Hedging Commodity Procurement in a Bilateral Supply Chain

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Abstract

This paper explores the merits of hedging stochastic input costs (i.e., reducing the risk of adverse changes in costs) in a decentralized, risk neutral supply chain. Specifically, we consider a generalized version of the well-known ‘selling-to-the-newsvendor’ model in which both the upstream and the downstream firms face stochastic input costs. The firms’ operations are intertwined – i.e., the downstream buyer depends on the upstream supplier for delivery and the supplier depends on the buyer for purchase. We show that if left unmanaged, the stochastic costs that reverberate through the supply chain can lead to significant financial losses. The situation could deteriorate to the point of a supply disruption if at least one of the supply chain members cannot profitably make its product. To the extend that hedging can ensure continuation in supply, hedging can have value to at least some of the members of the supply chain. We identify conditions under which the risk of the supply chain breakdown will cause the supply chain members to hedge their input costs: (i) The downstream buyer’s market power exceeds a critical threshold; or (ii) the upstream firm operates on a large margin, there is a high baseline demand for downstream firm’s final product, and the downstream firm’s market power is below a critical threshold. In absence of these conditions there are equilibria in which neither firm hedges. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. There are also situations when firms hedge in equilibrium although hedging reduces their expected payoff.

1 Introduction

Much of the extant supply chain and inventory literature takes demand as stochastic and production costs as fixed. The latter assumption, however, is in contrast to many industrial settings in which manufacturing firms are exposed to price changes of commodities that affect the direct costs of raw materials, components, sub-assemblies, and packaging materials as well as indirect costs from the energy consumed in operations and transportation. According to a survey of large US non-financial firms (e.g., Bodnar et al., 1995), approximately 40% of the responding firms routinely
purchase options or futures contracts in order to hedge price risks such as those mentioned above. Why do these firms hedge? (Definition. Throughout the paper, to ‘hedge’ means to buy or sell commodity futures as a protection against loss or failure due to price fluctuation. For details, see Van Mieghem, 2003, p.271.)

In order to provide rationale for hedging, extant theories have relied on the existence of taxes (e.g., Smith and Stulz, 1985), asymmetric information (e.g., DeMarzo and Duffie, 1991), hard budget constraints and costly external capital (e.g., Froot et al., 1993; Caldentey and Haugh, 2009), and risk aversion (e.g., Gaur and Seshadri, 2005; Chod et al., 2010). All these papers take a firm as the basic ‘unit of analysis’ and, broadly speaking, analyze situations in which the firm plans to either purchase or sell output at a later date while facing one of the above frictions. When viewed from today, the output’s price and consequently the firm’s transaction proceeds are random. However, by somehow eliminating the randomness in price – or by hedging – the firm is able to increase its (expected) payoff, which makes hedging beneficial to the firm.

The thesis behind this paper is that when several firms are embedded together in a supply chain while simultaneously facing the situation we described above, there will be an added level of complexity, which the existing theories cannot necessarily address. First, in a supply chain environment, firms will likely find themselves exposed not only to their own price risks, but also to their supply chain partner’s price risk. This is illustrated with an example from the auto industry (Matthews, 2011). In 2011, steelmakers have increased prices six times, for a total increase of 30%. However, consistent with the extant theory, many car and appliance manufactures did not necessarily pay attention to steel prices because they did not directly purchase steel from steelmakers, only to find themselves in difficulty with some of their financially weaker suppliers, for whom steel represents their biggest cost. In a supply chain, price increases can therefore result in the supplier experiencing significant financial losses if it does not have the resources to manage price risk. The situation could deteriorate to the point of a supply disruption if the supplier cannot profitably make products for its buyer. While firms can try to mitigate the risk of disruption by contractually imposing penalties, as will be seen, default penalties are often not a credible guarantee of supply unless they are also supported with hedging.

Second, unlike in the standalone firm’s setting, supply chain firms cannot necessarily control any price risk they want by hedging alone. Consequently, firms will care whether and how their supply chain partners hedge and any hedging activity will be impounded in transfer prices. Finally, the hedging behavior of their supply chain partners, will affect their own hedging behavior, as will be seen.

Following the seminal contribution of Lariviere and Porteus (2001) – who point out that there is an intimate relationship between wholesale price, demand uncertainty, and the distribution of market power in the supply chain – this paper explores the merits of hedging stochastic input costs in order to ensure continuity of supply in a decentralized supply chain. Specifically, we consider a generalized version of the well-known ‘selling-to-the-newsvendor’ model (Lariviere and Porteus, 2001) in which both the upstream and the downstream firms face stochastic production costs. (In
contrast to our paper, in Lariviere and Porteous, 2001, production costs are linear with constant coefficients.) The intuition that underlies our model of hedging in a supply chain is as follows. By agreeing to sell their output at (contractually) pre-determined prices before all factors affecting their productivity are known, both the upstream supplier and the downstream buyer subject themselves to default risk. The fact that a supply chain member may default in some states of the world will be, ex ante, impounded into the wholesale price as well as into the order quantity. However, if the firms could somehow ‘guarantee’ that they will fulfill the supply contract (no matter what happens to their inputs costs), their supply chain counterpart may respond with a more favorable price or a more favorable order quantity. Either firm would like to guarantee supply contract performance if the expected profit associated with the guarantee is greater than the expected profit associated with default in some states. In such circumstances, the firm can commit to the guarantee by purchasing futures contracts on the underlying input commodity. The futures contracts will pay off precisely at the time when the firm’s resources are strained. Hence, futures contracts have value because they prevent the firm from defaulting on a contractual obligation when not defaulting is (ex ante) important.

We identify the following conditions under which the supply chain firms should be expected to hedge in equilibrium: (i) The downstream buyer’s market power exceeds a critical threshold; or (ii) the upstream firm operates on a large margin, there is a high baseline demand for downstream firm’s final product, and the downstream firm’s market power is below a critical threshold. In absence of these conditions there are equilibria in which neither firm hedges. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. There are also situations when firms hedge in equilibrium although hedging reduces their expected payoff. (A partial hedging policy leaves the hedger exposed to some residual price risk. A full hedging policy eliminates all price risk.)

To summarize, we provide a rationale for hedging, which is based on a non-cooperative behavior in a supply chain. We illustrate this point with an anecdotal example from a diversified global manufacturing company. The company’s Residential Solutions Division produces a collection of home appliances and power tools, which are sold worldwide. Its Commercial Division manufactures industrial automation equipment and climate control technology. Many of these products are raw-material-intense. When it comes to supply chain management, the firm plays a variety of roles: Sometimes the firm is a supplier; other times, the firm plays a role of an end-product assembler. Recently, the company saw an increased volatility in the prices of metals, which affected its margins. To combat this, the firm hedged its raw material inputs, but only some. When asked when the firm hedges, a manager who presented the case replied that the firm’s hedging decisions partly depend both on the product and on the firm’s supply chain partners. For example, as a supplier, the firm fully hedges the raw material it uses to produce garbage disposals, which are sold through a major U.S. home-improvement store. This is because the firms’ bargaining power against this retailer is small and it is the retailer who dictates both production quantities and pricing. As a buyer,
however, the firm tends not to hedge, when it knows that it may face glitches in supply because then it is exposed to risk of financial losses from its hedging activities. The existing risk-management literature does not always provide clear guidance on hedging in cases like these. Either because it requires frictions (e.g., risk-aversion), which do not apply to large multi-nationals (Carter et al., 2006; Jin and Jorion, 2006); or because it does not allow hedgers to vary their hedging decisions across products and channel relationships. While our model is a simple selling-to-the-news-vendor setting, this example illustrates that the equilibrium that we advance seem to mirror the facts of these cases (see Part i of Proposition 2).

In what follows, we review related literature, present the model (§§3 and 4) and derive the firms’ equilibrium behavior under the unhedged and hedged contracts (§§5.1 and 5.2). Then, by comparing the expected payoffs both firms can achieve under the unhedged and hedged contracts we make predictions as to when offers of hedged wholesale contract should be expected in equilibrium (§5.3). §6 concludes.

2 Related Literature

Recent research in the finance literature has greatly improved our understanding of why and how individual firms should hedge. For instance, it has established that for hedging to be beneficial, the firm’s payoff must be a concave function of the stochastic hedge-able exposure metric. In spirit, this result follows from basic convex-analysis theory: It means that the firm’s expected payoff, a weighted average of a concave function, is lower when the level of variability of the exposure metric is higher. Thus, since hedging reduces variability, then it increases the firm’s expected payoff.

Focusing on a single-firm setting, Smith and Stulz (1985) demonstrate that managerial risk-aversion or market frictions such as corporate taxes cause a firm’s payoff to become a concave function of its hedge-able exposure. One of the implications of this paper’s result is that firms should hedge fully or not at all. Froot et al. (1993) expand the discussion to settings in which firms are exposed to multiple sources of uncertainty that are statistically correlated. They show that in such type of situations, it may become optimal for a firm to utilize partial hedging. Building upon this observation, a large body of the finance literature has studied hedging by taking a firm as the basic ‘unit of analysis’ and by exogenously imposing the afore-mentioned concavity property. This property has been commonly justified by assuming that the hedger’s preferences on the set of its payoffs are consistent with some form of concave utility function. This literature provides insights into the mechanics of hedging using financial instruments such as futures and options. For instance, Neuberger (1999) show how a risk-averse supplier may use short-maturity commodity futures contracts to hedge a long-term commodity supply commitment. A comprehensive summary of this research is given in Triantis (1999).

While the finance literature focuses on mitigating pricing risk, the operations management literature predominantly considers demand and supply risks. An excellent survey of the early operations literature can be found in Van Mieghem (2003). More recent and influential papers in
this area include Gaur and Seshadri (2005) and Caldentey and Haugh (2009). Gaur and Seshadri (2005) address the problem of using market instruments to hedge stochastic demand. The authors consider a news vendor who faces demand that is a function of the price of an underlying financial asset. They show how firms can use price information in financial assets to set optimal inventory levels, and demonstrate that financial hedging of demand risk can lead to higher return on inventory investment. Caldentey and Haugh (2009) show that financial hedging of stochastic demand can increase supply chain output. Note, however, that there are several important differences between the supply chain considered in Caldentey and Haugh, 2009 and the supply chain in this paper. Their supply chain is centralized, the hedge-able risk in their chain is stochastic demand, and the risk is faced by only one of the firms in the supply chain. In order to provide a rationale for the hedging behavior, Gaur and Seshadri (2005) rely upon risk-aversion; Caldentey and Haugh (2009) require market frictions, which lead to credit rationing.

Other papers in the operations management literature discuss and analyze hedging strategies for managing risky supply and ask when it makes a difference if firms hedge financially or operationally. For example, while intuition may suggest that operational and financial hedging may be substitutes, surprisingly, Chod et al. (2010) show that they can be complements. Emergency inventory (e.g., Schmitt et al., 2010); emergency sourcing (e.g., Yang et al., 2009); and dual sourcing (e.g., Gümüş et al., 2012) are examples of operational hedging strategies capable of mitigating adverse supply shocks. Supplier subsidy (e.g., Babich, 2010) is an example of a financial strategy designed to reduce the likelihood of supply contract abandonment. Haksöz and Seshadri (2007), in turn, put financial value on the supplier’s ability to abandon a supply contract. We differ from the aforementioned literature by focusing on the management of input cost risk (rather than demand risk) and by extending the discussion of why and how risk-neutral firms hedge from the single-firm setting into a decentralized supply chain.

3 Preamble: A Model of a Vertically Integrated Supply Chain

Consider the problem of a risk-neutral manufacturer (final good assembler), $M$, who faces the newsvendor problem in that at time $t_0$ it must choose a production quantity, $q$, before it observes a single realization of stochastic demand, $D$, for its final product. Stochastic demand for the final product, $D$, has a distribution $F$ and density $f$. $F$ has support on $[\underline{d}, \overline{d}]$, i.e., there is always some demand (Cachon, 2004, p.225). The selling price of the final product is normalized to 1.

To assemble $q$ units of the final product, $M$ requires $q$ units of commodity 2 as well as $q$ components (sub-assemblies), which are produced by a component supplier, $S$. To produce $q$ components, $S$ requires $q$ units of commodity 1. (Figure 1 summarizes the production inputs required to assemble 1 unit of the final product.) For simplicity, unmet demand is lost, unsold stock is worthless, and per unit production and holding costs are zero.

Due to operational frictions (e.g., capacity constraints and lead times), the sub-assembly production cannot be completed before time $t_1 > t_0$. Consequently, the final assembly cannot be
completed before time $t_2 \geq t_1$. $M$’s sales revenue is realized at time $t_3 \geq t_2$ with the proviso that $M$ cannot sell more at time $t_3$ than what it produced for at time $t_2$ and what the supplier, $S$, produced for at time $t_1$. Neither firm can produce more than what both firms committed to at time $t_0$.

The commodity $i = 1, 2$ per-unit price, $S_i$, is revealed at time $t_i$. Viewed from $t_0$, $S_i$ is a random variable modeled on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with filtration $\{\mathcal{F}_t\}, t \in [t_0, t_3]$. $S_1$ and $S_2$ have a joint density function $h$. For simplicity, the discount rate between times $t_0$ and $t_3$ is taken to be zero. From CAPM, this is equivalent to assuming that the risk-free rate is zero and that both input commodities are zero beta assets. Relaxing these assumptions can be done without changing the main message of the results presented here.

We begin by observing that, in the absence of any working capital constraints, a vertically integrated supply chain in this context would actually take advantage of input price volatility and suspend production whenever input costs are unfavorable. To see this, for the case when $t_0 < t_1 = t_2 = t_3$, the time $t_0$ expected payoff of the integrated system is:

$$E_{t_0} \Pi(q) = E_{t_0} \min\{q, D\} (1 - S_1 - S_2)^+.$$  

The right side of Equation (1) says that at time $t_0$, the integrated supply chain holds $\min\{q, D\}$ of put options, each with a time $t_3$ payoff of $(1 - S_1 - S_2)^+$. Because of this optionality, the integrated system’s expected payoff increases both in $q$ and in input cost volatility. Consequently, the system’s optimal production quantity will be $q = \bar{d}$. (When $t_0 < t_1 \leq t_2 \leq t_3$, the expression for $E_{t_0} \Pi(q)$ becomes more complicated, but no new important insights are added.)

The vertically integrated supply chain design is contrasted with a decentralized supply chain design where $M$ and $S$ operate independently. Then:

- Each firm will exercise its market power by pushing its price above its (expected) marginal cost, which will lead to double-marginalization.

- Each firm will face a stochastic input cost, which will lead to an incentive to default when defaulting is not necessarily optimal for the integrated channel.

Under some conditions, the integrated channel payoff given in (1) can be replicated with the use of a state-contingent contract. A generalized version of the well-known revenue-sharing contract $\{w_r, \phi\}$ is an important example of such a contract, where $w_r = \phi S_1 - (1 - \phi) S_2$ is a wholesale price the assembler pays to the supplier at time $t_2$ and $0 \leq \phi \leq 1$ is the fraction of the supply chain
revenue the assembler keeps at time \(t_3\); so \((1 - \phi)\) is the fraction the supplier gets.

Notice that \(w_r\) is a function of both \(S_1\) and \(S_2\), which are revealed at times \(t_1\) and \(t_2\). Pricing this way is what allows the firms to share their production costs and thereby achieve the integrated channel outcome. As such, \(w_r > 0\) if and only if \(S_1 > S_2(1 - \phi)/\phi\); otherwise \(w_r \leq 0\), which means that the supplier pays the assembler. If \(t_1 = t_2 = t_3\), then the supplier’s and the assembler’s time \(t_3\) payoffs respectively are as follows:

\[
\Pi^S(q) = (1 - \phi)\min\{q, D\} + q(w_r - S_1), \quad \text{and} \quad \Pi^M(q) = \phi\min\{q, D\} - q(w_r + S_2). \tag{2}
\]

A special case of this generalized revenue sharing contract is the well-known pass-through contract, which is obtained by setting \(\phi = 1\). Under the pass-through contract, raw material charges are simply passed-through in full onto the downstream assembler.

**Example 1.** Let \(\phi = 1/2\) and suppose that viewed from time \(t_0, (S_1, S_2, D)\) is a Bernoulli random vector that takes on values \((4/5, 2/5, 5), (2/5, 2/5, 5)\) with respective probabilities \(P\{(4/5, 2/5, 5)\} = 1/2, P\{(2/5, 2/5, 5)\} = 1/2\). Then both the supplier’s and the assembler’s time \(t_3\) payoffs are either 0 or 1/2, depending on the outcome of \(S_1\). The total supply chain payoff is 0 or 1, which matches the integrated channel payoff given by the right side of (1). \(\square\)

Example 1 illustrates a situation where the state-contingent contract addresses both double-marginalization and the adverse default incentive. That is, under the terms of the contract, the forward-looking supplier defaults exactly when it is optimal to default for the integrated chain and the sum of both firm’s expected payoffs is the same as the expected payoff of the integrated chain. However, as it seen next, there are situations where the contract fails.

**Example 2** (Continuation of Example 1). Everything is the same as in Example 1, except that viewed from time \(t_0, (S_1, S_2, D)\) is a Bernoulli random vector that takes on values \((2/5, 2/5, 5), (2/5, 0, 5)\) with respective probabilities \(P\{(2/5, 2/5, 5)\} = 1/2, P\{(2/5, 0, 5)\} = 1/2\), and \(\phi = 2/5\).

Now, the state contingent contract of Example 1 no longer matches the integrated chain payoff for all outcomes of \((S_1, S_2, D)\). To see this, suppose \((S_1, S_2, D) = (2/5, 2/5, 5)\). With \(q = 5\), the integrated chain payoff will be \((\min\{D, q\} - (4/5)q) = 1 > 0\) while the decentralized chain’s total payoff will be 0. The latter follows because \(w_r = \phi S_1 - (1 - \phi) S_2 = -(2/25)\). Therefore by not producing, the assembler will collect \((2/25)q = 2/5\ ( = -w_r)\) at time \(t_2\). At the same time, the highest payoff the assembler can achieve by producing at time \(t_2\) is \((2/25)q = 2/5\). The assembler therefore has no incentive to produce at time \(t_2\). To avoid losing \((12/25)q = 12/5\ ( = qS_1 - qw_r)\), the forward-looking supplier therefore breaks the contract at time \(t_1\). \(\square\)

An obvious resolution to the situation described in Example 2 is to impose contractual default penalties that would induce the firms not to shirk when producing is profitable for the integrated supply chain. Had the assembler faced a default penalty greater than \(2/5\) at time \(t_2\), the state-contingent contract in Example 2 would again achieve the integrated channel payoff. A state-contingent contract with penalties, however, is still not a guarantee that all supply chain members will default only when it is optimal to default for the integrated supply chain.
Example 3 (Continuation of Example 2). Everything is the same as in Example 2, except that $t_1 \leq t_2$; $S$ has zero assets; and when viewed from time $t_0$, $(S_1, S_2, D)$ is a Bernoulli random vector that takes on values $(2/5, 2/5, 5), (2/5, 4/5, 5)$ with respective probabilities $\mathbb{P}\{(2/5, 2/5, 5)\} = 1/2, \mathbb{P}\{(2/5, 4/5, 5)\} = 1/2$.

Under these assumptions, the supplier will require $(2/5)q$ in cash-on-hand to procure production inputs at time $t_1$ and either $(2/25)q$ or $(8/25)q$ (depending on the state of the world at time $t_2$) to pay the assembler at time $t_2$.

Facing a hard budget constraint, the upstream supplier, $S$, could do one of the following: (a) Borrow from a bank and incur an interest payment, which is a cost that the integrated supply chain could avoid. (b) Borrow from the downstream assembler. To overcome the supplier’s budget constraint, the assembler could simultaneously procure commodity 1 at time $t_1$, worth $(2/5)q$, and put over the payment of $w_rq$ to time $t_2$. The problem is that option (b) is not incentive compatible because if $(S_1, S_2, D) = (2/5, 4/5, 5)$, then the assembler will earn a payoff of $-(2/5)q$. If, on the other hand, $(S_1, S_2, D) = (2/5, 2/5, 5)$, then the supplier will earn $2/5$. Since $(1/2)(2/5) - (1/2)(2/5)q \leq 0$ for all $q = 1, 2, \ldots$, then the assembler has no incentive to lend to the supplier. At the same time, the integrated channel can achieve a payoff of 1 with probability $1/2$. □

In summary, characteristics of the industry or channel relationship could make implementing of the state-contingent contract either expensive or ineffective. Since a price-only contract (Lariviere and Porteus, 2001) is a natural alternative to the state-contingent contract, evaluating performance under such a contract is a first step in determining whether the additional costs of a more sophisticated policy are worth bearing.

Similarly to the state-contingent, the traditional price-only contract too does not address the adverse default issue. The idea that we develop in the remainder of this paper is that hedging together with contractually imposed default penalties are a way to protect the entire supply chain from failing when one of its members faces high realized input costs while its supply chain partner would like to produce.

4 Model of a Decentralized Supply Chain

Our assumptions are exactly the same as in §3 with the provision that the supply chain is decentralized, implying that the supplier, $S$, is now selling to the newsvendor, $M$. Both firms are risk-neutral and are protected by limited liability. Each firm has a reservation payoff $R_j \geq 0$ and $c_j \geq 0$, $j \in \{M, S\}$ dollars in cash-on-hand. Firms have no outside wealth. (As explained in Cachon, 2003, p.234, exogenous reservation payoffs, $R_j$, are a standard way to model a firm’s market power.)

One can interpret the firms as playing a game over times $t_0 < t_1 \leq t_2 \leq t_3$. The supplier offers the assembler a wholesale contract at time $t_0$ when only the distribution function of future commodity input costs is known. Due to production lead times, the supplier produces the subassembly at time $t_1$ and the assembler performs the final assembly at time $t_2$. As before, an outcome
of the supplier’s commodity input cost, $S_1 \geq 0$, is revealed at time $t_1$; an outcome of an assembler’s commodity input cost, $S_2 \geq 0$, is revealed at time $t_2$; the assembler’s sales revenue is realized at time $t_3$. The discount rate between times $t_0$ and $t_3$ is taken to be zero.

As explained in §3, both firms may have separate incentives to default on the supply contract when their time $t_i$ commodity input costs turn out to be too high in relation to their pre-determined selling prices. We assume that if a firm defaults on the supply contract, then it incurs a contractually pre-determined default penalty $L_j, j \in \{M, S\}$. Throughout the paper, we assume that the penalty, $L_j$, is payable to firm $j$’s supply chain partner. (It is, however, rather straightforward to change this to a model, in which $L_j$ represents an indirect, reputation cost.) If both firms choose to produce, then $q$ is the highest payoff either firm can achieve, as will be seen. To prevent a situation in which one firm experiences a financial windfall when its supply chain partner defaults, throughout the analysis we therefore assume that $L_S \leq q$ and $L_M \leq q$. (As an example of a supply contract penalty, see the Plymouth Rubber Company contract in Appendix A – there, $L_j$ is set as a percentage of the wholesale price. In the Davita, Inc. contract, the penalty can be more than 100% of the wholesale price.)

One of the goals of this paper is to investigate whether firms should be expected to hedge their commodity input costs in order to avoid default on the underlying supply contract. For this reason we suppose that for each commodity $i = 1, 2$ there exists a futures contract to purchase a specific amount of commodity $i$ at time $t_i$ for a pre-specified (futures) price, $F_i$. As is convention, the futures price is set so that the value of the futures contract at inception is zero. The payoff to a futures contract on commodity $i = 1, 2$ is realized at time $t_i$, where the payoff is the difference between the futures price, $F_i$, and the time $t_i$ spot price, $S_i$. We use the variable $n_j, j \in \{M, S\}$ to represent firm $j$’s position in the futures market: $n_j > 0$ indicates that firm $j$ has a ‘long position’ of $n_j$ futures contracts and $n_j < 0$ indicates that it has a ‘short position.’ If the firm $j \in \{M, S\}$ buys – or is long in – $n_j$ futures contracts, its time $t_i$ payoff is $n_j(S_i - F_i)$, which will be positive when input prices are high and negative when input prices are low. In our setting, both firms will be hedging by taking long positions. Therefore, hereafter, $n_j \geq 0, j \in \{M, S\}$.

The futures contracts are negotiated at a futures exchange, which acts as an intermediary. One of the key roles of the exchange is to organize trading so that futures contract defaults are completely avoided (for additional discussion, see §2–3 in Hull, 2009). For this reason, the exchange will limit the number of contracts a firm $j \in \{M, S\}$ can buy if it knows that firm $j$ will not have sufficient ex post resources to pay off its long futures contract position when the realized input prices at time $t_i, i = 1, 2$ are low.

In summary, there are three key assumptions about penalties and futures contracts that underlie our model of supply chain hedging:

Assumption 1. Any default on the underlying supply contract is costly; neither firm, however, stands to unusually profit from a default of its supply chain partner.

Assumption 2. To avoid pathological cases, the futures prices $F_i, i = 1, 2$ are low enough so that by hedging, neither firm effectively locks in a negative (expected) profit.
Assumption 3. The futures market is credible, meaning that neither firm \( j \in \{M, S\} \) defaults on its futures contract position in all possible futures states of the world (Hull, 2009, §2–3).

Later in the paper, we’ll translate Assumptions 1 – 3 into mathematical conditions. It is worth mentioning that all three assumptions are quite standard in the hedging literature, which includes Smith and Stulz, 1985; Froot et al., 1993.

4.1 Game Sequence

What follows is a sequence of stages in the game between the supplier and the assembler. At time \( t_0 \):

- Firms choose to take positions \( N_j \geq 0, j \in \{M, S\} \) in the futures contracts.
- The futures exchange accepts \( 0 \leq n_j \leq N_j, j \in \{M, S\} \).
- Supplier, \( S \), sets a wholesale price, \( w \).
- Assembler, \( M \), sets an order quantity, \( q \).

This ends time \( t_0 \). Before time \( t_1 \) begins the future spot price of commodity 1 price is revealed. At time \( t_1 \):

- The supplier produces \( q \) units for the assembler and incurs a production cost \( q S_1 \). (If the supplier chooses not to produce, then it incurs a penalty \( L_S \).)

This ends time \( t_1 \). Before time \( t_2 \) begins the future spot price of commodity 2 and the supplier’s production decision at time \( t_1 \) are revealed. If the supplier produced at time \( t_1 \), then at time \( t_2 \):

- The assembler produces \( q \) units of the final product and incur a production cost \( q (w + S_2) \).
  (If the assembler chooses not to produce, then it incurs a penalty \( L_M \).)

This ends time \( t_2 \). Before time \( t_3 \) begins the value of demand \( D \) is revealed. If the assembler produced at time \( t_2 \), then at time \( t_3 \) the sales revenue \( \min \{q, D\} \) is realized.

In terms of the information structure, we assume that all market participants can observe the actions taken by all players and can observe all market outcomes, i.e., information is complete. This assumption can best be justified with emerging empirical research in finance, which reveals that companies commonly incorporate covenant restrictions into supply contracts (e.g., see Costello, 2011) that help them reduce or eliminate asymmetric information and moral hazard.

5 Analysis of the Decentralized Supply Chain

As mentioned earlier, one of the objectives of this paper is to identify conditions under which the supply chain firms should be expected to hedge their commodity input costs. We will proceed with the analysis by first assuming that neither firm hedges its input costs and show that in this case both firms subject themselves to default risk since they agree to sell their output at prices that are fixed before all factors affecting their productivity are known. We’ll refer to this situation as the ‘ unhedged wholesale contract.’ Where useful, the subscript ‘\( U \)’ will be used to indicate association with this subgame.
We then analyze the case in which the supplier avoids defaulting on the supply contract by entering into \( n_S > 0 \) futures contracts to buy commodity 1 at time \( t_1 \) and the assembler avoids defaulting on the supply contract by entering into \( n_M > 0 \) futures contracts to buy commodity 2 at time \( t_2 \). We’ll refer to this subgame as the ‘hedged wholesale contract’ and use the subscript ‘\( H \)’ to represent it. In analyzing this subgame, we determine the number of futures contracts, \( n_j, j \in \{ M, S \} \), both firms will be purchasing in equilibrium and consider what happens if \( N_j > 0 \) and \( N_k = 0, j, k \in \{ M, S \}, j \neq k \), which is a case in which one of the firms attempts to hedge while its supply chain partner does not hedge.

As is standard, the model is solved in two stages using backward induction: First stage characterizes the equilibrium behavior of the assembler; second stage characterizes the equilibrium behavior of the supplier and identifies the equilibrium in each subgame. The final equilibrium in which both firms decide whether to hedge can be determined by simply comparing the expected profits the supply chain firms generate with and without hedging.

The cash flows reveal that, in general, each firm \( j \in \{ M, S \} \) will choose not to produce if the firm \( j \)’s future spot price, \( S_i, i = 1, 2 \), is either too high in relation to firm \( j \)’s selling price (retail or wholesale price) or too low in relation to firm \( j \)’s futures price, \( F_i, i = 1, 2 \). We evaluate the issue of default in detail in Lemma 1 and present the results graphically in Figure 2. Given arbitrary futures contract positions, \( N_M \geq 0 \) and \( N_S \geq 0 \), cash flows to both firms are:

\[
\Pi^M =  \begin{cases} 
N_M (S_2 - F_2) + \min \{ q, D \} - q S_2 - w q & \text{if } u^S_{L1} < S_1 \leq u^S_{H1}, u^M_L < S_2 \leq u^M_H, t_2 < t_3 \\
N_M (S_2 - F_2) + \min \{ q, D \} (1 - S_2) - w q & \text{if } u^S_{L1} < S_1 \leq u^S_{H1}, u^M_L < S_2 \leq u^M_H, t_2 = t_3 \\
\max \{ N_M (S_2 - F_2) - L_M, -c_M \} & \text{if } u^S_{L2} < S_1 \leq u^S_{H2}, S_2 > u^M_H, t_2 \leq t_3 \\
\max \{ N_M (S_2 - F_2) - L_M, -c_M \} & \text{if } u^S_{L1} < S_1 \leq u^S_{H1}, S_2 \leq u^M_L, t_2 \leq t_3 \\
\max \{ N_M (S_2 - F_2) + \ell_S, -c_M \} & \text{otherwise,}
\end{cases}
\]

\[
\Pi^S =  \begin{cases} 
N_S (S_1 - F_1) + q (w - S_1) & \text{if } u^S_{L1} < S_1 \leq u^S_{H1}, u^M_L < S_2 \leq u^M_H, t_1 \leq t_2 \\
N_S (S_1 - F_1) + \ell_M - q S_1 & \text{if } u^S_{L2} < S_1 \leq u^S_{H2}, S_2 > u^M_H, t_1 \leq t_2 \\
N_S (S_1 - F_1) + \ell_M - q S_1 & \text{if } u^S_{L1} < S_1 \leq u^S_{H1}, S_2 \leq u^M_L, t_1 \leq t_2 \\
\max \{ N_S (S_1 - F_1) - L_S, -c_S \} & \text{otherwise,}
\end{cases}
\]

where \( \ell_S \equiv \min \left\{ L_S, (c_S + N_S (S_1 - F_1))^+ \right\} \) and \( \ell_M \equiv \min \left\{ L_M, (c_M + N_M (S_2 - F_2))^+ \right\} \).

Above, \( \ell_j, j \in \{ M, S \} \) are expressions for penalties that each firm \( j \) incurs for failing to fulfill a contract for the years the futures contract is held. In (3c), \( N_j (S_i - F_i), j \in \{ M, S \}, i = 1, 2 \) is firm \( j \)’s payoff from the futures contract position and \( c_j + N_j (S_i - F_i) \) is firm \( j \)’s total capital at time \( t_i \). Taken together, the expressions for \( \ell_j, j \in \{ M, S \} \) reflect the fact that both firms \( j \) have limited liability. The \( u \)’s in Equations (3a)–(3b) are spot prices of commodities 1 and 2 at which the supplier and the assembler will break the underlying supply contract at times \( t_1 \) and \( t_2 \) respectively. Their values are as follows.

**Lemma 1.** If \( t_0 < t_1 \leq t_2 \leq t_3 \) and \( 0 \leq N_j \leq q, j \in \{ M, S \} \), then in (3): \( u^S_{L1} = u^S_{L2} = 0 \), \( u^M_L = 0 \), \( \ell_S = 0 \), \( \ell_M = 0 \).
and

\[
\begin{align*}
    u^S_{H1} &= \begin{cases} 
    \min \left\{ \frac{w+q+c_S-N_S F_1}{q-N_S}, w + \frac{L_S}{q}, 1 \right\} & \text{if } t_1 = t_2, \\
    \min \left\{ \frac{c_S-N_S F_1}{q-N_S} + q \cdot w \cdot \mathbb{P}\{S_2 \leq u^H_q\} + \mathbb{P}\{S_2 > u^H_q\} + L_S, 1 \right\} & \text{if } t_1 < t_2,
    \end{cases} \\
    u^S_{H2} &= \begin{cases} 
    \min \left\{ \frac{L_M+L_S}{q}, \frac{(L_M+c_S-N_S F_1)}{q-N_S}, 1 \right\} & \text{if } t_1 = t_2, \\
    u^S_{H1} & \text{if } t_1 < t_2,
    \end{cases} \\
    u^M_H &= \begin{cases} 
    \min \left\{ \frac{\min(q,D)+c_M-N_M F_2-w q}{\min(q,D)-N_M}, \frac{\min(q,D)+L_M-w q}{\min(q,D)} + 1 \right\} & \text{if } t_2 = t_3, \\
    \min \left\{ \frac{c_M-N_M F_2-w q}{q-N_M} + \mathbb{E}_q \min(q,D) + L_M - w q, \mathbb{E}_q \min(q,D) \right\} & \text{if } t_2 < t_3.
    \end{cases}
\end{align*}
\]

If \( t_0 < t_1 \leq t_2 \leq t_3 \) and \( 0 \leq q < N_j \), \( j \in \{M,S\} \), then in (3):

\[
\begin{align*}
    u^S_{H1} &= \begin{cases} 
    \max \left\{ u^S_{L1}, \min \left\{ w + \frac{L_S}{q}, 1 \right\} \right\} & \text{if } t_1 = t_2, \\
    \max \left\{ u^S_{L1}, \min \left\{ \frac{q \cdot w \cdot \mathbb{P}\{S_2 \leq u^H_q\} + \mathbb{P}\{S_2 > u^H_q\} + L_S}{q}, 1 \right\} \right\} & \text{if } t_1 < t_2,
    \end{cases} \\
    u^S_{L1} &= \begin{cases} 
    \left( \frac{N_S F_1-w q-c_S}{N_S-q} \right) & \text{if } t_1 = t_2, \\
    \left( \frac{N_S F_1-c_S}{N_S-q} \right) & \text{if } t_1 < t_2,
    \end{cases} \\
    u^S_{H2} &= \begin{cases} 
    \max \left\{ u^S_{L2}, \frac{L_M+L_S}{q}, 1 \right\} & \text{if } t_1 = t_2, \\
    u^S_{H1} & \text{if } t_1 < t_2,
    \end{cases} \\
    u^S_{L2} &= \begin{cases} 
    \left( \frac{N_S F_1-c_S-t_M}{N_S-q} \right) & \text{if } t_1 = t_2, \\
    u^S_{L1} & \text{if } t_1 < t_2,
    \end{cases} \\
    u^M_H &= \begin{cases} 
    \max \left\{ u^M_{L1}, \min \left\{ \frac{\min(q,D)+L_M-w q}{\min(q,D)}, 1 \right\} \right\} & \text{if } t_2 = t_3, \\
    \max \left\{ u^M_{L1}, \min \left\{ \mathbb{E}_q \min(q,D) + L_M - w q, \mathbb{E}_q \min(q,D) \right\} \right\} & \text{if } t_2 < t_3,
    \end{cases} \\
    u^M_L &= \begin{cases} 
    \left( \frac{w q+N_M F_2-\min(q,D)-c_M}{N_M-\min(q,D)} \right) & \text{if } t_2 = t_3, \\
    \left( \frac{w q+N_M F_2-c_M}{N_M-q} \right) & \text{if } t_2 < t_3.
    \end{cases}
\end{align*}
\]

(1) Firm \( j \)'s expected profit from production is strictly less than the penalty \( L_j \), \( j \in \{M,S\} \) firm \( j \) agreed to pay for breaking the underlying supply contract. That is, if input prices at time \( t_i \) turn out to be high, and the cost of default, \( L_j \), is low, then firm \( j \) will choose not to produce owing to insufficient expected payoff.

(2) Firm \( j \) lacks sufficient resources to produce. To illustrate, suppose \( t_1 < t_2 \). Then, the supplier, \( S \), will inevitably default at time \( t_1 \) if \( c_S + N_S (S_1 - F_1) < q S_1 \) because, in such a case, the time \( t_1 \) cost of production inputs, \( q S_1 \), exceeds the time \( t_1 \) available capital, \( c_S + N_S (S_1 - F_1) \).

Figure 2, which illustrates the functions \( u \) graphically confirms that the supplier, \( S \), will rationally choose to produce if it knows that the assembler will want to produce and the time \( t_1 \) spot price of commodity 1 is between \( u^S_{L1} \) and \( u^S_{H1} \). In this case the supplier will earn a payoff of \( q (w - S_1) + N_S (S_1 - F_1) \). However, the figure also reveals that the supplier will choose to produce
Figure 2: Supplier’s and Assembler’s Equilibrium Strategies

if it knows that the assembler will not produce and if its time $t_1$ spot price of commodity 1 is between $u_{L1}^S$ and $u_{H2}^S$. In this case, the supplier will receive a payoff of $N^S (S_1 - F_1) + \ell_M - qS_1$, which is in excess of the penalty payment $\ell_S$. The assembler, on the other hand, will only produce if its commodity input cost is between $u_{L1}^M$ and $u_{H1}^M$ and if the supplier produces. (This is because the assembler’s production process requires an input from the supplier.)

A closer examination at the functions $u$ given in the lemma also reveals that the exogenously given capital endowment, $c_j \geq 0$, $j \in \{M, S\}$, causes ‘parallel shifts’ to the functions $u$, but it does not qualitatively change how each firm $j$ behaves as its endogenously chosen position in the futures market, $N_j$, and the spot price of commodity $i = 1, 2$ change. (Except when $c_j$ is very large; then the role of the firm $j$’s futures contract position, $N_j$, diminishes.) It is for this reason that hereafter we analyze the model in the analytically simplest way by taking $c_j = 0$. Moreover, to allow firms to produce without having to borrow, we take $0 < t_1 = t_2 = t_3$. Then both firms incur revenues and costs simultaneously and production is feasible without cash-on-hand and without borrowing. (Strictly speaking, to avoid borrowing, all we require is that $(t_3 - t_1) \geq 0$ be no more than the standard contractual grace period, which allows payments to be received for a certain period of time after a payment due date. This is usually up to 21 days.)
5.1 Analysis of the Unhedged Contract

Suppose now that neither firm hedges its commodity input cost. The assembler’s and supplier’s time to payoffs, $\Pi_U^M$ and $\Pi_S^M$, can be recovered from Equations (3) and (4) and by setting $N_M = N_S = 0$. Since $c_M = c_S = 0$, then Equations (3c) imply $\ell_M = \ell_S = 0$ for all $L_S \geq 0$, $L_M \geq 0$, implying that default penalties in the unhedged case are irrelevant.

\[
\max_{\frac{d}{d} \leq q \leq \bar{d}} \int_0^{u_{H_H}^q} \int_0^{\bar{d}} \int_0^{u_{H_H}^M(q,q)} q (1 - y + w_U) f(x) h(y, z) dy dx dz \\
+ \int_0^{u_{H_H}^q} \int_0^{\bar{d}} \int_0^{u_{H_H}^M(x,q)} (x - x y - w_U) f(x) h(y, z) dy dx dz. \tag{6}
\]

Using the necessary optimality conditions with respect to $q$, (e.g., Bertsekas, 2003, Proposition 1.1.1) and the Leibniz’s rule we can obtain the implicit inverse demand curve, $w_U(q)$, that the supplier faces:

\[
w_U(q) = \frac{\int_0^{u_{H_H}^q} \int_0^{\bar{d}} \int_0^{u_{H_H}^M(q,q)} (1 - y) f(x) h(y, z) dy dx dz}{\mathbb{P}\{S_1 \leq u_{H_H}^S, S_2 \leq u_{H_H}^M\}}.
\tag{7}
\]

By substituting the right side of (7) for the wholesale price, $w_U$, into the maximand in (6), we can obtain an expression for the assembler’s expected payoff (in terms of $q$):

\[
E_{t_0} \Pi_U^M(q) = \int_0^{u_{H_H}^q} \int_0^{\bar{d}} \int_0^{u_{H_H}^M(x,q)} x (1 - y) f(x) h(y, z) dy dx dz. \tag{8}
\]

Since the distribution of demand, $F$, has support on $[d, \bar{d}]$, then $E_{t_0} \Pi_U^M(q) : [d, \bar{d}] \to \mathbb{R}$, given by (8), is a continuous real-valued mapping, where $[d, \bar{d}]$ is non-empty subset of $\mathbb{R}$. There exists a quantity $q^*_M$ such that $E_{t_0} \Pi_U^M(q) \leq E_{t_0} \Pi_U^M(q^*_M)$ for all $q \in [d, \bar{d}]$ (Weierstrass proposition). It follows that if $E_{t_0} \Pi_U^M(q^*_M) \leq R_M$, then the assembler will place no order.

\[
E_{t_0} \Pi_S^S(q) = q \left(w_U(q) - E_{t_0} \left(S_1 \mid S_1 \leq u_{H_H}^S, S_2 \leq u_{H_H}^M\right)\right) \mathbb{P}\{S_1 \leq u_{H_H}^S, S_2 \leq u_{H_H}^M\}, \text{ or, equivalently,}
\]

\[
E_{t_0} \Pi_S^S(q) = q \int_0^{u_{H_H}^M(q,q)} \int_0^{\bar{d}} \int_0^{u_{H_H}^S(q,q)} (1 - y) f(x) h(y, z) dz dx dy - q \int_0^{u_{H_H}^M(q,q)} \int_0^{\bar{d}} \int_0^{u_{H_H}^S(q,q)} z h(y, z) dz dy. \tag{9}
\]

Since $F$ has support on $[d, \bar{d}]$, then $E_{t_0} \Pi_S^S(q) : [d, \bar{d}] \to \mathbb{R}$ is a continuous real-valued mapping where $[d, \bar{d}]$ is a non-empty subset of $\mathbb{R}$. There exists a quantity $q^*_S$ such that $E_{t_0} \Pi_S^S(q) \leq E_{t_0} \Pi_S^S(q^*_S)$ for all $q \in [d, \bar{d}]$ (Weierstrass proposition). The supplier’s optimal order quantity $q^*_U$ is given by $\arg \max_{q \geq 2} E_{t_0} \Pi_S^S(q)$ s.t. $E_{t_0} \Pi_U^M(q) \geq R_M$ and $E_{t_0} \Pi_S^S(q) \geq R_S$, where $R_S, R_M$ are the firms’ reservation payoffs (note that if $R_M$ and $R_S$ are excessively large, then $q^*_U$ may not exist). It follows that if the supplier were able to choose any wholesale price, it would choose $w_U(q^*_S)$, where $w_U$ solves Equation (7).
5.2 Analysis of the Hedged Contract

Suppose now that both firms hedge by entering into $n_j \geq 0$, $j \in \{M, S\}$ futures contracts to purchase commodity $i = 1, 2$ at time $t_i$. Each firm $j$’s time $t_i$ payoff from the futures contract position will be $n_j (S_i - F_i)$, which is positive when the realized future spot price, $S_i$, is high (i.e., when $0 \leq F_i < S_i \leq \infty$) and negative when it is low (i.e., when $0 \leq S_i < F_i < \infty$). Before we begin the Stage 1 analysis of the supply chain contract with hedging, we present a preliminary result that deals with firms’ equilibrium long futures contract positions.

**Proposition 1.** Suppose Assumption 3 holds and $c_j = 0$, $j \in \{M, S\}$. The number of futures contracts, $n_j \geq 0$, that can be supported as a subgame equilibrium (SE) must satisfy the following constraints:

$$\frac{qL_S}{L_S + q(w_H - F_1)} \leq n_{S} \leq \min \left\{ \frac{qw_H}{F_1}, \frac{L_M}{F_1}, q \right\}, \quad (10a)$$

$$\frac{dL_M}{d(1 - F_2) + L_M - w_H q} \leq n_{M} \leq \min \left\{ \frac{d - qw_H}{F_2}, \frac{L_S(S_2 = 0)}{F_2}, q \right\}, \quad (10b)$$

where $L_S(S_2 = 0)$ denotes the value penalty $L_S$ when $S_2 = 0$. (Of course, this is only relevant in cases when $L_S$ is a function of $S_2$.)

Reading from the left, the first upper bound on $n_j$ given in (10) is in force when both supply chain members choose to produce at time $t_i$, $i = 1, 2$ and firm $j$’s realized input price, $S_i$, at time $t_i$ is less than the futures price, $F_i$. The bound ensures that firm $j$’s operating profit is sufficient to offset any loss from firm $j$’s futures contract position, $n_j (S_i - F_i) < 0$, $j \in \{M, S\}$, $i = 1, 2$. Otherwise firm $j$ would be at risk of defaulting on its futures contract position due to insufficient resources.

The second upper bound on $n_j$ given in (10) is in force when firm $j$’s realized time $t_i$ input price is again low (i.e., $S_i < F_i$) and its supply chain counterpart, firm $k \in \{M, S\}$, $j \neq k$, chooses not to produce at time $t_l$, $l = 1, 2$, $l \neq i$. This will occur exactly when firm $k$’s realized input cost, $S_l$, at time $t_l$ is high (infinite). In this situation firm $k$ will be contractually obligated to use the payoff from its futures contract position, $n_k (S_l - F_l) > 0$, to compensate firm $j$ with a default penalty, $L_k \geq 0$. (Note that if firm $k$ breaks the supply contract, the penalty payment, $L_k$, is firm $j$’s only revenue.) As before, the upper bound on $n_j$ ensures that $L_k$ is sufficient to offset any loss from firm $j$’s futures contract position, $n_j (S_i - F_i) < 0$. At the same time, the lower bound on $n_k$ given in (10) ensures that firm $k$’s payoff from its futures contract position, $n_k (S_l - F_l) > 0$, is enough to pay at least $L_k$ to firm $j$.

In summary, Proposition 1 can be viewed as a set of necessary and sufficient conditions under which neither supply chain member defaults on its futures contract position in all possible futures states of the world – as specified in our Assumption 3.

Since the lower bounds on $n_M$ and $n_S$ are strictly positive, then Proposition 1 also reveals that, in equilibrium, either both firms $j \in \{M, S\}$ simultaneously enter into futures contract positions $n_j > 0$ or they do not hedge at all. This reflects the fact that a situation in which one firm hedges
while its supply chain partner does not hedge, makes the hedger susceptible to default on the futures contract. This default occurs exactly when the realized future spot price of the firm that did not hedge is high (causing it to default on the underlying supply contract) and the realized future spot price of the firm that did hedge is low (causing it to default on the futures contract). Corollary 1 summarizes the result.

Corollary 1. If \( c_j = 0 \), \( j \in \{M, S\} \), then in equilibrium, either both firms \( j \) simultaneously enter into futures contract positions \( n_j > 0 \) or they don’t hedge at all.

Finally, to ensure that the ranges for \( n_j \), \( j \in \{M, S\} \) given in Proposition 1 are non-empty we require the following two conditions:

**Condition 1.** \( w_H q \leq L_M \leq q \) and \( \min\{q, D\}(1 - S_2)^+ - w_H q \leq L_S \leq q \).

**Condition 2.** \( F_1 \leq w_H \) and \( F_2 \leq 1 - \frac{q w_H}{d} \).

Condition 1 corresponds to Assumption 1, which requires that a default on the underlying supply contract be costly; neither firm, however, should unusually profit from a default of its supply chain partner. Interestingly, the penalties that satisfy the bounds given in Condition 1 not only ensure that neither firm \( j \in \{M, S\} \) defaults on its futures contract position, but also that each firm \( j \) is indifferent as to whether its supply chain partner chooses to produce or not. To see this, consider the case when \( L_M = w_H q \) and \( L_S = \min\{q, D\}(1 - S_2)^+ - w_H q \) and when both firms hedge. Then the supplier is guaranteed to earn a payoff of \( q w_H(q) - \min\{q S_1, \min\{q, D\}(1 - S_2)^+\} + n_S(S_1 - F_1) \), and the assembler is guaranteed to receive a payoff of \( \min\{q, D\}(1 - S_2)^+ - q w_H + n_M(S_2 - F_2) \), whatever action its supply chain counterpart takes. It is worth mentioning that penalties that satisfy Condition 1 are not inconsistent with what one can observe in empirical practice. For example, in the Davita, Inc. contract included in Appendix A, the penalty can be more than 100% of the wholesale price.

Condition 2 corresponds to Assumption 2, which requires upper bounds on the futures prices of commodities 1 and 2. Without these upper bounds, firms cannot make money in expectation and hedging is essentially infeasible.

**Lemma 2.** If Conditions 1 – 2 hold, then the range for \( n_j \), \( j \in \{M, S\} \) given in (10) is non-empty for all \( 0 \leq S_1 < \infty \), \( 0 \leq S_2 < \infty \), \( d \leq D \leq \bar{d} \), and \( c_j = 0 \).

We illustrate Proposition 1 with the following simple example.

**Example 4.** Suppose that \( S_1 \) and \( S_2 \) are independent and log-normally distributed with volatilities \( \sigma_1 = \sigma_2 = 60\% \). Lead time, i.e., \( t_1 - t_0 \), is 6 weeks. \( F_1 = F_2 = \$0.25 \). Demand, \( D \), is uniformly distributed on the interval \([100,150]\). Using Condition 1, we assume \( L_M = w_H q \) and \( L_S = \min\{q, D\}(1 - S_2)^+ - w_H q \), where the (equilibrium) wholesale price \( w_H \) is given by (12), which will be derived later in the paper. Using the result in Proposition 1, the following table summarizes the range for \( n_S \) and \( n_M \) that can be supported as an SE.
Note that when \( q = 100 \), then the assembler faces no demand risk, but the range for \( n_M \) reduces to a single point, implying that the assembler can only hedge by purchasing \( q \) futures contracts. The range for \( n_M \), however, opens up as the order quantity, \( q \) increases. Therefore for \( q > 100 \), the assembler can hedge by adopting different hedging policies that achieve the same expected payoff.

Figure 3 illustrates Proposition 1 graphically for the case when \( q = 120 \). Figure 3a illustrates a hypothetical non-equilibrium situation in which neither of the firms purchased a sufficient number of futures contracts and as a consequence there exist \( S_1 \times S_2 \times D \) combinations for which at least one of the firms should be expected to default on its futures contract position (these combinations are represented by the hollow space in Figure 3a). In Figure 3b, both firms purchased a correct number of futures contracts and can rationally sustain not to default for all \( S_1 \times S_2 \times D \). □

<table>
<thead>
<tr>
<th>Order Quantity, ( q )</th>
<th>Range for ( n_S ) Required by (10a)</th>
<th>Range for ( n_M ) Required by (10b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>( \frac{100}{3} \leq n_S \leq 100 )</td>
<td>( n_M = 100 )</td>
</tr>
<tr>
<td>105</td>
<td>( \frac{2667}{29} \leq n_S \leq 105 )</td>
<td>( 91 \leq n_M \leq 105 )</td>
</tr>
<tr>
<td>110</td>
<td>( \frac{1738}{29} \leq n_S \leq 110 )</td>
<td>( \frac{242}{3} \leq n_M \leq 110 )</td>
</tr>
<tr>
<td>115</td>
<td>( \frac{4439}{27} \leq n_S \leq 115 )</td>
<td>( 69 \leq n_M \leq 115 )</td>
</tr>
<tr>
<td>120</td>
<td>( \frac{696}{17} \leq n_S \leq 120 )</td>
<td>( 56 \leq n_M \leq 120 )</td>
</tr>
</tbody>
</table>

Note. Assumes \( F_1 = F_2 = \$0.25 \), \( D \sim U[100, 150] \), \( q = 120 \), \( L_M = w_H q \), and \( L_S = \min\{q, D\}(1 - S_2^+ - w_H q) \).

Partial vs. Complete Hedging. Following SFAS 52 and 80, which are Generally Accepted Accounting Principles (GAAP) for the accounting treatment of hedged transactions (see Mian,
Forthcoming in *Manufacturing and Service Operations Management* 1996, p.425), one could adopt the assumption that a futures trade is not considered to be a hedge unless the underlying transaction is a firm commitment. Under this assumption, both firms would hedge by taking futures contract positions \( n_j = q, \ j \in \{M, S\} \) and thus completely insulate their market values from hedge-able risks.

Here, however, we allow that firms adopt the ‘take-the-money-and-run’ strategy under which firms default on the underlying supply contract and sell their commodity inputs on the open market – whenever it is profitable to do so. There is some empirical evidence for this conduct: Hillier et al. (2008, p.768), for example, describe a case where in 2001 an aluminum producer Alcoa temporarily shut down its smelters and sold its electricity futures contracts on the open market. (Since it could make more money by selling electricity than by selling aluminum.)

Allowing firms to adopt the take-the-money-and-run strategy implies that the hedged supply contract is no longer a firm commitment and the results in Proposition 1 reveal that in equilibrium firms may only hedge partially, meaning that \( 0 < n_j < q, \ j \in \{M, S\} \).

The hedged contract is operationalized as follows: In Stage 1, both firms infer the equilibrium quantity, say \( q_H^* \in H \), and adopt hedging positions \( n_j, \ j \in \{M, S\} \) that satisfy Proposition 1. The equilibrium quantity that firms infer can be derived by adapting results from the existing literature (for example, Lariviere and Porteus, 2001). Using Equations (3), the assembler’s expected payoff is given by:

\[
E_t \Pi_M^H (q, w_H) = \int_0^\infty \int_q^\infty \int_0^1 q (1 - y) f(x)h(y, z)dy dx dz
+ \int_0^\infty \int_q^\infty \int_0^1 (1 - y) x f(x)h(y, z)dy dx dz - q w_H + n_M \frac{(E_t S_Z - F_2)}{q}.
\]  

(11)

Using the fact that \( F_2 = E_t S_Z \), the assembler’s optimal order quantity implicitly defined by

\[
\int_0^\infty \int_q^\infty \int_0^1 (1 - y) f(x)h(y, z)dy dx dz - w_H = 0,
\]

from which we can solve for the inverse demand, say \( w_H(q) \), the supplier faces:

\[
w_H(q) = \min \left\{ \int_0^\infty \int_q^\infty \int_0^1 (1 - y) f(x)h(y, z)dy dx dz, \frac{d(1 - F_2)}{q} \right\}.
\]  

(12)

We can then use Equations (11) and (12) to yield

\[
E_t \Pi_M^H (q) = \max \left\{ \int_0^\infty \int_q^\infty \int_0^1 x (1 - y) f(x)h(y, z)dy dx dz, \int_0^\infty \int_q^\infty \int_0^1 q (1 - y) f(x)h(y, z)dy dx dz
+ \int_0^\infty \int_q^\infty \int_0^1 (1 - y) x f(x)h(y, z)dy dx dz - d(1 - F_2) \right\},
\]  

(13)

which is the assembler’s expected payoff in terms of \( q \). Differentiation reveals that the assembler’s payoff is increasing in \( q \). To see this, the derivatives of the first and the second expression on the right side of (13) are:

\[
\int_0^\infty \int_0^1 q (1 - y) f(q)h(y, z) dy dz \geq 0 \quad \text{and} \quad \int_0^\infty \int_q^\infty \int_0^1 (1 - y) f(x)h(y, z) dy dx dz \geq 0.
\]  

(14)
The supplier’s expected profit is:

\[ \mathbb{E}_{t_0} \Pi^S_H(q) = q w_H(q) - \mathbb{E}_{t_0} \min \{ q S_1, \min \{ q, D \} (1 - S_2)^+ \} + \eta_S(\mathbb{E}_{t_0} S_1 - F_1), \quad (15) \]

The supplier’s most preferred order quantity, \( q_H^* \), is given by \( \arg \max_{q \geq 0} \mathbb{E}_{t_0} \Pi^S_H(q) \); the supplier’s optimal order quantity \( q_H^{**} \) is given by \( \arg \max_{q \geq 0} \mathbb{E}_{t_0} \Pi^S_H(q) \) s.t. \( \mathbb{E}_{t_0} \Pi^S_H(q) \geq R_M \) and \( \mathbb{E}_{t_0} \Pi^S_H(q) \geq R_S \), where \( R_S, R_M \) are the firms’ reservation payoffs. In special case, the results in Lariviere and Porteus (2001) show that \( \mathbb{E}_{t_0} \Pi^S_H(q) \), given by (15), is unimodal in \( q \).

### 5.3 Equilibrium Among Unhedged and Hedged Contracts

We now allow the firms to enter into the supply chain contract by choosing whether they wish to hedge. Depending on the model parameters, it is possible to have equilibria where: (a) Neither firm hedges; or (b) both firms hedge. Due to Proposition 1, case (a) will occur if one of the firms in the model decides not to hedge at time \( t_0 \). If, on the other hand, there exists an SPNE in which

\[ \mathbb{E}_{t_0} \Pi^S_H(q_H^{**}) \geq \mathbb{E}_{t_0} \Pi^U_H(q_H^{**}), \quad (16a) \]

then the supplier, \( S \), will hedge, and as will be seen in Lemma 3, Part (i), the assembler will hedge as well. Similarly, the supplier will hedge, in cases where the unhedged wholesale contract may be infeasible. This occurs when:

\[ \mathbb{E}_{t_0} \Pi^M_S(q_M^*) \leq R_M, \text{ and there exists } q \in H \text{ for which } R_S \leq \mathbb{E}_{t_0} \Pi^S_H(q) \text{ and } R_M \leq \mathbb{E}_{t_0} \Pi^M_H(q). \quad (16b) \]

The following Lemma 3 is needed to establish that Conditions (16a) and (16b) can hold in equilibrium. Parts (i) and (ii) describe how the risk of supply contract default affects both the assembler’s expected payoff and the wholesale price. Parts (iii) and (iv) give sufficient conditions under which (for both the supplier and the assembler) there exist expected payoff levels that may only be achieved via the hedged wholesale contract.

**Lemma 3.** Let \( w_U(q), \mathbb{E}_{t_0} \Pi^M_U(q), \mathbb{E}_{t_0} \Pi^S_U(q), w_H(q), \mathbb{E}_{t_0} \Pi^M_H(q), \text{ and } \mathbb{E}_{t_0} \Pi^S_H(q) \) respectively be given by (7), (8), (9), (12), (13), and (15). If \( c_j = 0 \) and Conditions 1 – 2 hold then:

(i) \( 0 \leq \mathbb{E}_{t_0} \Pi^M_U(q) \leq \mathbb{E}_{t_0} \Pi^M_H(q) \).

(ii) \( w_H(q) \leq w_U(q) \) (if \( S_1 \) and \( S_2 \) are independent).

(iii) The set \( H_M := \{ q \in H \mid \mathbb{E}_{t_0} \Pi^M_U(q_M^*) \leq \mathbb{E}_{t_0} \Pi^M_H(q) \} \) is non-empty. (An example of the set \( H_M \) can be seen graphically in Figure 4a.)

(iv) \( H_S := \{ q \in H \mid \mathbb{E}_{t_0} \Pi^S_U(q_M^*) \leq \mathbb{E}_{t_0} \Pi^S_H(q) \} \) is non-empty if

\[ \mathbb{P} \{ S_1 \leq q_H^*, S_2 \leq u_H^M(q_U^*, q_V^*) \} + \left( \int_0^1 \frac{z h_Z(z)}{F(q_U^*)} dz + \int_0^1 y h_Y(y) dy \right) \]

\[ \leq \mathbb{P} \{ S_2 \leq 1 \} + \left( \int_0^{u_H^M(q_U^*, q_V^*)} \int_0^{u_H^S(q_U^*, q_V^*)} \left( \frac{z}{F(q_U^*)} + y \right) h(y, z) dy dz \right), \quad (17) \]
D follows a uniform distribution $U(30, 60)$; $S_1$ and $S_2$ are independent and log-normally distributed; time $t_0$ prices of commodities 1 and 2 are $0.3$; lead-time $t_1 = t_0 = 6$ weeks; $t_2 = t_1$; annual volatility of commodity 1 and 2 spot prices is $60\%$.

**Definitions.** (i) For the order quantities $\hat{q}_S$ and $\hat{q}_M$ respectively we have $\mathbb{E}_{t_0} \Pi_H^S[\hat{q}_S] = \mathbb{E}_{t_0} \Pi_H^U(\hat{q}_U^*)$ and $\mathbb{E}_{t_0} \Pi_H^M[\hat{q}_M] = \mathbb{E}_{t_0} \Pi_H^U(\hat{q}_U^*)$. (ii) For the order quantity $q = \hat{q}_S$ we have $\mathbb{E}_{t_0} \Pi_H^S[\hat{q}_S] = \mathbb{E}_{t_0} \Pi_H^U(\hat{q}_U^*)$; for the quantity $q = \hat{q}_M$ we have $\mathbb{E}_{t_0} \Pi_H^M[\hat{q}_M] = \mathbb{E}_{t_0} \Pi_H^U(\hat{q}_U^*)$. (iii) For the order quantity $q = \bar{q}$ we have $\mathbb{E}_{t_0} \Pi_H^S[\bar{q}] = 0$.

where $h_Y(y)$ and $h_Z(z)$ denote marginal densities. (An example of the set $H_S$ can be seen graphically in Figure 4b.)

For the case when $S_1$ and $S_2$ are independent, Part (ii) of the lemma formally establishes that the equilibrium wholesale price is lower when the downstream assembler hedges. This result reflects the fact that, by hedging, the assembler guarantees the supply contract performance and the rational supplier responds to this guarantee by reducing the wholesale price.

Parts (iii) and (iv) give sufficient conditions under which there exist expected payoff levels that can only be achieved via the hedged wholesale contract. As a consequence of Part (i) of the lemma, the set $H_M$ will always be non-empty.

The set $H_S$ will be non-empty if $d$ and the supplier’s margin are sufficiently large. The former is consistent with a situation in which there is a high baseline demand for the assembler’s final product. The latter is consistent with a situation in which $F_1$ is sufficiently low and $w_H$ is sufficiently high. Under these conditions, reducing the probability that the assembler defaults on the supply contract is ex-ante important to the supplier. Mathematically, this situation reduces to the Condition (17). We illustrate this in the following example, which reveals that the supplier can be better off or worse under the hedged contract:

**Example 5 (Continuation of Example 4).** If $F_1 = F_2 = 0.25$, $q_U^* = 105$ and Condition (17) reduces to $1.446 \leq 1.477$. In fact, with $R_M = R_S = 0$, the highest profit the supplier
can attain without hedging is $E_0 \Pi_S^U(q^*_U) = $39.89. However, if the supplier and the assembler respectively purchase $2667 \leq n_S \leq 105$ and $91 \leq n_M \leq 105$ futures contracts, the supplier’s expected payoff increases from $39.89 to $42.86. Clearly, hedging is beneficial to the supplier (Note that the assembler will join the supplier in hedging due to Lemma 3, Part (i).) If, however, $F_1 = F_2 = $0.45, then $q^*_U = 102$ and the Condition (17) reduces to $1.38001 \leq 1.3488$ and the highest profit the supplier can attain without hedging is $E_0 \Pi_S^U(q^*_U) = $8.22. With hedging, however, the supplier’s expected payoff would decrease from $8.22 to $4.83. The supplier is therefore better off without hedging. □

The primary use of Lemma 3 is in establishing the next Proposition 2.

**Proposition 2.** If $c_j = 0$ and Conditions 1 – 2 hold then there exist SPNEs in which both firms will hedge. Both firms will hedge if (i) $R_M \geq E_0 \Pi_M^H(q^*_M)$; or (ii) if $R_M = 0$ and the set $H_S$ is non-empty (see Part iv of Lemma 3).

**Prediction 1.** Offers of the hedged wholesale contract should be expected if the downstream firm’s market power exceeds a critical threshold.

Prediction 1 essentially identifies situations when hedging is mainly important to the downstream assembler. A second empirical prediction regarding the use of a hedged contract can be made using Part (ii) of Proposition 2. Prediction 2 identifies situations when hedging is mainly important to the upstream supplier.

**Prediction 2.** Offers of the hedged wholesale contract should be expected if the upstream firm operates on a large margin, there is a high baseline demand for downstream firm’s final product, and the downstream firm’s market power is below a critical threshold.

Table 1 presents some data on how much better the assembler can do with the hedged contract: If the unhedged equilibrium order quantity is $q$, then column (L) of Table 1 shows the percentage profit increase the assembler experiences if it is offered an alternative hedged contract that leaves the supplier no worse off than the original unhedged contract. The data shows that the assembler’s expected payoff can increase from 0.34% to 75%. Column G of the same table presents comparable data for the supplier. As might be expected, the value derived from using the hedged contract will depend on lead-times, spot price volatilities, and spot price correlation. In computing the results in Table 1, we assumed that the spot prices of commodities 1 and 2 were independent and chose annual volatilities ranging from 30%–60% (a range comparable to equities; e.g., see Hull, 2009, p.238). Interestingly, in practice, annual volatility of commodity prices can easily exceed 60%: To illustrate, silver, an industrial metal used in electrical contacts and in catalysis of chemical reactions, was trading at around $18 per ounce in April and May of 2010. Less than a year later, silver almost tripled in value to $49 per ounce during the final week of April 2011 – see Christian (2011). Our lead-times, namely the difference between $t_1$ and $t_0$, were 4 to 12 weeks.

We can also circle back some of our predictions both to results found in the research existing literature and to the existing industry practices. In Example 5, we demonstrate that hedging can
be unprofitable for the upstream supplier. However, such a supplier may still hedge in equilibrium if $\mathbb{E}_{t_0} \Pi_{U}^M (q_M^*) \le R_M$ (i.e., if the supplier’s counterpart has a high reservation payoff). This observation is not necessarily consistent with the existing theories of hedging, which imply that hedging should increase firms’ market values. It does, however, appear to be consistent with the empirical findings in Jin and Jorion (2006) who report that while many firms hedge, hedging does not necessarily increase their market values. Likewise, it appears to be consistent with anecdotal examples from auto parts supply chains (e.g., Hakim, 2003; Matthews, 2011), food-processing supply chains (e.g., Eckblad, 2012), and energy supply chains (e.g., Thakkar, 2013), which appeared in popular
business press.

6 Conclusion

The production processes of many firms depend on raw materials whose prices can be highly volatile (e.g., Carter et al., 2006). Moreover, many firms who purchase raw materials also depend on suppliers who purchase raw materials for their own production. Of course, suppliers experience price volatility, too, and they may request price increases and surcharges. In some cases, if commodity prices significantly increase, the suppliers may not be able to fulfill contractual requirements, causing a breakdown in the supply chain. It is possible to find examples of this phenomenon from various industries, including auto parts, e.g., Hakim, 2003; Matthews, 2011; food-processing, e.g., Eckblad, 2012; energy and utilities, e.g., Thakkar, 2013; and heavy manufacturing, e.g., Matthews, 2011. Given the prevalence of this issue, one might guess that the topic of risk management would command a great deal of attention from researchers in finance, and that practitioners would therefore have a well-developed body of wisdom from which to draw in formulating hedging strategies.

Such a guess would, however, be at best only partially correct. Finance theory does a good job of instructing standalone firms on the implementation of hedges. Finance theories, however, are generally silent on the implementation of hedges in situations where a firm operates in a supply chain and its operations are exposed to price fluctuations not only through direct purchases of commodities, but also through its suppliers and the suppliers of its suppliers. This is precisely the question that we study in this paper.

Specifically, we ask the following: (1) When should supply chain firms hedge their stochastic input costs? (2) Should they hedge fully or partially? (3) Do the answers depend on whether the firms’ supply chain partners hedge? To answer these questions, we consider a simple supply chain model – the ‘selling-to-the-newsvendor’ model (Lariviere and Porteus, 2001) – and generalize it by assuming that both the upstream and the downstream firms face stochastic production costs. The stochastic costs could represent the raw material costs. We show that the stochastic costs reverberate through the supply chain and will be, ex ante, impounded into the wholesale price. In some cases, if input costs significantly increase, one of the supply chain members may not be able to fulfill its contractual requirements, causing the entire supply chain to break down. We identify conditions under which the risk of the supply chain breakdown and its impacts on the firms’ operations will cause the supply chain members to hedge their input costs: (i) The downstream buyer’s market power exceeds a critical threshold; or (ii) the upstream firm operates on a large margin, there is a high baseline demand for downstream firm’s final product, and the downstream firm’s market power is below a critical threshold. In absence of these conditions there are equilibria in which neither firm hedges. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. There are also situations when firms hedge in equilibrium although hedging reduces their expected payoff.
As an extension, one could consider the case when firms' operations are financed with borrowing and show that hedging can be profitable even in the absence of breakdown risk. There, the equilibrium hedging policy would be a full hedging policy. Future research may consider the role coordinating contracts, financing, and hedging play in the effective management of decentralized supply chains in the presence of stochastic demands and stochastic input costs.

References


Forthcoming in *Manufacturing and Service Operations Management*


Online Supplement

A Sample Penalty Clauses

Taken from the Contract between Plymouth Rubber Company, Inc. (the Buyer) and Kleinewefers Kunststoffanlagen GmbH (the Supplier) (Source: Plymouth’s July 17, 1997, 10-K filing).

§10.1: If due to the responsibility of the Supplier components have not been delivered at the relevant dates according to Article 4.1, the Supplier shall be obliged to pay the Buyer penalty that shall not exceed 5% of the contract price.

Taken from the Contract between Davita, Inc. (the Buyer) and Rockwell Medical Technologies, Inc. (the Supplier) (Source: Rockwell’s March 28, 2003, 10-K filing).

Failure to Perform Supply Obligation. In the event ROCKWELL is unable to fulfill DAVITA’S orders at any time during the Term of this Agreement, DAVITA may, as its sole and exclusive remedy, upon prior notice to ROCKWELL, seek other suppliers to fill purchase orders for some or all Products. If DAVITA is required to purchase Products from a third party under this Section 8, ROCKWELL will provide DAVITA with a purchase credit equal to the difference, if any, in the then current purchase prices for the Products which ROCKWELL was unable to deliver and the purchase prices DAVITA is reasonably required to pay, including legitimate freight charges incurred, in order to obtain similar or equivalent products from a different supplier(s).
B Proofs

Proof of Lemma 1. For brevity, we’ll only show the derivation of the expression for \( u_{H1}^{S} \) and \( u_{H2}^{S} \) for the case when \( t_1 = t_2 \) and \( N_j \leq q, j \in \{M, S\} \). The other cases follow in a similar manner. Suppose that the supplier and the assembler respectively are long \( N_S \) and \( N_M \) futures contracts to purchase commodity \( i \) at time \( t_i, i = 1, 2 \). In order for the supplier to produce the following must hold:

1. The hedged supplier has sufficient cash to procure the required production inputs. That is:

   \[
   q w + c_S + N_S (S_1 - F_1) \geq q S_1 \quad \text{if} \quad S_2 \leq u_H^M, \quad (B.1a)
   \]

   \[
   \min \{L_M, w q_i (c_M + N_M (S_2 - F_2))^{+}\} + c_S + N_S (S_1 - F_1) \geq q S_1 \quad \text{if} \quad S_2 > u_H^M. \quad (B.1b)
   \]

2. The hedged supplier stands to earn a higher expected payoff by producing than by defaulting on the supply contract, selling commodity 1 on the open market and incurring the default penalty \( \max \{-L_S, -N_S (S_1 - F_1) - c_S\} \). That is:

   \[
   N_S (S_1 - F_1) + q w - q S_1 \geq \max \{N_S (S_1 - F_1) - L_S, -c_S\} \quad \text{if} \quad S_2 \leq u_H^M, \quad (B.2a)
   \]

   \[
   N_S (S_1 - F_1) + \min \{L_M, w q_i (c_M + N_M (S_2 - F_2))^{+}\} - q S_1 \\
   \geq \max \{N_S (S_1 - F_1) - L_S, -c_S\} \quad \text{if} \quad S_2 > u_H^M. \quad (B.2b)
   \]

3. \( S_1 \leq 1 \) for if \( S_1 > 1 \), the supplier can simply pay the assembler \( q (1 - w) \), which is the highest payoff the assembler can achieve if both firms produce. By receiving the payment \( q (1 - w) \), the assembler is no worse off than if the supplier produced and the supplier is better off by not producing.

To establish the proof, Conditions (B.1a) and (B.2a) imply that the supplier will commit to production if and only if:

\[
S_1 \leq \min \left\{ \left( \frac{q w + c_S - N_S F_1}{q - N_S} \right)^{+}, w + \frac{L_S}{q}, 1 \right\} \quad \Leftrightarrow \quad S_1 \leq u_{H1}^{S}.
\]

Similarly, Conditions (B.1b) and (B.2b) imply that the supplier will commit to production if and only if:

\[
S_1 \leq \min \left\{ \left( \frac{\ell_M + L_S}{q} \right), \left( \frac{\ell_M + c_S - N_S F_1}{q - N_S} \right)^{+}, 1 \right\} \quad \Leftrightarrow \quad S_1 \leq u_{H2}^{S}.
\]

The proof is analogous for the assembler.

\[ \Box \]

Proof of Proposition 1. As we explain in Section 4, as a guarantor, the futures exchange must organize trading in futures contracts so that defaults are completely avoided. This is accomplished by only accepting futures contract positions \( 0 \leq n_j \leq q \), that the exchange knows each firm \( j \) can sustain without default for all for all \( 0 \leq S_1 < \infty, 0 \leq S_2 < \infty, \) and \( d \leq D \leq d \). To establish the bounds on \( n_j \) given in (10), we begin by defining \( \bar{s}_1 \) and \( \bar{s}_2 \) respectively as values of \( S_1 \) and \( S_2 \) for
which:

\[ q(w_H - S_1) = -L_S \quad \text{and} \quad d(1 - S_2)^+ - qw_H = -L_M. \]

The proof now proceeds in cases:

**Case 1:** \( \bar{s}_1 < S_1 < \infty, 0 \leq S_2 < \infty. \) In this situation, the supplier does not produce. Consequently, that the assembler cannot produce either. To avoid the assembler’s default on the futures contract, the following must hold for all \((S_1, S_2): L_S + n_M(S_2 - F_2) \geq 0\) and \(q(w_H - \bar{s}_1) + n_S(\bar{s}_1 - F_1) \geq 0.\) This gives \(\frac{qL_S}{L_S + q(w_H - F_1)} \leq n_S \leq q \text{ and } 0 \leq n_M \leq \frac{L_S(S_2 = 0)}{F_2},\) where \(L_S(S_2 = 0)\) denotes the value penalty \(L_S\) when \(S_2 = 0.\) (Of course, this is only relevant in cases when \(L_S\) is a function of \(S_2.\))

**Case 2:** \(0 \leq S_1 \leq \bar{s}_1, 0 \leq S_2 \leq \bar{s}_2.\) In this situation, both firms produce. To eliminate the risk that either the assembler or supplier defaults on its futures contract position, the following must hold for all \((S_1, S_2): q(w_H - S_1) + n_S(S_1 - F_1) \geq 0\) and \(\min\{q, D\}(1 - S_2)^+ - q w_H + n_M(S_2 - F_2) \geq 0.\) Case 2 breaks down to the following three sub-cases in which at least one of the firms experiences a negative payoff on its long futures contract position (and is therefore at risk of default):

(i) If \(S_1 \geq F_1\) and \(S_2 \leq F_2,\) then \(\frac{qL_S}{L_S + q(w_H - F_1)} \leq n_S \leq q \text{ and } 0 \leq n_M \leq \frac{d - qw_H}{F_2}\) is required.

(ii) If \(S_1 \leq F_1\) and \(S_2 \geq F_2,\) then \(0 \leq n_S \leq \frac{qw_H}{F_1}\) and \(\frac{dL_M}{d(1 - F_2) + L_M - w_H q} \leq n_M \leq q\) is required.

(iii) If \(S_1 \leq F_1\) and \(S_2 \leq F_2,\) then \(0 \leq n_S \leq \frac{qw_H}{F_1}\) and \(0 \leq n_M \leq \frac{d - qw_H}{F_2}\) is required.

**Case 3:** \(0 \leq S_1 \leq \bar{s}_1, \bar{s}_2 < S_2 < \infty.\) In this situation, only the supplier produces. To avoid the supplier’s default on the futures contract, the following must hold for all \((S_1, S_2): L_M + n_S(S_1 - F_1) \geq 0\) and \(\min\{q, D\}(1 - \bar{s}_2)^+ - q w_H + n_M(\bar{s}_2 - F_2) \geq 0.\) This gives \(0 \leq n_S \leq \frac{L_M}{F_1}\) and \(\frac{dL_M}{d(1 - F_2) + L_M - w_H q} \leq n_M \leq q.\)

Together, Cases 1–3 give (10).

**Proof of Lemma 2.** In Lemma 1, Condition 1 ensures that at \(S_1 = 0\) and \(S_2 = 0\) (see Cases 1 and 3 in the proof of Proposition 1):

\[
\frac{qw_H}{F_1} \leq \frac{L_M}{F_1} \quad \text{and} \quad \frac{d - qw_H}{F_2} \leq \frac{L_S(S_2 = 0)}{F_2}. \tag{B.3}
\]
Condition 2 ensures that:

\[
\frac{q L_S}{L_S + q(w_H - F_1)} \leq q,
\]

and

\[
\frac{d L_M}{d (1 - F_2) + L_M - w_H q} \leq \frac{d - q w_H}{F_2},
\]

Together (B.3) and (B.4) imply that the range for \( R \) assembler can achieve under the unhedged contract and \( E \) is:

\[
\begin{align*}
0 & \leq \frac{q w_H}{F_1}, \\
& & \text{and} \quad \frac{d L_M}{d (1 - F_2) + L_M - w_H q} \leq q.
\end{align*}
\]

Proof of Lemma 3. (i) Equations (8) and (14) give:

\[
\mathbb{E}_{t_0} \Pi_H^M(q) = \int_0^{u_{t_1}^H} \int_0^q \int_0^{u_{t_1}^H(x,q)} x(1 - y) f(x) h_Y(y) h_Z(z) dy dx dz \\
\leq \min \left\{ \int_0^\infty \int_0^q \int_0^{u_{t_1}^H(x,q)} x(1 - y) f(x) h_Y(y) h_Z(z) dy dx dz, \int_0^\infty \int_0^1 q(1 - y) f(x) h_Y(y) h_Z(z) dy dx dz \\
+ \int_0^\infty \int_0^q \int_0^1 (1 - y) x f(x) h_y(y,z) dy dx dz - d(1 - F_2) \right\} = \mathbb{E}_{t_0} \Pi_H^M(q).
\]

(ii) Let \( h_Y(\cdot) \) and \( h_Z(\cdot) \) be the marginal densities of \( S_2 \) and \( S_1 \). Equations (7) and (12) give:

\[
w_H(q) = \min \left\{ \frac{1 - F(q)}{P \{ S_1 \leq \infty \} P \{ S_2 \leq \infty \}}, \frac{d(1 - F_2)}{q} \right\} \leq \min \left\{ \frac{1 - F(q)}{P \{ S_2 \leq 1 \} P \{ S_2 \leq u_{t_1}^H(q) \}}, \frac{d(1 - F_2)}{q} \right\} \\
\leq \frac{1 - F(q)}{P \{ S_2 \leq u_{t_1}^H(q) \}} \frac{d(1 - F_2)}{q} = \frac{(1 - F(q)) \int_0^{u_{t_1}^H(q)} (1 - y) h_Y(y) dy}{P \{ S_2 \leq u_{t_1}^H(q) \}} = w_U(q).
\]

(iii) Follows from Part (i), which asserts \( \mathbb{E}_{t_0} \Pi_H^M(q) \leq \mathbb{E}_{t_0} \Pi_H^M(q) \) for all \( q \in H \) and from the fact that \( \mathbb{E}_{t_0} \Pi_H^M(q) \) is strictly increasing in \( q \) (see 14). These properties imply that there exists an order quantity \( \hat{q}_M \) for which \( \mathbb{E}_{t_0} \Pi_H^M(\hat{q}_M) = \mathbb{E}_{t_0} \Pi_U^M(\hat{q}_M) \) and \( \mathbb{E}_{t_0} \Pi_U^M(q) \leq \mathbb{E}_{t_0} \Pi_H^M(q) \) for all \( \hat{q}_M \leq q \in H \).

(iv) The condition implies

\[
\mathbb{E}_{t_0} \Pi_U^S(q) = q \int_0^{u_{t_1}^H(q)} \int_0^{u_{t_1}^H} (1 - y) f(x) h_y(y,z) dz dx dy - q \int_0^{u_{t_1}^H(q)} \int_0^{u_{t_1}^H} h_y(y,z) dz dy \\
\leq \int_0^\infty \int_0^q \int_0^1 q(1 - y) f(x) h_y(y,z) dy dx dz - q \int_0^\infty \int_0^q h_y(y,z) dy dz \\
\leq \int_0^\infty \int_0^q \int_0^1 q(1 - y) f(x) h_y(y,z) dy dx dz - \mathbb{E}_{t_0} \min \left\{ q S_1, \min \{ q, D \} (1 - S_2)^+ \right\} = \mathbb{E}_{t_0} \Pi_H^S(q).
\]

Proof of Proposition 2. (i) Since the set \( H_M \) is non-empty, then there exists \( q \in H \) such that \( \mathbb{E}_{t_0} \Pi_U^M(q^*_M) < \mathbb{E}_{t_0} \Pi_H^M(q) \) and \( R_S = 0 \leq \mathbb{E}_{t_0} \Pi_H^S(q) \). Because \( \mathbb{E}_{t_0} \Pi_U^M(q^*_M) \) is the highest payoff the assembler can achieve under the unhedged contract and \( R_M \geq \mathbb{E}_{t_0} \Pi_U^M(q^*_M) \), then the supplier must choose a hedged contract with \( q \in \{ \hat{q}_M, \overline{q} \} \subset H_M \), where \( \mathbb{E}_{t_0} \Pi_H^M(\hat{q}_M) = \mathbb{E}_{t_0} \Pi_U^M(\hat{q}_M) \), which is
condition (16b). (The last claim follows because $E_{t_0} \Pi^M_H(q)$, given by (14), is strictly increasing in $q$.)

(ii) $H_S$ is non-empty implies that there exists some $q \in H$ such that $E_{t_0} \Pi^S_U(q_U) < E_{t_0} \Pi^S_H(q_H)$. Since $q^*_H \in [q, d]$ and $q^*_H = \arg \max_{q \geq d} E_{t_0} \Pi^S_H(q)$, then $E_{t_0} \Pi^S_U(q^*_U) < E_{t_0} \Pi^S_H(q^*_H)$. Now, $R_M = 0$ $\Rightarrow$ $E_{t_0} \Pi^S_U(q^*_U) = E_{t_0} \Pi^S_H(q^*_H)$ and $E_{t_0} \Pi^S_H(q^*_H) = E_{t_0} \Pi^S_H(q^*_H)$. Therefore $E_{t_0} \Pi^S_U(q^*_U) \leq E_{t_0} \Pi^S_H(q^*_H)$, which is condition (16a).