Operational hedging strategies and competitive exposure to exchange rates

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1. Introduction

1.1. Problem motivation

As firms are globalizing their supply chains, they are facing increasing exposure to currency fluctuations. The risks associated with currency exposure can be significant. Currency fluctuations can often be of 20–40% within a year, with the 1997 Asian Crisis resulting in even higher devaluations overnight for some of the weaker currencies (e.g., Indonesian Rupiah).

Robust profit maximizing performance and reasonable downside risk control. For example, Honda Motor Co., the Japanese auto maker, with 80% of its profits generated in the U.S. market, was predicted to reduce its income by $1.22 billion in 2004, because of the depreciation of dollar sales (Business Week 2004). German automakers suffered similar magnitude negative exposure effects in the face of the appreciating Euro during the 80s and early 90s (Harvard Business School case 9-796-030: Japan’s Automakers Face Endaka).

Unfortunately, it has always been extremely hard to predict not only the magnitude but even the direction of change (positive or negative) of economic exposure to exchange rate shocks, as we have witnessed even earlier in the days of “Endaka” and “Super-Endaka” periods (of consistently appreciating Yen relative to the dollar of the 80s and early 90s), with the auto-industry again the example (Harvard Business School case 9-796-030: Japan’s Automakers Face Endaka).

An exchange rate regime heavily influenced by the US government actions to favor the domestic car makers ended up leading to erosion of their own market shares, competitive position and profitability.
Exchange rate shocks substantially affect the relative competitive position of firms as they are changing their cost structures, with the exact magnitude depending on their supply and demand network structures and the product lines of the competing firms. As a result, economic exposures to exchange rate shocks are hard to counteract.

Generally speaking, there are two types of exposure to currency exchange rate, transaction (contractual) and competitive (operating) exposure (see Lessard and Lightstone, 1986 for a basic exposition of the concepts). Transaction exposure is caused by contracts denominated in a foreign currency. Competitive exposure explicitly captures the effects that currency fluctuations have on a company's future revenues and costs, as a result of the overall effect of such macroeconomic changes on the competitive position of the firm. Transaction exposure is easy to identify and estimate its magnitude as it appears in financial statements and contractual agreements, and under reasonable assumptions on the distribution of the future exchange rates, it can be effectively handled through financial hedging approaches (O'Brien, 1996). Competitive exposure is heavily dependent on the global supply chain structure and product markets of the competing firms, and it is hard to estimate, as we argued above. Managing competitive exposure needs a longer-term perspective, and cannot be dealt with solely through the use of financial hedging techniques. It is well recognized, and mostly anecdotally advocated, in the finance and international business literature (see Lessard and Lightstone, 1986 and Hertzell and Caspar, 1988 among others, with a more comprehensive listing left for our literature review, Section 1.2) that operational hedging, such as operational flexibility of a global network of production facilities, can be an effective long-term way to handle it. However, a counter arguing literature has been built around risk avoidance and variance reduction strategies predicated on “natural hedges” (and their implications for structuring the network of global facilities of the firm) and financial hedging approaches for handling such exposures.

A frequently adopted operational approach by global firms is to create a “natural hedge” (Pringle and Connolly, 1993; Pringle, 1995; Logue, 1995), which is also called “matching the currency footprints” (Harris et al., 1996), by locating production facilities in the country where sales are generated. This way, costs and revenues are matched in the same currency and the firm’s competitive exposure to currency fluctuation is eliminated. In an empirical study by Bodnar and Marston (2002), the authors document very low currency exchange exposure for a majority of firms in their sample that have used “natural hedges.” “Natural hedge” regains its popularity in the last few years. “Toyota Motor Corp., Renault SA (RNO) and Nissan Motor Co. are among carmakers widening their global production footprint to limit exposure to currency risk (Business Week, March 2013).” Meanwhile, US auto makers such as Ford and GM have increased their production in overseas markets despite their export-favoring weaker home currencies, but probably creating “natural hedges” in markets with increasing sales. However, the effectiveness of natural hedges is a point of heated debate (see Harris et al., 1996) to which our paper will add its unique, and reasonably conclusive, from a profit maximizing objective, viewpoint. The main thrust of our argument is as follows.

The “matching the currency footprints” approach typically results in flexible national or regional networks. As a result, it forgoes the operational flexibility of a global supply/production/distribution network in exchange for the false security of minimal risk of currency exposure of the firm’s own (preplanned, and non-anticipation of competitive reactions) cash flows. Consequently, a firm that pursues this “natural hedge” approach is less capable in improving its profit and controlling its downside risk. We will formally advance this point with a stylized model of a global firm competing with a local firm in a foreign market in the presence of exchange rate uncertainties.

The main focus of our paper is on the effects of operational flexibility, such as allocation flexibility, on the performance of firms facing global competition in the presence of currency uncertainties. “Allocation flexibility (defined in Ding et al., 2007) refers to the ability to supply two markets from one flexible facility and make market commitment ex post. We consider a global firm selling to both domestic and foreign markets. In the foreign market, the global firm encounters competition from a local supplier. We consider two distinct operational settings: the global firm (a) employs the “natural hedge” approach and (b) introduces allocation flexibility. For these operational settings, we explore (1) What are the effects of allocation flexibility on the global firm’s capacity strategy, and its deployment tactics, in response to the realized exchange rate shocks? and (2) How do allocation flexibility affect firms’ profits and risks in response to fluctuating exchange rates?

Our results show that operational flexibility improves the global firm’s expected profit and reduces its downside risk. Furthermore, the global firm’s flexibility increases the local firm’s downside risk, although it does not always decrease its expected profit. Moreover, we find that allocation flexibility mitigates the adverse effect of increasing competition in the foreign market on the global firm. These effects become more prominent as the volatility of exchange rate increases. Our analysis substantiates that allocation flexibility has robust performance in terms of both expected profit and downside risk in the presence of fluctuating exchange rates and foreign competition. Our results clearly disprove the effectiveness of “natural hedges” for dealing with competitive exposure for profit maximizing firms.

1.2. Literature review

Our work is related to the research of operational hedging in the operations management literature. In this literature, emphasis is given to modeling different types of operational hedging strategies, and the optimal operating (capacity, technology selection, inventory etc.) policy under operational flexibility in a single firm setting. Boyabatli and Toktay (2004) provide a survey of frequently used operational hedging strategies in operations management and identify two definitions of operational hedging in the literature: real options view and counterbalancing-action view. The real options view (Huchzermeier and Cohen, 1996) considers operational hedging strategies as real (compound) options that are exercised in response to demand, price and exchange rate contingencies. These real options have different forms: postponement of the allocation on foreign markets in Ding et al. (2007) and Kazaz et al. (2005), acquisitions in Hankins (2011), holding excess capacity in Cohen and Huchzermeier (1999), and switching options in Kogut and Kulatilaka (1994), Cohen and Huchzermeier (1999), Dasu and Li (1997), Li and Kouvelis (1999), and Aytekin and Birge (2004).

Specifically, the early influential work of Huchzermeier and Cohen (1996) values global manufacturing strategy options for a single firm (i.e., no competitive exposure considerations) under exchange rate risk. The operational hedge in their study is the use of excess capacity and production switching options. Their numerical study shows that operational hedging reduces the firm’s downside risk. Kogut and Kulatilaka (1994) develop a stochastic dynamic programming model to investigate option value of the costly switching production between two manufacturing plants located in different countries under exchange rate uncertainty. Aytekin and Birge (2004) generalize this work and show that financial hedging is preferred for low volatility exchange rates and operational hedging is preferred for high volatilities. Kouvelis et al. (2001) study the effects of real exchange rates on the choice and dynamic adjustment of ownership strategies of production facilities of global firms supplying foreign demand. Kazaz et al. (2005) examine two complementary forms of operational hedging, production hedging where the firm deliberately produces under
capacity before the realization of exchange rate uncertainty, and allocation hedging where the firm under-serves a market under unfavorable exchange rate realizations. They show that production and allocation hedging are features of a robust optimal policy under exchange rate uncertainty for a profit maximizing firm.

The counterbalancing-action view (Van Mieghem, 2007), which is much closer to the etymological meaning of the word “hedge”, defines operational hedging as “mitigating risk by counterbalancing actions in a processing network that do not involve financial instruments.” Strategies such as dual-sourcing, component commonality, transshipping, holding safety stock are subjects of study in an extensive literature (summarized in the above paper), often in a general supply chain setting with the main risks driven by demand uncertainties and without considering the effect of currency fluctuations. For the interested readers, see representative works of Harrison and Van Mieghem (1999) and Van Mieghem (2003), and more recent works of Chod et al. (2012) and Anupindi and Jiang (2008). Our paper, even though it bears conceptual similarities to the above literature, also differs from it in substantive ways. We study operational hedging approaches as counterbalancing acts to exchange rate shocks, with subsequent demand risks resulting from competitor reactions to the altered cost structure of the competing firms after the exchange rate realization. Within our work, the two viewpoints on operational hedging are reasonably unified, as our firm “hedges” its competitive exposure via appropriate exploitation of operational flexibility resulting from the exercise of operational options (resource pooling and allocation).

Our work is also related to research on firms’ economic exposure in the international business (finance/accounting) literature. This literature (Adler and Dumas, 1984; Lessard and Lightstone, 1986; Flood and Lessard, 1986; Hertzell and Caspar, 1988; Marston, 2001) defines and measures the firm’s different types of exposure to exchange rates, and investigates the hedging policies to mitigate it. The researchers have noticed that under global competition, the financial hedging restricted in pre-planned contracts based on exchange rate estimates, and isolated from strategic exploitation of real time managerial flexibility and relevant operating options, is not adequate to maintain firms’ competitive advantage. The literature has insightfully recognized the need for a joint use of operational hedges and financial hedges, but often advocates its use based on anecdotal stories of effectiveness for certain firms in certain instances. A recent research by Bartram et al. (2010) shows empirically that firms jointly use financial and operational hedges. Financial hedges decrease exposure by 40% and operational hedges reduce exposure by 10–15%.

A parallel stream of literature, with a strong preference for operational hedges, even though of very specific nature, to financial hedges advocates the use of the “matching currency footprints” approach (see Pringle and Connolly, 1993, Pringle, 1995, and Logue, 1995) as the way to manage large foreign currency exposures. Such an approach has implications on the structure and location of the global facility network for the firm, but after that it requires relatively simple currency contracts to hedge the risk associated with any remaining exposure. Bodnar and Marston (2002) provide empirical evidence supporting such an approach by showing that matching foreign currency revenues and costs would have been important in reducing the foreign exchange rate exposure of many U.S. firms. Harris et al. (1996) and Melumad et al. (2009) argue informally via numerical examples and anecdotal stories that global firms taking actions to match currency footprints to reduce variability in profitability may lose their strategic flexibility and lower expected profit, especially when their competitors can adjust quantities or prices. In our paper, we will formally illustrate, and prove within the confines of our stylized model, that the latter thesis is both correct and of serious implications for global firms. Furthermore, we will rigorously show that firms employing operational hedging via appropriate exploitation of operational flexibility (and exercise of appropriate operational options) can achieve both robust profit performance and control of downside risk in the presence of volatile exchange rates, while at the same time penalizing inflexible competitors.

Recently, researchers started to explore the integration of financial and operational hedging tools (for example, Dong and Liu, 2007, Caldentey and Haugh, 2006, Ding et al., 2007, Goel and Gutierrez, 2004, Gaur and Seshadri, 2005, Wong, 2007, and Zhu and Kapuscinski, 2011 and references therein). Most of this work is confined within models of single firm reacting to pricing, demand and/or currency risks, and is not concerned on the competitive exposure aspects of volatile exchange rates. Our paper models the impact on firm profit performance and risk of the competitive exposure to exchange rate shocks, and proves the effectiveness of the operational hedging approach in handling them.

The structure of the paper is as follows: in Section 2, we provide models of the two operational settings and the corresponding optimal capacity and selling decisions. In Section 3, we compare firms’ expected profits and downside risks associated with the two settings. In Section 4, we conduct sensitivity analysis with respect to the exchange rate volatility and the severity of competition in the foreign market (in terms of the established capacity). We conclude findings in Section 5.

2. Model: two operational settings

Consider a global firm (firm 1) selling to two markets: market 1, the domestic market, and market 2, a foreign market. In market 1, firm 1 is a monopolist; in market 2, firm 1 competes with an local firm (firm 2) that sells a perfectly substitutable product to market 2 exclusively. Each firm evaluates its own profit in its home currency.

We assume iso-elastic demand functions in both markets. The iso-elastic demand function is frequently used in econometric and empirical marketing research, due to its consistency with the consumer-utility-maximization theory, its unambiguous economic interpretation, and good statistical fit with available sales data. We refer readers to Monahan et al. (2004) for a thorough discussion of this assumption within a newsvendor setting and further references. The inverse demand function of the iso-elastic demand for market 1 is

$$ P_t(Q_t) = \theta Q_t^{-\epsilon} \epsilon_i, $$

(1)

where $P_t$ is the market clearing price for quantity $Q_t$ sold in market $i$, $\theta_i$ indicates market potential, $-1/q_i$ with $q_i \in (0,1]$ represents the elasticity of demand, and random variable $\epsilon_i$ represents the demand curve uncertainty, $i = 1, 2$. We assume that $\epsilon_i$’s are independently distributed with known cumulative distribution function (cdf) $G_i$, and probability density function (pdf) $g_i$, $i = 1, 2$. One interpretation of $\epsilon_i$ given by Petruzzi and Dada (1999) is that the demand curve is deterministic while the scaling parameter representing the market size is random. Thus, we refer to $\epsilon_i$, $i = 1, 2$, as demand scaling factors. We also assume that $\epsilon_i$ is independent of the market price $P_t$ and the currency exchange rate $s$. Without loss of generality we also assume that $E[\epsilon_i] = 1, i = 1, 2$. We make no distinction between the real exchange rate and the nominal exchange rate by assuming zero inflation in both countries. We assume that exchange rate $s$ is randomly distributed, according to a known cumulative distribution function (cdf) $F$, and corresponding probability density function (pdf) $f$. In order to deal with the currency exchange rate uncertainty and the demand uncertainty in both markets, firm 1 has the postponement option on the production decisions (defined in Van Mieghem and Dada, 1999). That is, the manufacturing process can be decomposed into two steps: Step 1 involves long-leadtime core production activities such as procuring and/or producing complex
components/subassemblies, and is performed long before the realization of uncertainties; and step 2 involves short-leadtime final assembly and configuration, and can be performed after the realization of uncertainties to meet the demand.

Firm 1 can employ different operational strategies depending on its product-process features. We consider two operational strategies (see Fig. 1):

**Strategy 1, natural hedge (N):** Firm 1 has one dedicated facility in each market serving exclusively that market. Strategy 1, locating manufacturing where the revenues are generated and thereby matching revenues and costs with little net exposure to exchange rate changes (also known as “matching currency footprints”), is a common practice among multinationals to handle the currency risk (Harris et al., 1996). Both facilities can perform the step 2 production after demand and exchange rate uncertainties are realized.

**Strategy 2, capacity pooling (C):** Firm 1 adds more operational flexibility upon strategy 1 through resource sharing between the two markets. Firm 1 has a single facility located in market 1, serving both markets. Firm 1’s products share the vanilla-middle-products and its production facility can delay the localization operation until the realization of uncertainties.

In practice, firms can also invest in various operational flexibilities such as multiple production facilities and switching options. In the scope of this paper, we consider allocation option and compare it with the natural hedge strategy (strategy 1) that has no allocation flexibility but creates natural hedge against the exchange rate fluctuations.

We divide the unit production cost of firm 1’s products into two parts: the time-consuming step 1 costs $C_1$ per unit for both markets, and the quick step 2 costs $\tau_i$ for market $i$ product, $i = 1, 2$. We assume $C_1$ and $\tau_i$ are the same for both strategies, which allows us to focus on the benefit of operational flexibility and their effects on firms’ exposures to exchange rate changes and profit risks. Clearly, under this assumption, the model offers upper bounds of the benefit of strategy. The benefits can then be compared to additional cost that might be required for its implementation. Without loss of generality, step 1 costs are measured in market 1 currency and the step 2 costs $\tau_i$ measured in market $i$ currency.

Following the lead of Dong et al. (2013), we assume that the local competitor, firm 2, in the foreign market pursues a local strategy and focuses on the local market only. Its local knowledge and close connection with local suppliers enable it to act more swiftly to capitalize the market opportunities. Therefore, the global firm, firm 1, and the local competitor, firm 2, are endowed with different flexibilities to cope with changes in market conditions (see Hill, 2012, Chapter 13: The Strategy of International Business). Firm 2’s production can be decomposed into two steps. We assume that step 1 production cost is $C_2$ per unit and step 2 production cost is negligible. Firm 2, being a local firm in market 2 without overseas operations or trade, has an established production facility of a fixed capacity. At step 2, firm 2 always sells full capacity to market 2, since the marginal revenue associated with the iso-elastic demand is always positive. The two firms’ joint production quantity determines the market clearing price at step 2 in market 2.

We use the following notation:

$s$ foreign market currency exchange rate at time 1, i.e., the value of one unit of the foreign currency measured in the domestic currency;

$\epsilon_i$ demand scaling factor for market $i$, $i = 1, 2$;

$\mu$ expected exchange rate, $\mu = E[s]$;

$K_{ii}$ firm 1’s time 0 production output level in market $i$, $i = 1, 2$ under strategy 1;

$K$ firm 1’s time 0 production output level, $K = (K_{11}, K_{12})$ under strategy 1, $K \equiv K_1$ under strategy 2;

$K_2$ firm 2’s existing capacity;

$C_1$ firm 1’s step 1 production cost per unit, measured in market 1 currency;

$\tau_i$ firm 1’s step 2 production cost per unit of producing market $i$ product, measured in market $i$ currency, $i = 1, 2$;

$q_i$ firm 1’s time 1 selling quantity to market $i$, $i = 1, 2$;

$f(\cdot), f(\cdot)$ the cumulative distribution function (cdf) and probability density function (pdf) of foreign exchange rate $s$;

$G_i(\cdot), g_i(\cdot)$ the cdf and pdf distributions of $\epsilon_i$, $i = 1, 2$;

$\pi_{i1}$ firm 1’s realized time 1 profit from market $i$, $i = 1, 2$.

We compare the two strategies in a two-stage stochastic program setting of time interval $[0, 1]$. For simplicity, we set the time discount rate to be zero. At time 0, firm 1 decides production output level $K$ and performs the step 1 production in the presence of the exchange rate and demand uncertainties. At time 1, after the realization of exchange rate and demand uncertainties, firm 1 determines selling quantity $q_1^*$ to each market ($q_1^*$ is firm 1’s optimal selling quantity to market $i$, $i = 1, 2$), given that firm 2 sells its full capacity to market 2.

Let $\Pi_i$, represent firm i’s time 0 expected profit, and $\pi_1, \pi_2$ the realized time 1 profit of firm 1 and firm 2, respectively. Expectation, with respect to the demand and exchange rate distributions, is denoted by $E$. For each strategy, firm 1’s two-stage decision can be formulated as

**Time 0:**

$$\max_{K \in K_1} \Pi_1(K, K_2),$$

where

$$\Pi_1(K, K_2) = [\pi_1(s, \epsilon_1, \epsilon_2, K, K_2, q_1^*, q_2^*)] - C_1 K_1$$

and

**Time 1:**

$$\pi_1(s, \epsilon_1, \epsilon_2, K, K_2, q_1^*, q_2^*) = \max(q_1(s, \epsilon_1, \epsilon_2, K, K_2, q_1, q_2)),$$

(2)

where $K \equiv (K_{11}, K_{12})$ under strategy 1 and $K \equiv K_1$ under strategy 2.

Firm 2 also has a two-stage production. At step 1, firm 2 has an established facility of fixed capacity $K_2$. At step 2, firm 2 always sells full capacity to market 2, since the marginal revenue associated with the iso-elastic demand is always positive. The market price $P_2$ is jointly determined by the total production quantity supplied by both firms. Firm 2’s expected profit under each strategy is expressed as follows:

$$\Pi_2(K, K_2) = [\pi_2(s, \epsilon_2, K, K_2, q_2^*)],$$

where

$$\pi_2(s, \epsilon_2, K, K_2, q_2^*) = [P_2(q_2^* + K_2) - C_2] K_2.$$ (3)

We now analyze the two strategies by deriving firm 1’s optimal time 1 selling decisions $q_1^*$ and $q_2^*$, and optimal time 0 capacity decision, $K_1^*$.

### 2.1. Strategy 1: natural hedge, one domestic and one foreign facility (N)

Firm 1 has two dedicated facilities in each country, serving markets 1 and 2 separately. At time 0, firm 1 produces "middle
products in quantities \( K_{11} \) and \( K_{12} \) for markets 1 and 2, respectively; at time 1, after the realization of the exchange rate and demand uncertainties, firm 1 decides the selling quantity to the two markets and produces finished products.

Firm 1's two-stage problem can be formulated as

\[
\text{Time 0:} \quad \max_{k_{11}, k_{12} \geq 0} \Pi_1^N(K_{11}, K_{12})
\]

where

\[
\Pi_1^N(K_{11}, K_{12}) = -C_1(k_{11} + K_{12}) + E_{11}[\pi_{11}(e_1, K_{11})] + E_{12}[\pi_{12}(s, e_2, K_{12})]
\]

and

\[
\text{Time 1:} \quad \pi_{11}(e_1, K_{11}) = \max_{0 \leq q_1 \leq k_{11}} (P_1(q_1) - r_1)q_1
\]

\[
\pi_{12}(s, e_2, K_{12}) = \max_{0 \leq q_2 \leq K_{12}} [sP_2(q_2 + K_{12}) - r_2q_2].
\]

Lemma 1 summarizes the firm 1's time 1 optimal selling decisions to markets 1 and 2.

**Lemma 1.** At time 1, given capacities \( K_{11} \) and \( K_{12} \) for markets 1 and 2, respectively, (1) firm 1's optimal selling quantity to market 1 is

\[
q_{11}^* = \begin{cases} k_{11}(e_1) & \text{if } e_1 \leq \tau_1(K_{11}) \\ k_{11} & \text{otherwise} \end{cases}
\]

where

\[
\tau_1(K_{11}) = \frac{K_{11}}{\theta_1(1 - a_1)} \quad \text{and} \quad k_{11}(e_1) = \left[ \frac{\theta_1 e_1}{\tau_1} (1 - a_1) \right]^{1/a_1}.
\]

(2) Firm 1's optimal selling quantity to market 2 is

\[
q_{12}^*(s, e_2, K_{12}) = \begin{cases} 0 & \text{if } e_2 \leq t_2(K_{12}) \\ \frac{q_{2U}(s, e_2, K_{12})}{p_2} & \text{if } t_2(K_{12}) < e_2 < t_2^*(K_{12}, K_{12}) \\ k_{12} & \text{if } t_2^*(K_{12}, K_{12}) \leq e_2 \end{cases}
\]

where

\[
t_2(K_{12}) = \frac{k_{2U}}{2} + \frac{r_2}{2}, \quad t_2^*(K_{12}, K_{12}) = \frac{\theta_2(k_{2U} + K_{12})^{a_2 + 1}}{\theta_2(k_{2U} + (1 - a_2)K_{12})}
\]

solves equation

\[
[\theta_2((1 - a_2)q_{2U} + K_{12})q_{2U} + K_{12}) - a_2 - 1]k_{2U} = \tau_2.
\]

Lemma 1 shows that the postponement option allows firm 1 to respond to the demand uncertainty. As illustrated in Table 1, for market 1: when demand is high in market 1, i.e., \( e_1 \leq \tau_1(K_{11}) \), firm 1 sells under-capacity at \( k_{11}(e_1) \); when demand is low in market 1, firm 1 sells at capacity \( K_{11} \). For market 2, when \( e_2 \) is small, it is not worthwhile to pursue market 2 because the low revenue cannot even recoup the localization cost \( r_2 \). As the value of \( e_2 \) increases and reaches threshold \( t_2(K_{12}) \), firm 1 starts to sell \( q_{2U} \) to market 2, and the selling quantity increases in \( e_2 \). When the value of \( e_2 \) exceeds threshold \( t_2^*(K_{12}, K_{12}) \), firm 1 sells at capacity \( K_{12} \) to market 2. Note that firm 1's selling decision in market 2 is not affected by the exchange rate under "natural hedge" because cost and revenue are matched in the foreign currency.

**Table 1.** Time 1 selling decisions under strategy 1.

<table>
<thead>
<tr>
<th>Market 1</th>
<th>Market 2</th>
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<tbody>
<tr>
<td>( e_1 \leq \tau_1(K_{11}) )</td>
<td>( q_{11}^* = k_{11}(e_1) )</td>
</tr>
<tr>
<td></td>
<td>( e_2 \leq t_2(K_{12}) )</td>
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<tr>
<td></td>
<td>( t_2(K_{12}) &lt; e_2 &lt; t_2^*(K_{12}, K_{12}) )</td>
</tr>
<tr>
<td></td>
<td>( q_{12}^* = 0 )</td>
</tr>
<tr>
<td>( e_1 &gt; \tau_1(K_{11}) )</td>
<td>( q_{11}^* = K_{11} )</td>
</tr>
<tr>
<td></td>
<td>( e_2 \geq t_2^*(K_{12}, K_{12}) )</td>
</tr>
<tr>
<td></td>
<td>( q_{12}^* = q_{2U} )</td>
</tr>
</tbody>
</table>

**Proposition 1.** Firm 1's expected profit \( \Pi_1^N(K_{11}, K_{12}) \) is concave in \( K_{11} \) and \( K_{12} \). The optimal capacity for market 1, \( K_{11}^* \), solves equation

\[
\int_{\tau_1(K_{11})}^{K_{11}} \theta_1 e_1 (1 - a_1)(K_{11}^{\theta_1})^{-\theta_1} - \theta_1 q_1(1 - 1/a_1) \, de_1 = \frac{1}{C_1},
\]

and the optimal capacity for market 2, \( K_{12}^* \), solves equation

\[
\int_{t_2(K_{12})}^{K_{12}} sP_2 K_{12} + (1 - a_2)K_{12} - r_2 q_2 \, de_2 = \frac{1}{C_1}.
\]

The optimal total capacity investment is \( K_{11}^* + K_{12}^* \).

Proposition 1 states that the market-specific optimal capacity should equalize the time 0 marginal production cost and expected time 1 market-specific marginal profit.

The following Proposition 2 states that firm 1's optimal capacity for market 2, \( K_{12}^* \), is not monotone in firm 2's established capacity. The rationale is similar to that under strategy 1.

**Proposition 2.** Firm 1's optimal capacity for market 2, \( K_{12}^* \), is concave in \( K_{2} \).

2.2. Strategy 2: capacity pooling strategy (C)

Firm 1 has a single facility located in his home country serving both markets. At time 0, before the realization of demand and exchange rate uncertainties, firm 1 produces vanilla-middle-products at quantity \( K_1 \). At time 1, after uncertainties are resolved, firm 1 decides selling quantities \( q_1 \) to market 1 and \( q_2 \) to market 2, with the total selling quantity being constrained by the capacity \( K_1 \).

Firm 1’s two-stage problem can be formulated as

\[
\text{Time 0:} \quad \max_{k_1} \Pi_1^C(K_1)
\]

where

\[
\Pi_1^C(K_1) = E_{e_1,e_2}[\pi_1^C(e_1, e_2, K_1)] - C_1 K_1
\]

and

\[
\text{Time 1:} \quad \pi_1^C(e_1, e_2, K_1, K_2) =\max_{0 \leq q_1 \leq \tau_1(K_1)} \left( (P_1(q_1) - r_1)q_1 + sP_2(q_2 + K_2) - r_2q_2 \right).
\]

Lemma 2 summarizes firm 1’s time 1 optimal selling decisions to markets 1 and 2.

**Lemma 2.** At time 1, given capacity \( K_1 \), (1) If market 1 has a low demand realization, \( e_1 \leq \tau_1(K_1) \), then firm 1's optimal selling quantities to markets 1 and 2 are

\[
q_{11}^C, q_{22}^C = \begin{cases} (K_1(\epsilon_1), 0), & e_2 \leq t_1(K_1) \\ (K_1(\epsilon_1), q_{2U}), & t_1(K_1) < e_2 < t_2^*(\epsilon_1, K_1, K_2) \\ (K_1 - q_{2U}, q_{2U}), & e_2 \geq t_2^*(\epsilon_1, K_1, K_2) \end{cases}
\]

where

\[
t_2^*(\epsilon_1, K_1, K_2) \equiv \frac{\theta_2 K_1 - k_{11}(\epsilon_1) + K_1(\epsilon_1)}{\theta_2 K_1 + (1 - a_2)(K_1 - k_{11}(\epsilon_1))}
\]

\( q_{2U} \) is as defined by (5) in Lemma 1, \( q_{2U} \) is the solution to equation

\[
(1 - a_1)\theta_1 K_1 - q_{2U} - \theta_1 = \frac{\theta_2 K_1 - k_{11}(\epsilon_1) + K_1(\epsilon_1)}{\theta_2 K_1 + (1 - a_2)(K_1 - k_{11}(\epsilon_1))} - \theta_1 q_{2U} - r_2
\]

(2) If market 1 has a high demand realization \( \epsilon_1 > \tau_1(K_1) \), then firm 1's optimal selling quantities to markets 1 and 2 are

\[
q_{11}^C, q_{22}^C = \begin{cases} (K_1(\epsilon_1), 0), & e_2 \leq t_1(\epsilon_1, s, K_1, K_2) \\ (K_1 - q_{2U}, q_{2U}), & t_1(\epsilon_1, s, K_1, K_2) < e_2 \end{cases}
\]
It is clear that $\pi_t^1 \leq \pi_t^2$ for any given $(s, \epsilon_1, \epsilon_2)$ and $(K_{11}, K_{12})$. Hence, $\pi_t^1 \leq \text{ISD} \pi_t^2$, where $\text{ISD}$ represents the first-order stochastic dominance. Intuitively, comparing strategies 1 and 2, the pooling of capacity in strategy 2 allows firm 1 to respond better to situations in which one market is more profitable than the other market by giving priority to the more profitable market when allocating the central capacity. Thus, strategy 2 dominates strategy 1. Immediately, we have the following proposition.

Proposition 4. (1) Given capacities $K_{11}$ and $K_{12}$, firm 1’s profits under the two strategies are ordered as: $\pi_t^1(K_{11}, K_{12}) \leq \text{ISD} \pi_t^2(K_{11} + K_{12})$.

(2) For any increasing function $u$, $\text{EU}(\pi_t^1) \leq \text{EU}(\pi_t^2)$ and the ranking of $u$-values under optimal capacities follows the same order.

(3) Under risk-neutral utility, the expected optimal profits are ordered as: $\Pi_t^1(K_{11}^\text{opt}) \leq \Pi_t^2(K_{12}^\text{opt})$.

Proposition 4 states that firm 1 will always prefer the strategy with more operational flexibility regardless of the utility function used. In particular, under risk neutral utility, firm 1’s optimal expected profit increases with more operational flexibility. However, the allocation option in strategy 2 does not necessarily lead to increased optimal capacity, and the reason is as follows. We consider firm 1’s optimal capacity under strategy 1, $K_{11}^\text{opt} + K_{12}^\text{opt}$, and we compare firm 1’s marginal revenue of capacity under strategy 1 and strategy 2 at time 0, which is the expectation, across all exchange rate and demand realizations, of his time 1 marginal revenue. Assuming demand is deterministic for simplicity, we first compute firm 1’s time 1 marginal revenue. When the exchange rate is relatively low and market 2 is not profitable enough for firm 1 to sell at large quantity, under strategy 2 firm 1 will sell more than $K_{11}^\text{opt}$ to market 1 and less than $K_{12}^\text{opt}$ to market 2. Thus, the capacity that would be unused under strategy 2 would be allocated to market 1 to earn profit under strategy 2, and the marginal revenue of capacity is higher under strategy 2 than under strategy 2. When the exchange rate is relatively high, firm 1 will sell less than $K_{11}^\text{opt}$ to market 1 and more than $K_{12}^\text{opt}$ to market 2. Since firm 1’s marginal revenue decreases in the selling quantity in market 2, in this case, the marginal revenue under strategy 2 is lower than that under strategy 2. In expectation, the time 0 marginal revenue of capacity under strategy 2 can be lower or higher than under strategy 2 depending on the relative effect of high and low exchange rates.

We use two measures to evaluate the downside risk associated with an operational strategy: VaR (value-at-risk) and EDR (expected downside risk). VaR, a widely used risk metric by financial institutions, regulators, nonfinancial corporations (e.g. multinationals), and asset managers (see Jorion, 2001), is defined as measuring the maximum expected loss over a given horizon under normal market conditions at a given confidence level $\alpha$. Specifically, if we define $V$ as $\Pr(\pi < V) = 1 - \alpha$, then $VaR = -V$. For example, if a bank says its daily VaR of its trading portfolio is $35$ million at the $99\%$ confidence level, it means there is $1\%$ chance, under normal market conditions, for a loss greater than $35$ million to occur. Given confidence level, the larger is the VaR value, the higher the downside risk. The EDR measure is defined by Huchzermeier and Cohen (1996) as the expected deviation of the firm’s profit over the planning horizon from a profit level $z$, $EDR(z) = \mathbb{E}[Z - \pi^*]$. Given target profit level, the higher is the EDR value, the higher the downside risk.

Proposition 5. If under strategy 1 firm 1 invests $K_{11}$ and $K_{12}$ for markets 1 and 2, respectively, and invests $K_{11} + K_{12}$ under strategy 2, then firm 1’s VaRs are ordered as $VaR^1 \geq VaR^2$, and EDRs are ordered as $EDR^1 \geq EDR^2$.
Proposition 5 states that for a given capacity, more operational flexibility enables firm 1 to respond more effectively in undesirable demand and exchange rate scenarios, and thus firm 1’s downside risk decreases. Numerical study (see examples in Section 4) shows that even when firm 1 adjusts capacity investment optimally, the downside risks associated with the two strategies follow the same ranking order.

The operational strategies employed by firm 1 also have impact on firm 2’s downside risks.

Proposition 6. Assume that firm 1 under strategy 1 invests $K_{11}$ and $K_{12}$ for markets 1 and 2, respectively, and invests $K_{11} + K_{12}$ under strategy 2. Firm 2’s profit is denoted as $\pi_2^f(t_2) = \{c_2 \theta_2(q_{2d}(t_2) + K_2) - z_2 - c_2[K_2, K_{12}]$.

1. Demand in market 1 is constant and low ($c_1 \leq T_1(K_{11})$), (1) If the confidence level for VaR satisfies $\alpha \geq G_2(t_2^R)$, firm 2’s VaRs are ordered as $\text{VaR}_R^2 \leq \text{VaR}_R^1$. (2) If the profit level for EDR measures $z \leq \pi_2^f(t_2^R)$, firm 2’s EDRs are ordered as $\text{EDR}_R^2 \leq \text{EDR}_R^1$.

2. Assume that there is no demand uncertainty, and demand in market 1 is high ($c_1 > T_1(K_{11})$). Let $s_{K_{12}} = (K_1 - K_{12})^{-1} \theta_1(1 - \theta_1) - r_1$

(1) If the confidence level for VaR satisfies $\alpha \geq G_2(s_{K_{12}})$, firm 2’s VaRs are ordered as $\text{VaR}_R^2 \leq \text{VaR}_R^1$. (2) If the profit level for EDR measures $z \leq \pi_2^f(s_{K_{12}})$, firm 2’s EDRs are ordered as $\text{EDR}_R^2 \leq \text{EDR}_R^1$.

Proposition 6 states that if market demand uncertainty is controlled, when the confidence level of VaR, $\alpha$, is high and when the benchmark profit level for EDR measure is low, firm 2’s downside risk increases as firm 1 employs more flexibility in its operations. Note that under strategy 1, when firm 1 matches his revenue and cost currencies, firm 2 is immune to the exchange rate fluctuation and her profit is only subject to the risk brought by high realization of market demand 2. Strategy 2 allows firm 1 to exploit the opportunity associated with the weakening of market 1’s currency by competing aggressively in market 2. Therefore, the more flexible is firm 1’s operation, the more aggressive he can compete in market 2, and the higher the loss firm 2 may potentially suffer.

Propositions 5 and 6 illustrate that a firm’s employment of operational flexibility affects both the firm’s and its competitor’s competitive positions in global markets. In particular, operational flexibility reduces a firm’s downside risk, and increases its competitor’s downside risk in the presence of the exchange rate uncertainty. The effect of operational flexibility on firm 2’s expected profit is more complex. Given that firm 2 always sells at capacity $K_2$, her profit is only affected by the market 2 price which in turn is affected by firm 1’s selling quantity to market 2. Under strategy 1, firm 1’s selling quantity to market 2 is only affected by market 2 demand realization $c_2$, and it increases in $c_2$ until reaching the capacity level. Under strategy 2, firm 1’s allocation to market 2 is jointly affected by the demand realizations in both markets and also increases in the exchange rate $s$ (Fig. 2).

4. Comparative statics

In this section, we examine, for each strategy, effects of variability of exchange rate and the severity of competition (reflected by the size of the established capacity $K_2$) in the foreign market on firm 1’s optimal capacity decisions, and on both firms’ profits and corresponding profit downside risks.

First, we adopt the definition of variability by Rothschild and Stiglitz (1970). We say that $Y$ has more variability than $X$ in the sense of RS if $E(X) = E(Y)$ and there exists a random variable $Z$ such that $Y = X + Z$ and $E(Z|X = x) = 0$ for all $x$. That is, $Y$ has the same distribution as $X$ plus a zero mean noise term, i.e., $Y$ can be obtained from $X$ by a sequence of mean-preserving spread of $X$, denoted as $X \leq \text{MPS}^2$. Intuitively, $Y$ has fatter tails on both ends than $X$. For some families of distributions, such as uniform, normal, and lognormal distributions, mean preserving spread can be obtained within the distribution family simply by adjusting distribution parameters.

The concept of mean-preserving spread is closely related to the concept of the second-order stochastic dominance (SSD), as shown by the following lemmas.

Lemma 3 (Rothschild and Stiglitz (1970)). Suppose $X$ and $Y$ are positive random variables with equal means. Then $X$ dominates $Y$ under SSD iff $Y$ has more variability in the sense of RS than $X$ iff $Y$ can be obtained from $X$ by a sequence of mean preserving spreads.

Lemma 4 (Milne and Neave (1994)). Suppose $E(X) = E(Y)$. Then $X$ dominates $Y$ under SSD iff $E_E(X) \geq E_E(Y)$ for every concave function $u$.

We provide a corollary that is useful for the subsequent analysis.

Corollary 1. If $X \leq \text{MPS}^2 Y$, then $E_E(X) \leq E_E(Y)$ for every convex function $v$.

4.1. Exchange rate risk

The effect of changes in the exchange rate volatility on a firm’s performance is largely determined by how effectively the firm responds to various exchange rate scenarios. Lemma 5 shows the partial sensitivity of firm 1’s time 1 profit to the exchange rate $s$ under each of the three operational strategies.

Lemma 5. (1) $\pi_2^{t_1}$ is independent of $s$ and $\pi_2^{t_1}$ is linear nondecreasing in $s$. (2) $\pi_2^{t_1}$ is convex nondecreasing in $s$.

Under strategy 1, by matching the currency footprints, firm 1’s time 1 profit increases at a constant rate as the exchange rate $s$ increases. In contrast, under strategy 2, firm 1’s profit increases in $s$ at an increasing rate. It shows that one of the important effects of allocation flexibility is to enable a firm to exploit favorable exchange rate realizations. Alternatively, one can view firm 1’s time 1 profit as a preference function over the exchange rate $s$, the
The convex increasing property suggests a risk-seeking preference over $s$ under strategy 2. Consequently, Proposition 7 shows the effect of exchange rate volatility on firm 1’s optimal capacity decision and optimal expected profit.

**Proposition 7.** As the variability of the exchange rate $s$ increases in the sense of RS, (1) under strategy 1, firm 1’s optimal capacities and expected profits in both markets remain constant; (2) under strategy 2, firm 1’s optimal capacity and expected profit increase.

Under strategy 1, matching of currency footprints decouples firm 1’s operational decision from the effect of the exchange rate fluctuations, thus firm 1’s optimal capacity decision and expected profit do not change when the exchange rate becomes more volatile. Under strategy 2, operational flexibility allows firm 1 to sell aggressively in market 2 when the exchange rate realization is favorable, thus the more volatile is the exchange rate, the more occurrences of favorable exchange rate that firm 1 can exploit, and the more profit firm 1 can gain from market 2. Therefore, firm 1 increases its optimal capacity as the exchange rate volatility increases and receives higher expected profit.

**Proposition 8.** Assume that there is no demand uncertainty in markets 1 and 2. Consider two exchange rates $s^{(1)}$, $i = 1, 2$, with $s^{(1)}$ having more variabilty than $s^{(2)}$ in the sense of RS, $s_2$ is the point such that $F^{(1)}(s) > F^{(2)}(s)$ for $s < s_2$, and $F^{(1)}(s) < F^{(2)}(s)$ for $s > s_2$. At a given capacity, (1) under both strategies, firm 1’s VaR is higher (resp. lower) under $s^{(1)}$ if the confidence level of VaR $\alpha$ is higher (resp. lower) than $F(s_2)$; (2) under Strategy 2, firm 2’s VaR is higher (resp. lower) under $s^{(1)}$ if the confidence level for VaR $\alpha$ is higher (resp. lower) than $F(s_2)$.

**Proposition 8** (1) states that for both strategies, given a capacity, under a reasonably high confidence level of VaR $\alpha$, the increase of the exchange rate volatility increases firm 1’s downside risk. This is because low realizations of exchange rate occur more often as the exchange rate becomes more volatile and firm 1’s profit increases in the currency exchange rate.

Under strategy 1, firm 2 has no exposure to the currency fluctuations. Proposition 8 (2) states that under strategy 2, for a reasonably high confidence level of VaR $\alpha$, when firm 1’s capacity is fixed, the increase of the exchange rate volatility also increases firm 2’s downside risk. Firm 2’s downside risk is associated with the presence of high exchange rate realizations under which firm 1 aggressively sells to market 2. The more volatile is the exchange rate, the more prevalent are occurrences of high exchange rate realizations.

Fig. 3 compares changes in firm 1’s optimal capacity decision, expected profit, and corresponding VaR and EDR as the exchange rate volatility increases, assuming there is no demand uncertainty in either market. Several observations are in order. First, firm 1’s optimal expected profit is higher under strategy 2 than under strategy 1. Second, firm 1’s downside risks, measured by VaR and EDR, are higher under strategy 1. Third, as the exchange rate volatility increases, firm 1’s optimal capacity and expected profit under strategy 1 stay the same, and they increase under strategy 2, as predicted in Proposition 7. Downside risks under both strategies increase. Finally, both VaR and EDR increase more slowly under strategy 2, because firm 1 can respond better at low exchange rate realizations by employing the allocation option. Fig. 4 compares changes in firm 2’s expected profit and downside risks as the exchange rate volatility increases, given that firm 1 adjusts capacity optimally and assuming that there is no demand uncertainty in either market. Interestingly, firm 2’s downside risks, both VaR and EDR, increase at a decreasing rate under strategy 2 and remain constant under strategy 1. This is a reflection of the changes of the market 2 clearing price at high exchange rates as the volatility of the exchange rate increases. At high exchange rates, under strategy 1, firm 1 sells to market 2 at full capacity $K^*_{21}$, whose value remains constant in the exchange rate volatility (as shown in Fig. 3). Similarly, the market 2 clearing price is constant in the volatility of exchange rates. Under strategy 2, although the full

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capacity $K_n^1$ is also increasing convexly in the exchange rate volatility, firm 1’s selling quantity to market 2 is strictly less than the full capacity (due to the market 1 effect) and is increasing concavely in the exchange rate volatility; consequently, the market 2 selling price increases concavely in the exchange rate volatility.

4.2. Firm 2’s capacity $K_2$

Firm 2’s capacity level $K_2$ represents the severity of the competition in market 2. The larger the value of $K_2$, the more severe the competition in the foreign market for global firm 1.
Proposition 9. Under both strategies, firm 1’s expected profit is convex nonincreasing in firm 2’s capacity \( K_2 \).

Proposition 9 states that the intuitive result that firm 1’s optimal profit decreases as competition in market 2 becomes more severe. Fig. 5 shows that for both strategies, the optimal capacity first increases then decreases in \( K_2 \) with the underlying rationale already given in the discussion of Proposition 2. Consequently, VaR first increases then decreases in \( K_2 \). Interestingly, strategy 2’s VaR starts decreasing earlier than for strategy 1. Fig. 6 shows that firm 2’s expected profit is first increasing then decreasing in \( K_2 \), while VaR first decreases then increases in \( K_2 \). Firm 2’s VaR under strategy 2 starts increasing earlier and faster than under strategy 1.

To study how operational flexibility mitigates the severity of market 2’s intensified competition on firm 1’s performance, we choose two values of \( K_2 = 5 \) and \( K_2 = 35 \), and plot the decrease of the optimal expected profit and the increase of VaR from \( K_2 = 5 \) to \( K_2 = 35 \) as functions of the exchange rate volatility in Fig. 7. The increase of VaR decreases under both strategies as the exchange rate volatility increases. The decrease of expected profit does not change under strategy 1, but decreases under strategy 2. This suggests that in environments of increasing volatility in exchange rates, operational flexibility becomes more effective in helping firm 1 to deal with the foreign market competition, as it enables firm 1 to effectively exploit the currency fluctuations.

5. Summary

In this paper, we establish a stylized model to advocate the use of operational hedging approach, such as allocation of shared resources among markets, in mitigating risks and enhancing long run profitability of global firms exposed to fluctuating exchange rates, we examine two different operational strategies of a global firm selling in the domestic market as a monopolist and competing in a foreign market against a local competitor. In the base case strategy, we use the often advocated paradigm of a “natural hedge” approach with the firm operating dedicated facilities for each market. The strategy provides little operational flexibility, but minimizes risks by matching currency footprints of revenues and costs. We introduce operational hedging via allocation option in strategy 2. Strategy 2 adopts risk pooling flexibility by allowing the two market products to share a vanilla middle product (or share capacity) and postponing the allocation of the middle product (or committing the capacity to market specific production) to after the realization of the exchange rate.

The operational flexibility, via the smart exercise of allocation option, enables the global firm to sell conservatively to the foreign market when the exchange rate is unfavorable and to sell aggressively when the exchange rate is favorable. Consequently, operational flexibility results in improving the firm’s expected profit and decreasing its downside risk, while at the same time exposing the local competitor to increased downside risks. Furthermore, for higher volatility exchange rate environments, operational flexibility effectively mitigates the adverse effects of increasing competition (mostly captured through increased capacity commitments of the local firm) in the foreign market on the firm’s expected profit and downside risks.

The global firm’s exchange rate driven selling decisions in the foreign market enabled by allocation option weaken the effectiveness of competitive responses in the, pp. 325-336 foreign market when the exchange rate is high (i.e., the firm’s
home currency is depreciated). However, the global firm is less well positioned to weather the adversity of competition when the exchange rate is low (i.e., appreciated home currency), but still operational flexibility can reduce the impact by diverting shared resources to the domestic market or even abandoning temporarily the servicing of the foreign market. As a result, the global firm’s flexibility not only improves its performance but also can adversely affect the local competitor’s performance. More specifically, it increases the local firm’s downside risk, and it can increase or decrease the local firm’s expected profit depending on the relative strength of effects from the low and high exchange rates.

Appendix A


Proof of Proposition 1. Take the first-order derivative of $\Pi^N_2$ w.r.t. $K_{11}$ and $K_{12}$, we have

$$
\frac{\partial \Pi^N_2(K_{11}, K_{12})}{\partial K_{11}} \bigg|_{K_{11} = K_{12}^0} = \int_{\tau_1(K_{11}^0)}^{+\infty} [\theta_1 e_1(1-a_1)(k^N_{11} - \epsilon_1 - \tau_1) + g_1(\epsilon_1) \, d\epsilon_1 - C_1 = 0,
$$

$$
\frac{\partial \Pi^N_2(K_{11}, K_{12})}{\partial K_{12}} \bigg|_{K_{12} = K_{12}^0} = \int_{\tau_1(K_{12}^0)}^{+\infty} [\theta_2 e_2(K_2 + k^N_{12}) - a_2 - 2(K_2 - a_2 k^N_{12}) - \tau_2 g_2(\epsilon_2) \, d\epsilon_2 - C_1 = 0.
$$

The concavity in $K_{11}$ and $K_{12}$ is straightforward by checking the second-order derivatives.

Proof of Proposition 2. Firm 1’s best response to $K_{2}, k^N_{12}(K_{2})$, is given in Eq. (7):

$$
C_1 = \frac{1}{\int_{\tau_1(K_{11})}^{+\infty}} s[e_2 \theta_2(K_2 + k^N_{12}) - a_2 - 2(K_2 - a_2 k^N_{12}) - \tau_2 g_2(\epsilon_2) \, d\epsilon_2 - C_1.
$$

We have

$$
\frac{\partial K^N_{12}}{\partial K_2} = \frac{\int_{\tau_1(K_{11})}^{+\infty} \theta_2 e_2(K_2 + k^N_{12}) \, d\epsilon_2(K_2 + k^N_{12}) - a_2 - 2(K_2 - a_2 k^N_{12}) - \tau_2 g_2(\epsilon_2) \, d\epsilon_2}{\int_{\tau_1(K_{11})}^{+\infty} \theta_2 e_2(K_2 + k^N_{12}) \, d\epsilon_2(K_2 + k^N_{12}) - a_2 - 2(K_2 - a_2 k^N_{12})} = \frac{K_2 - a_2 k^N_{12}}{2K_2 + (1-a_2)k^N_{12}}
$$

$$
\frac{\partial^2 K^N_{12}}{\partial K_2^2} = \frac{(a_2 + 1)(K_2 + k^N_{12}) (1-a_2)(k^N_{12} - K_2)}{(2K_2 + (1-a_2)k^N_{12})^2} < 0.
$$

Proof of Lemma 2. The Lagrangian of firm 1’s stage 2 optimization is $L_1 = \pi_1 - \lambda (q_1 + q_2 - K_1)$. Take the partial derivative of the Lagrangian, we have

$$
\frac{\partial L_1}{\partial q_1} = \theta_1 e_1 (1-a_1) - \tau_1 - \lambda = 0 \Rightarrow q^C_1 = q_1(s, \epsilon_1, \epsilon_2, K_1, K_2),
$$

$$
\frac{\partial L_1}{\partial q_2} = \left[\theta_2 e_2(q_2 + K_2) - a_2 \left(1 - \frac{a_2 q_2}{q_2 + K_2}\right) - \tau_2 \right] - \lambda = 0 \Rightarrow q^C_2 = q_2(s, \epsilon_1, \epsilon_2, K_1, K_2),
$$

$$
= \lambda \Rightarrow q_1 + q_2 - K_1 = 0.
$$

It is easy to see that the optimal allocation $(q^C_1, q^C_2)$ equalizes the marginal profits from two markets, i.e.,

$$
\frac{(q^C_1)^{-\alpha_1} \theta_1 e_1 (1-a_1) - \tau_1}{\text{marginal profit in market 1}} = \frac{(q^C_2)^{-\alpha_2} \theta_2 e_2(q_2 + K_2) - a_2 \left(1 - \frac{a_2 q_2}{q_2 + K_2}\right) - \tau_2}{\text{marginal profit in market 2}}.
$$

If $(q^C_1)^{-\alpha_1} \theta_1 e_1 (1-a_1) - \tau_1 < 0$, then $(q^C_1, q^C_2)$ can be obtained from (11) by setting $\lambda = 0$. That is, $q^C_1 = k_{11}(\epsilon_1), q^C_2 = q_{2u}$. Moreover, $q^C_1$ is non-increasing in $e_2$, and $q^C_2$ is non-decreasing in $e_2$. If $(q^C_1)^{-\alpha_1} \theta_1 e_1 (1-a_1) - \tau_1 > 0$, then $(q^C_1, q^C_2) = (q_{1b}, q_{2b})$.

1. Scenario 1 $(S_1) - e_1 \leq \tau_1(K_1)$: Firm 1 is not capacity constrained when he only sells to market 1, $q^C_1 = k_{11}(\epsilon_1)$. Firm 1 will not sell to market 2 until $e_2 > t_1(K_2)$, where $t_1(K_2)$ solves $e_2$ by setting $\lambda = 0$ and $q^C_2 = 0$. As $e_2$ increases, firm 1 increases the sales in market 2. Before firm 1 is binding in capacity, firm 1 treats the two markets separately, $q^C_1 = q_{2u}$ is given in Eq. (5). Firm 1’s capacity will be binding when $e_2 \geq t_2^+(e_1, K_1, K_2)$, where $q^C_2(e_2 = t_2^+) = K_1 - k_{11}(\epsilon_1)$, and $t_2^+(e_1, K_1, K_2)$ solves $e_2$ in Eq. (11) by setting $\lambda = 0$ and $q^C_1 = K_1 - k_{11}(\epsilon_1)$. For $e_2 > t_2^+$, $q^C_1 = K_1 - q_{2b}$, where $q_{2b}$ solves Eq. (11).

2. Scenario 2 $(S_2) - e_1 \leq \tau_1(K_1)$: Firm 1 is capacity constrained even when he only sells to market 1, $q^C_1 = K_1$ when $e_2 < t_2^+(e_1, s, K_1, K_2)$. As market 2 becomes more attractive, firm 1 starts to allocate to market 2 at $e_2 = t_3(e_1, s, K_1, K_2)$, where $t_3(e_1, s, K_1, K_2)$ solves $e_2$ in Eq. (11), by setting $q^C_1 = 0$ and $q^C_2 = K_1$. As $e_2$ increases further, $q^C_1 < K_1$, and $q^C_1 + q^C_2 = K_1$, thus, similar to (1), $q_{2b}$ can be obtained by solving Eq. (11) with $q^C_1 = K_1 - q_{2b}$. □
Proof of Proposition 3. We use the superscript $S_i$ (i = 1, 2) to denote firm $i$’s time 1 profit $\pi_i$ under Scenario $i$. Under Scenario 1,

$$\pi_1^{(i)}(s, e_1, e_2, K_1, K_2) = \begin{cases} 
- C_i K_1 - (\theta_i e_1 (q_{2i} - q_{1i}) - \pi_1 - \tau_1) q_{1i}^c, & e_2 \leq t_1(K_2) \\
- C_i K_1 - (\theta_i e_1 (q_{2i} - q_{1i}) - \pi_1 - \tau_1) q_{1i}^c + s(e_2 \theta_2 (K_2 + q_{2i}) - \pi_2 - \tau_2) q_{2i}, & t_1(K_2) < e_2 < t_2^c, \\
- C_i K_1 - (\theta_i e_1 (K_1 - q_{1i}) - \pi_1 - \tau_1) (K_1 - q_{1i}) + s(e_2 \theta_2 (K_2 + q_{2i}) - \pi_2 - \tau_2) q_{2i}, & e_2 \geq t_2^c
\end{cases}$$

and

$$\frac{\partial \pi_1^{(i)}(s, e_1, e_2, K_1, K_2)}{\partial K_1} = \begin{cases} 
- C_i, & e_2 \leq t_2^c \\
- C_i + \theta_i e_1 (1 - a_1)(K_1 - q_{1i}) - \pi_1 - \tau_1, & e_2 > t_2^c
\end{cases}$$

and

$$\frac{\partial^2 \pi_1^{(i)}(s, e_1, e_2, K_1, K_2)}{\partial K_1^2} = \begin{cases} 
0, & e_2 \leq t_2^c \\
- a_1 \theta_1 e_1 (1 - a_1)(K_1 - q_{1i}) - \pi_1 - \tau_1 - 1 \left( -1 - \frac{\partial q_{2i}}{\partial K_1} \right), & e_2 > t_2^c
\end{cases}$$

By implicit function theorem,

$$\frac{\partial q_{2i}}{\partial K_1} = \frac{a_1 \theta_1 (1 - a_1)(K_1 - q_{1i})^{-\pi_1 - \tau_1 - 1}}{a_1 \theta_1 (1 - a_1)(K_1 - q_{1i})^{-\pi_1 - \tau_1 - 1} + s e_2 \theta_2 (K_2 + q_{2i})^{-\pi_2 - \tau_2 - 2} (K_2 + (1 - a_2) q_{2i})} \in (0, 1).$$

Hence,

$$\frac{\partial^2 \pi_1^{(i)}(s, e_1, e_2, K_1, K_2)}{\partial K_1^2} < 0, \quad \text{for } e_2 \geq t_2^c.$$

Therefore,

$$\begin{align*}
(E_{e_1, e_2} [\pi_1^{(i)}(s, e_1, e_2, K_1, K_2)]_{K_1}) = & \mathbb{E} \int_0^{t_1(K_2)} \int_{t_1(K_2)}^{\infty} \frac{\partial \pi_1^{(i)}(s, e_1, e_2, K_1, K_2)}{\partial K_1} \bar{g}_2(e_2) \bar{g}_1(e_1) \, de_2 \, de_1, \\
(E_{e_1, e_2} [\pi_1^{(i)}(s, e_1, e_2, K_1, K_2)]_{K_1}) = & \mathbb{E} \int_0^{t_1(K_2)} \int_{t_1(K_2)}^{\infty} \frac{\partial \pi_1^{(i)}(s, e_1, e_2, K_1, K_2)}{\partial K_1} \bar{g}_2(e_2) \bar{g}_1(e_1) \, de_2 \, de_1 < 0.
\end{align*}$$

Similarly, under Scenario 2,

$$\begin{align*}
\pi_1^{(2)}(s, e_1, e_2, K_2) = & \begin{cases} 
- C_i K_1 + (\theta_i e_1 K_1^{-\pi_1 - \tau_1 - 1}) q_{1i}^c, & e_2 \leq t_3 \\
- C_i K_1 + (\theta_i e_1 (K_1 - q_{1i}) - \pi_1 - \tau_1) (K_1 - q_{1i}) + s e_2 \theta_2 (K_2 + q_{2i})^{-\pi_2 - \tau_2 - 2} (K_2 + (1 - a_2) q_{2i}) - \pi_2 - \tau_2, & e_2 > t_3
\end{cases}
\end{align*}$$

and

$$\frac{\partial \pi_1^{(2)}(s, e_1, e_2, K_1, K_2)}{\partial K_1} = \begin{cases} 
- C_i - \theta_i e_1 (1 - a_1)(K_1 - q_{1i})^{-\pi_1 - \tau_1}, & e_2 \leq t_3 \\
- C_i - \theta_i e_1 (1 - a_1)(K_1 - q_{1i})^{-\pi_1 - \tau_1} + s e_2 \theta_2 (K_2 + q_{2i})^{-\pi_2 - \tau_2 - 2} (K_2 + (1 - a_2) q_{2i}) - \pi_2 - \tau_2, & e_2 > t_3
\end{cases}$$

and

$$\frac{\partial^2 \pi_1^{(2)}(s, e_1, e_2, K_1, K_2)}{\partial K_1^2} = \begin{cases} 
0, & e_2 \leq t_3 \\
- a_1 \theta_1 e_1 (1 - a_1)(K_1 - q_{1i})^{-\pi_1 - \tau_1 - 1} - 1 \left( -1 - \frac{\partial q_{2i}}{\partial K_1} \right), & e_2 > t_3
\end{cases}$$

Therefore,

$$\begin{align*}
(E_{e_1, e_2} [\pi_1^{(2)}(s, e_1, e_2, K_1, K_2)]_{K_1}) = & \mathbb{E} \int_0^{t_1(K_2)} \int_{t_1(K_2)}^{\infty} \frac{\partial \pi_1^{(2)}(s, e_1, e_2, K_1, K_2)}{\partial K_1} \bar{g}_2(e_2) \bar{g}_1(e_1) \, de_2 \, de_1, \\
(E_{e_1, e_2} [\pi_1^{(2)}(s, e_1, e_2, K_1, K_2)]_{K_1}) = & \mathbb{E} \int_0^{t_1(K_2)} \int_{t_1(K_2)}^{\infty} \frac{\partial \pi_1^{(2)}(s, e_1, e_2, K_1, K_2)}{\partial K_1} \bar{g}_2(e_2) \bar{g}_1(e_1) \, de_2 \, de_1 < 0.
\end{align*}$$

At time 0,

$$\frac{\partial \Pi_1^{(1)}(K_1, K_2)}{\partial K_1} = - C_i + \int_0^{\tau_1} \left( E_{e_1, e_2} [\pi_1^{(1)}(s, e_1, e_2, K_1, K_2)]_{K_1} \bar{g}_1(e_1) \right) \, de_1 + \int_{\tau_1(K_1)}^{\infty} \left( E_{e_1, e_2} [\pi_1^{(1)}(s, e_1, e_2, K_1, K_2)]_{K_1} \bar{g}_1(e_1) \right) \, de_1,$n

$$\frac{\partial^2 \Pi_1^{(1)}(K_1, K_2)}{\partial K_1^2} = - C_i + \int_0^{\tau_1} \left( E_{e_1, e_2} [\pi_1^{(1)}(s, e_1, e_2, K_1, K_2)]_{K_1} \bar{g}_1(e_1) \right) \, de_1 + \int_{\tau_1(K_1)}^{\infty} \left( E_{e_1, e_2} [\pi_1^{(1)}(s, e_1, e_2, K_1, K_2)]_{K_1} \bar{g}_1(e_1) \right) \, de_1 + \int_{\tau_1(K_1)}^{\infty} \left( E_{e_1, e_2} [(\pi_1^{(1)}(s, e_1, e_2, K_1, K_2)]_{K_1} \bar{g}_1(e_1) \right) \, de_1,$n

$$+ \left( \tau_1(K_1) \right) \left( E_{e_1, e_2} [(\pi_1^{(1)}(s, e_1, e_2, K_1, K_2)]_{K_1} \bar{g}_1(e_1) \right) \, de_1.$$
Therefore, firm 1’s profit is concave in $K_1$ at time 1. It is trivially true that firm 1’s profit at time 0 is concave. □

**Proof of Proposition 4.**

(1) Straightforward from the arguments in Section 3.
(2) It follows Milne and Neave (1994).
(3) The expected profit comparison follows (2). □

**Proof of Proposition 5.** Omitted. It trivially holds by Proposition 4. □

**Proof of Proposition 6.** For the domestic firm 2, $\pi_2 = [e_2 \theta_2(q_2 + K_2)^{-\alpha_2} - C_2] K_2$. $\pi_2$ is non-increasing in s and $e_2$. Firm 2’s downside risk is associated with realization of large s or $e_2$.

1. Demand in market 1 is constant and low ($e_1 \leq \tau_1(K_1)$). Under strategy 1, $\pi_i^N = [e_2 \theta_2(K_1 + K_2)^{-\alpha_2} - C_2]K_2$, for $e_2 \geq t_2^N$. Under strategy 2, (1) If $\tau_1(K_1) \geq e_1$, $q_1^N \leq K_1$ for $e_2 \geq t_2^N$. Therefore, we have $\pi_2^N(e_2) \geq \pi_2^N(e_2)$ for $e_2 \geq t_2^N$. The definition of VaR, with VaR = $-V$ and $Pr(\tau_2(e_2) \leq V) = 1 - \alpha$ is equivalent to $Pr(\pi_2(e_2) \geq V) = \alpha$. Since $\pi_2$ is non-increasing in s, it follows that $\alpha = G_2(\pi_2^{-1}(V))$, where $G_2$ is the CDF of $e_2$. If $\alpha \geq G_2(t_2^N)$, then $\pi_2^{-1}(1 - \alpha) = G_2^{-1}(t_2^N)$ for both strategies, which implies $V^N \geq V^C$ and VaR$^N \leq$ VaR$^C$. (2) It follows that $EDR(\pi_2^N) = \frac{\partial}{\partial e_2} E\{\tau_2(e_2) = \tau_2([s - \pi_2^N(e_2)]^2G_2(e_2) \, de_2)\} = \frac{\partial}{\partial e_2} E\{\tau_2(s - \pi_2^N(e_2)]^2G_2(e_2) \, de_2\}$. Moreover, since $\pi_2^N(e_2) \geq \pi_2^N(e_2)$ for $e_2 \geq t_2^N$, it follows that $\pi_2^N(1 - \alpha) \leq \pi_2^N(1 - \alpha)$. Thus, we have for $s \geq \pi_2^N(e_2)$, $EDR(\pi_2^N) = \frac{\partial}{\partial e_2} E\{\tau_2(e_2) = \tau_2([s - \pi_2^N(e_2)]^2G_2(e_2) \, de_2\} \leq \frac{\partial}{\partial e_2} E\{\tau_2(e_2) = \tau_2([s - \pi_2^N(e_2)]^2G_2(e_2) \, de_2\} = EDR(\pi_2^N)$.

2. There is no demand uncertainty and market 1 demand is high ($e_1 \geq \tau_1(K_1)$), $\pi_1^N(s) = [e_2 \theta_2(q_2 + K_2)^{-\alpha_2} - C_2] K_2$, where $q_2 \geq K_2$ for $s \geq s_0$, where $s_0$ solves $s_0 = K_1$ in equation (7). The rest of the proof follows the Proof of Proposition 6.1. □

**Proof of Lemma 3.** Omitted. □

**Proof of Lemma 4.** Omitted. □

**Proof of Corollary 1.** Omitted. □

**Proof of Lemma 5.** (1) Under strategy 1, firm 1’s market 1 profit $\pi_{11}^N$ independent of s. Firm 1’s time 1 profit in market 2 is as follows:

$$
\pi_{12}^N = \begin{cases} 
0, & t_1(K_2) \geq e_2 \\
\theta_2 e_2(q_2 + K_2)^{-\alpha_2} - \tau_2 q_2, & t_1(K_2) < e_2 < t_2^N(K_1, K_2) \\
\theta_2 e_2(K_2 + K_2)^{-\alpha_2} - \tau_2 K_2, & e_2 \geq t_2^N(e_1, K_1, K_2) 
\end{cases}
$$

Therefore, firm 1’s profit $\pi_{12}^N$ is linear in s. (2) Under strategy 2, for the low demand scenario of market 1 ($S_1$), by Envelope Theorem:

$$
\frac{\partial^2 \pi_{12}^N}{\partial s^2} = \begin{cases} 
0, & t_1(K_2) \geq e_2 \\
0, & t_1(K_2) < e_2 < t_2^N(K_1, K_2) \\
\theta_2 e_2(q_2 + K_2)^{-\alpha_2} - \tau_2 q_2 \frac{\partial q_2}{\partial s} > 0, & e_2 \geq t_2^N(K_1, K_2) 
\end{cases}
$$

(13)

For the high demand scenario of market 1 ($S_2$):

$$
\frac{\partial^2 \pi_{12}^N}{\partial s^2} = \begin{cases} 
0, & e_2 \leq t_2^N(e_1, s, K_1, K_2) \\
\theta_2 e_2(q_2 + K_2)^{-\alpha_2} - \tau_2 q_2 \frac{\partial q_2}{\partial s} > 0, & e_2 \geq t_2^N(e_1, s, K_1, K_2) 
\end{cases}
$$

Therefore, $\pi_{12}^N$ and $\pi_{12}^N$ are convex nondecreasing in s. □

**Proof of Proposition 7.**

(1) Under strategy 1, firm 1’s capacity and production decisions are made based on expected exchange rate and demand. As long as the expectation remains the same, the optimal capacity level and expected profit remain the same for strategy 1.

(2) Under strategy 2, by Lemma 5 (2), firm 1’s time 1 profit is convex nondecreasing in s:

$$
\frac{\partial^2 \pi_1}{\partial K_1} = -C_1 + E_t \left[ \int_{c_1}^{c_2} \frac{\partial \pi_1^N}{\partial K_1} \, dc_1 \right]
$$

For the low demand scenario of market 1 ($S_1$):

$$
\frac{\partial^2 \pi_1}{\partial K_1} = \begin{cases} 
0, & e_2 \leq t_2^N(e_1, K_1, K_2) \\
\theta_2 e_2(q_2 + K_2)^{-\alpha_2} - \tau_2 q_2(1 - \alpha_2) - \tau_2, & e_2 > t_2^N(e_1, K_1, K_2) 
\end{cases}
$$

(14)
For the high demand scenario of market 1 (S₂):

\[
\frac{\partial^2 \pi_1^c}{\partial K_1 \partial s} = \begin{cases} 
0, & e_2 \leq t_2'(e_1, s, K_1, K_2) \\
\frac{\partial \pi_1}{\partial s}(K_2 + q_{2s}) \left( -a_2 - q_{2s} + \frac{\partial \pi_1}{\partial s} \right) + \frac{\partial^2 \pi_1}{\partial s^2} q_{2s}, & e_2 > t_2'(e_1, s, K_1, K_2).
\end{cases}
\]  

(15)

For \( e_2 > t_3(e_1, s, K_1, K_2) \) under \( S_1 \) and \( e_2 > t_2(e_1, s, K_1, K_2) \) under \( S_2 \):

\[
\frac{\partial^2 \pi_1}{\partial s^2} = a_1 \theta_1 (1-a_1)(K_1 - q_{2s})^{-a_1} \frac{\partial \pi_1}{\partial s}.
\]

By Lemma 2, \( q_{2s} \) solves

\[
H(q_{2s}, s) = \mathbb{P}(K_2 + (1-a_2)q_{2s}, q_{2s} + K_2 - a_2 - q_{2s} - e_2 - \tau_2) - (1-a_1)\theta_1(e_1)(K_1 - q_{2s})^{-a_1} - \tau_1 = 0.
\]

By Implicit function theorem,

\[
\frac{\partial q_{2s}}{\partial s} = -\frac{\partial H}{\partial q_{2s}} = -\frac{\partial \pi_1}{\partial s} \frac{\partial \pi_1}{\partial q_{2s}} \frac{\partial q_{2s}}{\partial s} = \frac{\partial^2 \pi_1}{\partial s^2} \frac{\partial q_{2s}}{\partial s} > 0
\]

\[
\frac{\partial H}{\partial q_{2s}} = -\frac{\partial \pi_1}{\partial s} \frac{\partial \pi_1}{\partial q_{2s}} \frac{\partial q_{2s}}{\partial s} = \frac{\partial^2 \pi_1}{\partial s^2} \frac{\partial q_{2s}}{\partial s} > 0.
\]

Therefore, \( \frac{\partial q_{2s}}{\partial s} > 0 \) and \( \frac{\partial^2 \pi_1}{\partial s^2} \frac{\partial q_{2s}}{\partial s} > 0 \). \( \frac{\partial \pi_1}{\partial K_1} \) is convex in \( s \).

\[
\frac{\partial^2 \pi_1^c}{\partial s \partial q_{2s}} \frac{\partial q_{2s}}{\partial s} \geq 0, \quad \frac{\partial \pi_1}{\partial K_1} \frac{\partial q_{2s}}{\partial s} > 0.
\]

which implies \( K_1^c(s) \geq K_1^c(s') \).

\( \square \)

**Proof of Proposition 8.**

(1) Without loss of generality, we prove the case of \( \alpha \geq 1 - F^{[1]}(s_2) \). By Muller and Stoyan (2002, Definition 1.5.25), if \( F^{[1]} \) differs from \( F^{[2]} \) by a mean preserving spread, then they cross at \( s = s_2 \) with \( F^{[1]} \geq F^{[2]} \) for \( s < s_2 \) and \( F^{[1]} < F^{[2]} \) for \( s > s_2 \). For a given confidence level \( \alpha \), we have \( 1 - \alpha = P_{\pi_1}(s \leq \pi_1^{-1}(V_{i}^{[1]})) = P_{\pi_1}(s \leq \pi_1^{-1}(V_{i}^{[2]})) \). For \( \alpha \geq 1 - F^{[1]}(s_2) \), then \( F^{[1]}(s_2) \geq 1 - \alpha = F^{[1]}(\pi_1^{-1}(V_{i}^{[1]})) = F^{[2]}(\pi_1^{-1}(V_{i}^{[2]})) \).

It follows that \( \pi_1^{-1}(V_{i}^{[1]}) < \pi_1^{-1}(V_{i}^{[2]}) \), \( V_{i}^{[1]} < V_{i}^{[2]} \), and \( \text{VaR}^{[1]} > \text{VaR}^{[2]} \).

(2) Without loss of generality, we prove the case of \( \alpha \geq F^{[1]}(s_2) \). For a given confidence level \( \alpha = P_{\pi_1}(\pi_2(s) \geq V_{i}^{[1]})) = P_{\pi_1}(\pi_2(s) \geq V_{i}^{[2]})) \), \( i = 1, 2 \). If \( \alpha \geq F^{[1]}(s_2) \), then \( F^{[1]}(\pi_2^{-1}(V_{i}^{[1]})) = F^{[2]}(\pi_2^{-1}(V_{i}^{[2]})) \) for \( \alpha \geq F^{[1]}(s_2) \). It follows that \( \pi_2^{-1}(V_{i}^{[1]}) < \pi_2^{-1}(V_{i}^{[2]}) \), \( V_{i}^{[1]} < V_{i}^{[2]} \), and \( \text{VaR}^{[1]} > \text{VaR}^{[2]} \).

\( \square \)

**Proof of Proposition 9.** We use strategy 2 as an example. The proof for strategy 1 follows the same type of analysis. Taking the first derivative of the firm 1’s optimal expected profit with respect to \( K_2 \), we have

\[
\frac{\partial \Pi_1^c}{\partial K_2} = E_{\pi_1} \left[ \int_0^{\tau_1} \left( \int_{t_1}^{t_2} \frac{\partial H}{\partial q_{2s}} \frac{\partial \pi_1}{\partial q_{2s}} \frac{\partial q_{2s}}{\partial s} \right) g_2(e_2) g_1(e_1) de_2 de_1 \
+ \int_{t_1}^{t_2} \frac{\partial H}{\partial q_{2s}} \frac{\partial \pi_1}{\partial q_{2s}} \frac{\partial q_{2s}}{\partial s} \left( \int_{t_1}^{t_2} \frac{\partial q_{2s}}{\partial q_{2s}} \right) g_2(e_2) g_1(e_1) de_2 de_1 \right] 
\]

where

\[
\frac{\partial q_{2s}}{\partial s} = \frac{\partial q_{2s}}{\partial s} \left( \frac{\partial \pi_1}{\partial s} \right) \frac{\partial q_{2s}}{\partial s},
\]

\[
\frac{\partial^2 \pi_1}{\partial s^2} \frac{\partial q_{2s}}{\partial s} > 0.
\]

Since

\[
\frac{\partial^2 \pi_1}{\partial s^2} < 0, \quad \frac{\partial q_{2s}}{\partial s} > 0,
\]

it follows \( H \geq 0 \). Similarly \( J \geq 0 \). Therefore,

\[
\frac{\partial^2 \Pi_1^c}{\partial K_2^2} = E_{\pi_1} \left[ \int_0^{\tau_1} \left( \int_{t_1}^{t_2} \frac{\partial \pi_1}{\partial q_{2s}} \frac{\partial q_{2s}}{\partial s} \right) g_2(e_2) g_1(e_1) de_2 de_1 + \int_{t_1}^{t_2} \frac{\partial \pi_1}{\partial q_{2s}} \frac{\partial q_{2s}}{\partial s} \left( \int_{t_1}^{t_2} \frac{\partial q_{2s}}{\partial q_{2s}} \right) g_2(e_2) g_1(e_1) de_2 de_1 \right] \geq 0.
\]

\( \square \)
References


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