Consumer Search Activities and the Value of Ad Positions in Sponsored Search Advertising

Tat Y. Chan and Young-Hoon Park*

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* Tat Y. Chan is Associate Professor of Marketing at the Olin School of Business, Washington University, St. Louis, MO 63130; phone: (314) 935-6096; fax: (314) 935-6359; email: chan@wustl.edu. Young-Hoon Park is Sung-Whan Suh Professor of Management and Associate Professor of Marketing at the Samuel Curtis Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853; phone: (607) 255-3217; fax: (607) 254-4590; email: yp34@cornell.edu. Earlier versions of this paper were distributed under the titles “Position Competition in Sponsored Search Advertising” and “The Value of Consumer Search Activities for Sponsored Search Advertisers”. The authors would like to thank the company, which wishes to remain anonymous, that provided the data used in this study.
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Abstract

Consumer search activities can be endogenously determined by the ad positions in sponsored search advertising. We model how advertisers compete for ad positions in sponsored listing and conditional on the list of sponsored ads, how online consumers search for information and make purchase decisions. On the consumer side, assuming that users browse information from top to bottom and adopt a sequential search strategy, we develop a two-stage model of consumer search (whether to click and whether to stop the search) that extends the standard sequential search framework in economics literature. On the advertiser side, it is very difficult to fully specify the optimal strategies of advertisers because the equilibrium outcome depends on variables that researchers do not observe. As we have an “incomplete” model of advertiser competition, we propose using the necessary condition that, at equilibrium, no advertiser will find another available ad position more valuable than the one it has chosen. Using a dataset obtained from a search engine, we find that consumers can be classified into two segments which exhibit distinct search behaviors. For advertisers, the value of search advertising comes primarily from terminal clicks which represent the last link (including organic results) clicked by an online user. We also demonstrate that the value of ad positions depends not only on the identities and the positions of the advertisers in sponsored listing but also the composition of online consumers who exhibit distinct search behaviors.

Keywords: Search advertising, Advertiser value, Advertising competition, Impressions, Clicks, Terminal clicks, Moment inequality
1. Introduction

Sponsored search advertising, like other advertising formats, can impact different stages of consumer decision-making process. When running sponsored search advertising campaigns, some advertisers might seek to increase their brand awareness and preferences from online users and thus value consumers’ browsing and clicking their sponsored ads. Other advertisers, on the other hand, could target optimizing the purchase conversions from consumers’ browsing and clicking behavior. The likelihoods which a consumer browses a sponsored ad, clicks the link, and purchases at the advertiser’s website vary across different ads in sponsored listing. They could depend on not only the identities and the positions of the advertisers in sponsored listing but also the characteristics of the consumers who may have different information needs and thus exhibit different search behaviors. By investigating how advertisers value consumers’ browsing, clicking, and purchasing behaviors, and what factors drive these types of consumer search activities in keyword search, one can advance our understanding of the underlying mechanism that drives the willingness-to-pay of advertisers for ad positions in search advertising markets.

This paper proposes a methodology to address the questions described above. We first model how consumers search on a search results page. Assuming that users browse information from top to bottom and adopt a sequential search strategy, we develop a two-stage model of consumer search (whether to click and whether to stop the search) that extends the standard sequential search framework in economics literature (McCall 1970). This structural model of consumer search allows us to estimate the likelihoods of three levels of consumer search activities, (1) effective impression, (2) click and, (3) terminal click, from data for each advertiser at each ad position. An effective impression implies that a user indeed browses the ad. A click indicates that the user not only browses the ad but also clicks the sponsored link to search for
information, which may enhance the consumer awareness and goodwill for the advertiser. A *terminal click*, which is a subset of clicks, represents the last link (including organic results) clicked by the user. If the user takes an action (e.g., purchase) following the search query, a terminal click is a precondition that indicates the link the user has chosen for the action. We also adopt a latent class approach to capture the heterogeneity in consumer search behavior; consumers in different segments may exhibit different click-through rates (CTR) and terminal click-through rates (TCTR). Furthermore, we explicitly model how advertisers at different ad positions compete against one another for consumer search activities. Our approach thus differs from standard “position competition” models in the literature (e.g., Athey and Ellison 2011), which assume that the value of an ad position is exogenous and independent from other positions.

Using estimation results from the consumer search model as an input, we employ a “revealed preference” approach to infer advertisers’ values for the three types of consumer search activities from their decisions of paying the prices for ad positions. To this end, we estimate a model of advertiser competition with a finite number of ad positions. The main methodological challenge for estimating such a model is that the optimal strategies of advertisers are very difficult to fully specify, mainly because the equilibrium outcome depends on variables that researchers do not observe. For example, the order of advertisers’ bidding for a keyword or paying prices for ad positions may impact which advertisers take what ad positions but such an order is typically not in data. We therefore have an “incomplete” model. To tackle this challenge, we propose using the necessary condition that, at equilibrium, no advertiser will find another available ad position more valuable than the one it has chosen. Derived from this condition, we construct lower bounds for the advertiser’s payoff function for a search query, and use the
method of moment inequalities (Pakes et al. 2014) in model estimation. This method allows us to estimate the advertiser competition model without imposing restrictive assumptions on how equilibrium outcomes are generated.

We apply the proposed methodology to estimate consumer search activities and advertiser values from a unique dataset obtained from a leading search engine in Korea. The search engine considered in this research adopts the cost-per-impression (CPM) pricing mechanism, and all advertisers purchase ad positions through exercising the buy-it-now (BIN) option in position auctions. Although we only study one particular keyword in a specific context, the proposed methodology can be used for a wide range of empirical applications. The two-stage consumer search model, for example, can be applied to other online and offline contexts where consumers decide in a sequential manner where to search for product or price information and where to purchase. The model helps predict what attracts website traffic (or store visits) and what drives purchase conversions, as well as how consumers differ in the search process. Moreover, the advertiser competition model and its estimation have general applications, allowing researchers not to make restrictive assumptions on the data-generating process. In the market of online display advertising, for example, ad slots are limited and the price for each slot is predetermined by publishers. Which advertisers get what ad slots at equilibrium depends on the order that advertisers make purchase decisions, but such order is typically unobserved by researchers. In this research, we take the path of utilizing necessary conditions for the equilibrium in model estimation and apply it under fixed prices in the context of sponsored search advertising. It can also be applied to other pricing mechanisms, including CPM, cost-per-click (CPC), and generalized second-price (GSP) auction mechanism that is commonly adopted
by search engines such as Google and Yahoo!. It is with ease to construct lower bounds for the advertiser’s payoff function based on the bids submitted from the advertiser and its competitors.

Applying the proposed methodology to a keyword in search advertising, we show that consumers can be classified into two segments which exhibit distinct search behaviors. Users in the larger, low-involvement segment are less likely to click sponsored links, and once they do, they are more likely to stop the search. In contrast, users in the smaller, high-involvement segment are more likely to click multiple links and less likely to stop the search. We also find that the value of an ad comes from terminal clicks only, neither from effective impressions nor from non-terminal clicks. Although our results may not be generalized to all other advertisers, it illustrates that TCTR can be an important metrics that search engines should provide to sponsored advertisers. Finally, we highlight how the value of a search query for advertisers depends on not only the identities and positions of the advertisers in sponsored listing but also the composition of online consumers who exhibit different search behaviors.

1.1 Literature Review

The commercial success of sponsored search advertising has motivated substantial research studying its various aspects. Theoretical research has offered important insights about optimal bidding strategies for sponsored links and appropriate designs for auction mechanisms (e.g., Edelman et al. 2007, Varian 2007, Katona and Sarvary 2010, Desai et al. 2014). Such literature typically treats each ad position as an auction item whose value is exogenously given. Recognizing this as a major limitation, a few studies (e.g., Athey and Ellison 2011, Chen and He 2011, Jerath et al. 2011) examine how the ad value comes from consumer information search that is determined by ad positions. Other studies (e.g., Aggarwal et al. 2008, Das et al. 2008, Kempe and Mahdian 2008) employ statistical approaches to capture the effect of competition
from other ads on the probabilities of consumer browsing and clicking an ad position. Different from these streams of research, this paper develops a structural model of consumer search that can be empirically applied. Our search model helps describe and predict how ad positions compete for effective impressions, clicks, and terminal clicks from consumers. While the model derives probabilities of these search activities in a manner similar to the click model in Aggarwal et al. (2008), the structural approach is useful when we use counterfactuals to estimate the bounds for advertiser values. Furthermore, empirical research in marketing (e.g., Ghose and Yang 2009, Agarwal et al. 2011, Rutz and Bucklin 2011, Rutz et al. 2012) have documented the effects of ad positions on consumer clicks and conversions. Due to data limitation, these studies do not consider the competition effect like we do; this paper thus extends the literature by studying how advertisers compete for consumer search activities in search advertising markets.

Economics literature has investigated both non-sequential (e.g., Stigler 1961) and sequential (e.g., McCall 1970, Rothschild 1974) search, in which the latter is in general considered as a better strategy since it does not require pre-commitment on the number of searches. Yet some researchers offer empirical evidence in support of both sequential (Zhang et al. 2012) and non-sequential (De los Santos et al. 2012, Honka and Chintagunta 2014) search, while others propose an optimal sequential process (Kim et al. 2010). In this tradition, our two-stage search model represents a modified version of the standard sequential search framework.

Empirical research has developed useful methodologies to study the advertiser competition in keyword auctions. Yao and Mela (2011), for example, model competition for ad positions and investigate strategic behaviors by advertisers. Athey and Nekipelov (2012) propose a homotopy-based method to compute equilibrium outcomes when advertisers face uncertainty and develop a means to estimate the advertiser value. Due to the complexity of modeling the
equilibrium conditions in GSP auctions (see Edelman et al. 2007), several empirical studies have adopted an approach proposed by Haile and Tamer (2003) to estimate the bounds of advertiser values for multiple ad positions (e.g., Edelman and Ostrovsky 2007, Varian 2007). Our approach of estimating the bounds of advertiser values is similar to Haile and Tamer (2003). While previous works treat the advertiser value for each ad position as exogenously given, we investigate how the value is determined by consumer search activities, which are endogenously determined by the competition across ad positions. This paper therefore represents a further development in the literature by considering both the consumers’ and advertisers’ decisions. Finally, we model how advertisers compete for ad positions, which is close in spirit to the location competition where inter-firm competition is resulted from the substitutability of demand among neighboring firms (e.g., Thomadsen 2005, Seim 2006, Chan et al. 2007). Yet our empirical context is rather more complicated. With an incomplete competition model, the equilibrium conditions are difficult to fully specify, so we adopt a unique estimation strategy.

The rest of the paper is organized as follows. We develop the consumer search model in section 2. In section 3, we describe the advertiser competition model in sponsored search advertising, characterize the necessary condition for equilibrium outcomes, and develop the estimation methodology. Section 4 describes the data of our empirical application, and discusses the results and implications. We conclude with some directions for future work in section 5.

2. Consumer Search

As an overview of our modeling approach, we model a two-stage game. In the first stage, advertisers decide what ad positions to purchase using auction or fixed price format. The decisions are based on their valuations of three types of consumer search activities (effective
impressions, clicks, and terminal clicks) that each ad position can attract. Consumers arrive in the second stage and decide how to optimally search for information on the listed sponsored links. After browsing a sponsored link, a consumer first decides whether to click the link to search for more information at the advertiser’s website and, if she does, then decides whether to terminate the search. This section develops the structural model of consumer search that helps infer the effective impressions, clicks, and terminal clicks each ad position will attract. In section 3, we build the model of advertiser competition and discuss the estimation strategy.

To develop the consumer search model, we make a few assumptions about how a consumer behaves in keyword search. We assume that she responds to search listings sequentially, starting with the sponsored ad listed at the topmost position, then the one at the second position, and so on. After searching for information in each sponsored link, the consumer starts to browse the organic results if she continues the search. Consistent with standard sequential search literature in economics (e.g., McCall 1970, Gastwirth 1976), we assume the consumer does not revisit links that has been clicked.

2.1 A Model of Sequential Search

Conditional on consumer $i$ browsing advertiser $j$’s link listed at a certain position, we assume that she makes decisions in two steps: whether to click the link and whether to stop the search. The selling propositions of an ad, which typically consist of a few phrases or words describing the products and offers to consumers, provide the consumer partial information about what would be revealed if she clicks the link. If she does not click the link, the consumer browses the next sponsored or organic links; if she clicks, the consumer finds full information from the advertiser’s website. If she does not stop the search, the consumer browses sponsored or organic
In day $t$, the utility of advertiser $j$’s offering for consumer $i$ who searches is:

$$u_{ijt} = X_{jt} \beta + w_{ijt},$$

(1)

where $X_{jt}$ is a vector of observed attributes at advertiser $j$’s sponsored link, including the advertiser’s identity and the selling propositions, that we assume to be exogenous. The stochastic component $w_{ijt}$ captures the advertiser’s offering to the consumer, unobserved to researchers, that may have different fits with individual needs. For example, if advertiser $j$ promotes running shoes that the consumer is searching for, $w_{ijt}$ will be higher than the average.

After browsing the link, the consumer observes the selling propositions. They may include other information ($I_{jt}$), unobserved to researchers, on the products and offers to consumers. For example, it may advertise that running shoes are on promotion at the website; the consumer’s expected utility will increase. Still, she has to click into the website to find out the exact prices for different models or styles of running shoes, and whether the advertiser carries the models or styles that she likes. We assume that the consumer uses the information to form her expectation $e_{ijt} \equiv E[w_{ijt}|X_{jt}, I_{jt}]$. Her expected utility after browsing the link will be:

$$E[u_{ijt}|X_{jt}, I_{jt}] = X_{jt} \beta + e_{ijt}.$$

(2)

If the consumer clicks the link and search for information, $w_{ijt}$ will be revealed. Defining $\varepsilon_{ijt} \equiv w_{ijt} - e_{ijt}$, Equation (1) can be rewritten as:

$$u_{ijt} = X_{jt} \beta + e_{ijt} + \varepsilon_{ijt}.$$

(3)

We assume that consumers are rational in forming expectations. $\varepsilon_{ijt}$ represents a shock to the consumer which has zero mean (i.e., no systematic bias) and is uncorrelated with $e_{ijt}$; otherwise she has not fully used the information. Define $F_X$, $F_e$, and $F_\varepsilon$ as the distributions of $X_{jt}$, $e_{ijt}$, and
\( \varepsilon_{ijt} \), respectively. We also assume that the distributions, but not the exact values, are known by the consumer prior to the search.

We assume an individual-specific marginal cost \( c_{it}^1 \) to browse a link, equal to the time and cognitive effort the consumer exerts to process the ad, and this cost remains constant throughout the search. We also assume a marginal cost \( c_{it}^2 \) to click a link, which remains constant during the search. The two search costs differ in the time and cognitive costs required for consumer decision-making. If a consumer clicks a link, she incurs both \( c_{it}^1 \) and \( c_{it}^2 \) because she would have browsed the link before clicking it.

Our model constitutes a variation of the dynamic sequential search model in economics (e.g., McCall 1970). We define \( V(c_{it}, c_{it}^2; X_{jt}, e_{ijt}) \) as the consumer’s value of the optimal search decision in the first stage, conditional on search costs \((c_{it}, c_{it}^2)\) and attributes \((X_{jt}, e_{ijt})\) obtained from browsing advertiser \( j \)’s link. We also define \( W(c_{it}, c_{it}^2; X_{jt}, e_{ijt}, e_{ijt}) \) as the value of the optimal search decision in the second stage, conditional on clicking advertiser \( j \)’s link and observing \( \varepsilon_{ijt} \). After clicking the link, the consumer decides whether to stop the search by maximizing the value in the following Bellman’s equation:

\[
\tilde{W}(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt}) = \max \{X_{jt} \beta + e_{ijt} + \varepsilon_{ijt}, EV_{it}\},
\]

where \( EV_{it} = \int_{X',e'} V(c_{it}^1, c_{it}^2; X', e')dF_X(X')dF_e(e') \) is the consumer’s expectation of the value of continuing the search, measured by integrating out the attributes \((X', e')\) of the next link that are yet to be observed by the consumer. The value of the optimal search in the second stage is

\[
W(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, e_{ijt}) \equiv \tilde{W}(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, e_{ijt}) - c_{it}^2.
\]

If we define \( z_{it}^2 = EV_{it} \) as the reservation utility for stopping the search, we can apply a simple optimal stopping rule in the second stage: If \( u_{ijt} \geq z_{it}^2 \), the consumer stops; otherwise, she continues the search. We assume that \( F_X, F_e, \) and

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$F_\epsilon$ are continuous and independent across advertisers. Previous research on sequential search models has shown that a unique $z^2_{it}$ exists under these conditions (e.g., McCall 1970).

Unlike sequential search models in literature, our approach also characterizes the optimal decision in the first stage. Conditional on browsing advertiser $j$’s link, the consumer has yet observed $\varepsilon_{ijt}$ and must decide whether to click the link by maximizing the expected utility through the following Bellman’s equation:

$$
V(c^1_{it}, c^2_{it}; X_{jt}, e_{ijt}) = \max \{ \int_{\epsilon} W(c^1_{it}, c^2_{it}; X_{jt}, e_{ijt}, e_{ijt})dF_\epsilon(e_{ijt}), EV_{it} \}.
$$

That is, if she clicks, the expected utility is $\int_{\epsilon} W(c^1_{it}, c^2_{it}; X_{jt}, e_{ijt}, e_{ijt})dF_\epsilon(e_{ijt})$, the value of optimal search in the second stage as defined in Equation (4) after integrating out $\varepsilon_{ijt}$; otherwise, the user continues to search the next link with an expected value of $EV_{it}$. The value of the optimal search decision in the first stage thus is $V(c^1_{it}, c^2_{it}; X_{jt}, e_{ijt}) = \tilde{V}(c^1_{it}, c^2_{it}; X_{jt}, e_{ijt}) - c^1_{it}$.

Given $z^2_{it}$, let $z^1_{it}$ be the reservation utility of clicking the link such that:

$$
-c^2_{it} + \int_{\tilde{z}^-_{it}}^{\infty} (z^1_{it} + e_{ijt})dF_\epsilon(e_{ijt}) - z^2_{it} \cdot [1 - F_\epsilon(z^2_{it})] = 0.
$$

In Appendix A, we derive the optimal stopping rule in the first stage: If $\bar{u}_{ijt} \equiv X_{jt} \beta + e_{ijt} \geq z^1_{it}$, the consumer clicks the link. Given $z^2_{it}$, Equation (6) offers a unique solution, so a unique reservation utility $z^1_{it}$ exists.

2.2 Model Estimation

Under the sequential search assumption, the distribution functions $F_X$, $F_\epsilon$, and $F_\epsilon$ for the next link that the consumer has not browsed remain unchanged in the expectation function during the search process. We also assume that the distributions are the same for every consumer. The heterogeneity in reservation utilities $z^1_{it}$ and $z^2_{it}$ is driven by the difference in consumer’s search costs. We show in Appendix A that the higher the search cost $c^2_{it}$, the higher is $z^1_{it}$ in the first
stage. It is also easy to see that the higher the search cost $c_{it}^1$, the lower is $z_{it}^2$ in the second stage.

To estimate the sequential search model, we adopt an approach similar to Kiefer and Neumann’s (1979), using a reduced-form approximation for $z_{it}^1$ and $z_{it}^2$, as follows:

$$
\begin{align*}
z_{it}^1 &= Z_{it} \gamma_1 + \nu_{it}^1 \quad \text{and} \quad z_{it}^2 = Z_{it} \gamma_2 + \nu_{it}^2,
\end{align*}
$$

(7)

where $Z_{it}$ is a vector of consumer characteristics and time-related variables of consumer searches, including month and weekday dummies that may influence consumer search costs and utility. The stochastic components $\nu_{it}^1$ and $\nu_{it}^2$ capture the unobserved individual-specific effects.\(^1\)

For notational simplicity, we observe $K$ sponsored links in day $t$, where $j = 1, \ldots, K$ represents the advertiser at the $j$-th position (“1” for the topmost, “2” for the second position, and so on). Let $y_{ijt}$ be an indicator that equals 1 if consumer $i$ clicks advertiser $j$’s link, and 0 otherwise. Let $y_{itol}$ be an indicator that equals 1 if consumer $i$ clicks any of organic links. For the data on consumer search activity in the full list, the assumption of top-down sequential search implies (1) an effective impression on link $j$ if $y_{ikt} = 1$ for $k \geq j$, or $y_{itol} = 1$; (2) a click on link $j$ if $y_{ijt} = 1$; and (3) a terminal click on link $j$ if $y_{ijt} = 1$ and $y_{ikt} = 0$ for all $k > j$, and $y_{itol} = 0$. From the optimal decision rules contained in our model, two conditions must be satisfied for a terminal click: $e_{ijt} - v_{1it} \equiv \tilde{\omega}_{ijt}^1 \geq Z_{it} \gamma_1 - X_{jt} \beta$ and $e_{ijt} + \epsilon_{ijt} - v_{2it} \equiv \tilde{\omega}_{ijt}^2 \geq Z_{it} \gamma_2 - X_{jt} \beta$.

For a non-terminal click, $\tilde{\omega}_{ijt}^1 \geq Z_{it} \gamma_1 - X_{jt} \beta$ and $\tilde{\omega}_{ijt}^2 < Z_{it} \gamma_2 - X_{jt} \beta$. If the consumer browses but chooses not to click, $\tilde{\omega}_{ijt}^1 < Z_{it} \gamma_1 - X_{jt} \beta$.

We next build a general formulation of the likelihood function. Define consumer $i$’s search as $y_{it} = \{y_{i1t}, y_{i2t}, \ldots, y_{iKt}; y_{itol}\}$. From definition, $\tilde{\omega}_{ijt}^1$ and $\tilde{\omega}_{ijt}^2$ are driven by the

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\(^1\) Since $z_{it}^1$ and $z_{it}^2$ capture the consumer’s expectation of the attributes from links that she has not browsed, as well as her search costs, $z_{it}^1$ and $z_{it}^2$ are not a function of $X_{jt}$ of the sponsored link that she has browsed.
advertiser’s offers (observed before and after clicking the link) and the consumer’s individual-specific preferences and search costs that are unobserved to researchers. Under general conditions these error terms may correlate with each other and across sponsored links. They may also correlate across consumers who browse link $j$ in the same day. To account for these correlations, we specify $\tilde{\omega}_{ijt}^1$ and $\tilde{\omega}_{ijt}^2$ as follows:

$$\tilde{\omega}_{ijt}^1 = \omega_{jt}^1 + \omega_{it}^1 + \omega_{jp}^1 \quad \text{and} \quad \tilde{\omega}_{ijt}^2 = \omega_{jt}^2 + \omega_{it}^2 + \omega_{jp}^2,$$

where $\omega_{jt}^1$ and $\omega_{jt}^2$ represent advertiser-specific attributes observed by consumers at time $t$, but not by researchers; $\omega_{it}^1$ and $\omega_{it}^2$ refer to the consumer preferences and time-cost shocks common to all sponsored links; and $\omega_{ijt}^1$ and $\omega_{ijt}^2$ are the idiosyncratic shocks, i.i.d. across consumers and sponsored links.

Suppose we observe $N$ consumers in day $t$. Let $\mathbf{y}_t = (y_{i1}, y_{i2}, \ldots, y_{iN})$ be the collection of consumer searches. From the assumptions for stochastic terms in Equation (8), we can write the likelihood of $\mathbf{y}_t$ as:

$$\Pr(\mathbf{y}_t) = \int \ldots \int \prod_{i=1}^{N} \left\{ \Pr(y_{i1} | \omega_{it}^1, \omega_{it}^2, \ldots, \omega_{it}^j, \omega_{it}^1; \omega_{jt}^1, \omega_{jt}^2) \right\} dF^1(\omega_{jt}^1, \omega_{jt}^2) \prod_{j=1}^{K} dF^2(\omega_{jt}^1, \omega_{jt}^2),$$

(9)$^2$

where $F^1$ is the joint distribution function of consumer-specific errors $(\omega_{it}^1, \omega_{it}^2)$, and $F^2$ is the joint distribution function of advertiser-specific errors $(\omega_{jt}^1, \omega_{jt}^2)$. In this expression, the errors are integrated out first in the evaluation of search likelihood.

We differentiate the search into two different types: one in which the search ends at the $j$-th link such that for all $k > j$, $y_{ikt} = 0$ and $y_{i0t} = 0$, and the second in which the search ends at an

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$^2$ For analytical simplicity, we assume that for advertiser $j$, $\omega_{jt}^1$ and $\omega_{jt}^2$ are independent over time. This assumption appears reasonable because we include indicators to capture advertiser-fixed effects as part of $X_j$. We assume each search was conducted by a separate consumer and that $\omega_{it}^1$ and $\omega_{it}^2$ are i.i.d. across consumers.
organic link such that \( y_{i0t} = 1 \). The conditional likelihood \( \Pr(y_{it} \mid \omega_{i1}^1, \omega_{i2}^2, \ldots, \omega_{Kt}^1, \omega_{Kt}^2; \omega_{it}^1, \omega_{it}^2) \) in Equation (9) can be written as:

\[
\Pr(y_{it} \mid \omega_{i1}^1, \omega_{i2}^2, \ldots, \omega_{Kt}^1, \omega_{Kt}^2; \omega_{it}^1, \omega_{it}^2) = \prod_{k=1}^{K} \left[ \Pr(\omega_{ikt}^1 \geq Z_{ikt} \gamma_1 - X_{ikt} \beta - \omega_{ikt}^1 - \alpha_1^1 \text{ and } \omega_{ikt}^2 < Z_{ikt} \gamma_2 - X_{ikt} \beta - \omega_{ikt}^2 - \alpha_2^1) \cdot 1\{y_{ikt} = 1\} \right] \\
+ \Pr(\omega_{ikt}^1 < Z_{ikt} \gamma_1 - X_{ikt} \beta - \omega_{ikt}^1 - \alpha_1^1) \cdot 1\{y_{ikt} = 0\} \\
\times \Pr(\omega_{ijt}^1 \geq Z_{ijt} \gamma_1 - X_{ijt} \beta - \omega_{ijt}^1 - \alpha_1^1 \text{ and } \omega_{ijt}^2 < Z_{ijt} \gamma_2 - X_{ijt} \beta - \omega_{ijt}^2 - \alpha_2^1) \\
\prod_{k=1}^{K} \left[ \Pr(\omega_{ikt}^1 \geq Z_{ikt} \gamma_1 - X_{ikt} \beta - \omega_{ikt}^1 - \alpha_1^1 \text{ and } \omega_{ikt}^2 < Z_{ikt} \gamma_2 - X_{ikt} \beta - \omega_{ikt}^2 - \alpha_2^1) \cdot 1\{y_{ikt} = 1\} \right] \\
+ \Pr(\omega_{ikt}^1 < Z_{ikt} \gamma_1 - X_{ikt} \beta - \omega_{ikt}^1 - \alpha_1^1) \cdot 1\{y_{ikt} = 0\} \right] \right) \tag{10}
\]

The first four lines in Equation (10) represent the first type of search,\(^3\) whereas the other lines represent the second type. Because \( \omega_{ikt}^1 \) and \( \omega_{ikt}^2 \) may correlate, the functions in this equation must be jointly estimated. For day \( t \), we evaluate the likelihood \( \Pr(y_i) \) by simulating \( NS \) times of \( \omega_{jt}^1 \) and \( \omega_{jt}^2 \) from \( F^2 \) for link \( j \), as well as simulating \( NS \) times of \( \omega_{it}^1 \) and \( \omega_{it}^2 \) from \( F^1 \) for consumer \( i \), following the distribution assumptions for those stochastic terms.\(^4\)

We posit that consumers who search the keyword come from several different latent segments (Kamakura and Russell 1989) which allow for the heterogeneity of search behavior among users. For a user who belongs to segment \( q \), her search behavior is captured by the segment-specific parameters \( (\beta^q, \gamma_1^q, \gamma_2^q) \).\(^5\) Based on those parameters, we can evaluate the simulated segment-specific likelihood function \( \Pr(y_{it} \mid q) \). Let \( Q \) be the number of consumer

\[^3\] A consumer, after the last click, may continue browsing additional links below without making further clicks. As a result, the likelihood function may be incomplete because it does not incorporate this behavior. The omission of this component from the likelihood, however, is unlikely to affect the result under (i) the i.i.d. assumption of error terms across links and (ii) the assumption that advertisers do not treat these additional browsing activities as effective impressions.

\[^4\] We assume \( F^1 \) and \( F^2 \) are joint normal distributions and fix \( NS \) to be 1,000 in the estimations.

\[^5\] We do not estimate \( c_{it}^1 \) and \( c_{it}^2 \) in the model. These parameters affect \( z_{it}^1 \) and \( z_{it}^2 \) in Equation (7). Estimated \( z_{it}^1 \) and \( z_{it}^2 \), together with estimated \( \beta \), give us the predicted probabilities of effective impression, click and terminal click.
segments and $\pi_q$ be the probability with which a consumer is in segment $q$. The model estimates 

$$(\beta^1, \gamma^1, \gamma^2; \ldots; \beta^0, \gamma^0, \gamma^0)$$

maximize the full likelihood function

$$\prod_{i,s} \sum_q (\pi_q \cdot \Pr(y_{it} | q))$$.

With the estimation results, we compute the probabilities of effective impression, click, and terminal click, for a specific link, as shown in Appendix B. It allows us to infer the value per effective impression, click, and terminal click for each advertiser.

2.3 Discussion of Model Assumptions

We have made a few important assumptions in the model of consumer search. These assumptions may be quite restrictive in describing consumer search behavior and, if they are invalid, the model is misspecified and estimation results could be biased. The top-down search assumption has been well supported in the literature. In particular, using eye-tracking data, Granka et al. (2004) show that online consumers generally investigate a list of ranked results from top down. Hoque and Lohse (1999) and Ansari and Mela (2003) also find supportive evidence in other contexts. Under the assumption that the distributions $F_X$ and $F_w$ are i.i.d. across advertisers, no other browsing strategies (e.g., bottom to top, random) will dominate top-down search. We thus believe that this is a reasonable assumption which is consistent with the optimal search strategy. This assumption has also been made in economics literature (e.g. Aggarwal et al. 2008, Das et al. 2008, Kempe and Mahdian 2008).

Sequential search models have been widely studied in economics (e.g., McCall 1970, Rothschild 1974, Gastwirth 1976, Arbatskaya 2007). These models assume no updating for the distribution of advertiser offers (e.g., product, price) from previous searches, which can be justified if consumers have sufficient search experience and knowledge of the distribution. Hence, consumers would not revisit a link that has been previously clicked. Several behavioral studies indicate that consumers process alternatives sequentially (e.g., Saad and Russo 1996,
We use these assumptions to infer effective impressions and terminal clicks. Although competing models, such as non-sequential (finite sample) search and sequential search with updates are available, they cannot help identify consumer search activities if the entire sequence of consumer clicks on sponsored and organic links is not observed. Suppose the actual sequence of clicks is observed; researchers can test the assumptions of no-revisit and top-down browsing with data. Still, a structural model of consumer search is needed to infer how advertisers value consumer search activities in the advertiser competition model, which is discussed in the next section. As shown in section 4, this model fits the data considered in this study very well. If advertisers use these assumptions to approximate consumer search behaviors and guide their decisions in search advertising, we can treat it as a good as-if model, since our main objective is to infer the advertiser values for consumer search activities.

We also note that the infinite horizon assumption in our sequential search model is an approximation that users typically find a large number of relevant results on the search results page. Since the probability of terminating the search at any link is positive, the probability that a user will continue the search quickly declines as more links have been browsed. We expect that the predicted search behavior is not too different from a finite horizon model. Finally, consumers likely take actions following their search, although different consumers arrive at the search engine with different information objectives. As data from search engines do not include post-click conversion behavior of consumers, we cannot distinguish purchase from no-purchase

---

6 Sequential search is a better strategy than non-sequential search, as long as users are not too impatient in getting results. Honka and Chintagunta (2014) test the two search models based on the prices of the searched options in buyers’ consideration set, and find evidence in support of non-sequential search. De Los Santos et al. (2012) test the no-revisit assumption from the sequential search model using the Comscore clickstream data. Although the test rejects the sequential search model, their data reveals that, among the 10% of transactions in which consumers visited more than one bookstore, 62% bought from the last one in the search sequence, which is consistent with the predictions from the proposed model. Only 38% revisited a previously searched bookstore. For the rest 90% of transactions, consumers only visited one bookstore. This suggests that the no-revisit assumption may be a reasonable approximation of the consumer search behavior in reality.
behavior following a search query. The value per terminal click for advertisers therefore constitutes the expected value, accounting for the probability of no purchase.
3. **Advertiser Competition**

Given the consumer search model, this section proposes a strategy to estimate an “incomplete” model of advertiser competition with a finite number of ad positions, in which the optimal strategies of players are not fully specified because of the complicated nature of the game. We focus on the necessary conditions that at equilibrium no advertisers can increase their payoffs by changing strategies. Based on the necessary conditions we develop an estimation strategy to infer how advertisers value the three types of consumer search activities. It can be applied to various pricing mechanisms, including fixed prices and GSP auctions, and CPM and CPC price formats.

Suppose that \( J \) advertisers compete for \( K \) ad positions, \( J > K \). We denote \( A_{jt} \) as the ad position that advertiser \( j \) acquires at day \( t \), where “1” indicates the topmost position. If the advertiser does not acquire any ad position, \( A_{jt} = 0 \). Let \( \bar{A}_t = (A_t, A_{t+1}, \ldots, A_T) \) be the collection of positions which advertisers purchased. Let \( c(A_{jt}) \) be the cost of advertiser \( j \) for a search at day \( t \) that depends on the ad position. Under CPM, \( c(A_{jt}) \) is the price the advertiser pays for a search query which may or may not bring an effective impression; under CPC, it is the price paid for a click, multiplied by the probability that the consumer will click the sponsored link. If the advertiser chooses not to advertise, \( c(A_{jt}) = 0 \).

Given \( \bar{A}_t \), let \( \pi_j^1 \) and \( \pi_j^2 \) be advertiser \( j \)’s expected value per effective impression and click, respectively. We denote the value per terminal click as \( \pi_{jt}^3 = \pi_j^3 + \xi_j \), where \( \pi_j^3 \) is the average value over the data period and \( \xi_j \) captures the time-varying profit shock for the advertiser. If a manufacturer runs a trade promotion in day \( t \), for instance, the margin of the advertiser selling the manufacturer’s products is higher; \( \xi_j \) is positive. This stochastic term may
be serially correlated and correlated across advertisers (e.g., the manufacturer offers the same promotion to all advertisers), and by definition $E(\xi_{jt}) = 0$ for advertiser $j$.

Let the probability of browsing link $j$ (i.e., effective impression), conditional on the nominal positions $\widetilde{A}_j$ and a vector of variables $S_t$, including observed attributes $X_{jt}$ of all advertisers and the set of variables $Z_{it}$ that affects reservation utilities, be $P^1_j(\widetilde{A}_j, S_t)$. Let the probability of clicking link $j$ be $P^2_j(\widetilde{A}_j, S_t)$. Finally, let the probability of the click being the terminal click be $P^3_j(\widetilde{A}_j, S_t)$. The three probabilities for advertiser $j$ at an ad position $L$ can be derived from (A4)-(A6) in Appendix B. If $A_{jt} = 0$, $P^1_j(\widetilde{A}_j, S_t) = P^2_j(\widetilde{A}_j, S_t) = P^3_j(\widetilde{A}_j, S_t) = 0$. The payoff function of advertiser $j$ for acquiring ad position $A_{jt}$ from a search query is:

$$V_j(\widetilde{A}_j, S_t) = P^1_j(\widetilde{A}_j, S_t) \cdot \pi^1_j + P^2_j(\widetilde{A}_j, S_t) \cdot \pi^2_j + P^3_j(\widetilde{A}_j, S_t) \cdot (\pi^3_j + \xi_{jt}) - c(A_{jt}).$$

(11)

If $A_{jt} = 0$, $V_j(\widetilde{A}_j, S_t) = 0$.

We assume that given $\widetilde{A}_j$, the probabilities of search activities attracted by each advertiser as well as the advertiser’s valuation of the activities are common knowledge. However, other advertisers only know the distribution, not the exact value of $\xi_{jt}$. Advertiser $j$ has to maximize its expected value in Equation (11) when it makes the purchase decision.

It is difficult to characterize the optimal strategies that advertisers employ in such a game. Advertisers in reality make their purchase decisions of ad positions in a sequential manner. We therefore think of the competition as a sequential game. The order of moves, however, is stochastic and unobserved by researchers. When ad positions are sold under the fixed-price

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7 We assume the advertiser cannot generate effective impressions, clicks, or terminal clicks from organic results. From the data that we use to estimate the model, we find little overlap between the two sets of search listings, which also differ in nature (i.e., information-based links in organic results versus commercial links in sponsored results).
format, suppose advertiser $j$ chooses $A_{jt}$ which generates the highest profit to the advertiser. This assumption may be invalid because the advertiser may want to but can’t buy another ad position as it has been taken by a competitor who moved earlier. When ad positions are sold by GSP auctions, advertisers may change bids frequently so that the advertiser competition can be treated as an infinitely repeated game. The optimal strategies are very complex since it does not have an equilibrium in dominant strategies. Furthermore, depending on how advertisers strategically change bids, the game has a large set of equilibria (Edelman et al. 2007). Because of this problem, it is challenging to estimate the game from the data to infer advertiser values.

3.1 Equilibrium Conditions and Inequalities

Our estimation strategy follows the recent literature in economics (e.g., Andrews et al. 2007, Chernozhukov et al. 2007, Pakes et al. 2014) that infers model parameters from “incomplete” econometric models. Researchers search for a set of model parameters (which can be a singleton) that are consistent with the necessary conditions for equilibrium. Any value within the set is an acceptable candidate for the estimated parameters. Although less precise than traditional econometric methods that generate point estimates, this approach allows for a model to be estimated without requiring that optimal strategies of players are fully specified.

We adopt the method of moment inequalities, as proposed by Pakes et al. (2014), in model estimation. This method applies a lower computational burden than other methods (e.g., Andrews et al. 2007, Chernozhukov et al. 2007), and it does not rely on the distributional assumption of the error term (i.e., the profit shock $\xi_{jt}$). Therefore, $\xi_{jt}$ can be serially correlated and correlated across advertisers. The consistency of estimators requires only that the

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8 This game can be applied to other contexts, e.g., online display advertising, and even traditional advertising media (e.g., TV, print) in which ad slots are sold at prices predetermined by publishers.
expectation of $\xi_{jt}$, conditional on instrumental variables, is zero. Estimation results are robust to various behavioral assumptions about the data-generation process in empirical context.

To adopt such estimation strategy, we assume that the ad positions acquired by advertisers observed in data are at equilibrium. We use this assumption and construct bounds for the payoff function in Equation (11) to estimate model parameters. We first derive the bounds under fixed prices and then GSP auctions. Under fixed prices, we assume that at equilibrium no advertiser can obtain a higher value by switching to an alternatively available position, including no advertising.\(^9\)

Conditional on observed $\tilde{A}_t$, let $A_{jt}^U$ be an alternative higher position for advertiser $j$ and $A_{jt}^L$ be an alternative lower position for it, which also includes no advertising. Let

$$\tilde{A}_{jt}^{U:j} = (A_{jt},...,A_{jt},...,A_{jt})$$ be the vector of advertisers’ positions in the higher position scenario, where advertiser $j$ moves up to position $A_{jt}^U$ and all others remain at their original positions.

Similarly, let $\tilde{A}_{jt}^{L:j} = (A_{jt},...,A_{jt},...,A_{jt})$ be the alternative scenario in the lower position scenario for advertiser $j$. Under the assumption, two conditions must be satisfied to ensure the observed advertisers’ final choices are at equilibrium. The first (higher position scenario) is as follows:

i. Suppose $A_{jt}^U$ exists. The value of $A_{jt}$ to advertiser $j$ is greater than the value of $A_{jt}^U$. That is, $V_j(\tilde{A}_t, S_t) \geq V_j(\tilde{A}_{jt}^{U:j} S_t)$.

Because $P_j^3(\tilde{A}_t, S_t) \leq P_j^3(\tilde{A}_{jt}^{U:j}, S_t)$, we can derive the following inequality condition:

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\(^9\) This assumption ignores the possibility of a class of perfect Bayesian equilibria, in which an advertiser may choose not to pay for a more valuable position because it expects that competitors will respond aggressively such that it will end up losing. It also does not allow collusive behavior among advertisers. Furthermore, it requires that the cost of switching does not dominate the increased profit of switching to a more valuable position.
\[
\frac{P_j^1(\bar{A}_t, S_t) - P_j^1(A_{t-j}^U, S_t)}{P_j^3(\bar{A}_t, S_t) - P_j^3(A_{t-j}^U, S_t)} \cdot \pi_j^1 + \frac{P_j^2(\bar{A}_t, S_t) - P_j^2(A_{t-j}^U, S_t)}{P_j^3(\bar{A}_t, S_t) - P_j^3(A_{t-j}^U, S_t)} \cdot \pi_j^2 - \frac{c(A_{t-j}) - c(A_{t-j}^U)}{[P_j^3(\bar{A}_t, S_t) - P_j^3(A_{t-j}^U, S_t)]} \geq \xi_{\mu}.
\]

(12)

If \( A_{t-j} = 0 \), i.e., advertiser \( j \) is not currently advertising, the inequality can be simplified to:

\[
\frac{P_j^1(\bar{A}_t, S_t)}{P_j^3(\bar{A}_t, S_t)} \cdot \pi_j^1 - \frac{P_j^2(\bar{A}_t, S_t)}{P_j^3(\bar{A}_t, S_t)} \cdot \pi_j^2 + \frac{c(A_{t-j})}{P_j^3(\bar{A}_t, S_t)} \geq \xi_{\mu}.
\]

(13)

The second necessary equilibrium condition, similar to the first, is as follows:

ii. Suppose \( A_{t-j} \) exists. The value of \( A_{t-j} \) to advertiser \( j \) is greater than the value of \( A_{t-j}^U \).

That is, \( V_j(\bar{A}_t, S_t) \geq V_j(A_{t-j}^U, S_t) \).

Because \( P_j^1(\bar{A}_t, S_t) \geq P_j^3(\bar{A}_t^U, S_t) \), we can derive the following inequality condition:

\[
\frac{c(A_{t-j}) - c(A_{t-j}^U)}{[P_j^3(\bar{A}_t^U, S_t) - P_j^3(\bar{A}_t, S_t)]} - \frac{P_j^1(\bar{A}_t, S_t) - P_j^3(A_{t-j}^U, S_t)}{P_j^3(\bar{A}_t, S_t)} \cdot \pi_j^1 + \frac{P_j^2(A_{t-j}^U, S_t) - P_j^3(A_{t-j}^U, S_t)}{P_j^3(\bar{A}_t, S_t)} \cdot \pi_j^2 + \pi_j^3 \geq -\xi_{\mu}.
\]

(14)

If \( A_{t-j}^U \) means no advertising, Equation (14) can be simplified:

\[
\frac{P_j^1(\bar{A}_t, S_t)}{P_j^3(\bar{A}_t, S_t)} \cdot \pi_j^1 + \frac{P_j^2(\bar{A}_t, S_t)}{P_j^3(\bar{A}_t, S_t)} \cdot \pi_j^2 + \pi_j^3 \geq -\xi_{\mu}.
\]

(15)

In GSP auctions, conditional on observed \( \bar{A}_t \), let \( A_{t-j}^U \) be one position higher than advertiser \( j \)'s current position, and let the alternative scenario \( \bar{A}_t^U \) denote that advertiser \( j \) exchanges the position with the advertiser at \( A_{t-j}^U \), while other ad positions remain the same. We make use of the following condition to ensure that \( \bar{A}_t \) is at equilibrium:

i. An advertiser cannot improve its playoff by exchanging bids with the advertiser ranked one position above (i.e., \( A_{t-j}^U \)).

This is the “locally envy-free” assumption in Edelman et al. (2007). This condition suggests that:

\[
V_j(\bar{A}_t, S_t) \geq V_j(A_{t-j}^U, S_t) = P_j^1(\bar{A}_t^U, S_t) \cdot \pi_j^1 + P_j^2(\bar{A}_t^U, S_t) \cdot \pi_j^2 + P_j^3(\bar{A}_t^U, S_t) \cdot (\pi_j^3 + \xi_{\mu}) - c(A_{t-j}^U),
\]

(16)
which will derive the same inequality as in Inequality (12).

Let \( b_{jt} \) be the observed final bid of the advertiser and \( c(b_{jt}) \) be the implied cost for each search query if the advertiser pays for the bid. Under CPM \( c(b_{jt}) \) is equal to \( b_{jt} \) and under CPC it is equal to \( P_j^2(\tilde{A}_j, S_j) \cdot b_{jt} \). In GSP auctions it is optimal for bidders to shade their bids. We therefore assume that at equilibrium the following condition will hold:

ii. Advertisers do not bid more than their valuation for ad positions.

This condition helps us construct an inequality that is analogous to Inequality (15):

\[
\frac{P_j^1(\tilde{A}_j, S_j)}{P_j^1(\tilde{A}_j, S_j)} \cdot \pi_{j1}^1 + \frac{P_j^2(\tilde{A}_j, S_j)}{P_j^2(\tilde{A}_j, S_j)} \cdot \pi_{j2}^2 + \pi_{j3}^3 - \frac{c(b_{jt})}{P_j^3(\tilde{A}_j, S_j)} \geq -\xi_{jt}.
\]  

\[(17)\]^{10}

3.2 Boundaries and Inequalities

Suppose \( A_{jt} = 0 \). Advertiser \( j \) cannot move to a lower position, \( A_{jt}^L = \phi \). Inequalities (14) and (15) cannot be constructed. Since the advertiser does not submit a bid, Inequality (17) also cannot be used. We are at a boundary condition. These observations cannot be ignored from model estimation; otherwise we have the selection issue and our estimates will be biased. When there is only one boundary, Pakes et al. (2014) propose using the symmetry assumption for the distribution of \( \xi_{jt} \). One may infer the bounds for the \( \xi \)'s when \( A_j = 0 \) from the distribution of high-valued \( \xi \)'s, under which \( A_j \neq 0 \) and one can derive upper bounds. This procedure is valid as long as the distribution of \( \xi \) is symmetric. In our case, however, we may have two-way boundaries: In

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10 This assumption is motivated by the concept of using the “Generalized English Auction” to approximate how in GSP auctions advertisers will converge to a long-run steady state in Edelman et al. (2007). They show that in such an auction the bid an advertiser submits will be lower than its valuation. Haile and Tamer (2003) estimate the bounds of the distribution of the valuation of bidders in English auctions, using two assumptions: (a) Bidders do not bid more than their valuation and (b) bidders do not let opponents win at a price they are willing to pay. These assumptions are very similar to the equilibrium conditions we use here for GSP auctions. Varian (2007) models the auction as a simultaneous move game with complete information and derives bounds for advertisers’ value of each consumer click based on the Nash equilibrium assumption. He constructs a bound based on the payoff that an advertiser can obtain by moving to lower positions, giving an inequality that is different from Inequality (17).
some periods, the advertiser cannot move down, and in other periods it cannot move up. The latter occurs when the advertiser is at the topmost position or, in the context of fixed prices when all higher positions are occupied by other advertisers. To address this problem, we assume that the value per terminal click is never negative for advertisers such that \( \pi^3_j = \pi^2_j + \xi_j \geq 0 \) for all \( t \). This means that to maximize profits, an advertiser never sells at a loss, and the value per terminal click is never negative. We then obtain the following inequality:

\[
\pi^3_j \geq -\xi_j, \tag{18}
\]

providing us the upper bound for the (negative of) low-valued \( \xi \)'s. When \( A_{jt} = 0 \), we use this inequality instead of Inequalities (14), (15), or (17). We then follow Pakes et al. (2014), assuming that the distribution of \( \xi_{jt} \) is symmetric with mean 0 which implies that the maximum of \( \xi_{jt} \) is equal to the maximum of \(-\xi_{jt} \). As \( \pi^3_j \geq \max_t (-\xi_{jt}) \geq \xi_{jt} \), this gives the following inequality:

\[
\pi^3_j \geq \xi_{jt}, \tag{19}
\]

providing us the upper bound for high-valued \( \xi \)'s. When \( A^T_{jt} = \phi \), we use this inequality instead of Inequalities (12) and (13).

Combining Inequalities (12), (13), and (19), and given \( E(\xi_{jt}) = 0 \), we recognize that for \( \bar{A} \) to be an equilibrium, we have the following inequality condition:
For GSP auctions, we combine Inequalities (12) and (19) to obtain a similar condition.

Combining Inequalities (14), (15), and (18), we obtain another inequality condition:

\[
\frac{1}{T} \sum_{j} \left[ \frac{P_j(A_j, S_j) - P_j(A_{-j}, S_j)}{P_j(A_{-j}, S_j)} \cdot \pi^1_j + \frac{P_j^2(A_j, S_j) - P_j^2(A_{-j}, S_j)}{P_j^2(A_{-j}, S_j)} \cdot \pi^2_j \right] + \sum_{A_j \neq \phi} \left[ \frac{c(A_j) - c(A_{-j})}{\kappa \cdot P_j^3(A_{-j}, S_j)} - \pi^3_j \right] \geq 0 \quad \text{as } T \to \infty
\]
\( \Delta^i V(\bar{A}_i, \bar{A}^{U,i}_i, S, \theta_j) \) denote the left-hand side of Inequality (21), where \( \bar{A}^{U,i}_i \) and \( \bar{A}^{L,i}_i \) are the collections of \( \bar{A}^{U,i}_i \) and \( \bar{A}^{L,i}_i \), respectively.

For the model estimation, we use instruments \( W_{jt} \), which provide a vector of variables such that \( E(\xi_j | W_{jt}) = 0 \). Define

\[
\Delta V(\bar{A}_i, \bar{A}^{U,i}_i, S, \theta_j) = [\Delta^U V(\bar{A}_i, \bar{A}^{U,i}_i, S, \theta_j) | \Delta^L V(\bar{A}_i, \bar{A}^{L,i}_i, S, \theta_j)], \quad \text{and} \quad W_{jt} = \begin{pmatrix} W_{jt} & 0 \\ 0 & W_{jt} \end{pmatrix}.
\]

Let

\[
m(W_{jt}, \theta_j) = W_{jt} ' \Delta V(\bar{A}_i, \bar{A}^{U,i}_i, S, \theta_j) \quad \text{and} \quad P_T m(W_{jt}, \theta_j) = \frac{1}{2T} \sum_{t=1}^{T} m(W_{jt}, \theta_j). \]

Furthermore,

\[
(D_{jt}^{1/2} P_T m(W_{jt}, \theta_j))_+ = \min\{0, (W_{jt} ' W_{jt})^{-1/2} P_T m(W_{jt}, \theta_j)\}. \]

The value thus is negative if the inequality conditions are violated, and zero otherwise. The estimator of the method of moment inequalities can be obtained from minimizing the violation of the inequality conditions, as follows:

\[
\hat{\theta}_j = \arg \min_{\theta \in \Theta} (D_{jt}^{1/2} P_T m(W_{jt}, \theta_j)) ' (D_{jt}^{1/2} P_T m(W_{jt}, \theta_j))_-
\]

There is no guarantee that \( \hat{\theta}_j \) is a singleton in the parameter space because there may be a set of estimates all can satisfy Inequalities (20) and (21). Therefore, we search for the set estimates in the parameter space.

Finally, to calculate the standard errors for estimators, Pakes et al. (2014) offer an analytical asymptotic distribution, though it cannot be applied directly because of the complexities in our study. First, \( \xi_j \) may be serially correlated and correlated across advertisers within any period. Second, we must account for the errors of our estimates in the first-stage

\[\text{\footnotesize{11}} \] We can include in \( W_{jt} \) all variables in \( X_{jt} \) (advertiser identities and selling propositions) in Equation (3), and variables in \( Z_{it} \) (month and day of week indicators) in Equation (7).
consumer search model, which affect the calculation of $P_j^1(\tilde{A}, S_t)$, $P_j^2(\tilde{A}, S_t)$, and $P_j^3(\tilde{A}, S_t)$. There is no existing analytical distribution that accounts for these issues. Instead, we propose using a semi-parametric overlapping-block bootstrapping procedure to calculate the standard errors for $\hat{\Theta}_j$. The estimation and bootstrapping procedures are discussed in Appendix C.

4. An Empirical Application

4.1 Data

We apply the proposed methodology of estimating the models of consumer search and advertiser competition to a dataset obtained from a leading search engine firm in Korea. We observe which sponsored ads are displayed in response to the consumer’s search query, and which sponsored ads are clicked. We also have data on whether the consumer clicks on any organic links. However, we observe neither the sequence of clicks nor the post-click conversion behavior.

The search engine uses CPM pricing, and offers potential advertisers up to five ad positions in the sponsored section of the search results page. Ad positions are sold through first-price position auctions with BIN option for each position. In our data, advertisers always exercised BIN options to acquire ad positions, implying that the market features are essentially fixed prices. Wang et al. (2008) show that, when the BIN option is adopted in auctions, the resulting price format at equilibrium could be fixed prices (i.e., buyers always exercise the BIN option) if participation costs for advertisers are sufficiently high. They also show that the BIN option can increase sellers’ revenue, relative to the pure auction mechanism. Other studies that

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12 Auctions open at 9:00 a.m. and close at 4:00 p.m. each day. All ad positions are available, as long as they have not been sold or the day has not passed. The BIN option remains in effect throughout the auction, as long as it has not been exercised. Advertisers can bid for any unsold ad position auction, and may buy a position for multiple days. We do not have data on the time at which advertisers make their decisions.
assume bidders to be risk averse (e.g., Budish and Takayama 2001, Reynolds and Wooders 2009) or impatient (e.g., Matthews 2004) also obtain similar results. We assume that fixed prices are the unique equilibrium in our empirical context because advertisers have high participation costs. Another important feature of the data is that the search engine adopts a “no regret” selling rule that allows advertisers, after purchasing ad positions, to change positions and pay different prices, as long as the positions have not been sold. This practice represents an effort to maintain long-term relationships with advertisers, rather than confronting them with regret. It supports our equilibrium assumption since advertisers would have an incentive to switch to an available position if doing so increases their payoffs.

We apply the proposed methods to a single keyword, related to a brand of sporting goods (e.g., footwear, apparel, accessories). During the data period (1/1/2008 to 9/10/2008), six advertisers, all popular retailers in Korea, purchased ad positions in sponsored listing. For each sponsored link, we observe the selling propositions of the advertiser, which typically describe the products and offers to consumers. Two variables, price discount and assortment information, may influence how users respond to an ad, so we control for them. 

On average, users conduct 211 search queries daily. We observed that 65.6% of searches led to clicks only on organic listings, compared with 3.7% of searches that produced only clicks on sponsored listings, and 5.5% with clicks for both. For searches that prompted clicks on sponsored links, approximately 30% involved clicking multiple links.

In Figure 1, we illustrate the BIN prices of the ad positions throughout the data period. The topmost position is the most expensive (mean = 2.8 cents) and the bottom position is the

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13 All of the six retailers sell products of the brand. By clicking a sponsored link, consumers are directed to the retailer’s website where branded products can be found. In addition, selling propositions are all related to the brand. This characterizes the distinctive feature of search advertising that gives advertisers the opportunity of reaching consumers at the time when they are ready to make purchases.
least expensive (mean = 1.5 cents).\textsuperscript{14} Prices have changed considerably over the data period. For the topmost position, the price started at 3.2 cents, dropped to 2.7 cents, and then fell to 2.3 cents. Lower positions also experienced price declines. Our model assumes that BIN prices are exogenously set. Figure 2 shows the ad positions obtained by advertisers. These advertisers purchase nominal ad positions, but the actual positions will move up if a high position has not been sold. The directional characteristics of the sponsored listings imply that the position of an advertiser’s ad on the search results page depends on the decisions of other advertisers, as well as its own. On average 2.98 ads are displayed each day, with a standard deviation of 1.45. Advertisers generally do not change their positions, indicating stickiness in the value of ad positions. However, we also observe some abrupt changes in positions (e.g., Advertiser 1).

\textbf{Insert Figures 1 and 2 about here}

\textit{4.2 Estimation Results}

Actual ad positions, denoted by $\tilde{a}_t = (a_1, a_2, \ldots, a_J)'$, can differ from the nominal ad positions $\tilde{A}_t$ which advertisers purchased. We infer $\tilde{a}_t$ from $\tilde{A}_t$ as $\tilde{a}_t = \text{rankindex}(\tilde{A}_t | \tilde{A}_t \neq 0)$, where rankindex is an operator that returns the rank of $\tilde{A}_t$ (conditional on $\tilde{A}_t \neq 0$) from smallest to largest. The likelihood function in Equation (10) is estimated based on the actual ad positions.

We estimate the proposed search model with various numbers of consumer segments, each with a different set of model parameters. The two-segment model performs best, in terms of the AIC and BIC. To determine the fit of this two-segment model, we computed the predicted CTR and TCTR from the model, compared with the observed values at each ad position over the data period. Figure 3 presents the average observed and predicted CTR and TCTR of the five ad positions.

\textsuperscript{14} All prices are in U.S. currency. During the data collection period, foreign exchange rates were highly volatile, but we simplify matters by equating 1,000 Korean won with approximately US$1.
positions, which are closely matched. Overall, the model predicts CTR at a 0.24% mean absolute error (MAE) and TCTR at a 0.21% MAE, which reinforces the validity of the proposed model.15 Moreover, our model can explain the nonmonotonic relationship in CTR and TCTR between the fourth and fifth ad positions. Therefore, though the top-down sequential search assumptions may be restrictive, our model captures consumer search behavior well.

**Insert Figure 3 about here**

The key parameter estimates of the two-segment consumer search model are presented in Table 1. The upper panel contains the coefficients for variables that influence the reservation utility for clicking a sponsored link ($z_1$); the middle panel features the coefficients for variables that influence the reservation utility for stopping the search ($z_2$). A positive coefficient indicates that the higher the value of the corresponding variable, the lower the probability of clicking a sponsored link or stopping the search after having clicked. Finally, the lower panel of Table 1 contains the coefficients of ad attributes that affect consumers’ expected utility of clicking the advertised website. A positive coefficient indicates that a higher value of the corresponding attribute increases the probability that a consumer will click and then terminate the search.16

**Insert Table 1 about here**

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15We also compared the observed and predicted CTR and TCTR of different ad positions daily; despite large daily fluctuations, the variations are highly correlated (e.g., observed and predicted CTR [TCTR] for the top position correlate at 0.68 [0.50]).

16In the model estimation, we assume $\omega$’s in Equation (8) are normally distributed and normalize the variances of $\omega_{jt1}$ and $\omega_{jt2}$ to 1. We also assume that they are uncorrelated. Estimated variances for consumer-specific $\omega_{jt}$ and advertiser-specific $\omega_{jt}$, and their covariances, are small and insignificant, probably due to the sparseness of clicking on sponsored links. To identify variances and covariance of $\omega_{jt1}$ and $\omega_{jt2}$, for example, we need sufficient variation in the number of clicks across consumers. As more than 90% of searches do not lead to any clicks on sponsored links, and only about 3% of searches click multiple links, it is difficult to obtain precise estimates for the variances and covariance. Advertisers (especially those at lower ad positions) only attract a few daily clicks on average, so it is also difficult to obtain precise estimated variances and covariances of $\omega_{jt1}$ and $\omega_{jt2}$. 


We have one large segment (94.8%) and one small segment (5.2%) of consumers.\textsuperscript{17} The low-involvement segment 1 is less likely to click any of the links, judging from the large positive constant coefficient in $z_1$. Once they click a link, these users are more likely to stop the search, since the constant coefficient in $z_2$ is much smaller than that for segment 2. In contrast, the smaller, high-involvement segment 2 is more likely to click any of the links, then less likely to stop the search.

With respect to the advertiser-specific attributes, we first note that for segment 1, the fixed effects for most advertisers are significantly smaller than that of Advertiser 1 (which is normalized to 0), indicating that the majority of consumers prefer Advertiser 1 over the others. Even though only about 5% of consumers fall into segment 2, they are more likely to click. Because they also have a high preference for Advertisers 2 and 4, this small group of consumers helps generate higher CTR and subsequently higher TCTR for the two advertisers than the others. Advertisers 3 and 6 reveal the lowest CTR and TCTR. These results illustrate that, depending on which segment consumers belong to, advertisers can be heterogeneous in their attractiveness for clicks and terminal clicks from the consumers.

Consumers in segment 1 are attracted by discount-related information, according to the positive and significant estimate for $\ln(\text{Discount}+1)$, but price discount does not have any effect on consumers in segment 2. Assortment-related information in the selling propositions reduces consumer clicks; the estimates for $\ln(\#\text{Categories}+1)$ are significantly negative for both segments. Perhaps consumers expect popular online retailers to carry a full line of products.

\textsuperscript{17} On average, users conduct 211 search queries daily, and the data period spans more than several months. Assuming they come from unique users, the sample size is large enough to obtain significant estimates even though the size of segment 2 is small.
associated with the keyword, so promoting only a few selected product categories (e.g., men’s shoes) fails to convey value to consumers.

To investigate the value of consumer search for advertisers, we estimate two model specifications, using the results from the two-segment consumer model. Model 1 (upper panel in Table 2) assumes that every advertiser has the same value per effective impression, click, and terminal click. We also investigate the possibility that the values differ across advertisers. Because Advertisers 1 and 2 adopt an open market format, their profits from sales may differ from other retailers. In Model 2, we estimate the value per effective impression, click, and terminal click of Advertisers 1 and 2 separately from the others (lower panel in Table 2).

**Insert Table 2 about here**

To estimate the advertiser values using moments of inequality, we use instruments including the advertisers’ identities and day of the week indicators. We also include the CPM for each ad position and ad attributes as instruments. The first two columns in Table 2 contain the lower and upper bounds of each estimated value in the two models, in which all instruments are employed. The lower and upper bounds are the same, which implies we have obtained point estimates. Surprisingly, the values per effective impression and click are all zero; advertisers do not appear to perceive any benefits from search activities that do not lead to immediate transactions. The value of ad positions comes from terminal clicks only. On average, one terminal click is valued at $34.4. The next two columns in Table 2 report the bootstrapped 90% confidence intervals of the estimates.18 We caution that the results may differ for keywords in which the market is highly fragmented, with a large number of small businesses, because visits

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18 We use the lowest 5th percentile of the bootstrapped lower-bound estimates to measure the lower intervals, and the highest 5th percentile of the bootstrapped upper-bound estimates to measure the upper intervals.
to these websites and ad browsing can help increase consumer awareness of these websites, which would be valuable to such advertisers.

Results from Model 2 again confirm that the value per effective impression and click are all zero. Although the upper 90% confidence interval for Advertisers 1 and 2 under Model 2 are greater than zero, their monetary values are trivially small, compared with the estimated value per terminal click. It is much higher for Advertisers 1 and 2 than other advertisers ($64.2 vs. $12), and the difference is statistically significant according to the bootstrapped 90% confidence intervals.

The validity of the instruments $W_{jt}$ requires $E(\xi_j \mid W_{jt}) = 0$. To test the exogeneity assumption of these variables, we estimate the models with different sets of instruments. We have more instruments than necessary to estimate advertisers’ values, and our test is similar to the overidentification test for instrument validity (Hansen 1982). To check whether the CPMs of ad positions correlate with $\xi_j$, such as when the search engine sets prices strategically, we exclude CPMs from $W_{jt}$ (see the “Without CPMs” columns in Table 2). We further exclude $X_{jt}$ from $W_{jt}$, because they include price discount that may correlate with $\xi_j$ (see the last two columns of Table 2). The estimation results are close to the estimates with the whole set of instruments, and such robustness implies that the exogeneity assumptions of CPMs and advertiser attributes are likely valid.19

These findings have important managerial and economic implications. Search engines typically report CTR and impressions of a keyword to advertisers as key performance metrics.

19 We believe that CPM can be exogenous to the profit shock because we have already incorporated month indicators (and weekday indicators) to capture macro demand shocks in the consumer model. Suppose the profit shocks are independent across advertisers. The search engine may not have information on how the profit of each individual advertiser, who carries tens of thousands of branded products, changes daily for the particular brand considered in this research. CPM for each sponsored position thus may not be affected by the shocks.
Our results suggest that search engines may also report the TCTR for each ad position, because it would help advertisers evaluate the returns of their search advertising investments. As typical clickstream data tracks the time of consumer clicks, search engines and advertisers do not have to rely on the sequential search model proposed in this study to infer terminal clicks. With such concrete performance information, search engines can attract advertisers by reducing their uncertainty about search advertising. We also highlight that one cannot infer TCTR just from CTR; as shown in Figure 3, for example, TCTR is not proportional to CTR across ad positions: CTR for the topmost position is 6.80%, with about 26% of TCTR. In contrast, for the fourth (fifth) position, CTR is 2.33% (2.40%), with about 37% (40%) of TCTR.

4.3 The Determinants of the Value of Ad Positions

The unique feature of search advertising is that it gives an advertiser the opportunity of reaching consumers when they are ready to make purchases. The value of an ad position is not exogenously given; it is the consumer search activities that the ad position can attract, multiplied by the advertiser values. Consumer search activities depend on not only the advertisers in sponsored listing at different ad positions but also the composition of users who exhibit different search behaviors.

To demonstrate how the value of ad positions is generated, we conduct simulations to compare each of the three metrics (i.e., rate of effective impression $P^1_j$, CTR $P^2_j$, and TCTR $P^3_j$) and the advertiser value under two scenarios, each with different advertisers in sponsored listing. In the first scenario, Advertiser 1 occupies the topmost position and, in the second scenario, Advertiser 2 is on top. Lower positions 2 to 5 are occupied by the same set of advertisers. The three metrics and advertiser value are calculated from the estimation results from Model 2 (see Table 2), where the values from a terminal click for Advertisers 1 and 2 are the same. Difference
in the value between the two scenarios, therefore, is not driven by the difference in the advertiser value for consumer search activities.

The top panel in Table 3 reports the results of the first scenario, and the bottom panel reports the results for the second. For each case, the first (second) four columns under Segment 1 (2) report the three metrics and the advertiser value from a search query in segment 1 (2), and the last four columns report the aggregated results weighted by segment sizes. Advertiser 1, as compared to Advertiser 2, generates more value by 27% ($3.679 for Advertiser 1 in scenario 1 vs. $2.725 for Advertiser 2 in scenario 2) by placing its ad at the top. This is because Advertiser 1 is more likely to generate terminal clicks than Advertiser 2. Because of the same reason, however, having Advertiser 1 on top implies that advertisers listed below can attract fewer terminal clicks. The total value for lower-ranked advertisers in the first scenario is smaller by 1% than that in the second scenario ($0.827 in scenario 1 vs. $0.833 in scenario 2). In comparison, the increased value for these advertisers, when we move up their ads by one position, is about 5%, suggesting that the change of the competing advertiser placed at the top has a non-negligible profit impact on the advertisers listed below.

Insert Table 3 about here

Table 3 also highlights the importance of different types of consumers on the advertiser value from search query. For each search query, a consumer in segment 2 on average clicks 2.6 sponsored links in the two scenarios, compared to 0.3 clicks from a consumer in segment 1. The probability that a consumer in segment 2 will terminate the search is around 30%, compared to around 10% for a consumer in segment 1. Their search behaviors are distinctly different. In line with these search behaviors, a search query from the high-involvement segment 2 generates much higher value than the low-involvement segment 1 ($5.274 vs. $4.464 in scenario 1; $7.740
vs. $3.33 in scenario 2). Yet Advertiser 1 generates more value from the latter segment ($3.748 from segment 1 vs. $2.429 from segment 2). Consumer type is especially important for Advertiser 4, which is placed at the third position in both scenarios. Its value from a search query from segment 2 is about ten times higher than from segment 1. Our results demonstrate that for advertisers the value of a search query depends on not only the identities and the positions of advertisers in the sponsored listing but also what type of consumers conducts keyword search. The search engine may use past search history to identify which segment a consumer belongs to and create targeted advertisements to maximize the value for advertisers.

5. Conclusions
This paper proposes a methodology to study how consumer search activities drive the value of ad positions in sponsored search advertising and what are the determinants for the search activities at ad positions. We develop a structural model of consumer search which allows us to construct measures of effective impression, click, and terminal click. This model also describes how consumer search activities at each position depend on the positions occupied by competitors. To estimate the model of advertiser competition, we make use of the necessary equilibrium conditions and adopt the method of moment inequalities (Pakes et al. 2014) to tackle the associated estimation challenges. While we apply this estimation strategy under fixed prices, it can be applied to other pricing mechanisms, e.g., GSP auctions, CPM and CPC pricing mechanisms. Using a dataset obtained from a search engine, we demonstrate that the value of ad positions depends on not only the identities and the positions of the advertisers in sponsored listing but also the composition of online consumers who exhibit very different search behaviors.
Several limitations of this study might be addressed by further research. Our empirical application only studies a particular branded keyword, and all the advertisers were popular online retailers. Future research can build on our modeling approach to explore how and why the advertiser values for consumer search activities may vary with other types of keywords and advertisers. Moreover, we used terminal clicks to proxy for consumer actions. Richer data, including sales data from advertisers and users’ search activities on the search results page, could further enhance understanding of consumer behavior in keyword search. Finally, online retailers usually purchase multiple, similar keywords to attract consumer demand. Future research should examine the competition among advertisers for multiple keywords; our study provides a possible framework for such further empirical explorations.
Appendix A: Consumer Search

1. If \( \bar{u} \geq z_1 \), the consumer clicks the link.

Proof: Substitute \( z_2 \) from the optimal decision rule in the second stage into Equation (3). Then,

\[
V(u) = -c_1 + \max \{-c_2 + \int_{z_1}^{\infty} (u - z_2) \, dF(u|X,e) + z_2, z_2\}
\]

\[
= -c_1 + z_2 + \max \{-c_2 + \int_{z_1}^{\infty} (u - z_2) \, dF(u|X,e), 0\}
\]

This expression indicates that, given \( z_2 \) for the optimal rule of stopping the search, the consumer clicks the link if

\[
-c_2 + \int_{z_1}^{\infty} (u - z_2) \, dF(u|X,e) \geq 0.
\]  \hspace{1cm} (A1)

Because \( u = X\beta + e + \varepsilon \), the left-hand side in Equation (A1) can be rewritten as:

\[
-c_2 + \int_{z_1}^{\infty} (X\beta + e + \varepsilon) \, dF(\varepsilon) - z_2 \left[1 - F(z_2)\right],
\]

using change of variable and \( d\varepsilon/du = 1 \). Let \( z_I \) be the reservation utility of the clicking decision, such that

\[
-c_2 + \int_{z_1-z_I}^{\infty} (z_I + \varepsilon) \, dF(\varepsilon) - z_2 \left[1 - F(z_2)\right] = 0.
\]  \hspace{1cm} (A2)

The optimal clicking rule in Equation (A1) can be rewritten as follows: If \( \bar{u} \geq z_1 \), the consumer clicks the link.

2. Equation (A2) has a unique solution.

Proof: Let \( \kappa(z_1 \mid z_2) \equiv \int_{z_1-z_2}^{\infty} (z_I + \varepsilon) \, dF(\varepsilon) \). Then we can derive

\[
\frac{\partial \kappa(z_1 \mid z_2)}{\partial z_1} = [1 - F(z_2 - z_1)] + z_2 \cdot f(z_2 - z_1) > 0.
\]

That is, \( \kappa(z_1 \mid z_2) \) is an increasing function of \( z_I \). Therefore, given \( z_2 \), Equation (A2) has a unique solution, and a unique \( z_I \) exists for the optimal clicking rule in the first stage.

3. \( \partial z_2 / \partial c_1 > 0 \).

Proof: Using Equation (3) for the second-stage decision, and \( z_2 = EV(u) \), we know:

\[
z_2 = -c_1 + E_{X,\xi} \max \{-c_2 + \int_{z_1}^{\infty} (u - z_2) \, dF(u|X,\xi) + z_2, z_2\}
\]

\[
\Rightarrow c_1 = E_{X,\xi} \max \{-c_2 + \int_{z_1}^{\infty} (u - z_2) \, dF(u|X,\xi), 0\}
\]

Furthermore, using the optimal clicking rule in the first stage, we have

\[
c_1 = \int_{z_1-z_2}^{\infty} \left[-c_2 + \int_{z_1-\bar{u}}^{\infty} (\bar{u} + \varepsilon - z_2) \, dF(\varepsilon)\right] \, dF(\bar{u}(X,\xi)).
\]  \hspace{1cm} (A3)

Let \( h(z_2) = \int_{z_1-z_2}^{\infty} (\bar{u} + \varepsilon - z_2) \, dF(\varepsilon) \). Then,

\[
\frac{\partial h(z_2)}{\partial z_2} = -[1 - F(z_2 - \bar{u})] - 0 \cdot f(z_2 - \bar{u}) < 0.
\]

To maintain equality in Equation (A3), a higher search cost in the first stage, conditional on \( c_2 \) and \( z_I \), leads to a lower reservation utility in the second stage. That is, \( \partial z_2 / \partial c_1 > 0 \).
Appendix B: Probabilities of Effective Impression, Click, and Terminal Click

1. Pr(Effective impression)
   For advertiser \( j \) at an ad position \( L \), the probability of effective impression is as follows:
   \[
   \text{Pr}(i \text{ browses } j) = \prod_{k=1}^{L-1} \text{Pr}(i \text{ does not terminate at } k)
   \]
   \[
   = \int_{a_{i}, a_{i}'} \cdots \int_{a_{i,L-1}, a_{i,L-1}'} \int_{a_{i}, a_{i}'}^{L-1} \left[ 1 - \Pr(\omega_{ik}^1 \geq Z_{i}y_{1} - X_{k} \beta - \omega_{k}^1 - \omega_{i}^1 \text{ and } \omega_{ik}^2 \geq Z_{i}y_{2} - X_{k} \beta - \omega_{k}^2 - \omega_{i}^2) \right]
   \]
   \[
   \cdot \text{dF}(\omega_{i}^1, \omega_{i}^2) \text{dF}(\omega_{i}^1, \omega_{i}^2, \ldots, \omega_{i, L-1}^1, \omega_{i, L-1}^2)
   \]
   \[
   \equiv P_{j}^{1}(X, Z; L)
   \]

2. Pr(Click)
   \[
   \text{Pr}(i \text{ browses and searches } j) = \prod_{k=1}^{L-1} \text{Pr}(i \text{ does not stop at } k) \cdot \text{Pr}(i \text{ clicks into } j | i \text{ does not stop at } k)
   \]
   \[
   = \int_{a_{i}, a_{i}'} \cdots \int_{a_{i,j}, a_{i,j}'} \int_{a_{i}, a_{i}'}^{L-1} \left[ 1 - \Pr(\omega_{ik}^1 \geq Z_{i}y_{1} - X_{k} \beta - \omega_{k}^1 - \omega_{i}^1 \text{ and } \omega_{ik}^2 \geq Z_{i}y_{2} - X_{k} \beta - \omega_{k}^2 - \omega_{i}^2) \right]
   \]
   \[
   \cdot \text{Pr}(\omega_{ij}^1 \geq Z_{i}y_{1} - X_{j} \beta - \omega_{j}^1 - \omega_{i}^1) \text{dF}(\omega_{i}^1, \omega_{i}^2) \text{dF}(\omega_{1}^1, \omega_{i}^2, \ldots, \omega_{j}^1, \omega_{j}^2)
   \]
   \[
   \equiv P_{j}^{2}(X, Z; L)
   \]

3. Pr(Terminal click)
   \[
   \text{Pr}(i \text{ browses, clicks and terminates search at } j)
   \]
   \[
   = \prod_{k=1}^{L-1} \left[ \text{Pr}(i \text{ does not terminate at } k) \right] \cdot \text{Pr}(i \text{ terminates at } j | i \text{ does not terminate at } k)
   \]
   \[
   = \int_{a_{i}, a_{i}'} \cdots \int_{a_{i,j}, a_{i,j}'} \int_{a_{i}, a_{i}'}^{L-1} \left[ 1 - \Pr(\omega_{ik}^1 \geq Z_{i}y_{1} - X_{k} \beta - \omega_{k}^1 - \omega_{i}^1 \text{ and } \omega_{ik}^2 \geq Z_{i}y_{2} - X_{k} \beta - \omega_{k}^2 - \omega_{i}^2) \right]
   \]
   \[
   \cdot \text{Pr}(\omega_{ij}^1 \geq Z_{i}y_{1} - X_{j} \beta - \omega_{j}^1 - \omega_{i}^1 \text{ and } \omega_{ij}^2 \geq Z_{i}y_{2} - X_{j} \beta - \omega_{j}^2 - \omega_{i}^2) \text{dF}(\omega_{i}^1, \omega_{i}^2) \text{dF}(\omega_{1}^1, \omega_{i}^2, \ldots, \omega_{j}^1, \omega_{j}^2)
   \]
   \[
   \equiv P_{j}^{3}(X, Z; L)
   \]
Appendix C: Estimation Algorithm and Bootstrapping Standard Errors

Model estimates $\hat{\theta}$ minimize the value of the criterion function in Equation (22). The criterion function is not smooth and differentiable everywhere; therefore, one should use non derivatives-based algorithms such as the simplex method to search for $\hat{\theta}$. When the dimensionality of model parameters is small (e.g., in our empirical application we only estimate the advertiser values $\pi^1, \pi^2, \text{and } \pi^3$ for each advertiser), one can also use the grid search method. After finding a point in the parameter space that minimizes the criterion function value, one should increase and decrease the value of each estimate to search for the upper bound and the lower bound of the estimate. This procedure will continue until the criterion function value becomes larger when the estimate value further increases or decreases. Any value within the range of the upper bound and the lower bound minimizes the criterion function value thus is in the set estimates.

Pakes et al. (2014) derived a closed-form asymptotic distribution for the estimator $\hat{\theta}$. However, the derivation is based on the i.i.d. assumption of the stochastic term $\xi$ across periods and across advertisers, which may not be valid in our context. The method of moment inequality estimator also is based on the estimation of the first-stage consumer search model. Correct standard errors of $\hat{\theta}$ must account for the standard errors of the first-stage estimators. Because of these complications, we use the following semi-parametric bootstrapping procedure to calculate the standard errors for $\hat{\theta}$:

1. Let $\hat{\mathbf{B}} = (\hat{\gamma}_1, \hat{\gamma}_2, \hat{\beta})$ be the estimated parameters, and $\hat{\Sigma}_n$ be the estimated variance-covariance matrix for $\hat{\mathbf{B}}$, in the first-stage estimation. During each bootstrapping, we simulate $\hat{\mathbf{B}}^b = (\hat{\gamma}_1^b, \hat{\gamma}_2^b, \hat{\beta}^b)$ from the distribution $\mathcal{N}(\hat{\mathbf{B}}, \hat{\Sigma}_n)$. It is the parametric component in the bootstrapping procedure.
2. We non-parametrically bootstrap advertisers’ purchasing decisions of ad positions from our data. We use the overlapping-blocked bootstrapping procedure. We randomly select (uniformly draw from the entire periods) a starting period and the following $l-1$ periods from our data (with $l$ fixed to 10 periods). The choices of all advertisers $A^b, c^b$, and other variables that affect consumer click behavior ($S^b$) during these $l$ periods become data for bootstrapping. We repeat this procedure $T/l$ times until we have simulated $T$ periods of data. By bootstrapping blocks of periods and choices of all advertisers during these periods, we take the temporal and cross-sectional correlations of the stochastic term $\xi$ into account.
3. With bootstrapped data $\hat{\mathbf{B}}^b$, $A^b$, $c^b$, and $S^b$, we reestimate the structural parameter $\theta$ using the method of moment inequalities and obtain $\hat{\theta}^b$.
4. We repeat the bootstrapping procedure $B$ times (fixed to 1,000 times) and obtain $(\hat{\theta}_1, \hat{\theta}, \ldots, \hat{\theta}^g)$, then calculate the confidence interval of the estimate $\hat{\theta}$.
References


Table 1: Consumer Search Model Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Two-segment Model</th>
<th>Segment 1</th>
<th>Std. Err.</th>
<th>Segment 2</th>
<th>Std. Err.</th>
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Notes: Parameter estimates for month dummies are not included. Numbers in bold are significant at the 95% significance level.
<table>
<thead>
<tr>
<th></th>
<th>All Instruments</th>
<th></th>
<th>Without CPMs</th>
<th>Without CPMs and $X_{jt}$</th>
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<tbody>
<tr>
<td></td>
<td>Estimates Lower bound</td>
<td>Estimates Upper bound</td>
<td>Estimates Lower bound</td>
<td>Estimates Upper bound</td>
</tr>
<tr>
<td><strong>Model 1</strong></td>
<td></td>
<td></td>
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<tr>
<td>Per effective impression</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Per click</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>Per terminal click</td>
<td>34.4</td>
<td>34.4</td>
<td>16.9</td>
<td>55.8</td>
</tr>
<tr>
<td><strong>Model 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Advertisers 1 and 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Per effective impression</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.04</td>
</tr>
<tr>
<td>Per click</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.05</td>
</tr>
<tr>
<td>Per terminal click</td>
<td>64.2</td>
<td>64.2</td>
<td>32.1</td>
<td>98.0</td>
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<tr>
<td>Advertisers 3, 4, 5, and 6</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Per effective impression</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Per click</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Per terminal click</td>
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<td>12.0</td>
<td>6.6</td>
<td>19.0</td>
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</table>
### Table 3: Search Activities and the Value of Ad Positions

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Segment 1</th>
<th>Segment 2</th>
<th>Two Segments Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>Advertiser</td>
<td>$p_j^1$</td>
<td>$p_j^2$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.000</td>
<td>0.121</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.942</td>
<td>0.094</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.900</td>
<td>0.049</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.885</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.882</td>
<td>0.006</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td></td>
<td><strong>4.609</strong></td>
<td><strong>0.286</strong></td>
</tr>
<tr>
<td><strong>Ranks 2-5</strong></td>
<td></td>
<td><strong>3.609</strong></td>
<td><strong>0.164</strong></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Scenario 2</th>
<th>Segment 1</th>
<th>Segment 2</th>
<th>Two Segments Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>Advertiser</td>
<td>$p_j^1$</td>
<td>$p_j^2$</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.000</td>
<td>0.095</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.960</td>
<td>0.096</td>
</tr>
<tr>
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<td>4</td>
<td>0.917</td>
<td>0.050</td>
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<tr>
<td>4</td>
<td>3</td>
<td>0.902</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.899</td>
<td>0.006</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td></td>
<td><strong>4.678</strong></td>
<td><strong>0.263</strong></td>
</tr>
<tr>
<td><strong>Ranks 2-5</strong></td>
<td></td>
<td><strong>3.678</strong></td>
<td><strong>0.167</strong></td>
</tr>
</tbody>
</table>
Figure 1: BIN Prices

![BIN Prices Graph](image)

Figure 2: Advertiser Positions

![Advertiser Positions Graph](image)
Figure 3: Model Fit

Ad position

CTR (Observed)
CTR (Predicted)
TCTR (Observed)
TCTR (Predicted)