PBM Competition in Pharmaceutical Supply Chain: Formulary Design and Drug Pricing

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We model the competition among multiple Pharmacy Benefit Managers (PBMs) for the patronage of a client organization. Each PBM selects a list of prices to be charged to the client organization for each of the branded and generic drugs within a therapeutical class (price decision) and a formulary list that assigns branded drugs to preferred or non-preferred tiers (formulary decision). Drug manufacturers offer rebates to PBMs for drugs on preferred tier of formularies. The individuals participating in the client’s pharmacy benefit plan are the ones consuming the drugs and making purchasing decisions, while the client organization is paying the majority of drug cost. The choices of the individuals and the client organization are governed by different utility measures. For this complex drug distribution setting and for competing PBMs, we show the existence and uniqueness of a pure Nash equilibrium on aggregate formulary and price decisions, which represent the welfare-adjusted cost and welfare-adjusted price of each PBM’s plan respectively. We characterize each PBM’s optimal formulary and equilibrium price decisions, and discuss the impact of various model primitives. We apply our model to gain insights on the impact of mergers in the PBM industry.

Key words: Pharmacy Benefit Manager, drug distribution, tiered-formulary, pricing, competition

History:

1. Introduction

This research is motivated by two factors. First, from a practical point of view, prescription drugs are an important part of the US healthcare market. Expenditure on prescription drugs accounts for more than 10% of the total healthcare spending in the US and roughly 2% of the GDP. Prescription drug costs have increased steadily over the past few decades, which has been a major factor behind the growing healthcare service spending. According to national health expenditure data from the Centers for Medicare and Medicaid Services (2012), prescription drug costs rose from about $2.7 billion in 1960 to $263.3 billion in 2012. This rising expenditure and decreasing affordability of prescription drugs is a big concern for families, employers, and the government.

Second, from the supply chain perspective, the pharmaceutical supply chain has many unique features. As discussed in Henry J. Kaiser Family Foundation (2005), the pharmaceutical supply chain is the means through which prescription medicines are delivered to patients. The flow of
products from manufacturers to consumers resembles the distribution of consumer goods: pharmaceuticals are produced in manufacturing sites; transferred to wholesale distributors; stocked and dispensed by retail or mail-order pharmacies; and delivered to patients. However, the pricing of prescription drugs and the flow of money in the pharmaceutical supply chain is much more complex than that of a traditional supply chain. In the pharmaceutical supply chain, while patients are the end consumers of prescription drugs, they only pay a nominal fee (copayment) for their prescription drugs and the majority of the drug cost is paid by their drug benefit plan sponsors (including employers, health care insurers, and government programs). Compared to the payment scheme in the traditional supply chain of consumer products, where customers pay full prices for the products or services they receive, the cost-sharing structure in the pharmaceutical supply chain dampens patient sensitivity to price, and allows patients to focus more on quality relative to price than they otherwise would. Moreover, as prescription drugs move from manufacturers to consumers, a complex set of market transactions involving prices and rebates occur along the supply chain. The price of prescription drugs paid by each party along the supply chain is determined by a constellation of negotiated contracts between various entities.

A key player in the financial flow of the pharmaceutical supply chain is Pharmacy Benefit Managers (PBMs). Since the 1970s, PBMs have emerged as third-party administrators of prescription drug programs, and served as intermediaries between upstream pharmaceutical manufacturers and downstream clients (including employers, health care insurers, and federal and state programs such as Medicaid). In 2008, PBMs managed more than 70% of the 3 billion prescriptions dispensed that year in the United States. This represents 95% of all prescriptions of all individuals with a managed pharmacy plan, see Navarro (2009, page 95). This large market is covered by a relatively small number of PBMs, giving rise to an oligopolistic industry: In the 2nd quarter of 2011, the largest four PBMs covered close to 60% of the PBM market, as measured by prescriptions filled. Medco and Express Scripts each covered 17.01% and 15.08% of the market in the 2nd quarter of 2011, see Pharmacy Benefit Management Institute (2014).

PBMs create value in three distinct ways. At the most basic level, they provide an administrative service, processing and paying prescription drug claims. However, their main raison d’être and ever increasing importance in the pharmaceutical supply chain arises from the following two additional services: First, as large purchasing organizations, they are able to negotiate better wholesale prices with drug manufacturers and retail pharmacies than individuals or individual employers are able to obtain. Second, they help design tiered formularies, tailored to the needs of individual clients.
A tiered formulary distinguishes among a given number of tiers, each with a specified copayment level, and assigns individual drugs to each of the tiers. Tiered formularies are used to encourage the choice of less expensive drugs. Two recent studies, see Claxton et al. (2011) and Hoadley et al. (2011), have estimated that, in 2011, 77% of privately insured employees and 91% of Medicare Part D beneficiaries have a plan with three or more tiers, up from a mere 27% in the year 2000. For a three-tier formulary, if a generic drug exists in a given therapeutic class, it represents tier 1 with the lowest copayment; branded drugs are assigned to tier 2 with an intermediate level of copayment or tier 3 with the highest such level. Indeed, copayment levels vary quite significantly. For example, within the therapeutical class of statins, i.e., blood-cholesterol-lowering drugs (with an annual sales volume of $19.6 billion in 2006), copayments for a month’s supply vary from $0 to $105 (across different plans), with an average of $22 for branded drugs and $9 for generics; see Carrera (2010). Moreover, a recent study in the Journal of the AMA (Goldman et al. 2007) has estimated that a mere 10% increase in the copayment level may result in a decrease in drug spending by anywhere between 2% and 6%, depending on the therapeutic class involved. Recognizing individuals’ price sensitivity with respect to the copayment level, pharmaceutical companies offer PBMs rebates if a particular drug is assigned to tier 2 (preferred branded drugs) as opposed to tier 3 (non-preferred branded drugs). These rebates represent one source of income for the PBMs, part of which is shared with the client organization.

Another source of income for the PBMs are price spreads (markups), i.e., a PBM charges the client organization a higher price for a drug than the wholesale price the PBM negotiated with the drug manufacturers and retail pharmacies. The markup in price varies among different drugs as well as different PBMs. Price spreads of up to $200 for a single prescription have been reported, see Garis and Clark (2004).

Since PBMs’ formulary and price decisions determine the costs for consumers and plan sponsors, PBMs play a critical role in managing the financial flow of the pharmaceutical supply chain. Our research serves as the first step to understand the critical role of PBMs and more efficiently manage the highly complex pharmaceutical supply chain. Based on our interviews with supply chain managers at a leading PBM, their core challenges were developing drug formularies and providing cost-effective quality care to win the business of potential clients in a competitive market. We model the competition among PBMs on prices and formularies for the patronage of a client organization. Typically, a client reaches out to several PBMs requesting a complete design of a coverage plan for the coming two or three years. This includes a specification, for each therapeutical class of drugs, of the prices charged per prescription as well as which of the drugs are assigned to
each of the tiers of the plan. We focus on three-tier plans, the most commonly used structure, as discussed above. In our base model, we confine ourselves to a single client organization and a single therapeutical class of drugs. All of our results can be extended to more general settings of multiple client organizations and multiple therapeutical classes of drugs, see Appendix B for details.

When designing the prescription plan, a PBM needs to consider the estimated out-of-pocket cost, for the client, as well as the consumer surplus of the individuals participating in the client’s pharmacy benefit plan. Companies are clearly concerned with their total cost exposure to the drugs purchased under their plan. They also care about the well-being of their employees for several reasons. The quality of a company’s benefit program is often a decisive factor in the recruitment and retention process of its employees, and the overall employee health contributes to the company’s overall productivity. To model the choice process of the client organization, we employ a MultiNomial Logit (MNL) model in which the client’s expected utility of contracting with a PBM is a function of the expected consumer surplus of plan enrollees and the client’s expected cost. The MNL model captures the probabilistic choice process of the client organization, due to the inability of PBMs to observe and measure all the factors that determine the client’s preferences, e.g., its preference of a PBM’s pharmacy network, program service on employee education and disease management.

Both the client’s total expected cost and the employees’ consumer surplus are derived from a second underlying consumer choice model, pertaining to the insured individuals who select which of the drugs to adopt. While almost all drugs are prescribed by a physician, patients play an important and often decisive role in the selection process. Henry J. Kaiser Family Foundation (2005, page 26) reports that “Historically, patient compliance with whatever treatment the doctor ordered was assumed as part of the physician-patient relationship; increasingly, however, patients are becoming more proactive in their interaction with physicians, particularly in the area of prescription drug treatment decisions. Greater access to health information (fueled, in part, by widespread use of the Internet), the loosening of “direct-to-consumer” (DTC) advertising restrictions on drug manufacturers, and a general increase in the public’s awareness of health care issues have helped transform many once-passive patients into inquiring and demanding consumers”. In our model, the choice of prescription drug is made by a patient in consultation with a physician to maximize the patient’s utility, which involves the tradeoff between cost and quality. To specify the choice process of individual employees, we employ a second MNL model in which an employee’s per prescription expected utility of a drug is a function of its copayment value and its quality. The MNL model captures statistical variations in patient utility for prescription drugs in a diverse population.
PBMs are thus engaged in a complex type of oligopolistic competition, requiring each of them to select a list of gross prices to be charged to the client for each of the branded and generic drugs (the *price vector*) and a formulary list that assigns drugs to preference tiers. Designing the formulary is equivalent to specifying a vector of binary variables indicating which of the branded drugs are assigned to tier 2 or tier 3 (the *formulary decision*), with generic drugs always assigned to tier 1. After the choice of a PBM by the client organization, the individual employees participating in the client’s pharmacy benefit plan make the ultimate drug purchasing decisions, and the choices of these individuals are governed by trade-offs different from those of the client organization itself.

For this complex drug distribution setting and for multiple competing PBMs, we are able to show the existence and uniqueness of a pure Nash equilibrium on aggregate formulary and price decisions. The PBM’s formulary is a dominant choice, because it is optimal irrespective of the choices made by the competing PBMs. More specifically, each PBM’s optimal formulary may be computed upfront as the assignment vector that minimizes the welfare-adjusted cost of its plan. With a PBM’s optimal formulary design determined, the remaining price competition among PBMs further reduces to a classical single-dimension MNL price competition model, which has a unique equilibrium on the aggregate price decision. The single aggregate price decision made by each PBM represents the welfare-adjusted price of its plan. While the aggregate formulary and price decisions are unique, the formulary assignment vector and price vector may not be unique and may be implemented in different ways by accounting for other considerations beyond the confines of our model. This result is consistent with the current practice in the PBM industry that PBMs manage the quality and the cost of their plans at the aggregate level of all drugs instead of each individual drug, as confirmed in our interview with a senior executive and healthcare strategist (Lang 2013).

As an application of our model, for the special case of symmetric PBMs, we study the combined impact of the increased negotiation leverage (“power” effect) and reduced competition intensity (“competitive” effect) of PBM mergers. Our analysis implies that there is no win-win situation for all parties affected by PBM mergers. As the power effect increases, the merger becomes progressively favorable to the merged PBM, the client organization, and the social welfare, and has an opposite, i.e., less favorable effect, to all non-merging PBMs.

Finally, we remark that the sequential choice process in our model has the same structure as the two-level nested MNL model, but differs from the latter in two major ways. First, the sequential choices are made by different entities with different utility measures. Second, the cost sharing structure between the decision makers of the sequential choices differentiates the pharmaceutical supply chain from the traditional supply chain of consumer products. Our model can be applied to study...
the competition in a market where the cost of the product is shared by decision makers at different levels. This cost-sharing structure is present in various employment-based benefit programs, such as housing subsidy programs, backup childcare benefit programs, etc.

2. Related Literature

The rapid growth of the PBM industry has received increasing attention from both media and academic literature, alike. Many industry reports and research articles examine the role and value of PBMs in processing prescription drug claims, pointing out that PBMs help employers better control drug benefit costs by developing formularies and negotiating rebates with drug manufacturers, see, e.g., Lipton et al. (1999), Olson (2003), Grabowski and Mullins (1997). In particular, Rentmeester and Garis (2008) describe two revenue-generating practices – rebates and price spreads – which significantly account for PBMs' profits but have been neglected in the health policy literature. They raise questions about transparency in contract agreements between PBMs and employers, draw attention to how rebates and price spreads may be considered in ethical terms, and encourage future research to better understand the role of PBMs in the US health care system.

Most of the existing literature on the role of PBMs in drug distribution is either descriptive or empirical. Limited analytical work has been conducted on this topic. Gür Ali and Mantrala (2010) employ a simulation model to investigate rebate contracts between two branded drug manufacturers and a PBM. Cui and Desai (2010) conduct an analytical research on drug distribution through PBMs, focusing on the impact of heterogeneity of insurance plan size and PBM's bargaining power on the cost savings provided by a PBM. Both of the above papers have a different research focus from ours, and neither considers competition among PBMs for a client organization’s business.

Our work is related to the vast literature on multiproduct pricing and revenue management, see, e.g., Gallego et al. (2006). Although there are different choices of demand models, the price-dependent MNL and the nested MNL models are most commonly used to model consumer behavior and to optimize prices for multiple products of a firm. The classic work of Anderson and De Palma (1992) shows the existence of a symmetric price equilibrium under the nested MNL model. A recent work of Li and Huh (2011) considers a multiproduct pricing problem with the nested logit model. They prove the concavity of profit functions with respect to the market share vector, and apply this result to compare the optimal monopoly solution to the oligopolistic equilibrium solutions. In the standard nested logit model, a consumer first chooses a group of products and then chooses a product within the group; hence the two levels of choices are made by the same decision maker. In our model, each PBM faces a nested demand, which has a structure similar to that of the nested
logit model with two levels of choices, but with these choices made by different decision makers, who have different utility functions and share the cost of prescription drugs.

Finally, our work is also related to the existing literature on mergers under price competition. Deneckere and Davidson (1985) study mergers under price competition with linear demand functions in the symmetric case. They show that in the absence of cost efficiencies resulting from a merger, aggregate profits of the merging firms increase as do equilibrium prices. The equilibrium profits of the non-merging firms increase, while the consumer welfare declines. Werden and Froeb (1994) extend Deneckere and Davidson (1985) to the case with MNL demands and linear costs. They find that mergers may enhance welfare even though they increase prices of all products. Federgruen and Pierson (2011) study mergers under price competition with differentiated goods and asymmetric firms allowing for general non-linear demand and cost functions. Their results also confirm that, in the absence of cost synergies, post-merger equilibrium prices increase, and the equilibrium profit of the merged firm exceeds the combined pre-merger equilibrium profits of the merging firms. The equilibrium profits of the non-merging firms increase as well. In all these models, the merger reduces the number of competing firms in the industry, but the number of products remain the same before and after the merger. Unlike the above models of mergers among manufacturers, our model considers mergers of PBMs, who are service providers to the client organization. The merger reduces the number of PBMs (i.e., the number of firms) in the industry as well as the set of drug prices and formulary offered to the client (i.e., the number of products). We analytically show the impact of the merger upon the market shares and profits of the merging and non-merging PBMs, as well as upon the client organization’s utility and social welfare. Our results show that the post-merger equilibrium profit of the merged PBM is less than the aggregate of the pre-merger equilibrium profits of the merging PBMs unless the merger is associated with sufficient savings due to cost synergies. This contrasts with the results made in Deneckere and Davidson (1985) and Federgruen and Pierson (2011). Moreover, the post-merger equilibrium market share of the merged PBM and the expected utility of the client organization exceed their counterparts in the pre-merger model only when the merger brings about significant savings.

3. Model

In this section, we model the competition among $M$ PBMs for the patronage of a client organization. Each PBM manages $N$ branded drugs and a generic drug within a single therapeutical class. PBMs compete by providing the potential client with a set of drug prices and formulary decisions.

Since tier 2 (preferred branded) drugs have lower copayments and might be preferred by employees of the client organization who purchase the drugs, formulary decisions play an important role
in deciding each drug’s market share. PBMs leverage their power in formulary decisions to negotiate rebates with branded drug manufacturers. There are two basic forms of rebate contracts: flat rebates and performance rebates. Our model focuses on the most common form, flat rebates, where the manufacturer pays the PBMs assigning his drug to tier 2 (preferred tier) a fixed amount of money per unit drug sale. PBMs then pass the client a portion of these rebates, which helps to lower the client’s drug benefit plan cost. PBMs differentiate themselves in terms of the wholesale prices and rebates they negotiate with drug manufacturers and retail pharmacies, and the set of prices and formulary decisions they offer to the client. We need the following notation to denote quality and cost characteristics of different drugs:

\[ q_i = \text{quality index of drug } i, \quad i = 0, \ldots, N, \text{ where drug 0 denotes the generic drug;} \]
\[ w_{ij} = \text{drug } i\text{'s wholesale price negotiated by PBM } j, \quad i = 0, \ldots, N, \quad j = 1, \ldots, M; \]
\[ r_{ij} = \text{rebate (in dollars) per unit drug sale offered by branded drug manufacturer } i \text{ to PBM } j. \]

We assume the above parameters are exogenously given for several reasons. First, the quality of a drug is determined by its manufacturer’s R&D capability and cannot be easily changed, and such quality is independent of the PBM choice. Second, PBMs typically negotiate drug wholesale prices and rebates every two to three years, except the special case of patent expiration, while different client organizations typically reach out to multiple PBMs requesting a complete design of a coverage plan for the next few years at different times during the calendar year. Therefore, PBMs do not negotiate drugs’ wholesale prices and rebates specifically for each client. Instead, when requested by a client organization, PBMs respond to the request by using the current wholesale prices and rebates already negotiated with drug manufacturers and retail pharmacies. We allow drug prices and rebates to vary among PBMs, so as to model the fact that some PBMs may be able to negotiate more favorable drug prices and rebates than others.

In practice, copayment values of all tiers of the plan are typically pre-specified by the client organization, reflecting how much the client wants its employees to share the cost of their care. Therefore, copayment values of each tier of the plan vary from client to client. In our model, we take the copayment values of all tiers as exogenous parameters pre-specified by the client organization, and denote them by:

\[ c_{gj}^j = \text{copayment of tier 1 (generic) drug on PBM } j\text{'s formulary, } j = 1, \ldots, M; \]
\[ c_{pj}^j = \text{copayment of tier 2 (preferred branded) drugs on PBM } j\text{'s formulary, } j = 1, \ldots, M; \]
\[ c_{nj}^j = \text{copayment of tier 3 (non-preferred branded) drugs on PBM } j\text{'s formulary, } j = 1, \ldots, M. \]
For generality, we allow the copayment values to be PBM-specific, and all our results hold when the client organization proposes the same copayment value to all PBMs. In addition, we assume the rebate pass-through rate from a PBM to the client organization is fixed at:

$$\rho_j = \text{rebate pass-through rate to the client by PBM } j, j = 1, \ldots, M.$$ 

This assumption is made without loss of generality, as all of our analysis continues to hold if the rebate pass-through rate is modeled as a decision variable instead of an exogenous parameter, see §4 and §5 for details. Finally, in all practical cases, the above model primitives are non-negative, and $r_{ij} \leq w_{ij}$ for all $i = 1, \ldots, N$ and $j = 1, \ldots, M$.

We introduce the following notation to model the price and formulary decisions of PBMs:

- $p_{ij} = \text{drug } i's \text{ resale price that PBM } j \text{ charges to the client}, i = 0, \ldots, N, j = 1, \ldots, M$;
- $\vec{p}_j = \{p_{0j}, p_{1j}, \ldots, p_{Nj}\}$, the price decision vector of PBM $j$, $j = 1, \ldots, M$;
- $y_{ij} \in \{0, 1\} = \text{assignment indicator of branded drug } i \text{ by PBM } j \text{ to tier 2 (preferred brand), with } 1$ denoting assignment to tier 2 (preferred brand), and 0 implying otherwise assignment to tier 3 (non-preferred brand), $i = 1, \ldots, N, j = 1, \ldots, M$;
- $\vec{y}_j = \{y_{1j}, \ldots, y_{Nj}\}$, the formulary decision vector of PBM $j$, $j = 1, \ldots, M$.

Given each PBM's formulary decisions, the copayment for a branded drug charged to each individual participating in its plan can be written as

$$c_{ij}(y_{ij}) = c^p_{ij}y_{ij} + c^n_{ij}(1 - y_{ij}) = \text{copayment of branded drug } i \text{ on PBM } j's \text{ formulary}, i = 1, \ldots, N, j = 1, \ldots, M,$$

while the copayment for the generic drug is

$$c_{0j} = c^g_j = \text{copayment of the generic drug on PBM } j's \text{ formulary}, j = 1, \ldots, M.$$

Given the client organization's choice of PBM, we adopt a MultiNomial Logit (MNL) model to specify the choice process of individual enrollees of the drug. While almost all drugs are prescribed by a physician, patients typically play an important and often decisive role in the selection process. The choice of prescription drug is often made by a patient in consultation with a physician. Thus, the patient and the physician act as one agent, or equivalently, the physician is the loyal agent for the patient such that the physician prescribes the drug to maximize the utility for the patient. This assumption is reasonable since physicians typically have no financial interest in the drugs selected. This is a common assumption in the literature on prescription drug choice, see Esposito (1995), Bhatia et al. (2006), Rizzo and Zeckhauser (2009), and Epstein and Ketcham (2010).
Individuals choose drugs to maximize their expected utility, which is a function of the copayment, the quality, and other brand-specific attributes of the selected drug. We denote the brand-specific attributes of drug $i$ as $\gamma_i$. If the client organization has contracted with PBM $j$, the enrollee’s utility of purchasing drug $i$ is given by

$$u_{ij} = \gamma_i - \alpha c_{ij} + \beta q_i + e_{ij}, \quad i = 0, \ldots, N,$$

where $\gamma_i - \alpha c_{ij} + \beta q_i$ represents the attractiveness of drug $i$, and $e_{ij}$ is a random variable whose value is influenced by unobservable characteristics. The quality index $q_i$ refers to the clinical efficacy of the drug, which is usually measured over a large group of patients. For example, the chronic drugs, statins, reduce blood cholesterol levels, and have been found effective in reducing the risk of coronary heart disease and heart attacks. The efficacy of statins is measured as expected percent reduction of lower density lipoprotein (LDL), see Carrera (2010). However, patients often vary in their medical and functional responsiveness to a medication, as captured by the random term $e_{ij}$. Assume $e_{ij}$’s are i.i.d., following a double exponential distribution with mean 0 and variance $\mu^2\pi^2/6 \ (\mu > 0)$. For the client organization that has contracted with PBM $j$, drug $i$’s market share among its plan enrollees is specified by the following MNL model:

$$d_{0j}(\vec{y}_j) = \frac{\exp\left((\gamma_0 - \alpha c_{0j} + \beta q_0)/\mu\right)}{\exp\left((\gamma_0 - \alpha c_{0j} + \beta q_0)/\mu\right) + \sum_{k=1}^{N} \exp\left((\gamma_k - \alpha c_{kj} (y_{kj}) + \beta q_k)/\mu\right)},$$

$$d_{ij}(\vec{y}_j) = \frac{\exp\left((\gamma_i - \alpha c_{ij} (y_{ij}) + \beta q_i)/\mu\right)}{\exp\left((\gamma_0 - \alpha c_{0j} + \beta q_0)/\mu\right) + \sum_{k=1}^{N} \exp\left((\gamma_k - \alpha c_{kj} (y_{kj}) + \beta q_k)/\mu\right)}, \quad i = 1, \ldots, N. \quad (1)$$

Note that the market share of each drug among the plan enrollees is dependent on the enrollees’ out-of-pocket cost (copayment), and hence, on the formulary decision of the PBM that manages the plan. As we will show, each PBM’s optimal formulary decision depends on the wholesale prices and rebates that the PBM has negotiated with drug manufacturers and retail pharmacies. In other words, drug wholesale prices and rebates implicitly affect the market share of each drug via PBM’s optimal formulary decision.

In the special case where a branded drug has a patent expiration date of several years into the future, the drug will be the only drug (maybe sharing the market with another branded drug) in the market without any generic presence for many years. Since the branded drug has no or little competition in this case, the drug manufacturer may not offer any rebates to PBMs and $r_{ij}$ may be zero. Our model captures this special case by setting $\gamma_0 = -\infty$ and $N = 1$ or 2.

Given the price and formulary decisions of all PBMs, the client organization selects a PBM to maximize its expected utility, which is a function of the expected consumer surplus of its employees.
and its own expected total cost. The expected consumer surplus of contracting with PBM $j$ is the total expected utility of the plan enrollees under PBM $j$'s plan, in dollar terms, as follows:

$$
\text{CS}_j(y_j) = \frac{1}{\alpha} E \left( \max_i u_{ij} \right) S = \mu \left( \frac{\ln \left( \sum_{i=0}^{N} \exp \left( \frac{\gamma_i - \alpha c_{ij} + \beta q_i}{\mu} \right) \right)}{\mu} \right) S
$$

(2)

where $S$ is the size of the client organization as measured by the number of prescriptions filled within the therapeutical class. Since each plan enrollee selects the drug which yields the highest utility, his/her expected utility is the expected maximum utility of all drugs on the plan. This definition of consumer surplus is widely used in the literature of discrete choice models, see Small and Rosen (1981), Anderson and De Palma (1992), Train (2003), etc. The detailed derivation of the second equality in (2) can be found in Williams (1977) and Small and Rosen (1981).

For each prescription filled, the client organization is charged the drug’s resale price set by the selected PBM, and this cost is partly covered by the patient’s copayment and by the rebate passed down from the PBM. Anticipating the plan enrollees’ choice of prescription drugs, the client organization’s expected total cost of contracting with PBM $j$ is given by:

$$
B_j(y_j, p_j) = \left( d_{0j} (p_{0j} - c_j^0) + \sum_{i=1}^{N} d_{ij} (p_{ij} - c_j^n + (c_j^n - c_j^0) y_{ij} - \rho_j r_{ij} y_{ij}) \right) S.
$$

(3)

Beyond the basic factors of consumer surplus and plan cost, the client organization considers several other factors when evaluating competing PBM plans. Such factors include each PBM’s program service, administration, drug distribution, etc. Since PBMs cannot observe and measure all the factors that determine the preferences of the client organization, we employ a standard MultiNomial Logit model (MNL) to describe the client organization’s choice process. The client’s utility of contracting with PBM $j$ is:

$$
v_j = A_j + \text{CS}_j - B_j + \epsilon_j,
$$

(4)

where the deterministic term $A_j$ captures the observable and measurable PBM-specific attributes (e.g., the accuracy and timeliness of claim payments and reporting), and the random term $\epsilon_j$ represents unobservable characteristics of the client’s preference (e.g., its preference of a PBM’s pharmacy network, program service on employee education and disease management). The client’s utility of not contracting with any PBM is $v_0 = u_c + \epsilon_0$, where $u_c$ denotes its expected value. Assume $\epsilon_j$’s ($j = 0, 1, \ldots, M$) are i.i.d., following a double exponential distribution with mean 0 and variance $\nu^2 \pi^2 / 6$ ($\nu > 0$). The probability that the client organization selects PBM $j$ is:

$$
n_j(y_j, p_j) = \frac{\exp((A_j + \text{CS}_j - B_j)/\nu)}{\exp(u_c/\nu) + \sum_{k=1}^{M} \exp((A_k + \text{CS}_k - B_k)/\nu)}, \quad j = 1, \ldots, M.
$$

(5)
We now formulate the competition of PBMs for winning the client organization’s drug benefit distribution business. We employ a game-theoretical model with complete information. We assume that the client organization knows its enrollees’ price- and quality- sensitivities and brand preferences, and that each PBM knows all the parameters of the decision-making process for the client organization and its enrollees, except for the unobservable characteristics captured by the random terms $\epsilon_j$ and $e_{ij}$. Therefore, the PBMs can only estimate the decision-making of the client organization in probabilistic terms, as in (5).

The objective of each PBM is to maximize its expected profit, which includes both price spreads and retained rebates. Price spread is the difference between a drug’s resale price the PBM charges to the client organization and the wholesale price the PBM pays to the drug manufacturer or retail pharmacy. Rebates are paid by branded drug manufacturers if and only if PBMs put their drugs on the preferred tier 2. After passing a portion of the rebate to the client, PBMs keep the remainder. Each PBM makes formulary decisions for all branded drugs, and price decisions for all branded and generic drugs. We can write PBM $j$’s formulary and pricing problem as

$$
\max_{\vec{y}_j \in \{0, 1\}^N, \vec{p}_j \in \mathbb{R}_+^{N+1}} \pi_j = n_j(\vec{y}_j, \vec{p}_j) \left( d_{0j}(\vec{y}_j)(p_{0j} - w_{0j}) + \sum_{i=1}^N d_{ij}(\vec{y}_j)(p_{ij} - w_{ij} + (1 - \rho_j)r_{ij}y_{ij}) \right) S. \quad (6)
$$

In our formulation, we do not constrain the price vector to be nonnegative, we will show that there always exists a nonnegative price vector that achieves the competitive equilibrium of our problem.

The objective function has the same structure as those of two-level nested MNL models, where relevant choices are part of a sequential process: First, the client organization adopts a plan provided by a PBM; Second, conditional on the choice of this plan, each plan enrollee selects a drug to maximize his/her expected utility according to the copayments specified by the plan. The two choice levels are described by MNL models. Our model differs from other nested MNL models in two major ways: (1) the choices at the two levels are made by different entities with different utility measures; (2) the prescription drug cost is shared between the decision makers at the two levels.

4. Equilibrium Analysis

In this section, we conduct an equilibrium analysis of the oligopolistic competition among PBMs. Each PBM designs the formulary for $N$ branded drugs and sets prices for all drugs. We will show that each PBM’s formulary and price decision, which consists of $2N + 1$ decision variables, can be projected into the following two aggregate decision variables: For $j = 1, \ldots, M$, let

$$
V_j(\vec{y}_j, \vec{p}_j) = B_j(\vec{y}_j, \vec{p}_j) - CS_j(\vec{y}_j), \quad (7)
$$

and

$$
U_j(\vec{y}_j) = W_j(\vec{y}_j) - CS_j(\vec{y}_j), \quad (8)
$$
where \( W_j(\vec{y}_j) = \left( d_{0j}(\vec{y}_j)(w_{0j} - c_j^0) + \sum_{i=1}^{N} d_{ij}(\vec{y}_j) (w_{ij} - c_{ij}(y_{ij}) - r_{ij}y_{ij}) \right) S. \) (9)

Note that \( W_j(\vec{y}_j) \) is the client’s expected total cost of working with an “internal” PBM \( j \), i.e., PBM \( j \) applies no markups to its wholesale prices and passes on the full rebates resulting from its formulary decisions. One can interpret \( W_j(\vec{y}_j) \) as the cost of PBM \( j \)’s plan, and the client organization’s expected total cost of contracting with PBM \( j \), \( B_j(\vec{y}_j, \vec{p}_j) \), as the price of PBM \( j \)’s plan. Hence, \( U_j(\vec{y}_j) \) can be interpreted as the welfare-adjusted cost of PBM \( j \)’s plan, and \( V_j(\vec{y}_j, \vec{p}_j) \) can be interpreted as the welfare-adjusted price of PBM \( j \)’s plan. The following theorem reformulates each PBM’s formulary design and pricing problem as a dual-decision problem, develops the structural property of the latter, and characterizes the equilibrium of PBMs’ competition.

**Theorem 1 (Existence and Uniqueness of Equilibrium).**  (a) For PBM \( j = 1, \ldots, M \), its formulary design and pricing problem can be reformulated as the following dual-decision problem:

\[
\max_{U_j \in \Omega_j, \vec{V}_j \in \mathbb{R}} \pi_j = \frac{\exp \left( \frac{(A_j - V_j)}{\nu} \right)}{\exp \left( u_0^0 / \nu \right) + \sum_{k=1}^{M} \exp \left( \frac{(A_k - V_k)}{\nu} \right)} (V_j - U_j),
\]

where \( \Omega_j := \{ W_j(\vec{y}_j) - CS_j(\vec{y}_j) \mid \vec{y}_j \in \{0, 1\}^N \} \) denotes the feasible set of \( U_j(\vec{y}_j) \).

(b) For PBM \( j = 1, \ldots, M \), its profit function, \( \pi_j \), is linearly decreasing in \( U_j \) for any given \( V_j \), and is strictly log-concave in \( V_j \) for any given \( U_j \). Furthermore, given any original formulary decision vector, \( \vec{y}_j \) (and hence \( U_j \)), any \( V_j \in \mathbb{R} \) can be obtained by varying the original price decision vector \( \vec{p}_j \). Conversely, given any \( V_j \in \mathbb{R} \), any \( U_j \in \Omega_j \) can be obtained by varying the original price decision vector \( \vec{p}_j \) and the original formulary decision vector \( \vec{y}_j \), while keeping \( V_j \) fixed.

(c) The PBMs’ competition has a unique Nash equilibrium \( \{(U_1^*, V_1^*), (U_2^*, V_2^*), \ldots, (U_M^*, V_M^*)\} \).

The proofs of Theorem 1 and all other analytical results of the paper are relegated to Appendix A. Theorem 1 shows that for PBM \( j \), its formulary and pricing problem can be solved in two sequential steps: First, its optimal formulary decision vector, \( \vec{y}_j^* \), can be obtained as the binary vector that minimizes \( U_j(\vec{y}_j) \), the welfare-adjusted cost of PBM \( j \)’s plan. Since \( U_j(\vec{y}_j) \) is independent of all other PBMs’ decisions, the optimal formulary decision is a dominant choice. Second, with each PBM’s optimal formulary decision determined, the competition among PBMs can be reduced to a classical single-dimension MNL price competition model. The single (aggregate) price value selected by PBM \( j \), \( V_j(\vec{y}_j^*, \vec{p}_j^*) \), refers to the welfare-adjusted price of PBM \( j \)’s plan.

We now characterize PBMs’ optimal formulary and equilibrium price decisions in Proposition 1, Proposition 2, and Theorem 2 below. As discussed, PBM \( j \)’s optimal and dominant formulary decision can be obtained as the binary vector that minimizes \( U_j(\vec{y}_j) \). The optimal formulary decision
\( \vec{y}_j^* \) may not be unique since multiple binary vectors may result in the same value of the optimal \( U_j^* \). In this case, PBM \( j \) is indifferent with such different formulary designs.

Minimizing \( U_j(\vec{y}_j) \) is a complex combinatorial optimization problem, with a feasible set of \( 2^N \) vector choices. Proposition 1 establishes a sufficient condition of assigning a drug to the preferred tier. First, we introduce the following additional notation: For \( j = 1, \ldots, M, i = 1, \ldots, N \), let
\[
\begin{align*}
x_{0j} &= \exp \left( \left( \gamma_0 - \alpha c_i^j + \beta q_i \right) / \mu \right), \\
x_{ij} &= \exp \left( \left( \gamma_i - \alpha c_i^j + \beta q_i \right) / \mu \right), \\
m_{0j} &= w_{0j} - c_i^j, \\
m_{ij} &= w_{ij} - c_i^j + (c_i^j - c_j^p - r_{ij})y_{ij}.
\end{align*}
\]

For \( i = 0, \ldots, N \), \( x_{ij} \) measures the attractiveness of drug \( i \) to enrollees of PBM \( j \)'s plan, and is proportional to drug \( i \)'s market share under PBM \( j \)'s plan, i.e., \( d_{ij}(\vec{y}_j) = \frac{x_{ij}}{\sum_{k=0}^{N} x_{kj}} \). For \( i = 0, \ldots, N \), \( m_{ij} \) is the client organization’s per prescription cost of drug \( i \) on an “internal” PBM \( j \)'s plan. Define the following cost change rate index of branded drug \( i \):
\[
\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)}, \quad i = 1, \ldots, N,
\]
which measures the change rate of the client’s cost by moving branded drug \( i \) from the non-preferred tier (tier 3) to the preferred tier (tier 2). Without loss of generality, PBM \( j \) ranks the set of branded drugs \( \{1, \ldots, N\} \) in ascending order of their cost change rate indices, i.e., \( \delta_{i1} \leq \delta_{i2} \leq \ldots \leq \delta_{iNj} \). We assume PBMs put a branded drug on tier 2 if they are indifferent about putting the drug on tier 2 or tier 3. The following proposition characterizes properties of PBMs’ optimal formulary decision.

**Proposition 1 (Sufficient Condition of PBMs’ Optimal Formulary Decision).** Define the threshold index
\[
I_j = \begin{cases} 
\max \{ i : \delta_{ij} \leq \frac{x_{0j}m_{0j} + \sum_{k=1}^{i-1} x_{kj}(1)m_{kj}(1) + \sum_{k=i}^{N} x_{kj}(0)m_{kj}(0)}{x_{0j} + \sum_{k=1}^{i} x_{kj}(1) + \sum_{k=i}^{N} x_{kj}(0)} \} , & \text{when such } i \text{ exists;} \\
0 , & \text{otherwise.}
\end{cases}
\]

If \( i \leq I_j \), it is optimal for PBM \( j \) to assign drug \( i \) to tier 2 (i.e., \( y_{ij}^* = 1 \)).

The index of branded drug \( i \) on PBM \( j \)'s plan can be computed upfront, as a function of the drug’s wholesale price and rebate, as well as the copayment values of PBM \( j \)'s plan. Since different PBMs often negotiate different wholesale prices and rebates with the drug manufacturers and retail pharmacies, the index values of the drugs may vary by PBM, as does their relative ranking. Therefore, the optimal formulary design differs by PBM.

In the special case when all branded drugs on a PBM’s plan have the same brand-specific attribute and quality, the following proposition explicitly characterizes PBMs’ optimal formulary decision, and enables each PBM to solve the complex combinatorial optimization problem of formulary assignment in \( O(N \log N) \) elementary operations and evaluations of exponential functions.
**Proposition 2 (Optimal Formulary Design with Equal Brand-specific Attribute and Quality).**

Consider the special case where all branded drugs on PBM $j$’s plan have the same brand-specific attribute $\gamma_i$ and quality $q_i$. The optimal set of branded drugs on tier 2 is consecutive in their cost change rate indices. The optimal formulary decision is given by the following threshold policy:

$$y^*_ij \equiv \begin{cases} 1, & \text{for } i = 1, \ldots, I^*_j; \\ 0, & \text{for } i = I^*_j + 1, \ldots, N, \end{cases}$$

where

$$I^*_j = \arg \min_{k \in \{I^*_j, \ldots, N\}} \frac{x_{0j}m_{0j} + \sum_{i=1}^{k} x_{ij}(1)m_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0)m_{ij}(0)}{x_{0j} + \sum_{i=1}^{k} x_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0)} - \frac{\mu}{\alpha} \ln \left( x_{0j} + \sum_{i=1}^{k} x_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0) \right).$$

Proposition 2 shows that, when all branded drugs have the same brand-specific attribute and quality, PBMs prioritize drugs that are most cost-effective to put on the preferred tier. This is consistent with current practice in the PBM industry: “in the absence of statistically and clinically significant differentiation among similar drugs, the net cost (which may be reduced by a rebate) can have an important impact on ultimate formulary positioning.” (Navarro 2009, page 40).

After the optimal formulary decision vector of PBM $j$, $\vec{y}^*_j$, and the optimal value of the welfare-adjusted cost of PBM $j$’s plan $U^*_j = U_j(\vec{y}^*_j)$ are determined, the remaining price competition among the PBMs reduces to a classical single-dimension MNL price competition model, where the single price value selected by PBM $j$, $V_j$, measures the welfare-adjusted price of PBM $j$’s plan. As in the classical single-dimension MNL price competition model, there exists a unique equilibrium, $\{V^*_1, \ldots, V^*_M\}$, which is the solution of the following system of equations (with details on how these equations emerge in the proof of Theorem 1, see Appendix A):

$$\frac{\exp \left( (A_j - V^*_j) / \nu \right)}{\exp \left( u^*_j / \nu \right) + \sum_{k=1}^{M} \exp \left( (A_k - V^*_k) / \nu \right)} = 1 - \frac{\nu}{V^*_j - U_j^*}, \quad j = 1, \ldots, M. \quad (13)$$

The equilibrium can be easily computed with a standard Round Robin tatonnement scheme, starting from an arbitrary set of welfare-adjusted prices $\{V_1, \ldots, V_M\}$, see Topkis (1998, §4.3.1).

Moreover, PBMs’ equilibrium welfare-adjusted prices can be characterized by the approach in Li and Huh (2011). First, let $H$ be a mapping from $(0, \infty)$ to $(0, 1)$ such that, for any $x \in (0, \infty)$, $H(x)$ is the unique solution $h \in (0, 1)$ satisfying

$$h \cdot \exp \left( \frac{h}{1 - h} \right) = x. \quad (14)$$

It can be verified that $H(x)$ is a strictly increasing function. The following theorem gives a closed-form expression of PBMs’ welfare-adjusted price equilibrium in terms of the $H$ function.
Theorem 2 (PBMs’ Equilibrium Welfare-adjusted Prices). Given each PBM’s optimal formulary decision determined, in the reduced price competition among PBMs, the equilibrium probability of the client not contracting with any PBM, $n_0^*$, is the unique solution to the following single-variable equation:

$$n_0 + \sum_{j=1}^{M} H\left(n_0 \cdot \exp\left(\frac{A_j - U_j^* - u_0^* - \nu}{\nu}\right)\right) = 1.$$  

(15)

The equilibrium probability of the client contracting with PBM $j$, i.e., the equilibrium expected market share of PBM $j$, is given by:

$$n_j^* = H\left(n_0 \cdot \exp\left(\frac{A_j - U_j^* - u_0^* - \nu}{\nu}\right)\right), \quad j = 1, \ldots, M.$$  

(16)

The equilibrium expected profit of PBM $j$ is given by:

$$\pi_j^* = \frac{\nu n_j^*}{1 - n_j^*}, \quad j = 1, \ldots, M.$$  

(17)

The equilibrium welfare-adjusted price of PBM $j$’s plan is given by:

$$V_j^* = A_j - u_0^* - \nu \left(\ln n_j^* - \ln n_0^*\right), \quad j = 1, \ldots, M.$$  

(18)

The client organization’s expected utility is given by:

$$v^* = \nu \ln \left(\exp\left(\frac{u_0^*}{\nu}\right) + \sum_{j=1}^{M} \exp\left(\frac{A_j - V_j^*}{\nu}\right)\right) = u_0^* - \nu \ln n_0^*.$$  

(19)

Theorem 2 characterizes the unique equilibrium in terms of the welfare-adjusted price. With the optimal formulary decision vector $\vec{y}_j^*$ determined, and the equilibrium welfare-adjusted price $V_j^*$ determined as in Theorem 2, PBM $j$ may select a vector of drug prices from the hyperplane defined by (7) and achieving the value $V_j^*$. Each PBM has ample opportunities to specify various pricing schemes by choosing prices on this hyperplane, and their final choices may reflect considerations beyond those captured in our model. This result is consistent with the current practice in the PBM industry that pricing is managed and competed at the aggregate level. The target welfare-adjusted price of each PBM’s plan is achieved by pricing individual drugs accordingly.

One plausible pricing scheme for a PBM is the “unified markup ratio pricing scheme”, where the PBM applies an identical markup ratio to all branded and generic drugs. Under this pricing scheme, PBM $j$ sets the price vector $\vec{p}_j^*$ with the additional constraints that $(p_{0j}^* - w_{0j})/w_{0j} = (p_{ij}^* - w_{ij} + (1 - \rho_j)r_{ij}y_{ij}^*)/w_{ij}, \quad i = 1, \ldots, N$. In this case, the equilibrium price vector $\vec{p}_j^*$ can be uniquely determined as follows:

$$p_{0j}^* = w_{0j} \left(1 + \frac{V_j^* - U_j^*}{S \sum_{i=0}^{N} w_{ij}d_{ij}(\vec{y}_j^*)}\right), \quad p_{ij}^* = w_{ij} \left(1 + \frac{V_j^* - U_j^*}{S \sum_{i=0}^{N} w_{ij}d_{ij}(\vec{y}_j^*)}\right) - (1 - \rho_j)r_{ij}y_{ij}^*, \quad i = 1, \ldots, N.$$  

(20)

It is easily verified from (20) that all drug prices are nonnegative under this pricing scheme.
5. Comparative Statics

In this section, we study how various primitives of the model impact the optimal welfare-adjusted cost of a PBM’s plan, the equilibrium welfare-adjusted price of a PBM’s plan, expected market share and expected profit of a PBM, as well as the client organization’s expected utility.

Recall from Theorem 1 that the formulary and price decisions of PBM \( j \) can be solved in two sequential steps by first determining its optimal formulary decision vector \( \vec{y}_j^* \) that minimizes the welfare-adjusted cost of its plan, and then analyzing the equilibrium of the reduced price competition in terms of the welfare-adjusted price of its plan. Moreover, the equilibrium analysis of the reduced price competition in the second step depends on the optimal formulary decision in the first step only via the optimal welfare-adjusted cost. Therefore, the comparative statics results on all the quantities of interest mentioned above with respect to various model primitives depend on the impact of the optimal welfare-adjusted cost upon these terms, as given in the following lemma:

**Lemma 1 (Impact of Optimal Welfare-Adjusted Cost).** Consider PBM \( j = 1, \ldots, M \).

(a) The equilibrium probability of the client not contracting with any PBM, \( n_0^* \), is strictly increasing in the optimal welfare-adjusted cost of PBM \( j \)’s plan \( U_j^* \).

(b) PBM \( j \)’s equilibrium expected market share and expected profit, \( n_j^* \) and \( \pi_j^* \), are strictly decreasing in the optimal welfare-adjusted cost of its own plan \( U_j^* \), and strictly increasing in the optimal welfare-adjusted cost of any competing PBM’s plan \( U_k^* (k = 1, \ldots, M \text{ and } k \neq j) \).

(c) The equilibrium welfare-adjusted price of PBM \( j \)’s plan \( V_j^* \) is strictly increasing in the optimal welfare-adjusted cost of its own plan \( U_j^* \), as well as in the optimal welfare-adjusted cost of any competing PBM’s plan \( U_k^* (k = 1, \ldots, M \text{ and } k \neq j) \).

(d) The client organization’s expected utility \( v^* \) is strictly decreasing in the optimal welfare-adjusted cost of any PBM’s plan.

Utilizing Lemma 1, we are ready to analyze the impact of various model primitives. Proposition 3 below shows the impact of wholesale prices.

**Proposition 3 (Impact of Wholesale Price).** Consider PBM \( j = 1, \ldots, M \).

(a) PBM \( j \)’s optimal formulary decision for branded drug \( i \), \( y_{ij}^* \), is weakly decreasing in drug \( i \)’s wholesale price on PBM \( j \)’s plan, non-monotone in any other (branded or generic) drug’s wholesale price on PBM \( j \)’s plan, and independent of any drug’s wholesale price on any competing PBM’s plan.

(b) The optimal welfare-adjusted cost of PBM \( j \), \( U_j^* \), is strictly increasing in [independent of] any (branded or generic) drug’s wholesale price charged to PBM \( j \) [any competing PBM].
(c) The equilibrium welfare-adjusted price of PBM $j$, $V_j^*$, is strictly increasing in the wholesale price charged on any (branded or generic) drug to any PBM.

(d) The client organization’s expected utility, $\bar{v}$, is strictly decreasing in the wholesale price charged on any (branded or generic) drug to any PBM.

(e) PBM $j$’s equilibrium expected market share and expected profit, $\pi_j^*$ and $\pi_j^*$, are strictly decreasing [increasing] in any drug’s wholesale price charged to PBM $j$ [any competing PBM].

Proposition 3 shows that an increase in any drug’s wholesale price on a PBM’s plan may result in the PBM reassigning the drug from tier 2 to tier 3, but a change in the opposite direction cannot occur. However, an increase in any competing drug’s wholesale price on the PBM’s plan may result in the PBM reassigning the focal drug to a different tier, either from tier 2 to tier 3, or from tier 3 to tier 2. Moreover, an increase in any drug’s wholesale price on a PBM’s plan raises the plan’s welfare-adjusted cost, and hence its welfare-adjusted price. Since the price competition among PBMs is a log-supermodular game, the equilibrium welfare-adjusted price of any competing PBM increases as well, which reduces the client organization’s expected utility. The PBM with an increased drug wholesale price gets lower market share and profit, while any competing PBM obtains higher market share and profit at equilibrium.

Since an increased rebate and a decreased wholesale price of a drug both result in a reduced per prescription cost of the drug on a PBM’s plan, all comparative statics results identified in Proposition 3 with respect to wholesale prices apply to the impact of rebates with reverse (weak) monotonicity properties. The weak monotonicity properties of the rebate come from the fact that a drug’s rebate affects a PBM’s per prescription cost of the drug only when the drug is on the preferred tier of the PBM’s plan. On the other hand, the quantities of interest discussed in Proposition 3 do not depend on the rebate pass-through rate, because the optimal formulary decision vector of the PBM is the binary assignment vector that minimizes the welfare-adjusted cost of the PBM’s plan, or equivalently, the welfare-adjusted price of the PBM’s plan when the PBM applies no markups to its wholesale prices and passes on the full rebates to the client.

Unlike the impact of a drug’s wholesale price and rebate, the impacts of a drug’s quality and other brand-specific attribute are much more involved. In our model, a drug’s quality ($q_i$) and its brand-specific attribute ($\gamma_i$) work together as the aggregate term $\gamma_i + \beta q_i$. The impact of drug’s brand-specific attribute ($\gamma_i$) on all quantities of interest differs from that of the drug’s quality ($q_i$) only by a factor of $\beta$, which measures the difference in consumers’ sensitivity with respect to these two terms. Therefore, we present the comparative statics results with respect to a drug’s quality, and all these results directly extend to the case with respect to the drug’s brand-specific attribute.
Proposition 4 (Impact of Drug Quality). (a) For each PBM $j$, there exist two thresholds of drug $i$'s quality, $q_{ij}$ and $\overline{q}_{ij}$ ($0 \leq q_{ij} \leq \overline{q}_{ij}$), such that the optimal welfare-adjusted cost of PBM $j$'s plan $U^*_j$ strictly increases in branded drug $i$'s quality $q_i$ if $0 \leq q_i < q_{ij}$, and strictly decreases in $q_i$ if $q_i > q_{ij}$. In particular, both thresholds weakly increase in $w_{ij}$ and weakly decrease in $r_{ij}$.

(b) Consider the case when all PBMs are symmetric with respect to the cost parameters, i.e., with identical wholesale price and rebate for each drug, and identical copayment for each formulary tier. A change in a drug's quality affects the optimal welfare-adjusted cost and the equilibrium welfare-adjusted price of any PBM's plan in the same direction, while affecting any PBM's equilibrium expected market share, equilibrium expected profit, and the client organization's expected utility in the opposite direction.

Proposition 4(a) characterizes a sufficient condition when the impact of drug quality is monotone. The impact of a drug's quality depends on the tradeoff between the consumer surplus and cost: An increase in a drug's quality alone, without any change in the formulary decision, increases the drug's market share and decreases any competing drug's market share. Depending on the drug's relative cost (wholesale price and rebate), such change in market share may increase or decrease the cost of the PBM's plan. On the other hand, without any change in the formulary decision, we are able to show that the consumer surplus of the plan enrollees is convex increasing in the drug's quality, and independent of the drug's wholesale price and rebate. When a drug's quality is sufficiently high, an increase in the drug's quality significantly increases the consumer surplus of the plan enrollees. At the same time, since the high-quality drug already captures a major market share, an increase in its quality only marginally increases its market share, and hence, marginally affects the cost of the plan. In this case, the increase in the consumer surplus dominates the possible increase in the cost, so an increase in the drug's quality decreases the welfare-adjusted cost of PBM $j$'s plan. In contrast, when a drug's quality is sufficiently low, an increase in the drug's quality marginally increases the consumer surplus of the plan enrollees and the drug's market share. When the low-quality drug is significantly more expensive than the competing drugs, such increase in its market share results in an increase in the plan's cost that dominates the increase in the consumer surplus. In this case, an increase in the drug's quality increases the welfare-adjusted cost of PBM $j$'s plan. Note that both thresholds are cost-dependent, as they weakly increase [decrease] in the drug's wholesale price [rebate]. When the lower threshold $q_{ij} = 0$, the range $[0, q_{ij}]$ is an empty set.

In the special case when there is no quality gap among the drugs, we are able to show that drug quality does not affect each PBM's optimal formulary assignment, which is determined based on cost parameters and brand preferences.
Except the special cases discussed above, the impact of drug quality is, in general, quite involved. A drug’s quality improvement may result in the drug being shifted to the non-preferred tier, as the plan enrollees may now choose to adopt the drug even through its copayment value has increased. However, the opposite switch may also occur. The following example shows that an increase in a branded drug’s quality has a non-monotone effect on a PBM’s optimal formulary decision. Consider

![Graphs showing impact of q₁](image)

**Figure 1** Impact of $q₁$.

PBM $j$, which manages two branded drugs and a generic drug within a therapeutical class. We set the default parameter values as follows: $\mu = 0.1$, $S = 100$, $\alpha = 0.5$, $\beta = 1$, $\gamma_i = 0$ ($i = 0, 1, 2$), $c_j^g = 5$, $c_j^p = 10$, $c_j^n = 15$, $q_0 = 8$, $q_1 = 20$, $q_2 = 12$, $w_{0j} = 8$, $w_{2j} = 20$, $r_{1j} = 4$, $r_{2j} = 6$. Under two different wholesale prices, Figures 1(a) and 1(b) show that the optimal formulary assignment of branded drug 1 and that of branded drug 2 are non-monotone, as the quality of branded drug 1 increases from 0 to 20. For these two cases, Figures 1(c) and 1(d) plot the PBM’s optimal welfare-adjusted cost. Figures 1(c) and 1(d) confirm the finding in Proposition 4(a) that when
the drug quality is sufficiently high, the PBM’s optimal welfare-adjusted cost strictly decreases in the drug quality. Note that \( q_{1j} = 0 \) in both cases. Moreover, Figure 1(c) shows that the PBM’s optimal welfare-adjusted cost is non-monotone in the quality of branded drug 1. Consequently, the equilibrium welfare-adjusted price, expected market share, expected profit of the PBM, and the client organization’s expected utility are non-monotone in the drug quality as well.

6. Impact of PBM Mergers

In view of recent mergers and acquisitions in the PBM industry, we use our model to better understand the impact of a merger of two PBMs. As discussed, PBMs operate in an oligopolistic industry with high entry barriers. In the past ten years, many mergers and acquisitions took place in the industry, creating a highly concentrated market. For example, Caremark acquired AdvancePCS in 2003, Express Scripts acquired Medco in April 2012, and SXC acquired Catalyst in July 2012. We study the impact of a merger on the merged PBM, non-merging PBMs and the client organization. For the tractability of analysis, we assume all \( M (M \geq 3) \) PBMs before the merger are symmetric, i.e., with identical wholesale price and rebate for each drug, identical copayment for each formulary tier, and identical PBM-specific attributes. Without loss of generality, we index the two merging PBMs by \( j = 1, 2 \) in the pre-merger model. After the merger, we index the merged PBM by \( j = 1 \), and \( j = 2 \) is not used. Non-merging PBMs are indexed by \( j = 3, \ldots, M \) in both pre-merger and post-merger models. We use superscript “m” to denote the post-merger model.

The PBM merger has two first order effects: increased negotiating leverage for the merged PBM and decreased competition intensity in the industry. The common argument in support of mergers and acquisitions in the PBM industry is the increased bargaining power of the merged PBM to negotiate lower wholesale prices and/or higher rebates from branded drug manufacturers and retail pharmacies. Therefore, for all \( i = 1, \ldots, N \), we assume \( w_{i1}^m \leq w_{i1} \) and \( r_{i1}^m \geq r_{i1} \) for the merged PBM, and \( w_{ij}^m = w_{ij} \) and \( r_{ij}^m = r_{ij} \) for each non-merging PBM \( j = 3, \ldots, M \). We define a negotiating leverage (or “power”) index, \( \Delta U = U_j^* - U_j^{m*} \), as a measure of the increased bargaining power effect for the merged PBM. It follows from Proposition 3 and the discussion after Proposition 3 that \( U_j^{m*} = U_j^* \) for all \( j = 3, \ldots, M \), and \( U_1^{m*} \leq U_1^* \), i.e., \( \Delta U \geq 0 \). Therefore, \( \Delta U \) also measures the saving in the welfare-adjusted cost, i.e., the cost synergy effect of the merger. On the other hand, the merger reduces the competition intensity among PBMs, as the total number of competing PBMs in the market decreases from \( M \) to \( M - 1 \). Define the equilibrium profit and market share of the PBM industry before the merger as:

\[
\pi_i^* = \sum_{j=1}^{M} \pi_j^* \quad \text{and} \quad n_j^* = 1 - n_0^*.
\]
Similarly, define the equilibrium profit and market share of the PBM industry after the merger as:

\[ \pi_1^{m*} \equiv \pi_1^{m*} + \sum_{j=3}^{M} \pi_j^{m*} \quad \text{and} \quad n_1^{m*} \equiv n_1^{m*} + \sum_{j=3}^{M} n_j^{m*} = 1 - n_0^{m*}. \]  \hfill (22)

The following proposition characterizes the impact of mergers on PBMs and the client organization.

**Proposition 5 (Impact of PBM Mergers on PBMs and the Client Organization).** Consider \( M \) symmetric PBMs. After the merger of PBM 1 and PBM 2, there exist three threshold values of the power index, \( \Delta_{\bar{U}}, \Delta_{\bar{U}} \) and \( \Delta_{\bar{U}} \) (0 \( \leq \Delta_{\bar{U}} \leq \Delta_{\bar{U}} < \Delta_{\bar{U}} \), such that:

(I) If \( 0 \leq \Delta_U \leq \Delta_{\bar{U}} \),

(ii) For the merged PBM, \( \pi_1^* < \pi_1^{m*} \leq \pi_2^* \) and \( n_1^* < n_1^{m*} \leq n_2^* \).

(iii) For the non-merging PBM \( j \) (\( j = 3, \ldots, M \)), \( \pi_j^* \leq \pi_j^{m*} \leq \pi_1^{m*} \) and \( n_j^* \leq n_j^{m*} \leq n_1^{m*} \).

(iv) For the PBM industry, \( \pi_j^{m*} \leq \pi_1^* \) and \( n_j^{m*} \leq n_1^* \).

(v) For the client organization, \( \pi^{m*} \leq \pi^* \).

(II) If \( \Delta_{\bar{U}} \leq \Delta_U \leq \Delta_{\bar{U}} \),

(i) For the merged PBM, \( \pi_1^* < \pi_1^{m*} \leq \pi_2^* \) and \( n_1^* < n_1^{m*} \leq n_2^* \).

(ii) For the non-merging PBM \( j \) (\( j = 3, \ldots, M \)), \( \pi_j^* \leq \pi_j^{m*} \leq \pi_1^{m*} \) and \( n_j^* \leq n_j^{m*} \leq n_1^{m*} \).

(iii) For the PBM industry, \( \pi_j^{m*} \geq \pi_1^* \) and \( n_j^{m*} \leq n_1^* \).

(v) For the client organization, \( \pi^{m*} \leq \pi^* \).

(III) If \( \Delta_{\bar{U}} \leq \Delta_U \leq \Delta_{\bar{U}} \),

(i) For the merged PBM, \( \pi_1^{m*} \geq \pi_1^* + \pi_2^* \) and \( n_1^{m*} \leq n_1^* + n_2^* \).

(ii) For the non-merging PBM \( j \) (\( j = 3, \ldots, M \)), \( \pi_j^{m*} \leq \pi_j^{m*} \leq \pi_1^{m*} \) and \( n_j^{m*} \leq n_j^{m*} \leq n_1^{m*} \).

(iii) For the PBM industry, \( \pi_j^{m*} \geq \pi_1^* \) and \( n_j^{m*} \leq n_1^* \).

(v) For the client organization, \( \pi^{m*} \leq \pi^* \).

(IV) If \( \Delta_U \geq \Delta_{\bar{U}} \),

(i) For the merged PBM, \( \pi_1^{m*} \geq \pi_1^* + \pi_2^* \) and \( n_1^{m*} \geq n_1^* + n_2^* \).

(ii) For the non-merging PBM \( j \) (\( j = 3, \ldots, M \)), \( \pi_j^{m*} \leq \pi_j^{m*} \leq \pi_1^{m*} \) and \( n_j^{m*} \leq n_j^{m*} \leq n_1^{m*} \).

(iii) For the PBM industry, \( \pi_j^{m*} \geq \pi_1^* \) and \( n_j^{m*} \geq n_1^* \).

(iv) For the client organization, \( \pi^{m*} \geq \pi^* \).

Proposition 5 shows the combined impact of the increased negotiating leverage and decreased competition effects of the merger. The equilibrium expected profit [market share] of the merged PBM always exceeds the pre-merger individual PBM’s equilibrium expected profit [market share]. However, if the merged PBM cannot obtain sufficient savings from lowered wholesale prices and/or additional rebates to guarantee a power index higher than the middle threshold (\( \Delta_{\bar{U}} \)), the post-merger equilibrium expected profit [market share] of the merged PBM is less than the combined
pre-merger equilibrium expected profits [market shares] of the two merging PBMs, and the post-merger expected utility of the client organization is also less than its pre-merger counterpart. At the same time, all non-merging PBMs benefit from the merger, in terms of both their equilibrium expected profits and expected market shares. On the other hand, if the power index is greater than the highest threshold ($\Delta_U$), the equilibrium expected profit [market share] of the merged PBM is greater than the combined pre-merger equilibrium expected profits [market shares] of the two merging PBMs, and the post-merger expected utility of the client organization is also greater than its pre-merger counterpart. At the same time, each non-merging PBM has a lower equilibrium expected profit [market share]. When the power index falls in between these two threshold levels ($\Delta U$ and $\Delta U$), the merged PBM has an equilibrium expected profit greater than the combined pre-merger equilibrium expected profits of the two merging PBMs, and an equilibrium expected market share less than the combined pre-merger equilibrium expected market shares of the two merging PBMs. In this case, each non-merging PBM benefits from the merger, in terms of both its equilibrium expected profit and expected market share, while the client organization incurs a loss in its expected utility. Moreover, for the PBM industry, the combined equilibrium profit [market share] is larger after the merger if and only if the power index is greater than the lowest [highest] threshold, $\Delta U$ [\Delta U]. To sum up, the client organization’s expected utility and the total market share of all PBMs move in the same direction after the merger, while the equilibrium expected profit and expected market share of each non-merging PBM move in the other direction. Therefore, there is no win-win situation for all parties involved in the merger.

In all four cases of Proposition 5, the merged PBM always has a higher profit and a larger market share than its non-merging competitors, because of its increased bargaining power with drug manufacturers and retail pharmacies after the merger. However, the equilibrium profit of the merged PBM exceeds the combined pre-merger equilibrium profits of the two merging PBMs if and only if the power index is greater than the middle threshold. This finding contrasts with the existing literature on mergers under price competition (e.g., Deneckere and Davidson (1985) and Federgruen and Pierson (2011)), which states that, in the absence of cost synergies, the post-merger equilibrium profit of the merged firm exceeds the combined pre-merger equilibrium profits of the merging firms. This is because the merger of PBMs is different from mergers of competing firms in the existing literature. In all the models that consider mergers with price competition in the literature, the merger reduces the number of competing firms in the industry, but the number of products remains the same before and after the merger. In other words, after the merger, the
merged firm sets prices for all the products that were supplied by the merging firms in the pre-merger model. Unlike the above models of mergers among manufacturers, our model considers mergers of PBMs, who are service providers to the client organization. Before the merger, each PBM provides the client a set of drug prices and formulary. When two PBMs merge, all drugs are managed by the merged PBM, and the merged PBM only needs to offer a single set of drug prices and formulary to the client. Therefore, the merger decreases both the number of competing firms (PBMs) and the number of products (the sets of drug prices and formulary) in our model.

To study the impact of PBM merger on social welfare, we define the equilibrium social welfare as the sum of the equilibrium profit of the PBM industry and the client’s expected utility. The latter term captures both the consumer surplus of the plan enrollees and the expected cost of the client. Specifically, we define the equilibrium social welfare before and after the merger as:

\[ \Pi^* \equiv \pi^*_I + \Pi^* \] and \[ \Pi^{m*} \equiv \pi^{m*}_I + \Pi^{m*} \] (23)

The following proposition characterizes the impact of PBM mergers on social welfare.

**Proposition 6 (Impact of PBM Mergers on Social Welfare).** Consider \( M \) symmetric PBMs. After the merger of PBM 1 and PBM 2, the equilibrium social welfare \( \Pi^{m*} \) increases in the power index \( \Delta U \). Hence, there exists a threshold value of the power index, \( \Delta \hat{U} \) \( (\Delta \bar{U} \leq \Delta \hat{U} < \Delta \bar{U}) \), such that \( \Pi^{m*} \leq \Pi^* \) if \( 0 \leq \Delta U \leq \Delta \hat{U} \), and \( \Pi^{m*} \geq \Pi^* \) if \( \Delta U \geq \Delta \bar{U} \).

Proposition 6 shows that as the bargaining power of the merged PBM increases, the equilibrium social welfare after the merger also increases. When the merged PBM obtains sufficiently high bargaining power after the merger, the increased negotiating leverage effect dominates the decreased competition effect of the merger, and the merger results in higher social welfare.

To study the impact of PBM merger on equilibrium welfare-adjusted prices, we define the share-weighted average welfare-adjusted price before and after the merger as:

\[ V^*_I = \frac{\sum_{j=1}^{M} n_j^* V^*_j}{\sum_{j=1}^{M} n_j^*}, \quad \text{and} \quad V^{m*}_I = \frac{n^*_1 V^{m*}_1 + \sum_{j=3}^{M} n^*_j V^{m*}_j}{n^*_1 + \sum_{j=3}^{M} n^*_j}. \] (24)

We now characterize the impact of PBM mergers on equilibrium welfare-adjusted prices:

**Proposition 7 (Impact of PBM Mergers on Equilibrium Welfare-Adjusted Prices).** Consider \( M \) symmetric PBMs. After the merger of PBM 1 and PBM 2, there exist three threshold values of the power index, \( \Delta \hat{U} \), \( \Delta \bar{U} \) and \( \Delta \bar{U} \) \( (0 < \Delta \hat{U} \leq \Delta \bar{U} < \Delta \bar{U}) \), such that:

(I) If \( 0 \leq \Delta U \leq \Delta \hat{U} \),

(i) For the merged PBM, \( V^{m*}_1 \geq V^*_1 \).
(ii) For the non-merging PBM \( j \) \((j = 3, \ldots, M)\), \( V_{jm}^* \geq V_j^* \).

(iii) For the PBM industry average, \( V_I^m \geq V_I^* \).

(II) If \( \Delta \widehat{U} \leq \Delta U \leq \Delta \overline{U} \),

(i) For the merged PBM, \( V_1^m \leq V_1^* \).

(ii) For the non-merging PBM \( j \) \((j = 3, \ldots, M)\), \( V_j^m \geq V_j^* \).

(iii) For the PBM industry average, \( V_I^m \geq V_I^* \).

(III) If \( \Delta \widehat{U} \leq \Delta U \leq \Delta \overline{U} \),

(i) For the merged PBM, \( V_1^m \leq V_1^* \).

(ii) For the non-merging PBM \( j \) \((j = 3, \ldots, M)\), \( V_j^m \geq V_j^* \).

(iii) For the PBM industry average, \( V_I^m \leq V_I^* \).

(IV) If \( \Delta U \geq \Delta \overline{U} \),

(i) For the merged PBM, \( V_1^m \leq V_1^* \).

(ii) For the non-merging PBM \( j \) \((j = 3, \ldots, M)\), \( V_j^m \leq V_j^* \).

(iii) For the PBM industry average, \( V_I^m \leq V_I^* \).

Proposition 7 confirms the finding in the current literature on mergers under price competition that, in the absence of cost synergies, equilibrium prices increase after the merger. Moreover, Proposition 7 completely characterizes three threshold values of the power index: The equilibrium welfare-adjusted price of the merged [non-merging] PBM decreases after the merger if and only if the power index is higher than the lowest [highest] threshold, while the middle threshold works for the industry average equilibrium welfare-adjusted price.

Finally, we comment that our results in this section are obtained under the assumption that all PBMs are symmetric before the merger. Although this assumption excludes the more general case with asymmetric PBMs, it greatly simplifies the analysis and enables us to deliver important managerial insights. Our results in Propositions 5–7 suggest that, when considering future mergers and acquisitions in the already concentrated PBM industry, policy makers need to carefully weigh the potential gains resulting from negotiation leverage on drug wholesale prices and rebates against the relaxed competition intensity among the remaining PBMs. In an environment with limited total drug manufacturing capacity and drug manufacturers able to withstand negotiation pressures due to an increased global drug demand, our model tends to predict that clients may not appropriate much value from a PBM merger. This prediction is confirmed by a recent empirical study on the Express Scripts/Medco merger, conducted by the economists at the Federal Trade Commission, Shelanski et al. (2012). This study employs two approaches to analyze the impact of PBM mergers on costs, and concludes that “Neither approach revealed significant incremental scale economies in the negotiation of rebates or pharmacy reimbursement” (Shelanski et al. 2012, page 306).
7. Conclusion

We have characterized the equilibrium behavior of a complex oligopolistic competition among multiple PBMs seeking the business of a client organization. Each PBM selects $2N + 1$ action variables: a list of gross prices to be charged to the client for each of the branded and generic drugs (the price decision) as well as a binary vector assigning the branded drugs to preferred or non-preferred formulary tier (the formulary decision). We have shown that each PBM’s formulary and price decision can be solved in two sequential steps. First, its optimal formulary decision can be obtained as the binary vector that minimizes the welfare-adjusted cost of its plan. Since this variable is independent of all other PBMs’ decisions, each PBM’s optimal formulary design is a dominant choice. Second, with each PBM’s optimal formulary design determined, the competition among PBMs can be reduced to a classical single-dimension MNL price competition model, where each PBM selects the welfare-adjusted price of its plan. We characterize the equilibrium of the reduced price competition, the resulting equilibrium expected market share and expected profit for each PBM, and the client organization’s expected utility. In addition, we discuss the impact of our model primitives. In particular, a change in drug quality has a non-monotone impact on a PBM’s optimal formulary decision, equilibrium welfare-adjusted price, expected market share and expected profit, as well as the client organization’s expected utility.

Employing our stylized model, we study the impact of PBM mergers. Our finding confirms the existing literature on mergers under price competition that, in the absence of cost synergies, equilibrium prices increase after the merger. In addition, we characterize the combined impact of the increased negotiating leverage and decreased competition effects of the merger. In particular, the post-merger equilibrium expected profit [market share] of the merged PBM is less than the combined pre-merger equilibrium expected profits [market shares] of the two merging PBMs, unless the merger brings about sufficient savings to the merged PBM from increased bargaining power with drug manufacturers and retail pharmacies. This finding contrasts with the existing literature on mergers under price competition, because the merger of PBMs is different from mergers of competing firms in the literature: the PBM merger reduces both the number of competing firms (PBMs) and the number of products (the sets of drug prices and formulary).

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References


Appendix A: Proofs

Proof of Theorem 1 (a). By (5) and (6), we have
\[
\max_{\nu_j \in \{0,1\}^N, \bar{y}_j \in \mathbb{R}^{N+1}} \pi_j = \frac{\exp((A_j + CS_j - B_j)/\nu)}{\exp(u_0^0/\nu) + \sum_{k=1}^M \exp((A_k + CS_k - B_k)/\nu)} (B_j - W_j),
\]
where \(W_j\) is given in (9). Using (7) and (8), we can rewrite PBM \(j\)'s problem as
\[
\max_{\nu_j \in \{0,1\}^N, \bar{y}_j \in \mathbb{R}^{N+1}} \pi_j = \frac{\exp((A_j - V_j)/\nu)}{\exp(u_0^0/\nu) + \sum_{k=1}^M \exp((A_k - V_k)/\nu)} (V_j - U_j).
\]
Note that \(U_j\) only depends on the formulary decision \(\bar{y}_j \in \{0,1\}^N\). Hence, the feasible set of \(U_j\) is given by \(\Omega_j\), and (10) follows directly.

(b). It directly follows from (10) that \(\pi_j\) linearly decreases in \(U_j\). Note that
\[
\frac{\partial \ln \pi_j}{\partial V_j} = -\frac{1}{\nu V_j - U_j} + \frac{1}{\nu} + \frac{\exp((A_j - V_j)/\nu)}{\nu (\exp(u_0^0/\nu) + \sum_{k=1}^M \exp((A_k - V_k)/\nu))},
\]
and
\[
\frac{\partial^2 \ln \pi_j}{\partial V_j^2} = -\frac{1}{(V_j - U_j)^2} - \frac{\exp((A_j - V_j)/\nu)}{\nu^2 (\exp(u_0^0/\nu) + \sum_{k=1}^M \exp((A_k - V_k)/\nu))^2} < 0,
\]
showing that \(\pi_j\) is strictly log-concave in \(V_j\). Further note that
\[
V_j = \left(d_{0j}(\bar{y}_j)(p_{0j} - c_{0j}^0) + \sum_{i=1}^N d_{ij}(\bar{y}_j)(p_{ij} - c_{ij}(y_{ij}) - \rho_j r_{ij}y_{ij})R_S \right) - \frac{\mu}{\alpha} \left(\ln \left(\exp\left(\frac{\gamma_0 - \alpha c_{0j}^0 + \beta q_0}{\mu}\right) + \sum_{i=1}^N \exp\left(\frac{\gamma_i - \alpha c_{ij}(y_{ij}) + \beta q_i}{\mu}\right)\right)\right) S.
\]
For any given original formulary decision vector \(\bar{y}_j \in \{0,1\}^N\) (and hence \(U_j\)), \(V_j\) is a linear combination of the \(N + 1\) original price decision variables \(p_{ij} \in \mathbb{R}\) (\(i = 0, \ldots, N\)), as shown by (27). Therefore, \(V_j\) can target any value by varying the original price decision vector \(\bar{y}_j\). Conversely, for any given \(V_j \in \mathbb{R}\), \(U_j\) can target any value in its feasible set \(\Omega_j\) by choosing the corresponding original formulary decision vector \(\bar{y}_j\). At the chosen \(\bar{y}_j\), we can keep \(V_j\) fixed at the given value by varying the original price decision vector \(\bar{y}_j\).

(c). Based on the proof of parts (a) and (b), PBM \(j\) can determine its best response sequentially by solving the optimal \(U_j\) first and then the optimal \(V_j\). For PBM \(j\), the minimization of \(U_j\) is a combinatorial optimization on a finite set: \(\min_{\bar{y}_j \in \{0,1\}^N} U_j(\bar{y}_j)\), and thus the optimal \(U_j^*\) is unique. Since each PBM’s optimal aggregate formulary decision \(U_j^*\) is independent of each other, PBMs’ competition on \((U_j, V_j)\) can be reduced to a competition on \(V_j\) only. With \(U_j^*\) determined, PBM \(j\) only needs to consider the value of \(V_j\) in a compact and convex set, \([U_j^*, \bar{V}_j]\), where \(\bar{V}_j\) is the unique root of \(-\frac{1}{V_j - U_j} + \frac{1}{\nu} + \frac{\exp((A_j - V_j)/\nu)}{\nu (\exp(u_0^0/\nu) + \exp((A_j - V_j)/\nu))} = 0\), because PBM \(j\) would make negative profit for any \(V_j < U_j^*\) and \(\partial \ln \pi_j / \partial V_j < 0\) for any \(V_j > \bar{V}_j\). The existence of equilibrium directly follows from the concavity of \(\ln \pi_j\) in \(V_j\). Furthermore,
\[
\frac{\partial^2 \ln \pi_j}{\partial V_j^2} + \sum_{k \neq j} \left|\frac{\partial^2 \ln \pi_j}{\partial V_j \partial V_k}\right| = -\frac{1}{(V_j - U_j)^2} - \frac{\exp((A_j - V_j)/\nu) \exp(u_0^0/\nu)}{\nu^2 (\exp(u_0^0/\nu) + \sum_{k=1}^M \exp((A_k - V_k)/\nu))^2} < 0.
\]
By Milgrom and Roberts (1990), a unique equilibrium is guaranteed, and satisfies the first order condition given by (25).
Proof of Proposition 1 Substituting (9) and (2) into (8), we can write PBM $j$'s formulary problem as
\[
\min_{\vec{y}_j \in \{0,1\}^N} U_j(\vec{y}_j) = \left( d_{00j}(\vec{y}_j)(w_{00} - c_j^0) + \sum_{i=1}^N d_{ij}(\vec{y}_j)(w_{ij} - c_i^0 + (c_i^0 - c_j^0 - r_{ij}) y_{ij}) \right) S
\]
\[\quad - \frac{\mu}{\alpha} \ln \left( \exp \left( \frac{\gamma_0 - \alpha c_j^0 + \beta q_j}{\mu} \right) + \sum_{i=1}^N \exp \left( \frac{\gamma_i - \alpha c_i^0 + \alpha (c_i^0 - c_j^0) y_{ij} + \beta q_i}{\mu} \right) \right) S. \quad (29)\]

By (11), we rewrite (29) into the following form:
\[
\min_{\vec{y}_j \in \{0,1\}^N} U_j(\vec{y}_j) = \left( \frac{x_0 m_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) m_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj})} \right) S
\]
\[\quad - \frac{\mu}{\alpha} \ln \left( \frac{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj})} \right) S. \quad (30)\]

First, we introduce the following lemma to characterize properties of PBMs' optimal formulary decision.

**Lemma 2.** For $j = 1, \ldots, M$, PBM $j$ puts branded drug $i$ on tier 2 (i.e., $y_{ij}^* = 1$) if for any $\vec{y}_j \in \{0,1\}^N$,
\[
\delta_{ij} \leq \frac{x_0 m_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) m_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj})}. \quad (31)\]

**Proof of Lemma 2** We prove by contradiction. Let $\vec{y}_j^*$ be PBM $j$'s optimal formulary decision vector. Assume to the contrary that (31) holds for any $\vec{y}_j \in \{0,1\}^N$ and $y_{ij}^* = 0$. We can construct another formulary decision $\vec{y}_j'$ by setting $y_{ij}' = 1$ and $y_{kj}' = y_{kj}^*$ for $k \neq i$. Then we have:
\[
U_j(\vec{y}_j') = \left( \frac{x_{0j} m_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}')(m_{ij}(y_{kj}')) + x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) + x_{ij}(1) - x_{ij}(0)} \right) S
\]
\[\quad - \frac{\mu}{\alpha} \ln \left( \frac{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) + x_{ij}(1) - x_{ij}(0)}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) + x_{ij}(1) - x_{ij}(0)} \right) S. \quad (32)\]

Since $\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)} \leq \frac{x_{0j} m_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) m_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj})}$ and $x_{ij}(1) > x_{ij}(0)$, we have $U_j(\vec{y}_j') < U_j(\vec{y}_j^*)$, contradicting the optimality of $\vec{y}_j^*$. This completes the proof.

Lemma 2 shows that when assigning branded drugs to tier 2, it is optimal for PBM $j$ to start by choosing the branded drug with the smallest cost change rate, then add the drug with the second smallest cost change rate to tier 2, and so on, until (31) no longer holds. Since PBM $j$ sorts branded drugs by their cost change rates in an ascending order, we can find the threshold index as follows:

Initialize $I_j = 0$;

FOR $i = 1, \cdots, N$,

IF $\delta_{ij} \leq \frac{x_0 m_{0j} + \sum_{k=1}^N x_{kj}(1)m_{kj}(1) + \sum_{k=1}^N x_{kj}(0)m_{kj}(0)}{x_{0j} + \sum_{k=1}^N x_{kj}(1) + \sum_{k=1}^N x_{kj}(0)}$,

$I_j = i$;

ELSE STOP;

ENDFOR

Note that
\[
\min_{\vec{y}_j \in \{0,1\}^N} \frac{x_0 m_{0j} + \sum_{k=1}^N x_{kj}(y_{kj}) m_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^N x_{kj}(y_{kj})} = \frac{x_0 m_{0j} + \sum_{k=1}^{I_{ij}+1} x_{kj}(1)m_{kj}(1) + \sum_{k=1}^N x_{kj}(0)m_{kj}(0)}{x_{0j} + \sum_{k=1}^{I_{ij}+1} x_{kj}(1) + \sum_{k=1}^N x_{kj}(0)}. \quad (29)\]

By Lemma 2, if $i \leq I_j$, it is optimal for PBM $j$ to assign drug $i$ on tier 2 (i.e., $y_{ij}^* = 1$).
Proof of Proposition 2 When all branded drugs on PBM $j$’s plan have the same brand-specific attribute $\gamma_i$ and quality index $q_i$, we first show that if PBM $j$ puts $k$ branded drugs on tier 2, it is optimal for PBM $j$ to assign $k$ branded drugs with the smallest cost change rate indices on tier 2.

We prove the above statement by contradiction. Let $\vec{y}_j^*$ be PBM $j$’s optimal formulary decision vector. Assume to the contrary of the statement, there exists an $y_{ij}^* = 0$ $(i \leq k)$ and an $y_{ij}^* = 1$ $(l > k)$. We can construct another formulary decision $\vec{y}_j^{**}$ by setting $y_{ij}^{**} = 1$ and $y_{ij}^{**} = 0$ while keeping other branded drugs’ formulary decision unchanged. Then we have:

$$U_j(\vec{y}_j^{**}) = \left( x_{0j} m_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^{*}) y_{kj}(y_{kj}^{*}) + x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) - x_{ij}(1)m_{ij}(1) + x_{ij}(0)m_{ij}(0) \right)$$

$$- \frac{\mu}{\alpha} \ln \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^{*}) + x_{ij}(1) - x_{ij}(0) - x_{ij}(1) + x_{ij}(0) \right) S.$$

Note that all branded drugs have the same brand specific attribute and quality index, we have $x_{ij}(1) - x_{ij}(0) = x_{ij}(1) - x_{ij}(0) > 0$. Since PBM $j$ ranks branded drugs in ascending order of their cost change rate, i.e., $\delta_{i1} \leq \delta_{i2} \leq \ldots \leq \delta_{Nj}$, we have $\delta_i = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)} \leq \delta_i = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)}$, and hence $x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) \leq x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)$. Therefore, $U_j(\vec{y}_j^{**}) \leq U_j(\vec{y}_j^*)$, contradicting the optimality of $\vec{y}_j^*$. This completes the proof of the statement.

By the above statement and Proposition 1, PBM $j$ only needs to compare $N - I_j + 1$ formulary decision vectors, each corresponding to putting $k$ ($k = I_j, \ldots, N$) branded drugs with smallest cost change rate indices on tier 2. This completes the proof.

Proof of Theorem 2 By the definition of $V_j$ in (7) and the expression of $n_j$ in (5), we can rewrite $V_j$ as a function of PBMs’ market share vector, $\vec{n} = (n_1, \ldots, n_M)$, as follows:

$$V_j = A_j - u_e^0 - \nu \left( \ln n_j - \ln(1 - \sum_{k=1}^{M} n_k) \right), \quad j = 1, \ldots, M. \quad (33)$$

Let $n_0$ denote the probability of the client not contracting with any PBM, we have $n_0 = 1 - \sum_{k=1}^{M} n_k$.

Given the optimal formulary decision, the equilibrium $V_j^*$ satisfies the set of first order conditions given by (13). Substituting (33) into (13) and rearranging the terms, we have

$$n_j^* \cdot \exp \left( \frac{n_j^*}{1 - n_j^*} \right) = n_0^* \cdot \exp \left( \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right), \quad j = 1, \ldots, M. \quad (34)$$

By the definition of $H(\cdot)$ function in (14), (16) directly follows. Combining with the fact that $\sum_{j=0}^{M} n_j^* = 1$, the equilibrium probability of not contracting with any PBM, $n_0^*$, satisfies the single-variable equation:

$$n_0 + \sum_{j=1}^{M} H \left( n_0 \cdot \exp \left( \frac{A_j - U_j^* - u_e^0 - \nu}{\nu} \right) \right) = 1.$$ 

Since $H(\cdot)$ strictly increases from 0 to 1, the left-hand-side of the above equation strictly increases from 0 to a value greater than 1 as $n_0$ increases from 0 to 1. Therefore, $n_0^*$ is the unique solution to (15).

Substituting (5), (7) and (13) into (10), PBM $j$’s equilibrium expected profit is given by $\pi_j^* = \frac{\nu n_0^*}{1 - n_j^*}$.

Substituting $n_0^* = 1 - \sum_{k=1}^{M} n_k^*$ into (33), (18) follows directly.
The client organization’s expected utility in the equilibrium is given by

$$\varphi = E \left( \max \left( \max n_j^*, v_0 \right) \right) = \nu \ln \left( \exp \left( \frac{u_0^*}{\nu} \right) + \sum_{j=1}^{M} \exp \left( \frac{A_j - V_j^*}{\nu} \right) \right)$$

$$= \nu \ln \left( \exp \left( \frac{u_0^*}{\nu} \right) \left( 1 + \sum_{j=1}^{M} n_j^* \right) \right) = u_0^* - \nu \ln n_0^*.$$

**Proof of Lemma 1** (a). Applying the implicit function theorem on the function $H(x)$, we have

$$H'(x) = \frac{(1 - H)^2}{(1 - H + H^2) \exp(\frac{H}{1 - H})} > 0.$$  

By (15), (34) and (35), and applying the implicit function theorem on (15), we have

$$\frac{\partial n_0^*}{\partial U_j^*} = \frac{(1 - n_j^*)^2 n_j^*}{\nu (1 - n_j^* + n_j^* n_0^*)} > 0.$$  

(b). By (16) and (36), we have

$$\frac{\partial n_j^*}{\partial U_j^*} = H' \left( n_j^* \cdot \exp \left( \frac{A_j - U_j^* - u_j^* - \nu}{\nu} \right) \right) \cdot \exp \left( \frac{A_j - U_j^* - u_j^* - \nu}{\nu} \right) \left( \frac{\partial n_0^*}{\partial U_j^*} - \frac{n_j^*}{\nu} \right)$$

$$= -H' \left( n_j^* \cdot \exp \left( \frac{A_j - U_j^* - u_j^* - \nu}{\nu} \right) \right) \cdot \exp \left( \frac{A_j - U_j^* - u_j^* - \nu}{\nu} \right) \frac{n_j^* + \sum_{k=1}^{M} n_k^*}{\nu \left( 1 + \sum_{k=1}^{M} (1 - n_k^* + n_k^* n_0^*) \right)} < 0,$$

$$\frac{\partial n_j^*}{\partial U_k^*} = H' \left( n_j^* \cdot \exp \left( \frac{A_j - U_j^* - u_j^* - \nu}{\nu} \right) \right) \cdot \exp \left( \frac{A_j - U_j^* - u_j^* - \nu}{\nu} \right) \frac{\partial n_0^*}{\partial U_k^*} > 0, \quad k \neq j.$$  

By (17), (37) and (38), $\frac{\partial n_j^*}{\partial U_k^*} < 0$ and $\frac{\partial n_k^*}{\partial U_k^*} > 0 \ (k \neq j)$ directly follow.

(c). By (18), (36) and (37), we have

$$\frac{\partial V_j^*}{\partial U_j^*} = -\nu \left( \frac{1}{n_j^*} \frac{\partial n_j^*}{\partial U_j^*} - \frac{1}{n_0^*} \frac{\partial n_0^*}{\partial U_j^*} \right) > 0.$$  

By (18), (34), (36) and (38), we have

$$\frac{\partial V_j^*}{\partial U_k^*} = -\nu \left( \frac{1}{n_j^*} \frac{\partial n_j^*}{\partial U_k^*} - \frac{1}{n_0^*} \frac{\partial n_0^*}{\partial U_k^*} \right) = \frac{\nu n_j^*}{(1 - n_j^* + n_j^* n_0^*)} \frac{\partial n_0^*}{\partial U_k^*} > 0, \quad k \neq j.$$  

(d). The result directly follows from (19) and part (c).

**Proof of Proposition 3** (a). Since formulary decision $y_{ij}$ is a binary variable, showing that it is weakly decreasing is equivalent to showing that an increase from 0 to 1 cannot happen when the wholesale price of branded drug $i$ increases. We prove by contradiction. Suppose that the wholesale price of branded drug $i$ increases from $w_{ij}$ to $w_{ij}'$ ($w_{ij}' > w_{ij}$), $y_{ij}|_{w_{ij}} = 0$ and $y_{ij}'|_{w_{ij}'} = 1$. Note that for any given $\bar{y}_j$, we have

$$\frac{\partial U_j(y_{ij})}{\partial w_{ij}} = \frac{x_{ij}(y_{ij})S}{x_0 + \sum_{k=1}^{N} x_{kj}(y_{kj})} > 0, \quad \text{and} \quad \frac{\partial^2 U_j(y_{ij})}{\partial w_{ij}^2} = 0.$$  

Therefore,

$$U_j(\bar{y}_j|_{w_{ij}}) \mid_{w_{ij}} \leq U_j(\bar{y}_j'|_{w_{ij}'}) \mid_{w_{ij}} < U_j(\bar{y}_j'|_{w_{ij}'}|_{w_{ij}'}) \leq U_j(\bar{y}_j'|_{w_{ij}}|_{w_{ij}'}) \mid_{w_{ij}'},$$

which shows that

$$U_j(\bar{y}_j|_{w_{ij}}) \mid_{w_{ij}} - U_j(\bar{y}_j'|_{w_{ij}}) \mid_{w_{ij}} \geq U_j(\bar{y}_j'|_{w_{ij}'}|_{w_{ij}'}) \mid_{w_{ij}'} - U_j(\bar{y}_j'|_{w_{ij}}) \mid_{w_{ij}}.$$  

(42)
By (41), we have
\[ U_j (\bar{y}_j^* | w_{ij}) - U_j (y_{ij}^* | w_{ij}) = \frac{x_{ij}(y_{ij}^* | w_{ij})(w_{ij}' - w_{ij})S}{x_{ij} + \sum_{k=1}^{N} x_{kj}(y_{kj}^* | w_{ij})}, \]
(43)
and
\[ U_j (y_{ij}^* | w_{ij}) - U_j (\bar{y}_j^* | w_{ij}) = \frac{x_{ij}(y_{ij}^* | w_{ij})(w_{ij}' - w_{ij})S}{x_{ij} + \sum_{k=1}^{N} x_{kj}(y_{kj}^* | w_{ij})}. \]
(44)

Substitute \( y_{ij} | w_{ij} = 0 \), \( y_{ij}^* | w_{ij} = 1 \), (43) and (44) into (42), we have
\[ x_{ij}(0) \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^* | w_{ij}) \right) \geq x_{ij}(1) \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^* | w_{ij}) \right) \]
(45)
The left hand side of (45) satisfies
\[ x_{ij}(0) \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^* | w_{ij}) \right) \leq x_{ij}(0) \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(1) \right) = x_{ij}(0)x_{0j} + \sum_{k=1}^{N} \exp \left( \frac{\gamma_i + \gamma_k (q_i + q_k) - \alpha \left( c_i^n + c_i^g \right)}{\mu} \right), \]
while the right hand side of (45) satisfies
\[ x_{ij}(1) \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^* | w_{ij}) \right) \geq x_{ij}(1) \left( x_{0j} + \sum_{k=1}^{N} x_{kj}(0) \right) = x_{ij}(1)x_{0j} + \sum_{k=1}^{N} \exp \left( \frac{\gamma_i + \gamma_k (q_i + q_k) - \alpha \left( c_i^n + c_i^g \right)}{\mu} \right). \]

By (45) and \( x_{0j} > 0 \), we have \( x_{ij}(0) \geq x_{ij}(1) \). This contradicts with the fact that \( x_{ij}(0) < x_{ij}(1) \).

In general, the impact of any other drug’s wholesale price on the focal drug’s optimal formulary decision is non-monotone, and we have observed this non-monotone effect in numerical examples.

Recall that the optimal formulary decision vector is a dominant choice, and is independent of all other PBMs’ decision and cost parameters. Therefore, PBM j’s optimal formulary decision is independent of any drug’s wholesale price on any competing PBM’s plan.

(b). Note that for any given \( \bar{y}_j \), (41) holds for any \( i = 0, \ldots, N \). Consider the case that the wholesale price of any (branded or generic) drug \( i \) increases from \( w_{ij} \) to \( w_{ij}' \) (\( w_{ij}' > w_{ij} \)). By (41), \( U_j (y_{ij}^* | w_{ij}) > U_j (\bar{y}_j^* | w_{ij}) \geq U_j (\bar{y}_j^* | w_{ij}) \) \( w_{ij} \geq U_j (\bar{y}_j^* | w_{ij}) \) \( w_{ij} \). Therefore, \( U_j^* \) strictly increases in the wholesale price of any (branded or generic) drug charged to PBM j.

Since each PBM’s formulary decision is independent of each other, the optimal welfare-adjusted cost of PBM j’s plan \( U_j^* \) is independent of any (branded or generic) drug’s wholesale price charged to any competing PBM.

(c). Since each PBM’s formulary decision is independent of each other, the optimal welfare-adjusted cost of PBM j’s plan \( U_j^* \) is independent of any (branded or generic) drug \( i \)’s wholesale price charged to PBM k (\( k \neq j \)), \( w_{ik} \). Therefore, the impact of \( w_{ik} \) on the equilibrium welfare-adjusted price of PBM j’s plan \( V_j^* \) is only through its impact on the optimal welfare-adjusted cost of PBM k’s plan \( U_k^* \). By part (c) of Lemma 1 and part (b) of Proposition 3, \( V_j^* \) strictly increases in \( w_{ik} \) and \( w_{ij} \).

(d). The impact of \( w_{ik} \) on the client’s expected utility \( \Phi \) directly follows from (19) and part (c) of Proposition 3.

(e). We will show this part in an analogous approach as the above proof in part (c). Since the optimal welfare-adjusted cost of PBM j’s plan \( U_j^* \) is independent of any (branded or generic) drug \( i \)’s (\( l = 0, \ldots, N \) and
wholesale price charged to any other PBM $k$ ($k \neq j$), $w_{ik}$. Therefore, the impact of $w_{ik}$ on PBM $j$’s equilibrium expected market share $n_j^*$ and expected profit $\pi_j^*$ is only through its impact on the optimal welfare-adjusted cost of PBM $k$’s plan $U_k^*$. By part (b) of Lemma 1 and part (b) of Proposition 3, the statement follows directly.

**Proof of Proposition 4** (a). For any given $\bar{y}_j$ and branded drug $i$,

$$
\frac{\partial U_j(\bar{y}_j)}{\partial q_i} = -\frac{\beta S x_{ij}(y_{ij})}{\mu (x_0 + \sum_{i=1}^N x_{ij}(y_{ij}))} \left( \frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) \right). 
$$

(46)

Therefore, $\text{sgn} \left( \frac{\partial U_j(\bar{y}_j)}{\partial q_i} \right) = -\text{sgn} \left( \frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) \right)$. Note that $\frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right)$ is convex increasing in $q_i$, and approaches to $+\infty$ when $q_i \to +\infty$. So $\frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right)$ crosses 0 at most once. Denote $q_{ij}(\bar{y}_j)$ as the unique root of $\frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) = 0$ when exists, and $q_{ij}(\bar{y}_j) = -\infty$ otherwise. Set $\overline{y}_{ij} = (\max_{q_i \in (0,1)N} q_{ij}(\bar{y}_j))$ and $\underline{y}_{ij} = (\min_{q_i \in (0,1)N} q_{ij}(\bar{y}_j))$. It directly follows that $\frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) > 0$ for any $\bar{y}_j \in \{0,1\}^N$ when $q_i > q_{ij}$, and $\frac{\rho}{\alpha} x_{ij}(y_{ij}) + x_0 \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\rho}{\alpha} - m_{ij}(y_{ij}) + m_0 \right) < 0$ for any $\bar{y}_j \in \{0,1\}^N$ when $0 < q_i < q_{ij}$. Therefore, PBM $j$’s optimal welfare-adjusted cost, $U_j^*$, strictly increases in branded drug $i$’s quality index, $q_i$, if $0 \leq q_i < q_{ij}$, and strictly decreases in $q_i$ if $q_i > q_{ij}$.

When $q_{ij}(\bar{y}_j) \neq -\infty$, applying the implicit function theorem, we have $q_{ij}(\bar{y}_j)$ increases in $w_{ij}$ and weakly decreases $r_{ij}$. Therefore, $\overline{y}_{ij} = (\max_{q_i \in (0,1)N} q_{ij}(\bar{y}_j))$ and $\underline{y}_{ij} = (\min_{q_i \in (0,1)N} q_{ij}(\bar{y}_j))$ both weakly increase in $w_{ij}$ and weakly decrease in $r_{ij}$.

The impact of generic drug’s quality $q_0$ can be shown in the same way.

(b). By Lemma 1, the impact of drug quality affects any PBM’s equilibrium expected welfare-adjusted price, market share, profit, and the client’s utility through its impact on all PBMs’ optimal welfare-adjusted costs. When all PBMs are symmetric with respect to the cost parameters, the impact of any drug’s quality on the optimal welfare-adjusted cost, $U_j^*$, is the same across all PBMs. Therefore, the impact of any drug’s quality on all the quantities of interest mentioned above depends on the aggregate impact of the drug’s quality on all PBMs’ optimal welfare-adjusted costs. For PBM $j$, by (37) and (38), we have

$$
\sum_{k=1}^M \frac{\partial n_j^*}{\partial U_k^*} = \frac{-H'}{n_j^* \exp \left( \frac{A_j - U_j^* - w_0^* - \nu}{\nu} \right)} \exp \left( \frac{A_j - U_j^* - w_0^* - \nu}{\nu} \right) \frac{n_j^*}{\nu} \left( 1 + \sum_{k=1}^M \frac{n_j^*}{\nu} \right) < 0.
$$

(47)

Therefore, the impact of $q_i$ on $n_j^*$ has the opposite directional change as the impact of $q_i$ on $U_j^*$.

The impact of $q_i$ on $V_j^*$, $\pi_j^*$, and $\overline{\pi}^*$ can be shown in the same way.

**Proof of Proposition 5** Substituting (16) into (15) and by the symmetry of PBMs in the pre-merger model, the equilibrium market share of PBM $j$, $n_j^*$, is the unique solution on $(0,1)$ of the following equation:

$$
n_j^* \left( M + \exp \left( \frac{n_j^*}{1-n_j^*} - \frac{A_j - U_j^* - w_0^* - \nu}{\nu} \right) \right) = 1,
$$

(48)
After the merger, it directly follows from (16) and the fact $U_1^{m^*} \leq U_j^* = U_j^{m^*}$ ($j = 3, \ldots, M$) that $n_1^{m^*} \geq n_j^{m^*}$ ($j = 3, \ldots, M$). All non-merging PBMs remain symmetric after the merger. By (17), $\pi_1^{m^*} \geq \pi_j^{m^*}$ ($j = 3, \ldots, M$) always holds. Apply (15) and (16) to the post-merger model, we have the following relationships:

$$n_1^{m^*} + n_j^{m^*} + (M-2)n_j^{m^*} = 1,$$  \hspace{1cm} (49)

$$n_j^{m^*} \exp\left(\frac{n_j^{m^*}}{1-n_j^{m^*}}\right) = n_0^{m^*} \exp\left(\frac{A_1 - U_1^{m^*} - u_0^0 - \nu}{\nu}\right),$$  \hspace{1cm} (50)

$$n_j^{m^*} \exp\left(\frac{n_j^{m^*}}{1-n_j^{m^*}}\right) = n_0^{m^*} \exp\left(\frac{A_j - U_j^{m^*} - u_0^0 - \nu}{\nu}\right), \quad j = 3, \ldots, M.$$  \hspace{1cm} (51)

Using $A_1 = A_j$ and $U_j^{m^*} = U_1^* = U_j^*$ ($j = 3, \ldots, M$), we have $n_j^{m^*}$ satisfies the following equation:

$$n_j^{m^*} \exp\left(\frac{n_j^{m^*}}{1-n_j^{m^*}} - \frac{A_j - U_1^* - u_0^0 - \nu}{\nu}\right) + H\left(n_j^{m^*} \exp\left(\frac{n_j^{m^*}}{1-n_j^{m^*}} + \frac{\Delta U}{\nu}\right)\right) + (M-2)n_j^* \leq 1,$$  \hspace{1cm} (52)

where $n_j^*$ is given by (48). (53) can be further reduced to $\Delta U \leq \Delta U^*$, where

$$\Delta U = \nu \left(\frac{2n_j^*}{1-2n_j^*} - \frac{n_j^*}{1-n_j^*} + \ln 2\right).$$  \hspace{1cm} (54)

By (17), it directly follows that $\pi_1^{m^*} \geq \pi_j^* \ (j = 3, \ldots, M)$ iff $\Delta U \leq \Delta U^*$.

Similarly, by (49), (50) and (51), we have $n_1^{m^*}$ satisfies the following equation:

$$n_1^{m^*} \exp\left(\frac{n_1^{m^*}}{1-n_1^{m^*}} - \frac{\Delta U + A_1 - U_1^* - u_0^0 - \nu}{\nu}\right) + n_1^{m^*} + (M-2)H\left(n_1^{m^*} \exp\left(\frac{n_1^{m^*}}{1-n_1^{m^*}} - \frac{\Delta U}{\nu}\right)\right) = 1,$$  \hspace{1cm} (55)

and $n_1^{m^*}$ is the unique solution of the above equation. When $n_1^{m^*} = n_1^*$, the left hand side of (55) is:

$$n_1^* \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{\Delta U + A_1 - U_1^* - u_0^0 - \nu}{\nu}\right) + n_1^* + (M-2)H\left(n_1^* \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{\Delta U}{\nu}\right)\right)$$

$$\leq n_1^* \exp\left(\frac{n_1^*}{1-n_1^*} - \frac{A_1 - U_1^* - u_0^0 - \nu}{\nu}\right) + n_1^* + (M-2)H\left(n_1^* \exp\left(\frac{n_1^*}{1-n_1^*}\right)\right)$$

$$= n_1^* \left(M - 1 + \exp\left(\frac{n_1^* - A_1 - U_1^* - u_0^0 - \nu}{\nu}\right)\right) < n_1^* \left(M + \exp\left(\frac{n_1^* - A_1 - U_1^* - u_0^0 - \nu}{\nu}\right)\right) = 1.$$

Therefore, $n_1^{m^*} > n_1^*$ always holds. By (17), $\pi_1^{m^*} > \pi_1^*$ always holds as well.

Since the left hand side of (55) strictly increases in $n_1^{m^*}$, $n_1^{m^*} \geq n_1^* + n_2^* = 2n_2^*$ iff

$$2n_2^* \exp\left(\frac{2n_2^*}{1-2n_2^*} - \frac{\Delta U + A_1 - U_1^* - u_0^0 - \nu}{\nu}\right) + 2n_2^* + (M-2)H\left(2n_2^* \exp\left(\frac{2n_2^*}{1-2n_2^*} - \frac{\Delta U}{\nu}\right)\right) \leq 1.$$  \hspace{1cm} (56)

where $n_2^*$ is given by (48). Note that the left hand side of (56) is a decreasing function of $\Delta U$. Therefore, (56) is equivalent to $\Delta U \geq \Delta U^*$, where $\Delta U$ is the unique solution of

$$2n_2^* \exp\left(\frac{2n_2^*}{1-2n_2^*} - \frac{\Delta U + A_1 - U_1^* - u_0^0 - \nu}{\nu}\right) + 2n_2^* + (M-2)H\left(2n_2^* \exp\left(\frac{2n_2^*}{1-2n_2^*} - \frac{\Delta U}{\nu}\right)\right) = 1.$$  \hspace{1cm} (57)

Note that when $\Delta U = \Delta U^*$, the left hand side of (57) is given by

$$2n_2^* \exp\left(\frac{n_2^*}{1-n_2^*} - \ln 2 - \frac{A_1 - U_1^* - u_0^0 - \nu}{\nu}\right) + 2n_2^* + (M-2)H\left(2n_2^* \exp\left(\frac{n_2^*}{1-n_2^*} - \ln 2\right)\right).$$
\[ n_j^* \left( M + \exp \left( \frac{n_j^* - A_j - U_j^* - u_\nu^0}{\nu} \right) \right) = 1. \]

Since the left hand side of (57) is a decreasing function of \( \Delta U \), the above equality proves that \( \Delta \bar{U} = \Delta \bar{U} \).

By (17), \( \pi_1^{m*} \geq \pi_1^* + \pi_2^* = 2\pi_j^* \) is equivalent to \( n_1^{m*} \geq \frac{2n_j^*}{1+\pi_j^*} \). Therefore, \( \pi_1^{m*} \geq \pi_1^* + \pi_2^* \) iff

\[
\frac{2n_j^*}{1+n_j^*} \exp \left( \frac{2n_j^* - \Delta U + A_j - U_j^* - u_\nu^0}{\nu} \right) + \frac{2n_j^*}{1+n_j^*} + (M-2) \frac{\Delta U - \nu}{\nu} \leq 1. 
\]

where \( n_j^* \) is given by (48). Note that the left hand side of (58) is a decreasing function of \( \Delta U \). Therefore, (58) is equivalent to \( \Delta U \geq \Delta \underline{U} \), where \( \Delta \underline{U} \) is the unique solution of

\[
\frac{2n_j^*}{1+n_j^*} \exp \left( \frac{2n_j^* - \Delta U + A_j - U_j^* - u_\nu^0}{\nu} \right) + \frac{2n_j^*}{1+n_j^*} + (M-2) \frac{\Delta U - \nu}{\nu} = 1. 
\]

(59)

It follows from (57) that \( n_1^{m*} \) increases in \( \Delta U \). Since \( \frac{2n_j^*}{1+n_j^*} < 2n_j^* \), we have \( \Delta U < \Delta \bar{U} = \Delta \bar{U} \).

By (19), \( \pi_1^{m*} \geq \pi_j^* \) iff \( n_1^{m*} \leq n_0^* \). Using (51), \( n_0^{m*} \leq n_0^* \) is equivalent to

\[ n_j^{m*} \exp \left( \frac{n_j^{m*} - A_j - U_j^{m*} - u_\nu^0}{\nu} \right) \leq n_j^* \exp \left( \frac{n_j^* - A_j - U_j^* - u_\nu^0}{\nu} \right), \quad j = 3, \ldots, M. \]

(60)

It directly follows that the above inequality is equivalent of \( n_j^{m*} \leq n_j^* \) (\( j = 3, \ldots, M \)). Therefore, \( \pi_1^{m*} \geq \pi_j^* \) iff \( \Delta U \geq \Delta \bar{U} \).

For the industry profit, we will show that \( \pi_1^{m*} \) increases in \( \Delta U \). By (16), we have \( n_j^{m*} = H \left( n_j^{m*} \exp \left( \frac{n_j^{m*}}{1+n_j^{m*}} + \frac{\Delta U}{\nu} \right) \right) \) for \( j = 3, \ldots, M \). Take derivative with respect to \( \Delta U \), and use (35), we have

\[
\frac{\partial n_j^{m*}}{\partial \Delta U} = \left( \frac{1-n_j^{m*}}{1+n_j^{m*}} \right) \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \frac{n_j^{m*}}{\nu} + \left( \frac{1-n_j^{m*}+n_j^{m*}+\Delta U}{1-n_j^{m*}} \right) \frac{\partial n_j^{m*}}{\partial \Delta U}. 
\]

(61)

Apply Implicit Function Theorem to (52), we have for \( j = 3, \ldots, M \)

\[
\frac{\partial n_j^{m*}}{\partial \Delta U} = \left( \frac{1-n_j^{m*}}{1+n_j^{m*}} \right) \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \frac{n_j^{m*}}{\nu} + \left( \frac{1-n_j^{m*}+n_j^{m*}+\Delta U}{1-n_j^{m*}} \right) \frac{\partial n_j^{m*}}{\partial \Delta U}. 
\]

(62)

Substitute (17), (61) and (62) into (22), we have: \( \partial \pi_1^{m*} / \partial \Delta U = \)

\[
\frac{n_j^{m*} \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \left( \frac{1-n_j^{m*}}{1+n_j^{m*}} \right) \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \frac{n_j^{m*}}{\nu} + \left( \frac{1-n_j^{m*}+n_j^{m*}+\Delta U}{1-n_j^{m*}} \right) \frac{\partial n_j^{m*}}{\partial \Delta U} \right) \]

\[ \quad = 0. \]

At \( \Delta U = 0 \), \( \pi_j^* \) is the unique solution of \( \frac{\pi_j^*}{1-n_j^{m*}} \left( \frac{M + \exp \left( \frac{\pi_j^*}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \left( \frac{1-n_j^{m*}}{1+n_j^{m*}} \right) \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \frac{n_j^{m*}}{\nu} \right) \right) = 1 \), and \( n_j^{m*} \) is the unique solution of \( \frac{n_j^{m*}}{\pi_j^*} \left( \frac{M + \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \left( \frac{1-n_j^{m*}}{1+n_j^{m*}} \right) \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} + \frac{\Delta U}{\nu} \right) \frac{n_j^{m*}}{\nu} \right) \right) = 1 \). If \( \pi_1^{m*} \geq \pi_1^* \) at \( \Delta U = 0 \), then \( \pi_1^{m*} \geq \pi_1^* \) for any \( \Delta U \geq 0 \), and we set \( \Delta \underline{U} = 0 \). Otherwise, if \( \pi_1^{m*} \leq \pi_1^* \) at \( \Delta U = 0 \), there exists a threshold \( \Delta \bar{U} \geq 0 \), such that \( \pi_1^{m*} \leq \pi_1* \) if \( \Delta U \leq \Delta \bar{U} \), and \( \pi_1^{m*} \geq \pi_1^* \) if \( \Delta U \geq \Delta \bar{U} \). Since \( \pi_1^{m*} \geq \pi_1^* \) if \( \Delta U \in [\Delta \bar{U}, \Delta \bar{U}] \), we have \( \Delta \bar{U} \leq \Delta \bar{U} \).
Proof of Proposition 6 As shown in the proof of Proposition 5, \( \partial \pi^{m*}/\partial \Delta U > 0 \). By (19), we have
\[
\frac{\partial \pi^{m*}/\partial \Delta U = - (\nu/n_0^{m*}) (\partial n_0^{m*}/\partial \Delta U)}{\partial n_0^{m*}}.
\]
By (15), \( n_0^{m*} \) satisfies the equation
\[
n_0^{m*} + H \left( n_0^{m*} \exp \left( \frac{A_j - u_0^* - \frac{\nu U}{\partial n_0^{m*}}} - \nu \right) \right) = 1,
\]
and thus \( \partial n_0^{m*}/\partial \Delta U < 0 \). Therefore, \( \pi^{m*} \) increases in \( \Delta U \). Combine with part (I) and (IV) of Proposition 5, there exists a threshold values of the power index, \( \Delta \hat{U} \) which lies between \( \Delta \hat{U} \) and \( \Delta \hat{U} \), such that \( \pi^{m*} \leq \pi^* \) if \( 0 \leq \Delta U \leq \Delta \hat{U} \), and \( \pi^{m*} \geq \pi^* \) if \( \Delta U \geq \Delta \hat{U} \).

Proof of Proposition 7 By (18), we have
\[
V_j^* = A_j - u_0^* - \nu \ln \left( \frac{n_j^*}{n_0^{m*}} \right)
\]
for \( j = 1, \ldots, M \), \( V_1^{m*} = A_1 - u_0^* - \nu \ln \left( \frac{n_1^*}{n_0^{m*}} \right) \), and \( V_j^{m*} = A_j - u_0^* - \nu \ln \left( \frac{n_j^{m*}}{n_0^{m*}} \right) \) for \( j = 3, \ldots, M \). Use (16), we have \( V_j^{m*} \geq V_j^* \) (\( j = 3, \ldots, M \)) iff \( n_j^* \leq n_j^{m*} \) (\( j = 3, \ldots, M \)). By the result of Proposition 5, it follows that \( V_j^{m*} \geq V_j^* \) if \( \Delta U \leq \Delta \hat{U} \) for \( j = 3, \ldots, M \).

Similarly, \( V_1^{m*} \geq V_1^* \Leftrightarrow n_1^{m*} \geq \frac{\nu n_1^* + (1 - n_1^*) \Delta U}{\nu + (1 - n_1^*) \Delta U} \), which is equivalent to the following condition using (55):
\[
\frac{\nu n_1^* + (1 - n_1^*) \Delta U}{\nu + (1 - n_1^*) \Delta U} \bigg( \exp \left( \frac{n_1^*}{1 - n_1^*} - \frac{A_j - u_0^* - \nu}{\nu} \right) + 1 \bigg) + (M - 2) H \left( \frac{\nu n_1^* + (1 - n_1^*) \Delta U}{\nu + (1 - n_1^*) \Delta U} \right) \left( \frac{n_1^*}{1 - n_1^*} \right) \leq 1.
\]
Note that the left-hand-side of (63) increases in \( \Delta U \). Therefore, \( V_1^{m*} \geq V_1^* \Leftrightarrow \Delta U \leq \Delta \hat{U} \), where \( \Delta \hat{U} \) is the unique solution of
\[
\frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu + (1 - n_j^*) \Delta U} \bigg( \exp \left( \frac{n_j^*}{1 - n_j^*} - \frac{A_j - u_0^* - \nu}{\nu} \right) + 1 \bigg) + (M - 2) H \left( \frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu + (1 - n_j^*) \Delta U} \right) \left( \frac{n_j^*}{1 - n_j^*} \right) = 1.
\]
Note that at \( \Delta U = \Delta \hat{U} \) given by (54), and use (48), the left-hand-side of (63) is
\[
\frac{2n_j^* + (1 - 2n_j^*) \ln 2}{1 + (1 - 2n_j^*) \ln 2} \left( \frac{1}{n_j^*} - M + 1 \right) + (M - 2) H \left( \frac{2n_j^* + (1 - 2n_j^*) \ln 2}{1 + (1 - 2n_j^*) \ln 2} \right) \left( \frac{n_j^*}{1 - n_j^*} \right)
\]
\[
> 2n_j^* \left( \frac{1}{n_j^*} - M + 1 \right) + (M - 2) H \left( n_j^* \left( \frac{n_j^*}{1 - n_j^*} \right) \right) = 2 - M n_j^* > 1.
\]
Therefore, we have \( \Delta \hat{U} < \Delta \hat{U} \).

For the share-weighted industry average aggregate price decision, it directly follows that \( V_j^{m*} \geq V_j^* \) if \( \Delta U \leq \Delta \hat{U} \), and \( V_j^{m*} \leq V_j^* \) if \( \Delta U \geq \Delta \hat{U} \). When \( \Delta \hat{U} \leq \Delta U \leq \Delta \hat{U} \), \( V_1^{m*} \leq V_1^* \) while \( V_j^{m*} \geq V_j^* \) (\( j = 3, \ldots, M \)). By (18) and (24), we have \( V_j^{m*} \geq V_j^* \) iff
\[
n_1^{m*} \ln \left( \frac{n_1^{m*} n_j^*}{n_0^{m*} n_0^*} \right) - (M - 2)n_j^{m*} \ln \left( \frac{n_1^{m*} n_j^*}{n_0^{m*} n_0^*} \right) \leq 0, \quad j = 3, \ldots, M.
\]
It follows from (55), (52) and (15) that \( n_1^{m*} \) increases in \( \Delta U \), \( n_j^{m*} \) decreases in \( \Delta U \) (\( j = 3, \ldots, M \)) and \( n_0^{m*} \) decreases in \( \Delta U \). By (16), we have
\[
n_0^{m*} = \exp \left( \frac{n_0^{m*}}{1 - n_0^{m*}} - \frac{A_j - u_0^* - \nu}{\nu} \right),
\]
which decreases in \( \Delta U \). Therefore, the left-hand-side of (65) increases in \( \Delta U \). Since the left-hand-side of (65) is less than 0 at \( \Delta U = \Delta \hat{U} \), and greater than 0 at \( \Delta U = \Delta \hat{U} \), there exists a threshold value \( \Delta \hat{U} \in [\Delta \hat{U}, \Delta \hat{U}] \), such that \( V_j^{m*} \geq V_j^* \) if \( \Delta U \leq \Delta \hat{U} \), and \( V_j^{m*} \leq V_j^* \) if \( \Delta U \geq \Delta \hat{U} \).
Appendix B: Extension

We now extend our base model to the more general setting of multiple client organizations, each with drugs from multiple therapeutical classes. First, we consider the setting of one client organization with drugs from multiple therapeutical classes. When there are \( L \) therapeutical classes, PBM \( j \) needs to charge the same copayment for all drugs on the same formulary tier and pass the same percentage of rebate to the client organization for all branded drugs that are on the preferred tier. Therefore, the copayment values \((c_g^j, c_p^j, c_n^j)\) and rebate pass-through rates \((\rho_j)\) do not vary across different therapeutical classes. All other model primitives and decision variables depend on each drug’s therapeutical class, so we add subscript \( k \) \((k = 1, \ldots, L)\) to each of them to denote the drug’s therapeutical class. It can be verified that Theorem 1 continues to hold with

\[
U_j(\bar{y}_{j1}, \ldots, \bar{y}_{jL}) = \sum_{k=1}^{L} U_{jk}(\bar{y}_{jk}) = \sum_{k=1}^{L} (W_{jk}(\bar{y}_{jk}) - CS_{jk}(\bar{y}_{jk})) \tag{66}
\]

and

\[
V_j(\bar{y}_{j1}, \ldots, \bar{y}_{jL}, \bar{p}_{j1}, \ldots, \bar{p}_{jL}) = \sum_{k=1}^{L} V_{jk}(\bar{y}_{jk}, \bar{p}_{jk}) = \sum_{k=1}^{L} (B_{jk}(\bar{y}_{jk}, \bar{p}_{jk}) - CS_{jk}(\bar{y}_{jk})) \tag{67}
\]

It follows from Theorem 1 that each PBM obtains its optimal formulary decision vector by minimizing the welfare-adjusted cost of its plan in (66). Note from (66) that the formulary decisions for different therapeutical classes are separable, because drugs in different therapeutical classes are not substitutable. Therefore, each PBM can solve for its optimal formulary decision vector of each therapeutical class separately. With the optimal formulary decision of each PBM determined, the PBMs compete in the reduced price competition by choosing the welfare-adjusted price of its plan, as defined in (67). The equilibrium welfare-adjusted price, the equilibrium expected market share and the expected profit of each PBM, as well as the client organization’s expected utility, can be obtained by Theorem 2. It can be verified that all comparative statics results, as well as the result on the impact of PBM mergers, continue to hold in the case with multiple therapeutical classes.

Next, we consider the case with multiple client organizations. It is common in the PBM industry for a PBM to offer a customized plan for each of its client organizations. Since a PBM does not have to commit to the same formulary and pricing schemes for different clients, its formulary and price decisions are completely separable across different clients. Therefore, a PBM can obtain its formulary and price decisions for a client organization, without considering other client organizations, by the procedure outlined above. This result is consistent with the current practice in the PBM industry that different client organizations typically reach out to PBMs requesting a complete design of a coverage plan at different times of a calendar year, and PBMs respond to such requests by customizing a plan for each client organization.