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Notes, Comments, and Letters to the Editor

Competitive Equilibrium in the Credit Market under Asymmetric Information

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We study a competitive credit market equilibrium in which all agents are risk neutral and lenders a priori unaware of borrowers' default probabilities. Admissible credit contracts are characterized by the credit granting probability, the loan quantity, the loan interest rate and the collateral required. The principal result is that in equilibrium lower risk borrowers pay higher interest rates than higher risk borrowers; moreover, the lower risk borrowers get more credit in equilibrium than they would with full information. No credit is rationed and collateral requirements are higher for the lower risk borrowers. *Journal of Economic Literature* Classification Numbers: 022, 314. © 1987 Academic Press, Inc.

1. INTRODUCTION

The celebrated result of economic theory that the competitive equilibrium resulting from the exchange of capital between borrowers and lenders is Pareto optimal has underlying assumptions with little real-world support. Individual borrowers and lenders rarely exchange capital directly. Borrowers frequently cannot borrow precisely what they desire. And lenders use extensively non-interest items like collateral. Such credit practices may be attributed to asymmetric information, a serious impediment to free borrowing and lending. Borrowers with varying default risks simultaneously seek credit, often without meaningful distinguishing signals. Lenders may thus be a priori incapable of sorting borrowers out.

The purpose of this paper is to develop a model of a competitive credit

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market under asymmetric information. The model focuses on the majority of credit policy instruments considered important by lenders: loan quantity, loan interest rate, collateral, and the possibility of rationing.

In Section 2 we begin by considering a single-period, risk-neutral economy with potential borrowers, each of whom can invest in a project that either succeeds or fails. Each borrower knows its own success probability, but borrower differences are ex ante imperceptible to lenders. A lender designs a credit policy consisting of an interest rate, the loan amount, collateral, and the credit granting probability, so as to sort borrowers through their contract choices. Lenders operate in competitive asset and liability markets, and are initially assumed to face a perfectly elastic deposit supply at an exogenous, market-determined rate. The equilibrium concept studied is Riley's [8] reactive equilibrium (RRE). The RRE, characterized and discussed in Section 3, exists and is unique. Section 4 takes up endogenous deposit rate determination by assuming lenders compete for loans and for a limited quantity of deposits that can either exceed or be less than loan demand. In equilibrium, loan demand and deposit supply are equated and the deposit rate is endogenously determined. This notion of competition is similar to that of Stiglitz and Weiss [10], but unlike their result, we find *no* equilibrium credit rationing. Section 5 concludes. No formal proofs of results are given in the paper. These appear in our working paper [2].

Three of our results differ from the literature on asymmetrically informed credit markets (e.g., Jaffee and Russell [4], Keeton [6], Stiglitz and Weiss [10, 11], Wette [12]). First, under asymmetric information, low-risk borrowers obtain more credit than under full information. These borrowers also obtain more credit and pay higher interest rates than high-risk borrowers. Second, low-risk borrowers put up more collateral than high-risk borrowers. Third, although asymmetric information induces the allocational distortion of suboptimally high investments by some borrowers, there is no credit rationing. The reason for the difference between this result and Stiglitz and Weiss [10], where lenders may prefer rationing to an upward price adjustment, lies in the loan interest rate being the only initially admissible credit instrument in Stiglitz and Weiss. Collateral is later examined, but only for risk-averse borrowers. Wette [12] extends the argument to permit risk-neutral borrowers, but focuses on just one credit instrument since collateral requirements are varied with the interest rate held *fixed*. Thus, both models have a single contract for all borrowers, precluding sorting of borrowers into risk classes. This means a higher interest rate or collateral requirement could attract entry of lower quality borrowers and reduce bank profits. In contrast, the lender in our model uses a sufficiently large set of credit instruments to sort borrowers into as many risk classes as there are types. Since the contract for each class generates at least zero expected profit for the lender, rationing is avoided, albeit at the expense of distorting some investments away from first-best levels.

2. Model Description

Consider a market consisting of many risk-neutral borrowers and lenders. A potential borrower may invest I dollars in a project yielding a gross wealth of $R(I, \delta)$ with probability δ and zero with probability $1-\delta$. (The variable δ denotes both a borrower's type and its success probability.) The function R: $\mathbb{R}_+ \times [\delta_1, \delta_2] \to \mathbb{R}_+$ is twice continuously differentiable, with $R(0, \delta)=0\ \forall \delta$, $R_I(\cdot, \cdot)>0$, $R_{II}(\cdot, \cdot) \leqslant 0$, $R_\delta(\cdot, \cdot)>0$ for $I>0$, and $R_{I\delta}(\cdot, \cdot)>0$. Subscripts denote partial derivatives. Throughout, $\mathbb{R}_+ \equiv (0, \infty)$ and $\mathbb{R} \equiv (-\infty, \infty)$. These assumptions imply that higher δ borrowers have higher total and marginal returns in the successful state. In addition to the wealth that can be generated by investment, each borrower has an identical non-random end-of-period wealth W. No borrower, however, has any initial endowment and each must approach a lender for a one-period loan. These loans must be repaid with interest at the end of the period. The function $R(\cdot, \cdot)$ is common knowledge.[1] The success probability δ, however, varies across borrowers and is known only to the borrower. We assume each lender knows the cross-sectional distribution $G(\delta)$ of δ's in the pool of credit applicants. G is concentrated on $[\delta_1, \delta_2]$ with a continuous density function, $g(\delta)$, which is positive over $[\delta_1, \delta_2]$. Each lender's liabilities are fully insured[2] and loanable deposit funds are in perfectly elastic supply at the (for now exogenous) riskless interest factor, r. (Note $r \geqslant 1$ since it is one plus the riskless rate.)

Lenders compete on four dimensions: (i) the interest factor $F \in [1, \infty]$ (one plus the interest rate on the loan); (ii) the loan size $I \in \mathbb{R}_+$; (iii) the collateral requirement, $k \in [0, 1]$, that indicates the fraction of W that the bank is given title to; and (iv) the probability, $\Pi \in [0, 1]$, that credit will be granted. A *credit contract* $C \equiv (F, I, k, \Pi)$ is a vector that specifies a value of each of these variables. A *credit policy* is the set of all credit contracts offered by a lender. Given a credit contract C, a type-δ borrower's expected utility is $U(C, \delta) = \Pi\{\delta[R(I, \delta) - FI] - (1-\delta)[FI \wedge kW] + W\} + (1-\Pi) W$, where "$\wedge$" denotes the min operator.[3] The expected

[1] Banks are assumed to be able to observe ex post whether a borrower's project failed or succeeded, but not the actual realized return. This rules out the possibility that sorting could be achieved costlessly through a forcing contract based on R.

[2] Lenders are institutional depository financial intermediaries with federally (de facto) fully insured liabilities. Relaxing the full-insurance assumption does little to alter the analysis.

[3] The repayment in the unsuccessful state is written as $FI \wedge kW$ to indicate that a borrower will default whenever optimal.

profit of a lender offering contract C to a type-δ borrower is $P(C, \delta) = \delta FI + (1-\delta)[FI \wedge kW] - rI$.

Our goal is to characterize equilibrium credit policies in a competitive credit market with free entry. The equilibrium concept used is Riley's Reactive equilibrium (RRE) (Riley [8]).[4] RRE relies on the notion of strongly informationally consistent (SINC) policies. A credit policy is SINC if there is a contract $C(\delta)$ for every δ such that

(i) $C(\delta) \gtrsim_\delta C(\hat{\delta}),\ \forall \delta, \hat{\delta} \in [\delta_1, \delta_2]$,

(ii) $P(C(\delta), \delta) = 0,\ \forall \delta \in [\delta_1, \delta_2]$,

where $\gtrsim_\delta$ denotes the preference ordering of a type-δ borrower. Riley has demonstrated that the Pareto-dominant SINC policy is the unique RRE.

The Pareto-dominating SINC credit policy may be found by solving

PROBLEM I

$$\max \int_{\delta_1}^{\delta_2} U(\delta)\, g(\delta)\, d\delta, \tag{1}$$

subject to ($\forall \delta, \hat{\delta}, \in [\delta_1, \delta_2]$):

$$U(\delta) \equiv \Pi(\delta)\{\delta[R(I(\delta), \delta) - F(\delta)\, I(\delta)] - [1-\delta]\, T(\delta) + W\} + [1 - \Pi(\delta)]\, W, \tag{2}$$

$$U(\delta) \geqslant \Pi(\hat{\delta})\{\delta[R(I(\hat{\delta}), \delta) - F(\hat{\delta})\, I(\hat{\delta})] - [1-\delta]\, T(\hat{\delta}) + W\} + [1 - \Pi(\hat{\delta})]\, W, \tag{3}$$

$$\delta F(\delta)\, I(\delta) + [1-\delta]\, T(\delta) = rI(\delta), \tag{4}$$

$$T(\delta) \equiv F(\delta)\, I(\delta) \wedge k(\delta)\, W, \tag{5}$$

$$F(\delta) \in [1, \infty], \tag{6}$$

$$k(\delta) \in [0, 1], \tag{7}$$

$$\Pi(\delta) \in [0, 1], \tag{8}$$

where $U(\delta)$ is the equilibrium expected utility of a type-δ borrower.[5,6]

[4] An alternative is Wilson's [13] anticipatory E2 equilibrium. However, with a continuum of types, the E2 equilibrium will not in general exist. With discrete types, the E2 equilibrium concept would be attractive because Myerson [7] has shown that it is the appropriate generalization of the core concept to asymmetric information environments.

[5] The solution to Problem I is independent of $g(\delta)$, implying that the Pareto-dominating policy is independent of the "welfare weights." The reason is Problem I has an equivalent representation in the form of a nested sequence of smaller maximization problems (Spence [9]), none linked to $g(\delta)$.

[6] Borrowers' equilibrium utilities are assumed throughout to exceed reservation levels.

Conditions (2)–(8) define the SINC credit policies. Note (3) is the self-selection contraint, (4) is the zero profit condition, (2) and (5) are definitional contraints, and (6)–(8) are feasibility conditions. To characterize the solution to this problem, Lemma 1 replaces the global self-selection constraint in (3) with an equivalent local representation.

LEMMA 1. *Any credit policy that satisfies* (3) *also satisfies*

$$U'(\delta) = \Pi(\delta)\{R(I(\delta), \delta) + \delta R_\delta(I(\delta), \delta) - F(\delta)\, I(\delta) + T(\delta)\} \tag{9}$$

for almost every $\delta \in [\delta_1, \delta_2]$.

The above lemma characterizes only a (necessary) local condition for incentive compatibility. We will select the optimal solution from among those policies that satisfy this condition. We will then demonstrate that this optimal solution satisfies the incentive compatibility constraint globally. (See Lemma 6 in the Appendix.) For later use, we define

$$u(\hat{\delta}, \delta) \equiv \Pi(\hat{\delta})\{\delta[R(I(\hat{\delta}), \delta) - F(\hat{\delta})\, I(\hat{\delta})] - [1-\delta]\, T(\hat{\delta})\} + W \tag{10}$$

as the expected utility of a type-δ borrower reporting itself to be type-$\hat{\delta}$.

Using Lemma 1 and some algebra, we restate Problem I. Note that (4) is equivalent to

$$T(\delta) = [1-\delta]^{-1}[r - \delta F(\delta)]\, I(\delta), \tag{11}$$

and that (5) is equivalent to

$$F(\delta) \geqslant r \tag{12a}$$

$$[r - \delta F(\delta)]\, I(\delta) \leqslant [1-\delta]\, k(\delta)\, W, \tag{12b}$$

with either (12a) or (12b) holding as an equality. Substituting (11) into (2) and (9), respectively, results in

$$U(\delta) = \Pi(\delta)\{\delta R(I(\delta), \delta) - rI(\delta)\} + W, \tag{13}$$

$$U'(\delta) = \Pi(\delta)\{R(I(\delta), \delta) + [1-\delta]^{-1}\, I(\delta)[r - F(\delta)] + \delta R_\delta(I(\delta), \delta)\}. \tag{14}$$

Problem I can now be restated as follows. Maximize (1) subject to: (7), (8), (12a), (12b) (with either holding as an equality), (13), and (14). (Note that because of (12a) and $r \geqslant 1$, constraint (6) is superfluous.) In Section 3, we characterize and discuss the solution to this problem. As a benchmark, however, we first discuss the full-information Pareto-optimum.

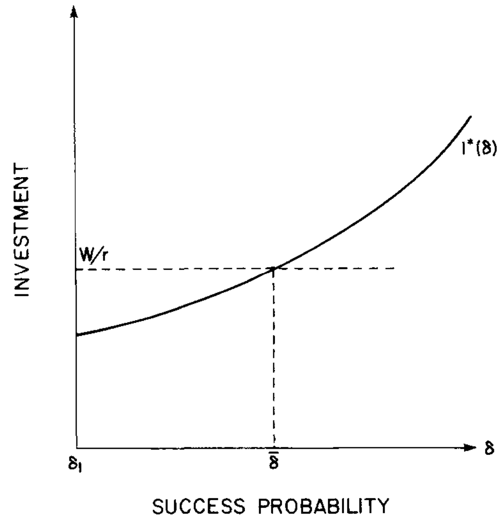

FIG. 1. First-best investment as a function of success probability.

The full-information Pareto-optimal policy solves Problem I without constraint (14). The solution (identified by stars) is given by[7]

$$I(\delta) = I^*(\delta) \equiv \underset{I}{\operatorname{argmax}}\ \delta R(I, \delta) - rI, \tag{15}$$

$$F^*(\delta) = \begin{cases} r & \text{for } \delta_1 \in [\delta, \bar{\delta}] \\ r\delta^{-1} - [1-\delta]\ W[\delta I^*(\delta)]^{-1} & \text{for } \delta \in (\bar{\delta}, \delta_2], \end{cases} \tag{16}$$

$$\Pi^*(\delta) = 1, \qquad \forall \delta \in [\delta_1, \delta_2], \tag{17}$$

$$k^*(\delta) = \begin{cases} rI^*(\delta)\ W^{-1} & \text{for } \delta \in [\delta_1, \bar{\delta}] \\ 1 & \text{for } \delta \in (\bar{\delta}, \delta_2], \end{cases} \tag{18}$$

where $\bar{\delta}$ satisfies

$$I^*(\bar{\delta}) = W/r. \tag{19}$$

Throughout we assume that $\delta_1 < \bar{\delta} < \delta_2$. Hereafter, the cutoff success probability, $\bar{\delta}$, refers to the δ that solves (19). Figure 1 sketches the determination of $\bar{\delta}$. The full-information equilibrium entails each borrower receiving a loan that maximizes the net expected social surplus of its project. Borrowers with low success probabilities ($\delta < \bar{\delta}$) make small investments and pay the riskless rate because they offer enough collateral to ensure full loan repayment. Borrowers with larger success probabilities

[7] An interior solution is assumed to exist for every δ. Given $R_{II} < 0$, a sufficient condition for this is $\delta_1 R_I(0, \delta_1) > r$. We assume this condition holds.

$(\delta > \tilde{\delta})$ make larger investments in equilibrium and put up all of their wealth as collateral. Given wealth constraints, these borrowers have a positive probability of default. A higher interest rate compensates the lender for this default risk.

The full-information Pareto-optimum cannot be achieved under asymmetric information. This can be seen by noting that, at the full-information optimum, the expected utility of a type $\delta > \tilde{\delta}$ would be higher if that borrower type chose the contract designed for a type $\hat{\delta} > \delta$. To see this, note that

$$u_{\hat{\delta}}(\hat{\delta}, \delta)\,|_{\hat{\delta}=\delta} = [\delta R_I(I^*(\delta), \delta) - r]\, dI^*(\delta)/d\delta + \delta^{-1}[rI^*(\delta) - W]$$
$$= \delta^{-1}[rI^*(\delta) - W] > 0.$$

The above inequality follows because $rI^*(\delta) > W$ for $\delta > \tilde{\delta}$.

3. Equilibrium under Asymmetric Information

The equilibrium under asymmetric information is the solution to Problem I and is described below.

THEOREM. *The RRE has the following properties*:

(i) $$I(\delta) = \begin{cases} I^*(\delta), & \forall \delta \in [\delta_1, \tilde{\delta}] \\ \hat{I}(\delta) > I^*(\delta), & \forall \delta \in (\tilde{\delta}, \delta_2], \end{cases}$$

$$I'(\delta) > 0, \qquad \forall \delta \in [\delta_1, \delta_2].$$

(ii) $$F(\delta) = \begin{cases} r, & \forall \delta \in [\delta_1, \tilde{\delta}) \\ r + [rI(\delta) - W][1 - \delta][\delta I(\delta)]^{-1} > F^*(\delta) \geq r, & \forall \delta \in (\tilde{\delta}, \delta_2]. \end{cases}$$

(iii) $\Pi(\delta) = 1, \qquad \forall \delta \in [\delta_1, \delta_2].$

(iv) $$k(\delta) = \begin{cases} rI^*(\delta)/W, & \forall \delta \in [\delta_1, \tilde{\delta}] \\ 1, & \forall \delta \in (\tilde{\delta}, \delta_2]. \end{cases}$$

The idea in proving this theorem is to examine the optimal control solution to Problem I separately over $[\delta_1, \tilde{\delta}]$ and $(\tilde{\delta}, \delta_2]$ and then join the two solutions. This is straightforward for $[\delta_1, \tilde{\delta}]$ because every δ in that interval receives its first-best allocation. Over $(\tilde{\delta}, \delta_2]$, we formulate the appropriate Lagrangian, characterize the transversality and complementary slackness conditions, and derive the first-order optimality conditions. To ensure global incentive compatibility, we check that the continuity of the optimal functions is not broken at $\tilde{\delta}$. Finally, we prove existence of equilibrium by verifying that the necessary conditions for optimality are sufficient using the Seirstad and Sydsaeter sufficiency theorem for optimal control problems with constraints on state and control variables (Kamien and Schwartz [5]). Finally, we demonstrate that the optimal solution, obtained to satisfy local incentive compatibility, is globally incentive compatible. The steps required to arrive at the theorem above are stated as lemmas in the appendix, and rigorous proofs appear in Besanko and Thakor [2]. The fact that the Pareto-dominant SINC credit policy derived in the theorem is indeed the RRE is established later in this section.

We now interpret the theorem. It indicates that, absent binding wealth constraints, lenders can screen borrowers without welfare loss by lending $I^*(\delta)$ at the riskless rate to a type-δ borrower and asking for title to $rI^*(\delta)$ of the borrower's terminal wealth in the event the project fails. This contract is feasible as long as the borrower's terminal wealth exceeds the required collateral, i.e., $W \geqslant rI^*(\delta)$ or $\delta \leqslant \hat{\delta}$. This maximum collateralization implication differs from Wette's [12] result that demanding more collateral may deter safer borrowers' participation. This difference arises because, in Wette's model, credit contracts extract all borrower surplus, which in equilibrium is passed on to depositors. Thus, elevating collateral requirements at a fixed interest rate may induce some safe borrowers to exit the market. By contrast, credit contracts in our model maximize borrower surpluses which, by assumption, exceed reservation levels. Consequently, collateral does not cause adverse selection as in Wette.

The low-risk borrowers, those with $\delta \in (\hat{\delta}, \delta_2]$, commit all their terminal wealth as collateral, giving them a zero payoff in the unsuccessful state. This is the case in which the wealth constraint on collateral is binding, but the theorem shows that, despite this, a randomized loan granting strategy—as in Besanko and Thakor [3]—does not emerge in equilibrium; borrowers with $\delta > \hat{\delta}$ receive loans for sure. Finally, the theorem indicates that a borrower with $\delta \in (\hat{\delta}, \delta_2]$ receives a larger loan and pays more interest than under full information.

Two features of the equilibrium appear counterintuitive. First, the collateral requirements and interest rates are nondecreasing in borrowers' success probabilities. The rationale is that the equilibrium loan size is *increasing* in the borrower's (truthfully) revealed success probability. This positive cross-sectional relationship ensures that there is *no net default* for the lower quality types who borrow relatively small amounts, but there could be *net default* for the higher quality types. Maximum collateralization is, therefore, necessary for higher quality borrowers, and they must also be charged interest rates exceeding the riskless rate, as well as the rate they would pay under full information, to compensate the lender for the risk borne. This result should not be taken to imply that, in practice, all high-risk borrowers pay lower interest rates than all low-risk borrowers. As an extreme representation of asymmetric information, we have assumed the absence of observable characteristics that distinguish borrowers according to risk class. The conclusion of this model is that if two borrowers share the same observable characteristics, we should want low-risk borrowers to borrow more, and to satisfy incentive compatibility, this must be at interest rates higher than those paid by the ex ante indistinguishable higher risk borrowers.[8]

A second surprising property of the equilibrium is that higher quality borrowers (i.e., borrowers with $\delta > \bar{\delta}$) overinvest relative to the full-information Pareto-optimum. To understand this result, note that all borrowers in the interval $[\bar{\delta}, \delta_2]$ post the maximum available collateral. Thus, collateral cannot be used as an instrument for sorting borrowers. Lenders can, however, sort borrowers by adjusting the investment level and the interest rate. This is possible because when $R_{I\delta} > 0$, the marginal rate of substitution between investment and interest rate is an increasing function of δ. That is, a borrower with a higher δ is willing to pay more for an incremental amount of investment. This implies that a contract specifying a suboptimally high investment in conjunction with a high interest rate is relatively more attractive for a borrower with a higher δ. By offering a set of such contracts lenders can thus deter borrowers with lower δ's from exaggerating their success probabilities.

A remaining issue that deserves attention is the viability of the credit policy stated in the theorem. Does it indeed represent the RRE as claimed? To examine this question, we focus initially on contracts for $\delta \geqslant \bar{\delta}$. Some of these contracts are depicted in Figure 2 in $I-Q$ space, where $Q \equiv F \cdot I$. The locus of points between $\bar{C}$ and C_b is an indifference curve for a type-$\bar{\delta}$ borrower. The dashed curve through C_b is an indifference curve of a type-δ_b borrower where $\delta_b > \bar{\delta}$. The slope of a type-δ indifference curve when $I \geqslant I^*(\bar{\delta})$ and $Q \geqslant W$ can be shown to equal $R_I(I, \delta)$. Because $R_{I\delta} > 0$, a type-δ_b borrower has a steeper indifference curve at any point than does a type-$\bar{\delta}$ borrower. The zero profit line for a type-δ borrower has slope $r\delta^{-1}$. The points $\bar{C}$ and C_b represent the candidate equilibrium contracts for types $\bar{\delta}$ and δ_b, respectively.

Suppose an entrant offers a pooling contract $\hat{C}$ in an attempt to disturb the equilibrium. This contract is preferred by both types $\bar{\delta}$ and δ_b to their candidate equilibrium contracts. Suppose that the zero profit line corresponding to the set of types attracted to $\hat{C}$ lies below $\hat{C}$ so that $\hat{C}$ is a profitable contract. In order for $\bar{C}$ and C_b to constitute part of the *RRE* contract set, another entrant must be able to lure the δ_b borrowers away from $\hat{C}$. To show that this is indeed possible, note from Figure 3 that an entrant offering the contract $\tilde{C}$ will attract all the δ_b's away from $\hat{C}$

[8] We are grateful to an associate editor for this interpretation.

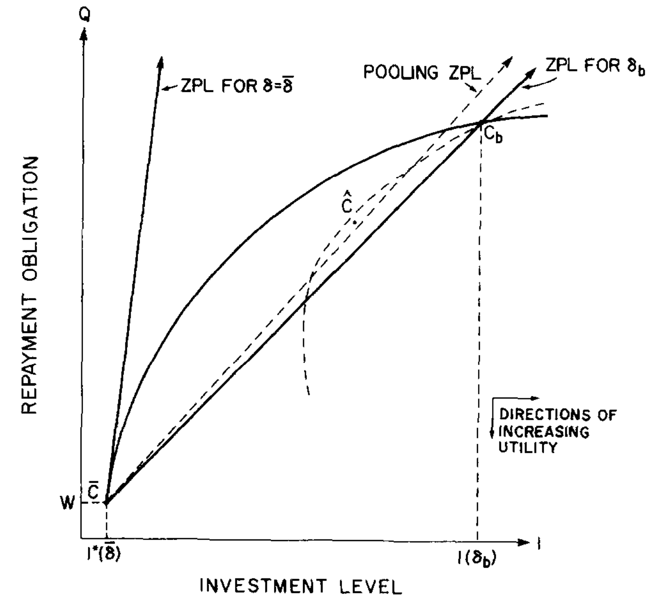

FIG. 2. Asymmetric information equilibrium contracts for borrowers with high success probabilities.

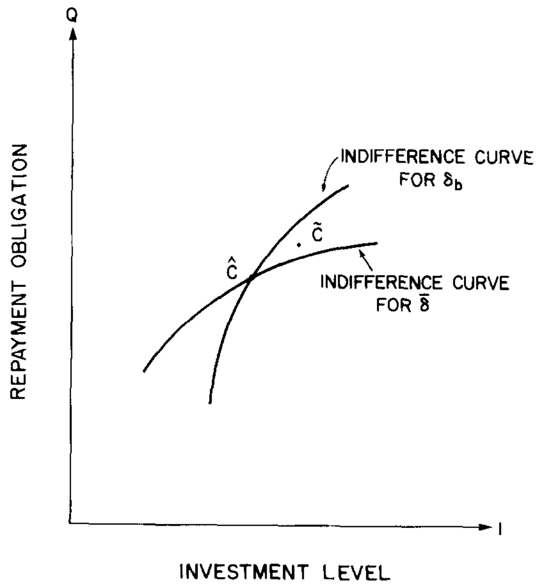

FIG. 3. Inability of a pooling contract to disturb equilibrium.

but none of the $\bar{\delta}$'s. This imposes losses on the deviating lender offering $\hat{C}$, and given the possible reactions of other lenders, the worst outcome for the lender offering $\tilde{C}$ is the loss of all its customers. But this is no worse than its outcome if it did not respond to $\hat{C}$. The *RRE* implies that $\hat{C}$ would not be offered in the first place and thus could not upset $(\bar{C}, C_b)$.[9] Although given for a two-type example, this argument applies to any pair of types in $[\bar{\delta}, \delta_2]$ and can thus be extended to a continuum.

Similar sustainability arguments hold for $\delta < \bar{\delta}$. In this region these arguments can be made with even more force. This is because not only can lenders use investment levels and interest rates to sort borrowers but can also resort to collateral alterations. This as well as the above existence arguments rely on the assumption that $R_{I\delta} > 0$. If, instead, R_I is independent of δ, then existence of the *RRE* may fail altogether.

4. Aggregate Credit Supply and Demand Considerations

Thus far we have assumed each lender faces a perfectly elastic supply of funds at an *exogenously* determined riskless interest factor r, and thus in equilibrium all credit demand is satisfied. We have also assumed all borrower types enter the market in equilibrium, i.e., $U(\delta; r) \geqslant U^0(\delta)$ $\forall \delta \in [\delta_1, \delta_2]$, where $U(\delta; r)$ is the utility of a type-δ borrower that accepts a credit contract when the riskless interest factor is r, and $U^0(\delta)$ is its utility if it exits the credit market. We now relax these assumptions, particularly the assumption that r is exogenous.

To endogenize r, we follow Stiglitz and Weiss [10] and assume that lenders face an upward sloping aggregate supply curve for deposits, $S(r)$. Moreover, we allow for the possibility that not all borrowers may be in the market at some rate r. Let $\mu(\cdot)$ be the Lebesgue measure on $\mathbb{R}$, and define

$$\mathscr{F}(r) \equiv \{\delta \mid U(\delta; r) \geqslant U^0(\delta)\} \subseteq [\delta_1, \delta_2]$$

and

$$D(r) \equiv \int_{\mathscr{F}(r)} I(\delta, r)\, dN(\delta),$$

where $I(\delta, r)$ is a type-δ borrower's investment under the RRE contracts,

[9] This argument assumes that borrowers cannot search sequentially, so that even a rejected borrower can not approach a second lender.

$D(r)$ is the loan demand when the riskless interest factor is r, and $N(\delta)$ is the number of borrowers with success probabilities $\leqslant \delta$. Lenders now compete for borrowers and for a limited deposit quantity. Discussed below is how this affects equilibrium.

Fix a deposit rate $r = \bar{r}$ and assume, first, that $S(\bar{r}) > D(\bar{r})$. Here lenders obtain all the deposits they require, and depositors compete by offering deposits at a lower rate. Existing lenders will accept the lower rate because all the credit contracts that were earning zero expected profits at $\bar{r}$ now earn positive expected profits. These excess profits engender a new round of competition for existing borrowers, whose credit contracts are consequently modified to reflect the lower r, thus increasing these borrowers' expected utilities. Assuming that $\mathscr{F}(\bar{r})$ is a strict subset of $[\delta_1, \delta_2]$, the new contracts that are offered will also attract some borrowers with $\delta \in [\delta_1, \delta_2] \backslash \mathscr{F}(\bar{r})$.[10] Consequently, to restore incentive compatibility, lenders offer *additional* contracts designed for the low δ types who were previously not in the market but now find participation profitable. (Because $U(\delta; r)$ is increasing in δ by incentive compatibility and can be shown to be decreasing in r, the types that exit when r rises are the lowest δ types.) Note that the independence of equilibrium contracts from $G(\delta)$ implies that a lender's credit policy can be viewed as a set of contracts parameterized by r such that changes in r do not alter the *functional relationships* between the policy instruments and δ. Thus, competition continues in this fashion until r drops sufficiently to clear the market for deposits and loans. This should clarify why, unlike Stiglitz and Weiss [10] and Wette [12], no rationing occurs in our model despite constrained deposit supply and an endogenously determined r.

Now suppose that $D(\bar{r}) > S(\bar{r})$. In this case some lenders announce a set of credit contracts, which they cannot fulfill because of insufficient deposits. Now suppose a new lender enters the market and offers the set of contracts offered by existing lenders but parameterized by a riskless factor $\hat{r} > \bar{r}$. Suppose, too, that this entrant offers to pay depositors a riskless return $\hat{r} - \varepsilon > \bar{r}$, where $\varepsilon > 0$ is a small, positive scalar. These contracts will be incentive compatible and will earn the lender positive expected profits on borrowers who remain in the market (borrowers with sufficiently low δ's will drop out if all other lenders eventually follow the entrant's lead). Moreover, the entrant will receive sufficient deposits. Under the RRE notion of competition, an entrant has an incentive to offer such a credit policy. Thus, with excess loan demand, competition among lenders for deposits will drive the riskless rate up to clear the market.

[10] The notation $x \in A \backslash B$ means $x \in \{x \mid x \in A, x \notin B\}$.

5. Concluding Observations

We have characterized the equilibrium in an asymmetrically informed, competitive credit market. Each borrower has one attribute—its default probability—that is a priori unknown to others. In the context of signaling models, if the loan interest rate is viewed as a "price," then we have a model in which three "signals"—loan size, collateral and rationing probability—are used to distinguish among agents with one unknown attribute each.[11] Our principal results differ significantly from the existing credit markets literature.

Our analysis relies, however, on two key assumptions. One is that the credit market is not dynamic and the other is that there is no moral hazard.[12] Extending our work by relaxing these assumptions appears worthwhile since it would enable us to know if the "no-rationing" result holds up under moral hazard in a dynamic context involving privately informed borrowers who develop reputations and competitive lenders who learn borrower attributes through time.

Appendix

We outline here, without proofs, the main steps involved in reaching the theorem stated in Section 3. The first step is to demonstrate that, for every $\delta \in [\delta_1, \bar{\delta}]$, the equilibrium contract will be the full-information Pareto-optimal contract for that type. This result is stated as

Lemma 2. *In the* RRE, *a borrower with type* $\delta \in [\delta_1, \bar{\delta}]$ *is awarded the same credit contract it would be under full information.*

The next step is to find the solution to Problem I over $(\bar{\delta}, \delta_2]$. The solution is developed in a sequence of lemmas and is stated in Lemma 5. But first we establish that each borrower's collateral equals its repayment in the unsuccessful state:

Lemma 3. *In the optimal solution to Problem* I *over* $(\bar{\delta}, \delta_2]$, (12b) *is binding.*

Assuming that (12a) holds, Lemma 3, in conjunction with the zero-profit

[11] There appears to be recent interest in models involving an agent with one unknown attribute using more than one signal. The models we are aware of employ two signals (e.g., Ambarisha, John, and Williams [1]).

[12] Stiglitz and Weiss [11] examine a two-period bank-customer model with moral hazard.

condition (11), implies that the solution to Problem I over $(\bar{\delta}, \delta_2]$ is the solution to:

PROBLEM Ia.

$$\underset{U,I,k,\Pi}{\text{Maximize}} \int_{\bar{\delta}}^{\delta_2} U(\delta)\, g(\delta)\, d\delta,$$

subject to

$$U'(\delta) = \Pi(\delta)\{R(I(\delta), \delta) + \delta R_\delta(I(\delta), \delta) - r\delta^{-1} I(\delta) + k(\delta)\,\delta^{-1} W\}. \quad \text{(A.1)}$$

$$U(\delta) = \Pi(\delta)\{\delta R(I(\delta), \delta) - rI(\delta)\} + W. \quad \text{(A.2)}$$

$$k(\delta) \in [0, 1]. \quad (7)$$

$$\Pi(\delta) \in [0, 1]. \quad (8)$$

$$U(\bar{\delta}) = U^*(\bar{\delta}). \quad \text{(A.3)}$$

It is clear that the solution to Problem Ia will be unchanged if (A.2) is replaced by

$$U(\delta) \leqslant \Pi(\delta)\{\delta R(I(\delta), \delta) - rI(\delta)\} + W, \quad \text{(A.2')}$$

because (A.2′) will never be slack given that the objective function is strictly increasing in U. (For any allocation such that (A.2′) was slack, one could always add a constant such that (A.2′) would be binding, and doing this would not violate (A.1)).

Letting U be the state variable, θ the co-state variable, λ the multiplier for (A.2′), τ and Ψ the multipliers for (7), and ξ and ζ the multipliers for (8), the Lagrangian for Problem Ia is

$$\mathbb{L} = Ug(\delta) + \theta\{\Pi[R(I, \delta) + \delta R_\delta(I, \delta) - r\delta^{-1} I + k\delta^{-1} W]\}$$
$$- \lambda\{U - \Pi[\delta R(I, \delta) - rI] - W\} + \tau k - \Psi[k-1] + \xi\Pi - \zeta[\Pi - 1].$$

$$\mathbb{L}_I = [\lambda + \theta\delta^{-1}][\delta R_I - r] + \theta\delta R_{I\delta} = 0. \quad \text{(A.4)}$$

$$\mathbb{L}_k = \theta\delta^{-1} W + \tau - \Psi = 0. \quad \text{(A.5)}$$

$$\mathbb{L}_\Pi = \theta\{R + \delta R_\delta - r\delta^{-1} I + k\delta^{-1} W\} + \lambda\{\delta R - rI\} + \xi - \zeta = 0. \quad \text{(A.6)}$$

$$\mathbb{L}_U = -\theta' \Rightarrow \theta'(\delta) = \lambda(\delta) - g(\delta). \quad \text{(A.7)}$$

The transversality condition is

$$\theta(\delta_2) = 0,$$

and the complementary slackness conditions include

$$\tau(\delta)\, k(\delta)=0; \quad \tau(\delta)\geqslant 0.$$

$$\Psi(\delta)[k(\delta)-1]=0; \quad \Psi(\delta)\geqslant 0.$$

$$\xi(\delta)\, \Pi(\delta)=0; \quad \xi(\delta)\geqslant 0.$$

$$\zeta(\delta)[\Pi(\delta)-1]=0; \quad \zeta(\delta)\geqslant 0.$$

Moreover, note that

$$\lambda(\delta)\geqslant 0.$$

The next lemma characterizes the necessary conditions for optimality.

LEMMA 4. *A solution to the necessary optimality conditions for Problem Ia has the following properties*:

$$I(\delta)>I^*(\delta) \qquad \forall\, \delta\in(\bar{\delta}, \delta_2]. \tag{A.8}$$

$$k(\delta)=1 \qquad \forall\, \delta\in(\bar{\delta}, \delta_2]. \tag{A.9}$$

$$\Pi(\delta)=1 \qquad \forall\, \delta\in(\bar{\delta}, \delta_2]. \tag{A.10}$$

$$\theta(\delta)>0 \qquad \forall\, \delta\in(\bar{\delta}, \delta_2).$$

$$\lambda(\delta)>0 \qquad \forall\, \delta\in(\bar{\delta}, \delta_2).$$

$$\lambda(\delta_2)=0.$$

Sufficiency of these first-order conditions is now established in Lemma 5 which also presents the credit policy that solves Problem I over $(\bar{\delta}, \delta_2]$.

LEMMA 5. *The optimal solution to Problem I for* $\delta\in(\bar{\delta}, \delta_2]$ *is such that*

(i) $I(\delta)>I^*(\delta)$. (A.11)

(ii) $F(\delta)=r+[rI(\delta)-W][1-\delta][\delta I(\delta)]^{-1}>F^*(\delta)\geqslant r$. (A.12)

(iii) $\Pi(\delta)=1$. (A.13)

(iv) $k(\delta)=1$. (A.14)

The overall RRE credit policy, stated in the theorem in Section 3, is now obtained by combining Lemmas 2 and 5. As a final step, we check that the above solution is globally incentive compatible. This is necessary because in obtaining the optimal solution, we used only a local representation of the incentive compatibility constraint.

LEMMA 6. *The solution stated in the theorem is globally incentive compatible.*

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