Financial Wealth, Investment and Sentiment in a Bayesian DSGE Model

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Abstract

This paper is a quantitatively oriented theoretical study into interactions among macroeconomic fluctuations, stock price dynamics and investor sentiment variations, with a focus on two episodes, namely, the 2008 Great Financial Crisis and the 2020 COVID-19 Pandemic. Our objectives are fivefold, namely, household turnover and firm turnover mechanisms are introduced to capture financial wealth effect and capital reallocation effect respectively, variable capital utilization and the idiosyncratic marginal efficiency of investment shock are incorporated to study the interplay between the stock market and the production sector, the labor utilization shock is specified to examine impacts of pandemics on labor utilization, and stock price fluctuations induced by the financial shock are disentangled from those triggered by the sentiment shock, fiscal and monetary policies are evaluated to examine responses to financial instability and coronavirus recession. We formulate a Bayesian Dynamic Stochastic General Equilibrium (DSGE) model for the U.S. economy, our methodologies include Chib and Ramamurthy (2010)’s TaRB-MH algorithm, impulse response analysis, forecast error variance decomposition and Bayesian model comparison. Our findings contribute to the literature along four dimensions. Household turnovers, firm turnovers, preference, investment and sentiment jointly influence financial wealth distribution and financial resource allocation, inducing subsequent stock price swings and macroeconomic fluctuations. These are not only due to households’ interactions with the stock market through financial wealth and consumption, but also triggered by firms’ interactions with the stock market through financial resources and investment. Higher household turnover rate increases financial wealth effect and aggregate demand, whereas higher firm turnover rate enhances capital reallocation effect at the cost of contaminating financial wealth effect. Monetary policy responds counteractively and significantly to financial slack at business cycle frequencies.

Keywords

Financial wealth effect; Capital reallocation effect; Labor Utilization Shock;

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1. Introduction

History has not only witnessed financial markets’ function in signaling economic uncertainty and financial fragility, but also revealed the growing dependence between financial variability and macroeconomic fluctuations. For instance, the 2008 Great Financial Crisis sparked the 2009 Great Recession, whereas the 2020 COVID-19 Pandemic triggered the 2020 Stock Market Crash and the Deep Recession. Consequently, it is crucial to understand the mechanism through which stock price fluctuations propagate to the real economy, the channels through which macroeconomic aggregates influence stock price swings, and the monetary policy rules through which central banks respond to financial instability. These shed light on prevention of financial crises’ propagation to the real economy, mitigation of economic uncertainty’s impacts on financial markets, and development of monetary policy tools to ensure financial stability.

The literature has mainly investigated demand-side interaction between stock price fluctuations and the real economy in the Dynamic Stochastic General Equilibrium (DSGE) framework. Castelnuovo et al. (2010), Funke et al. (2011), and Nisticò (2012) introduce the household turnover mechanism and examine financial wealth effect through demand-side of the real economy. They not only find that stock prices are essential in influencing real activities and driving business cycles, but also identify significant and counteractive responses of central banks to stock price misalignments. However, on one hand, they incorporate neither the firm turnover mechanism nor physical capital accumulation, neglecting the stock market’s influence through supply-side of the real economy, on the other hand, their models do not account for labor utilization and therefore cannot capture pandemics’ impacts upon production through labor utilization. We not only include household turnover mechanism and firm turnover mechanism to capture macroeconomic consequences of financial wealth effect and capital reallocation effect respectively, but also specify the labor utilization shock and incorporate capital capacity utilization data to study impacts of pandemics on macroeconomic fluctuations. From the demand-side, stock price fluctuations, which reflect household expectations about future wealth, influence household intertemporal substitution of consumption and decisions through household financial wealth. Specifically, incumbent household traders with financial wealth incur a probability of exiting financial markets and being replaced by new traders without financial wealth, their financial wealth is redistributed among surviving households. Incumbent household traders use accumulated financial wealth to smooth consumption, whereas new traders without financial wealth cannot smooth consumption. Nonetheless, individual consumption smoothing does not imply the same level of aggregate consumption smoothing. Therefore, when household turnover rate increases, the difference between current and expected future average consumption is greater. These influence stochastic discount factor and subsequently affect household behavior. In consistence
with Castelnuovo et al. (2010)’s analysis, we find increases in household financial wealth convey higher purchasing power, creating a financial wealth effect. From the supply-side, stock price swings, which signal expectation about future market capitalization, influence wholesale firms’ investment, production and dividends through credit constraints, physical capital accumulation and firm budgets respectively. Concretely, when firm turnover rate $\gamma_W$ increases, more new wholesale firms replace incumbent wholesale firms, and new wholesale firms are more likely to contain stock bubbles than incumbent wholesale firms, consequently, more stock bubbles lessen credit constraints and generate more financial resources for new wholesale firms. Higher firm turnover rate amplifies capital reallocation effect, leading investment to more productive wholesale firms. Since investment may crowd out consumption, capital reallocation effect contaminates financial wealth effect, and larger financial wealth effect is associated with lower firm turnover rate.

Castelnuovo et al. (2010), Funke et al. (2011), Nisticò (2012), and Miao et al. (2015) elucidate macroeconomic consequences of stock price swings under the assumption that central banks target at zero inflation, although zero inflation targeting is rare in the real economy. However, in our model, central banks use a positive inflation target in the monetary policy rule, and a proportion of wholesale firms update prices according to past inflation and the positive inflation target.

Furthermore, along the lines of Castelnuovo et al. (2010), Funke et al. (2011) and Nisticò (2012), in order to investigate the interplay between the stock market and the production sector, we not only specify our DSGE model with variable capital utilization, physical capital accumulation and credit constraints, but also buffet our DSGE model with the labor utilization shock, the idiosyncratic marginal efficiency of investment shock, the financial shock and the sentiment shock. On one hand, these specifications provide the channles through which pandemics influence labor employment, capital utilization, market capitalization and financial slackness in the production sector, as well fluctuations in the stock market. On the other hand, these shocks capture the impacts of stock market uncertainty upon investment efficiency, financial flexibility, dividend prospects and investor expectation in the production sector.

Last but not least, the DSGE models of Castelnuovo et al. (2010), Funke et al. (2011), and Nisticò (2012) do not explicitly distinguish between stock price misalignments explained by macroeconomic fundamentals and stock price fluctuations induced by investor sentiment. Our DSGE model, however, elucidates that forces driving episodes of stock market booms and busts encapsulate both economic fundamentals and sentiment. On one hand, economic fundamentals explain procyclicality of stock market indices. On the other hand, sentiment not only drives evolution of speculative stock bubbles, but also amplifies procyclicality of stock market indices. Castelnuovo et al. (2010) do not disentangle the effect of the financial shock from that of the sentiment shock. However, we decompose stock market index into stock market fundamental value perturbed by the financial shock and speculative stock bubble driven by the sentiment shock. The financial shock captures firms’ external financing risk in the stock market. We not only formulate a microfoundation for speculative stock bubbles to enter the stock market, but also replace Castelnuovo et al. (2010)’s stock market index measurement error with the sentiment shock. The sentiment shock reflects proportional size change between old and new speci-
ulative stock bubbles, particularly agents’ optimistic or pessimistic beliefs about future stock market. According to impulse response analysis and forecast error variance decomposition, we find both the financial shock and the sentiment shock play essential roles in triggering financial crisis and magnifying economic recessions. In consistence with Miao et al. (2015)’s analysis, we find rises in stock bubbles signal more firm budget flexibility and higher investment profitability. Therefore, financial resources move to more profitable and efficient firms, strengthening capital reallocation effect. Excess stock market fluctuations and asset price misalignments contradict central banks’ goal of financial stability. Therefore, central banks generally respond to stock price misalignments through the monetary policy rule.

The remaining of this paper is organized as follows. Section 2 specifies the model structure. Section 3 summarizes the data, performs estimation and conducts model comparison. Section 4 elucidates the model dynamics. Section 5 concludes the paper.

2. The Baseline DSGE Model

The economy consists of households, capital good firms, final good firms, retail firms, wholesale firms, financial intermediaries, the central bank and the government. Households supply labor to wholesale firms, purchase final goods from final good firms, lend to financial intermediaries and purchase stocks from wholesale firms. Capital good firms transform final goods purchased from final good firms into investment goods, and sell investment goods to wholesale firms. Wholesale firms borrow from financial intermediaries, issue stocks to households, purchase investment goods from capital good firms, employ labor from households, combine investment goods with undepreciated physical capital to accumulate installed physical capital, produce and sell wholesale goods to retail firms. Retail firms transform homogenous wholesale goods into heterogeneous retail goods, and sell retail goods to final good firms. Final good firms assemble retail goods purchased from retail firms to produce final goods, and sell final goods to households, capital good firms and the government. Financial intermediaries absorb deposits from households and issue corporate loans to wholesale firms. The government imposes taxes on households and issues government bonds to finance government spending. The central bank adjusts interest rate in response to inflation gap, output gap and financial slack. Nominal variables, which include final good prices and retail prices, are denominated in currency.

2.1. Households

We specify household behavior along the lines of Castelnuovo et al. (2010), Funke et al. (2011) and Nisticò (2012). In period 0, the household sector is populated by a unit continuum of cohorts of households, and i-period-old households are categorized into cohort i, because they enter economic life at age i. Although households are heterogeneous in terms of the ages that they enter financial markets, households belonging to the same cohort are homogenous, and the aggregate behavior of each cohort is modelled by the decision-making process of the representative household within each cohort. Assuming no population growth, from period 1 onwards, incumbent household traders incur a constant
probability $\gamma_H$ of dying, exiting financial markets and being replaced by a commensurate fraction of new household traders without financial wealth. Since cohort size is $\gamma_H$, its implied effective economic decision horizon is $\frac{1}{\gamma_H}$.

Both incumbent and new household traders purchase consumption goods from final good firms, supply labor to wholesale firms and pay taxes to the government. However, only incumbent household traders hold financial assets.

### 2.1.1. Human Wealth

Since household traders affiliated with cohort $i$ own labor, retail firms and capital good firms, they obtain wages from supplying labor $[H_t(i)]$ to wholesale firms, as well as gaining profits $[\Gamma_t(i)]$ from owning retail firms and capital good firms. At the beginning of each period, cohort $i$ possesses nominal human wealth $[P_{Y,t}HW_t(i)]$, which is the present discounted value of cohort $i$’s future cash inflows generated by utilized labor wages $[P_{Y,t}W_t(i)H_t(i)]$, wholesale firms’ profits $[\int_0^1 \Gamma_{Y,t+i}(l)dl]$ and capital good firms’ profits $[P_{Y,t}\Gamma_{K,t}(i)]$, net of cash outflows induced by lump-sum taxes $[P_{Y,t}T_t(i)]$. Concretely, human wealth originating from endowment of labor and human wealth originating from ownership of capital good firms are the present discounted values of nominal wages and nominal profits respectively accruing in the future to cohorts currently alive. Assuming cohorts exhibit homogeneity in labor productivity and ownership of capital good firms, wage incomes, profits and taxes are all uniformly distributed across cohorts so that they are independent of cohort age $i$.

$$P_{Y,t}HW_t(i) = E_t \sum_{i=0}^{+\infty} \eta_{H,t+i} (1 - \gamma_H)^i P_{Y,t+i}[W_{t+i}H_{t+i}H_t(i)] + \int_0^1 \Gamma_{Y,t+i}(l)dl + \Gamma_{K,t+i} - T_t(i)$$

where $\eta_{H,t+i}$ is equilibrium nominal stochastic discount factor. $(1 - \gamma_H)$ is household survival probability in each period and captures uncertain economic life span.

### 2.1.2. Financial Wealth

Households of cohort $i$ begin trading in financial markets at the age of $i$. Specifically, incumbent households not only obtain payments from government bond holdings and deposit holdings, but also receive dividends from stock holdings. At the beginning of each period, cohort $i$ obtains bond wealth $[BW_t(i)]$ from holding government bonds $[B_t(i)]$:

$$P_{Y,t}BW_t(i) = P_{Y,t}B_t(i)$$

gains deposit wealth $[DW_t(i)]$ from holding deposits $[DE_t(i)]$ in financial intermediaries:

$$P_{Y,t}DW_t(i) = P_{Y,t}DE_t(i)$$

and receives dividends $[\int_0^1 D_t(j)dj]$ from holding stocks $[\int_0^1 S_t(i,j)j]dj$ issued by wholesale firms survived. Concretely speaking, cohort $i$’s nominal stock wealth $[P_{Y,t}SW_t(i)]$ is the

1In period $t$, aggregate household population $= \sum_{i=0}^{\infty} \gamma_H(1 - \gamma_H)^{t-i} = \gamma_H \sum_{i=0}^{t} (1 - \gamma_H)^{t-i} = 1$, where $\gamma_H$ is cohort $i$’s size, and $(1 - \gamma_H)^{t-i}$ is cohort $i$’s accumulated survival probability to period $t$.

2In period 0, cohort $i$’s effective economic decision horizon $= \sum_{i=0}^{+\infty} (1 - \gamma_H)^{i} = \frac{1}{\gamma_H}$. 

5
present discounted value of ex-dividend stock value $\int_0^1 P_{Y,t}P_{si}(j)S_t(i,j) dj$ and nominal dividends $\int_0^1 P_{Y,t}D_t(j)S_t(i,j) dj$:

$$P_{Y,t}SW_t(i) = \int_0^1 P_{Y,t}[P_{si}(j) + D_t(j)]S_t(i,j) dj$$  \hspace{1cm} (4)

### 2.1.3. Budget Constraints

For cohorts of incumbent household traders in financial markets, their intertemporal budget constraints require that nominal expenditure is funded by nominal income. Household traders purchase financial assets at the end of period $t$. On one hand, nominal expenditure includes nominal consumption of final goods $[P_{Y,t}C_t(i)]$, payment of taxes $(P_{Y,t}T_t)$, bank deposits $[P_{Y,t+1}L_{t+1}(i)]$, purchase of government bonds $[B_{t+1}(i)]$ and stocks $[\int_0^1 P_{Y,t}P_{si}(j)S_{t+1}(i,j) dj]$. On the other hand, nominal income includes nominal wages $[P_{Y,t}W_t\nu_{H,t}H_t(i)]$, profits $(P_{Y,t}\Gamma_{K,t})$, government bond wealth $[P_{Y,t}BW_t(i)]$, deposit wealth $[P_{Y,t}DW_t(i)]$ and stock wealth $[P_{Y,t}SW_t(i)]$. Due to the unconnectedness between incumbent and new household traders, cohorts have no bequest motive, triggering private insurance firms to offer insurance risklessly. Specifically, in period $t$, cohort $i$ receives a proportionate financial wealth$^3$ $[\gamma_HFW_t(i)]$ from insurance firms conditional on survival, whereas cohort $i$ pays the whole financial wealth $[FW_t(i)]$ to insurance firms conditional on death. Therefore, the reciprocal of household survival probability $\frac{1}{\gamma_H}$ represents the insurance contract’s gross yield, and this gross yield originates from redistributing financial wealth of households who exit financial markets among households who remain in financial markets within the same cohort. Wholesale firm exit probability $\gamma_M$ captures capital reallocation effect, which stems from reallocating eliminated wholesale firms’ financial resources to wholesale firms survived. Since households have no bequest motive, cohort $i$ consumes all resources and household budget constraint is binding in each period.

$$P_{Y,t}C_t(i) + \int_0^1 P_{Y,t}[P_{si}(j) + D_t(j)]S_{t+1}(i,j) dj + E_t[\eta_{t+1}P_{Y,t+1}B_{t+1}(i)] + \frac{1}{R_{D,t}}E_t[P_{Y,t+1}L_{t+1}(i)] + P_{Y,t}T_t \leq P_{Y,t}W_t\nu_{H,t}H_t(i) + P_{Y,t}\Gamma_{K,t}$$  \hspace{1cm} (5)

$$P_{Y,t}C_t(i) + P_{Y,t}T_t = P_{Y,t}W_t\nu_{H,t}H_t(i) + P_{Y,t}\Gamma_{K,t}$$  \hspace{1cm} (6)

### 2.1.4. Utility Maximization

Since households derive utility from consumption of final goods but suffer disutility from supplying labor to wholesale firms, cohort $i$’s additively separable utility function

$^3$Financial wealth $[FW_t(i)]$ includes government bond wealth $[BW_t(i)]$, deposit wealth $[DW_t(i)]$ and $(1 - \gamma_W)$ proportion of stock wealth $[SW_t(i)]$
is increasing and concave in real consumption \([C_t(i)]\), but is decreasing and convex in labor supply \([H_t(i)]\). The logarithmic utility specification ensures the existence of a balanced-growth path. The intertemporal preference shock \((\nu_{P,t})\) influences periodical utility through exhibiting intertemporal effects upon two adjacent periods’ utilities. The labor utilization shock \((\nu_{H,t})\) perturbs utility by reducing working hours during unexpected public health events. In period 0, the representative household trader of cohort \(i\) chooses optimal sequences of real consumption \([C_t(i)]\), labor supply \([H_t(i)]\), deposit holdings \([L_{t+1}(i)]\), government bond holdings \([B_{t+1}(i)]\) and stock holdings \([\int_0^1 S_{t+1}(i,j)\text{d}j]\) to maximize the expected lifetime utility \([U_0(i)]\), taking into consideration that cohort \(i\) survives to period \(t\) with probability \((1 - \gamma_H)^t\):

\[
U_0(i) = E_0 \sum_{t=0}^{+\infty} \beta^t (1 - \gamma_H)^t \nu_{P,t}\{lnC_t(i) + \varrho ln[1 - \nu_H H_t(i)]\} \tag{7}
\]

subject to a sequence of cohort \(i\)’s intertemporal budget constraints in equation (5) and borrowing constraint of non-negative deposits\(^4\) \([DE_t(i) \geq 0]\), where \(\beta\) is the intertemporal discount factor and captures consumption impatience. \(\varrho \in (0, +\infty)\) is leisure weight.

The intertemporal preference shock \((\nu_{P,t})\) follows an autoregressive process in logs with independently, identically and normally distributed preference innovation \((\varepsilon_{P,t})\), whose mean is 0 and standard deviation is \(\sigma_P\):

\[
\ln\nu_{P,t} = \ln\nu_P + \rho_P (\ln\nu_{P,t-1} - \ln\nu_P) + \varepsilon_{P,t} \varepsilon_{P,t} \sim i.i.d. N(0, \sigma^2_P) \tag{8}
\]

where \(\rho_P \in (0, 1)\) is autoregressive parameter and \(\nu_P\) is steady-state preference shock.

The labor utilization shock captures unexpected public health events’ impacts on labor supply, for instance, the recent COVID-19 epidemic, which is represented by a labor utilization shock with size less than one, makes it unsafe for households to work if their jobs require close interaction with people, and these households stay at home either by choice or by government containment policies. The labor utilization shock \((\nu_{H,t})\) follows an autoregressive process in logs with independently, identically and normally distributed labor utilization innovation \((\varepsilon_{H,t})\), whose mean is 0 and standard deviation is \(\sigma_H\):

\[
\ln\nu_{H,t} = \ln\nu_H + \rho_H (\ln\nu_{H,t-1} - \ln\nu_H) + \varepsilon_{H,t} \varepsilon_{H,t} \sim i.i.d. N(0, \sigma^2_H) \tag{9}
\]

where \(\rho_H \in (0, 1)\) is autoregressive parameter and \(\nu_H\) is steady-state labor utilization shock. Household utility maximization leads to the first order conditions in Appendix A.

Intratemporal condition between real consumption \([C_t(i)]\) and labor \([H_t(i)]\) indicates that the ratio of marginal utility of leisure \([\varrho H_t(i)]\) to marginal utility of consumption \([1/C_t(i)]\) equals the ratio of nominal wage \((P_{Y,t} W_t)\) to nominal final good price \((P_{Y,t})\):

\[
\frac{C_t(i) \varrho H_t(i)}{1 - \varrho H_t(i)} = W_t \tag{10}
\]

\(^4\)Since cohort \(i\) has no collateral assets, it can neither borrow nor issue stocks.
nominal payoffs, as intertemporal substitution in consumption \[ \Delta C_t = \nu \eta \text{, counted by intertemporal discount factor } \eta \text{.} \]

Where \( \nu \) is the intertemporal preference shock, \( \eta \) is marginal utility of consumption, \( \nu P_t \) is final good price.

Intertemporal condition of deposit holdings \( [L_{t+1}(i)] \) conveys that the discount factor for deposits \( (\frac{1}{R_{D,t}}) \) is not smaller than the nominal stochastic discount factor \( (\eta_{t,t+1}) \):

\[
\frac{1}{R_{D,t}} \geq \eta_{t,t+1} \tag{12}
\]

Intertemporal condition of stock holdings \( [S_{t+1}(i,j)] \) elucidates that nominal ex-dividend price \( [P_{Y,t}P_{S,t}(j)] \) equals the present discounted value of future nominal payoffs, which include future ex-dividend stock price \( [P_{Y,t+1}P_{I,t+1}(j)] \) and nominal dividends \( [P_{Y,t+1}D_{t+1}(j)] \):

\[
P_{Y,t}P_{S,t}(j) = (1 - \gamma_W)E_{t}\left\{ \eta_{t,t+1}P_{Y,t+1}[P_{S,t+1}(j) + D_{t+1}(j)] \right\} \tag{13}
\]

Where \( \eta_{t,t+1} \) is nominal stochastic discount factor, \((1 - \gamma_W)\) is firm survival probability.

Since government bonds pay one unit of currency in the next period with full probability, its return equals ex ante gross nominal interest rate \( (R_{n,t}) \). Following no-arbitrage condition, government bonds’ price equals nominal stochastic discount factor \( (\eta_{t,t+1}) \):

\[
R_{n,t}E_{t}\eta_{t,t+1} = E_{t}R_{t+1}E_{t}\pi_{t,t+1}E_{t}\eta_{t,t+1} = 1 \tag{14}
\]

Where \( E_{t}R_{t+1} \) is ex post gross real interest rate, \( E_{t}\pi_{t,t+1} \) is expected gross inflation. Aggregation across cohorts yields generation-specific per capita variables in Appendix B.

**Financial Wealth Effects**

The product of aggregate consumption \( (C_t) \) and the component \( \left( \frac{1}{MPC_t} - 1 \right) \) is the weighted average of expected future government bond market wealth \( E_{t}\left( \eta_{t,t+1}\pi_{t,t+1}BW_{t+1} \right) \) with weight of \( \gamma_H \), expected future deposit wealth \( E_{t}\left( \eta_{t,t+1}\pi_{t,t+1}DW_{t+1} \right) \) with weight of \( \gamma_H \), expected future stock market wealth \( E_{t}\left( \eta_{t,t+1}\pi_{t,t+1}SW_{t+1} \right) \) with weight of \( \gamma_H \), and expected future aggregate consumption \( E_{t}\left( \frac{\eta_{t,t+1}}{MPC_{t+1}}\pi_{t,t+1}C_{t+1} \right) \) with weight of \( (1 - \gamma_H) \).

\[
\left( \frac{1}{MPC_t} - 1 \right)C_t = \gamma_H E_{t}\left( \eta_{t,t+1}\pi_{t,t+1}BW_{t+1} \right) + \gamma_H E_{t}\left( \eta_{t,t+1}\pi_{t,t+1}DW_{t+1} \right) + \gamma_H \left(1 - \gamma_W\right) E_{t}\left( \eta_{t,t+1}\pi_{t,t+1}SW_{t+1} \right) + \left(1 - \gamma_H\right) E_{t}\left( \frac{\eta_{t,t+1}}{MPC_{t+1}}\pi_{t,t+1}C_{t+1} \right) \tag{15}
\]
where $\gamma_H$ is household turnover rate. $\gamma_W$ is firm turnover rate. $MPC_t$ is marginal propensity to consume. $\eta_{t,t+1}$ is nominal stochastic discount factor. $\pi_{t,t+1}$ is future inflation. $C_{t+1}$ is future real aggregate consumption. $BW_{t+1}$ is future real bond market wealth. $DW_{t+1}$ is future real deposit wealth. $SW_{t+1}$ is future real stock market wealth.

Based on the dynamics of aggregate consumption, equation (15) characterizes financial wealth effect originating from bond, deposit and stock markets. Financial wealth effect measures financial wealth’s impact on current aggregate consumption relative to expected future aggregate consumption. On one hand, in financial markets, when household turnover rate $\gamma_H$ increases, more new household traders, who have no financial wealth and cannot smooth consumption, replace incumbent household traders, who possess financial wealth and smooth consumption. Nevertheless, individual consumption smoothing does not imply aggregate consumption smoothing, because households exhibit heterogeneity in financial wealth and consumption. Less consumption smoothing indicates less expected future aggregate consumption ($C_{t+1}$) in comparison with current aggregate consumption ($C_t$). Therefore, larger financial wealth effects are associated with higher household turnover rate $\gamma_H$. On the other hand, larger financial effects are related to more financial wealth expected for the future. Consequently, financial wealth plays essential roles in influencing aggregate consumption, as well as establishing a channel through which financial wealth feeds back into the real economy.

On the flip side of the coin, when firm turnover rate $\gamma_W$ increases, more new wholesale firms replace incumbent wholesale firms. New wholesale firms are more likely to contain stock bubbles than incumbent wholesale firms, because new wholesale firms enter the stock market with a probability of containing stock bubbles, and a stock bubble cannot reemerge in the same incumbent wholesale firm after bursting. More stock bubbles imply looser credit constraints and more financial resources for new wholesale firms in comparison with incumbent wholesale firms. Consequently, higher firm turnover rate amplifies capital reallocation effect, which makes investment move to more productive wholesale firms. Because investment may crowd out consumption, capital reallocation effect contaminates stock market wealth effect by reducing the wedge between current and expected future consumption. Therefore, stock market wealth effect is weakened by capital reallocation effect to some extent, and larger financial wealth effect is associated with lower firm turnover rate $\gamma_W$. Compared with Castelnuovo et al. (2010)’s analysis, financial wealth effect shrinks, because the weight $\gamma_H$ for expected future stock market wealth $E_t(\eta_{t,t+1}\pi_{t,t+1}SW_{t+1})$ is dampened by firm turnover rate $\gamma_M$. Intuitively, as firm turnover rate $\gamma_M$ increases, expected future financial wealth is discounted further to have a smaller weight of $[\gamma_H(1-\gamma_M)]$. Since household turnover rate $\gamma_H$ is usually greater than firm turnover rate $\gamma_M$, stock market wealth effect generally dominates over capital reallocation effect.

### 2.2. Wholesale Firms

We model wholesale firms’ behavior along the lines of Miao et al. (2015) and Ikeda (2013). Wholesale sector is inhabited by a unit continuum of wholesale firms, which exhibit heterogeneity in ages. The $\tau$-period-old wholesale firm, which is established in
period $t-\tau$ and indexed by $j$, enters the stock market by issuing new stocks in period $t$. Wholesale firm $j$ not only purchases investment goods $[I_t(j)]$ from capital good firms at real investment good price ($P_{I,t}$) in a perfectly competitive investment good market, but also employs utilized labor $[\nu_{H,t} H_t(j)]$ from households at real wage ($W_t$) in a perfectly competitive labor market. Wholesale firms produce homogenous wholesale goods and sell them to retail firms in a perfectly competitive wholesale market. Incumbent wholesale firms face a constant probability $\gamma_W$ of exiting financial markets and being replaced by new wholesale firms. Financial resources of exiting wholesale firms are reallocated to wholesale firms survived. Endowed with initial physical capital ($K_{0t}$), new wholesale firms enter the stock market without entrance cost, operate identically to incumbent wholesale firms, and bring new speculative stock bubbles into the stock market with probability $\gamma_0$. Wholesale firm $j$’s production $[M_t(j)]$ combines the total factor productivity shock ($\nu_{A,t}$), utilized labor $[\nu_{H,t} H_t(j)]$ and utilized real physical capital $[U_{K,t}(j) K_t(j)]$ using Cobb-Douglas technology:

$$M_t(j) = \nu_{A,t}[U_{K,t}(j) K_t(j)]^\alpha[\nu_{H,t} H_t(j)]^{1-\alpha}$$

(16)

where $U_{K,t}(j)$ is physical capital utilization rate, $\nu_{H,t}$ is labor utilization shock, $\alpha \in (0, 1)$ and $(1 - \alpha) \in (0, 1)$ are elasticities of wholesale production $[M_t(j)]$ with respect to utilized physical capital $[U_{K,t}(j) K_t(j)]$ and utilized labor $[\nu_{H,t} H_t(j)]$ respectively.

The total factor productivity shock ($\nu_{A,t}$), which measures aggregate efficiency, follows an autoregressive process in logs with independently, identically and normally distributed technology innovation ($\varepsilon_{A,t}$), whose mean is 0 and standard deviation is $\sigma_A$:

$$\ln \nu_{A,t} = \ln \overline{\nu_A} + \rho_A (\ln \nu_{A,t-1} - \ln \overline{\nu_A}) + \varepsilon_{A,t}$$

$$\varepsilon_{A,t} \sim i.i.d. N (0, \sigma_A^2)$$

(17)

where $\rho_A \in (0, 1)$ is autoregressive parameter and $\overline{\nu_A}$ is steady-state total factor productivity shock.

Wholesale firm $j$, as owner of physical capital $[K_t(j)]$, accumulates physical capital $[K_{t+1}(j)]$ by combining undepreciated physical capital $\{1 - \delta_t[U_{K,t}(j)]\} K_t(j)$ with investment $[I_t(j)]$ perturbed by the idiosyncratic marginal efficiency of investment shock $[\nu_{I,t}(j)]$. Physical capital depreciation rate $\{\delta_t[U_{K,t}(j)]\}$ is an increasing, convex and twice continuously differentiable function of physical capital utilization rate $[U_{K,t}(j)]$ with function range $[0,1]$, because more intensive utilization of physical capital induces extra cost and higher depreciation. The optimal physical capital utilization is chosen before the realization of the marginal efficiency of investment shock, consequently, it does not depend on the idiosyncratic marginal efficiency of investment shock.

$$K_{t+1}(j) = \{1 - \delta_t[U_{K,t}(j)]\} K_t(j) + I_t(j) \nu_{I,t}(j)$$

(18)

The marginal efficiency of investment shock $[\nu_{I,t}(j)]$ creates a wedge between current investment $[I_t(j)]$ and future investment $[I_{t+1}(j)]$. When a positive marginal efficiency of investment shock arrives, marginal efficiency of investment increases, firms prepone future investment, and current investment increases. Marginal efficiency of investment shocks are independently, identically and normally distributed across wholesale firms and
over periods, and are drawn from a fixed normal cumulative probability density \((\Phi)\) with mean one and probability density function \(\Phi(\nu_{t, j})\):

Prior to production, wholesale firms finance working capital and investment through issuing stocks \([S_t(j)]\) to households and corporate loans \([L_t(j)]\) to financial intermediaries. We define the \(\tau\)-period-old wholesale firm \(j\)'s value \(V_{t, \tau}[K_t(j), L_t(j), \nu_{t, \tau}(j)]\) as a function of physical capital asset \([K_t(j)]\), corporate loan liability \([L_t(j)]\) and the marginal efficiency of investment shock \(\nu_{t, \tau}(j)\) in period \(t\). Wholesale firm \(j\) invests by purchasing investment goods from capital good firms, obtains working capital by employing labor from households, and pledges a fraction \((\nu_{C, t})\) of physical capital assets \([K_t(j)]\) as a collateral for borrowing from households. The collateral fraction \((\nu_{C, t})\), which represents the collateral shock, captures the magnitude of wholesale firms' financing risk, reflects the extent of financial markets' frictions, and measures the degree of credit markets' imperfections. Specifically, wholesale firm \(j\) faces a credit constraint such that maximum cost of investment \([P_t, I_t(j)]\) and wages of utilized labor \([W_t, \nu_{H, t} H_t(j)]\) equals the present discounted value of wholesale firm \(j\) adjusted by firm survival probability \((1 - \gamma_W)\):

\[
P_{t, I_t(j)} + W_t \nu_{H, t} H_t(j) \leq (1 - \gamma_W) E_t [V_{t+1, \tau+1}[\nu_{C, t} K_t(j), L_{t+1}(j), \nu_{t+1, \tau+1}(j)] \tag{19}\]

Wholesale firm \(j\)'s inflow-of-funds equals outflow-of-funds. Specifically, fund outflows include dividend payout \([D_t(j)]\), corporate loan repayment \([L_t(j)]\), investment \([P_{Y, t} P_{t, I_t(j)}]\) and utilized labor wages \([P_{Y, t} W_t \nu_{H, t} H_t(j)]\), while fund inflows incorporate wholesale production \([P_{W, t} M_t(j)]\) and corporate loan issuance \([\frac{L_{t+1}(j)}{R_{L, t}}]\). Negative dividends represent new equity issuance, whereas negative corporate loans represent corporate saving.

\[
D_t(j) + L_t(j) + P_{Y, t} P_{t, I_t(j)} + P_{Y, t} W_t \nu_{H, t} H_t(j) = P_{W, t} M_t(j) + \frac{L_{t+1}(j)}{R_{L, t}} \tag{20}\]

Wholesale firm \(j\) chooses optimal labor \([H_t(j)]\) to maximize operating profits in terms of wholesale income \([P_{W, t} M_t(j)]\) net of working capital \([P_{Y, t} W_t \nu_{H, t} H_t(j)]\), taking into account of wage \((W_t)\), physical capital utilization rate \((U_{K, t})\), wholesale price \((P_{W, t})\), final good price \((P_{Y, t})\) and the realized labor utilization shock \((\nu_{H, t})\):

\[
\max_{H_t(j)} [P_{W, t} M_t(j) - P_{Y, t} W_t \nu_{H, t} H_t(j)] \tag{21}\]

subject to wholesale production \([M_t(j)]\) in equation (16). Wholesale firms' profit maximization leads to the first order conditions in Appendix C.

The first order condition with respect to labor yields optimal labor \([H_t(j)]\):

\[
H_t(j) = \left[\frac{P_{W, t}(1 - \alpha) \nu_{A, t}}{W_t}\right]^{1/\gamma} U_{K, t}(j) K_t(j) \tag{22}\]

Substituting optimal labor \([H_t(j)]\) into equation (21) yields operating profits generated by physical capital \([R_{K, t} U_{K, t}(j) K_t(j)]\):

\[
R_{K, t} U_{K, t}(j) K_t(j) = \alpha \left[\frac{1 - \alpha \nu_{A, t}^{1/\gamma} P_{W, t}^{1/\gamma}}{W_t}\right]^{1/\gamma} U_{K, t}(j) K_t(j) = P_{W, t} M_t(j) - W_t H_t(j) \tag{23}\]
where return on wholesale firm j’s physical capital $R_{K,t} = \frac{\alpha}{1-\alpha} \left( \frac{1}{\bar{w}_t} \frac{\nu_{E,t}^{1-\alpha} R_{L,t}^{\alpha}}{} \right)^{\frac{1}{1-\alpha}}$ is independent of wholesale firm index j and age $\tau$.

After wage payment, investment is funded by internal funds in terms of physical capital returns and external borrowing, which consists of corporate loans and new equity issuance. Operating profits originating from physical capital $[R_{K,t} U_{K,t}(j) K_t(j)]$ and issuance of corporate loans $[\frac{L_{t+1}(j)}{R_{L,t}}]$ generate fund inflows, while investment $[P_{I,t} I_t(j)]$ and corporate loan repayment $[L_t(j)]$ incur fund outflows. Positive $D_t(j)$ represents dividend payout, which brings about fund outflows, whereas negative $D_t(j)$ represents new equity issuance, which leads to fund inflows.

$$D_t(j) + L_t(j) + P_{I,t} I_t(j) = R_{K,t} U_{K,t}(j) K_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} \tag{24}$$

Wholesale firm j is subject to new equity issuance constraint such that maximum new equity issuance $[-D_t(j)]$ equals physical capital $[K_t(j)]$ perturbed by the equity issuance shock $[\nu_{E,t}]$:

$$D_t(j) \geq -\nu_{E,t} K_t(j) \tag{25}$$

Substituting new equity issuance constraint into wholesale firm j’s flow-of-funds constraint yields:

$$L_t(j) + P_{I,t} I_t(j) \leq R_{K,t} U_{K,t}(j) K_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + \nu_{E,t} K_t(j) \tag{26}$$

Wholesale firm j’s credit constraint mirrors the incentive constraint between wholesale firm j and financial intermediaries in contracting corporate loans. In period t, wholesale firm j applies for corporate loans $[L_{t+1}(j)]$ from financial intermediaries at loan interest rate $(R_{L,t})$. However, conditional on survival, wholesale firm j may default on corporate loans at the beginning of period $t+1$ before the realization of the marginal efficiency of investment shock $[\nu_{I,t+1}(j)]$. Non-default ensures wholesale firm j’s continuation value $E_t\{\eta_{t+1} \int V_{t+1,\tau+1} [K_{t+1}(j), L_{t+1}(j), \nu_{I}(j)] d\Phi[\nu_I(j)]\}$, whereas default results in wholesale firm j’s remaining value after debt renegotiation and repayment relief. In the event of default, wholesale firm j incurs an efficiency loss of $(1 - \nu_{C,t})$ proportion of physical capital assets $[K_t(j)]$, while financial intermediaries, as debt holders, are entitled to seize the collateral fraction $(\nu_{C,t})$ of physical capital assets $[K_t(j)]$, they do not liquidate physical capital assets immediately, but reorganize debts with wholesale firm j and keep running its business for the next period. As long as continuation value of repaying debts is not smaller than continuation value of not repaying debts, wholesale firm j has no incentive to default. Therefore, equation (27) ensures wholesale firms do not default in equilibrium.

$$E_t\{\eta_{t+1} \int V_{t+1,\tau+1} [K_{t+1}(j), L_{t+1}(j), \nu_{I}(j)] d\Phi[\nu_I(j)]\} \geq E_t\{\eta_{t+1} \int V_{t+1,\tau+1} [K_{t+1}(j), 0, \nu_{I}(j)] d\Phi[\nu_I(j)]\} - E_t\{\eta_{t+1} \int V_{t+1,\tau+1} [\nu_{C,t} K_t(j), 0, \nu_{I}(j)] d\Phi[\nu_I(j)]\} \tag{27}$$

where $V_{t+1,\tau+1} [K_{t+1}(j), L_{t+1}(j), \nu_{I,t+1}(j)]$ is the $\tau + 1$-period-old wholesale firm j’s cum-dividend value as a function of physical capital $[K_{t+1}(j)]$, corporate loans $[L_{t+1}(j)]$ and
the marginal efficiency of investment shock \([\nu_{i,t+1}(j)]\) in period \(t+1\).

\[ \int V_{t+1,\tau+1} [\nu_C(j), L_{t+1}(j), \nu_t(j)] d\Phi[\nu_t(j)] \] is wholesale firm j’s ex ante value after integrating over the continuum of idiosyncratic marginal efficiency of investment shock \([\nu_t(j)]\).

The irreversibility of investment ensures its positivity \([I_t(j) \geq 0]\). Wholesale firm j chooses optimal purchase of investment goods \([I_t(j)]\) and issuance of corporate loans \([L_{t+1}(j)]\) to maximize dividends \([D_t(j)]\) represented by value function \(V_{t,\tau}[K_t(j), L_t(j), \nu_{t,\tau}(j)]\):

\[
V_{t,\tau}[K_t(j), L_t(j), \nu_{t,\tau}(j)] = \max_{\{I_t(j) \geq 0, L_{t+1}(j) \geq 0\}} R_{K_t}U_{K_t}(j)K_t(j) - P_t, I_t(j) \\
- L_t(j) + \frac{L_{t+1}(j)}{R_{L_t}} + (1 - \gamma W)E \eta_{t,\tau+1} V_{t+1,\tau+1}[K_{t+1}(j), L_{t+1}(j), \nu_{t,\tau+1}(j)]
\]

subject to physical capital accumulation in equation (18), flow-of-funds constraint:

\[
P_t, I_t(j) + \frac{1 - \alpha}{\alpha} R_{K_t}U_{K_t}(j)K_t(j) \leq (1 - \gamma W)E \eta_{t,\tau+1} V_{t+1,\tau+1}[K_{t+1}(j), L_{t+1}(j), \nu_{t,\tau+1}(j)]
\]

and credit constraint in equation (27).

Given perfectly competitive wholesale market and constant returns to scale production technology, we conjecture that wholesale firm j’s value function \(V_{t,\tau}[K_t(j), L_t(j), \nu_{t,\tau}(j)]\) takes a linear form, specifically, the sum of installed physical capital \([K_t(j)]\) multiplied by wholesale firm j’s marginal value of physical capital \(\{O_{t,\tau}[\nu_{t,\tau}(j)]\}\) net of corporate loans \(\{L_t[\nu_{t,\tau}(j)]\}\). In particular, the \(\tau\)-period-old wholesale firm j’s value may contain a nonzero speculative stock bubble \(\{O_{t,\tau}[\nu_{t,\tau}(j)]\}\) due to its credit constraint, and stock bubble depends on its age \(\tau\).

\[
V_{t,\tau}[K_t(j), L_t(j), \nu_{t,\tau}(j)] = Q_t[\nu_{t,\tau}(j)]K_t(j) + O_{t,\tau}[\nu_{t,\tau}(j)] - \eta_{t,\tau}[\nu_{t,\tau}(j)]L_t(j)
\]

Substituting firm value’s conjectured form into credit constraint of equation (27) conveys that maximum corporate loan should not be greater than collateralized physical capital value \(Q_t^*[\nu_C, K_t(j)]\) and stock bubble \(O_{t,\tau}^*\), because the existence of stock bubble raises firm j’s collateral value and relaxes its credit constraint:

\[
\frac{L_{t+1}(j)}{R_{L,t}} \leq Q_t^*[\nu_C, K_t(j)] + O_{t,\tau}^*
\]

Substituting equation (31) into equation (26) yields new credit constraint:

\[
P_t, I_t(j) \leq R_{K_t}U_{K_t}(j)K_t(j) + Q_t^*[\nu_C, K_t(j)] + \nu_{E,1}K_t(j) + O_{t,\tau}^* - L_t(j)
\]

Substituting firm value’s conjectured form and physical capital accumulation into wholesale firm j’s maximization problem yields:

\[
Q_t(j)K_t(j) + O_{t,\tau}(j) - \eta_{t,\tau}(j)L_t(j) = \max_{\{I_t(j) \geq 0, L_{t+1}(j) \geq 0\}} [R_{K_t}U_{K_t}(j) \\
+(1 - \delta K)Q_t(j)K_t(j) + \frac{Q_t[\nu_{t,\tau}(j)]}{P_t, I_t(j)} - 1]P_t, I_t(j) - L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + O_{t,\tau}^* - L_t
\]

subject to new credit constraint in equation (32).
Wholesale firm j’s ex-dividend stock price \( [P_{St}(j)] \) is the present discounted firm value \( V_{t+\tau+1}[K_{t+1}(j), L_{t+1}(j), \nu_{t,\tau+1}(j)] \) adjusted by wholesale firm survival probability \((1 - \gamma_W)\). Substituting its conjectured form into stock price yields:

\[
P_{St}(j) = (1 - \gamma_W)E_t\{\eta_{t+1}V_{t+1,\tau+1}[K_{t+1}(j), L_{t+1}(j), \nu_{t,\tau+1}(j)]\} = Q_t^* K_{t+1}(j) + O_{t,\tau}^* - L_t^*  
\]

(34)

where we define the average price of installed capital \((Q_t^*)\):

\[
Q_t^* = (1 - \gamma_W)E_t\{\eta_{t+1}Q_{t+1}[\nu_{t,\tau+1}(j)]\}  
\]

(35)

the average value of \(\tau\)-period-old stock bubbles \((O_{t,\tau}^*)\):

\[
O_{t,\tau}^* = (1 - \gamma_W)E_t\{\eta_{t+1}O_{t+1,\tau+1}[\nu_{t,\tau+1}(j)]\}  
\]

(36)

and the average value of corporate loans \((L_t^*)\):

\[
L_t^* = (1 - \gamma_W)E_t\{\eta_{t+1}\eta_{L,t+1}[\nu_{t,\tau+1}(j)]L_{t+1}(j)\}  
\]

(37)

as independent of wholesale firm index j, because idiosyncratic marginal efficiency of investment shock \([\nu_{t,\tau+1}(j)]\) has been integrated out.

Setting up wholesale firm j’s Lagrangean function \([L_{W,t}(j)]\):

\[
L_{W,t}(j) = \{R_{K,t} U_{K,t}(j) + \{1 - \delta(U_{K,t}(j))\}Q_t^* \} K_t(j) + [\frac{Q_t^* \nu_{t,\tau}(j)}{P_t} - 1]P_{I,t} I_t(j) - L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + O_{t,\tau}^* - L_t^* + \lambda_{W,t}(j)\{[R_{K,t} U_{K,t}(j) + Q_t^* \nu_{C,t} + \nu_{E,t}] K_t(j) + O_{t,\tau}^* - L_t(j) - P_{I,t} I_t(j)\}  
\]

(38)

where \(Q_t\) is shadow price of installed physical capital \([K_t(j)]\) and \(\lambda_{W,t}(j)\) is Lagrange multiplier on flow-of-funds constraint.

The first order condition with respect to wholesale firm j’s investment \([I_t(j)]\) yields Lagrange multiplier \([\lambda_{W,t}(j)]\) on flow-of-funds constraint:

\[
\lambda_{W,t}(j) = \frac{Q_t^* \nu_{t,\tau}(j)}{P_t^*} - 1 = \frac{\nu_{t,\tau}(j)}{\nu_{t,\tau}^*} - 1 \geq 0  
\]

(39)

where \(\nu_{t,\tau}^* = \frac{P_t^*}{Q_t^*}\) is marginal investment efficiency threshold. Marginal cost of investment is investment good price \((P_{I,t})\), while marginal benefit of investment is the expected physical capital value \((Q_t^*)\) perturbed by the marginal efficiency of investment shock \([\nu_{t,\tau}(j)]\). The threshold \((\nu_{t,\tau}^*)\) of marginal efficiency of investment shock is the ratio of investment good price \((P_{I,t})\) to the expected physical capital value \((Q_t^*)\). When marginal efficiency of investment shock \([\nu_{t,\tau}(j)]\) is larger than its threshold \((\nu_{t,\tau}^*)\), marginal benefit \([Q_t^* \nu_{t,\tau}(j)]\) is greater than marginal cost \((P_{I,t})\), wholesale firm j invests at full capacity and credit constraint is binding. Otherwise, wholesale firm j makes zero investment. Aggregate stock bubble \((O_t^*)\) influences the average price of installed capital, the threshold of the marginal efficiency of investment shock and therefore the number of investing wholesale firms \([\int_{\nu_t \geq \nu_{t,\tau}^*} d\Phi(\nu_t)]\), creating a capital reallocation effect.
Proposition 1. Because of linearity in investment, optimal investment \( I_t(j) \) is lumpy according to a bang-bang solution, and stock bubble \( (O_{t,t}^*) \) enters optimal investment rule such that higher investment is associated with larger stock bubble.

\[
P_{I,t}I_t(j) = \begin{cases} 
[R_K U_{K,t}(j) + Q_t^* \nu_{C,t} + \nu_{E,t}]K_t(j) + O_{t,t}^* - L_t(j) & \text{if } \nu_{I,t}(j) \geq \nu_{I,t}^* \\
0 & \text{if } \nu_{I,t}(j) < \nu_{I,t}^* 
\end{cases}
\]  

(40)

Proposition 2. Stock bubbles’ expected benefit \( G_t \) includes additional dividends \((\frac{\nu_I}{\nu_{I,t}^*} - 1)\) generated by investment, when the marginal efficiency of investment shock \( [\nu_{I,t}(j)] \) is larger than its threshold \((\nu_{I,t}^*)\):

\[
G_t = \int_{\nu_I \geq \nu_{I,t}^*} \left( \frac{\nu_I}{\nu_{I,t}^*} - 1 \right) d\Phi(\nu_I) \quad (41)
\]

Increases in physical capital utilization rate bring about both cost and benefits. Intuitively, higher physical capital utilization generates investment benefits \((R_{K,t}U_{K,t})\) and additional dividends \((G_t R_{K,t})\) at the cost of faster physical capital depreciation \([\delta_t(U_{K,t})Q_t]\).

Proposition 3. The non-arbitrage condition of installed physical capital \((K_t)\) conveys that wholesale firms choose the same physical capital utilization rate \((U_{K,t})\):

\[
(1 + G_t)R_{K,t} = \delta_t(U_{K,t})Q_t^* \quad (42)
\]

On one hand, some wholesale firms borrow by issuing corporate loans to financial intermediaries. On the other hand, the remaining wholesale firms save by purchasing corporate loans from financial intermediaries, these wholesale firms receive corporate loan interest \((R_{L,t})\) in period t+1 for every one dollar saved in period t, use loan interest to invest, and receive additional dividends \([\frac{\nu_{I,t}(j)}{\nu_{I,t}^*} - 1]\) when the marginal efficiency of investment shock \([\nu_{I,t}(j)]\) is larger than its threshold \((\nu_{I,t}^*)\).

Proposition 4. The asset pricing rule for corporate loan interest \((R_{L,t})\) elucidates that wholesale firms’ one dollar saving equals associated corporate loans’ return, which is the present discounted value of loan interest \((R_{L,t})\) times additional dividends generated by investment \((G_{t+1} R_{L,t})\) conditional on wholesale firm survival with probability \((1 - \gamma_W)\):

\[
\frac{1}{R_{L,t}} = (1 - \gamma_W)E_t[\eta_{t+1}(G_{t+1} + 1)] \quad (43)
\]

Proposition 5. The asset pricing rule for physical capital conveys that the cost equals the expected return of physical capital. Physical capital payoffs include rental returns \((R_{K,t+1}U_{K,t+1})\), undepreciated physical capital \([1 - \delta_t(Q_{t+1})]\), and additional investment’s dividends \([G_{t+1}(R_{K,t+1}U_{K,t+1} + Q_{t+1}^* \nu_{C,t+1} + \nu_{E,t+1})]\).

\[
Q_t^* = (1 - \gamma_W)E_t[\eta_{t+1}R_{K,t+1}U_{K,t+1} + (1 - \delta_t(Q_{t+1})^* + G_{t+1}(R_{K,t+1}U_{K,t+1} + Q_{t+1}^* \nu_{C,t+1} + \nu_{E,t+1})] \quad (44)
\]
Stock bubbles mitigate wholesale firms’ credit constraints and improve borrowing capacity, thus stimulating investment and generating additional benefits. Since these benefits are identical to dividends, stock bubbles emerge and coexist with fundamental assets which yield interest.

**Proposition 6.** The non-arbitrage condition determines the stock bubble size of wholesale firms born in period \( t - \tau \) by ensuring the expected benefits offset the cost \((O_{t, \tau}^*)\) of sustaining stock bubbles. Benefits include expected reselling value \((O_{t+1, \tau+1}^*)\) and associated dividends \((G_{t+1}O_{t+1, \tau+1}^*)\), both of which are discounted by stochastic discount factor \((\eta_{t, t+1})\) and conditional on firm survival with probability \((1 - \gamma_{W})\).

\[
O_{t, \tau}^* = (1 - \gamma_{W})E_t[\eta_{t+1}(G_{t+1} + 1)O_{t+1, \tau+1}^*] \tag{45}
\]

Stock bubble \((O_{t, \tau}^*)\) is not predetermined. One one hand, if no one believes bubbles, then stock bubbles \(\{O_{t+1, \tau+1}^*\}_{i=0}^{\infty}\) cannot exit and remain zero. On the other hand, there exits a non-zero stock bubble in equilibrium. Transversality conditions cannot rule out bubbles, because stock bubbles mitigate credit constraint by increasing firm value and generate additional dividends \((G_t)\) by stimulating firm investment. We model household beliefs about movements of stock bubbles by introducing the sentiment shock \((\nu_{S,t})\).

Households believe stocks may contain a bubble of size \((O_{t, 0}^*)\) with probability \((\gamma_{O})\) in period \( t \), and the total new bubble is \(\gamma_{W} \gamma_{O} o_t^*\). Specifically, households believe the relative size of bubbles in period \( t + \tau \) for any firms established in period \( t \) and period \( t+1 \) is driven by the sentiment shock \((\nu_{S,t})\):

\[
\frac{O_{t+\tau, \tau}^*}{O_{t+\tau, \tau-1}^*} = \nu_{S,t} \tag{46}
\]

The sentiment shock \((\nu_{S,t})\) follows an autoregressive process in logs with independently, identically and normally distributed sentiment innovation \((\varepsilon_{S,t})\), whose mean is 0 and standard deviation is \(\sigma_S\):

\[
\ln\nu_{S,t} = \ln\nu_{S} + \rho_S (\ln\nu_{S,t-1} - \ln\nu_{S}) + \varepsilon_{S,t} \quad \varepsilon_{S,t} \sim i.i.d.N(0, \sigma_S^2) \tag{47}
\]

where \(\rho_S \in (0, 1)\) is autoregressive parameter and \(\nu_{S}\) is steady-state sentiment shock. The sentiment shock, which links the sizes of new and old stock bubbles, captures households’ time-varying beliefs about the evolution of stock bubbles. The bubble growth rate \((\frac{O_{t+1, \tau+1}^*}{O_{t, \tau}^*})\) of wholesale firms born in period \( t - \tau \) satisfies the equilibrium condition in Proposition 6. Wholesale firms’ aggregate behavior is derived in Appendix D.

\[
O_{t, 0}^* = o_t^*, \quad O_{t, 1}^* = \nu_{S,t-1}o_t^*, \quad O_{t, 2}^* = \nu_{S,t-1}o_{S,t-2}o_t^*, \quad \cdots, \quad O_{t, \tau}^* = \prod_{i=1}^{\tau} \nu_{S,t-i}o_t^*, \quad t \geq 0 \quad (48)
\]

Combining new equity issuance of equation (25) and credit constraint of equation (31) yields wholesale firms’ aggregate capacity of external financing, which is the sum of maximum new equity issuance \((\nu_{E,t}K_t)\) and maximum corporate loan issuance \((Q_t^* \nu_{C,t}K_t + O_{t, \tau}^*)\). Defining the financial shock \((\nu_F = \frac{\nu_{E,t}}{Q_t} + \nu_{C,t})\) as the sum of the equity issuance
shock ($\nu_{E,t}$) divided by the expected physical capital value ($Q_{t}^{*}$) and the collateral shock ($\nu_{C,t}$), and substituting the financial shock into aggregate borrowing capacity yield:

$$\nu_{E,t}K_{t} + Q_{t}^{*} \nu_{C,t} K_{t} + O_{t,\tau}^{*} = \left( \frac{\nu_{E,t}}{Q_{t}^{*}} + \nu_{C,t} \right) Q_{t}^{*} K_{t} + O_{t,\tau}^{*} = \nu_{F,t} Q_{t}^{*} K_{t} + O_{t,\tau}^{*} \quad (49)$$

The financial shock captures financial frictions of both new equity issuance constraint in the stock market and collateral constraint in the credit market, and follows an autoregressive process in logs with independently, identically and normally distributed financial innovation ($\varepsilon_{F,t}$), whose mean is 0 and standard deviation is $\sigma_{F}$:

$$\ln \nu_{F,t} = \ln \nu_{F} + \rho_{F} \left( \ln \nu_{F,t-1} - 1 \right) + \varepsilon_{F,t} \quad (50)$$

where $\rho_{F} \in (0, 1)$ is autoregressive parameter and $\overline{\nu_{F}}$ is steady-state financial shock.

Aggregating stock bubbles across all wholesale firms of all ages yields the evolution of aggregate stock market bubble ($O_{t}^{*}$). Exogenous entries with new speculative stock bubbles and exogenous exits with collapsed speculative stock bubbles ensure stationarity of aggregate speculative stock market bubble:

$$O_{t}^{*} = (1 - \gamma_{W}) E_{t} \left[ \eta_{t,t+1} \left( G_{t+1} + 1 \right) u_{S,t} \frac{\nu_{t}}{\nu_{t+1}} O_{t+1}^{*} \right] \quad (51)$$

where $\nu_{t}$ is the mass of wholesale firms with stock bubbles.

Substituting the financial shock ($\nu_{F,t}$) into Proposition 5 yields the evolution of aggregate marginal physical capital value ($Q_{t}^{*}$):

$$Q_{t}^{*} = (1 - \gamma_{W}) E_{t} \eta_{t,t+1} \left[ R_{K,t+1} U_{K,t+1} + (1 - \delta_{t+1}) Q_{t+1}^{*} + G_{t+1} \left( R_{K,t+1} U_{K,t+1} + Q_{t+1}^{*} \nu_{F,t+1} \right) \right] \quad (52)$$

Substituting the financial shock ($\nu_{F,t}$) into Proposition 1 yields the optimal aggregate investment ($I_{t}$):

$$P_{I,t} I_{t} = \left[ \frac{1}{\nu} \left( R_{K,t} U_{K,t} + Q_{t}^{*} \nu_{F,t} \right) K_{t} + O_{t,\tau}^{*} \right] \int_{\nu \geq \nu_{t}} d\Phi(\nu_{t}) \quad (53)$$

2.3. Final Good Firms

Final good firms purchase retail goods at retail price [$P_{Y,t}(l)$] from retail firms, bundle heterogeneous retail goods to assemble homogenous final goods. Final goods are sold to households, capital good firms and the government. Final good production ($Y_{t}$) is a Dixit-Stiglitz aggregator of all retail goods [$Y_{t}(l)$] with retail good index $l \in (0, 1)$:

$$Y_{t} = \left[ \int_{0}^{1} Y_{t}(l)^{\frac{1}{\rho_{Y,t}}} dl \right]^{\rho_{Y,t}} \quad (54)$$

where $\frac{\nu_{Y,t}}{1 - \nu_{Y,t}}$ governs the degree of substitution among differentiated retail goods.

According to Del Negro and Schorfheide (2006), Justiniano, Primiceri and Tambalotti (2010), the final good price markup shock ($\nu_{Y,t}$) reflects time variations in the elasticity of substitution among differentiated retail goods [$Y_{t}(l)$] with retail good index $l \in (0, 1)$.
and captures random variations in retail firms’ market power. Higher price markup shock \( (\nu_{Y,t}) \) signals more inelastic retail good demand and motivates retail firms to charge higher retail price \([P_{Y,t}(l)]\). The final good price markup shock \((\nu_{Y,t})\) follows an autoregressive moving average process in logs with independently, identically and normally distributed price markup innovation \((\varepsilon_{Y,t})\), whose mean is 0 and standard deviation is \(\sigma_Y\), the moving average component captures high frequency variations in inflation:

\[
\ln \nu_{Y,t} = \ln \nu_Y + \rho_Y (\ln \nu_{Y,t-1} - \ln \nu_Y) + \varepsilon_{Y,t} - \xi_Y \varepsilon_{Y,t} \\
\varepsilon_{Y,t} \sim i.i.d.N(0, \sigma^2_Y) \tag{55}
\]

where \(\rho_Y \in (0, 1)\) is autoregressive parameter, \(\xi_Y \in (0, 1)\) is moving average parameter and \(\nu_Y\) is steady-state price markup shock.

In the perfectly competitive final good market, the representative final good firm’s profit \((P_{Y,t}\Gamma_{Y,t})\) is income net of expenditure. Income includes sales of final goods \((P_{Y,t}Y_t)\), and expenditure incorporates purchase of retail goods \(\int_{0}^{1} P_{Y,t}(l)Y_t(l)dl\). The representative final good firm chooses optimal continuum of retail goods \(Y_t(l)\) with retail good index \(l \in (0, 1)\) to maximize its profit \((P_{Y,t}\Gamma_{Y,t})\):

\[
P_{Y,t}\Gamma_{Y,t} = P_{Y,t}Y_t - \int_{0}^{1} P_{Y,t}(l)Y_t(l)dl \tag{56}
\]

subject to final good production in equation (54). Profit maximization and zero profit conditions in Appendix E indicate that final good price \((P_{Y,t})\) is a Dixit-Stiglitz aggregator of all retail prices \((P_{Y,t}(l))\) with retail good index \(l \in (0, 1)\):

\[
P_{Y,t} = \left[ \int_{0}^{1} P_{Y,t}(l) \frac{1}{1-\nu_{Y,t}} dl \right]^{-\nu_{Y,t}} \tag{57}
\]

In equilibrium, retail good \(l\)’s production equals demand \([Y_t(l)]\), which depends on retail good \(l\)’s price \([P_{Y,t}(l)]\), final good price \((P_{Y,t})\), final good production \((Y_t)\) and the final good price markup shock \((\nu_{Y,t})\):

\[
Y_t(l) = \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right]^{\nu_{Y,t}} Y_t \tag{58}
\]

2.4. Retail Firms

There is a unit continuum of infinitely lived and heterogenous retail firms indexed by \(l\). Retail firm \(l\) purchases wholesale goods \((M_t)\) at wholesale price \((P_{W,t})\) from wholesale firms, package homogenous wholesale goods into specialized retail goods \([Y_t(l)]\), and sell them to final good firms at retail price \([P_{Y,t}(l)]\). Calvo price-setting mechanism indicates that there is \((1 - \gamma_P)\) proportion of retail firms reoptimize retail price \([P_{Y,t}(l)]\) and maximize profits subject to demand, however, the remaining \(\gamma_P\) proportion of retail firms cannot adjust retail price \([P_{Y,t}(l)]\) due to price rigidity, which introduces inflation inertia and captures endogenous persistence. The \(\gamma_P\) proportion of retail firms adjust retail price \([P_{Y,t}(l)]\) using the indexing rule, which is a Cobb-Douglas average of past
inflation \((\pi_{t-2,t-1})\) with weight of \(\xi\) and the central bank’s inflation target \(\pi^*\) with weight of \((1 - \xi)\):

\[
P_{Y,t}(l) = \pi_{t-2,t-1}^{\xi} \pi^*^{(1-\xi)} P_{Y,t-1}(l)
\]

where \(\xi\) is price indexation and weight for past inflation.

On the flip side of the coin, inspired by Nisticò (2012), Justiniano, Primiceri and Tambalotti (2010), in the monopolistic competitive retail market, the \((1 - \gamma_p)\) proportion of retail firms choose optimal retail price \([P_{Y,t}(l)]\) to maximize expected profits \([P_{Y,t}\Gamma_{Y,t}(l)]\), and adjust retail price \([P_{Y,t}(l)]\) using the indexing rule \(\prod_{i=0}^{t} \pi_{t+i-2,t+i-1}^{\xi} \pi^*^{(1-\xi)}\) up to period \(t + \tau\) with probability \(\gamma_p\). Since households are owners of retail firms, retail firms weigh profits using equilibrium nominal stochastic discount factor \((\eta_{t,t+\tau})\).

\[
P_{Y,t}\Gamma_{Y,t}(l) = E_t \sum_{i=0}^{+\infty} \beta^i \gamma_p \eta_{t+i} Y_{t+i}(l) \left[ P_{Y,t}(l) \prod_{i=0}^{t} \pi_{t+i-2,t+i-1}^{\xi} \pi^*^{(1-\xi)} - P_{Y,t+\tau} P_{W,t+\tau} \right]
\]

subject to the sequence of retail good \(l\)’s demand \(Y_{t+i}(l) = \left[ \frac{P_{Y,t+i}(l)}{P_{Y,t+i}} \right]^{\gamma/1-\gamma} Y_{t+i} \).

2.5. Capital Good Firms

Capital good sector is populated by a unit continuum of perfectly competitive capital good firms, which are owned by households and manufacture investment goods. Capital good firms purchase final goods \((Y_t)\) from final good firms at final good price \((P_{Y,t})\), transform final goods into investment goods \((I_t)\), and obtain income from selling investment goods \((I_t)\) to wholesale firms at nominal investment good price \((P_{Y,t}I_{t,t})\). However, capital good firms incur final good purchase cost and investment adjustment cost\(^5\) \([J\left(\frac{I_t}{I_{t-1}}\right)] = \Omega \left(\frac{I_t}{I_{t-1}} - 1\right)^2\), which represents accrued losses such as opportunity cost of underutilized goods, obsolescence cost and transition cost during transformation. Capital good firms produce optimal investment goods \((I_t)\) to maximize expected nominal profits \((P_{Y,t}\Gamma_{K,t})\). Since households are owners of capital good firms, capital good firms weigh profits using equilibrium nominal stochastic discount factor \((\eta_{t,t+\tau})\).

\[
P_{Y,t}\Gamma_{K,t} = E_t \sum_{i=0}^{+\infty} \eta_{t+i} \{ P_{Y,t+i} P_{I,t+i} - [1 + J(\frac{I_{t+i}}{I_{t+i-1}})] P_{Y,t+i} \} I_{t+i}
\]

Profit maximization in Appendix F gives the Euler’s equation for optimal production of investment goods \((I_t)\):

\[
[1 + J(\frac{I_{t-1}}{I_{t-1}})] + J'(\frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}} P_{Y,t} - E_t \eta_{t+1} J'(\frac{I_t}{I_{t-1}}) \frac{I_t^2}{I_{t-1}^2} P_{Y,t+1} = P_{Y,t} P_{I,t}
\]

2.6. Financial Intermediaries

Financial intermediaries absorb deposits from households at deposit interest rate \((R_{D,t})\) and issue corporate loans to wholesale firms at loan interest rate \((R_{L,t})\). The perfectly

\(^5\)\(J(\frac{I_t}{I_{t-1}})\) is increasing and concave in \(\frac{I_t}{I_{t-1}}\), and in the steady state, \(J(1) = 0, J'(1) = 0, J''(1) = 2J > 0.\)
competitive credit market implies that deposit interest rate \( (R_{D,t}) \) equals loan interest rate \( (R_{L,t}) \) adjusted by wholesale firm survival probability \( (1 - \gamma_W) \). The risk premium \( \gamma_W \) between deposit interest rate and loan interest rate lies in the fact that corporate loan holders should be compensated for bearing additional exit risk of wholesale firms.

\[ R_{D,t} = (1 - \gamma_W) R_{L,t} \quad (63) \]

As long as stock market bubble’s expected aggregate benefit \( (G_{t+1}) \) is positive, deposit interest rate \( (R_{D,t}) \) is smaller than government bond interest rate \( (R_{n_t}) \). Therefore, households tend to invest in government bonds instead of depositing in financial intermediaries, inducing binding borrowing constraint and zero aggregate deposits \( (DE_t) \). Credit market clearing implies that aggregate deposits \( (DE_t) \) equal aggregate corporate loans \( (L_t) \). Zero aggregate corporate loans \[ L_t = \int_0^1 L_t(j)\,dj \] indicate that some wholesale firms borrow and invest with binding credit constraints, because their marginal efficiency of investment shocks are larger than corresponding threshold, while the remaining wholesale firms save and lend with non-binding credit constraints, because their marginal efficiency of investment shocks are smaller than corresponding threshold.

\[ \frac{1}{R_{D,t}} = \frac{1}{(1 - \gamma_W) R_{L,t}} = E_t[\eta_{t+1}(G_{t+1} + 1)] > \frac{1}{R_{n_t}} \quad (64) \]

In case that stock market bubble’s expected aggregate benefit \( (G_{t+1}) \) is zero, no arbitrage and transversality condition rule out stock bubbles. Since borrowing constraint is not binding and deposit interest rate \( (R_{D,t}) \) equals government bond interest rate \( (R_{n_t}) \), households deposit in financial intermediaries as well as investing in government bonds.

### 2.7. The Central Bank

The real economy is neither perfectly competitive nor perfectly monopolistic, leading to inefficient business cycles and non-neutrality of money. This phenomenon creates an economic stabilizer role for the central bank in the short-term. Although effective banking supervision and credit market regulation moderate stock price fluctuations, they are inefficient in eliminating excess stock market volatility. Therefore, the central bank should incorporate financial slack into monetary policy rule to stabilize stock market volatility. On one hand, the central bank monitors the distortions that induce economic dynamics to deviate from the steady state, as well as implementing monetary policy to pursue stability in output, inflation and the stock market. On the other hand, the central bank influences household and firm behavior through interest rate. The central bank adjusts nominal interest rate* \( (r_{n_t}) \) in response to inflation gap \( (\pi_t) \), output gap \( (y_t) \) and financial slack \( (s_t) \) according to the log-linearized augmented Taylor-type rule:

\[ r_{n_t} = \rho_R r_{n_t-1} + (1 - \rho_R) \left( \Phi_x \pi_t + \Phi_Y y_t + \Phi_S s_t \right) + \nu_{R,t} \quad (65) \]

---

*Interest rate \( (r_{n_t} = R_{n_t} - R_{n}) \) is deviation of nominal interest rate \( (R_{n_t}) \) from its steady state \( R_{n} \).

Inflation gap \( (\pi_t = \pi_{t-1,t} - \pi^*) \) is deviation of gross inflation \( (\pi_{t-1,t}) \) from its target \( \pi^* \).

Output gap \( (y_t = lnY_t - ln\bar{Y}) \) is percentage deviation of output \( (Y_t) \) from its frictionless benchmark \( \bar{Y} \).

Financial slack \( (s_t = lnPs_t - lnPs) \) is percentage misalignment of real stock market index \( (Ps_t) \) from its frictionless level \( \bar{Ps} \).
where the autoregressive parameter $\rho_R$ captures interest rate inertia. $\Phi_\pi$, $\Phi_Y$ and $\Phi_S$ are elasticities of nominal interest rate ($r_n_t$) with respect to inflation gap ($\pi_t$), output gap ($y_t$) and financial slack ($s_t$) respectively.

The Taylor-type interest rate rule is perturbed by the monetary policy shock ($\nu_{R,t}$), which reflects unsystematic monetary policy. A positive monetary policy shock signals a contractionary monetary policy, leading to a rise in nominal interest rate that discourages consumption and investment, whereas a negative monetary policy shock conveys an expansionary monetary policy, leading to a fall in nominal interest rate that stimulates consumption and investment. Monetary policy shocks ($\nu_{R,t}$) are independently, identically and normally distributed innovations ($\varepsilon_{R,t}$) with mean 0 and standard deviation $\sigma_R$:

$$\nu_{R,t} = \varepsilon_{R,t} \sim \text{i.i.d.} N(0, \sigma_R^2)$$  

(66)

2.8. The Government

Market imperfections and price rigidities create a stabilizer role of minimizing economic frictions and price distortions for the government. The government consumes fiscal goods and imposes taxes. Fiscal expenditure includes government spending ($P_{Y,t}G_t$) and buyback of government bonds ($P_{Y,t}G_t$), while fiscal revenue includes taxes ($P_{Y,t}T_t$) and new issuance of government bonds ($P_{Y,t+1}G_{t+1}$) discounted by gross nominal interest rate ($R_{n,t}$). The government budget constraint is:

$$P_{Y,t}G_t + P_{Y,t}G_t = P_{Y,t}T_t + \frac{1}{R_{n,t}}P_{Y,t+1}G_{t+1}$$  

(67)

The government spending shock ($\nu_{G,t}$) is defined as the ratio of government consumption to other sectors’ consumption of final goods ($\frac{G_t}{Y_t - G_t}$). Government spending ($G_t$) is a time-varying fraction ($\frac{\nu_{G,t}}{1+\nu_{G,t}}$) of final good production ($Y_t$). Government spending shocks influence the real economy through government budget constraint. A positive government spending shock stimulates an increase in the ratio of government consumption to other sectors’ consumption of final goods, inducing an increase in government spending ($G_t$) and an increase in taxes ($T_t$). The government spending shock ($\nu_{G,t}$) follows an autoregressive process in logs with independently, identically and normally distributed government spending innovation ($\varepsilon_{G,t}$), whose mean is 0 and standard deviation is $\sigma_G$:

$$\ln \nu_{G,t} = \ln (\overline{\nu_G}) + \rho_G (\ln \nu_{G,t-1} - \ln \overline{\nu_G}) + \varepsilon_{G,t}$$

$$\varepsilon_{G,t} \sim \text{i.i.d.} N(0, \sigma_G^2)$$  

(68)

where $\rho_G \in (0, 1)$ is autoregressive parameter and $\overline{\nu_G}$ is steady-state government spending shock.

2.9. Resource Allocation

In general equilibrium, final good production ($Y_t$) equals demand ($Y_t^D$), which includes consumption ($C_t$), government spending ($G_t$), investment ($I_t$) and investment adjustment
cost $\left[ \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 I_t \right]$

\[ Y_t = Y_t^D = C_t + G_t + \left[ 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t \]

(69)

The clearance of all markets leads to the general equilibrium. Although our DSGE model at the individual level features inequalities, we have not only shown that household borrowing constraints are always binding, but also illustrated the co-existence of wholesale firms with binding credit constraints and those with non-binding credit constraints. Our DSGE model involves no inequalities at the aggregate level. The steady state is calculated and the non-linear system is log-linearized around its steady state.

3. Bayesian Estimation of the DSGE Model

3.1. Data Structure

Our DSGE model is perturbed by nine structural shocks, and we use nine observable variables to identify them. We focus on quarterly U.S. data spanning from Quarter 2 1967 to Quarter 1 2020 covering post-World War II periods. Nine observable variables include nonfarm business sector output index growth rate ($\Delta lnY_t$), personal consumption expenditure of nondurable goods growth rate ($\Delta lnC_t$), gross private domestic investment growth rate ($\Delta lnI_t$), total industry capacity utilization rate ($KU_t$), nonfarm business sector hours of all persons growth rate ($\Delta lnH_t$), Consumer Sentiment Index growth rate ($\Delta lnCS_t$), real S&P 500 Index per capita growth rate ($\Delta lnSP_t$), inflation ($\Delta lnPD_t$), and effective federal funds rate ($FR_t$). Barsky and Sims (2012) indicate that consumer sentiment, which reflects changes in financial situations, should include both ‘news’ component and ‘animal spirits’ component. Inspired by Barsky et al. (2012), Miao et al. (2015), we define Consumer Sentiment Index growth rate ($\Delta lnCS_t$) as a linear function of sample mean of consumer sentiment index growth rate ($\Delta lnCS$), the sentiment shock ($\nu_{ST,t}$), and log-linearized output growth rate ($y_t - y_{t-1}$). The sentiment

---

10 Quarterly and monthly U.S. data originate from Federal Reserve Bank of St. Louis, University of Michigan and Yahoo Finance. Monthly data is transformed into quarterly data by taking mean values. In order to obtain normalized real per capita values, nonfarm business sector output index, personal consumption expenditure of nondurable goods, gross private domestic investment, Consumer Sentiment Index and S&P 500 Index are deflated by both Nonfarm Business Sector Implicit Price Deflator ($PD_t$) and civilian labor force over 16, while nonfarm business sector hours of all persons are divided by civilian labor force over 16. All data series are seasonally adjusted. All variables except effective federal funds rate are transformed into log-differences to ensure stationarity. All data are not percentualized.

11 Since Miao et al. (2015) indicate a high correlation between Consumer Sentiment Index and the smoothed sentiment shock, missing values of Consumer Sentiment Index are interpolated, and real Consumer Sentiment Index per capita growth rate ($\Delta lnCS_t$) is log-difference of Consumer Sentiment Index adjusted by Nonfarm Business Sector Implicit Price Deflator ($PD_t$) and civilian labor force.

12 Real S&P 500 Index per capita growth rate ($\Delta lnSP_t$) is log-difference of ex-dividend closing price of S&P 500 Index, which is adjusted by Nonfarm Business Sector Implicit Price Deflator ($PD_t$) and civilian labor force. ARCH test and Breush-Pagan test are conducted upon quarterly S&P 500 Index growth rate and no significant conditional heteroscedasticity is found.

13 Inflation ($\Delta lnPD_t$) is log-difference of Non-farm Business Sector Implicit Price Deflator ($PD_t$).
shock ($\nu_{ST,t}$) corresponds to 'animal spirits' component, while output deviation ($y_t - y_{t-1}$) corresponds to 'fundamental news' component.

$$\Delta \ln CS_t = \Delta \ln CS + \rho_V \nu_{ST,t} + \rho_Y (y_t - y_{t-1})$$  \hspace{1cm} (70)

Matrix of linearized measurement equations is:

$$\begin{bmatrix}
\Delta \ln Y_t \\
\Delta \ln C_t \\
\Delta \ln I_t \\
\Delta \ln H_t \\
\Delta \ln CS_t \\
\Delta \ln SP_t \\
\Delta \ln PD_t \\
FR_t
\end{bmatrix} = \begin{bmatrix}
\Delta \ln Y \\
\Delta \ln C \\
\Delta \ln I \\
\Delta \ln H \\
\Delta \ln CS \\
\Delta \ln SP \\
\Delta \ln PD \\
FR
\end{bmatrix} + \begin{bmatrix}
y_t - y_{t-1} \\
c_t - c_{t-1} \\
i_t - i_{t-1} \\
u_t - u_{t-1} \\
h_t - h_{t-1} \\
s_t \\
\pi_t \\
\tau n_t
\end{bmatrix}$$  \hspace{1cm} (71)

where observable variables with lines above heads represent their corresponding sample means. $y_t$ is log-linearized final good production. $c_t$ is log-linearized real consumption. $i_t$ is log-linearized real investment. $u_t$ is log-linearized physical capital utilization rate. $h_t$ is log-linearized real labor. $\nu_{ST,t}$ is the sentiment shock. $\rho_V$ and $\rho_Y$ are coefficients of the sentiment shock ($\nu_{ST,t}$) and output growth rate ($y_t - y_{t-1}$) respectively in consumer sentiment measurement equation. $s_t$ is percentage deviation of real stock market index from its frictionless level. $\pi_t$ is deviation of inflation ($\pi_{t-1,t}$) from its target $\pi^*$. $\tau n_t$ is deviation of interest rate from its steady state.

### 3.2. Calibration of Parameters

We calibrate a subset of parameters at quarterly frequency based on the literature, microeconomic data and long-term averages of macroeconomic aggregates in Table 1. We set prior distributions of the remaining parameters in Table 2.

<table>
<thead>
<tr>
<th>Structural Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Quarterly gross inflation target</td>
<td>$\pi^*$</td>
<td>1.005</td>
</tr>
<tr>
<td>Physical capital share</td>
<td>$\alpha$</td>
<td>0.3</td>
</tr>
<tr>
<td>Physical capital depreciation rate</td>
<td>$\delta_K$</td>
<td>0.025</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$\iota$</td>
<td>2</td>
</tr>
<tr>
<td>Fraction of new wholesale entrants with stock bubbles</td>
<td>$\gamma_O$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: As annual gross inflation target is $1.02=\pi^{*4}$, quarterly gross inflation target $\pi^* = 1.005$.

### 3.3. Prior and Posterior Distributions of Parameters

We estimate DSGE models with Bayesian techniques and implement them using Dynare in MatLab. Specifically, we initially evaluate the likelihood function using the Kalman
filter, then combine the likelihood function and the prior distributions to calculate the posterior distributions, finally simulate the posterior kernel using Chib and Ramamurthy (2010)’s Tailored Randomized Block Metropolis-Hastings (TaRB-MH) sampling algorithm. According to trace plots and multivariate MCMC diagnostics, all structural parameters converge to their ergodic distributions. Table 2 presents prior means, prior standard deviations, posterior means and 90% highest probability density intervals for all parameters obtained using Metropolis-Hastings algorithm.

Our estimated interest rate inertia $\rho_R$ is 0.873 with 90% confidence interval of [0.856, 0.890], which are significantly higher than those of Castelnovo et al. (2010)\(^\text{14}\). The higher persistence in interest rate dynamics indicates that the central bank exhibits a stronger tendency to smooth interest rate. In accordance with the Taylor principle, our estimated monetary policy response to inflation gap $\Phi_\pi$ is 1.177 with 90% confidence interval of [1.114, 1.259], which are smaller than those of Castelnovo et al. (2010)\(^\text{15}\). Our estimated monetary policy response to output gap $\Phi_Y$ is 0.723 with 90% confidence interval of [0.625, 0.873], which are significantly larger than those of Castelnovo et al. (2010)\(^\text{16}\). This is presumably because our DSGE model accounts for investment variations induced by variable capital utilization and the marginal efficiency of investment shock. Our estimated monetary policy response to financial slack $\Phi_S$ is 0.134 with 90% confidence interval of [0.108, 0.168], which are larger than those of Castelnovo et al. (2010)\(^\text{17}\). This is because our DSGE model captures stock price fluctuations induced by the sentiment shock, in addition to stock price misalignments triggered by the financial shock.

Our estimated firm turnover rate $\gamma_W$ is 0.018 with 90% confidence interval of [0.012, 0.023]. These indicate that approximately 1.8% of incumbent wholesale firms are replaced by new wholesale firms in the stock market, as well as implying that wholesale firms’ effective decision horizon ranges approximately between 44 quarters (11 years) and 84 quarters (21 years) on average. Our estimated firm turnover rate is lower than Miao et al. (2015)’s calibrated firm exit rate, although the increment is not statistically significant\(^\text{18}\). Our estimated household turnover rate $\gamma_H$ is 0.138 with 90% confidence interval of [0.131, 0.145], which are lower than those of Castelnovo et al. (2010), although the difference is not statistically significant\(^\text{19}\). These not only indicate that approximately 13.8% of incumbent household traders with financial wealth are replaced by new household traders without financial wealth in financial markets, but also imply that household effective decision horizon ranges between 7 quarters (1.8 years) and 8 quarters (2 years) on average. Intuitively, these are consistent with the fact that household turnover rate is larger than firm turnover rate, because firms generally incur higher cost of entering or exiting the

\(^{14}\)Their estimated interest rate inertia is 0.753 with 90% confidence interval of [0.707, 0.803].

\(^{15}\)Their estimated monetary policy response to inflation gap is 1.675 with 90% confidence interval of [1.449, 1.883].

\(^{16}\)Their estimated monetary policy response to output gap is 0.023 with 90% confidence interval of [0.006, 0.040].

\(^{17}\)Their estimated monetary policy response to financial slack is 0.1181 with 90% confidence interval of [0.0716, 0.1655].

\(^{18}\)Their calibrated firm exit rate of 0.02 is inside our estimated firm turnover rate’s 90% confidence interval of [0.012, 0.023].

\(^{19}\)Their estimated household turnover rate is 0.129 with 90% confidence interval of [0.080, 0.183].
stock market.
<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Prior Distribution</th>
<th>Posterior Mean</th>
<th>Posterior Bands [5th, 95th]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household turnover rate</td>
<td>$\gamma_H$</td>
<td>U[0,1]</td>
<td>0.138</td>
<td>[0.131, 0.145]</td>
</tr>
<tr>
<td>Firm turnover rate</td>
<td>$\gamma_W$</td>
<td>U[0,1]</td>
<td>0.018</td>
<td>[0.012, 0.023]</td>
</tr>
<tr>
<td>Leisure weight</td>
<td>$\varrho$</td>
<td>G(1.3,1)</td>
<td>1.179</td>
<td>[1.158, 1.199]</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calvo probability</td>
<td>$\gamma_P$</td>
<td>B(0.75,0.05)</td>
<td>0.782</td>
<td>[0.766, 0.804]</td>
</tr>
<tr>
<td>Price indexation</td>
<td>$\xi$</td>
<td>B(0.5,0.15)</td>
<td>0.274</td>
<td>[0.226, 0.321]</td>
</tr>
<tr>
<td><strong>Central Bank</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rate inertia</td>
<td>$\rho_R$</td>
<td>B(0.7,0.15)</td>
<td>0.873</td>
<td>[0.856, 0.890]</td>
</tr>
<tr>
<td>Response to inflation gap</td>
<td>$\Phi_\pi$</td>
<td>G(1,0.25)</td>
<td>1.177</td>
<td>[1.114, 1.259]</td>
</tr>
<tr>
<td>Response to output gap</td>
<td>$\Phi_Y$</td>
<td>G(0.5,0.25)</td>
<td>0.723</td>
<td>[0.624, 0.873]</td>
</tr>
<tr>
<td>Response to financial slack</td>
<td>$\Phi_S$</td>
<td>N(0,0.25)</td>
<td>0.134</td>
<td>[0.108, 0.168]</td>
</tr>
<tr>
<td><strong>AR(1) Coefficients of Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intertemporal preference</td>
<td>$\rho_P$</td>
<td>B(0.5,0.2)</td>
<td>0.500</td>
<td>[0.468, 0.533]</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>$\rho_A$</td>
<td>B(0.5,0.2)</td>
<td>0.804</td>
<td>[0.725, 0.875]</td>
</tr>
<tr>
<td>Final good price markup</td>
<td>$\rho_Y$</td>
<td>B(0.6,0.2)</td>
<td>0.759</td>
<td>[0.736, 0.783]</td>
</tr>
<tr>
<td>Labor utilization</td>
<td>$\rho_H$</td>
<td>B(0.5,0.2)</td>
<td>0.841</td>
<td>[0.827, 0.854]</td>
</tr>
<tr>
<td>Financial</td>
<td>$\rho_F$</td>
<td>B(0.5,0.2)</td>
<td>0.348</td>
<td>[0.278, 0.399]</td>
</tr>
<tr>
<td>Government spending</td>
<td>$\rho_G$</td>
<td>B(0.5,0.2)</td>
<td>0.825</td>
<td>[0.807, 0.841]</td>
</tr>
<tr>
<td>Sentiment</td>
<td>$\rho_{ST}$</td>
<td>N(0,0.25)</td>
<td>0.706</td>
<td>[0.674, 0.743]</td>
</tr>
<tr>
<td><strong>MA(1) Coefficients of Innovations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final good price markup</td>
<td>$\rho_Y$</td>
<td>B(0.5,0.2)</td>
<td>0.349</td>
<td>[0.309, 0.398]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Deviation of Innovations</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal preference</td>
<td>$\sigma_P$</td>
<td>IG(0.01,Inf)</td>
<td>0.076</td>
<td>[0.069, 0.083]</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>$\sigma_A$</td>
<td>IG(0.01,Inf)</td>
<td>0.007</td>
<td>[0.006, 0.009]</td>
</tr>
<tr>
<td>Final good price markup</td>
<td>$\sigma_M$</td>
<td>IG(0.01,Inf)</td>
<td>0.029</td>
<td>[0.022, 0.036]</td>
</tr>
<tr>
<td>Labor utilization</td>
<td>$\sigma_H$</td>
<td>IG(0.01,Inf)</td>
<td>0.015</td>
<td>[0.013, 0.017]</td>
</tr>
<tr>
<td>Investment</td>
<td>$\sigma_I$</td>
<td>IG(0.01,Inf)</td>
<td>0.056</td>
<td>[0.564, 0.742]</td>
</tr>
<tr>
<td>Financial</td>
<td>$\sigma_F$</td>
<td>IG(0.01,Inf)</td>
<td>0.074</td>
<td>[0.068, 0.081]</td>
</tr>
<tr>
<td>Government spending</td>
<td>$\sigma_G$</td>
<td>IG(0.01,Inf)</td>
<td>0.061</td>
<td>[0.056, 0.066]</td>
</tr>
<tr>
<td>Sentiment</td>
<td>$\sigma_{ST}$</td>
<td>IG(0.1,Inf)</td>
<td>0.162</td>
<td>[0.053, 0.072]</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$\sigma_R$</td>
<td>IG(0.01,Inf)</td>
<td>0.015</td>
<td>[0.013, 0.017]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Measurement Equations’ Coefficients</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of sentiment shock</td>
<td>$\rho_V$</td>
<td>U(1, 2.89)</td>
<td>1.321</td>
<td>[1.197, 1.426]</td>
</tr>
<tr>
<td>Coefficient of output growth</td>
<td>$\rho_Y$</td>
<td>U(1, 2.89)</td>
<td>1.136</td>
<td>[1.050, 1.251]</td>
</tr>
</tbody>
</table>

| Log-marginal Likelihood: | 3715.1 |

Note: B is beta distribution. U is uniform distribution. N is normal distribution. G is gamma distribution. IG is inverse gamma distribution. Prior means and prior standard deviations are in brackets. Inf denotes infinity. [5th,95th] posterior percentiles represent 90% highest probability densities. Prior information is based on the literature, microeconomic data and long-term averages of macroeconomic aggregates.
### 3.4. Model Comparison

Table 3: Baseline DSGE Model and Its Variants

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline</th>
<th>Variant 1</th>
<th>Variant 2</th>
<th>Variant 3</th>
<th>Variant 4</th>
<th>Variant 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm turnover rate $\gamma_W$</td>
<td>0.018</td>
<td>–</td>
<td>0.020</td>
<td>0.022</td>
<td>0.025</td>
<td>0.027</td>
</tr>
<tr>
<td>Household turnover rate $\gamma_H$</td>
<td>0.138</td>
<td>0.168</td>
<td>–</td>
<td>0.147</td>
<td>0.156</td>
<td>0.162</td>
</tr>
<tr>
<td>Calvo probability $\gamma_P$</td>
<td>[0.766, 0.804]</td>
<td>0.886</td>
<td>0.791</td>
<td>0.753</td>
<td>0.882</td>
<td>0.745</td>
</tr>
<tr>
<td>Price indexation $\xi$</td>
<td>0.274</td>
<td>0.119</td>
<td>0.204</td>
<td>0.217</td>
<td>0.266</td>
<td>0.213</td>
</tr>
<tr>
<td>Leisure weight $\varrho$</td>
<td>[1.158, 1.199]</td>
<td>1.243</td>
<td>1.213</td>
<td>1.285</td>
<td>1.294</td>
<td>1.186</td>
</tr>
<tr>
<td>Interest rate inertia $\rho_R$</td>
<td>0.873</td>
<td>0.835</td>
<td>0.869</td>
<td>0.884</td>
<td>0.807</td>
<td>0.863</td>
</tr>
<tr>
<td>Response to inflation gap $\Phi_\pi$</td>
<td>[1.114, 1.259]</td>
<td>0.992</td>
<td>1.108</td>
<td>0.998</td>
<td>1.110</td>
<td>1.101</td>
</tr>
<tr>
<td>Response to output gap $\Phi_Y$</td>
<td>0.723</td>
<td>0.831</td>
<td>0.698</td>
<td>0.615</td>
<td>0.846</td>
<td>0.653</td>
</tr>
<tr>
<td>Response to financial slack $\Phi_S$</td>
<td>[0.108, 0.168]</td>
<td>0.190</td>
<td>0.139</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Response to the sentiment shock ($\nu_{ST,t}$)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.202</td>
<td>–</td>
</tr>
<tr>
<td>Response to the financial shock ($\nu_{F,t}$)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.231</td>
</tr>
<tr>
<td>Logmarginal Likelihood</td>
<td>3715.1</td>
<td>3695.1</td>
<td>3672.7</td>
<td>3708.9</td>
<td>3710.2</td>
<td>3712.7</td>
</tr>
</tbody>
</table>

Note: Variant 1: Firm turnover rate $\gamma_W = 0$. Variant 2: Household turnover rate $\gamma_H = 0$.
Variant 3: No monetary policy response to financial slack $\Phi_S = 0$.
Variant 4: Monetary policy response to the sentiment shock ($\nu_{ST,t}$).
Variant 5: Monetary policy response to the financial shock ($\nu_{F,t}$).
Posterior means in bold face indicate significance. Variants use the same observables so that log-marginal likelihoods are comparable.

We not only comparatively assess performances of baseline DSGE model and its variants, but also evaluate financial wealth effect, capital reallocation effect, monetary policy responses to financial slack, the sentiment shock and the financial shock in Table 3. The
second column describes our baseline DSGE model, which is characterized by the central bank’s responsiveness to financial slack, positive household turnovers and positive wholesale firm turnovers.

With the aim of capturing capital reallocation effect through firm turnover rate $\gamma_W$, the third column describes the baseline DSGE model’s variant with the constraint of zero firm turnover rate ($\gamma_W = 0$), which indicates that wholesale firms live infinitely and trade forever in the stock market. The difference between the second and third columns evaluates capital reallocation effect. When firm turnover rate $\gamma_W$ shrinks to 0, price indexation $\xi$, interest rate inertia $\rho_R$, and monetary policy response to inflation gap $\Phi_\pi$ all decrease, whereas Calvo probability $\gamma_P$, monetary policy response to output gap $\Phi_Y$ and monetary policy response to financial slack $\Phi_S$ all increase. The increase of monetary policy response to financial slack $\Phi_S$ accounts for the missing direct transmission channel of capital reallocation effect from stock market index ($P_{st}$) to investment ($I_t$) and physical capital ($K_t$). The decreases of Calvo probability $\gamma_P$ and price indexation $\xi$ originate from more uncertainty in aggregate demand, higher motivation in changing prices and lower stickiness to past inflation. Log-marginal likelihood is 20 log points smaller than that of our baseline DSGE model, and the difference supports our baseline DSGE model with capital reallocation effect.

With the aim of capturing household financial wealth effect through household turnover rate $\gamma_H$, the fourth column describes the baseline DSGE model’s variant with the constraint of zero household turnover rate ($\gamma_H = 0$), which indicates that households live infinitely and trade forever in financial markets. The difference between the second and fourth columns evaluates financial wealth effect. When household turnover rate $\gamma_H$ shrinks to 0, Calvo probability $\gamma_P$, price indexation $\xi$, monetary policy response to inflation gap $\Phi_\pi$ and monetary policy response to output gap $\Phi_Y$ all decrease significantly, whereas interest rate inertia $\rho_R$ and monetary policy response to financial slack $\Phi_S$ all increase. The increase of monetary policy response to financial slack $\Phi_S$ accounts for the missing direct transmission channel of financial wealth effect from stock market index ($P_{st}$) to consumption ($C_t$) and aggregate demand ($Y_t^D$). The decreases of Calvo probability $\gamma_P$ and price indexation $\xi$ originate from higher uncertainty in aggregate demand, higher motivation in changing prices and lower stickiness to past inflation. Log-marginal likelihood is 27.7 log points smaller than that of our baseline DSGE model, and the difference supports our baseline DSGE model with financial wealth effect.

For the purpose of evaluating monetary policy response to financial slack $\Phi_S$, the fifth column describes the baseline DSGE model’s variant with the constraint that the central bank does not respond to financial slack ($\Phi_S = 0$), which assumes that the central bank conducts monetary policy regardless of stock price misalignments. The difference between the second and fifth columns evaluates monetary policy response to financial slack. When the central bank disregards stock price misalignments, Calvo probability $\gamma_P$, price indexation $\xi$, monetary policy response to inflation gap $\Phi_\pi$ and monetary policy response to output gap $\Phi_Y$ all decrease significantly, firm turnover rate $\gamma_W$ and interest rate inertia $\rho_R$ both increase substantially. The non-negligible increase of interest rate inertia $\rho_R$ acts as the replacement of monetary policy response to financial slack ($s_t$). The substantial decrease in Calvo probability $\gamma_P$ originates from higher uncertainty in
aggregate demand and higher motivation in reoptimizing prices. Log-marginal likelihood is 6.2 log points smaller than that of our baseline DSGE model, and the difference supports our baseline DSGE model with monetary policy response to financial slack.

With the intention of assessing to what extent the central bank responds to sentiment, the sixth column describes the DSGE model with the monetary policy rule that the central bank responds to the sentiment shock \( \nu_{ST,t} \) instead of financial slack \( s_t \):

\[
\begin{align*}
   r_{nt} &= \rho_R r_{nt-1} + (1 - \rho_R) \left( \Phi_\pi \pi_t + \Phi_Y y_t + \Phi_S \nu_{ST,t} \right) + \nu_{R,t}
\end{align*}
\]

The difference between the second and sixth columns evaluates monetary policy response to the sentiment shock \( \nu_{ST,t} \). When the central bank responds to the sentiment shock \( \nu_{ST,t} \), firm turnover rate \( \gamma_W \) and household turnover rate \( \gamma_H \) both increase significantly, monetary policy response to the sentiment shock \( \nu_{ST,t} \) increases moderately than to financial slack \( s_t \), Calvo probability \( \gamma_P \), price indexation \( \xi \) and monetary policy response to inflation gap \( \Phi_\pi \) all decrease significantly, whereas interest rate inertia \( \rho_R \) both increase substantially. These changes originate from more uncertainties in monetary policy and aggregate demand. Log-marginal likelihood is 5.1 log points smaller than that of our baseline DSGE model, and the difference supports our baseline DSGE model with monetary policy response to financial slack.

In the hope of assessing to what extent the central bank responds to the financial shock, the seventh column describes our baseline DSGE model with the monetary policy rule that the central bank responds to the financial shock \( \nu_{F,t} \) instead of financial slack \( s_t \):

\[
\begin{align*}
   r_{nt} &= \rho_R r_{nt-1} + (1 - \rho_R) \left( \Phi_\pi \pi_t + \Phi_Y y_t + \Phi_S \nu_{F,t} \right) + \nu_{R,t}
\end{align*}
\]

The difference between the second and seventh columns evaluates monetary policy response to the financial shock \( \nu_{F,t} \). When the central bank responds to the financial shock \( \nu_{F,t} \), household turnover rate \( \gamma_H \) and firm turnover rate \( \gamma_M \) both increase significantly, monetary policy response to the financial shock \( \nu_{F,t} \) increases moderately than to financial slack \( s_t \), Calvo probability \( \gamma_P \), price indexation \( \xi \) and monetary policy response to inflation gap \( \Phi_\pi \) all decrease significantly, whereas interest rate inertia \( \rho_R \) both increase substantially. These changes originate from higher uncertainties in monetary policy and aggregate demand. Log-marginal likelihood is 2.4 log points smaller than that of our baseline DSGE model, and the difference supports our baseline DSGE model with monetary policy response to financial slack.

All DSGE models’ monetary policy responses to stock price misalignments are significantly different from zero. Our baseline DSGE model has the largest log-marginal likelihood and is therefore supported by data. We conclude that a systematic component of monetary policy reacts to financial instability. We use log marginal likelihoods in Table 3 to formulate twice the logs of Bayes factors for model comparison in Table 4. According to Kass and Raftery (1995)’s criterion, our baseline DSGE model exhibits best performance in comparison with its five variants, and their differences in terms of Bayes Factors are significant. Therefore, we stick to our baseline DSGE model.

\footnote{Log-linearized sentiment shock \( \nu_{ST,t} = \ln \nu_{ST,t} - \ln \nu_{ST} = \ln \nu_{ST,t} \) is percentage deviation of the sentiment shock \( \nu_{ST,t} \) from its steady state \( \nu_{ST} \).}
Table 4: Bayesian Model Comparison between Baseline DSGE Model and Its Variants

<table>
<thead>
<tr>
<th>Models</th>
<th>Twice the Log of Bayes Factor</th>
<th>Strength of Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>$2\ln(BF_{Baseline,Variant1})$</td>
<td>Baseline Model is more supported by data than Variant 1 and strength is very strong</td>
</tr>
<tr>
<td>and Variant 1</td>
<td>$=2[\ln P(X</td>
<td>Baseline) - \ln P(X</td>
</tr>
<tr>
<td>Baseline</td>
<td>$2\ln(BF_{Baseline,Variant2})$</td>
<td>Baseline Model is more supported by data than Variant 2 and strength is very strong</td>
</tr>
<tr>
<td>and Variant 2</td>
<td>$=2[\ln P(X</td>
<td>Baseline) - \ln P(X</td>
</tr>
<tr>
<td>Baseline</td>
<td>$2\ln(BF_{Baseline,Variant3})$</td>
<td>Baseline Model is more supported by data than Variant 3 and strength is very strong</td>
</tr>
<tr>
<td>and Variant 3</td>
<td>$=2[\ln P(X</td>
<td>Baseline) - \ln P(X</td>
</tr>
<tr>
<td>Baseline</td>
<td>$2\ln(BF_{Baseline,Variant4})$</td>
<td>Baseline Model is more supported by data than Variant 4 and strength is strong</td>
</tr>
<tr>
<td>and Variant 4</td>
<td>$=2[\ln P(X</td>
<td>Baseline) - \ln P(X</td>
</tr>
<tr>
<td>Baseline</td>
<td>$2\ln(BF_{Baseline,Variant5})$</td>
<td>Baseline Model is more supported by data than Variant 5 and strength is positive</td>
</tr>
<tr>
<td>and Variant 5</td>
<td>$=2[\ln P(X</td>
<td>Baseline) - \ln P(X</td>
</tr>
</tbody>
</table>

Note: 'BF' denotes 'Bayes Factor'. 'X' denotes actual observations. 'ln' denotes natural logarithm. Kass and Raftery (1995) suggest twice the log of Bayes Factor criterion:
- $2\ln BF \in (0, 2)$: Strength of evidence is only worth a bare mention.
- $2\ln BF \in (2, 6)$: Strength of evidence is positive.
- $2\ln BF \in (6, 10)$: Strength of evidence is strong.
- $2\ln BF \in (10, +\infty)$: Strength of evidence is very strong.

3.5. Model Validation

For the purpose of evaluating empirical performance, we compute model moments using simulated data over 100000 periods from the estimated baseline DSGE model and its variants by taking posterior means as parameter values, and compare simulated moments implied by DSGE models with actual moments measured from observable data. Table 5 summarizes standard deviations, ratios of standard deviations, autocorrelations and cross-correlations of simulated observable variables. We find that our baseline DSGE model matches data closely and its five variants replicate data reasonably well except for four moments, namely, stock price volatility, consumer sentiment volatility, correlation between stock price and output, correlation between consumer sentiment and output. Model moments of our baseline DSGE model and its variants accord well with conventional wisdom. According to standard deviations, consumption ($\Delta \ln C_t$), investment ($\Delta \ln I_t$), labor supply ($\Delta \ln H_t$), sentiment ($\Delta \ln CS_t$) and stock price ($\Delta \ln SP_t$) all exhibit more volatility in comparison with output ($\Delta \ln Y_t$). According to cross-correlations, output ($\Delta \ln Y_t$), consumption ($\Delta \ln C_t$), investment ($\Delta \ln I_t$) and labor supply ($\Delta \ln H_t$) display procyclicality, exhibits acyclicality, whereas inflation ($\Delta \ln PD_t$) and interest rate ($FR_t$)
convey countercyclicality.
Table 5: Business Cycle Statistics

<table>
<thead>
<tr>
<th>Observables</th>
<th>Data</th>
<th>Baseline</th>
<th>Variant 1</th>
<th>Variant 2</th>
<th>Variant 3</th>
<th>Variant 4</th>
<th>Variant 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔlnYt</td>
<td>1.06</td>
<td>1.14</td>
<td>0.91</td>
<td>0.85</td>
<td>0.89</td>
<td>1.20</td>
<td>1.22</td>
</tr>
<tr>
<td>ΔlnCt</td>
<td>1.10</td>
<td>1.25</td>
<td>0.93</td>
<td>0.91</td>
<td>0.87</td>
<td>1.29</td>
<td>1.31</td>
</tr>
<tr>
<td>ΔlnIt</td>
<td>3.77</td>
<td>3.85</td>
<td>3.56</td>
<td>3.35</td>
<td>3.64</td>
<td>4.07</td>
<td>4.01</td>
</tr>
<tr>
<td>KT</td>
<td>4.17</td>
<td>4.29</td>
<td>4.01</td>
<td>4.08</td>
<td>4.04</td>
<td>5.03</td>
<td>5.10</td>
</tr>
<tr>
<td>ΔlnHt</td>
<td>0.78</td>
<td>1.25</td>
<td>1.14</td>
<td>1.18</td>
<td>1.03</td>
<td>1.30</td>
<td>1.41</td>
</tr>
<tr>
<td>ΔlnCs</td>
<td>6.23</td>
<td>7.01</td>
<td>0.91</td>
<td>6.87</td>
<td>7.40</td>
<td>8.08</td>
<td>8.16</td>
</tr>
<tr>
<td>ΔlnSp</td>
<td>6.44</td>
<td>7.90</td>
<td>7.58</td>
<td>7.85</td>
<td>7.74</td>
<td>8.59</td>
<td>8.33</td>
</tr>
<tr>
<td>ΔlnPd</td>
<td>0.67</td>
<td>0.73</td>
<td>0.79</td>
<td>0.74</td>
<td>0.76</td>
<td>0.87</td>
<td>0.90</td>
</tr>
<tr>
<td>Fr</td>
<td>3.82</td>
<td>3.47</td>
<td>3.23</td>
<td>3.92</td>
<td>3.34</td>
<td>3.91</td>
<td>4.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>Standard Deviations (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.06 1.10 3.77 4.17 0.78 6.23 6.44 0.67 3.82</td>
</tr>
<tr>
<td>Baseline</td>
<td>1.14 1.25 3.85 4.29 1.25 7.01 7.90 0.73 3.47</td>
</tr>
<tr>
<td>Variant 1</td>
<td>0.91 0.93 3.56 4.01 1.14 0.91 7.58 0.79 3.23</td>
</tr>
<tr>
<td>Variant 2</td>
<td>0.85 0.91 3.35 4.08 1.18 6.87 7.85 0.74 3.92</td>
</tr>
<tr>
<td>Variant 3</td>
<td>0.89 0.87 3.64 4.04 1.03 7.04 7.74 0.76 3.34</td>
</tr>
<tr>
<td>Variant 4</td>
<td>1.20 1.29 4.07 5.03 1.30 8.08 8.59 0.87 3.91</td>
</tr>
<tr>
<td>Variant 5</td>
<td>1.22 1.31 4.01 5.10 1.41 8.16 8.33 0.90 4.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>Standard Deviations Relative to Standard Deviation of ΔlnYt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.06 1.10 3.77 4.17 0.78 6.23 6.44 0.67 3.82</td>
</tr>
<tr>
<td>Baseline</td>
<td>1.14 1.25 3.85 4.29 1.25 7.01 7.90 0.73 3.47</td>
</tr>
<tr>
<td>Variant 1</td>
<td>0.91 0.93 3.56 4.01 1.14 0.91 7.58 0.79 3.23</td>
</tr>
<tr>
<td>Variant 2</td>
<td>0.85 0.91 3.35 4.08 1.18 6.87 7.85 0.74 3.92</td>
</tr>
<tr>
<td>Variant 3</td>
<td>0.89 0.87 3.64 4.04 1.03 7.04 7.74 0.76 3.34</td>
</tr>
<tr>
<td>Variant 4</td>
<td>1.20 1.29 4.07 5.03 1.30 8.08 8.59 0.87 3.91</td>
</tr>
<tr>
<td>Variant 5</td>
<td>1.22 1.31 4.01 5.10 1.41 8.16 8.33 0.90 4.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>First Order Autocorrelations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.23 0.17 0.27 0.96 0.55 -0.02 0.29 0.82 0.97</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.24 0.19 0.22 1.01 0.54 -0.07 0.30 0.82 0.95</td>
</tr>
<tr>
<td>Variant 1</td>
<td>0.25 0.21 0.28 0.24 0.54 -0.08 0.43 0.81 0.95</td>
</tr>
<tr>
<td>Variant 2</td>
<td>0.35 0.33 0.24 0.43 0.46 -0.03 0.47 0.87 0.94</td>
</tr>
<tr>
<td>Variant 3</td>
<td>0.29 0.26 0.17 0.18 0.59 -0.04 0.99 0.86 0.97</td>
</tr>
<tr>
<td>Variant 4</td>
<td>0.31 0.27 0.20 0.92 0.59 -0.11 0.27 0.86 0.96</td>
</tr>
<tr>
<td>Variant 5</td>
<td>0.35 0.37 0.25 0.97 0.59 -0.07 0.23 0.89 0.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>Correlations with $\Delta lnY_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1 0.44 0.69 0.05 0.72 0.30 0.33 -0.27 -0.16</td>
</tr>
<tr>
<td>Baseline</td>
<td>1 0.48 0.63 0.04 0.75 0.42 0.41 -0.22 -0.18</td>
</tr>
<tr>
<td>Variant 1</td>
<td>1 0.57 0.64 0.07 0.78 0.54 0.47 -0.21 -0.21</td>
</tr>
<tr>
<td>Variant 2</td>
<td>1 0.60 0.62 0.08 0.46 0.58 0.48 -0.15 -0.23</td>
</tr>
<tr>
<td>Variant 3</td>
<td>1 0.58 0.59 0.02 0.60 0.49 0.42 -0.32 -0.21</td>
</tr>
<tr>
<td>Variant 4</td>
<td>1 0.49 0.50 0.03 0.61 0.61 0.41 -0.31 -0.23</td>
</tr>
<tr>
<td>Variant 5</td>
<td>1 0.35 0.54 0.01 0.65 0.52 0.47 -0.25 -0.20</td>
</tr>
</tbody>
</table>

Note: $\Delta lnY_t$ is nonfarm business sector output index growth rate. $\Delta lnC_t$ is personal consumption expenditure of nondurable goods growth rate. $\Delta lnI_t$ is gross private domestic investment growth rate. $KT$ is total industry capacity utilization rate. $\Delta lnH_t$ is nonfarm business sector hours of all persons growth rate. $\Delta lnCs$ is consumer sentiment index growth rate. $\Delta lnSp$ is real S&P 500 Index per capita growth rate. $\Delta lnPd$ is inflation. $Fr$ is effective federal funds rate.
4. Dynamic and Cyclical Properties

4.1. Impulse Response Analysis

Since Bayesian impulse response functions capture percentage deviations of endogenous variables from their steady-state values in response to structural shocks, we have simulated main macroeconomic aggregates’ dynamic responses to investigate the interplay between monetary policies and unexpected stock price fluctuations in Figure 1. Intuitively, our impulse response functions, which are derived from the state space representation of the underlying DSGE model, elucidate expected future evolutionary trajectories of endogenous variables at given sizes of structural shocks over 20-quarter horizon. The black lines represent impulse responses originating from the baseline DSGE model with both financial wealth effect and capital reallocation effect, whereas the red lines represent impulse responses generated by its variant with only financial wealth effect. The dashed lines represent Bayesian 90% credible intervals associated with the black lines.

The first column depicts Bayesian impulse responses of effective federal funds rate, stock price gap, consumer sentiment index growth rate, real investment growth rate, output gap, capacity utilization rate and inflation gap to the monetary policy shock. Specifically, we initialize the size of monetary policy shock so that it induces a non-announced rise in interest rate, whose quarterly size equals 25 basis points of annual interest rate, and simulate the DSGE impulse responses to an unexpected 0.25% contractionary monetary policy shock. In the second, third, fourth, fifth and sixth columns, we initialize the sizes of financial, preference, investment, labor utilization and sentiment shocks respectively so that each of them produces a 1% increase in stock price gap, then we simulate the DSGE impulse responses to the 1% stock price gap.

In the first and second rows, we simulate percentage responses of effective federal funds rate to a 1% increase in financial slack represented by stock price gap, which is induced either by the financial shock, by the preference shock, by the investment shock, by the labor utilization shock or by the sentiment shock. Positive financial, preference, investment, labor utilization and sentiment shocks signal increases of stock price, demand, investment, labor utilization and sentiment respectively, inducing the central bank to increase interest rate and calm down the economy. Monetary policy exhibits significant responses to financial, preference and investment shocks except labor utilization and sentiment shocks. In addition, monetary policy responses to the labor utilization and the sentiment shock both decay slowly due to high persistence of the labor utilization shock process and the sentiment shock process respectively.
In the second and third rows, we simulate percentage responses of Consumer Sentiment Index growth rate to a 1% increase in stock price gap. A stock market boom originating from the sentiment or financial shock feeds back into consumer sentiment not only directly through financial wealth, but also indirectly through induced interest rate. These direct and indirect effects work in opposite directions. When a stock market boom emerges, financial wealth effect increases household expectation about future financial wealth and strengthens consumer confidence, leading to expansionary impacts on consumer sentiment. Rises in interest rates lower firms’ expectation about future investment profitability and weaken consumer confidence, resulting in contractionary impacts on consumer sentiment. The net impact depends on the relative strength between expansionary and contractionary impacts. The increment between estimated household turnover rate and firm turnover rate is large enough for expansionary impacts to dominate over con-
tractionary impacts. Impulse response of effective federal funds rate does not increase enough to lower firms’ expectation about future profitability due to relatively high degree of estimated monetary policy inertia. Impulse responses of Consumer Sentiment Index growth rate to sentiment and financial shocks are both positive and significant with associated credible intervals above zero. Therefore, monetary policy responses to financial and sentiment shocks do not sterilize the propagation of stock market booms’ effects on consumer sentiment.

In the second and fourth rows, we simulate percentage responses of real investment growth rate to a 1% increase in stock price gap. In the second and fifth rows, we simulate percentage responses of output gap to a 1% increase in stock price gap. A stock market boom originating from the sentiment or financial shock feeds back into the economy not only directly through household financial wealth effect on consumption and capital reallocation effect on investment, but also indirectly through induced interest rate variations that influence household intertemporal substitution of consumption and firms’ intertemporal substitution of investment. These direct and indirect effects work in opposite directions. When a stock market boom emerges, household financial wealth effect increases consumption and capital reallocation effect boosts investment, leading to expansionary impacts on output. Because stock market booms also cause rises in interest rates, households substitute more current consumption for future consumption and firms postpone more investment for future, resulting in contractionary impacts on output. The net impact hinges on the relative strength between expansionary and contractionary impacts. The increment between estimated household turnover rate and firm turnover rate is large enough for expansionary impacts to dominate over contractionary impacts. Due to relatively high degree of estimated monetary policy inertia, impulse response of interest rate does not increase enough to induce households to substitute more current consumption for future consumption and trigger firms to postpone more investment for future. Impulse responses of investment and output to financial shocks are positive and significant with associated credible intervals above zero. Therefore, monetary policy response to the financial shock does not sterilize the propagation of stock market booms’ effects on output.

In the second and sixth rows, we simulate percentage responses of capacity utilization rate to a 1% increase in stock price gap. When a positive stock market boom emerges, household financial wealth effect and capital reallocation effect induce inflation, and the concurrent rise in interest rate does not offset the rise in inflation due to relatively high degree of estimated monetary policy inertia. In addition, capacity utilization rate’s responses to the financial shock and the sentiment shock are significant with associated credible intervals above zero. Therefore, monetary policy response to the financial shock does not sterilize the propagation of stock market booms’ real effects on capacity utilization.

In the second and seventh rows, we simulate percentage responses of inflation gap to a 1% increase in stock price gap. When a positive stock market boom emerges, household financial wealth effect and capital reallocation effect provoke inflation, however, the rise in inflation is partially offset by the concurrent rise in interest rate, and inflation’s response to the financial shock is insignificant with associated credible intervals containing zero.
Therefore, monetary policy response to the financial shock sterilizes the propagation of stock market booms’ real effects on inflation. Preference shocks induce higher inflation through higher consumption and stock market boom, as well as putting upward pressure on interest rate through monetary policy rule. Preference shocks’ impacts on inflation are significant with associated credible intervals above zero. Therefore, monetary policy response to the preference shock does not sterilize the propagation of stock market booms’ effects on inflation.

4.2. Forecast Error Variance Decomposition

Table 6 elucidates unconditional forecast error variance decomposition of nine observable variables in terms of nine structural shocks, and evaluates the relative importance of these structural shocks in driving fluctuations of observable variables at business cycle frequencies. Effective federal funds rate fluctuations are essentially triggered by monetary policy, investment and financial shocks, and sentiment shocks only make marginal contributions. S&P 500 Index fluctuations are mostly induced by financial, sentiment, investment and preference shocks, because these shocks not only influence financial wealth and consumption, but also affect capital allocation and investment. Consumer sentiment variations are inherently driven by financial, sentiment, investment and preference shocks, because these shocks influence expectation of speculative stock market bubbles and evaluation of equity market capitalization. Investment fluctuations are generally stimulated by investment, preference, monetary policy and total factor productivity shocks, financial shocks make moderate contributions while sentiment shocks contribute marginally, because these shocks affect intertemporal substitution of investment and tightness of credit constraint. Capacity utilization variations are largely influenced by labor utilization, investment and financial shocks, because these shocks affect production scale and budget flexibility. Output fluctuations are primarily generated by preference, government spending, investment, labor utilization and total factor productivity shocks, financial shocks contribute moderately while sentiment shocks contribute marginally, because these shocks not only influence consumption, investment and government spending, but also affect household budget tightness and firm budget flexibility. Inflation variations are mainly provoked by price markup, investment and financial shocks, preference and sentiment shocks contribute moderately, because these shocks not only influence financial wealth, consumption and speculative stock market bubbles, but also affect market capitalization, investment and firm budget tightness. To summarize, financial, sentiment, preference, investment and labor utilization shocks bring more uncertainties into macroeconomies and financial markets.
Table 6: Forecast Error Variance Decomposition in the DSGE Model

<table>
<thead>
<tr>
<th>Observable Variable</th>
<th>$\nu_A$</th>
<th>$\nu_P$</th>
<th>$\nu_G$</th>
<th>$\nu_Y$</th>
<th>$\nu_R$</th>
<th>$\nu_H$</th>
<th>$\nu_I$</th>
<th>$\nu_S$</th>
<th>$\nu_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonfarm Business Sector Output Growth Rate</td>
<td>12.2</td>
<td>8.5</td>
<td>2.6</td>
<td>10.4</td>
<td>7.8</td>
<td>15.1</td>
<td>24.2</td>
<td>10.6</td>
<td>8.6</td>
</tr>
<tr>
<td>Personal Consumption Expenditure of Nondurable Goods</td>
<td>12.5</td>
<td>32.3</td>
<td>3.3</td>
<td>4.2</td>
<td>4.6</td>
<td>5.7</td>
<td>9.1</td>
<td>20.9</td>
<td>7.4</td>
</tr>
<tr>
<td>Gross Private Domestic Investment Growth Rate</td>
<td>11.4</td>
<td>12.4</td>
<td>8.3</td>
<td>3.3</td>
<td>12.4</td>
<td>3.5</td>
<td>33.5</td>
<td>6.1</td>
<td>9.1</td>
</tr>
<tr>
<td>Total Industry Capacity Utilization Rate</td>
<td>9.8</td>
<td>6.2</td>
<td>5.1</td>
<td>3.6</td>
<td>6.9</td>
<td>23.0</td>
<td>25.4</td>
<td>9.6</td>
<td>10.4</td>
</tr>
<tr>
<td>Nonfarm Business Sector Hours of All Persons Growth Rate</td>
<td>9.7</td>
<td>14.6</td>
<td>6.3</td>
<td>1.1</td>
<td>7.5</td>
<td>33.3</td>
<td>10.8</td>
<td>7.9</td>
<td>8.8</td>
</tr>
<tr>
<td>Consumer Sentiment Index Growth Rate</td>
<td>6.3</td>
<td>8.1</td>
<td>1.5</td>
<td>5.2</td>
<td>3.9</td>
<td>2.0</td>
<td>10.8</td>
<td>36.1</td>
<td>26.1</td>
</tr>
<tr>
<td>S&amp;P 500 Index Growth Rate</td>
<td>6.7</td>
<td>9.9</td>
<td>2.9</td>
<td>5.4</td>
<td>1.1</td>
<td>4.7</td>
<td>15.2</td>
<td>28.9</td>
<td>25.2</td>
</tr>
<tr>
<td>Inflation</td>
<td>7.2</td>
<td>8.4</td>
<td>2.3</td>
<td>30.5</td>
<td>3.4</td>
<td>2.0</td>
<td>16.2</td>
<td>14.1</td>
<td>15.9</td>
</tr>
<tr>
<td>Effective Federal Funds Rate</td>
<td>4.6</td>
<td>8.4</td>
<td>8.9</td>
<td>7.2</td>
<td>20.3</td>
<td>3.3</td>
<td>20.5</td>
<td>6.2</td>
<td>20.6</td>
</tr>
</tbody>
</table>

Note: $\nu_A$ is the total factor productivity shock. $\nu_P$ is the preference shock. $\nu_G$ is the government spending shock. $\nu_Y$ is the price markup shock. $\nu_R$ is the monetary policy shock. $\nu_H$ is the labor supply shock. $\nu_I$ is the marginal efficiency of investment shock. $\nu_S$ is the sentiment shock. $\nu_F$ is the financial shock.

Unconditional forecast error variance decompositions not only capture proportions of observables’ variances attributable to structural shocks in the long-term, but also signal relative importance of structural shocks as sources of observables’ variations. Sentiment shocks explain around 6.2% of interest rate fluctuations, 28.9% of stock price fluctuations, 36.1% of consumer sentiment variations, 6.1% of investment fluctuations, 9.6% of capacity utilization variations, 10.6% of output fluctuations and 14.1% of inflation variations. Financial shocks account for approximately 20.6% of interest rate fluctuations, 25.15% of stock price fluctuations, 26.1% of consumer sentiment variations, 15.1% of investment fluctuations, 10.4% of capacity utilization variations, 8.6% of output fluctuations and 15.9% of inflation variations. Preference shocks portray roughly 8.35% of interest rate fluctuations, 9.9% of stock price fluctuations, 8.1% of consumer sentiment variations, 12.4% of investment fluctuations, 6.2% of capacity utilization variations, 8.5% of output fluctuations and 8.4% of inflation variations. Investment shocks capture about 20.5% of interest rate fluctuations, 15.2% of stock price fluctuations, 10.8% of consumer sentiment variations, 33.5% of investment fluctuations, 25.4% of capacity utilization variations, 24.2% of output fluctuations and 16.2% of inflation variations. Labor utilization shocks explain about 16.3% of interest rate fluctuations, 4.7% of stock price fluctuations, 2.0% of consumer sentiment variations, 3.5% of investment fluctuations, 23.0% of capacity utilization variations, 15.1% of output fluctuations and 2.0% of inflation variations.
5. Conclusions

First, by decomposing stock market index into stock fundamental value and speculative stock bubble, our DSGE model identifies direct contributions of financial, sentiment, investment, preference and labor utilization shocks in driving stock price fluctuations. The financial shock reflects firms’ external financing risk in the stock market, whereas the sentiment shock drives the evolution of proportional size changes between current and new stock bubbles. We find that financial, sentiment, preference, investment and labor utilization shocks explain almost all the stock market booms and busts. In impulse response analysis, financial, sentiment, preference and investment shocks influence stock price fluctuations through stock fundamental values, speculative stock bubbles, nominal stochastic discount factor and dividends respectively. In forecast error variance decomposition, approximately 9.9%, 25.2%, 15.2%, 4.7% and 28.9% of stock market forecast error variances are attributable to preference, financial, investment, labor utilization and sentiment shocks respectively.

Further, Bayesian model comparison indicates that data supports our baseline DSGE model with financial wealth effect and capital reallocation effect. Estimated household turnover rate of 0.138, which indicates that households trade in financial markets for 7 quarters on average, is higher than that of Castelnuovo et al. (2010)’s DSGE model with neither firm turnovers nor capital reallocation effect. Higher household turnover rate amplifies the difference between current and future consumption, inducing more aggregate demand. Estimated firm turnover rate of 0.018, which indicates that firms trade in the stock market for 56 quarters on average, is higher than that of Miao et al. (2015)’s DSGE model with neither household turnovers nor financial wealth effect. Higher firm turnover rate diminishes the difference between current and future consumption, inducing less aggregate demand. According to impulse response analysis, output not only responds positively to sentiment shocks, but also responds more to stock price fluctuations induced by financial, preference and investment shocks, compared with those of the DSGE model without capital reallocation effect.

Furthermore, we detected a significant, counteractive and systematic response of the central bank to financial slack. Bayesian model comparison indicates that data supports our baseline DSGE model with monetary policy response to financial slack. Compared with the DSGE model without capital reallocation effect, our baseline DSGE model’s impulse response of interest rate to a positive financial shock increases more aggressively, this is due to higher estimated monetary policy response to output gap and higher simulated impulse response of output to the financial shock. Therefore, central banks use monetary policy tools to stabilize stock price fluctuations and display countercyclical responses to stock market cycles. These all suggest monetary policy’s prominent role in capturing stock price uncertainties and responding to financial slack.

In addition, output responds to positive financial and sentiment shocks not only directly through financial wealth, capital reallocation and investment efficiency, which exert effects on consumption and investment, but also indirectly through induced interest rate variations, which influence household intertemporal substitution of consumption and firms’ intertemporal substitution of investment. Specifically, when a stock market boom
emerges, financial wealth effect increases consumption, capital reallocation effect stimulates investment at the cost of decreasing consumption, and investment efficiency effect boosts investment, leading to expansionary impacts on output. Because stock market booms also cause rises in interest rates, households substitute more current consumption for future consumption and firms postpone more investment for future, resulting in contractionary impacts on output. Since estimated firm turnover rate is low enough, estimated household turnover rate and interest rate inertia are high enough, expansionary impacts dominate over contractionary impacts. Therefore, impulse responses of output gap to positive financial and sentiment shocks are positive, and in particular, output responds to positive financial shock significantly. These not only indicate that the central bank responds to the financial shock, but also imply that monetary policy response does not sterilize the propagation of stock market booms’ effects on output. When positive financial and sentiment shocks stimulate stock market booms, financial wealth effect and investment efficiency effect provoke inflation, and the concurrent rise in interest rate does not offset the increase in inflation due to high degree of estimated monetary policy inertia. Impulse responses of inflation to positive financial and sentiment shocks are positive and significant, suggesting that monetary policy responses to financial and sentiment shocks do not sterilize the propagation of stock market booms’ effects on inflation.

Last but not least, according to forecast error variance decomposition, financial shocks explain about 20.6% of interest rate fluctuations, 40% of stock price fluctuations, 25.15% of consumer sentiment variations, 14% of investment fluctuations, 15.11% of output fluctuations and 15.9% of inflation variations. Sentiment shocks explain around 6.22% of interest rate fluctuations, 28.96% of stock price fluctuations, 36.05% of consumer sentiment variations, 6.09% of investment fluctuations, 10.61% of output fluctuations and 14.13% of inflation variations. We can attribute about 28.96% of stock price fluctuations, which are captured by Castelnuovo et al. (2010)’s stock market index measurement errors, to sentiment shocks.

6. Further Research

First, some agents with limited cognitive ability use simple behavioral rules, while other agents learn from mistakes, therefore, we could characterize DSGE models with heterogeneous expectation. Second, we estimated a closed-economy DSGE model, but ignored features of international trade and exchange rates, hence, we could formulate open-economy DSGE models. Finally, we specified DSGE models with identically, independently and normally distributed structural shocks, which capture neither interdependence nor fat tail behavior, consequently, we could establish DSGE models with structural shocks, which are non-normally distributed or correlated or exhibit time-varying volatility.

Appendix
A. Derivation of Household Optimization Behavior

The representative household of cohort i chooses optimal real consumption $[C_t(i)]$, labor supply $[H_t(i)]$, government bond holdings $[B_{t+1}(i)]$, deposit holdings $[L_{t+1}(i)]$ and stock holdings $[\int_0^1 S_{t+1}(i,j) dj]$ to maximize the expected lifetime utility $[U_0(i)]$:

$$U_0(i) = E_0 \sum_{t=0}^{+\infty} \beta^t (1 - \gamma_H)^t \nu_{P_t} \{lnC_t(i) + \rho ln[1 - \nu_{H,t} H_t(i)]\}$$

subject to the sequence of cohort i’s intertemporal budget constraints:

$$C_t(i) + \int_0^1 [D_t(j) + P_s(j)] S_{t+1}(i,j) dj + E_t[\eta_{t,t+1}\pi_{t,t+1} B_{t+1}(i)] + \frac{E_t[\pi_{t,t+1} L_{t+1}(i)]}{R_{D,t}}$$

$$+ P_{Y,t} T_t \leq W_t \nu_{H,t} H_t(i) + \Gamma_{K,t} + \frac{B W_t(i)}{1 - \gamma_H} + \frac{D W_t(i)}{1 - \gamma_H} + \frac{1 - \gamma w}{1 - \gamma_H} S W_t(i)$$

$$t = 0, 1, \cdots, +\infty$$

(75)

Defining sub-utility functions:

$$\max_{\{C_t(i), B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj\}} \sum_{t=0}^{+\infty} \beta^t (1 - \gamma_H)^t \nu_{P_t} \{lnC_t(i) + \rho ln[1 - \nu_{H,t} H_t(i)]\}$$

subject to the sequence of cohort i’s intertemporal budget constraints in equation (75).

Due to utility function’s additive separability over time, value function $V[B_t(i), DE_t(i), \int_0^1 S_t(i,j) dj, \nu_{P_t}, \nu_{H,t}]$ is defined and the dynamic program is elucidated:

$$V[B_t(i), DE_t(i), \int_0^1 S_t(i,j) dj, \nu_{P_t}, \nu_{H,t}] = \max_{\{C_t(i), H_t(i), B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj\}} \nu_{P_t} U[C_t(i), H_t(i)] + \beta (1 - \gamma_H) V[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P_{t+1}}, \nu_{H,t+1}]$$

$$\nu_{P_t} U[C_t(i), H_t(i)] + \beta (1 - \gamma_H) V[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P_{t+1}}, \nu_{H,t+1}]$$

subject to the sequence of cohort i’s intertemporal budget constraints in equation (75).

Setting up cohort i’s Lagrangean function $[L_H(i)]$ generates:

$$L_H(i) = \nu_{P_t} U[C_t(i), H_t(i)] + \beta (1 - \gamma_H) V[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P_{t+1}}, \nu_{H,t+1}]$$

$$+ \lambda_H(i) \{W_t \nu_{H,t} H_t(i) + \Gamma_{K,t} + \frac{B_t(i)}{1 - \gamma_H} + \frac{DE_t(i)}{1 - \gamma_H} + \frac{1 - \gamma w}{1 - \gamma_H} \int_0^1 [P_s(j) + D_t(j)] S_t(i,j) dj$$

$$- C_t(i) - \int_0^1 P_s(j) S_{t+1}(i,j) dj - E_t[\eta_{t,t+1}\pi_{t,t+1} B_{t+1}(i)] + \frac{1}{R_{D,t}} E_t[\pi_{t,t+1} L_{t+1}(i)] - T_t\}$$

(78)

where $\lambda_H(i)$ is Lagrangean multiplier for cohort i.

Taking partial derivative of cohort i’s Lagrangean function $[L_H(i)]$ with respect to real consumption $[C_t(i)]$ yields:

$$\frac{\partial L_H(i)}{\partial C_t(i)} = \frac{\nu_{P_t}}{C_t(i)} - \lambda_H(i) = 0 \rightarrow \nu_{P_t} MU_{C_t}(i) = \frac{\nu_{P_t}}{C_t(i)} = \lambda_H(i)$$

(79)
Taking partial derivative of cohort i’s Lagrangean function \([L_H(i)]\) with respect to real labor supply \([H_t(i)]\) yields:

\[
\frac{\partial L_H(i)}{\partial H_t(i)} = -\frac{\nu_{p,t}\rho_{H,t}}{1 - \nu_{H,t}H_t(i)} + \lambda_H(i)W_t\nu_{H,t} = 0 \rightarrow -\nu_{p,t}MU_{H,t}(i) = \frac{\nu_{p,t}\rho_{H,t}}{1 - \nu_{H,t}H_t(i)} = \lambda_H(i)W_t\nu_{H,t}
\]

(80)

Taking partial derivative of cohort i’s Lagrangean function \([L_H(i)]\) with respect to government bond purchase \([B_{t+1}(i)]\) yields:

\[
\frac{\partial L_H(i)}{\partial B_{t+1}(i)} = 0 \rightarrow (1 - \gamma_H)V_B'[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i, j) dj, \nu_{p,t+1}] = \lambda_H(i)\eta_{t,t+1}\pi_{t,t+1}
\]

(81)

Cohort i equals marginal cost of working in consumption units \([\frac{C_t(i)q_{H,t}}{1 - \nu_{H,t}H_t(i)}]\) with marginal benefit in terms of real wage \((W_t)\), therefore, combining equations (79) and (80) generates:

\[
\frac{C_t(i)q_{H,t}}{1 - \nu_{H,t}H_t(i)} = W_t
\]

(82)

Rearranging budget constraint in equation (75) and leading forward for one period:

\[
C_{t+1}(i) = W_{t+1}\nu_{H,t+1}H_{t+1}(i) + \Gamma_{K,t+1} + \frac{BW_{t+1}(i)}{1 - \gamma_H} + \frac{DW_{t+1}(i)}{1 - \gamma_H} + \frac{1 - \gamma_H}{1 - \gamma_H}SW_{t+1}(i) - \int_0^1 P_{s_{t+1}(j)}S_{t+2}(i, j) dj - \frac{E_{t+1}[\pi_{t+1,t+2}L_{t+2}(i)]}{R_{D,t+1}} - E_{t+1}[\eta_{t+1,t+2}\pi_{t+1,t+2}B_{t+2}(i)] - T_{t+1}
\]

(83)

Leading value function in equation (77) forward for one period, substituting equations (83) and (2) into it, and taking partial derivative of value function \(V[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i, j) dj, \nu_{p,t+1}]\) with respect to \([B_{t+1}(i)]\) yield:

\[
V_B'[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i, j) dj, \nu_{p,t+1}] = \frac{\nu_{p,t+1}}{C_{t+1}(i)(1 - \gamma_H)} = \frac{\nu_{p,t+1}MU_{C,t+1}(i)}{1 - \gamma_H}
\]

(84)

Substituting equation (84) into equation (81) yields:

\[
\frac{\beta (1 - \gamma_H)\nu_{p,t+1}MU_{C,t+1}(i)}{1 - \gamma_H} = \lambda_H(i)\eta_{t,t+1}\pi_{t,t+1}
\]

(85)

Cohort i equals cost of marginal utility of consumption foregone in acquiring government bonds with interests, therefore, substituting equation (79) into equation (85) yields nominal stochastic discount factor \(\eta_{t,t+1}\):

\[
\eta_{t,t+1} = \frac{\beta\nu_{p,t+1}MU_{C,t+1}(i)}{\nu_{p,t}MU_{C,t}(i)\pi_{t,t+1}} = \frac{\beta\nu_{p,t+1}C_t(i)}{\nu_{p,t}C_{t+1}(i)\pi_{t,t+1}}
\]

(86)

Taking partial derivative of cohort i’s Lagrangean function \([L_H(i)]\) with respect to new deposits \([L_{t+1}(i)]\) yields:

\[
\frac{\partial L_H(i)}{\partial L_{t+1}(i)} = 0 \rightarrow (1 - \gamma_H)V_L'[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i, j) dj, \nu_{p,t+1}, \nu_{H,t+1}] = \lambda_H(i)\frac{\pi_{t,t+1}}{R_{D,t}}
\]

(87)
Under the assumption that households’ borrowing constraint is not binding in terms of positive deposits \( DE_t(i) > 0 \), leading value function in equation (77) forward for one period, substituting equations (75) and (3) into it, and taking partial derivative of value function \( V[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P,t+1}] \) with respect to \([L_{t+1}(i)]\) yield:

\[
V'_L[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P,t+1}, \nu_{H,t+1}] = \frac{\nu_{P,t+1}}{C_{t+1}(i)(1 - \gamma_H)}
\]  

(88)

Substituting equation (88) into equation (87) yields:

\[
\frac{\beta \nu_{P,t+1}}{C_{t+1}(i)} = \lambda_H(i) \frac{1}{R_{D,t}} \pi_{t,t+1}
\]  

(89)

Substituting equation (79) into equation (89) yields:

\[
\eta_{t,t+1} = \frac{1}{R_{D,t}}
\]  

(90)

Under the assumption that households’ borrowing constraint is binding in terms of zero deposits \( DE_t(i) = 0 \),

\[
\eta_{t,t+1} > \frac{1}{R_{D,t}}
\]  

(91)

Taking partial derivative of cohort i’s Lagrangean function \([L_H(i)]\) with respect to purchase of wholesale firm j’s stocks \([S_{t+1}(i,j)]\) yields:

\[
\frac{\partial L_H(i)}{\partial S_{t+1}(i,j)} = 0 \rightarrow \beta (1 - \gamma_H) V'_S[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P,t+1}] = \lambda_H(i) P_{s_t(j)}
\]  

(92)

Leading value function in equation (77) forward for one period, substituting equations (75) and (3) into it, and taking partial derivative of value function

\[ V[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P,t+1}, \nu_{H,t+1}] \] with respect to \([S_{t+1}(i,j)]\) yield:

\[
V'_S[B_{t+1}(i), L_{t+1}(i), \int_0^1 S_{t+1}(i,j) dj, \nu_{P,t+1}, \nu_{H,t+1}] = \frac{\nu_{P,t+1}(1 - \gamma_W)[P_{s_{t+1}(j)} + D_{t+1}(j)]}{C_{t+1}(i)(1 - \gamma_H)}
\]  

(93)

Substituting equation (93) into equation (92) yields:

\[
\beta(1 - \gamma_W) \nu_{P,t+1} M_{U_C,t+1}(i)[P_{s_{t+1}(j)} + D_{t+1}(j)] = \lambda_H(i) P_{s_t(j)}
\]  

(94)

Substituting equation (79) into equation (94) yields:

\[
\beta(1 - \gamma_W) \nu_{P,t+1} M_{U_C,t+1}(i)[P_{s_{t+1}(j)} + D_{t+1}(j)] = \nu_{P,t} M_{U_C,t}(i) P_{s_t(j)}
\]  

(95)

Substituting equation (86) into equation (95) yields:

\[
(1 - \gamma_W) \eta_{t,t+1} \pi_{t,t+1}[P_{s_{t+1}(j)} + D_{t+1}(j)] = P_{s_t(j)}
\]  

(96)
Substituting equation (96) into equation (75) yields:

\[
C_t(i) + E_t(\eta_{t+1} \pi_{t+1} [BW_{t+1}(i) + DW_{t+1}(i) + (1 - \gamma_W)SW_{t+1}(i)]) + T_t
= W_i \nu_{H,t} H_t(i) + \Gamma_{K,t} + \frac{1}{1 - \gamma_H}[BW_t(i) + DW_t(i) + (1 - \gamma_W)SW_t(i)]
\]

(97)

Incumbent household trader cohort i’s real financial wealth \([FW_t(i)]\) in period t includes government bond wealth \([BW_t(i)]\) in terms of real government bond holdings \([B_t(i)]\), deposit wealth \([DW_t(i)]\) in terms of real deposit holdings \([DE_t(i)]\) and stock wealth \([SW_t(i)]\) in terms of stock holdings \([\int_0^T S_t(i,j) dj]\), both of which are weighted by wholesale firm survival probability \((1 - \gamma_W)\):

\[
FW_t(i) = BW_t(i) + DW_t(i) + (1 - \gamma_W)SW_t(i)
\]

(98)

Substituting equation (98) into equation (97) yields:

\[
C_t(i) + E_t[\eta_{t+1} \pi_{t+1} FW_{t+1}(i)] = W_i \nu_{H,t} H_t(i) + \Gamma_{K,t} + T_t + \frac{1}{1 - \gamma_H}FW_t(i)
\]

(99)

Rearranging equation (99) and iteratively substituting \([FW_{t+1}(i)]\) forward yield:

\[
FW_t(i) = (1 - \gamma_H)C_t(i) + (1 - \gamma_H)E_t[\eta_{t+1} \pi_{t+1} FW_{t+1}(i)] - (1 - \gamma_H)[W_i \nu_{H,t} \nu_{H,t} H_t(i) + \Gamma_{K,t} - T_t]
\]

\[
\rightarrow \frac{1}{1 - \gamma_H}FW_t(i) = C_t(i) + E_t[\eta_{t+1} \pi_{t+1} (1 - \gamma_H)C_{t+1}(i)] + E_t[\eta_{t+2} \pi_{t+2} (1 - \gamma_H)C_{t+2}(i)]
\]

\[
+ \cdots - E_t \sum_{i=0}^{+\infty} (1 - \gamma_H)^i \eta_{t,t+i} \pi_{t,t+i} [W_i \nu_{H,t+i} H_{t+i}(i) + \Gamma_{K,t+i} - T_{t+i}]
\]

(100)

Defining cohort i’s human wealth \([HW_t(i)]\) as:

\[
HW_t(i) = HW_t = E_t \sum_{i=0}^{+\infty} (1 - \gamma_H)^i \eta_{t,t+i} \pi_{t,t+i} [W_i \nu_{H,t+i} H_{t+i}(i) + \Gamma_{K,t+i} - T_{t+i}]
\]

(101)

Rearranging equation (86) yields and leading it forward for i periods yield:

\[
C_{t+i}(i) = \frac{\beta \nu_{P,t+i} C_t(i)}{\eta_{t+i} \nu_{P,t} \pi_{t+i}}
\]

(102)

Substituting equations (101) and (102) into equation (100) yields:

\[
\frac{1}{1 - \gamma_H}FW_t(i) + HW_t(i) = C_t(i) \sum_{i=0}^{+\infty} \beta^i (1 - \gamma_H)^i \frac{\nu_{P,t+i}}{\nu_{P,t}}
\]

(103)

Defining \(MPC_t = \sum_{i=0}^{+\infty} [\frac{1}{1 - \gamma_H} \nu_{P,t+i}]\) as time-varying marginal propensity to consume out of financial wealth \([FW_t(i)]\) and human wealth \([HW_t(i)]\) yields:

\[
C_t(i) = MPC_t \{\frac{1}{1 - \gamma_H}[BW_t(i) + DW_t(i) + (1 - \gamma_W)SW_t(i)] + HW_t(i)\}
\]

(104)
B. Aggregation of Cohorts’ Behavior

Consumption

Aggregate real consumption \((C_t)\), labor \((H_t)\), physical capital \((K_{t-1})\), government bonds \((G_t)\), equity \(j\)’s shareholdings \([S_{t(j)}]\) and taxes \((T_t)\) are the weighted averages across all generations of cohort \(i\)’s real consumption \([C_t(i)]\), labor \([H_t(i)]\), physical capital \([K_{t-1}(i)]\), government bonds \([B_t(i)]\), deposits \([DE_t(i)]\), equity \(j\)’s shareholdings \([S_t(i,j)]\) and taxes \([T_t(i)]\) respectively weighted by \(\gamma_H (1 - \gamma_H)^{t-i}\) for each cohort \(i\). \(\gamma_H\) is cohort size and \([\gamma_H (1 - \gamma_H)^{t-i}]\) is cohort \(i\)’s cumulative survival probability.

Intratemporal Substitution between Consumption and Labor Supply

Aggregate real consumption \((C_t)\) equals leisure’s opportunity cost \([W_t (1 - \nu_{H,t} H_t)]\) over leisure weight \(\varrho\):

\[
C_t = \frac{W_t (1 - \nu_{H,t} H_t)}{\varrho}
\]  

(105)

Financial Wealth

New household traders entering the stock market in period \(t\) have no stock holdings \(S_t(t, j) = 0\) where \(j \in (0, 1)\). Therefore, aggregate real stock market wealth \((SW_t)\) is the weighted average of incumbent household traders’ stock wealth \([SW_t(i)]\) only:

\[
SW_t = \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} \int_0^1 P_s(j) S_t(i, j) dj = \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} SW_t(i) \]  

(106)

The product of aggregate stock market wealth \((SW_t)\) and household survival probability \((1 - \gamma_H)\) is the weighted average of household traders’ stock wealth \([SW_t(i)]\) with weights of cohort size \(\gamma_H\) and household cumulative survival probabilities \([(1 - \gamma_H)^{t-i}]\):

\[
(1 - \gamma_H)SW_t = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} \int_0^1 P_s(j) S_t(i, j) dj = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} SW_t(i) \]  

(107)

New household traders entering the bond market in period \(t\) have no bond holdings \([B_t(t) = 0]\). Therefore, aggregate real bond market wealth \((BW_t)\) is the weighted average of incumbent household traders’ bond wealth \([BW_t(i)]\) only:

\[
BW_t = \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} B_t(i) = \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} BW_t(i) \]  

(108)

The product of aggregate bond market wealth \((BW_t)\) and household survival probability \((1 - \gamma_H)\) is the weighted average of household traders’ bond wealth \([BW_t(i)]\) with weights of cohort size \(\gamma_H\) and household cumulative survival probabilities \([(1 - \gamma_H)^{t-i}]\):

\[
(1 - \gamma_H)BW_t = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} BW_t(i) \]  

(109)
New household traders entering the deposit market in period $t$ have no deposit holdings [$L_t(t) = 0$]. Therefore, aggregate real deposit wealth ($DW_t$) is the weighted average of incumbent household traders’ deposit wealth [$DW_t(i)$] only:

$$DW_t = \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} DE_t(i) = \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} DW_t(i) \quad (110)$$

The product of aggregate deposit wealth ($DW_t$) and household survival probability ($1 - \gamma_H$) is the weighted average of household traders’ deposit wealth [$DW_t(i)$] with weights of cohort size $\gamma_H$ and household cumulative survival probabilities [(1 - $\gamma_H$)$^{t-i}$]:

$$(1 - \gamma_H)DW_t = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} DW_t(i) \quad (111)$$

New household traders entering the stock market in period $t$ have neither stock wealth [$SW_t(t) = 0$] nor bond wealth [$BW_t(t) = 0$] nor deposit wealth [$DW_t(t) = 0$]. Therefore, the product of aggregate financial wealth ($FW_t$) and household survival probability ($1 - \gamma_H$) is the weighted average of household traders’ financial wealth [$FW_t(i)$] with weights of cohort size $\gamma_H$ and household cumulative survival probabilities [(1 - $\gamma_H$)$^{t-i}$]:

$$(1 - \gamma_H)FW_t = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} SW_t(i) + \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i}(1 - \gamma) BW_t(i)$$

$$+ \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} DW_t(i) = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} FW_t(i) \quad (112)$$

Aggregating equation (104) over household traders with weights of cohort size $\gamma_H$ and household cumulative survival probabilities [(1 - $\gamma_H$)$^{t-i}$] yields:

$$\sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} C_t(i) = MPC_t \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} HW_t(i)$$

$$+ \frac{MPC_t}{1 - \gamma_H} \sum_{i=-\infty}^{t-1} \gamma_H (1 - \gamma_H)^{t-1-i} [BW_t(i) + DW_t(i) + (1 - \gamma) SW_t(i)]$$

$$\rightarrow C_t = MPC_t [HW_t + BW_t + DW_t + (1 - \gamma) SW_t] = MPC_t (HW_t + FW_t) \quad (113)$$

**Equilibrium Budget Constraint**

Aggregating cohort $i$’s real financial wealth [$FW_t(i)$] in equation (68) over household traders to obtain intertemporal substitution between current aggregate financial wealth ($FW_t$) and future aggregate financial wealth ($FW_{t+1}$) in equation (81). Insurance contracts’ gross return $\frac{1}{1 - \gamma_H}$ disappears after aggregation because they have financial wealth.
redistributive effects on all cohorts.

\[
\sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} \{C_t(i) + E_t[\eta_{t,t+1} \pi_{t,t+1} FW_{t+1}(i)]\} = \sum_{i=-\infty}^{t} \gamma_H (1 - \gamma_H)^{t-i} [W_t \nu_{H,t} H_t(i) + \Gamma_{K,t} - T_t + \frac{1}{1 - \gamma_H} FW_t(i)]
\]

\[
\rightarrow C_t + E_t(\eta_{t,t+1} \pi_{t,t+1} FW_{t+1}) = W_t \nu_{H,t} H_t + \Gamma_{K,t} - T_t + FW_t
\]

Rearranging equation (114) yields:

\[
FW_t = C_t + E_t(\eta_{t,t+1} \pi_{t,t+1} FW_{t+1}) - W_t \nu_{H,t} H_t - \Gamma_{K,t} + T_t
\]

Substituting equation (115) into equation (113) yields:

\[
C_t = MPC_t[C_t + E_t(\eta_{t,t+1} \pi_{t,t+1} FW_{t+1}) + HW_t - W_t \nu_{H,t} H_t - \Gamma_{K,t} + T_t]
\]

Leading equation (113) forward for one period, multiplying equation by \(\frac{(1 - \gamma_H) \eta_{t,t+1}}{MPC_t}\), and taking expectation yield:

\[
E_t\left[\frac{(1 - \gamma_H) \eta_{t,t+1}}{MPC_t} C_{t+1}\right] = E_t\{(1 - \gamma_H) \eta_{t,t+1}[HW_{t+1} + BW_{t+1} + DW_{t+1} + (1 - \gamma_W)SW_{t+1}]\}
\]

Rearranging equation (1) yields:

\[
HW_t = W_t \nu_{H,t} \nu_{H,t} H_t + \Gamma_{K,t} - T_t + \eta_{t+1}(1 - \gamma_H) \sum_{i=1}^{+\infty} \eta_{t-1+i}(1 - \gamma_H)^{t-i} \pi_{t,t+1}(W_{t+i} \nu_{H,t+i} H_{t+i} + \Gamma_{K,t+i} - T_{t+i})
\]

\[
\eta_{t,t+1}(1 - \gamma_H) \pi_{t,t+1} HW_{t+1} = HW_t - (W_t \nu_{H,t} H_t + \Gamma_{K,t} - T_t)
\]

Multiplying equation (116) by \(\pi_{t,t+1}\) and substituting it into equation (118) yield:

\[
HW_t - (W_t \nu_{H,t} H_t + \Gamma_{K,t} - T_t) = E_t\left[\frac{(1 - \gamma_H) \eta_{t,t+1}}{MPC_t} \pi_{t,t+1} C_{t+1}\right] - E_t\{(1 - \gamma_H) \eta_{t,t+1} \pi_{t,t+1} BW_{t+1} + DW_{t+1} + (1 - \gamma_W)SW_{t+1}\}
\]

Substituting equation (119) into equation (116) yields:

\[
\left(\frac{1}{MPC_t} - 1\right)C_t = \gamma_H E_t\{\eta_{t+1} \pi_{t+1} BW_{t+1} + DW_{t+1} + (1 - \gamma_W)SW_{t+1}\}\]

\[
+(1 - \gamma_W) SW_{t+1}\} + (1 - \gamma_H) E_t\left(\frac{\eta_{t,t+1}}{MPC_t} \pi_{t,t+1} C_{t+1}\right)
\]

C. Derivation of Wholesale Firms’ Optimization Behavior

Wholesale firm \(j\) chooses optimal labor \([H_t(j)]\) to maximize:

\[
P_{W,t} M_t(j) - P_{Y,t} W_t \nu_{H,t} H_t(j) = P_{W,t} \nu_{A,t}[U_{K,t}(j) K_t(j)]^{\alpha}[\nu_{H,t} H_t(j)]^{1-\alpha} - P_{Y,t} W_t \nu_{H,t} H_t(j)
\]
The first order condition with respect to labor \([H_t(j)]\) yields optimal labor \([H_t(j)]\):

\[
P_{\text{W},t} \nu_{\text{A},t}[U_{K,t}(j)K_t(j)]^{(1-\alpha)}[1-H_t(j)]^{-\alpha} \nu_{H,t} = P_{\text{Y},t} W_t \nu_{\text{A},t} \rightarrow P_{\text{W},t}(1-\alpha)
\]

\[
\nu_{\text{A},t}[U_{K,t}(j)K_t(j)]^{\alpha} = P_{\text{Y},t} W_t \rightarrow H_t(j) = \frac{P_{\text{W},t}(1-\alpha) \nu_{\text{A},t}}{P_{\text{Y},t} W_t} \frac{U_{K,t}(j)K_t(j)}{\nu_{H,t}} \tag{122}
\]

Substituting optimal labor \([H_t(j)]\) into equation (121) yields wholesale firm \(j\)'s operating profits generated by physical capital \((R_{K,t})\):

\[
\frac{(1-\alpha) \frac{1-\alpha}{\nu_{\text{A},t}}}{W_t} \frac{P_{\text{W},t}^{\frac{1}{\nu_{\text{A},t}}}}{P_{\text{Y},t}^{\frac{1}{\nu_{\text{A},t}}}} U_{K,t}(j)K_t(j) \rightarrow \alpha \left[ \frac{(1-\alpha) \frac{1-\alpha}{\nu_{\text{A},t}}}{W_t} \frac{P_{\text{W},t}^{\frac{1}{\nu_{\text{A},t}}}}{P_{\text{Y},t}^{\frac{1}{\nu_{\text{A},t}}}} U_{K,t}(j)K_t(j) \right] \tag{123}
\]

Substituting wholesale good \(j\)'s optimal labor \([H_t(j)]\) into value function and flow-of-funds constraint respectively yields value function maximization problem:

\[
V_{t,\tau}[K_t(j), L_t(j), \nu_t(j)] = \max_{L_t(j) \geq 0, L_{t+1}(j) \geq 0} R_{K,t} U_{K,t}(j) K_t(j) - P_{t,t} I_t(j)
\]

\[-L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + (1-\gamma W) E_t \eta_{t,t+1} V_{t+1,\tau+1}[K_t(j), L_{t+1}(j), \nu_t(j)]\]

subject to physical capital accumulation in equation (18), flow-of-funds constraint:

\[
P_{t,t} I_t(j) + \frac{1-\alpha}{\alpha} \alpha \left[ \frac{(1-\alpha) P_{\text{W},t}^{\frac{1}{\nu_{\text{A},t}}} \nu_{\text{A},t}^{1-\alpha}}{W_t} \right] \frac{1-\alpha}{\nu_{\text{A},t}} U_{K,t}(j)K_t(j)
\]

\[
\leq (1-\gamma W) E_t \eta_{t,t+1} V_{t+1,\tau+1}[K_t(j), L_{t+1}(j), \nu_t(j)]
\]

\[
\rightarrow P_{t,t} I_t(j) + \frac{1-\alpha}{\alpha} R_{K,t} U_{K,t}(j) K_t(j) \leq (1-\gamma W) E_t \eta_{t,t+1} V_{t+1,\tau+1}[K_t(j), L_{t+1}(j), \nu_t(j)]\tag{124}
\]

and credit constraint:

\[
D_t(j) + L_t(j) + P_{t,t} I_t(j) + \frac{1-\alpha}{\alpha} \alpha \left[ \frac{(1-\alpha) P_{\text{W},t}^{\frac{1}{\nu_{\text{A},t}}} \nu_{\text{A},t}^{1-\alpha}}{W_t} \right] \frac{1-\alpha}{\nu_{\text{A},t}} U_{K,t}(j)K_t(j)
\]

\[
= \frac{\alpha}{\alpha} \left[ \frac{(1-\alpha) P_{\text{W},t}^{\frac{1}{\nu_{\text{A},t}}} \nu_{\text{A},t}^{1-\alpha}}{W_t} \right] \frac{1-\alpha}{\nu_{\text{A},t}} U_{K,t}(j)K_t(j) + \frac{L_{t+1}(j)}{R_{L,t}}
\]

\[
\rightarrow D_t(j) + L_t(j) + P_{t,t} I_t(j) = R_{K,t} U_{K,t}(j) K_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} \tag{125}
\]

Conjecturing wholesale firm \(j\)'s value function as \(V_{t,\tau}[K_t(j), L_t(j), \nu_t(j)] = Q_t[\nu_t(j)] \eta_t(j) + O_{t,\tau}[\nu_t(j)] - \eta_{L,t}[\nu_t(j)] L_t(j)\). Substituting value function’s conjectured form, physical capital accumulation, and flow-of-funds constraint yield:

\[
Q_t(j) K_t(j) + O_{t,\tau}(j) - \eta_{L,t}(j) L_t(j) = \max_{L_t(j) \geq 0, L_{t+1}(j) \geq 0} R_{K,t} U_{K,t}(j) K_t(j) - P_{t,t} I_t(j) - L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + Q_t^* K_{t+1}(j) + O_{t,\tau}^* L_{t+1}^* \tag{127}
\]

\[(1-\delta K) Q_t^* K_t(j) + \left[ \frac{Q_t^* \nu_t(j)}{P_{t,t}} - 1 \right] P_{t,t} I_t(j) - L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + O_{t,\tau}^* L_{t+1}^* \]
subject to physical capital accumulation in equation (18), flow-of-funds constraint:

\[P_{t,t} I_t(j) + \frac{1-\alpha}{\alpha} R_{K,t} U_{K,t}(j) K_t(j) \leq Q_t^* K_{t+1}(j) + O_{t,\tau}^* - L_{L,t}^* \] (128)

and credit constraint:

\[D_t(j) + L_t(j) + P_{t,t} I_t(j) = R_{K,t} U_{K,t}(j) K_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} \] (129)

where we define \(Q_t^* = (1 - \gamma_W) E_t \eta_{t+1} Q_{t+1}, O_{t,\tau}^* = (1 - \gamma_W) E_t \eta_{t+1} O_{t+1,\tau+1}(j), L_t^* = (1 - \gamma_W) E_t \eta_{t+1,\tau+1} L_{t+1}(j)\).

Wholesale firm \(j\) is subject to new equity issuance constraint:

\[D_t(j) \geq -\nu_{E,t} K_t(j) \] (130)

Substituting new equity issuance into wholesale firm \(j\)'s credit constraint yields:

\[P_{t,t} I_t(j) \leq R_{K,t} U_{K,t}(j) K_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} - L_t(j) + \nu_{E,t} K_t(j) \] (131)

Substituting firm value's conjectured form into equation (27) yields:

\[\frac{L_{t+1}(j)}{R_{L,t}} \leq Q_t^* \nu_{C,t} K_t(j) + O_{t,\tau}^* \] (132)

Substituting equation (132) into equation (131) yields:

\[P_{t,t} I_t(j) \leq R_{K,t} U_{K,t}(j) K_t(j) + Q_t^* \nu_{C,t} K_t(j) + \nu_{E,t} K_t(j) + O_{t,\tau}^* - L_t(j) \] (133)

Setting up wholesale firm \(j\)'s Lagrangean function \([L_{W,t}(j)]\):

\[L_{W,t}(j) = \{R_{K,t} U_{K,t}(j) + (1 - \delta_t[U_{K,t}(j)])\} Q_t^* K_t(j) + \left[\frac{Q_t^* \nu_{I,t}(j)}{P_{t,t}} - 1\right] P_{t,t} I_t(j) - L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} \]

\[+ O_{t,\tau}^* - L_t^* + \lambda_{W,t}(j)[R_{K,t} U_{K,t}(j) K_t(j) + Q_t^* \nu_{C,t} K_t(j) + \nu_{E,t} K_t(j) + O_{t,\tau}^* - L_t(j) - P_{t,t} I_t(j)] \] (134)

where \(Q_t\) is shadow price of installed physical capital \(K_t(j)\) and \(\lambda_{W,t}(j)\) is Lagrange multiplier on flow-of-funds constraint.

The first order condition with respect to wholesale firm \(j\)'s investment \([I_t(j)]\) yields Lagrange multiplier \([\lambda_{M,t}(j)]\) on flow-of-funds constraint:

\[\lambda_{M,t}(j) = \frac{Q_t^* \nu_{I,t}(j)}{P_{t,t}} - 1 = \frac{\nu_{I,t}(j)}{\nu_{I,t}^*} - 1 \geq 0 \] (135)

where \(\nu_{I,t}^* = \frac{P_{t,t}}{Q_t}\) is marginal investment efficiency threshold.

The first order condition with respect to wholesale firm \(j\)'s capital utilization rate \([U_{K,t}(j)]\) yields:

\[(1 + G_t) R_{K,t} = \delta_t[U_{K,t}(j)] Q_t^* \] (136)
Therefore, physical capital utilization rate \([U_{K,t}(j)] = U_{K,t}\) is independent of wholesale firm index \(j\) and age \(\tau\), substituting it into equation (122) yields:

\[
\frac{U_{K,t}(j)K_t(j)}{\nu_{H,t}H_t(j)} = \left[\frac{PW_t(1-\alpha)\nu_{A,t}}{P_{y,t}W_t}\right]^{-\frac{1}{\alpha}}
\] (137)

The ratio of utilized physical capital \([U_{K,t}(j)K_t(j)]\) to utilized labor \([\nu_{H,t}H_t(j)]\) is independent of wholesale firm index \(j\).

Wholesale firm \(j\)'s optimal investment \([I_t(j)]\) is as follows:

\[
P_{I,t}I_t(j) = \begin{cases} R_{K,t}U_{K,t}K_t(j) + Q_t^*\nu_{C,t}K_t(j) + \nu_{E,t}K_t(j) + O_{t,\tau}^* - L_t(j) & \text{if } \nu_{I,t}(j) \geq \nu_{I,t}^* \\ 0 & \text{if } \nu_{I,t}(j) < \nu_{I,t}^* \end{cases}
\] (138)

Substituting optimal investment into equation (134) yields:

\[
Q_t[\nu_{I,t}(j)]K_t(j) + O_{t,\tau}[\nu_{I,t}(j)] - \eta_{L,t}[\nu_{I,t}(j)]L_t(j) = [R_{K,t}U_{K,t} + (1 - \delta_t)Q_t^*]K_t(j) + \left[\frac{\nu_{I,t}(j)}{\nu_{I,t}^*}\right] - 1][R_{K,t}U_{K,t}K_t(j) + Q_t^*\nu_{C,t}K_t(j) + \nu_{E,t}K_t(j) + O_{t,\tau}^* - L_t(j)] - L_t(j) + \frac{L_{t+1}(j)}{R_{L,t}} + O_{t,\tau}^* - L_t^*
\] (139)

Extra dividends’ expected benefits are:

\[
G_t[\nu_{I,t}(j)] = \begin{cases} \frac{\nu_{I,t}(j)}{\nu_{I,t}^*} - 1 & \text{if } \nu_{I,t}(j) \geq \nu_{I,t}^* \\ 0 & \text{if } \nu_{I,t}(j) < \nu_{I,t}^* \end{cases} \rightarrow G_t = \int_{\nu_I \geq \nu_{I,t}^*} \left(\frac{\nu_I}{\nu_{I,t}^*} - 1\right)d\Phi(\nu_I)
\] (140)

Matching coefficients of physical capital \([K_t(j)]\) yields:

\[
Q_t[\nu_{I,t}(j)] = \begin{cases} R_{K,t}U_{K,t} + (1 - \delta_t)Q_t^* + [\frac{\nu_{I,t}(j)}{\nu_{I,t}^*} - 1](R_{K,t}U_{K,t} + Q_t^*\nu_{C,t} + \nu_{E,t}) & \text{if } \nu_{I,t}(j) \geq \nu_{I,t}^* \\ R_{K,t}U_{K,t} + (1 - \delta_t)Q_t^* & \text{if } \nu_{I,t}(j) < \nu_{I,t}^* \end{cases}
\] (141)

Substituting equation (141) into equation (35) yields:

\[
Q_t^* = (1 - \gamma_W)E_t[\eta_{t+1}\{R_{K,t+1}U_{K,t+1}(j) + (1 - \delta_{t+1})Q_{t+1}^* + G_{t+1}[R_{K,t+1}U_{K,t+1}(j) + Q_{t+1}^*\nu_{C,t+1} + \nu_{E,t+1}]\}]
\] (142)

Matching stock bubbles yields:

\[
O_{t,\tau}[\nu_{I,t}(j)] = \begin{cases} \frac{\nu_{I,t}(j)}{\nu_{I,t}^*} - 1 & \text{if } \nu_{I,t}(j) \geq \nu_{I,t}^* \\ O_{t,\tau}^* & \text{if } \nu_{I,t}(j) < \nu_{I,t}^* \end{cases} \rightarrow O_{t,\tau}[\nu_{I,t}(j)] = (G_t + 1)O_{t,\tau}^*
\] (143)

Substituting equation (143) into equation (36) yields:

\[
O_{t,\tau}^* = (1 - \gamma_W)E_t[\eta_{t+1}(G_{t+1} + 1)O_{t+1,\tau+1}^*]
\] (144)
Matching coefficients of outstanding corporate loans \([L_t(j)]\) yields:

\[
\eta_{L,t}[\nu_{I,t}(j)] = \begin{cases} 
\frac{\nu_{I,t}(j)}{\nu_{I,t}^*} - 1 & \text{if } \nu_{I,t}(j) \geq \nu_{I,t}^* \\
1 & \text{if } \nu_{I,t}(j) < \nu_{I,t}^*
\end{cases} \rightarrow \eta_{L,t}[\nu_{I,t}(j)] = G_t + 1 \tag{145}
\]

Matching coefficients of new corporate loans \([L_{t+1}(j)]\) yields:

\[
\frac{1}{R_{L,t}} = (1 - \gamma_W)E_t[\eta_{L,t+1}[\nu_{I,t+1}]] \tag{146}
\]

Combining equations (145) and (146) yield:

\[
\frac{1}{R_{L,t}} = (1 - \gamma_W)E_t[\eta_{L,t+1}(G_{t+1} + 1)] \tag{147}
\]

**D. Aggregation of Wholesale Firms’ Behavior**

Combining physical capital \(\int_0^1 K_t(j) \, dj\) across all survived wholesale firms in the end of period \(t-1\) with weight of survival probability \((1 - \gamma_W)\) and physical capital endowment \((K_{0t})\) of new wholesale firms with turnover probability \((\gamma_W)\) yield aggregate physical capital \((K_t)\) at the beginning of period \(t\), which occurs before realization of marginal efficiency of investment shocks and physical capital depreciation:

\[
K_t = (1 - \gamma_W) \int_0^1 K_t(j) \, dj + \gamma_W K_{0t} \tag{148}
\]

Since wholesale firms choose the same physical capital utilization rate \((U_{K,t})\) according to Proposition 3, equation (23) indicate that wholesale firms choose the same physical capital to labor ratio \(\frac{U_{K,i}K_i}{H_i(j)} = \frac{U_{K,i}K_i}{H_i} = \left(\frac{W_i}{\int_{0}^{1} (1 - \alpha) \nu_{A,i}}\right)^\frac{1}{\alpha}\). According to linear homogeneity property, aggregating across all labor \([H_i(j)]\) with \(j \in (0, 1)\) yields aggregate labor \((H_t)\). Substituting physical capital to labor ratio into wholesale production and aggregating across all wholesale production \([M_t(j)]\) with \(j \in (0, 1)\) yields aggregate wholesale production \((M_t)\):

\[
M_t = \int_0^1 M_t(j) \, dj = \nu_{A,i} \left(\frac{U_{K,i}K_i}{H_t}\right)^\alpha \int_0^1 H_t(j) \, dj = \nu_{A,i} \left(\frac{U_{K,i}K_i}{H_t}\right)^\alpha H_t \tag{149}
\]

Aggregating stock bubbles across all wholes firms yields aggregate stock market bubble \((O_t^*)\) in period \(t\). The weight for the \(\tau\)-period-old wholesale firm is the product of its accumulated survival probability \((1 - \gamma_W)^{t-\tau}\) to period \(t\), firm size\(^{21}\) \(\gamma_W\) and stock bubbles’ emergence probability \(\gamma_o\) in period 0.

\[
O_t^* = \sum_{\tau = -\infty}^{t} (1 - \gamma_W)^{t-\tau} \gamma_W \gamma_o O_{t-\tau} = \nu_t O_t^* \tag{150}
\]

\(^{21}\)In period \(t\), aggregate wholesale firm number=\(\sum_{\tau = -\infty}^{t} \gamma_W (1 - \gamma_W)^{t-\tau} = \gamma_W \sum_{\tau = -\infty}^{t} (1 - \gamma_W)^{t-\tau} = 1\).

In period 0, the \(\tau\)-period-old wholesale firm’s effective economic decision horizon=\(\sum_{t=0}^{\infty} (1 - \gamma_W)^t \tau = \frac{1}{\gamma_W}\).
The mass of wholesale firms with stock bubbles ($\nu_t$) evolves as a stationary recursive process in the neighborhood of its steady state ($\overline{\nu}$) as long as $(1 - \gamma_w)\overline{\nu} < 1$:

$$
\nu_t = (1 - \gamma_w)\nu_{S,t-1}\nu_{t-1} + \gamma_w \gamma_o
$$

(151)

where the initial mass of wholesale firms with stock bubbles $\nu_0 = \gamma_w \gamma_o$. The sentiment shock ($\nu_{S,t}$) influences aggregate stock market bubble ($O_t^*$) both directly and indirectly through the mass of wholesale firms with stock bubbles ($\nu_t$).

Combining Proposition 6 with the evolution of stock bubbles and aggregating across wholesale firms of all ages yield the equilibrium condition of aggregate new stock market bubble ($O_t^*$) both directly and indirectly through the mass of wholesale firms with stock bubbles ($\nu_t$).

$$
\prod_{i=1}^{\tau} \nu_{S,t-i}o_t^* = (1 - \gamma_w)E_t[\eta_{t,\tau}(G_{t+1} + 1)\prod_{i=1}^{\tau+1} \nu_{S,t+1-i}o_{t+1}^*] \rightarrow o_t^* = (1 - \gamma_w)E_t[\eta_{t,\tau}(G_{t+1} + 1)\nu_{S,t}o_{t+1}^*]
$$

(152)

Multiplying both sides of the equilibrium condition of aggregate new stock market bubble ($o_t^*$) yields the evolution of aggregate stock market bubble ($O_t^*$), preventing arbitrage conditions between aggregate new stock market bubbles in adjacent periods.

$$
\nu_t o_t^* = (1 - \gamma_w)E_t[\eta_{t,\tau}(G_{t+1} + 1)\nu_{S,t} \frac{\nu_t}{\nu_{t+1}} \nu_{t+1}o_{t+1}^*] \rightarrow O_t^* = (1 - \gamma_w)E_t[\eta_{t,\tau}(G_{t+1} + 1)\nu_{S,t} \frac{\nu_t}{\nu_{t+1}} O_{t+1}^*]
$$

(153)

Aggregating across wholesale firms’ stock prices yield aggregate stock market price ($P_{S,t}$), which includes fundamental value in terms of aggregate physical capital stock ($Q_t^* K_{t+1}$) and stock market bubble ($O_{t,t}^*$) net of aggregate liabilities ($L_t^*$).

$$
P_{S,t} = \int_0^1 P_{S,t}(j) dj = Q_t^* \int_0^1 K_{t+1}(j) dj + O_{t,t}^* - L_t^* = Q_t^* K_{t+1} + O_{t,t}^* - L_t^*
$$

(154)

Aggregation of Proposition 1 across all wholesale firms yields aggregate real investment ($I_t$):

$$
P_{I,t}I_t = P_{I,t} \int_0^1 I_t(j) dj = [(R_{K,t} U_{K,t} + Q_t^* \nu_{C,t} + \nu_{E,t}) K_t + O_t^* - L_t] \int_{\nu_{I,t} \geq \nu_{I,t}} d\Phi(\nu_{I,t})
$$

(155)

Based on law of large numbers and the independence between investment $[I_t(j)]$ and the marginal efficiency of investment shock $[\nu_{I,t}(j)]$, aggregating physical capital accumulation across all wholesale firms yields aggregate physical capital ($K_{t+1}$):

$$
K_{t+1} = \int_0^1 K_{t+1}(j) dj = (1 - \delta_t U_{K,t}) K_t + \int_0^1 I_t(j) \nu_{I,t}(j) dj
$$

$$
K_{t+1} = (1 - \delta_t U_{K,t}) K_t + I_t \frac{\int_{\nu_{I,t} \geq \nu_{I,t}} \nu_{I,t} d\Phi(\nu_{I,t})}{\int_{\nu_{I,t} \geq \nu_{I,t}} d\Phi(\nu_{I,t})}
$$

(156)
E. Derivation of Final Goods Firms’ Optimization Behavior

The representative final good firm chooses optimal continuum of retail goods \([Y_t(l)]\) with \(l \in (0, 1)\) to maximize profit \((P_{Y_t} \Gamma_{Y_t})\):

\[
P_{Y_t} \Gamma_{Y_t} = P_{Y_t} \left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}} - \int_0^1 P_{Y_t}(l) Y_t(l) dl
\]

(157)

Taking partial derivative of the representative final good firm’s profit \((P_{Y_t} \Gamma_{Y_t})\) with respect to retail good \(l [Y_t(l)]\) yields:

\[
\frac{\partial P_{Y_t} \Gamma_{Y_t}}{\partial Y_t(l)} = P_{Y_t} \nu_{Y_t} \left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}-1} Y_t(l)^{\frac{1-\nu_{Y_t}}{\nu_{Y_t}}} - P_{Y_t}(l) = 0
\]

(158)

Multiplying both sides of equation (158) by \(\left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}}\) yields:

\[
P_{Y_t} \left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}} Y_t(l)^{\frac{1-\nu_{Y_t}}{\nu_{Y_t}}} = P_{Y_t}(l) \left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}}
\]

(159)

Taking the exponential of equation (159) to the power of \(\nu_{Y_t}\) and substituting \(Y_t = \left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}}\) into it yields:

\[
P_{Y_t}^{\nu_{Y_t} Y_t^{\nu_{Y_t}}} Y_t(l)^{1-\nu_{Y_t}} = P_{Y_t}(l)^{\nu_{Y_t}} Y_t \rightarrow Y_t(l) = \left[ \frac{P_{Y_t}(l)}{P_{Y_t}} \right]^{\nu_{Y_t}} Y_t
\]

(160)

Taking the exponential of equation (160) to the power of \((\frac{1}{\nu_{Y_t}})\), integrating it from 0 to 1, taking the exponential of it again to the power of \((\nu_{Y_t})\), and substituting \(Y_t = \left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}}\) into it yields:

\[
\left[ \int_0^1 Y_t(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}} = Y_t = \left[ \int_0^1 \left[ \frac{P_{Y_t}(l)}{P_{Y_t}} \right]^{\frac{1}{\nu_{Y_t}}} Y_t^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}} = \left[ \int_0^1 P_{Y_t}(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{\nu_{Y_t}} Y_t^{\frac{\nu_{Y_t}}{\nu_{Y_t}}}
\]

(161)

Therefore, final good price \((P_{Y_t})\) is a Dixit-Stiglitz aggregator of retail prices \(P_{Y_t}(l)\) with \(l \in (0, 1)\):

\[
P_{Y_t} = \left[ \int_0^1 P_{Y_t}(l)^{\frac{1}{\nu_{Y_t}}} dl \right]^{1-\nu_{Y_t}}
\]

(162)

F. Derivation of Retail Firms’ Optimization Behavior

Retail firms face a probability of \(\gamma_P\) to adjust retail price \([P_{Y_t}(l)]\) according to the indexing rule, and incur a probability of \((1 - \gamma_P)\) to reoptimize retail price \([P_{Y_t}(l)]\) by maximizing expected retail profit:

\[
\text{Maximize}_{P_{Y_t}(l)} E_t \sum_{i=0}^{\infty} \gamma_p \eta_{t+i} Y_{t+i}(l) \left[ P_{Y_t}^{\nu_{Y_t}}(l) \prod_{i=0}^{\infty} \pi_{t+i, t+i+1}^{\xi} - P_{Y_{t+i}} P_{W_{t+i}} \right]
\]

(163)
subject to demand of retail good $l$:

$$Y_{t+i}(l) = \left[ \frac{P_{Y,t+1}^*(l)}{P_{Y,t+1}} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i}$$  \hspace{1cm} (164)$$

Substituting retail good $l$’s demand of equation (164) into (163) yields:

$$\text{Maximize} \prod_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} - P_{Y,t+i} P_{W,t+i}$$  \hspace{1cm} (165)$$

Substituting $[P_{Y,t+i}^*(l)] = \prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)$ into (165) yields:

$$\text{Maximize} \prod_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} - P_{Y,t+i} P_{W,t+i}$$  \hspace{1cm} (166)$$

Rearranging equation (166) yields:

$$\text{Maximize} \prod_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} - \sum_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} P_{Y,t+i}^* Y_{t+i} P_{W,t+i}$$  \hspace{1cm} (167)$$

Taking derivative of (167) with respect to retail good $l$’s optimal price $[P_{Y,t+1}^*(l)]$:

$$\prod_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} - \sum_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} P_{Y,t+i}^* Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} = 0$$  \hspace{1cm} (168)$$

Rearranging equation (168) and multiplying it by $[P_{Y,t+1}^*(l)]^2$ yields:

$$\prod_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} - \sum_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} P_{Y,t+i}^* Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} = 0$$  \hspace{1cm} (169)$$

Rearranging equation (169) yields:

$$\prod_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} - \sum_{i=0}^{\infty} \gamma_P^i \eta_{t+i} \left[ \frac{P_{Y,t+1}^*(l)}{\prod_{i=0}^{\infty} \pi_{t+i-1,t+i+1} P_{Y,t+1}^*(l)} \right]^{\frac{\gamma_{t+i+1}}{1-\nu Y,t+i}} P_{Y,t+i}^* Y_{t+i} \prod_{i=0}^{\infty} \pi_{t+i-2,t+i+1}^{\pi^*(1-\xi)} = 0$$  \hspace{1cm} (170)$$
Since wholesale price \((P_{W,t})\) is independent of retail firm’s age \(l\), and retail firms set marginal revenues equal to marginal cost in monopolistic competitive retail market, optimal retail price \([P^*_{Y,t}(l)]\) is also independent of retail firm index \(l\). Replacing \(P_{Y,t}(l)\) with \((P^*_{Y,t})\) and integrating over the continuum of retail firms yields:

\[
E_t \sum_{i=0}^{+\infty} \gamma_P \eta_{t,+i} P_{Y,t,+i}^{-1} - \gamma_P \eta_{t,+i} P_{Y,t,+i}^{-1} \left[ \prod_{i=0}^{+\infty} \pi_{t+i-1,t+i}^{-1} Y_{t+i} P_{Y,t} \sum_{i=0}^{+\infty} \pi_{t+i-2,t+i-1}^{-1} \pi^* \right] \pi^{*\xi} \pi_{t+1} \pi_{1} P_{W,t+1} = 0
\]

Substituting \(P_{Y,t+1} = [\int_0^1 P_{Y,t+1}^* (l) \frac{1}{1-\nu_{Y,t+1}} dl]^{1-\nu_{Y,t+1}}\) into equation (171) yields:

\[
E_t \sum_{i=0}^{+\infty} \gamma_P \eta_{t,+i} P_{Y,t,+i}^{-1} - \gamma_P \eta_{t,+i} P_{Y,t,+i}^{-1} \left[ \prod_{i=0}^{+\infty} \pi_{t+i-1,t+i}^{-1} Y_{t+i} P_{Y,t} \sum_{i=0}^{+\infty} \pi_{t+i-2,t+i-1}^{-1} \pi^* \right] \pi^{*\xi} \pi_{t+1} \pi_{1} P_{W,t+1} = 0
\]

Solving for retail good \(l\)’s optimal price \((P^*_{Y,t})\) yields:

\[
P^*_{Y,t} = \frac{E_t \sum_{i=0}^{+\infty} \gamma_P \eta_{t,+i} P_{Y,t,+i}^{-1} - \gamma_P \eta_{t,+i} P_{Y,t,+i}^{-1} \left[ \prod_{i=0}^{+\infty} \pi_{t+i-1,t+i}^{-1} Y_{t+i} P_{Y,t} \sum_{i=0}^{+\infty} \pi_{t+i-2,t+i-1}^{-1} \pi^* \right] \pi^{*\xi} \pi_{t+1} \pi_{1} P_{W,t+1}}{E_t \sum_{i=0}^{+\infty} \gamma_P \eta_{t,+i} P_{Y,t+1}^{-1} - \gamma_P \eta_{t,+i} P_{Y,t+1}^{-1} \left[ \prod_{i=0}^{+\infty} \pi_{t+i-1,t+i}^{-1} Y_{t+i} P_{Y,t} \sum_{i=0}^{+\infty} \pi_{t+i-2,t+i-1}^{-1} \pi^* \right] \pi^{*\xi} \pi_{t+1} \pi_{1} P_{W,t+1}}
\]

where \(\gamma_P\) is the probability that retail firms choose optimal prices by profit maximization. \(\eta_{t,+i}\) is nominal stochastic discount factor. \(Y_{t+i}\) is final good price. \(\nu_{Y,t+1}\) is price markup shock. \(P_{W,t+1}\) is wholesale price. \(\pi_{t+i-2,t+i-1}\) is past gross inflation. \(\pi^*\) is gross inflation target. \(\xi\) is price indexation.

G. Aggregation of Retail Firms’ Behavior

Calvo Pricing Setting

Rearranging aggregate price index in terms of final good price \((P_{Y,t})\) in equation (162):

\[
1 = \int_0^1 \frac{P_{Y,t}(l)}{P_{Y,t}}^{-1} dl
\]

Based on Calvo price-setting mechanism and the law of large numbers, \(\gamma_P\) proportion of retail firms adjust retail prices according to the indexing rule \([\pi_{t-2,j-1} \pi^{*\xi} \xi]\),
while \((1 - \gamma_P)\) proportion of retail firms reoptimize retail price \((P_{Y,t}^*)\), equation (174) is approximately equivalent to the following:

\[
1 = \gamma_P \int_0^1 \left[ \frac{\pi_{t-2,t}^{1,1}(1-\xi)P_{Y,t-1}(l)}{P_{Y,t}} \right] \frac{1}{1-\nu_{Y,t}} dl + (1 - \gamma_P) \int_0^1 \left[ \frac{P_{Y,t}^*(l)}{P_{Y,t}} \right] \frac{1}{1-\nu_{Y,t}} dl
\]  

(175)

Defining optimal final good price \((P_{Y,t}^*)\) as a Dixit-Stiglitz aggregator of optimal retail prices \(\{P_{Y,t}^*(l)\}_{l=0}^1\) yields:

\[
P_{Y,t}^* = [ \int_0^1 P_{Y,t}^*(l) \frac{1}{1-\nu_{Y,t}} dl ]^{1-\nu_{Y,t}}
\]  

(176)

Rearranging equation (175) and substituting equation (177) into it yield:

\[
1 = \gamma_P \left( \frac{\pi_{t-1,t}^{1,1}(1-\xi)}{\pi_{t-1,t}} \right) \frac{1}{1-\nu_{Y,t}} + (1 - \gamma_P) \left( \frac{P_{Y,t}^*}{P_{Y,t}} \right) \frac{1}{1-\nu_{Y,t}}
\]  

(177)

Defining indexed inflation \([\pi_{t-1,t}^{1,1} = \frac{\pi_{t-1,t}}{\pi_{t-2,t}^{1,1}(1-\xi)}]\) and substituting into equation (177):

\[
\gamma_P \left( \frac{\pi_{t-1,t}^{1,1}}{\pi_{t-1,t}} \frac{1}{1-\nu_{Y,t}} \right) + (1 - \gamma_P) \left( \frac{P_{Y,t}^*}{P_{Y,t}} \right) \frac{1}{1-\nu_{Y,t}} = 1
\]  

(178)

Substituting retail good \(l\)'s production \([Y_{t}(l)]\) of equation (16) into equation (164) and rearranging it yield:

\[
Y_{t}(l) = \nu_{A,t}[U_{K,t}(j)K_{t}(j)]^{\alpha}[\nu_{H,t}H_{t}(j)]^{1-\alpha} = \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right] \frac{\nu_{Y,t}}{1-\nu_{Y,t}} Y_{t}
\]  

(179)

Since utilized physical capital to utilized labor ratio \(\frac{U_{K,t}(j)K_{t}(j)}{\nu_{H,t}H_{t}(j)}\) is independent of wholesale firm index \(j\) and retail firm index \(l\), rearranging equation (179) and integrating over the continuum of retail firms yield:

\[
\nu_{A,t} \left( \frac{U_{K,t}(j)K_{t}(j)}{\nu_{H,t}H_{t}(j)} \right) \frac{\nu_{Y,t}}{1-\nu_{Y,t}} = \int_0^1 L_{t}(l) dl = \nu_{A,t} K_{t}^{\alpha} L_{t}^{1-\alpha} = \int_0^1 \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right] \frac{\nu_{Y,t}}{1-\nu_{Y,t}} dl Y_{t}
\]  

(180)

Defining retail price dispersion \((DELTA_A)\) as a Dixit-Stiglitz aggregator of the ratio of retail price to final good price \(\{\int_0^1 \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right] \frac{\nu_{Y,t}}{1-\nu_{Y,t}} dl\}\) yields:

\[
DELTA_A = \int_0^1 \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right] \frac{\nu_{Y,t}}{1-\nu_{Y,t}} dl
\]  

(181)

where \(\pi^*\) is inflation target. \(\xi\) is price indexation and weight for past inflation.

**Aggregate Good Production**

Substituting equation (181) into equation (180) yields aggregate production in terms of final good production \((Y_{t})\):

\[
\nu_{A,t} K_{t}^{\alpha} L_{t}^{1-\alpha} = DELTA_A Y_{t}
\]  

(182)
Retail Price Dispersion

Based on Calvo price-setting mechanism and the law of large numbers, $\gamma_P$ proportion of retail firms adjust retail prices according to the indexing rule $[\pi_{t-2, t-1}^{\xi} \pi^{1-\xi}]$, while $(1 - \gamma_P)$ proportion of retail firms reoptimize retail price $(P_{Y,t}^*)$, equation (180) is approximately equivalent to the following:

$$\int_0^1 \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right]^{\gamma_Y} \pi_Y dl = \gamma_P \int_0^1 \left[ \frac{\pi_{t-2, t-1}^{\xi} \pi^{1-\xi}}{P_{Y,t}^*} \right]^{\gamma_Y} \pi_Y dl + (1 - \gamma_P) \int_0^1 \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right]^{\gamma_Y} \pi_Y dl \quad (183)$$

Since optimal final good price $(P_{Y,t})$ is independent of retail firm index $l$, defining intermediate good price dispersion $\{\text{DELT}A_t = \int_0^1 \left[ \frac{P_{Y,t}(l)}{P_{Y,t}} \right]^{\gamma_Y} \pi_Y dl \}$ and substituting into equation (182) yield the dynamic evolution of retail price dispersion $\{\text{DELT}A_t\}$:

$$\text{DELT}A_t = \gamma_P \left[ \frac{\pi_{t-2, t-1}^{\xi} \pi^{1-\xi}}{\pi_{t-1, t}} \right]^{\gamma_Y} \pi_Y \text{DELT}A_{t-1} + (1 - \gamma_P) \left( \frac{P_{Y,t}^*}{P_{Y,t}} \right)^{\gamma_Y} \pi_Y \quad (184)$$

Substituting indexed inflation $\left[ \frac{\pi_{t-1, t}}{\pi_{t-2, t-1}^{\xi} \pi^{1-\xi}} \right]$ into equation (183) yield:

$$\text{DELT}A_t = \gamma_P \left[ \frac{\pi_{t-2, t-1}^{\xi} \pi^{1-\xi}}{\pi_{t-1, t}} \right]^{\gamma_Y} \pi_Y \text{DELT}A_{t-1} + (1 - \gamma_P) \left( \frac{P_{Y,t}^*}{P_{Y,t}} \right)^{\gamma_Y} \pi_Y \quad (185)$$

H. Derivation of Capital Good Firms’ Optimization Behavior

The representative capital good firm produces optimal investment goods $(I_t)$ to maximize its expected nominal profits, taking final good price $(P_{Y,t})$ and investment good price $(P_{I,t})$ as given:

$$\max_{\{I_{t+1}\}_{t=0}^{\infty}} E_t \sum_{i=0}^{\infty} \eta_{t,t+i} \{ P_{I,t+i} - [1 + \Omega \left( \frac{I_{t+i}}{I_{t+1}} - 1 \right)^2] P_{Y,t+i} \} \nu_{I,t+i} \nu_{I,t+i} \quad (186)$$

Taking partial derivative of capital good firm’s expected nominal profits $(L_K)$ with respect to investment goods $(I_t)$ yields:

$$\{ [1 + \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right)^2] P_{Y,t} + \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} P_{Y,t} \} \nu_{I,t} - E_t \eta_{t,t+1} \Omega \left( \frac{I_t}{I_{t-1}} - 1 \right) I_{t+1}^2 P_{Y,t+1} \nu_{I,t+1} = P_{I,t} \nu_{I,t} \quad (187)$$

Physical capital return $(R_{K,t})$ is the sum of installed physical capital’s share of production $(\alpha Y_t)$ and undepreciated installed physical capital $[(1 - \delta_K)Q_t K_t]$ divided by physical capital $(Q_{t-1} K_{t-1})$, and equals real interest rate $(R_t)$ in the steady state.

$$R_{K,t} = \frac{\alpha Y_t + (1 - \delta_K)Q_t K_t}{Q_{t-1} K_{t-1}} \quad (188)$$
References


