PBM Competition in Pharmaceutical Supply Chain: Formulary Design and Drug Pricing

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In this paper, we model the competition among multiple Pharmacy Benefit Managers (PBMs) for the patronage of a client organization. Each PBM selects a list of prices to be charged to the client organization for each of the branded and generic drugs within a therapeutical class (price decision) and a formulary list that assigns branded drugs to preferred or non-preferred tiers (formulary decision). Drug manufacturers offer rebates to PBMs for drugs on preferred tier of formularies. The individuals participating in the client’s pharmacy benefit plan are the ones consuming the drugs and making purchasing decisions, while the client organization is paying the majority of drug cost. The choices of the individuals and the client organization are governed by different utility measures. For this complex drug distribution setting and for competing PBMs, we show the existence and uniqueness of a pure Nash equilibrium on aggregate price and formulary decisions. Moreover, the formulary list for each PBM is a dominant choice, in the sense that it is optimal irrespective of the choices made by the competing PBMs. We characterize each PBM’s optimal formulary and equilibrium price decisions, and discuss the impact of various model primitives on these decisions. As an application of our model, we use it to gain insights on the impact of mergers in the PBM industry for both PBMs and the client. Finally, we extend our base model to the more general setting of multiple client organizations, each with drugs from multiple therapeutical classes.

Key words: Pharmacy Benefit Manager, drug distribution, tiered-formulary, pricing, competition

History:

1. Introduction

Since the 1970s, Pharmacy Benefit Managers (PBMs) have emerged as third-party administrators of prescription drug programs. As such, PBMs act as intermediaries between upstream pharmaceutical manufacturers and downstream clients (including employers, health care insurers, and federal and state programs such as Medicaid). PBMs are primarily responsible for processing and paying prescription drug claims, developing formularies, contracting with pharmacies, and negotiating discounts and rebates with drug manufacturers. In 2008, PBMs managed more than 70% of the 3 billion prescriptions dispensed that year in the United States. This represents 95% of all prescriptions of all individuals with a managed pharmacy plan, see Navarro (2009, page 95). This large
market is covered by a relatively small number of PBMs, giving rise to an oligopolistic industry: In the 2nd quarter of 2011, the largest four PBMs covered close to 60% of the PBM market, as measured by prescriptions filled. Medco and Express Scripts each covered 17.01% and 15.08% of the market in the 2nd quarter of 2011, see Pharmacy Benefit Management Institute (2014).

PBMs create value in three distinct ways. At the most basic level, they provide an administrative service, processing and paying prescription drug claims. However, their main raison d’être and ever increasing importance in the pharmaceutical market arises from the following two additional services: First, as large purchasing organizations, they are able to negotiate better wholesale prices with drug manufacturers and retail pharmacies than individuals or individual employers are able to obtain. Second, they help design tiered formularies, tailored to the needs of individual clients, i.e., employers, health care insurers, or government programs.

A tiered formulary distinguishes among a given number of tiers, each with a specified copayment level, and assigns individual drugs to each of the tiers. Tiered formularies are used to encourage the choice of less expensive drugs. Two recent studies, see Claxton et al. (2011) and Hoadley et al. (2011), have estimated that, in 2011, 77% of privately insured employees and 91% of Medicare Part D beneficiaries have a plan with three or more tiers, up from a mere 27% in the year 2000. For a three-tier formulary, if a generic drug exists in a given therapeutic class, it represents tier 1 with the lowest copayment; branded drugs are assigned to tier 2 with an intermediate level of copayment or tier 3 with the highest such level. Indeed, copayment levels vary quite significantly. For example, within the therapeutical class of statins, i.e., blood-cholesterol-lowering drugs (with an annual sales volume of $19.6 billion in 2006), copayments for a month’s supply vary from $0 to $105 (across different plans), with an average of $22 for branded drugs and $9 for generics; see Carrera (2010). Moreover, a recent study in the Journal of the AMA, see Goldman et al. (2007), has estimated that a mere 10% increase in the copayment level may result in a decrease in drug spending by anywhere between 2% and 6%, depending on the therapeutic class involved. Recognizing individuals’ price sensitivity with respect to the copayment level, pharmaceutical companies offer PBMs rebates if a particular drug is assigned to tier 2 (preferred branded drugs) as opposed to tier 3 (non-preferred branded drugs). These rebates represent one source of income for the PBMs, part of which is shared with the client organization.

Another source of income for the PBMs are price spreads (markups), i.e., a PBM charges the client organization a higher price for a drug than the wholesale price the PBM negotiated with the drug manufacturers and retail pharmacies. The markup in price varies among different drugs as
well as different PBMs. Price spreads of up to $200 for a single prescription have been reported, see Garis and Clark (2004).

In recent years, the rapid growth of the PBM industry has aroused increasing discussion on the role and effectiveness of PBMs in lowering prescription drug costs for their client organizations. In April 2012, Express Scripts and Medco Health Solutions completed a $29 billion merger, which was widely debated in the media. Express Scripts claimed that the merger would bring in significant cost synergies. Despite the Federal Trade Commission’s decision approving the merger, some (including the National Community Pharmacists Association and the National Association of Chain Drug Stores, etc.) still believe that the merger is anti-competitive. The controversy over the merger may not end any time soon, and the impact of the merger on PBMs and their client organizations is not quite clear. We develop the first analytical model to understand the complex role of PBMs in the financial flow of the pharmaceutical supply chain, and apply our model to investigate the impact of PBM mergers.

More specifically, we model the competition among PBMs on prices and formularies for the patronage of a client organization. Typically, a client reaches out to several PBMs requesting a complete design of a coverage plan for the coming two or three years. This includes a specification, for each therapeutical class of drugs, of the prices charged per prescription as well as which of the drugs are assigned to each of the tiers of the plan. We focus on three-tier plans, the most commonly used structure, as discussed above. In our base model, we confine ourselves to a single client organization and a single therapeutical class of drugs. All of our results can be extended to more general settings of multiple client organizations and multiple therapeutical classes of drugs, see §7 for details.

When designing the prescription plan, a PBM needs to consider the estimated out-of-pocket cost, for the client, as well as the consumer surplus of the individuals participating in the client’s pharmacy benefit plan. Companies are clearly concerned with their total cost exposure to the drugs purchased under their plan. They also care about the well-being of their employees for several reasons. The quality of a company’s benefit program is often a decisive factor in the recruitment and retention process of its employees, and the overall employee health contributes to the company’s overall productivity. To model the choice process of the client organization, we employ a MultiNomial Logit (MNL) model in which the client’s expected utility of contracting with a PBM is a function of the expected consumer surplus of plan enrollees and the client’s expected cost. The MNL model captures the probabilistic choice process of the client organization, due to the
inability of PBMs to observe and measure all the factors that determine the preferences of the client organization.

Both the client’s total expected cost and the employees’ consumer surplus are derived from a second underlying consumer (employee) choice model, pertaining to the insured individuals who select which of the drugs to adopt. While almost all drugs are prescribed by a physician, patients play an important and often decisive role in the selection process. Henry J. Kaiser Family Foundation (2005, page 26) reports that “Historically, patient compliance with whatever treatment the doctor ordered was assumed as part of the physician-patient relationship; increasingly, however, patients are becoming more proactive in their interaction with physicians, particularly in the area of prescription drug treatment decisions. Greater access to health information (fueled, in part, by widespread use of the Internet), the loosening of “direct-to-consumer” (DTC) advertising restrictions on drug manufacturers, and a general increase in the public’s awareness of health care issues have helped transform many once-passive patients into inquiring and demanding consumers”. In our model, the choice of prescription drug is made by a patient in consultation with a physician to maximize the patient’s utility, which involves the tradeoff between cost and quality. To specify the choice process of individual employees, we employ a second MNL model in which an employee’s per prescription expected utility of a drug is a function of its copayment value and its perceived quality. The MNL model captures statistical variations in patient utility for prescription drugs in a diverse population.

PBMs are thus engaged in a complex type of oligopolistic competition, requiring each of them to select a list of gross prices to be charged to the client for each of the branded and generic drugs (the price vector) and a formulary list that assigns drugs to preference tiers. Designing the formulary is equivalent to specifying a vector of binary variables indicating which of the branded drugs are assigned to tier 2 or tier 3 (the formulary decision), with generic drugs always assigned to tier 1. After the choice of a PBM by the client organization, the individual employees participating in the client’s pharmacy benefit plan make the ultimate drug purchasing decisions, and the choices of these individuals are governed by trade-offs different from those of the client organization itself. The sequential choice process has the same structure as the two-level nested MNL model, but differs from the latter in two major ways. First, the sequential choices are made by different entities with different utility measures. Second, the cost sharing structure between the decision makers of the sequential choices differentiates the pharmaceutical supply chain from the traditional supply chain of consumer products. While the plan enrollees make the ultimate drug choices, they only pay a nominal fee (copayment) for their prescription drugs and the majority of the drug cost is
paid by the client organization. The cost-sharing structure dampens patient sensitivity to price, and allows patients to focus more on quality relative to price than they otherwise would.

For this complex drug distribution setting and for multiple competing PBMs, we are able to show the existence and uniqueness of a pure Nash equilibrium on aggregate formulary and price decisions. The PBM’s formulary is a dominant choice, in the sense that it is optimal irrespective of the choices made by the competing PBMs. More specifically, each PBM’s optimal formulary may be computed upfront as the vector of assignment variables which maximizes the client’s utility measure when the PBM applies no markups to its wholesale prices and passes on the full rebates resulting from its formulary decisions. With a PBM’s optimal formulary design determined, the remaining price competition among PBMs further reduces to a classical single-dimension MNL price competition model, which has a unique equilibrium on the aggregate price decision. The single price value selected by each of the PBMs refers to the client’s expected utility under the offered plan. While the aggregate formulary and price decisions are unique, the formulary assignment vector and price vector may not be unique and may be implemented in different ways by each PBM by accounting for other considerations beyond the confines of our model. This result is consistent with the current practice in the PBM industry that PBMs manage the quality and the cost of their plans at the aggregate level of all drugs instead of each individual drug, as confirmed in our interview with a senior executive and healthcare strategist, who has over 20-year experience in healthcare industry and over 5-year experience as Senior Vice President and Chief Supply Chain Officer at a major PBM (Lang 2013).

As an application of our model, for the special case of symmetric PBMs, we study the combined impact of the increased negotiation leverage (“power” effect) and reduced competition intensity (“competitive” effect) of PBM mergers for both PBMs and the client organization. Our analysis implies that there is no win-win situation for all parties affected by PBM mergers. As the power effect increases, the merger becomes progressively favorable to the merged PBM and the client organization, and has an opposite, i.e., less favorable effect, to all non-merging PBMs.

The remainder of the paper is organized as follows: In §2, we review the related literature. In §3, we specify our base model. In §4, we show the existence and uniqueness of equilibrium in the competition among PBMs, and characterize each PBM’s optimal formulary design and equilibrium price decisions. In §5, we discuss the impact of various model primitives. In §6, we study the impact of PBM mergers on PBMs and the client organization. §7 summarizes and extends the base model to the more general setting with multiple client organizations and multiple therapeutical classes of drugs. All proofs are relegated to Appendix A.
2. Related Literature

Recently, the rapid growth of the PBM industry has received increasing attention from both media and academic literature, alike. A lot of industry reports and research articles examine the role and value of PBMs in processing prescription drug claims, pointing out that PBMs help employers better control drug benefit costs by developing formularies and negotiating rebates with drug manufacturers (see Lipton et al. (1999), Olson (2003), Grabowski and Mullins (1997), Abrams (2003), etc.). Among them, Rentmeester and Garis (2008) describe two PBM specific revenue-generating practices – rebates and price spreads – which significantly account for PBMs’ profits but have been neglected in the health policy literature. They raise questions about transparency in contract agreements between PBMs and employers, draw attention to how rebates and price spreads may be considered in ethical terms, and encourage future research to better understand the role of PBMs in the US health care system.

Most of the existing literature on the role of PBMs in drug distribution is either descriptive or empirical. Limited theoretical and modeling work has been conducted on rebate contracts between drug manufacturers and PBMs, and the impact of these contracts on PBMs’ formulary and price decisions. One such work is Gür Ali and Mantrala (2010), which employs a simulation model to investigate rebate contracts between two branded drug manufacturers and a PBM. Another analytical research on drug distribution through PBMs is Cui and Desai (2010), which focuses on the impact of heterogeneity of insurance plan size and PBM’s bargaining power on the cost savings provided by a PBM. Both of the above papers have a different research focus from ours, and neither considers competition among PBMs for a client organization’s business. To the best of our knowledge, our paper is the first to develop a theoretical model that characterizes PBMs’ formulary and pricing decisions in a competitive setting.

Our work is related to the vast literature on multiproduct pricing and revenue management, see Dong et al. (2009), Gallego et al. (2006), etc. Although there are different choices of demand models, the price-dependent MNL and the nested MNL models are most commonly used to model consumer behavior and to optimize prices for multiple products of a firm. The classic work of Anderson and De Palma (1992) shows the existence of a symmetric price equilibrium under the nested MNL model. A recent work of Li and Huh (2011) considers a multiproduct pricing problem with the nested logit model. They prove the concavity of profit functions with respect to the market share vector, and apply this result to compare the optimal monopoly solution to the oligopolistic equilibrium solutions. In the standard nested logit model, a consumer first chooses a group of products and then chooses a product within the group; hence the two levels of choices are made by
the same decision maker. In our model, each PBM faces a nested demand, which has a structure similar to that of the nested logit model, only the two levels of choices are made by different decision makers, who share the cost of prescription drugs and have different utility functions: First, a client organization chooses a plan offered by a PBM; Second, given the choice of the client organization, each insured individual within the organization selects a drug to maximize his/her expected utility according to the copayment scheme specified by the plan, while the client organization pays the majority of the drug cost.

Finally, our work is also related to the existing literature on mergers under price competition. Deneckere and Davidson (1985) study mergers under price competition with linear demand functions in the symmetric case. They show that in the absence of cost efficiencies resulting from a merger, aggregate profits of the merging firms increase as do equilibrium prices. The equilibrium profits of the non-merging firms increase, while the consumer welfare declines. Werden and Froeb (1994) extend Deneckere and Davidson (1985) with MNL demands and linear costs, exploring the impact of mergers on prices and welfare both analytically and numerically. They find that prices of all products increase as a result of a merger, but the magnitudes of the price increases are very different, depending on whether they are offered by the merging firm or the non-merging firm and the firm’s size. They also find that mergers may enhance welfare even though they increase prices. Federgruen and Pierson (2011) study mergers in the context of price competition model with differentiated goods and asymmetric firms allowing for general non-linear demand and cost functions. Their results also confirm that, in the absence of cost synergies, post-merger equilibrium prices increase, and the equilibrium profit of the merged firm exceeds the combined pre-merger equilibrium profits of the merging firms. The equilibrium profits of the non-merging firms increase as well. Among all the above models that consider mergers with price competition, the merger results in the decrease of the number of competing firms in the industry, but the number of products remain the same before and after the merger. Unlike the above models of mergers among manufacturers with price competition, our model considers mergers of PBMs, who are service providers to the client organization. The merger reduces both the number of PBMs (i.e., the number of firms) in the industry as well as the set of drug prices and branded drugs’ formulary offered to the client (i.e., the number of products). We analytically show the impact of the merger upon the market shares and profits of the merging and non-merging PBMs, as well as upon the client organization’s utility and social welfare. Our results show that the post-merger equilibrium profit of the merged PBM is less than the aggregate of the pre-merger equilibrium profits of the merging PBMs unless the merger is associated with sufficient savings due to cost synergies. This contrasts with the results made in
Deneckere and Davidson (1985) and Federgruen and Pierson (2011). Moreover, the post-merger equilibrium market share of the merged PBM and the expected utility of the client organization exceed their counterparts in the pre-merger model only when the merger brings about significant savings.

3. Model

In this section, we model the competition among $M$ PBMs for the patronage of an individual client organization. Each PBM manages $N$ branded drugs and a generic drug within a single therapeutic class and administers them to the individuals who participate in the client’s plan. Our assumptions of a single client and therapeutic class are without loss of generality, and can be easily extended to multiple client organizations and multiple therapeutic classes of drugs, see §7.

PBMs compete by providing the potential client with a set of drug prices and formulary decisions. Since tier 2 (preferred branded) drugs have lower copayments and might be preferred by employees of the client organization who purchase the drugs, formulary decisions play an important role in deciding each drug’s market share. PBMs leverage their power in formulary decisions to negotiate rebates with branded drug manufacturers. There are two basic forms of rebate contracts: flat rebates and performance rebates. Our model focuses on the most common form, flat rebates, where the manufacturer pays the PBMs assigning his drug to tier 2 (preferred tier) a fixed amount of money per unit drug sale. PBMs then pass the client a portion of these rebates, which helps to lower the client’s drug benefit plan cost. PBMs differentiate themselves in terms of the wholesale prices and rebates they negotiate with drug manufacturers and retail pharmacies, and the set of prices and formulary decisions they offer to the client. We need the following notation to denote quality and cost characteristics of different drugs:

$$q_i = \text{quality index of drug } i, \ i = 0, \ldots, N, \ \text{where drug } 0 \text{ denotes the generic drug;}$$
$$w_{ij} = \text{drug } i\text{'s wholesale price negotiated by PBM } j, \ i = 0, \ldots, N, \ j = 1, \ldots, M;$$
$$r_{ij} = \text{rebate (in dollars) per unit drug sale offered by branded drug manufacturer } i \text{ to PBM } j.$$ 

We assume the above parameters are exogenously given for several reasons. First, the quality of a drug is determined by its manufacturer’s R&D capability and cannot be easily changed, and such quality is independent of the PBM choice. Second, PBMs typically negotiate drug wholesale prices and rebates every two to three years, except the special case of patent expiration, while different client organizations typically reach out to multiple PBMs requesting a complete design of a coverage plan for the next few years at different times during the calendar year. Therefore, PBMs
do not negotiate drugs’ wholesale prices and rebates specifically for each client. Instead, when
requested by a client organization, PBMs respond to the request by using the current wholesale
prices and rebates already negotiated with drug manufacturers and retail pharmacies. We allow
drug prices and rebates to vary among PBMs, so as to model the fact that some PBMs may be
able to negotiate more favorable drug prices and rebates than others.

In practice, copayment values of all tiers of the plan are typically pre-specified by the client
organization, reflecting how much the client wants its employees to share the cost of their care.
Therefore, copayment values of each tier of the plan vary from client to client. In our model of
PBM competition, we take the copayment values of all tiers as exogenous parameters pre-specified
by the client organization, and denote them by:

\[ c^g_j = \text{copayment of tier 1 (generic) drug on PBM } j\text{'s formulary}, \quad j = 1, \ldots, M; \]
\[ c^p_j = \text{copayment of tier 2 (preferred branded) drugs on PBM } j\text{'s formulary}, \quad j = 1, \ldots, M; \]
\[ c^n_j = \text{copayment of tier 3 (non-preferred branded) drugs on PBM } j\text{'s formulary}, \quad j = 1, \ldots, M. \]

For generality, we allow the copayment values to be PBM-specific, and all our results hold when
the client organization proposes the same copayment value to all PBMs. In addition, we assume
the rebate pass-through rate from a PBM to the client organization is fixed at:

\[ \rho_j = \text{rebate pass-through rate to the client by PBM } j, \quad j = 1, \ldots, M. \]

This assumption is made without loss of generality, as all of our analysis continues to hold if the
rebate pass-through rate is modeled as a decision variable instead of an exogenous parameter, see
§4 and §5 for details. Finally, in all practical cases, the above model primitives are non-negative,
and \( r_{ij} \leq w_{ij} \) for all \( i = 1, \ldots, N \) and \( j = 1, \ldots, M \).

We introduce the following notation to model the price and formulary decisions of PBMs:

\[ p_{ij} = \text{drug } i\text{'s resale price that PBM } j\text{ charges to the client}, \quad i = 0, \ldots, N, \quad j = 1, \ldots, M; \]
\[ \vec{p}_j = \{p_{0j}, p_{1j}, \ldots, p_{Nj}\}, \text{the price decision vector of PBM } j, \quad j = 1, \ldots, M; \]
\[ y_{ij} \in \{0, 1\} = \text{assignment indicator of branded drug } i\text{ by PBM } j\text{ to tier 2 (preferred brand), with 1}
\text{denoting assignment to tier 2 (preferred brand), and 0 implying otherwise assignment to tier 3}
\text{(non-preferred brand), } i = 1, \ldots, N, \quad j = 1, \ldots, M; \]
\[ \vec{y}_j = \{y_{1j}, \ldots, y_{Nj}\}, \text{the formulary decision vector of PBM } j, \quad j = 1, \ldots, M. \]

Given each PBM’s formulary decisions, the copayment for a branded drug charged to each indi-
vidual participating in its plan can be written as

\[ c_{ij}(y_{ij}) = c^p_j y_{ij} + c^n_j (1 - y_{ij}) = \text{copayment of branded drug } i\text{ on PBM } j\text{'s formulary}, \quad i = 1, \ldots, N, \quad j = 1, \ldots, M. \]
while the copayment for the generic drug is

\[ c_{0j} = c_{g}^j = \text{copayment of the generic drug on PBM } j\text{'s formulary, } j = 1, \ldots, M. \]

Given the client organization’s choice of PBM, we adopt a MultiNominal Logit (MNL) model to specify the choice process of individual enrollees of the drug. While almost all drugs are prescribed by a physician, patients typically play an important and often decisive role in the selection process. The choice of prescription drug is often made by a patient in consultation with a physician. Thus, the patient and the physician act as one agent, or equivalently, the physician is the loyal agent for the patient such that the physician prescribes the drug to maximize the utility for the patient. This assumption is reasonable since physicians typically have no financial interest in the drugs selected. This is a common assumption in the literature on prescription drug choice, see Esposito (1995), Bhatia et al. (2006), Rizzo and Zeckhauser (2009), and Epstein and Ketcham (2010).

Individuals choose drugs to maximize their expected utility, which is a function of the copayment, the quality, and other brand-specific attributes of the selected drug. We denote the brand-specific attributes of drug \( i \) as \( \gamma_i \). If the client organization has contracted with PBM \( j \), the enrollee’s utility of purchasing drug \( i \) is given by

\[ u_{ij} = \gamma_i - \alpha c_{ij} + \beta q_i + e_{ij}, \quad i = 0, \ldots, N, \]

where \( \gamma_i - \alpha c_{ij} + \beta q_i \) represents the attractiveness of drug \( i \), and \( e_{ij} \) is a random variable whose value is influenced by unobservable characteristics. The quality index \( q_i \) refers to the clinical efficacy of the drug, which is usually measured over a large group of patients. For example, the chronic drugs, statins, reduce blood cholesterol levels, and have been found effective in reducing the risk of coronary heart disease and heart attacks. The efficacy of statins is measured as expected percent reduction of lower density lipoprotein (LDL), see Carrera (2010). However, patients often vary in their medical and functional responsiveness to a medication, as captured by the random term \( e_{ij} \). Assume \( e_{ij} \)'s are i.i.d., following a double exponential distribution with mean 0 and variance \( \mu^2 \pi^2 / 6 (\mu > 0) \). For the client organization that has contracted with PBM \( j \), drug \( i \)'s market share among its plan enrollees is specified by the following MNL model:

\[
\begin{align*}
d_{0j}(\mathbf{y}_j) &= \frac{\exp((\gamma_0 - \alpha c_{0j} + \beta q_0)/\mu)}{\exp((\gamma_0 - \alpha c_{0j} + \beta q_0)/\mu) + \sum_{k=1}^{N} \exp((\gamma_k - \alpha c_{kj}(y_{kj}) + \beta q_k)/\mu)} , \\
d_{ij}(\mathbf{y}_j) &= \frac{\exp((\gamma_i - \alpha c_{ij}(y_{ij}) + \beta q_i)/\mu)}{\exp((\gamma_0 - \alpha c_{0j} + \beta q_0)/\mu) + \sum_{k=1}^{N} \exp((\gamma_k - \alpha c_{kj}(y_{kj}) + \beta q_k)/\mu)}, \quad i = 1, \ldots, N. \quad (1)
\end{align*}
\]

Note that the market share of each drug among the plan enrollees is independent of the drug’s resale price that PBM \( j \) charges to the client organization, and dependent on the out-of-pocket
cost (copayment) of the plan enrollees. Therefore, the market share of each drug depends on the formulary decision of the PBM that manages the plan. As we will show, each PBM’s optimal formulary decision depends on the wholesale prices and rebates that the PBM has negotiated with drug manufacturers and retail pharmacies. In other words, drug wholesale prices and rebates implicitly affect the market share of each drug via PBM’s optimal formulary decision.

In the special case where a branded drug has a patent expiration date of several years into the future, the drug will be the only drug (maybe sharing the market with another branded drug) in the market without any generic presence for many years. Since the branded drug has no or little competition in this case, the drug manufacturer may not offer any rebates to PBMs and $r_{ij}$ may be zero. Our model captures this special case by setting $\gamma_0 = -\infty$ and $N = 1$ or 2.

Given the price and formulary decisions of all PBMs, the client organization selects a PBM to maximize its expected utility, which equals to the expected consumer surplus of its employees minus its own expected total cost. The expected consumer surplus of contracting with PBM $j$ is the total expected utility of the plan enrollees under PBM $j$’s plan, in dollar terms, as follows:

$$CS_j(\bar{y}_j) = \frac{1}{\alpha} E \left( \max_i u_{ij} \right) S = \frac{\mu}{\alpha} \left( \ln \sum_{i=0}^{N} \exp \left( \frac{\gamma_i - \alpha c_{ij} + \beta q_i}{\mu} \right) \right) S$$

$$= \frac{\mu}{\alpha} \left( \ln \left( \exp \left( \frac{\gamma_0 - \alpha c_0^j + \beta q_0}{\mu} \right) + \sum_{i=1}^{N} \exp \left( \frac{\gamma_i - \alpha c_i^j + \alpha (c_i^j - c_p^j) y_{ij} + \beta q_i}{\mu} \right) \right) \right) S, \quad (2)$$

where $S$ is the size of the client organization as measured by the number of prescriptions filled within the therapeutical class.

Anticipating the plan enrollees’ choice of prescription drugs, the client organization’s expected total cost of contracting with PBM $j$ is given by:

$$B_j(\bar{y}_j, \bar{p}_j) = \left( d_{0j}(\bar{y}_j)(p_{0j} - c_0^j) + \sum_{i=1}^{N} d_{ij}(\bar{y}_j) (p_{ij} - c_i^j + (c_i^j - c_p^j) y_{ij} - \rho_j r_{ij} y_{ij}) \right) S. \quad (3)$$

For each prescription filled, the client organization is charged the drug’s resale price set by the selected PBM, and this cost is partly covered by the patient’s copayment and by the rebate passed down from the PBM.

Since PBMs cannot observe and measure all the factors that determine the preferences of the client organization, we employ a standard MultiNomial Logit model (MNL) to describe the client organization’s choice process. The client’s utility of contracting with PBM $j$ is:

$$v_j = CS_j - B_j + \epsilon_j, \quad (4)$$

where $\epsilon_j$ represents unobservable characteristics of the client’s preference. The client’s utility of not contracting with any PBM is $v_0 = u_e^0 + \epsilon_0$, where $u_e^0$ denotes its expected value. Assume $\epsilon_j$’s
(j = 0, 1, . . . , M) are i.i.d., following a double exponential distribution with mean 0 and variance $\nu^2 \pi^2/6$ ($\nu > 0$). The probability that the client organization selects PBM $j$ is:

$$n_j(\vec{y}_j, \vec{p}_j) = \frac{\exp((CS_j - B_j) / \nu)}{\exp(u_0 / \nu) + \sum_{k=1}^{M} \exp((CS_k - B_k) / \nu)}, \quad j = 1, \ldots, M. \tag{5}$$

We now formulate the competition of PBMs for winning the client organization’s drug benefit distribution business. We employ a game-theoretical model with complete information. We assume that the client organization knows its enrollees’ price- and quality-sensitivities and brand preferences, and that each PBM knows all the parameters of the decision-making process for the client organization and its enrollees, except for the unobservable characteristics captured by the random terms $\epsilon_j$ and $e_{ij}$. Therefore, the PBMs can only estimate the decision-making of the client organization in probabilistic terms, as in (5).

The objective of each PBM is to maximize its expected profit, which includes both price spreads and retained rebates. Price spread is the difference between a drug’s resale price the PBM charges to the client organization and the wholesale price the PBM pays to the drug manufacturer or retail pharmacy. Rebates are paid by branded drug manufacturers if and only if PBMs put their drugs on the preferred tier 2. After passing a portion of the rebate to the client, PBMs keep the remainder. Each PBM makes formulary decisions for all branded drugs, and price decisions for all branded and generic drugs. We can write PBM $j$’s formulary and pricing problem as

$$\max_{\vec{y}_j \in \{0,1\}^N, \vec{p}_j \in \mathbb{R}^{N+1}} \pi_j = n_j(\vec{y}_j, \vec{p}_j) \left( d_{0j}(\vec{y}_j)(p_{0j} - w_{0j}) + \sum_{i=1}^{N} d_{ij}(\vec{y}_j)(p_{ij} - w_{ij} + (1 - \rho_j)r_{ij}y_{ij}) \right) S. \tag{6}$$

In our formulation, we do not constrain the price vector to be nonnegative, we will show that there always exists a nonnegative price vector that achieves the competitive equilibrium of our problem.

The objective function has the same structure as those of nested MNL models. As in a two-level nested MNL model, relevant choices are part of a sequential process: First, the client organization adopts a plan provided by a PBM; Second, conditional on the choice of this plan, each plan enrollee selects a drug to maximize his/her expected utility according to the copayments specified by the plan. The two choice levels are described by MultiNomial Logit models. Our model differs from other nested MNL models in two major ways: (1) the choices at the two levels are made by different entities with different utility measures; (2) the prescription drug cost is shared between the decision makers at the two levels.
4. Equilibrium Analysis

In this section, we conduct an equilibrium analysis of the oligopolistic competition among PBMs. Each PBM designs the formulary for \( N \) branded drugs and sets prices for all drugs. We will show that each PBM’s formulary and price decision, which consists of \( 2N + 1 \) decision variables, can be projected into the following two aggregate decision variables: For \( j = 1, \ldots, M \), let \( V_j \) denote the client’s expected utility of contracting with PBM \( j \). We denote by \( U_j \) the client’s expected utility of working with PBM \( j \) when the PBM applies no markups to its wholesale prices and passes on the full rebates resulting from its formulary decisions. One can expect the case of no markups and full rebates when the client organization operates its own drug benefit distribution program (i.e., owns or acts as a PBM-like organization), and thus we often refer to \( U_j \) as the client’s expected utility when PBM \( j \) is an “internal” one. The formal definitions of \( V_j \) and \( U_j \) are:

\[
V_j(\vec{y}_j, \vec{p}_j) = CS_j(\vec{y}_j) - B_j(\vec{y}_j, \vec{p}_j),
\]

and

\[
U_j(\vec{y}_j) = CS_j(\vec{y}_j) - W_j(\vec{y}_j),
\]

where

\[
W_j(\vec{y}_j) = \left( d_{0j}(\vec{y}_j)(w_{0j} - c_j^2) + \sum_{i=1}^{N} d_{ij}(\vec{y}_j) (w_{ij} - c_{ij}(y_{ij}) - r_{ij}y_{ij}) \right) S,
\]

denotes the client’s expected total cost of working with the internal PBM \( j \). Since the formulary decision is a vector of binary variables, we define the feasible set of \( U_j(\vec{y}_j) \) as

\[
\Omega_j = \{ CS_j(\vec{y}_j) - W_j(\vec{y}_j) | \vec{y}_j \in \{0,1\}^N \}.
\]

The following theorem reformulates each PBM’s formulary design and pricing problem as a dual-decision problem, develops the structural property of the latter, and characterizes the existence and uniqueness of the equilibrium of PBMs’ competition.

**Theorem 1 (Existence and Uniqueness of Equilibrium).** (a) For PBM \( j = 1, \ldots, M \), its formulary design and pricing problem can be reformulated as the following dual-decision problem:

\[
\max_{U_j \in \Omega_j, V_j \in \mathbb{R}} \pi_j = \frac{\exp(V_j/\nu)}{\exp(w_0^0/\nu) + \sum_{k=1}^{M} \exp(V_k/\nu)}(U_j - V_j).
\]

(b) For PBM \( j = 1, \ldots, M \), its profit function, \( \pi_j \), is linearly increasing in \( U_j \) for any given \( V_j \), and is strictly log-concave in \( V_j \) for any given \( U_j \). Furthermore, given any original formulary decision vector, \( \vec{y}_j \) (and hence \( U_j \)), any \( V_j \in \mathbb{R} \) can be obtained by varying the original price decision vector \( \vec{p}_j \). Conversely, given any \( V_j \in \mathbb{R} \), any \( U_j \in \Omega_j \) can be obtained by varying the original price decision vector \( \vec{p}_j \) and the original formulary decision vector \( \vec{y}_j \), while keeping \( V_j \) fixed.

(c) The PBMs’ competition has a unique Nash equilibrium \{ \((U_1^*, V_1^*), (U_2^*, V_2^*), \ldots, (U_M^*, V_M^*)\) \}. 

Theorem 1 shows that for PBM \( j = 1, \ldots, M \), its formulary and pricing problem can be solved in two sequential steps: First, its optimal formulary decision vector, \( \vec{y}_j^* \), can be obtained as the binary vector that maximizes \( U_j(\vec{y}_j) \), the client’s expected utility of working with an internal PBM \( j \). The optimal formulary decision \( \vec{y}_j^* \) may not be unique since multiple binary vectors may result in the same value of the optimal \( U_j^* \). In this case, PBM \( j \) is indifferent with such different formulary designs. Since \( U_j(\vec{y}_j) \) is independent of all other PBMs’ decisions, the optimal formulary decision is a dominant choice. Second, with each PBM’s optimal and dominant formulary decision determined, the competition among PBMs can be reduced to a classical single-dimension MNL price competition model. The single (aggregate) price value selected by PBM \( j \) in this reduced price competition model, \( V_j(\vec{y}_j^*, \vec{p}_j) \), refers to the client’s expected utility under the plan offered by PBM \( j \). Since the formulary and price decisions of PBM \( j \) can be solved in two sequential steps by first maximizing \( U_j(\vec{y}_j) \) and then analyzing the equilibrium \( V_j(\vec{y}_j^*, \vec{p}_j) \), we refer to \( U_j(\vec{y}_j) \) as the aggregate formulary decision variable and \( V_j(\vec{y}_j^*, \vec{p}_j) \) as the aggregate price decision variable.

We now characterize PBMs’ optimal formulary and equilibrium price decisions in Proposition 1, Proposition 2 and Theorem 2 below. As discussed, PBM \( j \)’s optimal and dominant formulary decision can be obtained as the binary vector that maximizes \( U_j(\vec{y}_j) \). Maximizing \( U_j(\vec{y}_j) \) is a complex combinatorial optimization problem, with a feasible set of \( 2^N \) vector choices. Proposition 1 establishes a sufficient condition of assigning a drug to the preferred tier. First, we introduce the following additional notation: For \( j = 1, \ldots, M \), \( i = 1, \ldots, N \), let

\[
\begin{align*}
x_{ij} &= \exp \left( \left( \gamma_0 - \alpha c_i^p + \beta q_i \right) / \mu \right), \\
x_{ij}(y_{ij}) &= \exp \left( \left( \gamma_0 - \alpha c_i^n + \alpha(c_i^p - c_i^p)y_{ij} + \beta q_i \right) / \mu \right), \\
m_{ij}(y_{ij}) &= w_{ij} - c_i^n, \\
m_{ij}(y_{ij}) &= w_{ij} - c_i^n + (c_i^n - c_i^p - r_{ij})y_{ij}.
\end{align*}
\]

(12)

For \( i = 0, \ldots, N \), \( x_{ij} \) measures the attractiveness of drug \( i \) to enrollees of PBM \( j \)’s plan, and is proportional to drug \( i \)’s market share under PBM \( j \)’s plan, i.e., \( d_{ij}(\vec{y}_j) = \frac{x_{ij}}{\Sigma_{k=0}^{N} x_{kj}} \). For \( i = 0, \ldots, N \), \( m_{ij} \) is the client organization’s per prescription cost of drug \( i \) on an internal PBM \( j \)’s plan. Define the following cost change rate index of branded drug \( i \):

\[
\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1)-x_{ij}(0)m_{ij}(0)}{x_{ij}(1)-x_{ij}(0)}, \quad i = 1, \ldots, N,
\]

(13)

which measures the change rate of the client’s cost by moving branded drug \( i \) from the non-preferred tier (tier 3) to the preferred tier (tier 2). Without loss of generality, PBM \( j \) ranks the set of branded drugs \( \{1, \ldots, N\} \) in ascending order of their cost change rate indices, i.e., \( \delta_{ij} \leq \delta_{2j} \leq \ldots \leq \delta_{Nj} \). We assume PBMs put a branded drug on tier 2 (preferred tier) if they are indifferent about putting the drug on tier 2 or tier 3. The following proposition characterizes properties of PBMs’ optimal formulary decision.
**Proposition 1 (Sufficient Condition of PBMs’ Optimal Formulary Decision).** Define the threshold index

\[ I_j \equiv \begin{cases} \max\{i : \delta_{ij} \leq \frac{x_{0j}m_{0j} + \sum_{k=1}^{i-1} x_{kj}(1)m_{kj}(1) + \sum_{k=1}^{N} x_{kj}(0)m_{kj}(0)}{x_{0j} + \sum_{k=1}^{i-1} x_{kj}(1) + \sum_{k=1}^{N} x_{kj}(0)}\}, & \text{when such } i \text{ exists;} \\ 0, & \text{otherwise.} \end{cases} \]

If \( i \leq I_j \), PBM \( j \) assigns drug \( i \) on tier 2 (i.e., \( y_{ij}^* = 1 \)).

The index of branded drug \( i \) on PBM \( j \)’s plan can be computed upfront, as a function of the drug’s wholesale price and rebate, as well as the copayment values of PBM \( j \)’s plan. Since different PBMs often negotiate different wholesale prices and rebates with the drug manufacturers and retail pharmacies, the index values of the drugs may vary by PBM, as does their relative ranking. Therefore, the optimal formulary design differs by PBM.

In the special case when all \( N \) branded drugs on a PBM’s plan have the same brand-specific attribute and quality, the following proposition explicitly characterizes PBMs’ optimal formulary decision, and enables each PBM to solve the complex combinatorial optimization problem of formulary assignment in \( O(N \log N) \) elementary operations and evaluations of exponential functions.

**Proposition 2 (Optimal Formulary Design with Equal Brand-specific Attribute and Quality).** Consider the special case where all \( N \) branded drugs on PBM \( j \)’s plan have the same brand-specific attribute \( \gamma_i \) and quality \( q_i \). The optimal set of branded drugs on tier 2 of PBM \( j \)’s plan is a subset of adjacent indices in the above mentioned ranked set of indices. The optimal formulary decision is given by the following threshold policy:

\[ y_{ij}^* \equiv \begin{cases} 1, & \text{for } i = 1, \ldots, I_j^*; \\ 0, & \text{for } i = I_j^* + 1, \ldots, N, \end{cases} \]

where

\[ I_j^* = \arg \max_{k \in \{I_j, \ldots, N\}} \frac{\mu \ln \left( x_{0j} + \sum_{i=1}^{k} x_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0) \right)}{x_{0j} + \sum_{i=1}^{k} x_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0)} - \frac{x_{0j}m_{0j} + \sum_{i=1}^{k} x_{ij}(1)m_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0)m_{ij}(0)}{x_{0j} + \sum_{i=1}^{k} x_{ij}(1) + \sum_{i=k+1}^{N} x_{ij}(0)} \]

Proposition 2 shows that, when all branded drugs have the same brand-specific attribute and quality, PBMs prioritize drugs that are most cost-effective to put on the preferred tier. This is consistent with current practice in the PBM industry, as mentioned in Navarro (2009, page 40) that “in the absence of statistically and clinically significant differentiation among similar drugs, the net cost (which may be reduced by a rebate) can have an important impact on ultimate formulary positioning.”

After the optimal formulary decision vector of PBM \( j \), \( \vec{y}_j^* \), and the optimal value of the aggregate formulary decision variable \( U_j^* = U_j(\vec{y}_j^*) \) are determined, the remaining price competition among
the PBM’s reduces to a classical single-dimension MNL price competition model. As in the classical single-dimension MNL price competition model, there exists a unique equilibrium, \( \{ V_1^*, \ldots, V_M^* \} \), which is the solution of the following system of equations (with details on how these equations emerge in the proof of Theorem 1, see Appendix A):

\[
\frac{\exp \left( \frac{V_j^*}{\nu} \right)}{\exp \left( \frac{U_0^e}{\nu} \right) + \sum_{k=1}^{M} \exp \left( \frac{V_k^*}{\nu} \right)} = 1 - \frac{\nu}{U_j^* - V_j^*}, \quad j = 1, \ldots, M.
\]

(14)

The equilibrium can be easily computed with a standard Round Robin tatonnement scheme, starting from an arbitrary set of expected utility values \( \{ V_1, \ldots, V_M \} \), see Topkis (1998, §4.3.1).

Moreover, PBMs’ equilibrium aggregate price decision can be characterized by the approach in Li and Huh (2011). First, let \( H \) be a mapping from \((0, \infty)\) to \((0, 1)\) such that, for any \( x \in (0, \infty) \), \( H(x) \) is the unique solution \( h \in (0, 1) \) satisfying

\[
h \cdot \exp \left( \frac{h}{1 - h} \right) = x.
\]

(15)

It can be verified that \( H(x) \) is a strictly increasing function. The following theorem gives a closed-form expression of PBMs’ aggregate price equilibrium in terms of the \( H \) function.

**Theorem 2 (PBMs’ Equilibrium Price Decision).** Given each PBM’s optimal formulary decision already determined, in the reduced price competition among PBMs, the equilibrium probability of the client not contracting with any PBM, \( n_0^* \), is the unique solution to the following single-variable equation:

\[
n_0 + \sum_{j=1}^{M} H \left( n_0 \cdot \exp \left( \frac{U_j^* - U_0^e - \nu}{\nu} \right) \right) = 1.
\]

(16)

The equilibrium probability of the client contracting with PBM \( j \), i.e., the equilibrium expected market share of PBM \( j \), is given by:

\[
n_j^* = H \left( n_0^* \cdot \exp \left( \frac{U_j^* - U_0^e - \nu}{\nu} \right) \right), \quad j = 1, \ldots, M.
\]

(17)

The equilibrium expected profit of PBM \( j \) is given by:

\[
\pi_j^* = \frac{\nu n_j^*}{1 - n_j^*}, \quad j = 1, \ldots, M.
\]

(18)

The client organization’s expected utility of contracting with PBM \( j \), i.e., the equilibrium aggregate price decision of PBM \( j \), is given by:

\[
V_j^* = U_0^e + \nu \left( \ln n_j^* - \ln n_0^* \right), \quad j = 1, \ldots, M.
\]

(19)

The client organization’s overall expected utility is given by:

\[
\overline{v}^* = \nu \ln \left( \exp \left( \frac{U_0^e}{\nu} \right) + \sum_{j=1}^{M} \exp \left( \frac{V_j^*}{\nu} \right) \right) = U_0^e - \nu \ln n_0^*.
\]

(20)
Theorem 2 characterizes the price equilibrium in terms of the aggregate price decision variable only. Although the reduced price competition has a unique equilibrium in terms of the aggregate price decision variable, the corresponding equilibrium price vector of each PBM has infinitely many choices. As specified in (7), the equilibrium aggregate price decision variable $V_j^*$ is a linear combination of the original price vector $\vec{p}_j$. With the optimal formulary decision vector $\vec{y}_j^*$ determined, and the equilibrium aggregate price decision variable $V_j^*$ determined as in Theorem 2, PBM $j$ may select a vector of drug prices from the hyperplane of such price vector choices, defined by (7) and achieving the value $V_j^*$. Each PBM has ample opportunities to specify various pricing schemes by choosing prices on this hyperplane, and their final choices may reflect considerations beyond those captured in our model. This result is consistent with the current practice in the PBM industry that pricing is managed and competed at the aggregate level. The target aggregate price is achieved by pricing individual drugs accordingly.

One plausible pricing scheme for a PBM is the “unified markup ratio pricing scheme”, where the PBM applies an identical markup ratio to all branded and generic drugs. Under this pricing scheme, PBM $j$ sets the price vector $\vec{p}_j^*$ with the additional constraints that

$$
\frac{p_{0j}^* - w_{0j}}{w_{0j}} = \frac{p_{ij}^* - w_{ij} + (1 - \rho_j)r_{ij}y_{ij}^*}{w_{ij}}, \quad i = 1, \ldots, N.
$$

In this case, the equilibrium price vector $\vec{p}_j^*$ can be uniquely determined as follows:

$$
p_{0j}^* = w_{0j} \left( 1 + \frac{U_j^* - V_j^*}{S \sum_{l=0}^{N} w_{lj} d_{lij}(\vec{y}_j^*)} \right), \quad p_{ij}^* = w_{ij} \left( 1 + \frac{U_j^* - V_j^*}{S \sum_{l=0}^{N} w_{lj} d_{lij}(\vec{y}_j^*)} \right) - (1 - \rho_j)r_{ij}y_{ij}^*, \quad i = 1, \ldots, N.
$$

(21)

It is easily verified from (21) that all drug prices are nonnegative under this pricing scheme.

5. Comparative Statics

In this section, we discuss how various primitives of the model impact the optimal aggregate formulary decision variable, the equilibrium aggregate price decision variable, expected market share and expected profit of a PBM, as well as the client organization’s overall expected utility.

Recall from Theorem 1 that the formulary and price decisions of PBM $j$ can be solved in two sequential steps by first determining its optimal formulary decision vector $\vec{y}_j^*$ that maximizes its aggregate formulary decision variable $U_j(\vec{y}_j)$, and then analyzing the equilibrium of the reduced price competition in terms of the aggregate price decision variable, $V_j(\vec{y}_j^*, \vec{p}_j^*)$. Moreover, the equilibrium analysis of the reduced price competition in the second step depends on the optimal formulary decision in the first step only via the optimal aggregate formulary decision variable. Therefore, the
comparative statics results on all the quantities of interest mentioned above with respect to various model primitives depend on the impact of the optimal aggregate formulary decision variable upon these terms, as given in the following lemma:

**Lemma 1 (Impact of Optimal Aggregate Formulary Decision).** Consider PBM $j = 1, \ldots, M$.

(a) The equilibrium probability of the client not contracting with any PBM, $n^*_0$, is strictly decreasing in PBM $j$’s optimal aggregate formulary decision $U^*_j$.

(b) PBM $j$’s equilibrium expected market share and expected profit, $n^*_j$ and $\pi^*_j$, are strictly increasing in its own optimal aggregate formulary decision $U^*_j$, and strictly decreasing in any competing PBM’s optimal aggregate formulary decision $U^*_k$ ($k = 1, \ldots, M$ and $k \neq j$).

(c) PBM $j$’s equilibrium aggregate price decision $V^*_j$ is strictly increasing in its own optimal aggregate formulary decision $U^*_j$, as well as in any competing PBM’s optimal aggregate formulary decision $U^*_k$ ($k = 1, \ldots, M$ and $k \neq j$).

(d) The client organization’s overall expected utility $\overline{\tau}^*$ is strictly increasing in any PBM’s optimal aggregate formulary decision.

Utilizing Lemma 1, we are ready to analyze the impact of various model primitives. Proposition 3 below shows the impact of wholesale prices.

**Proposition 3 (Impact of Wholesale Price).** Consider PBM $j = 1, \ldots, M$.

(a) PBM $j$’s optimal formulary decision for branded drug $i$, $y^*_ij$, is weakly decreasing in drug $i$’s wholesale price on PBM $j$’s plan, non-monotone in any other (branded or generic) drug’s wholesale price on PBM $j$’s plan, and independent of any drug’s wholesale price on any competing PBM’s plan.

(b) PBM $j$’s optimal aggregate formulary decision variable, $U^*_j$, is strictly decreasing in any (branded or generic) drug’s wholesale price charged to PBM $j$, and independent of any drug’s wholesale price charged to any competing PBM.

(c) PBM $j$’s equilibrium aggregate price decision variable $V^*_j$ and the client organization’s overall expected utility $\overline{\tau}^*$ are strictly decreasing in the wholesale price charged on any (branded or generic) drug to any PBM.

(d) PBM $j$’s equilibrium expected market share and equilibrium expected profit, $n^*_j$ and $\pi^*_j$, are strictly decreasing [increasing] in any drug’s wholesale price charged to PBM $j$ [any competing PBM].

Proposition 3 shows that an increase in any drug’s wholesale price on a PBM’s plan may result in the PBM reassigning the drug from tier 2 to tier 3, but a change in the opposite direction cannot
occur. However, an increase in any competing drug’s wholesale price on the PBM’s plan may result in the PBM reassigning the focal drug to a different tier, either from tier 2 to tier 3, or from tier 3 to tier 2. Moreover, an increase in any drug’s wholesale price on a PBM’s plan strictly increases any competing PBM’s equilibrium expected market share and expected profit, while strictly decreasing its own equilibrium expected market share and expected profit, as well as the client organization’s expected utility of working with any PBM. Therefore, the client organization’s overall expected utility strictly decreases in any drug’s wholesale price on any PBM’s plan.

Since an increased rebate and a decreased wholesale price of a drug both result in a reduced per prescription cost of the drug to the client organization, all comparative statics results identified in Proposition 3 with respect to wholesale prices apply to the impact of rebates with reverse (weak) monotonicity properties. The weak monotonicity properties of the rebate come from the fact that a drug’s rebate affects the client organization’s per prescription cost of the drug only when the drug is on the preferred tier.

Although the PBM’s optimal formulary decision, each PBM’s equilibrium aggregate price decision variable, expected market share and expected profit, as well as the client organization’s overall expected utility, depend on a drug’s rebate offered to the PBM, these quantities do not depend on the rebate pass-through rate, because the optimal formulary decision vector of the PBM is the binary assignment vector that maximizes the client organization’s expected utility of working with the PBM as an internal one (i.e., the PBM applies no markups to its wholesale prices and passes on the full rebates to the client.)

Unlike the impact of a drug’s wholesale price and rebate, the impact of a drug’s quality is much more involved. An increase in a branded drug’s quality has multiple effects on the market shares of all generic and branded drugs, and on the consumer surplus of the employees of the client organization. The combined effect may result in the non-monotone impact of a drug’s quality upon these quantities. The following proposition characterizes a sufficient condition when the impact of drug quality is monotone.

**Proposition 4 (Impact of Drug Quality).** (a) For each PBM $j$, there exist two threshold values of drug $i$’s quality, $\underline{q}_{ij}$ and $\overline{q}_{ij}$ ($0 \leq \underline{q}_{ij} \leq \overline{q}_{ij}$), such that PBM $j$’s optimal aggregate formulary decision variable $U^*_j$ strictly decreases in branded drug $i$’s quality index $q_i$ if $0 \leq q_i < \underline{q}_{ij}$, and strictly increases in $q_i$ if $q_i > \overline{q}_{ij}$. In particular, both thresholds weakly increase in $w_{ij}$ and weakly decrease in $r_{ij}$.

(b) Consider the case when all PBMs are symmetric, i.e., with identical wholesale price and rebate for each drug, and identical copayment for each formulary tier. The impact of a drug’s
quality on any PBM’s equilibrium aggregate price decision variable, equilibrium expected market share, equilibrium expected profit, and the client organization’s overall expected utility has the same directional change as the impact of the drug’s quality on any PBM’s optimal aggregate formulary decision.

Proposition 4 shows that the impact of a drug’s quality results from the tradeoff between the consumer surplus and cost: An increase in a drug’s quality alone, without any change in the formulary decision, increases the drug’s market share and decreases any competing drug’s market share. Depending on the drug’s relative cost (wholesale price and rebate), such change in market share may increase the cost of the PBM’s plan. On the other hand, without any change in the formulary decision, an increase in the drug’s quality always increases the consumer surplus of the enrollees. When a drug’s quality is sufficiently high [low] (higher [lower] than the high [low] threshold), the increase in the consumer surplus dominates [is dominated by] the possible increase in the cost, so an increase in the drug’s quality increases [decreases] the client organization’s expected utility of working with an internal PBM $j$. Note that when the lower threshold $q_{ij} = 0$, the range $[0, q_{ij})$ is an empty set. Moreover, both thresholds are cost-dependent, as they weakly increase in the drug’s wholesale price charged to the PBM, and weakly decrease in the drug’s rebate offered to the PBM. In the special case when all PBMs are symmetric, a change in a drug’s quality impacts all quantities of interest in the same direction.

When all drugs have the same quality index, the following proposition characterizes the impact of a uniform increase in the drug quality. Note that this proposition does not require symmetric PBMs.

**Proposition 5 (Impact of Drug Quality When Quality is Identical).** Consider the special case where all drugs (branded and generic) have the same quality index, i.e., $q_i = q$ for $i = 0, \ldots, N$. For each PBM $j = 1, \ldots, M$,

(a) PBM $j$’s optimal formulary decision, $y^*_j$, is independent of the drug quality index, $q$.

(b) An increase in the drug quality index $q$ increases PBM $j$’s optimal aggregate formulary decision variable $U^*_j$, equilibrium aggregate price decision variable $V^*_j$, equilibrium expected market share $n^*_j$ and equilibrium expected profit $\pi^*_j$, as well as the client organization’s overall expected utility $\nu^*$.

Proposition 5 shows that, when there is no quality gap among the drugs, drug quality does not affect each PBM’s optimal formulary assignment, which is determined based on cost parameters
and brand preferences. In this special case, a uniform increase in the drug quality improves all quantities of interest.

Except the special cases discussed in Propositions 4 and 5, the impact of drug quality is, in general, quite involved. To maximize the aggregate formulary decision variable, the PBM may respond to the increased quality of a drug by reassigning the drug and/or other branded drugs on its plan to a different tier. The reassignment can go in both directions, either from tier 2 to tier 3, or from tier 3 to tier 2. The following example shows that an increase in a branded drug’s quality has a non-monotone effect on the optimal formulary assignment of the drug as well as other drugs.

Consider the optimal formulary decision of a PBM $j$, which manages two branded drugs and a generic drug within a therapeutical class. We set the default parameter values as follows: $\mu = 0.1$, $S = 100$, $\alpha = 0.5$, $\beta = 1$, $\gamma_i = 0$ ($i = 0, 1, 2$), $c_i^2 = 5$, $c_i^3 = 10$, $c_i^4 = 15$, $q_0 = 8$, $q_1 = 20$, $q_2 = 12$, $w_0j = 8$, $w_2j = 20$, $r_1j = 4$, $r_2j = 6$. Figure 1 plots the optimal formulary assignment of the two branded drugs and the optimal aggregate formulary decision variable as the quality of branded drug 1 increases from 0 to 20 for two different wholesale prices. Figure 1(a) shows that the optimal formulary assignment of branded drug 1 is non-monotone in its quality. Under a different wholesale price for branded drug 1, Figure 1(b) shows that the optimal formulary assignment of branded drug 2 is not monotone in branded drug 1’s quality either. Since an increase in a branded drug’s quality has non-monotone effect on the PBM’s optimal formulary decision vector, its effect on the PBM’s optimal aggregate formulary decision variable is non-monotone as well. For the two cases above, Figure 1(c) and 1(d) plot the PBM’s optimal aggregate formulary decision variable as the quality of branded drug 1 increases. Figure 1(c) and 1(d) confirm the finding in Proposition 4(a) that when the drug quality is sufficiently high, the PBM’s optimal aggregate formulary decision variable strictly increases in the drug quality. Note that $q_{ij} = 0$ in both cases.

Similarly, a change in the copayment of a formulary tier on a PBM’s plan affects the market share of all drugs on the plan, the client organization’s per prescription cost of all the drugs on the formulary tier, and employees’ consumer surplus. We have observed non-monotone impact of the copayment values of a formulary tier on the PBM’s optimal formulary decision vector and optimal aggregate formulary decision variable in numerical study.

Our numerical study confirms that the impact of drug quality and copayment is quite involved, and has non-monotone effects on the optimal formulary assignment of all drugs and the optimal aggregate formulary decision variable. Consequently, the equilibrium aggregate price decision variable, expected market share, expected profit of the PBM, and the client organization’s overall expected utility are non-monotone in the drug quality or the copayment of a formulary tier.
6. Impact of PBM Mergers

In view of recent mergers and acquisitions in the PBM industry, we use our model to better understand the impact of a merger of two PBMs in our stylized competitive setting. As discussed, PBMs operate in an oligopolistic industry with high entry barriers. In the past ten years, many mergers and acquisitions took place in the industry, creating a highly concentrated market. For example, Caremark acquired AdvancePCS in 2003, Express Scripts acquired Medco in April 2012, and SXC acquired Catalyst in July 2012.

We study the impact of a merger on the merged PBM, non-merging PBMs and the client organization. For the tractability of analysis, we assume all $M$ ($M \geq 3$) PBMs before the merger are symmetric, i.e., with identical wholesale price and rebate for each drug, and identical copayment for each formulary tier. Without loss of generality, we index the two merging PBMs by $j = 1, 2$ in the pre-merger model. After the merger, we index the merged PBM by $j = 1$. Non-merging PBMs are indexed by $j = 3, \ldots, M$ in both pre-merger and post-merger models, and $j = 2$ is not used in
the post-merger model. We use superscript “m” to represent the post-merger model.

The PBM merger has two first order effects: increased negotiating leverage for the merged PBM and decreased competitive intensity in the industry. The common argument in support of mergers and acquisitions in the PBM industry is the increased bargaining power of the merged PBM to negotiate lower wholesale prices and/or higher rebates from branded drug manufacturers and retail pharmacies. Therefore, for all \( i = 1, \ldots, N \), we assume \( w_{i1}^m = w_{i1} \) and \( r_{i1}^m = r_{i1} \) for the merged PBM, and \( w_{ij}^m = w_{ij} \) and \( r_{ij}^m = r_{ij} \) for each non-merging PBM \( j = 3, \ldots, M \). We define a negotiating leverage (or “power”) index, \( \Delta U = U_1^{m*} - U_1^* \), as a measure of the increased bargaining power effect for the merged PBM. It follows from Proposition 3 and the discussion after Proposition 3 that \( U_{ij}^m = U_j^* \) for all \( j = 3, \ldots, M \), and \( U_1^{m*} \geq U_1^* \), i.e., \( \Delta U \geq 0 \). On the other hand, the merger reduces the competition intensity among PBMs, as the total number of competing PBMs in the market decreases from \( M \) to \( M - 1 \). Define the equilibrium profit and market share of the PBM industry before the merger as:

\[
\pi_i^* \equiv \sum_{j=1}^{M} \pi_j^* \quad \text{and} \quad n_i^* \equiv \sum_{j=1}^{M} n_j = 1 - n_0^*.
\]  

Similarly, define the equilibrium profit and market share of the PBM industry after the merger as:

\[
\pi_i^{m*} \equiv \pi_1^{m*} + \sum_{j=3}^{M} \pi_j^{m*} \quad \text{and} \quad n_i^{m*} \equiv n_1^{m*} + \sum_{j=3}^{M} n_j = 1 - n_0^{m*}.
\]

The following proposition characterizes the impact of mergers on PBMs and the client organization.

**Proposition 6 (Impact of PBM Mergers on PBMs and the Client Organization).** Consider \( M \) symmetric PBMs. After the merger of PBM 1 and PBM 2, there exist three threshold values of the power index, \( \Delta U, \Delta U, \) and \( \Delta U \) (0 \( \leq \Delta U < \Delta \)), such that:

(I) If \( 0 \leq \Delta U < \Delta \).

(i) For the merged PBM, \( \pi_1^* < \pi_1^{m*} \leq \pi_1^{*} + \pi_2^{*} \) and \( n_1^* < n_1^{m*} \leq n_1^{m*} + n_2^{m*} \).

(ii) For the non-merging PBM \( j \) (\( j = 3, \ldots, M \)), \( \pi_j^* \leq \pi_j^{m*} \leq \pi_j^{m*} \) and \( n_j^* \leq n_j^{m*} \leq n_j^{m*} \).

(iii) For the PBM industry, \( \pi_1^{m*} \leq \pi_1^* \) and \( n_1^{m*} \leq n_1^* \).

(iv) For the client organization, \( \pi^{m*} \leq \pi^* \).

(II) If \( \Delta U < \Delta U \).

(i) For the merged PBM, \( \pi_1^* < \pi_1^{m*} \leq \pi_1^{*} + \pi_2^{*} \) and \( n_1^* < n_1^{m*} \leq n_1^{m*} + n_2^{m*} \).

(ii) For the non-merging PBM \( j \) (\( j = 3, \ldots, M \)), \( \pi_j^* \leq \pi_j^{m*} \leq \pi_j^{m*} \) and \( n_j^* \leq n_j^{m*} \leq n_j^{m*} \).

(iii) For the PBM industry, \( \pi_1^{m*} \geq \pi_1^* \) and \( n_1^{m*} \leq n_1^* \).

(iv) For the client organization, \( \pi^{m*} \leq \pi^* \).
(III) If $\Delta U \leq \Delta U \leq \Delta \overline{U}$,

(i) For the merged PBM, $\pi_1^{m*} \geq \pi_1^* + \pi_2^*$ and $n_1^* < n_1^{m*} \leq n_1^* + n_2^*$.

(ii) For the non-merging PBM $j$ ($j = 3, \ldots, M$), $\pi_j^* \leq \pi_j^{m*} \leq \pi_1^{m*}$ and $n_j^* \leq n_j^{m*} \leq n_1^{m*}$.

(iii) For the PBM industry, $\pi_1^{m*} \geq \pi_1^*$ and $n_1^{m*} \leq n_1^*$.

(iv) For the client organization, $\pi_1^{m*} \leq \pi_1^*$.

(IV) If $\Delta U \geq \Delta \overline{U}$,

(i) For the merged PBM, $\pi_1^{m*} \geq \pi_1^* + \pi_2^*$ and $n_1^{m*} \geq n_1^* + n_2^*$.

(ii) For the non-merging PBM $j$ ($j = 3, \ldots, M$), $\pi_j^{m*} \leq \pi_j^* < \pi_1^{m*}$ and $n_j^{m*} \leq n_j^* < n_1^{m*}$.

(iii) For the PBM industry, $\pi_1^{m*} \geq \pi_1^*$ and $n_1^{m*} \geq n_1^*$.

(iv) For the client organization, $\pi_1^{m*} \geq \pi_1^*$.

Proposition 6 shows the combined impact of the increased negotiating leverage and decreased competition effects of the merger. The equilibrium expected profit [market share] of the merged PBM always exceeds the pre-merger individual PBM’s equilibrium expected profit [market share]. However, if the merged PBM cannot obtain sufficient savings from lowered wholesale prices and/or additional rebates to guarantee a power index higher than the middle threshold ($\Delta \overline{U}$), the post-merger equilibrium expected profit [market share] of the merged PBM is less than the combined pre-merger equilibrium expected profits [market shares] of the two merging PBMs, and the post-merger overall expected utility of the client organization is also less than its pre-merger counterpart. At the same time, all non-merging PBMs benefit from the merger, in terms of both their equilibrium expected profits and expected market shares. On the other hand, if the power index is greater than the highest threshold ($\Delta \overline{U}$), the equilibrium expected profit [market share] of the merged PBM is greater than the combined pre-merger equilibrium expected profits [market shares] of the two merging PBMs, and the post-merger overall expected utility of the client organization is also greater than its pre-merger counterpart. At the same time, each non-merging PBM has a lower equilibrium expected profit [market share]. When the power index falls in between these two threshold levels ($\Delta \underline{U}$ and $\Delta \overline{U}$), the merged PBM has an equilibrium expected profit greater than the combined pre-merger equilibrium expected profits of the two merging PBMs, and an equilibrium expected market share less than the combined pre-merger equilibrium expected market shares of the two merging PBMs. In this case, each non-merging PBM benefits from the merger, in terms of both its equilibrium expected profit and expected market share, while the client organization incurs a loss in its expected utility. Moreover, for the PBM industry, the combined equilibrium profit [market share] is larger after the merger if and only if the power index is greater than the lowest [highest] threshold, $\Delta \underline{U}$ [\Delta \overline{U}]$. To sum up, the client organization’s expected utility and the total market
share of all PBMs move in the same direction after the merger, while the equilibrium expected profit and expected market share of each non-merging PBM move in the other direction. Therefore, there is no win-win situation for all parties involved in the merger.

In all four cases of Proposition 6, the merged PBM always has a higher profit and a larger market share than its non-merging competitors, because of its increased bargaining power with drug manufacturers and retail pharmacies after the merger. However, the equilibrium profit of the merged PBM exceeds the combined pre-merger equilibrium profits of the two merging PBMs if and only if the power index is greater than the middle threshold. This finding contrasts with the existing literature on mergers under price competition (e.g., Deneckere and Davidson (1985) and Federgruen and Pierson (2011)), which states that, in the absence of cost synergies, the post-merger equilibrium profit of the merged firm exceeds the combined pre-merger equilibrium profits of the merging firms. This is because the merger of PBMs is different from mergers of competing firms in the existing literature. In all the models that consider mergers with price competition in the literature, the merger results in a decrease of the number of competing firms in the industry, but the number of products remains the same before and after the merger. In other words, after the merger, the merged firm sets prices for all the products that were supplied by the merging firms in the pre-merger model. Unlike the above models of mergers among manufacturers with price competition, our model considers mergers of PBMs, who are service providers to the client organization. Before the merger, each PBM provides the client organization a set of drug prices and formulary. When two PBMs merge, all drugs are managed by the merged PBM, and the merged PBM only needs to offer a single set of drug prices and formulary to the client. Therefore, the merger decreases both the number of competing firms (PBMs) and the number of products (the sets of drug prices and formulary) in our model.

To study the impact of PBM merger on the social welfare, we define the equilibrium social welfare as the sum of the equilibrium profit of the PBM industry and the client organization’s overall expected utility. Note that the latter term captures both the consumer surplus of the plan enrollees and the expected cost of the client organization. More specifically, we define the equilibrium social welfare before and after the merger as:

\[ \Pi^* \equiv \pi_{I}^* + v^* \quad \text{and} \quad \Pi^{m*} \equiv \pi_{I}^{m*} + \overline{v}^{m*}. \]  

(24)

The following proposition characterizes the impact of mergers on social welfare.

**Proposition 7 (Impact of PBM Mergers on Social Welfare).** Consider \( M \) symmetric PBMs. After the merger of PBM 1 and PBM 2, the equilibrium social welfare \( \Pi^{m*} \) increases in the
power index $\Delta U$. Hence, there exists a threshold value of the power index, $\Delta \hat{U}$ ($\Delta \bar{U} \leq \Delta \hat{U} < \Delta \bar{U}$), such that $\Pi^{m*} \leq \Pi^*$ if $0 \leq \Delta U \leq \Delta \hat{U}$, and $\Pi^{m*} \geq \Pi^*$ if $\Delta U \geq \Delta \bar{U}$.

Proposition 7 shows that as the bargaining power of the merged PBM increases, the equilibrium social welfare after the merger also increases. When the merged PBM obtains sufficiently high bargaining power after the merger, the increased negotiating leverage effect dominates the decreased competition effect of the merger, and the merger results in higher social welfare.

A common finding in the current literature on mergers under price competition is that, in the absence of cost synergies, equilibrium prices increase after the merger. Comparing the formulation of the standard MNL model to our model, we have

$$\text{Profit}_j = \frac{\exp((\text{Quality}_j - \text{Price}_j)/\nu)}{\exp(u_0^0/\nu) + \sum_{k=1}^M \exp((\text{Quality}_k - \text{Price}_k)/\nu)} \text{Profit}_j$$

in the standard MNL model, and

$$\pi_j = \frac{\exp(-(-V_j)/\nu)}{\exp(u_0^0/\nu) + \sum_{k=1}^M \exp(-(-V_k)/\nu)} ((-V_j) - (-U_j))$$

in our model.

In our model, the aggregate formulary decision, $U_j$, measures the client’s expected utility of working with an “internal” PBM $j$. Hence, $-U_j$ can be interpreted as the cost of PBM $j$, and $\Delta U$ measures the cost synergy resulting from the merger of PBM 1 and PBM 2. Similarly, the aggregate price decision, $V_j$, measures the client’s expected utility of working with PBM $j$, and $-V_j$ can be interpreted as the price charged by PBM $j$. Define the share-weighted average aggregate price decision before and after the merger as:

$$V^*_I = \frac{\sum_{j=1}^M n^*_j V^*_j}{\sum_{j=1}^M n^*_j}, \quad \text{and} \quad V^{m*}_I = \frac{n^{m*}_1 V^{m*}_1 + \sum_{j=3}^M n^{m*}_j V^{m*}_j}{n^{m*}_1 + \sum_{j=3}^M n^{m*}_j}.$$ (25)

Note that $V^*_I$ and $V^{m*}_I$ denote the additive inverse of the share-weighted average price of PBM industry before and after the merger. We characterize the impact of PBM mergers on the equilibrium aggregate price decisions in the following proposition:

**Proposition 8 (Impact of PBM Mergers on Equilibrium Aggregate Price Decisions).** Consider $M$ symmetric PBMs. After the merger of PBM 1 and PBM 2, there exist three threshold values of the power index, $\Delta \hat{U}$, $\Delta \bar{U}$ and $\Delta \bar{U}$ ($0 < \Delta \hat{U} \leq \Delta \bar{U} < \Delta \bar{U}$), such that:

1. **If** $0 \leq \Delta U \leq \Delta \hat{U}$,
   1. For the merged PBM, $V^{m*}_1 \leq V^*_1$.
   2. For the non-merging PBM $j$ ($j = 3, \ldots, M$), $V^{m*}_j \leq V^*_j$.
2. **If** $\Delta U \geq \Delta \bar{U}$,
   1. For the PBM industry average, $V^{m*}_I \leq V^*_I$.
(II) If $\Delta \hat{U} \leq \Delta U \leq \Delta \bar{U}$,

(i) For the merged PBM, $V_{1}^{m*} \geq V_1^*$.

(ii) For the non-merging PBM $j$ ($j = 3, \ldots, M$), $V_{j}^{m*} \leq V_j^*$.

(iii) For the PBM industry average, $V_{m*}^{I} \leq V_I^*$.

(III) If $\Delta \bar{U} \leq \Delta U \leq \Delta \bar{U}$,

(i) For the merged PBM, $V_{1}^{m*} \geq V_1^*$.

(ii) For the non-merging PBM $j$ ($j = 3, \ldots, M$), $V_{j}^{m*} \leq V_j^*$.

(iii) For the PBM industry average, $V_{m*}^{I} \geq V_I^*$.

(IV) If $\Delta U \geq \Delta \bar{U}$,

(i) For the merged PBM, $V_{1}^{m*} \geq V_1^*$.

(ii) For the non-merging PBM $j$ ($j = 3, \ldots, M$), $V_{j}^{m*} \geq V_j^*$.

(iii) For the PBM industry average, $V_{m*}^{I} \geq V_I^*$.

Proposition 8 confirms that, in the absence of cost synergies, equilibrium prices ($-V_j^*$) increase after the merger. Moreover, Proposition 8 completely characterizes three threshold values of the power index: The equilibrium price of the merged [non-merging] PBM decreases after the merger if and only if the power index is higher than the lowest [highest] threshold, while the middle threshold works for the industry average equilibrium price.

Our results in Propositions 6–8 suggest that, when considering future mergers and acquisitions in the already concentrated PBM industry, policy makers need to carefully weigh the potential gains resulting from negotiation leverage on drug wholesale prices and rebates against the relaxed competitive intensity among the remaining PBMs. In an environment with limited total drug manufacturing capacity and drug manufacturers able to withstand negotiation pressures due to an increased global drug demand, our model tends to predict that clients may not appropriate much value in an after-PBM-merger environment. This prediction is confirmed by a recent empirical study on the Express Scripts/Medco merger, conducted by the economists at the Federal Trade Commission, Shelanski et al. (2012). This study employs two approaches to analyze the impact of PBM mergers on costs, and concludes that “Neither approach revealed significant incremental scale economies in the negotiation of rebates or pharmacy reimbursement” (Shelanski et al. 2012, page 306).

7. Conclusion and Extension

We have characterized the equilibrium behavior of a complex oligopolistic competition among multiple PBMs seeking the business of a client organization. Each PBM selects $2N + 1$ action variables:
a list of gross prices to be charged to the client for each of the branded and generic drugs (the price vector) as well as an assignment vector of binary variables assigning the branded drugs to formulary tiers 2 or 3 (the formulary decision). We have shown that each PBM’s formulary and price decision can be solved in two sequential steps. First, its optimal formulary decision vector can be obtained as the binary vector that maximizes its aggregate formulary decision variable. Since this variable is independent of all other PBMs' decisions, its optimal formulary design is a dominant choice. Second, with each PBM’s optimal formulary design determined, the competition among PBMs can be reduced to a classical single-dimension MNL price competition model. The single (aggregate) price value, $V_j$, selected by PBM $j$ in this reduced price competition model, refers to the client’s expected utility under the plan offered by PBM $j$. As in the classical single-dimension MNL price competition model, there exists a unique equilibrium of the (aggregate) price value. We characterize the equilibrium of the reduced price competition, the resulting equilibrium expected market share and expected profit for each PBM, and the client organization’s expected utility. In addition, we discuss the comparative statics results of our model primitives. In particular, the impact of drug quality and copayment values is quite involved. A change in drug quality or copayment value of a formulary tier has a non-monotone impact on PBM’s optimal formulary decision, as well as the equilibrium aggregate price decision variable, expected market share, expected profit of the PBM and the client organization’s overall expected utility.

Employing our stylized model, we study the impact of PBM mergers on both PBMs and the client organization. Our finding confirms the existing literature on mergers under price competition that, in the absence of cost synergies, equilibrium prices ($-V_j^*$) increase after the merger. In addition, we characterize the combined impact of the increased negotiating leverage and decreased competition effects of the merger upon the merged PBM, non-merging PBMs and the client organization. In particular, the post-merger equilibrium expected profit [market share] of the merged PBM is less than the combined pre-merger equilibrium expected profits [market shares] of the two merging PBMs, unless the merger brings about sufficient savings to the merged PBM from increased bargaining power with drug manufacturers and retail pharmacies. This finding contrasts with the existing literature on mergers under price competition, because the merger of PBMs is different from mergers of competing firms in the literature: the merger decreases both the number of competing firms (PBMs) and the number of products (the sets of drug prices and formulary) in our model.

We now extend our base model to the more general setting of multiple client organizations, each with drugs from multiple therapeutical classes. First, we consider the setting of one client
organization with drugs from multiple therapeutical classes. When there are $L$ therapeutical classes, PBM $j$ needs to charge the same copayment for all drugs on the same formulary tier and pass the same percentage of rebate to the client organization for all branded drugs that are on the preferred tier. Therefore, the copayment values ($c_{g_j}^p$, $c_{p_j}^p$, and $c_{n_j}^p$) and rebate pass-through rates ($\rho_j$) do not vary across different therapeutical classes. All other model primitives and decision variables depend on each drug’s therapeutical class, so we add subscript $k$ ($k = 1, \ldots, L$) to each of them to denote the drug’s therapeutical class. It can be verified that Theorem 1 continues to hold with

$$U_j(\vec{y}_{j1}, \ldots, \vec{y}_{jL}) = \sum_{k=1}^{L} U_{jk}(\vec{y}_{jk}) = \sum_{k=1}^{L} (CS_{jk}(\vec{y}_{jk}) - W_{jk}(\vec{y}_{jk}))$$

(26)

and

$$V_j(\vec{y}_{j1}, \ldots, \vec{y}_{jL}, \vec{p}_{j1}, \ldots, \vec{p}_{jL}) = \sum_{k=1}^{L} V_{jk}(\vec{y}_{jk}, \vec{p}_{jk}) = \sum_{k=1}^{L} (CS_{jk}(\vec{y}_{jk}) - B_{jk}(\vec{y}_{jk}, \vec{p}_{jk})).$$

(27)

It follows from Theorem 1 that each PBM obtains its optimal formulary decision vector by maximizing its aggregate formulary decision variable in (26). Note from (26) that the formulary decisions for different therapeutical classes are separable, because drugs in different therapeutical classes are not substitutable. Therefore, each PBM can solve for its optimal formulary decision vector of each therapeutical class separately. With the optimal formulary decision of each PBM determined, the PBMs compete in the reduced price competition by choosing its aggregate price decision variable, as defined in (27). The equilibrium price decision variable, the equilibrium expected market share and the expected profit of each PBM, as well as the client organization’s expected utility, can be obtained by Theorem 2. It can be verified that all comparative statics results, as well as the result on the impact of PBM mergers, continue to hold in the case with multiple therapeutical classes.

Next, we consider the case with multiple client organizations. It is common in the PBM industry for a PBM to offer a customized plan for each of its client organizations. Since a PBM does not have to commit to the same formulary and pricing schemes for different clients, its formulary and price decisions are completely separable across different clients. Therefore, a PBM can obtain its formulary and price decisions for a client organization, without considering other client organizations, by the procedure outlined above. This result is consistent with the current practice in the PBM industry that different client organizations typically reach out to PBMs requesting a complete design of a coverage plan at different times of a calendar year, and PBMs respond to such requests by customizing a plan for each client organization.

To sum up, the intent of our research is to understand the complex role of PBMs in the financial flow of the pharmaceutical supply chain. We model the competition among PBMs on prices and formularies for the patronage of a client organization, and apply our model to investigate the impact
of PBM mergers. In our model, each PBM faces a nested demand, which has a structure similar to that of a two-level nested MNL model. The relevant choices in our model are part of a sequential process: First, the client organization adopts a plan provided by a PBM; Second, conditional on the choice of this plan, each plan enrollee selects a drug to maximize his/her expected utility according to the copayments specified by the plan. Our model differs from other nested MNL models in two major ways. First, the choices at the two levels are made by different entities with different utility measures. Second, the prescription drug cost is shared between the decision makers at the two levels. The plan enrollees are the end consumers of prescription drugs and make the ultimate drug choices. However, they only pay a nominal fee (copayment) for their prescription drugs and the majority of the drug cost is paid by the client organization. Compared to the payment scheme in the traditional supply chain of consumer products, where customers pay full prices for the products or services they receive, the cost-sharing structure in the pharmaceutical supply chain dampens patient sensitivity to price, and allows patients to focus more on quality relative to price than they otherwise would. Our model can be applied to study the competition in a market where the cost of the product is shared by decision makers at different levels. This cost-sharing structure is present in various employment-based benefit programs, such as housing subsidy programs, backup childcare benefit programs, etc.

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**Appendix A: Proofs**

*Proof of Theorem 1* (a). By (5) and (6), we have

\[
\max_{\bar{y}_j \in \{0,1\}^N, \bar{p}_j \in \mathbb{R}^{N+1}} \pi_j = \frac{\exp((CS_j - B_j) / \nu)}{\exp(u_0^0 / \nu) + \sum_{k=1}^M \exp((CS_k - B_k) / \nu)} (B_j - W_j),
\]

where \( W_j \) is given in (9). Using (7) and (8), we can rewrite PBM \( j \)'s problem as

\[
\max_{\bar{y}_j \in \{0,1\}^N, \bar{p}_j \in \mathbb{R}^{N+1}} \pi_j = \frac{\exp(V_j / \nu)}{\exp(u_0^0 / \nu) + \sum_{k=1}^M \exp(V_k / \nu)} (U_j - V_j).
\]

Note that \( U_j \) only depends on the formulary decision \( y_j \in \{0,1\}^N \), and \( \Omega_j \), the feasible set of \( U_j \) is given in (10). Hence, (11) follows directly.

(b). It directly follows from (11) that \( \pi_j \) linearly increases in \( U_j \). Note that

\[
\frac{\partial \ln \pi_j}{\partial V_j} = \frac{1}{\nu} \frac{1}{U_j - V_j} \frac{\exp(V_j / \nu)}{\nu \left( \exp(u_0^0 / \nu) + \sum_{k=1}^M \exp(V_k / \nu) \right)},
\]

and

\[
\frac{\partial^2 \ln \pi_j}{\partial V_j^2} = -\frac{1}{(U_j - V_j)^2} - \frac{\exp(V_j / \nu) \left( \exp(u_0^0 / \nu) + \sum_{k \neq j} \exp(V_k / \nu) \right)}{\nu^2 \left( \exp(u_0^0 / \nu) + \sum_{k=1}^M \exp(V_k / \nu) \right)^2} < 0,
\]
By (12), we rewrite (32) into the following form:

\[ V_j = \frac{\mu}{\alpha} \left( \ln \left( \exp \left( \frac{\gamma_0 - \alpha c_j^0 + \beta q_j}{\mu} \right) + \sum_{i=1}^{N} \exp \left( \frac{\gamma_i - \alpha c_j^i(y_{ij}) + \beta q_j}{\mu} \right) \right) \right) S - \left( d_{0j}(\tilde{y}_j)(p_{0j} - c_j^0) + \sum_{i=1}^{N} d_{ij}(\tilde{y}_j)(p_{ij} - c_j^i(y_{ij}) - \rho_j r_{ij} y_{ij}) \right) S. \]  

(30)

For any given original formulary decision vector \( \tilde{y}_j \in \{0,1\}^N \) (and hence \( U_j \)), \( V_j \) is a linear combination of the \( N + 1 \) original price decision variables \( p_{ij} \in \mathbb{R} \) (\( i = 0, \ldots, N \)), as shown by (30). Therefore, \( V_j \) can target any value by varying the original price decision vector \( \tilde{y}_j \). Conversely, for any given \( V_j \in \mathbb{R} \), \( U_j \) can target any value in its feasible set \( \Omega_j \) by choosing the corresponding original formulary decision vector \( \tilde{y}_j \). At the chosen \( \tilde{y}_j \), we can keep \( V_j \) fixed at the given value by varying the original price decision vector \( \tilde{y}_j \).

(c). Based on the proof of parts (a) and (b), PBM \( j \) can determine its best response sequentially by solving the optimal \( U_j \) first and then the optimal \( V_j \). For PBM \( j \), the maximization of \( U_j \) is a combinatorial optimization on a finite set: \( \max_{\tilde{y}_j \in \{0,1\}^N} U_j(\tilde{y}_j) \), and thus the optimal \( U_j \) is unique. Since each PBM’s optimal aggregate formulary decision \( U_j \) is independent of each other, PBMs’ competition on \( (U_j, V_j) \) can be reduced to a competition on \( V_j \) only. The existence of equilibrium directly follows from the concavity of \( \ln \pi_j \) in \( V_j \). Furthermore,

\[ \frac{\partial^2 \ln \pi_j}{\partial V_j^2} + \sum_{k \neq j} \frac{\partial^2 \ln \pi_j}{\partial V_j \partial V_k} \left( \frac{\exp (V_j/V)}{V^2} \frac{\exp (u_j^0/V)}{\sum_{k=1}^{M} \exp (V_k/V)} \right) < 0. \]  

(31)

By Milgrom and Roberts (1990), a unique equilibrium is guaranteed, and satisfies the first order condition given by (28).

**Proof of Proposition 1** Substituting (9) and (2) into (8), we can write PBM \( j \)’s formulary problem as

\[ \max_{\tilde{y}_j \in \{0,1\}^N} U_j(\tilde{y}_j) = \frac{\mu}{\alpha} \left( \ln \left( \exp \left( \frac{\gamma_0 - \alpha c_j^0 + \beta q_j}{\mu} \right) + \sum_{i=1}^{N} \exp \left( \frac{\gamma_i - \alpha c_j^i(y_{ij}) + \beta q_j}{\mu} \right) \right) \right) S - \left( d_{0j}(\tilde{y}_j)(w_{0j} - c_j^0) + \sum_{i=1}^{N} d_{ij}(\tilde{y}_j)(w_{ij} - c_j^i(y_{ij}) + (c_j^0 - c_j^i - r_{ij}) y_{ij}) \right) S. \]  

(32)

By (12), we rewrite (32) into the following form:

\[ \max_{\tilde{y}_j \in \{0,1\}^N} U_j(\tilde{y}_j) = \frac{\mu}{\alpha} \ln \left( x_{0j} + \sum_{i=1}^{N} x_{ij}(y_{ij}) \right) - \frac{x_{0j} m_{0j}}{x_{0j} + \sum_{i=1}^{N} x_{ij}(y_{ij})} \]  

(33)

First, we introduce the following lemma to characterize properties of PBMs’ optimal formulary decision.

**Lemma 2.** For \( j = 1, \ldots, M \), PBM \( j \) puts branded drug \( i \) on tier 2 (i.e., \( y_{ij}^* = 1 \)) if for any \( \tilde{y}_j \in \{0,1\}^N \),

\[ \delta_{ij} \leq \frac{x_{0j} m_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}) m_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj})}. \]  

(34)

**Proof of Lemma 2** We prove by contradiction. Let \( \tilde{y}_j^* \) be PBM \( j \)’s optimal formulary decision vector. Assume to the contrary that (34) holds for any \( \tilde{y}_j \in \{0,1\}^N \) and \( y_{ij}^* = 0 \). We can construct another formulary decision \( \tilde{y}_j' \) by setting \( y_{ij}' = 1 \) and \( y_{ij}' = 0 \) for \( k \neq i \). Then we have:

\[ U_j(\tilde{y}_j') = \frac{\mu}{\alpha} \ln \left( x_{0j} + \sum_{i=1}^{N} x_{kj}(y_{kj}) + x_{ij}(1 - x_{ij}) \right) - \frac{x_{0j} m_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}) m_{kj}(y_{kj}) + x_{ij}(1 - x_{ij}) + x_{ij}(0) m_{ij}(0)}{x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}) + x_{ij}(1 - x_{ij})} S. \]  

(35)
Since $\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)} \leq \frac{x_{0j}m_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^*)m_{kj}(y_{kj}^*)}{x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^*)}$ and $x_{ij}(1) > x_{ij}(0)$, we have $U_j(y_j^*) > U_j(y_j^*)$, contradicting the optimality of $y_j^*$. This completes the proof.

Lemma 2 shows that when assigning branded drugs to tier 2, it is optimal for PBM $j$ to start by choosing the branded drug with the smallest cost change rate, then add the drug with the second smallest cost change rate to tier 2, and so on, until (34) no longer holds. Since PBM $j$ sorts branded drugs by their cost change rates in an ascending order, we can find the threshold index as follows:

Initialize $I_j = 0$;

FOR $i = 1, \cdots, N$;

IF $\delta_{ij} \leq \frac{x_{0j}m_{0j} + \sum_{k=1}^{i-1} x_{kj}(1)m_{kj}(1) + \sum_{k=i}^{N} x_{kj}(0)m_{kj}(0)}{x_{0j} + \sum_{k=1}^{i-1} x_{kj}(1) + \sum_{k=i}^{N} x_{kj}(0)}$, $I_j = i$;

ELSE STOP;

ENDIF

ENDFOR

Note that

$$\min_{\vec{y} \in \{0,1\}^N} \frac{x_{0j}m_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj})m_{kj}(y_{kj})}{x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj})} = \frac{x_{0j}m_{0j} + \sum_{k=1}^{I_j} x_{kj}(1)m_{kj}(1) + \sum_{k=I_j+1}^{N} x_{kj}(0)m_{kj}(0)}{x_{0j} + \sum_{k=1}^{I_j} x_{kj}(1) + \sum_{k=I_j+1}^{N} x_{kj}(0)}.$$

By Lemma 2, it is optimal for PBM $j$ to assign drug $i$ ($i \leq I_j$) on tier 2 (i.e., $y^*_j = 1$).

Proof of Proposition 2 When all branded drugs on PBM $j$’s plan have the same brand-specific attribute $\gamma_i$ and quality index $q_i$, we first show that if PBM $j$ puts $k$ branded drugs on tier 2, it is optimal for PBM $j$ to assign $k$ branded drugs with the smallest cost change rate indices on tier 2.

We prove the above statement by contradiction. Let $\vec{y}_j^*$ be PBM $j$’s optimal formulary decision vector. Assume to the contrary of the statement, there exists an $y^*_j = 0$ ($i \leq k$) and an $y^*_j = 1$ ($l > k$). We can construct another formulary decision $\vec{y}_j'$ by setting $y^*_j = 1$ and $y^*_j = 0$ while keeping other branded drugs’ formulary decision unchanged. Then we have:

$$U_j(y_j') = \left(\mu \ln \left(\frac{\int x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^*)m_{kj}(y_{kj}^*) + x_{ij}(1) - x_{ij}(0) - x_{ij}(1) + x_{ij}(0)}{\int x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^*) + x_{ij}(1) - x_{ij}(0) - x_{ij}(1) + x_{ij}(0)}\right) - \frac{x_{0j}m_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^*)m_{kj}(y_{kj}^*) + x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) - x_{ij}(1)m_{ij}(1) + x_{ij}(0)m_{ij}(0)}{\int x_{0j} + \sum_{k=1}^{N} x_{kj}(y_{kj}^*) + x_{ij}(1) - x_{ij}(0) - x_{ij}(1) + x_{ij}(0)}\right),$$

Note that all branded drugs have the same brand specific attribute and quality index, we have $x_{ij}(1) - x_{ij}(0) = x_{ij}(1) - x_{ij}(0) > 0$. Since PBM $j$ ranks branded drugs in ascending order of their cost change rate, i.e., $\delta_{ij} \leq \delta_{2j} \leq \cdots \leq \delta_{Nj}$, we have $\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)} \leq \delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)}$, and hence $x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) \leq x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)$, we have $\delta_{ij} = \frac{x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)}{x_{ij}(1) - x_{ij}(0)}$, and hence $x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0) \leq x_{ij}(1)m_{ij}(1) - x_{ij}(0)m_{ij}(0)$. Therefore, $U_j(y_j') \geq U_j(y_j^*)$, contradicting the optimality of $y_j^*$. This completes the proof of the statement.

By the above statement and Proposition 1, PBM $j$ only needs to compare $N - I_j + 1$ formulary decision vectors, each corresponding to putting $k$ ($k = I_j, \ldots, N$) branded drugs with smallest cost change rate indices on tier 2. This completes the proof.
Proof of Theorem 2 By the definition of $V_j$ in (7) and the expression of $n_j$ in (5), we can rewrite $V_j$ as a function of PBMs’ market share vector, $n = (n_1, \ldots, n_M)$, as follows:

$$V_j = u^0_j + \nu \left( \ln n_j - \ln \left(1 - \sum_{k=1}^M n_k \right) \right), \quad j = 1, \ldots, M.$$  \hfill (36)

Let $n_0$ denote the probability of the client not contracting with any PBM, we have $n_0 = 1 - \sum_{k=1}^M n_k$.

Given the optimal formulary decision, the equilibrium $V^*_j$ satisfies the set of first order conditions given by (14). Substituting (36) into (14) and rearranging the terms, we have

$$n^*_j \cdot \exp \left( \frac{n^*_j}{1-n^*_j} \right) = n_0 \cdot \exp \left( \frac{U^*_j - u^0_j - \nu}{\nu} \right), \quad j = 1, \ldots, M.$$  \hfill (37)

By the definition of $H(\cdot)$ function in (15), (17) directly follows. Combining with the fact that $\sum_{j=0}^M n^*_j = 1$, the equilibrium probability of not contracting with any PBM, $n^*_0$, satisfies the single-variable equation:

$$n_0 + \sum_{j=1}^M H \left( n_0 \cdot \exp \left( \frac{U^*_j - u^0_j - \nu}{\nu} \right) \right) = 1.$$  

Since $H(\cdot)$ strictly increases from 0 to 1, the left-hand-side of the above equation strictly increases from 0 to a value greater than 1 as $n_0$ increases from 0 to 1. Therefore, $n^*_0$ is the unique solution to (16).

Substituting (5), (7) and (14) into (11), PBM $j$’s equilibrium expected profit is given by $\pi^*_j = \frac{\nu n^*_j}{1-n^*_j}$.

Substituting $n^*_0 = 1 - \sum_{k=1}^M n^*_k$ into (36), (19) follows directly.

The client organization’s overall expected utility in the equilibrium is given by

$$\mathbf{\pi} = E \left( \max_{j} \left( \max_{j} v^*_j, v_0 \right) \right) = \nu \ln \left( \exp \left( \frac{u^0_0}{\nu} \right) + \sum_{j=1}^M \exp \left( \frac{V^*_j}{\nu} \right) \right) = \nu \ln \left( \exp \left( \frac{u^0_0}{\nu} \right) \left( 1 + \sum_{j=1}^M \frac{n^*_j}{n_0} \right) \right) = u^0_0 - \nu \ln n_0.$$  

Proof of Lemma 1 (a). Applying the implicit function theorem on the function $H(x)$, we have

$$H'(x) = \frac{(1-H)(1-H^2)\exp(\frac{n^*_j}{1-n^*_j})}{(1-H+H^2)\exp(\frac{n^*_j}{1-n^*_j})} > 0.$$  \hfill (38)

By (16), (37) and (38), and applying the implicit function theorem on (16), we have

$$\frac{\partial n^*_0}{\partial U^*_j} = -\frac{n^*_0}{1 + \sum_{k=1}^M \frac{(1-n^*_0)^2 n^*_j}{(1-n^*_0)^2 n^*_j}} < 0.$$  \hfill (39)

(b). By (17) and (39), we have

$$\frac{\partial n^*_j}{\partial U^*_j} = H' \left( n^*_0 \cdot \exp \left( \frac{U^*_j - u^0_j - \nu}{\nu} \right) \right) \cdot \exp \left( \frac{U^*_j - u^0_j - \nu}{\nu} \right) \cdot \frac{\partial n^*_0}{\partial U^*_j} \left( \frac{n^*_0 + \sum_{k=1}^M \frac{(1-n^*_0)^2 n^*_j}{(1-n^*_0)^2 n^*_j}}{\nu (1 + \sum_{k=1}^M \frac{(1-n^*_0)^2 n^*_j}{(1-n^*_0)^2 n^*_j})} \right) > 0, \quad \text{and} \quad \text{(40)}$$

$$\frac{\partial n^*_j}{\partial U^*_k} = H' \left( n^*_0 \cdot \exp \left( \frac{U^*_j - u^0_j - \nu}{\nu} \right) \right) \cdot \exp \left( \frac{U^*_j - u^0_j - \nu}{\nu} \right) \frac{\partial n^*_0}{\partial U^*_k} < 0, \quad k = 1, \ldots, M \quad \text{and} \quad k \neq j.$$

By (18), (40) and (41), $\frac{\partial \pi^*_j}{\partial U^*_j} > 0$ and $\frac{\partial \pi^*_j}{\partial U^*_k} < 0 \quad (k \neq j)$ directly follow.

(c). By (19), (39) and (40), we have

$$\frac{\partial V^*_j}{\partial U^*_j} = \nu \left( \frac{1}{n^*_j \partial U^*_j} - \frac{1}{n^*_0 \partial U^*_j} \right) > 0.$$  \hfill (42)
By (19), (37), (39) and (41), we have
\[
\frac{\partial V^*_i}{\partial U^*_k} = \nu \left( \frac{1}{n^*_j} \frac{\partial n^*_j}{\partial U^*_k} - \frac{1}{n^*_0} \frac{\partial n^*_0}{\partial U^*_k} \right) = -\frac{\nu n^*_j}{(1-n^*_j + n^*_j x^*_j) n^*_0} \frac{\partial n^*_0}{\partial U^*_k} > 0, \quad k \neq j. \tag{43}
\]
(d). The result directly follows from (20) and part (c).

**Proof of Proposition 3** (a). Since formulary decision \(y_{ij}\) is a binary variable, showing that it is weakly decreasing is equivalent to showing that an increase from 0 to 1 cannot happen when the wholesale price of branded drug \(i\) increases. We prove by contradiction. Suppose that the wholesale price of branded drug \(i\) increases from \(w_{ij}\) to \(w'_{ij}\) \((w'_{ij} > w_{ij})\), \(y^*_{ij}|w_{ij} = 0\) and \(y^*_{ij}|w'_{ij} = 1\). Note that for any given \(\bar{y}_j\), we have
\[
\frac{\partial U_j(\bar{y}_j)}{\partial w_{ij}} = -\frac{x_{ij}(y^*_{ij})S}{x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj})} < 0, \quad \text{and} \quad \frac{\partial^2 U_j(\bar{y}_j)}{\partial w_{ij}^2} = 0. \tag{44}
\]
Therefore,
\[
U_j(\bar{y}_j|w_{ij})|w_{ij} \geq U_j(\bar{y}_j|w'_{ij})|w_{ij} > U_j(\bar{y}_j|w'_{ij})|w'_{ij} \geq U_j(\bar{y}_j|w_{ij})|w'_{ij},
\]
which shows that
\[
U_j(\bar{y}_j|w_{ij})|w_{ij} - U_j(\bar{y}_j|w_{ij})|w_{ij} \geq U_j(\bar{y}_j|w_{ij})|w_{ij} - U_j(\bar{y}_j|w_{ij})|w_{ij}.
\]
By (44), we have
\[
U_j(\bar{y}_j|w_{ij})|w_{ij} - U_j(\bar{y}_j|w_{ij})|w'_{ij} = \frac{x_{ij}(y^*_{ij}|w_{ij})(w'_{ij} - w_{ij})S}{x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj}|w_{ij})}, \tag{46}
\]
and
\[
U_j(\bar{y}_j|w_{ij})|w_{ij} - U_j(\bar{y}_j|w_{ij})|w'_{ij} = \frac{x_{ij}(y^*_{ij}|w_{ij})(w'_{ij} - w_{ij})S}{x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj}|w_{ij})}. \tag{47}
\]
Substitute \(y^*_{ij}|w_{ij} = 0\), \(y^*_{ij}|w'_{ij} = 1\), (46) and (47) into (45), we have
\[
\begin{align*}
x_{ij}(0) \left( x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj}|w'_{ij}) \right) & \geq x_{ij}(1) \left( x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj}|w_{ij}) \right) \tag{48}
\end{align*}
\]
The left hand side of (48) satisfies
\[
x_{ij}(0) \left( x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj}|w'_{ij}) \right) \leq x_{ij}(0) \left( x_{0j} + \sum_{k=1}^N x_{kj}(1) \right) = x_{ij}(0)x_{0j} + \sum_{k=1}^N \exp \left( \frac{\gamma_i + \gamma_k + \beta(q_i + q_k) - \alpha(c^*_j + c^*_p)}{\mu} \right),
\]
while the right hand side of (48) satisfies
\[
x_{ij}(1) \left( x_{0j} + \sum_{k=1}^N x_{kj}(y^*_{kj}|w_{ij}) \right) \geq x_{ij}(1) \left( x_{0j} + \sum_{k=1}^N x_{kj}(0) \right) = x_{ij}(1)x_{0j} + \sum_{k=1}^N \exp \left( \frac{\gamma_i + \gamma_k + \beta(q_i + q_k) - \alpha(c^*_j + c^*_p)}{\mu} \right).
\]
By (48) and \(x_{0j} > 0\), we have \(x_{ij}(0) \geq x_{ij}(1)\). This contradicts with the fact that \(x_{ij}(0) < x_{ij}(1)\).

In general, the impact of any other drug’s wholesale price on the focal drug’s optimal formulary decision is non-monotone, and we have observed this non-monotone effect in numerical examples.

Recall that the optimal formulary decision vector is a dominant choice, and is independent of all other PBMs’ decision and cost parameters. Therefore, PBM \(j\)’s optimal formulary decision is independent of any drug’s wholesale price on any competing PBM’s plan.

(b). Note that for any given \(\bar{y}_j\), (44) holds for any \(i = 0, \ldots, N\). Consider the case that the wholesale price of any (branded or generic) drug \(i\) increases from \(w_{ij}\) to \(w'_{ij}\) \((w'_{ij} > w_{ij})\). By (44), \(U_j|w_{ij} = \)
\( U_j(\tilde{g}_j^*|w_{ij}^*)|w_{ij}^* < U_j(\tilde{g}_j'|w_{ij}^*)|w_{ij}^* \leq U_j(\tilde{g}_j^*|w_{ij}^*) = U_j^*|w_{ij}^* \). Therefore, \( U_j^* \) strictly decreases in the wholesale price of any (branded or generic) drug charged to PBM \( j \).

Since each PBM’s formulation decision is independent of each other, PBM \( j \)’s optimal aggregate formulary decision \( U_j^* \) is independent of any (branded or generic) drug’s wholesale price charged to any competing PBM.

(c). Since each PBM’s formulation decision is independent of each other, PBM \( j \)’s optimal aggregate formulary decision \( U_j^* \) is independent of any (branded or generic) drug \( l \)’s wholesale price charged to PBM \( k \) \((k \neq j)\), \( w_{lk} \). Therefore, the impact of \( w_{lk} \) on PBM \( j \)’s equilibrium aggregate price decision \( V_j^* \) is only through its impact on PBM \( k \)’s optimal aggregate formulary decision \( U_k^* \). By part (c) of Lemma 1 and part (b) of Proposition 3, \( V_j^* \) strictly decreases in \( w_{lk} \) and \( w_{ij} \).

The impact of \( w_{lk} \) on the client’s overall expected utility \( \pi^* \) directly follows from (20).

(d). We will show this part in an analogous approach as the above proof in part (c). Since PBM \( j \)’s optimal aggregate formulary decision \( U_j^* \) is independent of any (branded or generic) drug \( l \)’s \((l = 0, \ldots, N)\) wholesale price charged to any other PBM \( k \) \((k \neq j)\), \( w_{lk} \). Therefore, the impact of \( w_{lk} \) on PBM \( j \)’s equilibrium expected market share \( n_j^* \) and expected profit \( \pi_j^* \) is only through its impact on PBM \( k \)’s optimal aggregate formulary decision \( U_k^* \). By part (b) of Lemma 1 and part (b) of Proposition 3, the statement follows directly.

Proof of Proposition 4 (a). For any given \( \tilde{y}_j \) and branded drug \( i \),

\[
\frac{\partial U_j(\tilde{y}_j)}{\partial q_i} = \frac{\beta S x_{ij}(y_{ij})}{\mu (x_{ij} + \sum_{j=1}^N x_{ij}(y_{ij}))^2} \left( \frac{\mu}{\alpha} x_{ij}(y_{ij}) + x_{ij} \left( \frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j} \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{ij}(y_{ij}) \right) \right).
\]

Therefore, \( sgn \left( \frac{\partial U_j(\tilde{y}_j)}{\partial q_i} \right) = sgn \left( \frac{\partial x_{ij}(y_{ij})}{\partial q_i} \right) = sgn \left( \frac{\alpha}{\alpha} x_{ij}(y_{ij}) + x_{ij} \left( \frac{\alpha}{\alpha} - m_{ij}(y_{ij}) + m_{0j} \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\alpha}{\alpha} - m_{ij}(y_{ij}) + m_{ij}(y_{ij}) \right) \right) \) is convex increasing in \( q_i \) and approaches to \(+\infty\) when \( q_i \to +\infty \). So \( \frac{\partial x_{ij}(y_{ij})}{\partial q_i} = x_{ij} \left( \frac{\alpha}{\alpha} - m_{ij}(y_{ij}) + m_{0j} \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\alpha}{\alpha} - m_{ij}(y_{ij}) + m_{ij}(y_{ij}) \right) \) crosses 0 at most once. Denote \( q_{ij}(\tilde{y}_j) \) the unique root of \( \frac{\partial x_{ij}(y_{ij})}{\partial q_i} = x_{ij} \left( \frac{\alpha}{\alpha} - m_{ij}(y_{ij}) + m_{0j} \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\alpha}{\alpha} - m_{ij}(y_{ij}) + m_{ij}(y_{ij}) \right) = 0 \) when exists, and \( q_{ij}(\tilde{y}_j) = -\infty \) otherwise. Set \( \tilde{q}_{ij} = \left( \max_{q_{ij} \in [0, 1]} q_{ij}(\tilde{y}_j) \right)^+ \) and \( \underline{q}_{ij} = \left( \min_{q_{ij} \in [0, 1]} q_{ij}(\tilde{y}_j) \right)^+ \). It directly follows that \( \frac{\partial x_{ij}(y_{ij})}{\partial q_i} = x_{ij} \left( \frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j} \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{ij}(y_{ij}) \right) > 0 \) for any \( \tilde{y}_j \in [0, 1]^N \) when \( q_i > \tilde{q}_{ij} \), and \( \frac{\partial x_{ij}(y_{ij})}{\partial q_i} = x_{ij} \left( \frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{0j} \right) + \sum_{i \neq i} x_{ij}(y_{ij}) \left( \frac{\mu}{\alpha} - m_{ij}(y_{ij}) + m_{ij}(y_{ij}) \right) < 0 \) for any \( \tilde{y}_j \in [0, 1]^N \) when \( 0 < q_i < \underline{q}_{ij} \). Therefore, PBM \( j \)'s optimal aggregate formulary decision variable, \( U_j^* \), strictly decreases in branded drug \( i \)'s quality index, \( q_i \), if \( 0 \leq q_i < \underline{q}_{ij} \), and strictly increases in \( q_i \) if \( q_i > \tilde{q}_{ij} \).

When \( q_{ij}(\tilde{y}_j) \neq -\infty \), applying the implicit function theorem, we have \( q_{ij}(\tilde{y}_j) \) increases in \( w_{ij} \) and weakly decreases \( r_{ij} \). Therefore, \( \tilde{r}_{ij} = \left( \max_{q_{ij} \in [0, 1]} q_{ij}(\tilde{y}_j) \right)^+ \) and \( \underline{r}_{ij} = \left( \min_{q_{ij} \in [0, 1]} q_{ij}(\tilde{y}_j) \right)^+ \) both weakly increase in \( w_{ij} \) and weakly decrease in \( r_{ij} \).

The impact of generic drug’s quality \( q_0 \) can be shown in the same manner.

(b). By Lemma 1, the impact of drug quality affects any PBM’s equilibrium aggregate price decision, expected market share, profit, and the client’s overall utility through its impact on all PBMs’ optimal
aggregate formulary decisions. When all PBMs are symmetric, the impact of any drug’s quality on the optimal aggregate formulary decision, $U^*_j$, is the same across all PBMs. Therefore, the impact of any drug’s quality on all the quantities of interest mentioned above has the same directional change as the aggregate impact of the drug’s quality on all PBMs’ optimal aggregate formulary decisions. For PBM $j$, by (40) and (41), we have

$$\sum_{k=1}^{M} \frac{\partial n_j^*}{\partial U_k^*} = H' \left( n_0^* \exp \left( \frac{U_j^* - u_0^* - \nu}{\nu} \right) \right) \exp \left( \frac{U_j^* - u_0^* - \nu}{\nu} \right) \nu \left( 1 + \sum_{k=1}^{M} \frac{(1-n_k^*)^2 n_j^*}{(1-n_k^*+n_j^*)n_0} \right) > 0. \quad (50)$$

Therefore, the impact of $q_i$ on $n_j^*$ has the same directional change as the impact of $q_i$ on $U_j^*$.

The impact of $q_i$ on $V_j^*\pi_j^*$ and $\pi^*$ can be shown in the same manner.

Proof of Proposition 5 When all (branded or generic) drugs have the same quality, i.e., $q_i = q_i (i = 0, \ldots, N)$.

$$U_j(\tilde{y}_j) = \left( \frac{\beta}{\alpha} q + \frac{\mu}{\mu} \right) \ln \left( \exp \left( \frac{\gamma_0 - \alpha c_j}{\mu} \right) + \sum_{i=1}^{N} \exp \left( \frac{(\gamma_i - \alpha c_j + \alpha (c_j - c_j^*) y_{ij})}{\mu} \right) \right) - \exp \left( \frac{\gamma_0 - \alpha c_j}{\mu} \right) + \sum_{l=1}^{N} \exp \left( \frac{(\gamma_i - \alpha c_j + \alpha (c_j - c_j^*) y_{ij})}{\mu} \right) S.$$  

PBM $j^*$’s optimal formulary decision $\tilde{y}_j$ is independent of $q_i$.

$$\frac{\partial U_j}{\partial q} = \frac{\partial U_j(\tilde{y}_j)}{\partial q} = \frac{\beta}{\alpha} S > 0, \quad \frac{\partial V_j}{\partial q} = \frac{\partial V_j(\tilde{y}_j)}{\partial q} > 0,$$

$$\frac{\partial n_j^*}{\partial q} = \sum_{k=1}^{M} \frac{\partial n_j^*}{\partial U_k^*} \frac{\partial U_k^*}{\partial q} = H' \left( n_0^* \exp \left( \frac{U_j^* - u_0^* - \nu}{\nu} \right) \right) \cdot \exp \left( \frac{U_j^* - u_0^* - \nu}{\nu} \right) \nu \left( 1 + \sum_{k=1}^{M} \frac{(1-n_k^*)^2 n_j^*}{(1-n_k^*+n_j^*)n_0} \right) \beta S > 0,$$

$$\frac{\partial n_0^*}{\partial q} < 0, \quad \frac{\partial \pi_j^*}{\partial q} > 0, \quad \frac{\partial \pi^*}{\partial q} > 0.$$  

Note that we do not require all PBMs are symmetric in this case.

Proof of Proposition 6 Substituting (17) into (16) and by the symmetry of PBMs in the pre-merger model, the equilibrium market share of PBM $j$, $n_j^*$, is the unique solution on $(0,1)$ of the following equation:

$$n_j^* \left( M + \exp \left( \frac{n_j^*}{1-n_j^*} - \frac{U_j^* - u_0^* - \nu}{\nu} \right) \right) = 1, \quad j = 1, \ldots, M. \quad (51)$$

After the merger, it directly follows from (17) and the fact $U_j^{m*} > U_j^* (j = 3, \ldots, M)$ that $n_j^{m*} > n_j^*$ ($j = 3, \ldots, M$). All non-merging PBMs remain symmetric after the merger. By (18), $\pi_j^{m*} > \pi_j^*$ ($j = 3, \ldots, M$) always holds. Apply (16) and (17) to the post-merger model, we have the following relationships:

$$n_0^{m*} + n_1^{m*} + (M-2)n_j^{m*} = 1, \quad (52)$$

$$n_j^{m*} \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} \right) = n_0^{m*} \exp \left( \frac{U_j^{m*} - u_0^* - \nu}{\nu} \right), \quad (53)$$

$$n_j^{m*} \exp \left( \frac{n_j^{m*}}{1-n_j^{m*}} \right) = n_0^{m*} \exp \left( \frac{U_j^{m*} - u_0^* - \nu}{\nu} \right), \quad j = 3, \ldots, M. \quad (54)$$
Using $U_j^{m*} = U_j^*$ ($j = 3, \ldots, M$), we have $n_j^{m*}$ satisfies the following equation:

$$
n_j^{m*} \exp \left( \frac{n_j^{m*}}{1 - n_j^{m*}} - \frac{U_j^* - u_0 - \nu}{\nu} \right) + H \left( n_j^{m*} \exp \left( \frac{n_j^{m*}}{1 - n_j^{m*}} + \frac{\Delta U}{\nu} \right) \right) + (M - 2)n_j^{m*} = 1, \quad j = 3, \ldots, M.
$$

(55)

Note that the left hand side of (55) strictly increases from 0 to a number greater than 1, as $n_j^{m*}$ increases on $[0, 1)$, so $n_j^{m*}$ is the unique solution of the above equation. Therefore, $n_j^{m*} \geq n_j^*$ ($j = 3, \ldots, M$) iff

$$
n_j \exp \left( \frac{n_j^*}{1 - n_j^*} - \frac{U_j^* - u_0 - \nu}{\nu} \right) + H \left( n_j^* \exp \left( \frac{n_j^*}{1 - n_j^*} + \frac{\Delta U}{\nu} \right) \right) + (M - 2)n_j^* \leq 1,
$$

where $n_j^*$ is given by (51). (56) can be further reduced to $\Delta U \leq \Delta \bar{U}$, where

$$
\Delta \bar{U} = \nu \left( \frac{2n_j^*}{1 - 2n_j^*} - \frac{n_j^*}{1 - n_j^*} + \ln 2 \right).
$$

(57)

By (18), it directly follows that $\pi_j^{m*} \geq \pi_j^*$ ($j = 3, \ldots, M$) iff $\Delta U \leq \Delta \bar{U}$.

Similarly, by (52), (53) and (54), we have $n_1^{m*}$ satisfies the following equation:

$$
n_1^{m*} \exp \left( \frac{n_1^{m*}}{1 - n_1^{m*}} - \frac{\Delta U + U_1^* - u_0 - \nu}{\nu} \right) + n_1^{m*} + (M - 2)H \left( n_1^{m*} \exp \left( \frac{n_1^{m*}}{1 - n_1^{m*}} - \frac{\Delta U}{\nu} \right) \right) = 1,
$$

and $n_1^{m*}$ is the unique solution of the above equation. When $n_1^{m*} = n_1^*$, the left hand side of (58) is:

$$
n_1^* \exp \left( \frac{n_1^*}{1 - n_1^*} - \frac{\Delta U + U_1^* - u_0 - \nu}{\nu} \right) + n_1^* + (M - 2)H \left( n_1^* \exp \left( \frac{n_1^*}{1 - n_1^*} - \frac{\Delta U}{\nu} \right) \right)
\leq n_1^* \exp \left( \frac{n_1^*}{1 - n_1^*} - \frac{U_1^* - u_0 - \nu}{\nu} \right) + n_1^*(M - 1 + \exp \left( \frac{n_1^*}{1 - n_1^*} - \frac{U_1^* - u_0 - \nu}{\nu} \right)) < n_1^* \left( M + \exp \left( \frac{n_1^*}{1 - n_1^*} - \frac{U_1^* - u_0 - \nu}{\nu} \right) \right) = 1.
$$

Therefore, $n_1^{m*} \geq n_1^*$ always holds. By (18), $\pi_1^{m*} \geq \pi_1^*$ always holds as well.

Since the left hand side of (58) strictly increases in $n_1^{m*}$, $n_1^{m*} \geq n_1^* + n_2^* = 2n_2^*$ iff

$$
2n_2^* \exp \left( \frac{2n_2^*}{1 - 2n_2^*} - \frac{\Delta U + U_2^* - u_0 - \nu}{\nu} \right) + 2n_2^* + (M - 2)H \left( 2n_2^* \exp \left( \frac{2n_2^*}{1 - 2n_2^*} - \frac{\Delta U}{\nu} \right) \right) \leq 1.
$$

(59)

where $n_2^*$ is given by (51). Note that the left hand side of (59) is a decreasing function of $\Delta U$. Therefore, (59) is equivalent to $\Delta U \geq \Delta \bar{U}$, where $\Delta \bar{U}$ is the unique solution of

$$
2n_2^* \exp \left( \frac{2n_2^*}{1 - 2n_2^*} - \frac{\Delta U + U_2^* - u_0 - \nu}{\nu} \right) + 2n_2^* + (M - 2)H \left( 2n_2^* \exp \left( \frac{2n_2^*}{1 - 2n_2^*} - \frac{\Delta U}{\nu} \right) \right) = 1.
$$

(60)

Note that when $\Delta U = \Delta \bar{U}$, the left hand side of (60) is given by

$$
2n_2^* \exp \left( \frac{n_2^*}{1 - n_2^*} - \ln 2 - \frac{U_2^* - u_0 - \nu}{\nu} \right) + 2n_2^* + (M - 2)H \left( 2n_2^* \exp \left( \frac{n_2^*}{1 - n_2^*} - \ln 2 \right) \right)
= n_2^* \left( M + \exp \left( \frac{n_2^*}{1 - n_2^*} - \frac{U_2^* - u_0 - \nu}{\nu} \right) \right) = 1.
$$

Since the left hand side of (60) is a decreasing function of $\Delta U$, the above equality proves that $\Delta \bar{U} = \Delta \bar{U}$.

By (18), $\pi_1^{m*} \geq \pi_1^* + 2n_2^*$ is equivalent to $n_1^{m*} \geq \frac{2n_2^*}{1 + n_2^*}$. Therefore, $\pi_1^{m*} \geq \pi_1^* + n_2^*$ iff

$$
\frac{2n_2^*}{1 + n_2^*} \exp \left( \frac{2n_2^*}{1 - n_2^*} - \frac{\Delta U + U_2^* - u_0 - \nu}{\nu} \right) + \frac{2n_2^*}{1 + n_2^*} + (M - 2)H \left( \frac{2n_2^*}{1 - n_2^*} \exp \left( \frac{2n_2^*}{1 - n_2^*} - \frac{\Delta U}{\nu} \right) \right) \leq 1.
$$

(61)
where \( n_j^* \) is given by (51). Note that the left hand side of (61) is a decreasing function of \( \Delta U \). Therefore, (61) is equivalent to \( \Delta U \geq \Delta U \), where \( \Delta U \) is the unique solution of
\[
\frac{2n_j^*}{1 + n_j^*} \exp \left( \frac{2n_j^* - \Delta U + U_j^* - u_j^0 - \nu}{\nu} \right) + \frac{2n_j^*}{1 + n_j^*} + (M - 2)H \left( \frac{2n_j^*}{1 + n_j^*} \exp \left( \frac{2n_j^* - \Delta U}{\nu} \right) \right) = 1. \tag{62}
\]
It follows from (58) that \( n_{1m}^* \) increases in \( \Delta U \). Since \( \frac{2n_j^*}{1 + n_j^*} < 2n_j^* \), we have \( \Delta U \leq \Delta U^\ast \).

By (20), \( \pi^{m*} \geq \pi^{m*} \) if \( n_{0m}^* \leq n_0^* \). Using (54), \( n_{0m}^* \leq n_0^* \) is equivalent to
\[
n_{jm}^{m*} \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) \leq n_j^* \exp \left( \frac{n_j^* - U_j^* - u_j^0 - \nu}{\nu} \right), \quad j = 3, \ldots, M. \tag{63}
\]
It directly follows that the above inequality is equivalent of \( n_{jm}^{m*} \leq n_j^* \) \( (j = 3, \ldots, M) \). Therefore, \( \pi^{m*} \geq \pi^{m*} \) iff \( \Delta U \geq \Delta U^\ast \) By (22) and (23), we have \( n_{jm}^{m*} \geq n_j^* \) iff \( \Delta U \geq \Delta U^\ast \).

For the industry profit after the merger, we will show that \( \pi_j^{m*} \) increases in \( \Delta U \). By (17), we have \( n_{jm}^{m*} = H \left( n_{jm}^{m*} \exp \left( \frac{n_{jm}^{m*} + \Delta U^\ast}{\nu} \right) \right) \) for \( j = 3, \ldots, M \). Take derivative with respect to \( \Delta U \), and use (38), we have
\[
\frac{\partial n_{jm}^{m*}}{\partial \Delta U} = \frac{(1 - n_{jm}^{m*})^2}{M - 2 + \frac{1}{n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}} \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) + \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right)} - \frac{\nu (1 - n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*})}{\nu (1 - n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}) + \Delta U^\ast} \tag{64}
\]
Apply Implicit Function Theorem to (55), we have for \( j = 3, \ldots, M \)
\[
\frac{\partial n_{jm}^{m*}}{\partial \Delta U} = \frac{1}{M - 2 + \frac{1}{n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}} \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) + \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right)} - \frac{\nu (1 - n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*})}{\nu (1 - n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}) + \Delta U^\ast} \tag{65}
\]
Substitute (18), (64) and (65) into (23), we have:
\[
\partial \pi_j^{m*} / \partial \Delta U = \frac{n_{jm}^{m*} \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right)}{(1 - n_{jm}^{m*} + n_{jm}^{m*})} \left( M - 2 + \frac{1}{n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}} \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) + \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) \right) \tag{66}
\]
At \( \Delta U = 0 \), \( \pi_j^* \) is the unique solution of \( \frac{n_{jm}^{m*}}{n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}} \left( M - 1 + \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) \right) = 1 \), and \( \pi_j^{m*} \) is the unique solution of \( \frac{n_{jm}^{m*}}{n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*} + n_{jm}^{m*}} \left( M - 1 + \exp \left( \frac{n_{jm}^{m*} - U_j^* - u_j^0 - \nu}{\nu} \right) \right) = 1 \). If \( \pi_j^{m*} \geq \pi_j^* \) at \( \Delta U = 0 \), then \( \pi_j^{m*} \geq \pi_j^* \) for any \( \Delta U \geq 0 \). Otherwise, if \( \pi_j^{m*} \leq \pi_j^* \) at \( \Delta U = 0 \), then there exists a threshold \( \Delta U^\ast \) such that \( \pi_j^{m*} \leq \pi_j^* \) if \( \Delta U \leq \Delta U^\ast \), and \( \pi_j^{m*} \geq \pi_j^* \) if \( \Delta U \geq \Delta U^\ast \). Since \( \pi_j^{m*} \geq \pi_j^* \) if \( \Delta U \in [\Delta U, \Delta U] \), we have \( \Delta U \leq \Delta U \).

Proof of Proposition 7 As shown in the proof of Proposition 6, \( \partial \pi_j^{m*} / \partial \Delta U > 0 \). By (20), we have
\[
\partial \pi^{m*} / \partial \Delta U = \left( -\nu / n_{jm}^{m*} \right) \left( \partial n_{jm}^{m*} / \partial \Delta U \right). \tag{67}
\]
By (16), \( n_{jm}^{m*} \) satisfies the equation \( n_{jm}^{m*} = H \left( n_{jm}^{m*} \exp \left( \frac{U_j^* + \Delta U^\ast - u_j^0}{\nu} \right) \right) + (M - 2)H \left( n_{jm}^{m*} \exp \left( \frac{U_j^* - u_j^0}{\nu} \right) \right) \) and \( \pi_j^{m*} \) increases in \( \Delta U \). Combine with part (I) and (IV) of Proposition 6, there exists a threshold values of the power index, \( \Delta U \) which lies between \( \Delta U \) and \( \Delta U \), such that \( \Pi^{m*} \leq \Pi^* \) if \( 0 \leq \Delta U \leq \Delta U \), and \( \Pi^{m*} \geq \Pi^* \) if \( \Delta U \geq \Delta U \).

Proof of Proposition 8 By (19), we have \( V_j^* = u_j^0 + \ln \left( n_j^* \nu \right) \) for \( j = 1, \ldots, M \), \( V_j^{m*} = u_j^0 + \ln \left( n_{jm}^{m*} \nu \right) \) for \( j = 3, \ldots, M \). Use (17), we have \( V_j^{m*} \geq V_j^* \) \( (j = 3, \ldots, M) \) if \( n_j^* \geq n_{jm}^{m*} \) \( (j = 3, \ldots, M) \).

By the result of Proposition 6, it follows that \( V_j^{m*} \geq V_j^* \) if \( \Delta U \geq \Delta U \) for \( j = 3, \ldots, M \).

Similarly, \( V_1^{m*} \geq V_1^* \) \( \iff \) \( n_{1m}^{m*} \leq \frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu (1 - n_j^*) \Delta U} \), which is equivalent to the following condition using (58):
\[
\frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu (1 - n_j^*) \Delta U} \left( \exp \left( \frac{n_j^* - U_j^* - u_j^0 - \nu}{\nu} \right) + 1 \right) + (M - 2)H \left( \frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu (1 - n_j^*) \Delta U} \exp \left( \frac{n_j^* - U_j^* - u_j^0 - \nu}{\nu} \right) \right) \geq 1. \tag{66}
\]
Note that the left-hand-side of (66) increases in $\Delta U$. Therefore, $V_1^{m*} \geq V_1^* \Leftrightarrow \Delta U \geq \Delta \hat{U}$, where $\Delta \hat{U}$ is the unique solution of

$$\frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu + (1 - n_j^*) \Delta U} \left( \exp \left( \frac{n_j^*}{1 - n_j^*} \cdot \frac{U_j^* - U_0^* - \nu}{\nu} \right) + 1 \right) + (M - 2) H \left( \frac{\nu n_j^* + (1 - n_j^*) \Delta U}{\nu + (1 - n_j^*) \Delta U} \exp \left( \frac{n_j^*}{1 - n_j^*} \right) \right) = 1. \tag{67}$$

Note that at $\Delta U = \Delta \hat{U}$ given by (57), and use (51), the left-hand-side of (66) is

$$\frac{2n_j^* + (1 - 2n_j^*) \ln 2}{1 + (1 - 2n_j^*) \ln 2} \left( \frac{1}{n_j^*} - M + 1 \right) + (M - 2) H \left( \frac{2n_j^* + (1 - 2n_j^*) \ln 2}{1 + (1 - 2n_j^*) \ln 2} \exp \left( \frac{n_j^*}{1 - n_j^*} \right) \right) > 2n_j^* \left( \frac{1}{n_j^*} - M + 1 \right) + (M - 2) H \left( n_j^* \exp \left( \frac{n_j^*}{1 - n_j^*} \right) \right) = 2 - Mn_j^* > 1.$$

Therefore, we have $\Delta \hat{U} < \Delta \overline{U}$.

For the share-weighted industry average aggregate price decision, it directly follows that $V_I^{m*} \leq V_I^*$ if $\Delta U \leq \Delta \hat{U}$, and $V_I^{m*} \geq V_I^*$ if $\Delta U \geq \Delta \overline{U}$. When $\Delta \hat{U} \leq \Delta U \leq \Delta \overline{U}$, $V_I^{m*} \geq V_I^*$ while $V_I^{m*} \leq V_I^*$ ($j = 3, \ldots, M$). By (19) and (25), we have $V_I^{m*} \geq V_I^*$ iff

$$\frac{n_j^{m*} \ln \left( \frac{n_j^{m*} n_0^*}{n_j^{m*} n_0^*} \right)}{(M - 2)n_j^{m*} \ln \left( \frac{n_j^{m*} n_0^*}{n_j^{m*} n_0^*} \right)} \geq 1, \quad j = 3, \ldots, M. \tag{68}$$

It follows from (58), (55) and (16) that $n_1^{m*}$ increases in $\Delta U$, $n_j^{m*}$ decreases in $\Delta U$ ($j = 3, \ldots, M$) and $n_0^{m*}$ decreases in $\Delta U$. By (17), we have $\frac{n_j^{m*}}{n_j^{m*}} = \exp \left( \frac{n_j^{m*}}{1 - n_j^{m*}} \cdot \frac{U_j^* - U_0^* - \nu}{\nu} \right)$, which decreases in $\Delta U$. Therefore, the left-hand-side of (68) increases in $\Delta U$. Since the left-hand-side of (68) is less than 1 at $\Delta U = \Delta \hat{U}$, and greater than 1 at $\Delta U = \Delta \overline{U}$. There exists a threshold value $\Delta \tilde{U} \in [\Delta \hat{U}, \Delta \overline{U}]$, such that $V_I^{m*} \leq V_I^*$ if $\Delta U \leq \Delta \tilde{U}$, and $V_I^{m*} \geq V_I^*$ if $\Delta U \geq \Delta \tilde{U}$. 

Note that the left-hand-side of (67) increases in $\Delta U$. Therefore, $\Delta \hat{U} < \Delta \overline{U}$. Therefore, we have $\Delta \hat{U} < \Delta \overline{U}$.