Each of these ingredients has empirical support in the literature, some of which we discuss below.

With respect to (i), Geradi et al. (2013) find that individual unemployment is the strongest predictor of default. Similarly, Herkenhoff (2012) finds that unemployment (and not negative equity) is the primary reason for household default, implying that households default mainly when they fail to find employment. With respect to (ii), household limited liability in the event of default is salient in the United States, where debtors can dissolve debt obligations by filing for personal bankruptcy (see, e.g., Dobbie and Goldsmith-Pinkham (2015), and Mahoney (2015)). With respect to (iii), Mahoney (2015) establishes that households do indeed take limited liability into account—they use the protection afforded by it as informal insurance. Further, Melzer (2017) demonstrates that limited liability in the form of asset exemptions in mortgage default leads to distortions in households' investment decisions. Households with negative equity cut back substantially on home improvements, but continue to invest in durable assets that can be retained in the event of default.

Finally, consider (iv). Research on the effects of unemployment insurance provides evidence that firms respond to household preference distortions when posting vacancies. Notably, Hagedorn, Manovskii, and Mitman (2015) exploit variation in unemployment insurance policies across U.S. states to show that increasing unemployment insurance causes firms to post fewer vacancies. They estimate that cuts to unemployment insurance created about 1.8 million jobs in the United States in 2014 due to increased job creation by firms. As we show in Section III.C, in our model, unemployment insurance has the same distortionary effect as household debt. This is because household debt is effectively a "tax" for finding employment—households repay their debts out of their wages—whereas unemployment insurance is a subsidy for *not* finding employment. The labor market distortions resulting from household leverage are likely to be even more important than those resulting from employment insurance. This is because personal bankruptcy results in more effective transfers than all state unemployment insurance programs combined (Lefgren, McIntyre, and Miller (2010)). Moreover, household limited liability is not limited to debt that is discharged in bankruptcy; in fact, bankruptcies constitute only about one-sixth of household defaults (Herkenhoff (2012)).

III. Extensions

In this section, we consider three extensions to our model. In each case, we add a realistic ingredient to the model in reduced form to generate new results. Specifically, taking Date 0 debt contracts as given, we add aggregate productivity shocks at Date 1 in the first extension, household collateral at Date 1 in the second extension, and default penalties/unemployment insurance at Date 1 in the third extension.

A. Aggregate Shocks and Wage Dynamics

Here, we examine the effects of changes in firm output *y* on employment and wages. We argue that household debt may be a source of sticky wages, and we discuss the complementarities between our household debt externality channel of unemployment and the aggregate demand channel.

In this extension, we include two possible aggregate states: a boom in which firm output is y_H and a recession in which firm output is $y_L < y_H$. Thus, given household debt with face value *F*, Proposition 2 specifies the labor market outcomes in the boom and recession states. In particular, the equations for the wages are

$$w_H = \frac{y_H + F + c}{2} \quad \text{and} \quad w_L = \frac{y_L + F + c}{2}. \tag{31}$$

The following proposition says that the fluctuation in wages across macroeconomic states decreases as household debt increases, which suggests that high levels of household debt represent a potential source of wage rigidity (see Bewley (1999)).

Proposition 7. The percentage change in wages across macroeconomic states,

$$\frac{w_H - w_L}{w_H} = \frac{y_H - y_L}{y_H + F + c}, \tag{32}$$

is decreasing in the level of household debt *F*.

Now turn to the employment rates. We have

$$\alpha_H = \frac{a^2}{2k}\Big(y_H - F - c\Big) \quad \text{and} \quad \alpha_L = \frac{a^2}{2k}\Big(y_L - F - c\Big), \tag{33}$$

which suggests that high levels of household debt may decrease employment in booms and, more importantly, amplify employment slumps in recessions. Thus, while our channel of unemployment—based on the effect of household debt on the labor market—is novel, it is complementary to channels based on varying aggregate output. In particular, when aggregate demand decreases, firm revenues decrease. In our model, this corresponds to a decrease in *y*. This shock to *y* has a more severe effect on the labor market when households are more highly levered (*F* is higher). This result is consistent with evidence in studies of the aggregate demand channel such as Mian and Sufi (2014a).

B. The Inclusion of Collateral

Next, we examine the extent to which our results are affected by the inclusion of collateral on household balance sheets. We argue that depressed collateral values may amplify the vacancy posting effect.[8]

Suppose households have collateral in place with value *h*. If $h \geq F$, a household can always repay its debt by liquidating its collateral, even if it is unemployed. In contrast, if $h < F$, a household defaults on its debt and gets zero if it is unemployed. Thus, it prefers to work at wage *w* as long as

$$w - F - c + h \geq \max \{h - F, 0\}. \tag{34}$$

Proposition 2 then gives the wage

$$w = \frac{y + c + \max \{F - h, 0\}}{2}. \tag{35}$$

Proposition 8. When collateral values are low, $h < F$, limited liability leads to a distortion in households' behavior, which induces high wages and low employment via the vacancy posting effect.

In contrast, when collateral values are high, $h \geq F$, limited liability does not lead to a distortion in households' behavior.

This extension yields the additional empirical prediction that the vacancy posting effect should be strongest when collateral values are low (or liquidation discounts are high), that is, when $h < F$. This result explains why the connection between household debt and unemployment is strongest in economic downturns (i.e., in periods during which assets values are depressed and asset illiquidity is low), for example, during the Great Recession when household collateral values were low due to the decline in house prices. This result is consistent with evidence in Mian and Sufi (2014b).

C. Default Penalties and Unemployment Insurance

Next, we extend our model to include default penalties. We show that default penalties attenuate the vacancy posting effect and therefore may help boost employment. We also discuss the role of unemployment insurance, which is analogous to a negative default penalty.

Here, we assume that a household that defaults on its debt suffers a penalty *d*. Thus, it prefers to work at wage *w* as long as

$$w - F - c \geq -d. \tag{36}$$

Proposition 2 then gives the wage

$$w = \frac{y + c + F - d}{2}. \tag{37}$$

Proposition 9. Increasing the default penalty *d* decreases wages and increases employment, that is, default penalties attenuate the vacancy posting effect.

This result may help us test our model empirically, since there is significant cross-state variation in default penalties.[9] Notably, Dobbie and Goldsmith-Pinkham (2015) find that the postcrisis employment slump was deeper in states with limited recourse for mortgage debt, consistent with our finding that higher default penalties mitigate the vacancy posting effect.

Note that a negative default penalty exacerbates the vacancy posting effect. This can be interpreted as unemployment insurance. Denoting the transfer to unemployed households by UI, the household's incentive compatibility constraint (IC) reads

$$w - F - c \geq \mathrm{UI}. \tag{38}$$

Thus, given Proposition 2,

$$w = \frac{y + c + F + \mathrm{UI}}{2}. \tag{39}$$

Thus, an increase in household debt induces the same distortion as an increase in unemployment insurance, amplifying the vacancy posting effect. The literature has established that unemployment insurance can distort labor market search, decreasing employment (Acemoglu and Shimer (1999)). We show that, given household limited liability, household debt induces the same distortion—*more* household leverage corresponds to *more* insurance, in contrast to other models in the literature (see, e.g., Rampini and Viswanathan (2018)). Further, given the size of transfers to defaulting households discussed in Section II.E, the negative effects of household debt for the labor market are likely to be even larger than those of unemployment insurance.[10] Additionally, the household debt externality suggests that by levering up too much, households are effectively "overinsuring" employment risk.

IV. Robustness

In our baseline model, we make a number of modeling assumptions to simplify the analysis. Perhaps most importantly, we assume that (i) households and firms have equal bargaining power, that is, each proposes the wage with probability one-half, (ii) the matching probabilities take a simple square-root form (see Assumptions 1 and 2), (iii) banks are perfectly competitive, and (iv) the contracts that banks offer are nondecreasing in wages. In this section, we relax each of these assumptions one at a time. We show that our baseline results continue to hold and we also obtain several new insights.

A. Generalized Bargaining between Firms and Households

In this section we generalize the bargaining protocol between the household and the firm, assuming that the firm offers the wage with probability β, as opposed to one-half as in the baseline model. This allows us to find a condition on households' bargaining power for which the labor market equilibrium is efficient. This is an extension of the Hosios condition in the search literature to the case in which households have debt on their balance sheets.

Proposition 10. The equilibrium in the labor market is efficient if firms' bargaining power is

$$\beta = -\eta \, \frac{y - c}{y - F - c}, \tag{40}$$

where η is the elasticity of households' matching probability, $\eta \equiv \frac{\partial \alpha / \partial q}{\alpha / q}$.

If $F = 0$, expression 40 reduces to the standard Hosios condition. When households have debt on their balance sheets, firms should have more bargaining power relative to the case with no debt. Intuitively, debt works as a commitment device for households, which allows them to extract too much rent from firms, discouraging firm entry/vacancy posting.

B. Alternative Matching Function

The square root matching probability in Assumption 1 is attractive because it allows us to solve the model in closed form, but it requires that we impose some restrictions on parameters to keep matching probabilities between zero and one. Here, we consider an alternative matching probability, which derives from the so-called "telephone" matching technology:

$$\alpha(q) = \frac{a_0}{1 + a_1 q}, \tag{41}$$

where $a_0, a_0/a_1 \in [0, 1]$. This probability has the attractive feature that both households' and firms' matching probabilities, α and $q\alpha$, are between zero and one for all $q > 0$. With this matching probability, the analysis is almost as simple as the baseline model. We find basically the same results as with the baseline model. In particular, there are still two equilibria, which we can solve for in closed form. Moreover, in this case, we have to make only minimal restrictions on the parameters.

Proposition 11. In lieu of Assumptions 1 and 2, suppose that the matching probability is given by $\alpha(q) = a_0/(1 + a_1 q)$ as described above and that the entry cost k is not too large,

$$2ka_1 < B + a_0(y - c). \tag{42}$$

There are two equilibria, which correspond to the two face values of debt F that solve the quadratic equation

$$-a_0 F^2 + \big(B + a_0(y - c) - 2k\big)F - B(y - c) = 0. \tag{43}$$

Note that the quadratic equation in 43 always has two positive roots given the assumption in 42 and that each root corresponds to well-defined matching probabilities given the function α (as a result of the assumptions above that $a_0, a_0/a_1 \in [0, 1]$).

Overall, this result implies that our multiplicity result (Proposition 4) is robust, and it is not driven by the specific matching function we choose.

C. Imperfectly Competitive Banks

We next relax the assumption that banks are perfectly competitive, assuming now that they can extract some surplus from households. To capture this idea, we assume that households bargain with banks over the face value of debt the same way they bargain with firms over wages: Each household is matched with a bank and offers it a face value F_h with probability π and gets an offer F_b from the bank with probability $1 - \pi$.

Here, we let u denote the utility a household gets from borrowing B.[11]

Given *u*, the bank offers the household the highest face value that induces the household to borrow rather than not:

$$u + \alpha\left(\overline{w}_F - F - c\right) = \alpha\left(\overline{w}_0 - c\right), \tag{44}$$

where $\overline{w}_F$ denotes the household's average wage if it has debt *F* and $\overline{w}_0$ denotes its average wage if it has no debt. Substituting in for the wages from Proposition 2 gives the equilibrium face values.

Proposition 12. In the extension in which banks offer contracts to households as described above, the face values are $F_b = 2u/\alpha$ and $F_h = B/\alpha$. The average face value is

$$\overline{F} = \frac{\pi u + (1-\pi)B}{\alpha}. \tag{45}$$

Since the average face value has the same form as in the baseline model— $\overline{F} = \text{const.} \times \frac{1}{\alpha}$ —the qualitative results are unchanged if the bank has some market power. However, the average face value here is higher than in the baseline model, which implies that household debt is increasing in banks' market power. Since increasing household debt exacerbates the vacancy posting effect (Corollary 1), this suggests that banking competition might help mitigate debt-driven employment slumps.

D. Nonincreasing Financial Contracts

We now show that limiting attention to increasing financial contracts is not restrictive, as nonincreasing contracts are subject to manipulation by the household and the firm. To demonstrate this, we allow for a simple side contract between households and firms. In particular, once matched, a household can borrow from a firm in the following sense: The firm increases the wage by *z* and the household repays $(1+r)z$ to the firm after it works, gets paid, and makes repayments to the bank. Thus, if financial contracts with banks are decreasing in wages, then the household always borrows from the firm to boost its wage upward and hence decrease its repayment to the bank. In other words, decreasing repayments are effectively not implementable.

Proposition 13. Suppose that the financial contract *R* is decreasing in some region. That is, $R\left(w_H\right) < R\left(w_L\right)$ for some $w_H > w_L$. Then, the household never makes the repayment $R(w_H)$.

This result is basically an implementation of Innes's (1990) argument that entrepreneurs' financial contracts must be increasing: if repayments were decreasing in output, entrepreneurs could secretly borrow, report higher output, make low repayments, and then repay their secret debt.

V. Conclusion

This paper examines the effect of household credit on the labor market. We find that debt on household balance sheets leads to a debt-overhang problem, which results in households requiring relatively high wages to work. The reason is that households' wages net of debt repayments must compensate them for the cost of working. This result is established in a setting in which debt is the optimal contract with which households finance current liquidity needs. Firms respond to households' distorted preferences by posting high wages but few vacancies. This vacancy posting effect explains why high levels of household debt precede unemployment slumps. Further, we show that households fail to internalize this negative effect that they have on the labor market. This household debt externality leads to excessive household debt in equilibrium. A regulation capping household debt can mitigate this externality. Thus, a central bank targeting unemployment can use such financial regulation to complement monetary policy.

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Appendix

Sufficiency of Bounds in Assumption 2: Here, we show the sufficiency of the bounds stated in Assumption 2 for the matching probabilities to be well defined. In order for the matching probabilities to be between zero and one, it must be the case that

$$a^2 < q < \frac{1}{a^2}. \tag{A.1}$$

We can substitute the equilibrium q from Proposition 3 into this expression to get

$$a^2 (y - F - c) < 2k < (y - F - c). \tag{A.2}$$

Plugging in for the smallest F from Proposition 4, that is, F_-, in the left-hand side of the equation and for the largest F from Proposition 4, that is, F_+, in the right-hand side of the equation, we obtain sufficient conditions for the inequality above to hold, namely,

$$a^2 \left(y - c + \sqrt{(y-c)^2 - \frac{8Bk}{a^2}} \right) < 4k < y - c - \sqrt{(y-c)^2 - \frac{8Bk}{a^2}}, \tag{A.3}$$

which is the condition in Assumption 2. □

Proof of Lemma 1. The result follows immediately from backward induction. The program just says that wages are determined optimally given *R*, and *R* is determined optimally in anticipation of the wages. The only subtlety is that households and banks take the employment probability α as given even though firms post vacancies contingent on financial contracts. This is because we have assumed that banks and households are indexed by continua and therefore are too small to affect α individually. □

Proof of Propositions 1 and 2. The proof has four main steps. In Step 1, we show that the wage is lower when the firm proposes than when the household proposes, $w_f \le w_h$, which implies, by monotonicity, that $R(w_f) \le R(w_h)$. In Step 2, we show that for any financial contract *R* the household's IC binds when the firm proposes, $w_f - R(w_f) - c = 0$, so the household gets surplus rent only when it proposes. In Step 3, we show that repayments to the bank are the same when the firm proposes and the household proposes, so the optimal financial contract is implementable with debt that has face value $F := R(w_f) = R(w_h)$. In Step 4, we find the optimal wage and face value of debt.

Before we start the main steps of the proof, we note that we can restrict attention to contracts in which the household always works, that is, its IC is satisfied for both $w = w_f$ and $w = w_h$, since output and repayments are all zero if the IC is violated.

Step 1: $w_f \leq w_h$. To see this, suppose (in anticipation of a contradiction) that $w_f > w_h$ in equilibrium. But then the firm can deviate to offer $w_f' = w_h$ and get profit $y - w_f' = y - w_h > y - w_f$ (since the IC is necessarily satisfied when $w = w_h$). This is a contradiction to the supposition that the firm offers $w_f > w_h$. We conclude that $w_f \leq w_h$.

Step 2: $w_f - R(w_f) - c = 0$. Intuitively, this says that when the firm makes the offer, it pushes the household to its binding IC. The subtlety is to prove that it holds for all admissible financial contracts *R*. Recall equation 9, which says that the firm chooses the smallest wage that satisfies the household's IC:

$$w_f \in \arg\max \left\{ y - w \,\middle|\, w - R(w) - c \geq 0 \right\}. \tag{A.4}$$

We must prove that the constraint binds (which requires a bit of work since *R* may be discontinuous). We now show that if

$$\widehat{w}_f := \inf \{ w \,|\, w - R(w) - c \geq 0 \}, \tag{A.5}$$

then $\widehat{w}_f - R(\widehat{w}_f) - c = 0$, so the infimum above is attained. Recall that *R* is increasing by assumption, so

$$\lim_{\varepsilon \to 0^+} R(\widehat{w}_f - \varepsilon) \leq R(\widehat{w}_f) \leq \lim_{\varepsilon \to 0^+} R(\widehat{w}_f + \varepsilon) \tag{A.6}$$

(note that the inequalities bind when *R* is continuous). We then have that

$$\begin{aligned} \lim_{\varepsilon \to 0^+} \widehat{w}_f \quad & - \varepsilon - R(\widehat{w}_f - \varepsilon) - c \geq \widehat{w}_f - R(\widehat{w}_f) \\ & - c \geq \lim_{\varepsilon \to 0^+} \widehat{w}_f + \varepsilon - R(\widehat{w}_f + \varepsilon) - c. \end{aligned} \tag{A.7}$$

(This follows since $\widehat{w}_f + \varepsilon$ is continuous in ε, so $\lim_{\varepsilon \to 0^+} \widehat{w}_f - \varepsilon = \lim_{\varepsilon \to 0^+} \widehat{w}_f + \varepsilon = \widehat{w}_f$.) We now proceed by contradiction to show that it cannot be that either $\widehat{w}_f - R(\widehat{w}_f) - c > 0$ or $\widehat{w}_f - R(\widehat{w}_f) - c < 0$, so equality must hold.

Suppose $\widehat{w}_f - R(\widehat{w}_f) - c > 0$. By equation A.7 there is an $\varepsilon > 0$ such that

$$\widehat{w}_f - \varepsilon - R(\widehat{w}_f - \varepsilon) - c \geq 0, \tag{A.8}$$

which says that the wage $w_f = \widehat{w}_f - \varepsilon < \widehat{w}_f$ satisfies the IC, contradicting the definition of $\widehat{w}_f$ as the infimum in equation A.5.

Suppose instead $\widehat{w}_f - R(\widehat{w}_f) - c < 0$. By the monotone convergence theorem, there is a decreasing sequence that satisfies that IC and converges to the infimum in equation A.5, that is, $\widehat{w}_f = \lim_{n \to \infty} \widehat{w}_f + \varepsilon_n$, where $\varepsilon_n > 0$, $\varepsilon_n \to 0$, and $\widehat{w}_f + \varepsilon_n$ satisfies the IC:

$$\widehat{w}_f + \varepsilon_n - R(\widehat{w}_f + \varepsilon_n) - c \geq 0. \tag{A.9}$$

From equation A.7, we know that for *n* sufficiently large (i.e., ε_n small and positive), we have that

$$w + \varepsilon_n - R(\widehat{w}_f + \varepsilon_n) - c < 0. \tag{A.10}$$

This contradicts the supposition that the IC is satisfied for the sequence $\widehat{w}_f + \varepsilon_n$.

It follows that the IC binds at $\widehat{w}_f$. This is the smallest wage satisfying the IC, and thus it is the optimal wage for the firm to propose, $w_f = \widehat{w}_f$.

Step 3:$R(w_f) = R(w_h) =: F$. Step 2 above says that the household's IC binds whenever the firm proposes the wage and thus the household gets zero utility whenever the firm proposes. Hence, the household maximizes its utility when it proposes the wage. That is, its optimization problem is thus to

$$\text{maximize} \quad w_h - R(w_h) \tag{A.11}$$

subject to

$$R(w_h) \geq R(w_f), \tag{A.12}$$

$$w_h \leq y, \tag{A.13}$$

$$\alpha\left(\frac{1}{2}R(w_h) + \frac{1}{2}R(w_f)\right) \geq B. \tag{A.14}$$

Since the objective is decreasing in $R(w_h)$, and $R(w_f)$ enters only in the constraints, the monotonicity constraint in equation A.12 binds, that is, $R(w_f) = R(w_h)$. In other words, the repayment is independent of the wage. We label this number *F*.

Step 4:Wage and face value. Given Step 3 above, we can rewrite the problem as

$$\text{maximize} \quad w_h - F \tag{A.15}$$

subject to

$$w_h \leq y, \tag{A.16}$$

$$\alpha F \geq B. \tag{A.17}$$

This is maximized when the constraints bind, so $w_h = y$ and $F = B/\alpha$.

To sum up, the optimal financial contract is $R\left(w_f\right) = R\left(w_h\right) \equiv F = B/\alpha$ and the corresponding labor contracts are $w_f = F + c$ and $w_h = y$. □

Proof of Proposition 3. The result follows from substituting in for the functional form of the matching probability α from Assumption 1 into the vacancy posting condition in equation 13. This gives

$$a\sqrt{q}\left(y - \overline{w}\right) \geq k. \tag{A.18}$$

Recalling that firms continue to post vacancies to compete away profits and that $\overline{w} = \left(y + F + c\right)/2$ from Proposition 2, we have

$$a\sqrt{q}\left(y - \frac{y + F + c}{2}\right) = k. \tag{A.19}$$

Rearranging gives the expressions in the proposition. □

Proof of Proposition 4. The result follows directly from the fixed-point condition $F(\alpha(F)) = F$ summarized in equation 17. The expressions for F_- and F_+ are the solutions of this quadratic equation and the corresponding employment levels α_- and α_+ follow from substituting the expressions for F_- and F_+ into the expression for α in Proposition 3. Assumption 3 (that *B* is not too large) ensures that both of the roots F_- and F_+ are real. □

Proof of Lemma 2. The result follows simply from substituting into the objective function in equation 23 for the functional form of α in Assumption 1. Thus, we must solve

$$\text{maximize } \frac{a}{\sqrt{q}}\left(y - c\right) - \frac{k}{q}. \tag{A.20}$$

The following first-order condition gives the global maximum q_{CE}:

$$-\frac{1}{2}a\left(y - c\right)q_{\text{CE}}^{-3/2} + kq_{\text{CE}}^{-2} = 0. \tag{A.21}$$

Solving for q_{CE} and substituting into α gives the expressions in the lemma. □

Proof of Proposition 5. The result follows from comparing α_{CE} from Lemma 2 with α_- from Proposition 4. We have that $\alpha_{\text{CE}} > \alpha_-$ whenever

$$\frac{a^2}{2k}(y - c) > \frac{a^2}{2k}\left(y - F_- - c\right), \tag{A.22}$$

which is always satisfied since $F_{-} > 0$. □

Proof of Proposition 6. To see that setting the wage equal to w_{CE} implements the constrained-efficient level of vacancy posting conditional on households working, substitute w_{CE} from equation 27 into firms' vacancy posting condition in equation 29, noting as before that firms continue to post vacancies until this inequality binds. Thus, we have that

$$q\alpha\left(y - w_{\mathrm{CE}}\right) = q\alpha\left(y - \frac{y+c}{2}\right) = k, \tag{A.23}$$

which, for $\alpha(q) = a/\sqrt{q}$, gives

$$q = \left(\frac{2k}{a(y-c)}\right)^2 \equiv q_{\mathrm{CE}}. \tag{A.24}$$

Therefore, setting the wage equal to w_{CE} implements the constrained-efficient outcome as long as it induces the household to work, that is, as long as the household's IC is satisfied. This is the case as long as

$$w_{\mathrm{CE}} - F - c \geq 0 \tag{A.25}$$

or

$$F \leq \frac{y-c}{2} \equiv F_{\mathrm{CE}}, \tag{A.26}$$

as stated in the proposition. □

Proof of Proposition 7. The result follows immediately from the argument in the text and Proposition 3. □

Proof of Proposition 8. The result follows immediately from the argument in the text and Proposition 3. □

Proof of Proposition 9. The result follows immediately from the argument in the text and Proposition 3. □

Proof of Proposition 10. We begin with the appropriately modified entry condition for firms and then compare it with the efficiency condition from the social planner's problem.

As in the baseline model, firm entry is given by equation 13:

$$k = q\alpha(y - \overline{w}). \tag{A.27}$$

But now $\overline{w}$ depends on firms' bargaining power β. The average wage is now weighted by the probability β that the firm makes the offer and the probability $1-\beta$ that the household makes the offer (the wages w_f

and w_h when each makes the offer are unchanged; see Proposition 2). Hence,

$$\overline{w} = \beta (F + c) + (1 - \beta) y. \tag{A.28}$$

After substituting in for $\overline{w}$ and doing a little manipulation, firms' entry condition becomes

$$k = q\alpha\beta(y - F - c). \tag{A.29}$$

We now compare the equilibrium expression above with the first-order condition for constrained efficiency (see equation 23):

$$\alpha' (y - c) + \frac{k}{q^2} = 0 \tag{A.30}$$

or, rearranging,

$$k = -q^2\alpha' (y - c). \tag{A.31}$$

Comparing equations A.29 and A.31, we see that the equilibrium is efficient if

$$q\alpha\beta (y - F - c) = -q^2\alpha' (y - c). \tag{A.32}$$

Rearranging and writing $\eta = q\alpha'/q$ gives the expression in the proposition. □

Proof of Proposition 11. Here, we follow the logic of the proof of Proposition 4 and look for the fixed point $F(\alpha(F))$, that is, the face value that solves both the banks' break-even condition and the firms' entry condition (see equations 16 and 13).

Banks' break-even condition reads

$$B = \alpha F = \frac{a_0}{1 + a_1 q} F \tag{A.33}$$

or, solving for q,

$$q = \frac{a_0 F - B}{a_1 B}. \tag{A.34}$$

Firms' entry condition reads

$$k = q\alpha\frac{1}{2}(y - F - c) = \frac{a_0 q}{1 + a_1 q}\frac{1}{2}(y - F - c)\,. \tag{A.35}$$

Substituting into this expression from equation A.34 above and rearranging, we get the quadratic in the proposition,

$$0 = A_0 + A_1 F + A_2 F^2, \tag{A.36}$$

where $A_0 = -B\,(y - c)\,, A_1 = B + a_0\,(y - c) - 2ka_1$, and $-a_0$. The solutions always have the same sign because $A_0 A_2 > 0$. They are positive as long as $A_1 > 0$, which is the case by the assumption in the proposition (equation 42). □

Proof of Proposition 12. To find F_h, observe that when the household makes the offer, the problem is exactly as in the baseline model, so $F_h = B/\alpha$.

To find F_b, substitute the wages from Proposition 2 into equation 44 to write

$$u + \alpha\left(\frac{1}{2}\Big(y + F_b + c\Big) - F_b - c\right) = \alpha\left(\frac{1}{2}\Big(y + c\Big) - c\right). \tag{A.37}$$

Rearranging gives the expression in the proposition. □

Proof of Proposition 13. Suppose (in anticipation of a contradiction) that the household gets the wage w_L and makes the repayment $R(w_L)$ in equilibrium. In this case, the payoffs to the household and firm are as follows:[12]

$$\text{household payoff}_0 \quad = w - R(w) - c, \tag{A.38}$$

$$\text{firm payoff}_0 \quad = y - w_L. \tag{A.39}$$

Now consider the deviation in which the household "borrows" $z = w_H - w_L > 0$ from the firm to get wage $w_H = w_L + z$ and makes repayment $(1 + r)z$ to the firm. We show that whenever *R* is decreasing, there is an "interest rate" *r* that makes both the household and the firm better off (at the expense of the bank).

First, write the household's and the firm's payoffs given the deviation in terms of their initial payoffs:

$$\text{household payoff}' = w_H - R\,(w_H) - c - (1 + r)\,z \tag{A.40}$$

$$= w_H - R(w_H) - c - (1+r)(w_H - w_L) \tag{A.41}$$

$$= w_L - R(w_H) - c - r(w_H - w_L) \tag{A.42}$$

$$= w_L - R(w_L) - c + [R(w_L) - R(w_H) - r(w_H - w_L)] \tag{A.43}$$

$$= \text{household payoff}_0 + \left[R(w_L) - R(w_H) - r(w_H - w_L)\right] \tag{A.44}$$

and

$$\text{firm payoff}' = y - w_H + (1+r)z \tag{A.45}$$

$$= y - w_H + (1+r)(w_H - w_L) \tag{A.46}$$

$$= y - w_L + r(w_H - w_L) \tag{A.47}$$

$$> \text{firm payoff}_0 + r(w_H - w_L). \tag{A.48}$$

The household and the firm can both gain from deviating if household payoff ′ > household payoff and firm payoff ′ > firm payoff, or, immediately from the calculations above,

$$R(w_L) - R(w_H) - r(w_H - w_L) > 0 \tag{A.49}$$

and

$$r(w_H - w_L) > 0. \tag{A.50}$$

Combining these inequalities indicates that the deviation is profitable whenever

$$0 < r < \frac{R(w_L) - R(w_H)}{w_H - w_L}, \tag{A.51}$$

which exists whenever $R(w_L) > R(w_H)$. This condition holds since R is decreasing by assumption.

This contradicts the supposition that the household makes the repayment $R(w_L)$. Hence, *R* cannot be decreasing in equilibrium. □

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