Price Search and Periodic Price Discounts

Xing Zhang\(^1\)
Tat Y. Chan\(^2\)
Ying Xie\(^3\)

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Abstract

This paper empirically investigates the interplay between buyer search behavior and firm pricing strategies in a B-to-B market where many firms sell the same product with similar costs. We use the heterogeneity in buyers’ search costs to explain why a firm in such a market offers periodic price discounts, even when its supply cost is stable. We propose a structural model treating the distributions of selling prices and quantity sales as equilibrium outcomes and use the maximum empirical likelihood approach to estimate the distribution of buyers’ search costs non-parametrically from prices and sales data. The model allows buyers to use an either non-sequential or sequential search strategy. We demonstrate how the sales information in addition to the price information helps us estimate the search cost distribution non-parametrically in the sequential search model and also makes the estimates more robust in the non-sequential search model. Estimation results suggest that the sequential search model out-performs the non-sequential search model in terms of fitting with the observed price distributions and true supply costs. Finally, we use a “what-if” exercise to illustrate how higher market prices in equilibrium can be supported by lower buyer search costs in our proposed model.

Keywords: Price variation, price competition, search cost, mixed pricing strategy, non-parametric empirical likelihood method

\(^1\) Xing Zhang is Assistant Professor of Marketing, Fudan University (xingzhang@fudan.edu.cn).
\(^2\) Tat Y. Chan is Associate Professor of Marketing, Washington University in St Louis (chan@wustl.edu).
\(^3\) Ying Xie is Associate Professor of Marketing, University of Texas at Dallas (ying.xie@utdallas.edu).
1. Introduction

Over the past few decades there has been an increasing interest in studying the interplay between buyer search behavior and firm pricing strategies in academic research (e.g., Stigler 1961, Rothschild 1974). Before making a purchase decision, buyers spend time and effort searching for price information. The intensity of their search activities depends on their time cost and comfort level in the process. Price search determines firms’ pricing strategies. At one extreme, firms have to compete fiercely in price if buyers search prices extensively and switch just for a slightly lower price; at the other extreme, firms can charge a monopoly price if buyers always purchase from the first firm they search. In reality, however, individual buyers have heterogeneous search costs thus their search behaviors are different. Firms are constantly facing the trade-off between selling at a high margin targeting buyers with high search costs and offering a discount so that they can also sell to those with low search costs. Past literature has shown that, when buyers have heterogeneous price search costs, firms may adopt a mixed pricing strategy at the equilibrium. This explains why we observe price dispersion in many markets where there is little differentiation among products (e.g. Burdett and Judd 1983, Rob 1985). It also explains why the same firm offers periodic price discounts instead of charging a single optimal price over time (e.g. Varian 1980, Narasimhan 1984). Periodic price discounts are commonly observed in markets; therefore, it is important to investigate how buyers search for price information and how such behavior impacts the firm’s pricing strategy.

This paper empirically investigates the pricing strategy of a firm in a B-to-B market where many firms sell the same product with similar costs. We use the heterogeneity in buyers’ search costs to explain why a firm in such a market offers periodic price discounts even when its supply cost is stable. We first show from data how the pattern of price discounts is consistent with the properties of a mixed strategy equilibrium. We then propose a full equilibrium model treating the distributions of selling prices and quantity sales as equilibrium outcomes and estimate the model from observed prices and sales data to recover the distribution of buyers’ search costs non-parametrically. We estimate the model assuming that buyers can use an either non-sequential or sequential search strategy. The non-sequential search strategy assumes that a buyer first determines the total number of searches and then chooses to buy from the firm offering the lowest price. Under the sequential search strategy, after obtaining a price quote from a firm, a buyer will decide whether to buy from the firm or to search further for lower prices. Both strategies are well studied in the economics literature (for examples, see Stigler 1961 and Burdet and Judd 1983 for
non-sequential search strategy, and Gastwirth 1976 and McCall 1970 for sequential search strategy).

Our study makes the following three contributions. First, our study empirically identify which price search strategy better describes the pattern of periodic price discounts in our data. Based on our estimation results, the two price search models offer different explanations for why selling prices on average dropped substantially over time: the non-sequential model suggests that buyers’ search costs were falling over time, while the sequential model suggests that the firm’s selling cost has been declining. To distinguish whether buyers adopt a non-sequential or sequential search strategy, we collect additional data on the supply price, which according to the owner is a good proxy for the firm’s cost, over the sample period. We find that, while the marginal cost estimates under the sequential search assumption are very close to the observed supply price, the estimates from the non-sequential model are significantly off. Since supply prices are not used in model estimation, this result provides a strong validation test in support of the sequential search model. We then show that the predictions from the sequential search model fit the observed price distribution better than those from the non-sequential model. Specifically, the former can generate a “kinked” price distribution at equilibrium which is consistent with the data. We conclude that sequential search provides a more accurate description of buyers’ price search behavior at least in the market that we study.

Second, the findings in our study can have important managerial implications. With correct inference of buyers’ search cost distribution based on our proposed framework, a firm can determine the optimal price discount schedule for the coming season accounting for the expected competitive reactions. Our modeling approach also sheds light on the important question of how prices may vary across firms and over time in different markets, which is difficult without adopting our approach. For example, intuitively one expects market prices to fall when buyers have lower search costs. We conduct a “what-if” exercise to illustrate that this may not be true. Suppose the price distributions in two different markets have the same support, but one stochastically dominates the other so that it has a lower average price. We show that, if both markets are at equilibrium, the average search cost for buyers in the market with lower average price can be actually higher. This result is consistent with the findings of Moraga-González et al. (2013) that lower search cost can lead to higher price when the heterogeneity in search costs is large enough.

Third, this study also makes a methodological contribution to the literature. We adopt the maximum empirical likelihood estimator developed by Owen (1988) to non-parametrically estimate the buyers’ search cost distribution. Previous studies in economics that adopt the same
approach (e.g. Hong and Shum 2006) have only used the information on the price distribution to estimate the model. Through a series of simulation studies, we show that, if researchers only have price information, estimates from the non-sequential search model are sensitive to the assumption on the maximum number of price searches. In contrast, by using both price and sales information our estimates are robust to the assumed maximum number. When estimating the sequential search model, by using the price information alone researchers have to make parametric assumption on the search cost distribution. With both price and sales information we can non-parametrically estimate the search cost distribution. Consequently, our estimates can fit the data very well, even when the observed prices have “kinks” in the distribution that cannot be easily rationalized with standard parametric assumptions for the search cost distribution. This helps us identify which search strategy buyers are using in different markets.

1.1 Literature Review

Since the seminal work of Stigler (1961) a rich stream of research has used the theory of consumer information search to explain cross-firm or intertemporal price dispersions in different markets. In particular, the so-called “search theoretical” models as classified by Baye et al. (2005), explicitly relate buyers’ price search behavior to the price dispersion in the market place. Among them, Salop and Stiglitz (1977) demonstrate how prices are dispersed at the equilibrium with ex ante heterogeneity in buyers’ search costs. They assume that buyers choose either to pay a fixed search cost to obtain complete information or to remain uninformed of prices. Reinganum (1979) shows that price dispersion can arise in a sequential search setting under the assumption that firms have heterogeneous marginal cost. Burdett and Judd (1983) demonstrate how the mixed pricing strategy equilibrium exists in a non-sequential search setting even when firms have same production costs. Rob (1985) considers a sequential search model in a market with identical firms assuming ex ante heterogeneity in buyer search costs, and finds that a non-degenerate price dispersion exists in equilibrium. The setting in our study is similar to Burdett and Judd (1983) and Rob (1985).

Many empirical studies, mainly using reduced-form regressions, have been conducted to test whether the theory of consumer information search is supported by data (e.g. Dahlby and West 1986, Sorensen 2000, Brown and Goolsbee 2002). Researchers also test the existence of the mixed pricing strategy by investigating the change of a firm’s position in the distribution of prices over time (e.g. Lach 2002, Baye et al. 2004, Iyer and Pazgal 2003). These studies find evidence that buyers’ price search and firms’ pricing decisions are interdependent. The development of the
“search theoretical” models facilitates the structural modeling of consumer price search and firm pricing strategy in the empirical literature. Hortacsu and Syverson (2004) use a structural approach to model investors’ information search and firms’ fee decisions in the mutual fund industry. Hong and Shum (2006) infer the consumer search cost distribution from the observed price dispersion of online booksellers. Their work is the first empirical study that estimates buyers’ search costs under the mixed pricing strategy equilibrium. A few studies have built on their modeling approach but allowed for more general conditions. Moraga-Gonzalez and Wildenbeest (2008), for example, extend the non-sequential search model to the oligopoly case. Moraga-Gonzalez et al. (2010) propose a semi-nonparametric estimation method that can identify search costs under more general assumptions. Wildenbeest (2011) incorporates vertical differentiation among firms in the model and shows that both firms’ mixed pricing strategy and product differentiation help to explain the observed price dispersion. Unlike these studies which use price information only, our estimation relies on the information on both prices and sales quantities. As we discussed above, using the additional sales information benefits the estimation of both sequential and non-sequential search models.

Testing which search strategy is employed by consumers has been an important topic in the search literature. Behavioral researchers conduct experiments to understand the search rule adopted by participants. Their findings have been equivocal: some studies find evidence that participants search sequentially (Schotter and Braunstein 1981, Harrison and Morgan 1990), while other studies find a fair number of participants have violated the no-recall prediction in sequential search (e.g., Kogut 1990). Among empirical studies that use field data, De los Santos et al (2012) utilize consumer panel data on web browsing and purchases to test the two search models. They also find that the no-recall prediction can be rejected in their data. Without observing the search sequence, Honka and Chintagunta (2014) test the two search models by using prices of the searched options in buyers’ consideration sets and find evidence in support of non-sequential search. Using a nonparametric likelihood ratio test, Chen et al (2007) find that the sequential and non-sequential search models are difficult to be distinguished when researchers only use price information. In this paper, we also test the two search models but our identification strategies are different from those used in previous studies. Furthermore, we use data from a B-to-B market in which buyers’ search behavior may be substantially different from B-to-C markets that are studied in previous empirical works.

The paper is organized as follows. Section 2 describes the data and provides evidence that the firm in data follows the mixed pricing strategy. Section 3 derives the equilibrium distributions
of prices and quantity sales under both non-sequential and sequential search models. Section 4 discusses the proposed empirical estimation model and presents results from simulation studies. Section 5 presents the estimation result and a "what-if" experiment. Finally section 6 concludes.

2. Data

We study a small firm in a Midwestern city in the US. The firm acts as an independent distributor specialized in sourcing and selling biomedical and chemical lab supplies to the science community. The firm provided us data on daily selling prices and sales quantities for each of the 655 products sold from January 2004 to June 2010. As in a typical purchase process in the B-to-B context a buyer who has needs for such supplies first obtains price quotes from sellers by phone, fax, or website. If the buyer decides to purchase from a seller, she can also place the order through one of these channels. The owner of the firm revealed to us that price bargaining is very rare in the transaction process; however, he also stated that price is the top priority for most of his customers.

Among the 655 products, we pick D-Luciferin Potassium Salt (termed “LUCK” in the industry), a chemical product that is commonly used as a bioluminescent reporter in the biotechnology industry specifically for in vivo imaging. We are told that there are about 1,300 firms in the US selling similar products. There is no quality or brand differentiation for this product in the industry. All firms acquire the product from a few major producers in the US hence no firm has a cost advantage over its competitors. The role of most firms including the firm in our study is simply a distributor selling to small business clients. Furthermore, the owner informed us that minimal processing with negligible costs is required before the firm ships the product to buyers. We also obtain the firm’s supply price (i.e., the price the firm pays to producers) for LUCK for the entire sample period.

We choose this product for our study because, firstly, it is one of the best selling products of the firm. Furthermore, we observe large daily price fluctuations in the data. LUCK is sold in five different package sizes: 100 milligram, 300 milligram, 500 milligram, 1 gram and 2 grams. Because ensuring proper environment for storage is difficult, buyers usually order the product as needed. They typically buy a single unit of the product, and choose the package size

2 The focal firm conducted a survey asking their customers to name suppliers they also considered, based on which 72 companies were identified as its competitors. All of these companies acquired product from same producers and operated at a similar scale.

3 LUCK is water soluable. It is usually stored in a freezer under -20°C and protected from light to ensure its stability. The stability of the dissolved luciferin is subject to pH value and the amount of oxgen in the solution. The common practice therefore is to purchase the product as needed.
based on the quantity needed for the project at hand. We focus our analysis on single-unit transactions in the package size of 1 gram, which accounts for 64% of all transactions. Figure 1 plots the movement of the selling prices as well as supply prices of LUCK, where dots represent daily selling prices on above and circles represent daily supply prices at below. The firm charges regular selling prices on most of the days, but also offers frequent price discounts as deep as 25% off. The supply price for LUCK only changed three times during the entire sample period. Given that other costs are negligible, the difference between the two prices indicates that, despite of the large number of suppliers in the industry, the firm is enjoying a large gross profit margin (between 40 to 45%). We divide the sample period into four periods based on the different levels of supply prices. Table 1 reports some summary statistics of selling prices for these four time periods. The median price dropped from $529 in period 1 to $299 in period 4. The lowest discounted price also decreased from $419 to $269 over the four periods.

Due to the price fluctuation, the quantity of daily sales also varies. We plot the daily number of transactions for the entire sample period in Figure 2. On 69% of the days there was no transaction and on 22% of the days there was only one transaction. Table 2 reports summary statistics of daily transactions during the four time periods. Firm’s sales have been growing steadily perhaps due to the drop in regular prices.

2.1 Mixed Pricing Strategy and Alternative Explanations for Periodic Price Discounts

Employing the mixed pricing strategy, a firm will set a pricing schedule at the beginning of the season. The schedule specifies the frequency of the regular price and price discounts at different levels, which mimics a price distribution. The actual daily price is then determined in a random manner based on this schedule. In this section, we first test several alternative explanations for the observed periodic discount pattern in daily prices and then provide a test on the intertemporal independence of daily prices as predicted by the mixed pricing strategy.

The first alternative explanation for the observed price discounts is that they are driven by changes in production or supply costs. Figure 1, however, shows that the supply price is stable with only three changes during the entire sample period. These changes seem to provide a good explanation for the movements of the median or regular prices over time as illustrated in Table 1, but they do not explain the within-period large fluctuation in daily selling prices when the supply price was constant.
The Edgeworth cycle model provides another possible explanation. The model predicts that, under the Bertrand price competition, firms will successively undercut each other’s price to compete for demand until reaching the marginal cost level, then raise the price to the highest level and start a new cycle of price cutting (Maskin and Tirole 1988). Yet, Figure 1 shows that the daily selling price fluctuates randomly between the regular price and discounted price at various depths, a pattern that is inconsistent with the Edgeworth cycle prediction. To further test the model, we compare selling prices on two consecutive days. We count how many times the daily selling price increases, decreases, or remains unchanged on the next day. The Edgeworth cycle model predicts that the frequency of price decreases should be much larger than the frequency of price increases. Yet, among the 1,404 observations, we find 99 price increases, 108 price decreases and for the remaining observations prices remain unchanged. Therefore the observed pattern of price changes is not consistent with the prediction based on the Edgeworth cycle model.

The third potential alternative explanation is related to price discrimination. The firm may offer discounts to some buyers but not to others. This practice can lead to the observed price fluctuation if the arrival of buyers is random. To test this hypothesis, we make use of the fact that the firm sells through both online and offline channels. Though the firm may offer buyer-specific discounts in the offline channel, it poses a single price on its website every day. Buyers who purchase through the website are exposed to and charged the same price on the same day; hence, price discrimination is infeasible in the online channel. We find that the standard deviation of prices online and offline are 12.33 and 13.63, respectively, which are not statistically significantly different from each other. Furthermore, through a regression analysis we find that buyer characteristics such as the type of buyer (non-profit vs. for-profit organization), the tenure length of the buyer, and the purchase channel (online vs. offline) do not have a significant effect on the purchase prices for LUCK-1g. Based on this test we believe that price discrimination is unlikely to be the cause of the observed periodic price discounts.

The final alternative explanation is that the firm offers quantity discounts to buyers. If buyers arrive in a stochastic manner, we will observe the stochastic distribution of discount prices in data. This explanation obviously does not apply to our empirical context since we only focus on 1-gram single-unit transactions. It is also plausible that price discounts are offered if buyers also purchase a bundle of other products from the firm. In our data, however, buyers rarely purchase other products in the same transaction. Only 7 out of 196 transactions for LUCK 1-gram include other products in the same order. We therefore believe quantity discounts is not the reason for the observed price fluctuation.
After ruling out the above alternative explanations, we conduct a test to examine whether the observed price variation is consistent with a unique prediction of the mixed pricing strategy that price discounts are independent over time. We use a method proposed by Ghoudi and Kulperger (2001) which can be generalized to non-parametrically test for serial independence in time series data. Under the null hypothesis of independence between two consecutive prices $p_t$ and $p_{t-1}$, a closed-form asymptotic distribution of the Cramér–von Mises test statistics can be derived (see Appendix A for details). The test statistic for our data is 0.052 with a p-value of 0.925. Therefore we can not reject the null hypothesis that prices over consecutive days are distributed independently, which is consistent with the prediction of a mixed pricing strategy.

We only obtain data from one firm, and do not observe selling prices and sales quantities of the firm’s competitors. The model developed in the next section is based on the assumption that the selling cost of LUCK is homogenous across firms and the price distribution of each firm is identical. This is a standard assumption in most of the theoretical papers that have shown the existence of a mixed pricing strategy as equilibrium outcome when consumers have heterogenous search costs (e.g., Burdett and Judd 1983, Rob 1985, Salop and Stiglitz 1977). Under this assumption, in a corresponding empirical context the price distribution observed from a single firm across time should be identical to the price distribution observed across multiple firms in a single period. Our study uses time series price and sales data from a single firm. In contrast, Hong and Shum (2006) utilized cross-sectional price data from multiple firms in a single period. We note our assumption is not more restrictive than Hong and Shum (2006), because both studies rely on the assumption that firms follow the same mixed pricing strategy at equilibrium. To examine whether we can assume firms to be homogeneous, we collect pricing data from several competitors of the firm, which are identified by the owner of the firm, for about half a year either through their websites or by calling the companies. Intertemporal price fluctuations of these firms are very similar to what we observe in the data. The price range is also similar to that of our focal firm during the same period. This offers some support that the homogeneity across firms is a reasonable assumption for our empirical context.

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4 We acknowledge that the price variation may also be driven by a change of inventory holding. With a large order coming in, the firm may temporarily lower the price to clear its inventory to save space. We cannot test this hypothesis because we do not have data on inventory. However, given that the volume of LUCK is very small (measured in milligrams and grams), we do not think that this is a plausible explanation in our empirical setting.

5 The data collection is after our sample period, therefore it cannot be used as a direct test of our data.
3. Model

We model how buyers search for price information in a market with homogenous products based on two previous theoretical works, Burdett and Judd (1983) for non-sequential search and Rob (1985) for sequential search. Both models assume a large number of firms with the same cost structure selling homogenous products and a large number of buyers each purchasing at most one unit of product in each period. To simplify the analysis, we assume that buyers have the same willingness-to-pay \( V \). This assumption is strong but reasonable in our context, considering that LUCK is a standardized chemical product. To capture the heterogeneity, we assume that each buyer has individual marginal search cost \( c \) that is distributed as \( F_c \) in the population. In such a model setting, Rob (1985) shows that a pure price equilibrium rarely exists under the sequential search assumption. Burdett and Judd (1983) also find that for non-sequential search a mixed pricing strategy exists at the equilibrium. We denote the equilibrium price distribution as \( F(p) \), and the lowest and the highest price in the distribution as \( p_l \) and \( \bar{p} \), respectively. At equilibrium, buyers know \( F(p) \) but they do not know which firm charges what price. Depending on their search cost buyers will search for prices with different degrees of intensity to minimize the total cost of search and price to pay. The equilibrium condition requires that the expected profits at all price points with positive support in the price distribution are equal. Compared with previous empirical studies that focused on the pure price equilibrium (e.g. Hortacsu and Syverson 2004), the key difference is that the first-order condition for a single optimal price cannot be used in our setting, since multiple optimal prices exist. Below we will follow Hong and Shum (2006) to derive the conditions and constraints at the mixed pricing strategy equilibrium.

3.1 Non–Sequential Search and Equilibrium Constraints

During a non-sequential search a buyer decides on the number of price quotes she will obtain at the beginning and commits to buy from the seller with the lowest price quote. Her objective is to minimize the sum of the total search cost and the expected lowest price, \( p_l \), among \( k \) price quotes. We assume that \( F(p) \) is continuous and the buyer conducts at least one search. With marginal search cost \( c \) she will choose the number of searches that minimizes the following total cost

\[
TC(k, c) = c(k - 1) + \int_{p_l}^{\bar{p}} kp (1 - F(p))^{(k-1)} f(p) dp
\]

(1)
The first term on the right side of the equation represents the total search cost (without accounting for the free first search) and the second term represents the expected lowest price to pay. As the number of searches increases, total search cost increases while the expected lowest price decreases. Let $K$ be the maximum number of searches among all buyers, and $F_c$ the distribution of search costs of all buyers. Also let $c_k$ be the cut-off point such that a buyer with search cost $c_k$ is indifferent between searching $k$ and $k + 1$ times. Equation (1) implies that $TC(k, c_k) = TC(k + 1, c_k)$. These definitions imply that $q_k$, the proportion of buyers obtaining $k$ price quotes, satisfies the following equation.

$$q_k = F_c(c_{k-1}) - F_c(c_k)$$  

(2)

For a price $p$ with positive support in the price distribution, the firm’s expected sales quantity can be derived as

$$Q(p) = \mu \sum_{k=1}^{K} q_k k(1 - F(p))^{k-1}$$  

(3)

where $\mu$ is the average sales for each firm in each period or the potential demand. At the equilibrium, for any two prices $p$ and $p'$ with positive support in the price distribution, the firm’s expected profits are equal, i.e.,

$$\pi(p) \equiv (p - r)Q(p) = (p' - r)Q(p') \equiv \pi(p')$$  

(4)

where $r$ is the marginal unit cost for the firm. If the firm charges the highest price $\bar{p}$, equation (3) suggests that the profit is $\pi(\bar{p}) = \mu(\bar{p} - r)q_1$.

We discretize the price distribution into $M$ quantiles such that $p < p_1 < \cdots < p_M = \bar{p}$. At each $p_m$, we can derive the following equation using equation (4):

$$p_m = r + \frac{(\bar{p} - r)q_1}{\sum_{k=1}^{K} q_k k(1 - F(p_m))^{k-1}}$$  

(5)

We define the empirical counterpart of $F(p_m)$ as $s_m = \frac{1}{n} \sum_{i=1}^{n} 1(p_i \leq p_m)$, where $n$ is the total number of observations and $p_i$ is an observed price in data. If the equilibrium price distribution predicted from the non-sequential search model is consistent with our data, we have the following equilibrium constraint:

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6 This is because we assume that $F(p)$ is continuous; therefore, there is zero probability that a buyer, after obtaining two or more price quotes, will find all prices to be $\bar{p}$.

7 The distribution can also be discretized based on unique price points observed in the data.
\[
E \left\{ 1(p \leq r + \frac{(\bar{p} - r)q_1}{\sum_{k=1}^{K} q_kk(1 - s_m)^{k-1}}) - s_m \right\} = 0
\]  

(6)

where \(1(.)\) is an indicator function which is equal to 1 if the logical expression inside the parentheses is true and zero otherwise.

Let \(Q_i\) be the observed sales (number of units sold) on day \(i\), the empirical distribution that the sales are smaller than \(j\) is 
\[
v_j = \frac{1}{n} \sum_{i=1}^{n} (Q_i < j),
\]
where \(j = 1 \ldots J\), and \(J\) is the maximum units of daily sales observed in the data. If the sales predicted by the non-sequential search model under the equilibrium \(F(p)\) are consistent with the data, we will have the second equilibrium constraint as follows:

\[
E \left\{ 1(Q(p) < j) - v_j \right\} = 0
\]

(7)

### 3.2 Sequential Search Model and Equilibrium Constraints

Under the sequential search assumption, a buyer decides whether or not to continue searching after a price quote is obtained. It has been shown in the literature (see Lippman and McCall 1976) that the optimal stopping rule for the buyer depends on the reservation price \(z\). If the price is below \(z\), she will accept the offer; otherwise she will continue to search for new prices. For a buyer with search cost \(c\), her reservation price is determined by the following equation:

\[
c = \int_p^z (z - p)f(p)(dp) = \int_p^z F(p)dp
\]

(8)

Let \(z(c)\) be the reservation price. Conditional on the price distribution, \(z(c)\) is an increasing function of \(c\). We define \(\mu\) as the average number of buyers requesting price quotes each day. Suppose the firm charges price \(p\). Let \(c(p)\) denote the cut-off point in the search cost distribution \(F_c\) such that \(z(c_p) = p\). The firm’s expected sales are

\[
Q(p) = \mu \left( 1 - F_c(c(p)) \right)
\]

(9)

Again we discretize the price space into \(M\) quantiles. For any price \(p_m\), let \(c_m\) be the search cost for a buyer who is indifferent between accepting the price and continuing to search, and let \(\tau_m = F_c(c_m)\) be the proportion of buyers whose reservation prices are below \(p_m\). From equation (8) we have \(c_m = \int_{p}^{p_m} F(p)dp\). To compute the equilibrium price distribution, we
differentiate both sides of the above equation with respect to \( \tau_m \) and rearrange terms. Thus we have the following equation

\[
F(p_m) = \frac{\partial c_m / \partial \tau_m}{\partial p_m / \partial \tau_m}
\]  

(10)

Let \( \alpha \) denote the proportion of buyers who will buy at the highest price \( \bar{p} \). The mixed pricing strategy equilibrium implies that

\[
p_m - r = \frac{(\bar{p} - r)\alpha}{1 - \tau_m}
\]  

(11)

Differentiating both sides with respect to \( \tau_m \), we obtain \( \frac{\partial p_m}{\partial \tau_m} = \frac{\alpha(\bar{p} - r)}{(1 - \tau_m)^2} \). We also use \( \frac{c_m - c_{m-1}}{\tau_m - \tau_{m-1}} \) to approximate \( \frac{\partial c_m}{\partial \tau_m} \). Substituting both derivatives into equation (10), we obtain the equilibrium distribution at each \( p_m \) as

\[
F(p_m) \approx \frac{c_m - c_{m-1}}{\tau_m - \tau_{m-1}} \cdot \frac{(1 - \tau_m)^2}{\alpha(\bar{p} - r) (1 - \tau_m)}
\]  

(12)

If the model is correctly specified, the predicted price distribution is an unbiased estimate of the price distribution constructed from the data. Therefore we have the following equilibrium constraint

\[
E\{1(p_i \leq p_m) - F(p_m)\} = 0
\]  

(13)

For sales, the sequential search model predicts that \( Q(p_m) = \hat{\mu}(1 - \tau_m) \). For every observed \( p_i \), if the model prediction is consistent with the data, we will have the second equilibrium constraint as follows

\[
E\{1(Q(p_i) < f) - v_j\} = 0
\]  

(14)

where \( v_j \) is the empirical distribution of sales as defined in the non-sequential search model.

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8 As a distribution function, \( F(p_m) \) has to be a non-decreasing function of \( p_m \). Also, it has to be between 0 and 1, i.e., \( 0 < c_m - c_{m-1} < \frac{(\tau_m - \tau_{m-1})^2(\bar{p} - r)}{(1 - \tau_m)^2} \). We impose these restrictions when estimating the model.
4. Model Estimation

In structural model estimation, researchers typically use optimality conditions to model the data generating process of endogenous variables which, in our case, include prices and sales quantities. When a pure price strategy equilibrium exists, optimal prices are typically based on the first-order conditions for profit maximization and, together with the assumptions regarding the unobservables (e.g. demand and cost shocks), well-established methods such as generalized methods of moments (GMM) and maximum likelihood (ML) can be applied to estimate model parameters. In this study, however, we model the data generating process under the mixed pricing strategy equilibrium assumption, where multiple or even a continuum of optimal prices may be charged. Since changing from one to another price with positive support in the equilibrium price distribution implies the same expected profit for the firm, we cannot directly employ the first-order conditions. Instead, our estimation is based on the equilibrium constraints that we derived in Section 3 under the equilibrium price distribution, $F(p)$. The GMM or ML method can still be used in the estimation, but doing so is computationally very expensive and time-consuming since under either sequential or non-sequential search $F(p)$ is a complicated function of $F_c$. More importantly, the observed price distribution in the data is usually irregular and complex. For example, a firm may charge a regular price most of the time and offer different levels of price discounts with different frequencies (see Figure 1). There may exist multiple modes in the price distribution, which is difficult to be rationalized with any assumed parametric distribution function for $F_c$. The non-parametric approach we propose in this study is useful to recover the true $F_c$ and helps to avoid model mis-specification and biased estimation. The key idea behind our estimation strategy is to maximize a joint empirical likelihood of the “events” (i.e., the daily prices and sales quantity) we observe in the data subject to the constraints that the distributions of these “events” are consistent with the equilibrium conditions.

4.1 Maximum Empirical Likelihood Estimator

We obtain the empirical distributions $s_m = F(p \leq p_m)$, where $m = 1, \cdots, M$, and $v_j = F(Q \leq j)$, where $j = 1, \cdots, J$, from data. Let $\theta$ be a vector of the structural model parameters. For the non-sequential search model we denote the parameter set as $\theta = \{q_1, q_2, \cdots, q_K, r, \mu\}$. For identification it is required that the maximum number of searches $K$ is smaller or equal to the number of quantiles in the price distribution $M$. In the sequential search model, the parameter set is $\theta = \{c_1, c_2, \cdots, c_M, \tau_1, \tau_2, \cdots, \tau_M, r, \tilde{\mu}\}$. 
Let \( w_i \) and \( \pi_i \) be the probabilities for an observed price \( p_i \) and sales quantity \( Q_i \), respectively. To estimate the model we maximize the following empirical likelihood as suggested by Owen (1988, 2001).

\[
I(p, Q) = \sum_{i=1}^{n} n \cdot \log(w_i) + \sum_{i=1}^{n} n \cdot \log(\pi_i) \tag{15}
\]

subject to

\[
\sum_{i=1}^{n} w_i = 1; \ w_i \geq 0 \ \text{and} \ \sum_{i=1}^{n} \pi_i = 1; \ \pi_i \geq 0 \tag{16}
\]

The empirical likelihoods in the estimation have to further satisfy the equilibrium constraints. For the non-sequential search model, define a set of \( M \) functions \( g_1(p_i, \theta) \), where the \( m \)-th function is

\[
1 \left( p_i \leq r + \frac{(\overline{p}-r)q_m}{\sum_{k=1}^{K} q_k(1-s_m)^{k-1}} \right) - s_m \quad \text{(see equation (6))}
\]

for each observed price \( p_i \), and define a set of \( J \) functions for quantity sales as \( g_2(Q(p_i), \theta) \), where the \( j \)-th function is

\[
1(Q(p_i) < j) - s_j \quad \text{(see equation (7))}
\]

For sequential search, define \( g_1(p_i, \theta) \) with the \( m \)-th function as

\[
1(p_i \leq p_m) - \frac{c_m-c_{m-1}}{r_m-r_{m-1}} \frac{(1-r_m)^2}{\alpha(\overline{p}-r)} \quad \text{(see equation (13))}
\]

and also define \( g_2(Q(p_i), \theta) \) with its \( j \)-th function as

\[
1(Q(p_i) < j) - s_j \quad \text{(see equation (14))}
\]

Since \( w_i \) and \( \pi_i \) are the probabilities of each event, \( g_1(p_i, \theta) \) implies a set of \( K \) constraints:

\[
E(g_1) = \sum_{i=1}^{n} w_i g_1(p_i, \theta) = 0 \tag{17}
\]

and \( g_2(Q(p_i), \theta) \) implies another set of \( J \) constraints:

\[
E(g_2) = \sum_{i=1}^{n} \pi_i g_2(Q(p_i), \theta) = 0 \tag{18}
\]

Maximizing \( I(p, Q) \) in equation (15) subject to the constraints in (17) and (18) is similar to the MPEC approach (Su and Judd 2011). Though multiple equilibria may exist in our model, our estimation approach does not require researchers to know which equilibrium is chosen. All it requires is that, conditional on structural parameters, the observed data is consistent with the necessary conditions imposed by the equilibrium. In the case of prices, the condition is that the expected profit at every price point, conditional on \( F(p) \) and \( F_c \), is equal.

In the actual estimation, we first use the Lagrangian method to solve for \( w_i \) and \( \pi_i \) by maximizing the following function:
\[ H(w, \pi, t_1, t_2, \lambda_1, \lambda_2, \theta) \]
\[ = \sum_{i=1}^{n} \log(w_i) + \sum_{i=1}^{n} \log(\pi_i) + nt_1 \sum_{i=1}^{n} w_i g_1(p_i, \theta) + n t_2 \sum_{i=1}^{n} \pi_i g_2(Q_i, p, \theta) \]
\[ + \lambda_1 \left( \sum_{i=1}^{n} w_i - 1 \right) + \lambda_2 \left( \sum_{i=1}^{n} \pi_i - 1 \right) \]

where \( t_1 = (t_{1,1}, \ldots, t_{1,M}) \) is an \( M \)-vector of Lagrangian multipliers for equilibrium constraints on the price distribution, \( t_2 = (t_{2,1}, \ldots, t_{2,J}) \) is a \( J \)-vector of Lagrangian multipliers for the equilibrium constraints on sales quantity, and \( \lambda_1 \) and \( \lambda_2 \) are the two other scalar Lagrangian multipliers.

Using the first-order conditions the optimal \( w_i \) and \( \pi_i \) can be derived from equation (19) as
\[ w_i = \frac{1}{n} \left( 1 - e^{-t_1 \cdot g_1(p_i, \theta)} \right) \]
\[ \pi_i = \frac{1}{n} \left( 1 - e^{-t_2 \cdot g_2(Q(p_i), \theta)} \right) \]

Plugging these expressions into the joint likelihood function in equation (15), we can estimate \( \theta \) as the following:
\[ \hat{\theta} = \arg\max_{\theta} \left\{ \min_{t_1} \left[ \sum_{i=1}^{n} (-\log(1 - t_1 \cdot g_1(p_i, \theta))) \right] \right. \]
\[ + \left. \min_{t_2} \left[ \sum_{i=1}^{n} (-\log(1 - t_2 \cdot g_2(Q(p_i), \theta))) \right] \right\} \]

where \( \hat{\theta} \) is our proposed Maximum Empirical Likelihood (MEL) estimator. The parameter search involves two steps. Conditional on any \( \theta \), we search for \( t_1 \) and \( t_2 \) that minimize the function value inside the squared brackets. At the outer level of the search algorithm, we search for the set of parameters \( \theta \) that maximizes the joint empirical likelihood function. Since the optimal \( w_i \) and \( \pi_i \) are substituted out, we only need to estimate \( \theta \), \( t_1 \) and \( t_2 \) using this algorithm. When the number of observations is large, this approach vastly reduces the number of parameters to be estimated.

The calculation of the standard errors for the estimator \( \hat{\theta} \) is very complicated. Following the literature (Efron 1979, Jeong and Maddala 1993), we use the bootstrap method to compute the standard errors. For each bootstrap \( b \), we randomly draw with replacement the observations of
price and sales quantity from our data and re-estimate our model to obtain an estimate $\hat{\theta}_b$. We repeat the procedure 100 times and use the estimates $\hat{\theta}_b$ to construct confidence intervals for $\theta$.

### 4.2 Simulation Studies

We conduct a series of simulation studies to examine the performance of our estimator. To illustrate the value of using both price and sales data to estimate the model, we also compare our results to another (price-only) model, assuming that we only observe the price distribution data (Hong and Shum 2006).

**Simulation Studies for Non-Sequential Search.** We assume a set of parameters $q$’s denoting the proportion of buyers conducting different numbers of searches. We also assume that the buyers’ willingness-to-pay $V$ is $499$, the firm marginal selling cost $r$ is $214$ and the average demand per day $\mu$ is one. Based on these parameters we simulate the distribution of equilibrium prices and sales quantities. We choose the “true” parameters $q_1 = 0.4$, $q_2 = 0.2$, and $q_3 = 0.4$ implying a bi-modal search cost distribution among buyers. We fix the number of quantiles in the price distribution $M = 8$ and assume that the maximum number of searches is $K = 3$.\(^9\)

The upper panel of Table 3 compares the true $q$’s to the estimates (confidence intervals we obtain from bootstrapping are in parentheses). The estimates from the price-only model and our proposed model perform equally well as the estimated $q$’s are very close to the true values. Based on the price distribution and estimated $q$’s we can infer the search cost distribution (from equation (2)) which is reported in the lower panel of Table 3. The estimated search cost distributions from both models are very close to the true one.

[Insert Table 3 Around Here]

In reality, $K$, the maximum number of searches buyers will ever make, is unknown to researchers. To test the sensitivity of the estimation results with regard to $K$, we vary it from 2 to 5 and estimate the model again using the above simulated data (where the true $K$ is 3). Table 4 reports the results. The second column of the table shows that the estimates of our proposed model which uses both prices and sales data are robust to different values of $K$: when we assume $K = 4$ or $K = 5$, the estimated proportions of buyers who search more than three times are close to zero implying that researchers can recover the true search cost distribution even under the wrong assumption of $K$. The estimates from the price-only model, however, are sensitive to the

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\(^9\) We have chosen multiple sets of $q$’s to conduct the simulation study and results are qualitatively similar.
assumption about $K$. For example, the estimate of $q_4$ (when $K = 4$ or $K = 5$) is significantly positive and $q_1, q_2$ and $q_3$ are quite different from the true parameter values. These results demonstrate that using the additional sales quantity information can help recover structural model parameters even when researchers have to make the assumption regarding $K$.

[Insert Table 4 Around Here]

**Simulation Studies for Sequential Search.** Using price information only, we do not have sufficient degrees of freedom from the data to non-parametrically estimate the search cost distribution. This is because we have to estimate $M$ search cost cut-off point $c_1, c_2, \cdots, c_M$ and their distributions $\tau_1, \tau_2, \cdots, \tau_M$, together with the marginal selling cost $r$. With price data we can use $F(p \leq p_1), \cdots, F(p \leq p_M)$, a total of $M$ observations from data, to estimate the parameters. After imposing the $M$ equilibrium constraints that expected profits are equivalent at different price points, we still do not have the degree of freedom from data that is necessary for estimating the $2M + 1$ model parameters. In practice, we find that there are different combinations of $\tau$'s and $r$ that return the same likelihood function value. To solve this problem parametric assumptions have to be imposed on the search cost distribution to reduce the dimensionality of the parameter space. That is, we assume $F_c = F_c(\theta_c)$ where the dimension of $\theta_c$ is smaller than $M$, to guarantee that there are sufficient degrees of freedom in the model estimation. In our study, however, by using sales quantity data we have additional observations $F(q \leq 1), \cdots, F(q \leq J)$. As a result, the search cost distribution can be non-parametrically identified.

We fix the number of quantiles in $F(p)$, $M$, to 8, assume 7 cut-off points, $c_1, \cdots, c_7$ in the search cost distribution and the corresponding probability mass $Pr(c_j < c < c_{j+1})$ (see Table 5). Other model parameters are set to the same values as in the simulation studies for the non-sequential search model. We then simulate the equilibrium price and quantity sales distributions under the sequential search assumption. Since we cannot non-parametrically estimate the price-only model, we focus on the results from our proposed model in which we use both price and sales information. Table 5 reports the results. The parameter estimates on search cost cut-off points and their corresponding probabilities are all very close to their true values. This exercise demonstrates how the proposed non-parametric estimation method can recover the true search cost distribution from the sequential search model.

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10 Hong and Shum (2006) also normalize the average potential demand in each day to one, since sales are not observed in their data.
5. Results

In this section, we will present estimation results first from the non-sequential model and then from the sequential search model. We observe a declining trend of selling prices in Figure 1. This trend could be due to the fluctuations in cost, or could be caused by the change in buyers’ search costs. To investigate what economic determinants that lead to the significant price decline trend, we estimate the search models separately for each of the four sub-sample periods during which a unique supply price was observed, allowing the search cost distribution and the firm’s selling cost to be period specific. We then compare the data fit of the two models and use the supply prices as a model validation check. We find that sequential search is a better model to describe our data. We will discuss how researchers may be able to distinguish sequential search from non-sequential search using the mixed pricing strategy equilibrium concept. Finally, we conduct a “what-if” analysis to show how our proposed modeling approach can shed light on some important policy issues.

5.1 Estimation Results

To estimate the non-sequential search model, we try different combinations of $K$ (2, 3, 4 and 5) and $M$ (6, 7, 8 and 9). To determine the best specification, we calculate the sum of the absolute deviations between the actual and the predicted price and sales distributions from each specification over the four data periods and select the specification that returns the smallest deviations. This model selection criterion is consistent with the principle of our non-parametric empirical likelihood estimation method that, at true parameters, the predicted distributions of the model should be consistent with the observed distributions. Based on this criterion we choose $K = 5$ and $M = 8$. We discuss below the results of the non-sequential search model with this chosen specification.

The upper panel of Table 6 reports the estimated $q$, the proportion of customers searching from one to four times, together with the estimated marginal selling cost $r$ and the average demand in each day $\mu$. The 90% confidence intervals for the estimated parameters are reported in parentheses. We then convert the estimated $q$’s to the corresponding distribution of search costs, which is reported in the lower panel of the table. The estimated search costs have dropped substantially over time. For example, in period 1 search cost is greater than $13.6 for about 90% of the buyers, while in period 4 search cost is lower than $0.50 for 46% of the buyers. Figure 3
graphically illustrates the distributions in the four periods. A distribution stochastically dominating the others (i.e., higher probability at each price level) is the one with the lowest average cost. Figure 3 shows that the average search cost has declined over time from period 1 to period 4. The reason that the distribution on the right side of the distributions cannot be further distinguished is because we observe regular prices during most of the days in the data. Most buyers purchasing at these price levels only search once, therefore, we can only make the inference that their search cost is greater than the highest cut-off point. In summary, results from the non-sequential search model suggest that the drop in the average selling prices in data (see Table 1) was driven by the decrease in buyers’ search costs.

Table 7 reports estimation results from the sequential search model. We estimate the search cost cut-off points \(c_1\) to \(c_4\) and the proportion of buyers whose costs are smaller than each of the cut-off points \(\tau_1\) to \(\tau_4\). Figure 4 graphically illustrates the search cost distributions for the four time periods. Search costs are high in period 1, however, there is no obvious evidence that search costs declined in later time periods. For example, 45% of buyers in period 2 have search cost lower than $1.36 and only 25% of the buyers in period 4 have search costs lower than $1.60. Therefore, the decrease in buyers’ search costs does not seem to be the primary driver for the decline in the average price, in constrast to what the estimation results from the non-sequential search model suggest. The main reason, instead, is the continuously declining selling cost. Estimated selling cost has dropped from $284.2 in period 1 to $178.4 in period 4, which is also about $80 and $40 lower than in period 2 and period 3, respectively. As cost decrease the firm will find it more costly to lose customers and therefore will assign a lower probability for high prices. For the non-sequential model, the estimated marginal selling cost in period 4 is higher than in period 3 (see Table 3); hence, selling cost is unlikely to be the driver of the declining price.

5.2 Identifying Buyers’ Search Strategy

The non-sequential and sequential search models offer different explanations for the price changes we observe in the data. Estimated changes in buyer search costs and the firm’s selling costs over time are significantly different between these two models. So which model provides a better description of buyer search behavior in our empirical setting? We use two methods to help answer this question. The first method relies on supply prices during the sample period that are reported in Table 1. As discussed before, supply price is a good proxy of the selling cost because not much re-processing is required for selling LUCK. This data has not been used in the model estimation
so it is useful for the validation purpose. Table 8 compares the reported supply prices and estimated selling costs from the two models. We find that in every period the estimate from the sequential search model closely matches the supply price. The trend of decline is also similar. It is interesting to note that the estimated selling costs are all just slightly higher than the supply prices. Given that the additional costs for selling LUCK are negligible, these estimates have a high face validity. The estimates from the non-sequential search model, on the other hand, seem to be far off. For example, the estimated selling cost in period 4 is about $55 higher than in period 3, yet the selling price fell from $213 in period 3 to $172 in period 4, moving in the opposite direction. In conclusion, this validation test offers a clear support for the sequential search model.

Second, we compare the in-sample data fit of the two models. Figure 5 illustrates the distribution of sales quantity observed in the data and the predicted distribution of sales quantity from the models. Both models fit with the data in a similar way, with the non-sequential model performing slightly better. The main difference is the fit with the actual price distribution as illustrated in Figure 6. The non-sequential search model predicts a “smooth” price distribution and therefore does not fit the actual price distributions during the four periods, where we observe multiple “kinks” because some prices are charged more frequently than others (e.g. in period 2 the product was priced at $425 in most days). The predicted price distribution in the sequential search model, on the other hand, replicates the kinks in the data, as the two price distributions overlap almost perfectly in each period in Figure 6.

[Insert Figures 5 and 6 Around Here]

A good fit with data from the non-parametric model estimation is not surprising given that we do not impose any parametric distributional assumption in the model. What is surprising is that the two models, both with non-parametric estimation, perform so differently. To investigate the reason we examine the properties of the predicted equilibrium price distribution in these two models. For the sequential search model, equation (12) shows the relationship between \( F(p_m) \) and the model parameters. Suppose that there is a kink in the distribution at \( p_m \). Given \( c_{m-1} \) and \( \tau_{m-1} \), one can vary \( c_m \) or \( \tau_m \) such that \( F(p_m) \) is much larger than \( F(p_{m-1}) \). Therefore the maximum empirical likelihood procedure will obtain estimates that fit the data well. For the non-sequential search model, equation (5) defines the equilibrium condition. We estimate \( q_1 \) to \( q_K \) and then infer the search cost distribution. Differentiating both sides of equation (5) with respect to \( p_m \), the density function can be derived as

\[
f_p(p_m) = \frac{[\sum_{k=1}^{K} q_k k(1 - F(p_m))^{k-1}]^2}{(\bar{p} - r)q_1 \sum_{k=2}^{K}(k-1)q_k k(1 - F(p_m))^{k-2}}
\] (21)
The density function of the equilibrium price distribution is a continuous and smooth function of all of the $q$’s. Since it is unlikely to obtain estimates from the $q$’s such that $f_p(p_{m-1})$ and $f_p(p_m)$ are very different, the sequential search model cannot replicate the kink at any price point in the data.

To further investigate whether the non-sequential search model can only predict a smooth price distribution, we conduct additional simulation studies to examine how the two search models may fit different price distributions. Figure 7 reports two results. The left panel displays a linear price distribution where each price level has the same number of occurrences and the right panel is a bi-modal distribution where high and low price levels occur more frequently. We estimate the two search models with these prices and use the estimated parameters to predict the equilibrium price distribution. Both models fit the linear distribution on the left well, but the non-sequential search model does not provide a good fit for the bi-modal distribution. This result further demonstrates that the two search strategies have different implications for the equilibrium price distribution. If some prices (e.g. regular prices) occur more frequently than the others in the data, the sequential search model fits better than the non-sequential search model.\footnote{Chen et al (2007) examined the two search strategies using textbook price data. They found that it is difficult to distinguish the two strategies in terms of data fit. There are two possible reasons that their finding is different from ours. First, the price distribution in their data may be different from what we observe in this study. If the distribution is smooth both non-sequential search and sequential search models may fit well with the data. Furthermore, since they only have price data, they have to assume a parametric search cost distribution for the sequential search model. It may be difficult to find a good candidate distribution function to fit with the price distribution in their data.}

Our findings stand in contrast to De los Santos et al (2012), who use ComScore consumer browsing and transaction data to test the recall pattern, and Honka and Chintagunta (2014), who use the pattern of actual prices in consumers’ consideration sets to test consumer shopping behavior in the auto insurance industry. Both studies find evidence in support of non-sequential search. Our findings support sequential search, perhaps because we study a B-to-B market while their studies are in B-to-C markets. Sequential search is in general more efficient than non-sequential search but, as Morgan and Manning (1985) argued, the advantage of non-sequential search is that prices can be gathered faster. It is a preferred strategy if individual consumers are sufficiently impatient. Buyers in B-to-B markets may be more patient and thus search sequentially. Another potential explanation is that the equilibrium price distribution reflects firms’ belief of how buyers search for prices. If they believe that sequential search is employed and decide on periodic price discounts accordingly, the price distribution in the data will match with the sequential search model prediction. This belief is consistent with observed sales, even
though buyers in reality may search non-sequentially. This is because the predicted equilibrium distribution of quantity sales under either search assumption is close to the observed one.

5.3 A “What-If” Analysis

The interdependence between buyers’ search behavior and firms’ pricing strategy can yield important policy implications. We use a “what-if” exercise to demonstrate its value. We focus on the sequential search model since we have shown that it fits the data better. Suppose we observe the price distribution in two markets as shown in the left panel of Figure 812. The minimum price $p_-$ ($315) and the maximum price $\bar{p}$ ($450) are the same in these two markets. However, at any price level, the number of firms that charge lower prices in market 1 is larger than in market 2. That is, the price distribution of the former stochastically dominates that of the latter. The average price thus is lower in market 1 than in market 2. Suppose that the supply cost, product quality and the number of firms in the two markets are identical. If we assume that a pure pricing strategy equilibrium exists, the only explanation for such a difference in average price is that, on average, search costs of buyers in market 1 are lower than search cost of buyers in market 2, driving firms in the former to charge lower prices.

The mixed pricing strategy equilibrium in our model, however, leads to a different conclusion. Using the price data in the two markets, we estimate the search cost distribution under the sequential search assumption. The right panel of Figure 8 reports the results. Surprisingly, the estimated search cost distribution in market 1 is stochastically dominated by market 2; hence, the average search cost in the former is actually higher than in the latter!

What drives this counter intuitive result is that, when the market is at equilibrium, the expected profit at every price level with positive support is the same. Further, when charged the minimum price all buyers will purchase. Since both markets have the same minimum price $p_-$ at any price $p^0$ (see the right panel in the Figure 8), the following equation has to hold

$$(p^0 - r)Q^1(p^0) = \overline{p} - r = (p^0 - r)Q^2(p^0)$$

where $Q^1$ and $Q^2$ are the expected sales in the two markets, respectively.13 This implies that the demand at any price $p^0$ is the same in both markets. Let $c^1(p^0)$ and $c^2(p^0)$ be the cut-off points of search cost for buyers being indifferent to continue searching or buy at $p^0$. Buyers with search cost above these cut-off points are willing to pay the price $p^0$. Expected sales are $1 –$

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12 These distributions are replicated from the observed price distribution of period 3 in our data, with manipulated changes at the region of high prices.
13 The total potential demand is normalized to one.
$F_c(c^k(p^0))$ in market $k$ which is represented by the arrowed line in the right panel of Figure 8 (same for both markets). Under sequential search we can use equation (8) to show that

$$c^1(p^0) = \int_{p^0}^p F^1(p) dp > c^2(p^0) = \int_{p^0}^p F^2(p) dp$$

where $F^1$ and $F^2$ are the price distributions in market 1 and market 2, respectively. The inequality comes from the fact that $F^1$ is always larger than $F^2$ at any price level. As $c^1(p^0) > c^2(p^0)$ and expected sales are equal, the search cost distribution in market 1 has to be stochastically dominated by the search cost distribution in market 2 as shown in the left panel of Figure 8.

This exercise shows that the common belief of lower consumer search costs leading to lower market prices can be misleading. Figure 8 demonstrates how lower search costs in market 2 can be supported by a mixed pricing strategy equilibrium with higher average market prices. To fully understand the impact of reduced consumer search costs due to, for example, the diffusion of Internet use, policy makers can use the proposed model framework of this paper to compute the possible equilibrium price distribution. Firms may also use the same algorithm to compute the optimal discount schedule as buyer search costs change. For researchers, one key take-away from this exercise is that the average price is not a sufficient statistic for comparing search costs across markets. One has to investigate how the equilibrium distribution of prices maps to the distribution of buyers’ search costs inside a market.

6. Concluding Remark

This paper empirically investigates the pricing strategy of a firm in a B-to-B market where many firms are selling the same product with similar costs. The analysis focuses on how buyers’ costly price search drives the periodic price discounts we observe in the data. We propose a structural model treating the distributions of selling prices and sales quantity as equilibrium outcomes and estimate the model from observed prices and sales to recover the non-parametric distribution of buyers’ search costs. We estimate our model under both the non-sequential and the sequential search assumption. Estimation results suggest that buyers in our data search prices in a sequential manner because the sequential search model outperforms the non-sequential search model in terms of fitting the price distribution and the true supply costs.

14 We note that multiple equilibria may exist. For example, the same search distributions in the right panel in Figure 8 may be supported by other equilibrium price distributions different from the left panel.
Making correct inference of buyers’ search costs and their price search strategy is important for firms and policy makers. A firm can use our modeling framework to schedule optimal price discounts taking into account the expected pricing responses from its competitors in an undifferentiated market setting. The mixed pricing strategy equilibrium concept also helps to explain why prices may vary across firms and over time under different market environments, which may not be possible if we adopt other modeling concepts. For example, we use a “what-if” exercise to illustrate how lower consumer search costs may be supported by higher market prices in equilibrium. This finding can have important implications on consumer welfare. We hope that the exploration in this paper will enrich and enhance our understanding on how consumers search for and how firms compete in price in different markets. We also hope more research work will be conducted on this important topic in the future.

There are several major limitations in this study. First, given a distribution of buyer search costs multiple mixed pricing strategy equilibria may arise. Computing multiple equilibria and investigating how an equilibrium is chosen are a challenging task. Furthermore, the assumption that firms are homogeneous in production costs and product quality does not apply to many markets. It is important for future research to further explore consumer search behavior and competitive outcomes when firms are differentiated. Finally, we assume that buyers have perfect information of the price distribution and there is no updating on the distribution as they obtain new price quotes. An interesting extension of our current work is to study how buyers dynamically search and learn market prices, which may have an important impact on firms’ strategic decision-making.
References


### Table 1: Summary Statistics of Daily Prices

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<th>Period</th>
<th>Number of days</th>
<th>Number of Unique Prices</th>
<th>Standard Deviation ($)</th>
<th>Median Price ($)</th>
<th>Lowest Price ($)</th>
<th>Highest Price ($)</th>
<th>Supply Price ($)</th>
<th>Average Firm Profit Margin ($)</th>
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<td>529</td>
<td>419</td>
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<td>499</td>
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</tr>
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<td>Period 4</td>
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<td>299</td>
<td>172</td>
<td>126.05</td>
</tr>
</tbody>
</table>

### Table 2: Summary Statistics of Daily Transactions

<table>
<thead>
<tr>
<th>Period</th>
<th>Number of Days</th>
<th>Number of Unique Transactions</th>
<th>Average Number of Transactions</th>
<th>Standard Deviation</th>
<th>Median Number of Transactions</th>
<th>Lowest Number of Transactions</th>
<th>Largest Number of Transactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>524</td>
<td>5</td>
<td>0.22</td>
<td>0.54</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Period 2</td>
<td>187</td>
<td>4</td>
<td>0.50</td>
<td>0.73</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Period 3</td>
<td>552</td>
<td>5</td>
<td>0.49</td>
<td>0.78</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Period 4</td>
<td>141</td>
<td>7</td>
<td>1.04</td>
<td>1.24</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

### Table 3: A Simulation Study of the Non–Sequential Search Model

<table>
<thead>
<tr>
<th></th>
<th>(\text{Pr}(k = 1))</th>
<th>(\text{Pr}(k = 2))</th>
<th>(\text{Pr}(k = 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Parameters</td>
<td>0.40</td>
<td>0.20</td>
<td>0.40</td>
</tr>
<tr>
<td>Proposed Model</td>
<td>0.40</td>
<td>0.20</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>(0.37, 0.45)</td>
<td>(0.18, 0.27)</td>
<td>(0.31, 0.46)</td>
</tr>
<tr>
<td>Price-Only Model</td>
<td>0.41</td>
<td>0.20</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>(0.35, 0.48)</td>
<td>(0.17, 0.31)</td>
<td>(0.22, 0.45)</td>
</tr>
</tbody>
</table>

Implied Search–Cost Distribution

<table>
<thead>
<tr>
<th></th>
<th>True Parameters</th>
<th>Joint Model</th>
<th>Price-Only Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Pr}(c &lt; c_1))</td>
<td>0.40</td>
<td>0.40</td>
<td>0.41</td>
</tr>
<tr>
<td>((c_1 = 13.24))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\text{Pr}(c &lt; c_2))</td>
<td>0.60</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>((c_2 = 31.47))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: (a) The number of quantiles used with price empirical likelihood \((M)\) is 8.
(b) The maximum number of searches \((K)\) is 3.
(c) 90% confidence intervals are recorded in the parentheses.
### Table 4: A Comparison of Estimates from Non–Sequential Models under Different Assumption of K

<table>
<thead>
<tr>
<th>Assumed K</th>
<th>Price-Only Model</th>
<th>Joint Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Pr(q) = (0.63,0.37)</td>
<td>Pr(q) = (0.54,0.46)</td>
</tr>
<tr>
<td>3</td>
<td>Pr(q) = (0.41,0.20,0.39)</td>
<td>Pr(q) = (0.40,0.20,0.40)</td>
</tr>
<tr>
<td>4</td>
<td>Pr(q) = (0.37,0.21,0.29,0.13)</td>
<td>Pr(q) = (0.40,0.21,0.38,0.01)</td>
</tr>
<tr>
<td>5</td>
<td>Pr(q) = (0.34,0.20,0.28,0.15,0.03)</td>
<td>Pr(q) = (0.40,0.22,0.37,0.01,0.00)</td>
</tr>
</tbody>
</table>

### Table 5: A Simulation Study of the Sequential Search Model

<table>
<thead>
<tr>
<th></th>
<th>True Parameters</th>
<th>Estimates from Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>6.00</td>
<td>6.02</td>
</tr>
<tr>
<td>$\Pr(c \leq c_1)$</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$c_2$</td>
<td>10.00</td>
<td>10.13</td>
</tr>
<tr>
<td>$\Pr(c \leq c_2)$</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>$c_3$</td>
<td>17.00</td>
<td>17.20</td>
</tr>
<tr>
<td>$\Pr(c \leq c_3)$</td>
<td>0.30</td>
<td>0.32</td>
</tr>
<tr>
<td>$c_4$</td>
<td>27</td>
<td>27.11</td>
</tr>
<tr>
<td>$\Pr(c \leq c_4)$</td>
<td>0.40</td>
<td>0.41</td>
</tr>
<tr>
<td>$c_5$</td>
<td>41</td>
<td>41.10</td>
</tr>
<tr>
<td>$\Pr(c \leq c_5)$</td>
<td>0.50</td>
<td>0.51</td>
</tr>
<tr>
<td>$c_6$</td>
<td>62</td>
<td>61.76</td>
</tr>
<tr>
<td>$\Pr(c \leq c_6)$</td>
<td>0.60</td>
<td>0.61</td>
</tr>
<tr>
<td>$c_7$</td>
<td>95</td>
<td>95.94</td>
</tr>
<tr>
<td>$\Pr(c \leq c_7)$</td>
<td>0.70</td>
<td>0.71</td>
</tr>
</tbody>
</table>
Table 6: Estimation Results of the Non-Sequential Search Model

<table>
<thead>
<tr>
<th>Period</th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$q_3$</th>
<th>$q_4$</th>
<th>Selling Cost $r$</th>
<th>Average Demand $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.46, 0.53)</td>
<td>(0.33, 0.42)</td>
<td>(0.01, 0.06)</td>
<td>(0.04, 0.09)</td>
<td>(343.14, 370.16)</td>
<td>(0.21, 0.27)</td>
</tr>
<tr>
<td>Period 1</td>
<td>0.51</td>
<td>0.40</td>
<td>0.02</td>
<td>0.05</td>
<td>349.35</td>
<td>0.22</td>
</tr>
<tr>
<td>Period 2</td>
<td>0.53</td>
<td>0.38</td>
<td>0.05</td>
<td>0.02</td>
<td>230.11</td>
<td>0.40</td>
</tr>
<tr>
<td>Period 3</td>
<td>0.71</td>
<td>0.40</td>
<td>0.058</td>
<td>0.064</td>
<td>199.64</td>
<td>0.386</td>
</tr>
<tr>
<td>Period 4</td>
<td>0.54</td>
<td>0.33</td>
<td>0.02</td>
<td>0.05</td>
<td>254.73</td>
<td>1.045</td>
</tr>
</tbody>
</table>

*Inferred Search Cost Distribution:*

<table>
<thead>
<tr>
<th>$c_1$</th>
<th>$Pr(c \leq c_1)$</th>
<th>$c_2$</th>
<th>$Pr(c \leq c_2)$</th>
<th>$c_3$</th>
<th>$Pr(c \leq c_3)$</th>
<th>$c_4$</th>
<th>$Pr(c \leq c_4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>7.56</td>
<td>0.02</td>
<td>9.64</td>
<td>0.07</td>
<td>13.64</td>
<td>0.09</td>
<td>22.69</td>
</tr>
<tr>
<td>Period 2</td>
<td>1.77</td>
<td>0.02</td>
<td>1.82</td>
<td>0.04</td>
<td>2.00</td>
<td>0.09</td>
<td>4.89</td>
</tr>
<tr>
<td>Period 3</td>
<td>1.98</td>
<td>0.06</td>
<td>2.55</td>
<td>0.10</td>
<td>4.63</td>
<td>0.16</td>
<td>12.99</td>
</tr>
<tr>
<td>Period 4</td>
<td>0.47</td>
<td>0.03</td>
<td>0.48</td>
<td>0.07</td>
<td>0.49</td>
<td>0.13</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: We assume that $M = 8$ and $K = 5$. Only estimates for $q_1, q_2, q_3, q_4$ are reported. $q_5 = 1 - \sum_{k=1}^{4} q_k$.

Table 7: Estimation Results of the Sequential Search Model

<table>
<thead>
<tr>
<th>Period</th>
<th>$c_1$</th>
<th>$Pr(c \leq c_1)$</th>
<th>$c_2$</th>
<th>$Pr(c \leq c_2)$</th>
<th>$c_3$</th>
<th>$Pr(c \leq c_3)$</th>
<th>$c_4$</th>
<th>$Pr(c \leq c_4)$</th>
<th>Selling Cost $r$</th>
<th>Average Potential Demand $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>2.50</td>
<td>(0.10,0.17)</td>
<td>13.61</td>
<td>(0.43,0.49)</td>
<td>26.12</td>
<td>(0.49,0.53)</td>
<td>43.24</td>
<td>(0.53,0.58)</td>
<td>284.19</td>
<td>0.68</td>
</tr>
<tr>
<td>Period 2</td>
<td>1.36</td>
<td>(0.20,0.56)</td>
<td>1.58</td>
<td>(0.35,0.65)</td>
<td>23.87</td>
<td>(0.43,0.71)</td>
<td>59.76</td>
<td>(0.53,0.77)</td>
<td>263.30</td>
<td>1.25</td>
</tr>
<tr>
<td>Period 3</td>
<td>1.19</td>
<td>(0.04,0.27)</td>
<td>6.60</td>
<td>(0.34,0.51)</td>
<td>19.79</td>
<td>(0.40,0.57)</td>
<td>45.20</td>
<td>(0.49,0.63)</td>
<td>221.52</td>
<td>0.99</td>
</tr>
<tr>
<td>Period 4</td>
<td>1.19</td>
<td>(0.10,0.14)</td>
<td>1.42</td>
<td>(0.02,0.28)</td>
<td>1.49</td>
<td>(0.07,0.30)</td>
<td>1.61</td>
<td>(0.05,0.32)</td>
<td>178.39</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Table 8: Comparing Supply Prices and Estimated Selling Costs

<table>
<thead>
<tr>
<th>Period</th>
<th>Supply Price($)</th>
<th>Selling Cost from Sequential Search Model($)</th>
<th>Selling Cost from Non-Sequential Search Model($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 1</td>
<td>284</td>
<td>284.19</td>
<td>349.35</td>
</tr>
<tr>
<td>Period 2</td>
<td>242</td>
<td>263.30</td>
<td>230.11</td>
</tr>
<tr>
<td>Period 3</td>
<td>213</td>
<td>221.52</td>
<td>199.64</td>
</tr>
<tr>
<td>Period 4</td>
<td>172</td>
<td>178.39</td>
<td>254.73</td>
</tr>
</tbody>
</table>
Figure 1: Time Series Plot of Daily Price and Supply Price of LUCK (1 Gram)

Figure 2: Time Series Plot of the Daily Transactions of LUCK (1 Gram)
Figure 3: Estimated Search Cost Distribution of the Non-Sequential Search Model

Note: Middle curve in each period represents the estimated search cost distribution, and the upper and lower curves represent the 90% confidence intervals.

Figure 4: Estimated Search Cost Distribution of the Sequential Search Model

Note: Middle curve in each period represents the estimated search cost distribution, and the upper and lower curves represent the 90% confidence intervals.
Figure 5: Predicted Sales from Non-Sequential and Sequential Search Model and Actual Sales

Figure 6: Predicted Prices from Non-Sequential and Sequential Search Models and Actual Prices
Figure 7: The Fit of Non-Sequential and Sequential Search Model in a Simulation

Figure 8: Price and Search Cost Distribution in Two Markets