Horizontal Coordinating Contracts in the Semiconductor Industry

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Integrated device manufacturers (IDMs) and foundries are two types of manufacturers in the semiconductor industry. IDMs integrate both design and manufacturing functions while foundries solely focus on manufacturing. Since foundries often have cost advantage over IDMs due to their specialization and economies of scale, IDMs have incentives to source from foundries for the purpose of avoiding excessive capacity investment risk. As the IDM is also a potential capacity source, the IDM and foundry are in a horizontal setting rather than a purely vertical setting. In the absence of sophisticated contracts, the benchmark contract for the IDM and foundry is a wholesale price contract. We define “coordinating” contracts as those that improve both the IDM’s and foundry’s expected profits over the benchmark wholesale price contract and also lead to the maximum system profit. This paper examines if there exist coordinating capacity reservation contracts. It is found that wholesale price contracts in the horizontal setting cannot achieve the maximum system profit due to either double marginalization effect, or “misalignment of capacity-usage-priority”. In contrast, if the IDM’s capacity investment risk is not too low, there always exist coordinating capacity reservation contracts. Furthermore, under coordinating contracts, the IDM’s sourcing structure, either sole sourcing from the foundry or dual sourcing, is contingent on the firms’ cost structures.

Keywords: supply chain management, horizontal capacity coordination, reservation contract, wholesale price contract in horizontal setting, sourcing structure.

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1. Introduction

In the semiconductor industry, there are two types of manufacturers. Integrated device manufacturers (IDMs) both design and manufacture semiconductor devices, while foundries concentrate only on manufacturing and take orders from IDMs and/or fabless firms. For both firm types, the cost of establishing production capacity is extremely high, and equipment technology becomes obsolete quickly as new technologies and demanding product requirements emerge. Therefore, manufacturers have to keep a consistently high fab utilization in order to pay back the capital expenditure in a timely manner. On the demand side, the customers of such manufacturers are themselves established firms that produce products such as cell phones, personal computers, and automobiles, among others. To gain orders from these customers, the semiconductor manufacturers need to be cost-effective, responsive, and flexible in accommodating highly fluctuating and uncertain order quantities. Because of this hard-to-match supply and demand situation, only a small number of manufacturers – typically the larger ones in terms of capacity, such as Intel (IDM), Samsung (IDM) and TSMC (foundry) – are consistently profitable. The rest fare less well on their own and are in need of horizontal coordination to justify the needed investment to reach a competitive level of economies of scale and flexibility. In 2009, IDMs like Texas Instruments, Freescale Semiconductor, STMicroelectronics, and Renesas Electronics are estimated to outsource their production to foundries at percentages of 55%, 23%, 20%, and 10% to 20%, respectively (IC Insights 2011).

Due to the significant lead times required to add capacity, capacity decisions and related investment outlays are made well ahead of actual demand. Once the capacity is installed, the rest of the production activities are conducted in a make-to-order fashion. When an IDM and a foundry interact in a decentralized fashion, from the IDM’s perspective, it is important to secure enough capacity internally and/or externally to meet uncertain demand without incurring excessive costs. Since only the IDM has access to the end market, the foundry has to work with the IDM in order to be profitable. On the other hand, the foundry is concerned about over-committing capacity. The IDM has to offer the foundry incentives to induce sufficient capacity commitment.

In the absence of sophisticated contracts, the benchmark contract for the IDM and foundry is a wholesale price contract. In the vertical supply chain setting, a wholesale price contract cannot coordinate the supply chain due to the well known double marginalization effect (Spengler 1950). The IDM and foundry are not in a vertical supply chain relationship,
because the IDM himself can produce the product in addition to the option of sourcing the product from the foundry. Thus, the IDM and foundry are in a horizontal setting. This paper identifies new features of wholesale price contracts in the horizontal setting compared to the vertical setting, and investigates whether wholesale price contracts can achieve the maximum system profit (i.e., the maximum sum of the IDM’s and foundry’s expected profits).

A prevailing type of contract used in the semiconductor industry is the capacity reservation contract. According to the structure of reservation contracts, the IDM reserves capacity $R$ from the foundry by paying a reservation fee per unit. The reservation fee is refundable, which means the IDM pays to the foundry a wholesale price deducted by the reservation fee for a unit of product when the purchasing quantity is less than $R$. When $R = 0$, the IDM always pays the wholesale price. That is, the pathological case of $R = 0$ in reservation contracts involves a single contract parameter, “wholesale price”, and is actually a wholesale price contract in the horizontal setting. Hereafter, we refer to wholesale price contracts in the horizontal setting as zero reservation contracts (ZRCs). We emphasize two differences between ZRCs and traditional wholesale price contracts in the vertical setting. First, under ZRCs, the foundry may or may not build capacity, even in some cases with the wholesale price exceeding her cost. That is, even though the IDM is willing to pay the wholesale price, there may be no available supply from the foundry. In contrast, in traditional wholesale price contracts, the buyer can always buy whatever quantity he needs from the supplier for a reasonable wholesale price (e.g., a price that exceeds the supplier’s cost). Second, under ZRCs, the IDM may himself be a source of supply in addition to the foundry. But under traditional wholesale price contracts in the vertical setting, the buyer solely relies on the supplier. Due to these nuances of wholesale price contracts in the horizontal setting, we refer to them as “ZRCs”.

We refer to reservation contracts with positive capacity reservation ($R > 0$) as positive reservation contracts (PRCs). A PRC is coordinating if it satisfies the following three conditions.

(1) **Capacity investment coordination**: The resulting capacity investments are the same as in the centralized system that maximizes the system profit.

(2) **Production coordination**: The production decisions (i.e., the demand allocation decisions) are consistent with those in the centralized system.
(3) *Individual rationality*: The resulting IDM’s and foundry’s expected profits are both greater than what they can earn under the benchmark wholesale price contract. Conditions (1) and (2) are required to achieve the maximum system profit. Condition (3) guarantees that the contract is individually rational for both the IDM and foundry. We refer to coordinating *PRCs*, if they exist, as *CPRCs*.

In our paper, we pursue answers to the following questions: (1) What are the centralized capacity investments of the IDM and foundry that maximize the system profit? What are the main factors affecting such capacity investments? (2) Can *ZRCs* maximize the system profit? If not, are the underlying reason(s) the same as in the vertical setting? (3) Under what circumstances do *CPRCs* exist? What is the resulting sourcing structure for the IDM? The answers to these questions will offer us a better understanding of the distinctions between horizontal capacity coordination and traditional vertical supply chain coordination.

This paper is organized as follows. Section 2 reviews the relevant literature. Section 3 introduces the model assumptions and notation. Section 4 derives the centralized capacity investments. Sections 5 and 6 study *ZRCs* and *PRCs*, respectively, to investigate their roles in achieving the maximum system profit when the IDM and foundry interact in a decentralized scheme, and check if there exist *CPRCs*. Section 7 compares the firms’ expected profits under *PRCs* with those under *ZRCs* and illustrates the impact of profit margin. Section 8 provides concluding remarks.

### 2. Literature Review

Vertical supply chain coordination has received substantial attention in the operations literature. Various forms of contracts are proposed to align supply chain partners’ decisions with those of the centralized system. Among these are buy-back (Pasternack 1985), revenue-sharing (Cachon and Lariviere 2005), quantity-flexibility (Tsay 1999), and sales-rebate contracts (Taylor 2002). Cachon (2003) provides a comprehensive review of this literature before 2002. More recently, Tomlin (2003) studies a vertical supply chain with a supplier selling a key component to the manufacturer who processes the component into end product. Tomlin (2003) proves the existence of a class of price-only contracts that arbitrarily allocate the supply chain profit between the supplier and manufacturer. Chick et al. (2008) show that cost-sharing contract can coordinate the vertical influenza vaccine
supply chain with yield uncertainty. In contrast to the above papers, we emphasize horizontal capacity coordination between an IDM and a foundry using capacity reservation contracts. There are dual capacity sources in meeting end-demand (both the foundry and IDM may build capacity for the end product), but without direct access to the market, the foundry serves as a subcontractor to the IDM.

Reservation contracts have been studied as part of vertical supply chain coordination in Erkoc and Wu (2005) and Jin and Wu (2007). Wu et al. (2005) provide an excellent survey of the reservation contracts literature. Erkoc and Wu (2005) propose two variants of capacity reservation contracts: partially deductible reservation contract whose reservation fee is partially deducted if the reserved capacity from the supplier is used, and cost-sharing contract for which the buyer pays a portion of the capacity cost associated with her reservation. Jin and Wu (2007) consider deductible and take-or-pay reservation contracts for vertical supply chain coordination, and extend the model from one customer to two or more customers. Brown and Lee (1998) study “pay-to-delay” capacity reservation contracts and derive optimal policies for the buyer in the semiconductor industry. Our paper differs from the above work in that it has the buyer (IDM) not only decide how much capacity to reserve from the supplier (foundry), but also how much of his own capacity to build, thus injecting horizontal coordination concerns.

Another stream of literature combines reservation contracts and the spot market, such as Serel et al. (2001), Wu et al. (2002), Spinler et al. (2003), Wu and Kleindorfer (2005), Spinler and Huchzermeier (2006), Fu et al. (2010), and Inderfurth et al. (2013). In these papers, a buyer can reserve capacity from a supplier in addition to buying from the spot market. In our paper, the IDM can reserve capacity from the foundry in addition to his own capacity if built upfront. However, the supply from the spot market is infinite while in our paper, the supply is limited and the capacities at the IDM and foundry are all that can be used after the demand realization. Recently, Peng et al. (2012) propose a dual-mode equipment procurement framework, with both modes using reservation contracts. Both the base and flexible suppliers in their paper are in a vertical relationship with Intel, which differs from the horizontal setting in our paper that the IDM does not need to use any contract for own capacity. In addition, we focus on the coordinating capability of reservation contracts, but they are interested in the value of the added flexibility of dual-mode for the firm.
Capacity expansion is an important topic in operations management. Van Mieghem (2003) provides a comprehensive review of the literature. Our work differs from single-resource, multiagent capacity models (e.g., Cachon and Lariviere 1999) in that both the IDM and foundry can build core capacity. The paper that explicitly addresses horizontal capacity coordination and is most closely related to ours is Van Mieghem (1999). He considers horizontal capacity coordination between two firms to satisfy two distinct market demands, and the two firms make capacity decisions simultaneously. In our paper, both the IDM and foundry are potential supply sources to meet one market demand. The IDM is the only one with market access, and he usually builds his own capacity before subcontracting the rest of his needs to the foundry. Hence, a sequential-move capacity game is more appropriate for our setting. Furthermore, our work studies contracting between the IDM and foundry using reservation contracts besides linear price contracts.

Finally, there is a stream of literature on interfirm surplus inventory transshipment that addresses horizontal coordination between/among retailers by setting ex-ante transshipment prices before demand realizations (Rudi et al. 2001 and Hu et al. 2007). A common feature of transshipment papers is that firms first satisfy their own demand using their own inventory and then transship surplus inventories to others or receive inventories from others. Our capacity coordination differs from inventory coordination in that the IDM does not necessarily use his own capacity first. He may use the foundry’s first as long as it is economically justified.

To summarize, our paper is one of the earliest to study horizontal capacity coordination in the presence of uncertain demand using capacity reservation contracts. Our studied setting is unique in the asymmetric roles of the IDM and foundry: Only the IDM has access to the market while both serve as capacity sources. The capacity decisions of the IDM and foundry are modeled by a sequential-move game with the IDM as the leader. Our paper aims at deriving horizontal coordinating contracts as important references for the IDM and foundry in their choice of contractual relationship.

1 Nevertheless, we have also checked the results under simultaneous-capacity-investment setting, i.e., the setting with the IDM and foundry simultaneously deciding their capacity investments after the IDM’s capacity reservation decision. We have found that the insights derived in this paper about the coordinating capability of reservation contracts still hold under simultaneous-capacity-investment setting.
Step 1: Under ZRCs, the IDM commits his production capacity by $K_s$ and goes to Step 2.

Under PRCs, the IDM commits his production capacity by $K_s$ and reserves $R(R > 0)$ units of the foundry’s capacity.

Step 2: Observing the IDM’s capacity commitment decision, the foundry determines her own capacity commitment $K_f$, with $K_f \geq R$.

Step 3: The demand is realized.

Step 4: The IDM determines the production quantity of his own, $z$, and the production order to the foundry, $y$, subject to the capacity constraints.

Table 1

| Protocol of Capacity Investment and Production Decisions under PRCs and ZRCs |

3. Model Assumptions and Notation

The IDM (he) designs a device and sells it for a unit price of $p$ to customers. The demand is realized in one period and is expressed as a random variable $X$. $f(x)$ is its probability density function where $f(x) > 0$ for $x \geq 0$, and $f(x)$ is differentiable. $F(x)$ is its cumulative distribution function where $F(x)$ is differentiable and invertible for $x \geq 0$. Anticipating an uncertain demand to meet, the IDM first decides whether to do business with the foundry (she); if so, the IDM and foundry agree upon a contract to use. The contracts considered in this paper are positive reservation contracts (PRCs) and wholesale price contracts in the horizontal setting (ZRCs). Given the contract, the IDM and foundry execute the capacity investment and production decisions following the sequence specified in Table 1.

Before the demand is realized, the IDM and foundry commit their production capacities $K_s$ and $K_f$, respectively, and expend the corresponding capacity costs $v_sK_s$ and $v_fK_f$. Let $K = (K_s, K_f)$. After the demand $x$ is realized, the IDM internally produces $z$ units and places an order to the foundry of $y$ units so that the ex-post profit of the IDM, $\pi_s$ is maximized. The corresponding ex-post profit of the foundry is denoted by $\pi_f$. The per-unit production costs of the IDM and foundry are $c_s$ and $c_f$, respectively. The IDM subcontracts manufacturing to the foundry for a unit subcontract price of $w$.

In PRCs, the IDM proposes to reserve $R (R > 0)$ units of the foundry’s capacity, and the foundry must commit at least $R$ units of capacity to the IDM. In return, the IDM pays to the foundry $rR$ in total as a capacity reservation fee, where $r$ is the per-unit reservation fee with $0 < r \leq w$. When the foundry delivers $y$ units to the IDM, she receives a unit price of $w - r$ for the quantity less than or equal to $R$ and $w$ for any quantity above $R$. That is, the total revenue for the foundry is $yw + (R - y)r$ for $y \leq R$ and $yw$ for $y > R$. This
pricing scheme shifts part of the capacity investment risk from the foundry to the IDM. In return, the IDM secures a certain amount of capacity from the foundry. On the other hand, in ZRCs there is no capacity reservation decision.

In this paper, we assume the IDM and foundry hold symmetric information about \( p \), the cost structure \( v_s, v_f, c_s, c_f \), and the demand distribution. These assumptions are reasonable in the semiconductor industry. The selling price \( p \) is verifiable in a business-to-business setting. Cost structure can be effectively guessed by each other since most players in this industry are long-term partners with repeated dealings, and they have a good understanding of the other’s facility and technology; furthermore, some third party (e.g., iSuppli) provides products such as integrated circuit (IC) cost evaluator, which estimates the cost and price involved in IC procurement. The symmetric information about the demand distribution can be viewed as a result of joint forecasting based on common economic indicators.

Throughout this paper, in order to avoid trivial cases, we assume: \( p > c_s + v_s \), \( w > c_f + v_f \), \( p > w \), \( 0 < r \leq w \), and \( c_s, v_s, c_f, v_f > 0 \). As foundries often have the cost advantage over IDMs due to their dedicated focus on manufacturing, we assume \( c_f + v_f < c_s + v_s \). Let \( K^*_s \) and \( K^*_f \) denote the equilibrium capacity for the IDM and foundry, respectively, \( R^* \) denote the IDM’s equilibrium reservation capacity out of the capacity game specified in Table 1, and \( K^* = (K^*_s, K^*_f) \). All the notation is summarized in Table 2.

4. Centralized Capacity Investments

As assumed in the protocol of Table 1, the IDM and foundry are part of a decentralized system and take actions sequentially, with each trying to maximize its own expected profit. In contrast, when the IDM and foundry collaborate, with their capacity investment decisions dictated by a central planner to maximize the sum of their expected profits, we refer to the IDM and foundry forming a centralized system (CS). Note that the CS has the choice between two production technologies: producing at a cost of \( c_s \) if using the IDM’s capacity, and \( c_f \) if using the foundry’s capacity. Based on the relationship between \( c_s \) and \( c_f \), the CS gives priority of usage to the one with lower production cost. If \( c_s = c_f \), the CS is indifferent on the priority of use of the IDM’s or foundry’s capacity. If \( c_s > c_f \) \((c_s < c_f)\), then after the demand is realized, the foundry’s (IDM’s) capacity is used first. In contrast to the purely vertical coordination problem (only one firm can build capacity and serves as the supplier of the other firm), the horizontal capacity coordination problem between
$x$ realized demand
$X$ demand random variable
$F(\cdot)$ cumulative distribution function (cdf) of $X$; $\bar{F}(\cdot) = 1 - F(\cdot)$
$f(\cdot)$ probability density function (pdf) of $X$
$p$ per-unit price of product
$w$ per-unit subcontract price of product
$c_s, c_f$ variable production cost for the IDM and foundry, respectively
$v_s, v_f$ variable capacity cost for the IDM and foundry, respectively
$K_s, K_f$ capacity of the IDM and foundry, respectively
$r$ per-unit capacity reservation fee
$R$ reserved capacity
$y$ production quantity for the foundry
$z$ production quantity for the IDM
$MOF$ $\frac{p - c - v}{v}$, i.e., profit margin over fixed cost
$ZRC$ zero reservation contract
$ZRC_f$ $ZRC$ with the foundry building positive capacity
$PRC$ positive reservation contract
$CCPRC$ candidate coordinating positive reservation contract
$CPRC$ coordinating positive reservation contract
$\pi_s, \pi_f$ ex-post profits for the IDM and foundry, respectively
$\Pi_s, \Pi_f$ expected profits for the IDM and foundry, respectively
$\Pi_*'$ IDM’s expected profit under the “IDM independent” case
$\Pi_{cs}$ centralized system profit
$N_T^T(\mu, \sigma^2)$ truncated normal distribution with mean $\mu$ and variance $\sigma^2$ over $(0, \infty)$

<table>
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<tr>
<th>Table 2</th>
<th>Notation</th>
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the IDM and foundry leads to three objective functions depending on the values of $c_s$ and $c_f$. Let $\Pi_{cs}$ be the centralized system profit. The three cases are summarized in Lemma 1.

**Lemma 1.**

For $c_s > c_f$, $\Pi_{cs} = (p - c_f) \int_0^{K_f} (1 - F(x)) \, dx + (p - c_s) \int_{K_f}^{K_s + K_f} (1 - F(x)) \, dx - v_s K_s - v_f K_f.$

(1)
For $c_f > c_s$, \[ \Pi_{cs} = (p - c_f) \int_{K_s}^{K_s + K_f} (1 - F(x)) \, dx + (p - c_s) \int_0^{K_s} (1 - F(x)) \, dx - v_sK_s - v_fK_f. \] (2)

For $c_f = c_s$, \[ \Pi_{cs} = (p - c_s) \int_0^{K_s + K_f} (1 - F(x)) \, dx - v_sK_s - v_fK_f. \] (3)

In Proposition 1, we derive the centralized capacities for the IDM and foundry, denoted by $K_c = (K_{cs}, K_{cf})$, to maximize the system profit, where the subscript “c” denotes “centralized” decision. Let us call \( \frac{p-c-v}{v} \) as the profit margin over fixed cost (MOF). \[ MOF_s = \frac{p-c_s-v_s}{v_s} \] and \[ MOF_f = \frac{p-c_f-v_f}{v_f}. \]

**Proposition 1.** The centralized capacity investments $K_c = (K_{cs}, K_{cf})$ to maximize the system profit are summarized in the following table:

<table>
<thead>
<tr>
<th>$c_s &gt; c_f$</th>
<th>$MOF_s \leq MOF_f$</th>
<th>$MOF_s &gt; MOF_f$</th>
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</thead>
<tbody>
<tr>
<td>$c_f &gt; c_s$</td>
<td>$c_f + v_f &lt; c_s + v_s$ implies this case is impossible</td>
<td>$0, F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$</td>
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Proposition 1 indicates that the IDM’s (foundry’s) capacity should be positive if $MOF_s \geq MOF_f$ ($MOF_s < MOF_f$). MOF is an important measure for the optimization of the centralized capacity investments. This is because when using a single capacity source, either the IDM’s or foundry’s capacity to meet the demand, the news vendor critical fractile is \( \frac{p-c_s-v_s}{p-c_s} = \frac{MOF_s}{MOF_s+1} \) for the IDM and \( \frac{p-c_f-v_f}{p-c_f} = \frac{MOF_f}{MOF_f+1} \) for the foundry (please refer to Porteus 2002 for the definition of “news vendor critical fractile”). Thus, a larger MOF implies a larger news vendor critical fractile. The centralized total capacity is always determined by the larger news vendor critical fractile out of \( \frac{p-c_s-v_s}{p-c_s} \) and \( \frac{p-c_f-v_f}{p-c_f} \).

Dual sourcing occurs (i.e., both firms must commit positive capacity) when the IDM has a larger profit margin over fixed cost ($MOF_s \geq MOF_f$). In this case, although the total capacity is determined by the IDM’s news vendor critical fractile \( \frac{p-c_s-v_s}{p-c_s} \), it is optimal for the centralized system to have the foundry build part of the capacity since the foundry’s per-unit total cost and production cost are both smaller than the IDM’s. Given the total capacity is targeted at $F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)$, if the foundry builds one less unit of capacity, then the IDM has to build an additional unit. Thus, the underage cost of the foundry’s capacity for the CS is $c_s + v_s - c_f - v_f$ and the corresponding overage cost is $v_f - v_s$; it is optimal for the CS to have the foundry build the capacity of $F^{-1}\left(\frac{c_s + v_s - c_f - v_f}{c_s - c_f}\right)$, and have the IDM build the rest.
To summarize, there are two types of contracting relationship: If \( MOF_f > MOF_s \), then the resulting centralized capacity is \( (0, F^{-1} \left( \frac{w-c_f-v_f}{p-c_f} \right)) \), which is complete subcontracting, or outsourcing to the foundry case. If \( MOF_s \geq MOF_f \), then the resulting centralized capacity is \( \left( F^{-1} \left( \frac{w-c_s-v_s}{p-c_s} \right) - F^{-1} \left( \frac{c_s+v_s-c_f-v_f}{c_s-c_f} \right), F^{-1} \left( \frac{c_s+v_s-c_f-v_f}{c_s-c_f} \right) \right) \), which is dual sourcing from both the IDM and foundry.

The centralized capacity and production decisions lead to the maximum system profit. However, in the absence of a central planner, the IDM and foundry are part of a decentralized system and in need of contracts to coordinate their decisions. In the following, we discuss two types of contracts in the decentralized horizontal setting, wholesale price contracts (ZRCs) and PRCs, with the focus on the latter to investigate whether they are coordinating.

5. Wholesale Price Contract in the Horizontal Setting (ZRC)

In this section, we investigate whether wholesale price contracts in the horizontal setting (i.e., ZRCs) can achieve the maximum system profit. First, we derive firms’ expected profits under ZRCs. There are two cases: \( w \leq c_s \) and \( w > c_s \). When \( w \leq c_s \), the IDM always allocates the demand to the foundry first, because buying from the foundry is less expensive than producing by his own capacity. We call this the “foundry-first case”. \( w > c_s \) corresponds to the “IDM-first case”. That is, the IDM uses his own capacity first to produce. Hereafter, if necessary, we emphasize the expected profits under ZRCs using the superscript “\( z \)”; that is, \( \Pi^z_s \) and \( \Pi^z_f \) are expected profits for the IDM and foundry under ZRCs. They are expressed in Lemma 2.

**Lemma 2.** If \( w \leq c_s \), \( \Pi^z_f = (w-c_f) \int_0^{K_f} (1-F(x)) \, dx - v_f K_f \) and \( \Pi^z_s = (p-w) \int_0^{K_f} (1-F(x)) \, dx + (p-c_s) \int_{K_f}^{K_s+K_f} (1-F(x)) \, dx - v_s K_s \).

If \( w > c_s \), \( \Pi^z_f = (w-c_f) \int_{K_s}^{K_s+K_f} (1-F(x)) \, dx - v_f K_f \) and \( \Pi^z_s = (p-w) \int_{K_s}^{K_s+K_f} (1-F(x)) \, dx + (p-c_s) \int_0^{K_s} (1-F(x)) \, dx - v_s K_s \).

The capacities as a result of the IDM and foundry maximizing \( \Pi^z_s \) and \( \Pi^z_f \), respectively, are expressed in Proposition 2.

**Proposition 2.** The equilibrium capacities for the IDM and foundry \( (K^*_s, K^*_f) \) under ZRCs can be expressed as follows:

For \( w \leq c_s \), there are two cases:
\[
\begin{align*}
\frac{w-c_f}{v_f} &< \frac{p-c_s}{v_s} \quad \left( F^{-1}\left( \frac{w-c_s-v_s}{p-c_s} \right) - F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \\
\frac{p-c_s}{v_s} &< \frac{w-c_f}{v_f} \quad \left( 0, F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \\
\frac{w-c_f}{v_f} &< \frac{p-c_s}{v_s} \quad \left( 0, F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \text{ or } \left( F^{-1}\left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right)
\end{align*}
\]

For \( c_s < w \leq c_s + v_s \), there are two cases:

\[
\begin{align*}
\frac{w-c_f}{v_f} &< \frac{p-c_s}{v_s} \quad \left( 0, F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \text{ or } \left( F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) - F^{-1}\left( \frac{w-c_s-v_s}{w-c_s} \right) \right)
\end{align*}
\]

For \( w > c_s + v_s \), there are three cases:

\[
\begin{align*}
\frac{w-c_f}{v_f} &< \frac{w-c_s}{v_s} \quad \left( F^{-1}\left( \frac{w-c_s-v_s}{w-c_f} \right), 0 \right) \\
\frac{p-c_s}{v_s} &< \frac{w-c_f}{v_f} \quad \left( F^{-1}\left( \frac{w-c_s-v_s}{w-c_f} \right), F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) - F^{-1}\left( \frac{w-c_s-v_s}{w-c_s} \right) \right) \\
\frac{w-c_s}{v_s} &< \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \quad \left( F^{-1}\left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right) \text{ or } \left( F^{-1}\left( \frac{w-c_s-v_s}{w-c_s} \right), F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) - F^{-1}\left( \frac{w-c_s-v_s}{w-c_s} \right) \right)
\end{align*}
\]

Basically, there are four possible capacity equilibria under ZRC’s, depending on the relative magnitude of the IDM’s two possible critical fractiles, \( \frac{p-c_s-v_s}{p-c_s} \) and \( \frac{w-c_s-v_s}{w-c_s} \), and the foundry’s critical fractile \( \frac{w-c_f-v_f}{w-c_f} \). The unique equilibrium can be determined except in two cases in Proposition 2 with \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \). In the scenario of \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \) (i.e., \( \frac{w-c_f-v_f}{w-c_f} < \frac{p-c_s-v_s}{p-c_s} \)), the IDM’s ideal total capacity level is \( F^{-1}\left( \frac{p-c_s-v_s}{p-c_s} \right) \). One option is to build by himself the whole capacity, which leads to the equilibrium \( \left( F^{-1}\left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right) \), and the other option is to also use the foundry’s capacity. In the latter option, the foundry knows that the IDM will use his own capacity first in the production stage by \( w > c_s \), so the foundry will only bring the total capacity to her critical fractile \( F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \) by building the difference between \( F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \) and the IDM’s capacity \( K_s \). Thus, if \( K_s \geq F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \), the foundry will not build any capacity, in which case, again \( \left( F^{-1}\left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right) \) is the IDM’s optimal outcome. If \( K_s < F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \), the foundry will bring the total capacity to \( F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \), in which case, the demand exceeding the IDM’s capacity will be satisfied from the foundry’s capacity, and thus the IDM’s underage capacity investment cost is \( (w-c_s-v_s)^+ \) and overage cost is \( v_s \). When \( w \leq c_s + v_s \), the underage cost is 0, which means the IDM should not build any capacity and solely rely on the foundry to build \( F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \). When \( w > c_s + v_s \), the underage cost is \( w-c_s-v_s \), leading to the IDM’s critical fractile \( \frac{w-c_s-v_s}{w-c_s} \); This explains the possible equilibrium \( \left( F^{-1}\left( \frac{w-c_s-v_s}{w-c_s} \right), F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) - F^{-1}\left( \frac{w-c_s-v_s}{w-c_s} \right) \right) \). To summarize, in the scenario of \( c_s < w < c_s + v_s \) and \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \), the IDM’s tradeoff in choosing \( \left( 0, F^{-1}\left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \) or \( \left( F^{-1}\left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right) \) is between (i) achieving a suboptimal total capacity level but bearing
no capacity investment risk, and (ii) achieving the optimal total capacity level but meanwhile bearing high capacity risk. Intuitively, (i) will outperform (ii) when \( v_s \) is sufficiently high. Similar tradeoff presents in the scenario of \( w > c_s + v_s \) and \( \frac{w-c_s}{v_s} < \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \).

All capacity equilibria in Proposition 2 are candidates to replicate the centralized capacity investments for the coordination purpose. In the vertical supply chain setting, the wholesale price contract cannot coordinate the supply chain due to the well known double marginalization effect (Spengler 1950). In the horizontal setting, we find the wholesale price contracts cannot achieve the maximum system profit for two possible reasons: One is still the double marginalization effect, and the other is “misalignment of capacity-usage-priority”.

In the case of \( MOF_f > MOF_s \), the centralized system has only the foundry build capacity \( F^{-1} \left( \frac{w-c_f-v_f}{p-c_f} \right) \). In this case, the horizontal supply chain setting reduces to the traditional vertical supply chain setting because in both cases the IDM does not build any capacity. As in the vertical setting, the maximum capacity the foundry is willing to build facing a wholesale price \( w \) in the horizontal setting is \( F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \), which is always strictly less than \( F^{-1} \left( \frac{w-c_f-v_f}{p-c_f} \right) \) for \( w < p \). In such cases, no ZRC can coordinate capacity investment decisions, also due to the double marginalization effect.

In the other case with \( MOF_f \leq MOF_s \) and \( c_s > c_f \) (Proposition 1), it is optimal for both the IDM and foundry to build capacity in the centralized system and the total capacity is \( F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \). The ZRCs with \( w \leq c_s \) cannot coordinate capacity investment decisions also due to double marginalization effect, because the foundry’s capacity \( F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \) under ZRCs with \( w \leq c_s \) is always less than her capacity in the centralized system, \( F^{-1} \left( \frac{c_s+v_s-c_f-v_f}{c_s-c_f} \right) \). On the other hand, ZRCs with \( w > c_s \) cannot coordinate the production decision because under the condition of \( c_s > c_f \), the centralized system uses the foundry’s capacity first, but \( w > c_s \) implies the IDM will use his own capacity first under ZRCs. This is called the “misalignment of capacity-usage-priority”.

6. Positive Reservation Contracts (PRCs)
In this section, we are interested in whether there exist coordinating PRCs (CPRCs). We first derive the expected profits and optimal capacities for the IDM and foundry when they are part of a decentralized system and follow the decision protocol of Table 1 under a PRC. Then we derive in Section 6.1 candidates for coordinating PRCs that achieve the maximum system profit but may not be individually rational for both the IDM and
foundry. Finally, we check in Section 6.2 the IDM’s and foundry’s individual rationality, respectively.

We derive the objective functions used in Steps 1 and 2 of Table 1 backward. The foundry maximizes $\Pi_f$ in Step 2 and determines her optimal capacity $K_f^*$. In Step 1, anticipating the foundry’s optimal action in Step 2, the IDM determines $K_s^*$ and $R^* > 0$ to maximize $\Pi_s$. The IDM’s ex-post production decision in Step 4 depends on the cost structure. The functional forms of $\Pi_f$ and $\Pi_s$ are derived in Lemma 3. Lemma 3(a) corresponds to the foundry-first case, where in Step 4, the IDM uses the foundry’s capacity first and then his own capacity. This is because the per-unit subcontract price $w$ is less than or equal to the IDM’s per-unit production cost $c_s$. Lemma 3(b) corresponds to the IDM-first case where the IDM’s capacity is used first, and then the foundry’s. This is because the IDM’s per-unit production cost is less than the per-unit sourcing cost from the foundry even if the IDM can obtain the reservation fee $r$ back. Lemma 3(c) corresponds to the foundry-first up-to-$R$ case where the foundry’s capacity is used first up to $R$, followed by use of the IDM’s capacity, and finally by the foundry’s remaining capacity, if any. This is because under the condition of $c_s + r > w > c_s$, it is more expensive for the IDM to produce by himself if he can get the reservation fee back by sourcing from the foundry; however, if there is no further reservation fee to get back, the IDM prefers to produce on his own. Recall the production decisions under ZRCs. Since the capacity investments have been sunk after the demand gets realized, the IDM only needs to compare sourcing from the foundry at per-unit cost of $w$ and producing by himself at per-unit cost of $c_s$, and chooses the less expensive option. However, under PRCs the foundry’s capacity investment has not been fully sunk due to the reservation fee that the IDM pays ex ante to the foundry. In order to get the reservation fee back, the IDM has more incentives to use the foundry’s capacity, which results in the foundry-first up-to-$R$ case in Lemma 3(c).

**Lemma 3.** The expected profit functions for the foundry and IDM under PRCs are as follows:

(a) If $w \leq c_s$, then

$$\Pi_f = (w - c_f) \int_0^{K_f} (1 - F(x)) \, dx - v_f K_f + r \int_0^R F(x) \, dx,$$

$$\Pi_s = (p - w) \int_0^{K_f} (1 - F(x)) \, dx + (p - c_s) \int_{K_f}^{K_s + K_f} (1 - F(x)) \, dx - v_s K_s - r \int_0^R F(x) \, dx.$$ (4)
Recall that a contract is coordinating if it satisfies three conditions: capacity coordination, result for deriving positive reservation contracts that achieve the maximum system profit. This lemma serves as an intermediate capacity reservation fee. Thus, in a positive reservation contract, the foundry has the obligation to build at least the PRC; R

\[ K_f = \begin{cases} 
R, & \text{for } R > F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \\
F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s, & \text{for } R + K_s < F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right).
\end{cases} \]

(b) If \( w > c_s + r \), then

\[ \Pi_f = (w - c_f) \int_{K_s+K_f}^{R} (1 - F(x)) \, dx - v_f K_f + r \int_{K_s+R}^{K_s+K_f} F(x) \, dx, \]

\[ \Pi_s = (p - w) \int_{K_s}^{R} (1 - F(x)) \, dx + (p - c_s) \int_{0}^{K_s} (1 - F(x)) \, dx - v_s K_s - r \int_{K_s}^{R} F(x) \, dx, \]

where

\[ K_f = \begin{cases} 
R, & \text{for } R + K_s \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \\
F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s, & \text{for } R + K_s < F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right).
\end{cases} \]

(c) If \( c_s < w \leq c_s + r \), then

\[ \Pi_f = (w - c_f) \left( \int_{0}^{R} (1 - F(x)) \, dx + \int_{K_s+K_f}^{R} (1 - F(x)) \, dx \right) - v_f K_f + r \int_{K_s+R}^{R} F(x) \, dx, \]

\[ \Pi_s = (p - w) \left( \int_{0}^{R} (1 - F(x)) \, dx + \int_{K_s+K_f}^{R} (1 - F(x)) \, dx \right) + (p - c_s) \int_{R}^{K_s+R} (1 - F(x)) \, dx - v_s K_s - r \int_{K_s}^{R} F(x) \, dx, \]

where

\[ K_f = \begin{cases} 
R, & \text{for } R + K_s \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \\
F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s, & \text{for } R + K_s < F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right).
\end{cases} \]

In a positive reservation contract, the foundry has the obligation to build at least the capacity reserved by the IDM, i.e., \( K_f \geq R \). However, \( 0 < R < K_f \) will never emerge in the capacity equilibrium, because if the foundry is willing to build a capacity level above the reservation quantity, then it is optimal for the IDM not to reserve any capacity to save the capacity reservation fee. Thus, in a PRC, \( R^* = K_f^* \) must hold in the equilibrium. Lemma 4 in the appendix characterizes the possible capacity equilibria for the foundry-first, IDM-first, and foundry-first up-to-R cases, respectively. This lemma serves as an intermediate result for deriving positive reservation contracts that achieve the maximum system profit. Recall that a contract is coordinating if it satisfies three conditions: capacity coordination,
production coordination, and individual rationality. A contract that satisfies the first two conditions maximizes the system profit. To check these two conditions, we examine if there exist PRCs so that 1) the centralized capacity investments characterized in Proposition 1 can be replicated by the decentralized capacity investments under the PRCs (see Lemma 4), and 2) the production decisions are also consistent with that in the centralized system.

6.1. Candidate Coordinating Positive Reservation Contracts

The following proposition derives PRCs that replicate the centralized capacity investment and production decisions under both cases of $MOF_s \geq MOF_f$ and $MOF_s < MOF_f$, and characterizes the corresponding profit allocation between the IDM and foundry.

**Proposition 3.** There exist the following PRCs $(w,r)$ that coordinate the IDM’s and foundry’s capacity investment and production decisions.

(a) Assume $MOF_s \geq MOF_f$ holds. Then set

\[ w = c_f + v_f + \alpha \quad \text{and} \quad r = v_f - \frac{\alpha (v_f - c_s)}{c_s + v_s - c_f - v_f} \quad \text{for} \quad (c_s - c_f - v_f)^+ < \alpha < c_s + v_s - c_f - v_f. \]

The expected profits for the foundry and IDM are

\[ \Pi_f^* = \frac{\alpha}{c_s + v_s - c_f - v_f} \left( (c_s - c_f) \int_0^{K_{sf}} (1 - F(x)) \, dx - (v_f - v_s) K_{cf} \right), \]

\[ \Pi_s^* = \Pi_{cs} - \Pi_f^* \quad \text{where} \]

\[ \Pi_{cs} = (p - c_f) \int_0^{K_{sf}} (1 - F(x)) \, dx + (p - c_s) \int_{K_{cs}}^{K_{sf} + K_{cf}} (1 - F(x)) \, dx - v_f K_{cf} - v_s K_{cs}. \]

The corresponding capacity investments are

\[ (K_{cs}, K_{cf}) = \left( F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right), F^{-1} \left( \frac{c_s + v_s - c_f - v_f}{c_s - c_f} \right) \right). \]

(b) Assume $MOF_f > MOF_s$ holds. Then set

\[ w = c_f + v_f + \alpha \quad \text{and} \quad r = v_f - \frac{\alpha v_f}{p - c_f - v_f} \quad \text{for} \quad 0 < \alpha \leq c_s + v_s - c_f - v_f. \]

The expected profits for the foundry and IDM are

\[ \Pi_f^* = \frac{\alpha}{p - c_f - v_f} \Pi_{cs}, \quad \Pi_s^* = \left( 1 - \frac{\alpha}{p - c_f - v_f} \right) \Pi_{cs}, \quad \text{where} \]

\[ \Pi_{cs} = (p - c_f) \int_0^{K_{sf}} (1 - F(x)) \, dx - v_f K_{cf}. \]

The corresponding capacity investments are

\[ (K_{cs}, K_{cf}) = \left( 0, F^{-1} \left( \frac{p - c_f - v_f}{p - c_f} \right) \right). \]

In Proposition 3, each $\alpha$ (defined as $\alpha = w - c_f - v_f$) corresponds to a PRC($w, r$), and $\alpha$ takes values in a continuous range. So there are an infinite number of PRCs that can replicate the centralized capacity investment and production decisions and thus achieve the maximum system profit.

The case of $MOF_s \geq MOF_f$ leads to horizontal dual sources under PRCs that maximize the system profit. Therefore, the coordination problem in this case is clearly different from the coordination of a vertical supply chain. Contract parameters have to be set so
that the capacities built at each firm and the capacity usage priority are consistent with that in the centralized system. In the vertical supply chain, there is no need for coordinating capacity usage priority (or production decision), as only supplier builds capacity. The coordination problem for the case of $MOF_f > MOF_s$ in the horizontal setting looks similar to the coordination problem in the purely vertical supply chain setting because in the resulting capacity investment decision, it is optimal that only the foundry builds capacity, which essentially reduces the horizontal setting to a vertical supply chain. Despite this similarity, there is a key difference. Under a vertical supply chain framework, several different contract types are shown to maximize and arbitrarily divide system profit, such as buy-back contracts, revenue-sharing contracts, quantity-flexibility contracts, sales-rebate contracts and quantity-discount contracts (Cachon 2003). Arbitrary profit allocation is not possible in the horizontal setting where the IDM himself is able to build capacity and earns at least a profit of $\Pi^i_s$ when he acts independently (note that the upper bound of $\alpha$ is $c_s + v_s - c_f - v_f$, instead of $p - c_f - v_f$), whereas in the vertical supply chain, the buyer has to rely on the supplier for a key input or product that the buyer cannot produce by itself. Therefore, both parties will earn zero profit without collaboration.

The $PRCs$ derived in Proposition 3 maximize the system profit. However, they may not be coordinating if they do not satisfy the IDM’s or foundry’s individual rationality condition. Thus, all $PRCs$ in Proposition 3 are referred to as “candidate coordinating $PRCs$” ($CCPRCs$) hereafter. Proposition 3 also presents the expected profits of the IDM and foundry under the $CCPRCs$. In the following subsection, these profits are compared with the IDM’s and foundry’s expected profits when they use the benchmark wholesale price contracts ($ZRCs$) to check the individual rationality condition.

6.2. Individual Rationality Condition

Among $ZRCs$, there exists an optimal contract for the IDM to maximize his expected profit. Let $\Pi^*_z$ denote the IDM’s expected profit under his optimal $ZRC$, and $\Pi^*_f$ denote the foundry’s expected profit under this $ZRC$. That is, under the benchmark wholesale price contract, the foundry has a business opportunity to earn $\Pi^*_f$. The IDM’s individual rationality condition can be checked by comparing $\Pi^*_z$ under the $CCPRCs$ in Proposition 3 with $\Pi^*_z$. If the IDM has incentives to improve his expected profit over the wholesale price contract by adopting a $PRC$ with the foundry, although theoretically he can switch unilaterally, in most practical settings — especially in the semiconductor industry, due
to considerations such as long-term business relationship and mutual benefit — both the IDM and foundry have a say in the contracting stage. In order to make a smooth switch from the wholesale price contract to a more sophisticated one, both parties should have incentives for the switch. That is, the foundry should also earn a larger expected profit in the alternative contract than $\Pi_f^*$. To check the foundry’s individual rationality, we compare $\Pi^c_f$ under the CCPRCs in Proposition 3 with $\Pi_f^*$. The CCPRCs with $\Pi^c_s$ and $\Pi^c_f$ greater than $\Pi^*_s$ and $\Pi^*_f$ respectively satisfy both firms’ individual rationality condition, and thus are CPRCs.

Under the IDM’s optimal ZRC, the foundry may or may not build capacity in the capacity equilibrium. For further discussion purpose, $ZRC^f$ denotes a ZRC with the foundry building positive capacity. Denote the IDM’s expected profit under a $ZRC^f$ as $\Pi^c_f$. If the foundry does not build capacity $ZRC$ reduces to “IDM independent” case. In this case, $\Pi^*_s = \Pi^i_s$, where $\Pi^i_s$ is the IDM’s expected profit when he acts independently. Thus, $\Pi^*_s = \Pi^i_s$ or $\Pi^*_s = \Pi^i_f$.

As a first step to compare $\Pi^c_s$ and $\Pi^c_f$ under the CCPRCs with $\Pi^*_s$ and $\Pi^*_f$ respectively, Propositions 4 and 5 derive some properties of firms’ profits under CCPRCs.

**Proposition 4.** Consider all $\Pi^c_s$ under the CCPRCs specified in Proposition 3. (i) If $MOF_s \geq MOF_f$, $\Pi^c_s$ is greater than $\Pi^i_s$. As $w$ converges to $c_s + v_s$, $\Pi^c_s$ converges to $\Pi^i_s$. (ii) If $MOF_s < MOF_f$, $\Pi^c_s$ is strictly greater than $\Pi^i_s$.

Proposition 4 shows that by using any CCPRC, the IDM earns at least what he can earn by acting independently. Note that $\Pi^c_s$ under the CCPRCs defined in Proposition 3 decreases in $w$. The upper bound of $w$ is $c_s + v_s$, which is the IDM’s per-unit total cost and the corresponding reservation fee $r = v_s$ for $MOF_s \geq MOF_f$ and $r < v_s$ for $MOF_s < MOF_f$. In fact, when $w = c_s + v_s$, $r = v_s$, and the total capacity is fixed, the IDM is indifferent between using the positive reservation contract $(w, r)$ and relying on his own capacity. This is because securing one unit of capacity by either building his own capacity or reserving from the foundry costs the same $v_s$, and producing one unit of product costs the same $c_s$ by using either the foundry’s capacity or his own capacity. The fact that the IDM’s least expected profit under CCPRCs is greater than or equal to $\Pi^i_s$ leads to Proposition 4.

If the IDM prefers to act independently (i.e., $\Pi^*_s = \Pi^i_s$, $\Pi^*_f = 0$), all the CCPRCs satisfy both the IDM’s and the foundry’s individual rationality condition. Otherwise, if the foundry builds positive capacity (i.e., $\Pi^*_s = \Pi^*_f$), then we compare $\Pi^c_f$ and $\Pi^*_f$ with
Πcs and Πcf under the CCPRCs respectively to check the individual rationality. Πcsf and Πcf\* can be derived by substituting the capacity equilibria with positive foundry’s capacity specified in Proposition 2 to Lemma 2. Due to the complexity of the newsvendor formula, Πcsf cannot be directly compared with Πcs. However, Proposition 5 below shows that the foundry’s expected profit under any CCPRC is always greater than that under the ZRC for the same w. This is partially because under any CCPRC the demand is allocated to the foundry first, and further the foundry receives the reservation fee under CCPRCs, but not under ZRCs.

**Proposition 5.** The foundry’s expected profit under any CCPRC \((w,r)\) specified in Proposition 3 is greater than that under the ZRC for the same w.

Based on Propositions 4, 5, and further analysis, we derive sufficient but not necessary conditions for CPRCs to exist in Proposition 6.

**Proposition 6.** Coordinating PRCs (CPRCs) exist when (i) \(\text{MOF}_s \geq \text{MOF}_f\) and \(c_s \leq c_f + v_f\), or (ii) \(\text{MOF}_f > \text{MOF}_s\) and the IDM’s optimal ZRC has \(w \leq c_s + v_s\).

The condition (i) in Proposition 6 implies the IDM’s cost structure is more flexible and his unit production cost is not very high. In this case, for \(w\) converging to \(c_f + v_f\) the IDM can achieve the maximum system profit, which must be strictly greater than his profit under the optimal ZRC, \(\Pi_{s^*}\). As \(\Pi_s\) decreases in \(w\), there exists a \(\bar{w}\) \((c_f + v_f < \bar{w} \leq c_s + v_s)\) so that \(\Pi_s = \Pi_{s^*}\) at \(\bar{w}\) and all CCPRCs with \(w < \bar{w}\) is strictly preferred by the IDM to his optimal ZRC. On the other hand, for \(w\) converging to \(c_s + v_s\), the foundry must obtain a strictly greater profit than her profit under the IDM’s optimal ZRC, because at \(c_s + v_s\), the IDM’s expected profit under the CCPRC is exactly the same as when he acts independently, which is less than or equal to his profit under the optimal ZRC, \(\Pi_{s^*}\). However, the total profit under CCPRC must be greater than that under ZRC, so the foundry’s profit under the CCPRC with \(w\) converging to \(c_s + v_s\) must be greater than \(\Pi_{f^*}\). Since \(\Pi_f\) increases in \(w\), there exists a lower bound \(\underline{w}\) so that \(\Pi_f = \Pi_{f^*}\) at \(\underline{w}\), and all the CCPRCs with \(w > \underline{w}\) are preferred by the foundry to the IDM’s optimal ZRC. Based on the above discussion, if \(\bar{w} > \underline{w}\), CCPRCs with \(w \in (\underline{w}, \bar{w})\) satisfy both firms’ individual rationality condition, and hence are coordinating. In fact, \(\bar{w} > \underline{w}\) always holds: At \(\bar{w}\), \(\Pi_s = \Pi_{s^*}\), which implies the foundry’s profit at the CCPRC must be greater than \(\Pi_{f^*}\); at \(\underline{w}\), the foundry’s profit at the CCPRC equals \(\Pi_{f^*}\). Since the foundry’s profit under CCPRCs increases in \(w\), we have \(\bar{w} > \underline{w}\).
If $MOF_f > MOF_s$, $w$ can also be arbitrarily close to $c_f + v_f$ by the observation from Proposition 3. Following the same argument above, there exists a $\tilde{w}$ so that $CCPRCs$ with $w < \tilde{w}$ satisfy the IDM’s individual rationality condition. Suppose the IDM’s optimal $ZRC$ has $w^* \leq c_s + v_s$; then by Proposition 5, the foundry’s profit under the $CCPRC$ with $w^*$ is strictly greater than her profit under the IDM’s optimal $ZRC$. Then there exists a $w < w^*$ so that $\Pi^c_f = \Pi^c_f$ at $w$, and all the $CCPRCs$ with $w > w$ are preferred by the foundry to the IDM’s $ZRC$. However, if the IDM’s optimal $ZRC$ has $w^* > c_s + v_s$, then the foundry’s profit under the IDM’s optimal $ZRC$ might be so large that it is greater than her largest profit under the $CCPRC$ with $w = c_s + v_s$. In this case, the foundry’s individual rationality condition is not guaranteed. Similarly, it can be shown $\bar{w} > w$ holds, and $CCPRCs$ with $w \in (w, \bar{w})$ satisfy both firms’ individual rationality condition, and hence are $CPRCs$. This explains the sufficient conditions in Proposition 6 (ii).

If neither (i) nor (ii) in Proposition 6 holds, firms’ individual rationality condition is not guaranteed. Because under $MOF_s \geq MOF_f$ and $c_s > c_f + v_f$, the foundry’s profit is strictly greater than a positive value under $CCPRCs$ (since $\alpha$ is strictly greater than a positive value). This implies that the upper bound of $\Pi^c_s$ under all the $CCPRCs$ is less than the maximum system profit. In this case, it is not guaranteed that the largest $\Pi^c_s$ under $CCPRCs$ is greater than $\Pi^c_f$. To give an example, Figure 1 shows under the case of $MOF_s \geq MOF_f$ and $c_s > c_f + v_f$, there exist $ZRCs$ (to the right of the vertical dashed line) leading to larger IDM’s expected profits compared to all the $CCPRCs$. Then, the IDM prefers to use his optimal $ZRC$, and no $CCPRC$ satisfies the IDM’s individual rationality condition, i.e., no $PRC$ is coordinating.
The conditions of $MOF_s \geq MOF_f$ and $c_s > c_f + v_f$ imply the IDM’s per-unit capacity investment cost, $v_s$ is very low and so is the capacity investment risk. Under this case, the IDM may be better off by using a ZRC with a low subcontract price and mainly relying on his own capacity, so PRC cannot be coordinating. However, it is typical in most practical settings that the foundry has a more flexible cost structure than the IDM (i.e., $MOF_s < MOF_f$), and one party’s total cost is greater than the other’s production cost. Thus, it is rather extreme for both conditions $MOF_s \geq MOF_f$ and $c_s > c_f + v_f$ to hold. In almost all practical situations, we have either $MOF_f > MOF_s$ or $c_s < c_f + v_f$, which guarantees the existence of a continuous range of CCPRCs that satisfy the IDM’s individual rationality condition. In our numerical study, the IDM’s optimal ZRC has $w > c_s + v_s$ in most cases. $w < c_s + v_s$ in the IDM’s optimal ZRC does not occur frequently, but is possible; even for such less frequently encountered cases, we find that $\Pi_f$ at $w = c_s + v_s$ is much greater than $\Pi_f^*$, which implies the foundry’s individual rationality condition is also satisfied for a range of CCPRCs. Overall, these results imply that the conditions for CPRCs to exist can easily be met in practice.

7. Numerical Study

In the extensive numerical study conducted, we present two cases whose results are representative for those under $MOF_s \geq MOF_f$ and $MOF_f > MOF_s$, respectively. For the case of $MOF_s \geq MOF_f$, one example of cost structure is $\{c_s = 450, v_s = 150, c_f = v_f = 270\}$. Although in the semiconductor industry, the per-unit total production cost is around 50% of the revenue, i.e., the reasonable retail price is around 1200 for the considered cost structure, we consider four different retail prices $\{p = 650, 700, 800, 1300\}$ to study the impact of profit margin. In Figure 2, we plot for each case the IDM’s and foundry’s expected profits under both CCPRCs and ZRCs. $(\Pi_s, \Pi_f)$ under ZRCs with $w \in (c_f + v_f, c_s + v_s)$ and $w \in [c_s + v_s, p]$, respectively, are marked with different symbols. Note that for the considered cost structure, $w < c_s$ is impossible because $w > c_f + v_f > c_s$. The triangle signs denote $(\Pi_s, \Pi_f)$ under the CCPRCs with $w \in (c_f + v_f, c_s + v_s)$ as specified in Proposition 3. The arrow in each plot indicates the direction of increasing $w$. By representing $(\Pi_s, \Pi_f)$ under both CCPRCs and ZRCs in the same plot, we are able to show the efficiency loss

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2 In the semiconductor industry, both the equipment procurement and the device manufacturing present economies of scale. Due to foundries’ specialization and economies of scale, they often have cost advantage over IDMs; that is, $v_f < v_s$ and $c_f < c_s$ are likely to hold, both contributing to $\frac{p - c_f}{v_f} > \frac{p - c_s}{v_s}$. 

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There are two interesting observations from Figure 2. First, when the product price is large (e.g., $p = 800, 1300$), the IDM’s optimal choice under ZRCs is the “IDM independent case”, and all the CCPRCs are indeed coordinating (all the triangle signs are on the upper-right side of the round dot). Second, when the product price is low ($p = 680, 700$), the IDM’s optimal ZRC leads to positive $\Pi^*_s$ and $\Pi^*_f$ as indicated in the plot. In this case, only the CCPRCs with the corresponding triangle signs on the upper-right side of $\left(\Pi^*_s, \Pi^*_f\right)$ are coordinating. These observations hold in all the extensive examples we have studied.

Figure 2 also shows that the profit allocations under all CPRCs constitute the complete efficient frontier. Figure 3 shows the results for the case of $MOF_f > MOF_s$. Different from the case of $MOF_s > MOF_f$ shown in Figure 2, all the profit allocations under the
CPRCs in Figure 3 may not constitute the complete efficient frontier, as in the case of $p = 800, 1300$. This is because the lower bound of $\Pi^c_s$ is strictly greater than $\Pi^c_i$ when $MOF_f > MOF_s$, as shown in Proposition 4. Nevertheless, there exists a range of CPRCs. The identification of CPRCs is important for the IDM and foundry to understand if they can improve their expected profits over the wholesale price contract and the possible range of surplus allocation.

8. Conclusion
Motivated by the capacity expansion practice in the semiconductor industry, this paper studies horizontal capacity coordination in a one-market setting that differentiates from the purely vertical supply chain setting and the two-market horizontal setting. The frequent and expensive nature of capacity investment in the semiconductor industry often motivates IDMs to seek collaboration with foundries. In a decentralized decision-making scheme, the search for contracts that can achieve the maximum system profit is a worthwhile topic of study. We define “coordinating contracts” as those that can replicate the centralized
capacity investment and production decisions and also lead to greater expected profits for both firms compared with the benchmark wholesale price contract. In this paper, we search for coordinating contracts within the scope of capacity reservation contracts that are frequently used in the semiconductor and other high-tech industries.

In the absence of sophisticated coordinating contracts, the benchmark contract for the IDM and foundry is a wholesale price contract. By extending the study of wholesale price contract in the horizontal setting (referred to as ZRC in this paper), we find that in addition to the double marginalization effect, misalignment of capacity-usage-priority is another reason for a wholesale price contract to fail to maximize the system profit. This alternative reason is unique to the horizontal setting where both players can produce the end product and the one who produces may not be the more efficient one.

In contrast, we prove that there always exist PRCs that maximize the system profit under our studied scenario, where the foundry has total cost advantage over the IDM due to the foundry’s specialization and focus on manufacturing. However, it is possible none of PRCs that maximize the system profit satisfy the individual rationality condition. Under the extreme parameter settings with the IDM’s capacity investment risk being very low, the IDM may be better off using his optimal wholesale price contract. Nevertheless, in extensive practical parameter settings, there exist PRCs that both maximize the system profit and are individually rational for both firms. In particular, if the profit margin is reasonably large, any PRC that maximizes the system profit satisfies the individual rationality condition. If the profit margin is small, then both the IDM and foundry earn positive profit under the optimal ZRC for the IDM. In this case, CPRCs exist when the IDM’s capacity investment risk is not extremely low.

We observe that the resulting sourcing structure under CPRCs hinges upon the cost structures of the IDM and foundry. The condition of $MOF_s < MOF_f$ implies that the foundry has an advantageous cost structure, and thus it is optimal for the system to have only the foundry build capacity. In this case, the horizontal setting reduces to the vertical supply chain setting and sole sourcing from the foundry is the outcome. If $MOF_s \geq MOF_f$, the foundry has per-unit total cost advantage while the IDM has higher margin over fixed cost. Under this condition, both the IDM and foundry build positive capacity under CPRCs, which results in horizontal dual sourcing.

In our paper, the IDM and foundry do not compete in the end-product market. The horizontal capacity coordination between two parties that engage in end-product competition is an interesting research direction for future exploration.
References


Appendix

**Lemma 4.** The following PRCs \((w, r)\) with \(R = K_f\) may outperform ZRC \((w)\) in terms of the IDM’s expected profit:

(a) Assume \(w \leq c_s\) holds.

(a1) If \(\frac{w - c_f}{v_f} < \frac{p - c_s}{v_s}\) and \(r < \frac{v_s(p - w)}{p - c_s - v_s}\), then \(K = \left(0, F^{-1}\left(\frac{p - w}{p - w + r}\right)\right)\).

(a2) If \(\frac{w - c_f}{v_f} < \frac{p - c_s}{v_s}\), \(r \geq \frac{v_s(p - w)}{p - c_s - v_s}\) and \(r < \frac{v_f c_s - v_s c_f + w(v_s - v_f)}{w - c_f - v_f}\), then
K = \left( F^{-1} \left( \frac{p-c_a-v_s}{p-c_a} \right) - F^{-1} \left( \frac{c_a+v_s-w}{c_a+r-w} \right), F^{-1} \left( \frac{c_a+v_s-w}{c_a+r-w} \right) \right).

(a3) If \frac{p-c_a}{v_s} \leq \frac{w-c_f}{v_f} \text{ and } r < \frac{v_f(p-w)}{w-c_f-v_f}, \text{ then } K = \left( 0, F^{-1} \left( \frac{p-w}{p-w+r} \right) \right).

(b) Assume \( w \geq c_s + r \) holds.

(b1) If \( w \leq c_s + v_s, \text{ and } r < \frac{v_f(p-w)}{w-c_f-v_f}, \text{ then } K = \left( 0, F^{-1} \left( \frac{p-w}{p-w+r} \right) \right).

(b2) If \( w > c_s + v_s, r < \min \left( \frac{v_f(p-w)}{w-c_f-v_f}, \frac{v_s(p-w)}{p-c_s-v_s} \right), \text{ and } w > c_s + r, \text{ then } K = \left( F^{-1} \left( \frac{w-c_s-v_s}{w-c_s-r} \right), F^{-1} \left( \frac{p-w}{p-w+r} \right) - F^{-1} \left( \frac{w-c_s-v_s}{w-c_s-r} \right) \right).

(c) Assume \( c_s < w < c_s + r \) holds.

(c1) If \( w \leq c_s + v_s, \frac{w-c_f}{v_f} < \frac{p-c_a}{v_a} \text{ and } r < \frac{v_s(p-w)}{p-c_s-v_s}, \text{ then } K = \left( 0, F^{-1} \left( \frac{p-w}{p-w+r} \right) \right).

(c2) If \( w \leq c_s + v_s, \frac{w-c_f}{v_f} < \frac{p-c_a}{v_a} \text{ and } r \geq \frac{v_s(p-w)}{p-c_s-v_s}, \text{ then } K = \left( F^{-1} \left( \frac{p-c_a-v_s}{p-c_a} \right), F^{-1} \left( \frac{c_a+v_s-w}{c_a+r-w} \right), F^{-1} \left( \frac{c_a+v_s-w}{c_a+r-w} \right) \right).

(c3) If \frac{p-c_a}{v_a} \leq \frac{w-c_f}{v_f} \text{ and } r < \frac{v_f(p-w)}{w-c_f-v_f}, \text{ then } K = \left( 0, F^{-1} \left( \frac{p-w}{p-w+r} \right) \right).
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Proof of Lemma 1. If \( c_s > c_f \) holds, then after the demand is realized, the foundry’s capacity is used first. Therefore, by substituting \( w = p, K_f^* = K_f \) and \( R = 0 \) in Lemma 3(a), we obtain (1). If \( c_f > c_s \) holds, then after the demand is realized, the IDM’s capacity is used first. Therefore, by substituting \( w = p, K_f^* = K_f \) and \( R = 0 \) in Lemma 3(b), we obtain (2). If \( c_f = c_s \) holds, then after the demand is realized, there is no difference whether IDM’s or foundry’s capacity is used first. By substituting \( c_f \) with \( c_s \) in (1), we obtain (3).

Proof of Proposition 1. First, we consider the case of \( c_s > c_f \). Taking the first and second order derivatives of (1) and the corresponding Hessian, \( H \), with respect to \( K_s \) and \( K_f \), we obtain

\[
\partial \Pi_{cs}/\partial K_s = p - c_s - v_s - (p - c_s) F (K_s + K_f)
\]
\[
\partial \Pi_{cs}/\partial K_f = p - c_f - v_f - (c_s - c_f) F (K_f) - (p - c_s) F (K_s + K_f)
\]
\[
\partial^2 \Pi_{cs}/\partial K_s^2 = -(p - c_s) f (K_s + K_f)
\]
\[
\partial^2 \Pi_{cs}/\partial K_f^2 = -(c_s - c_f) f (K_f) - (p - c_s) f (K_s + K_f)
\]
\[
\partial^2 \Pi_{cs}/\partial K_s \partial K_f = -\partial^2 \Pi_{cs}/\partial K_f \partial K_s = -(p - c_s) f (K_s + K_f)
\]
\[
\det (H) = (p - c_s) (c_s - c_f) f (K_f) f (K_s + K_f)
\]

Since \( c_s > c_f \) and \( p > c_s \), \( \partial^2 \Pi_{cs}/\partial K_s^2 < 0 \), \( \partial^2 \Pi_{cs}/\partial K_f^2 < 0 \) and \( \det (H) > 0 \). Thus, \( \Pi_{cs} \) is jointly concave.

Case \( \frac{p-c_f}{v_f} \leq \frac{p-c_s}{v_s} \): \( \frac{p-c_f}{v_f} \leq \frac{p-c_s}{v_s} \) is equivalent to \( \frac{c_s+v_s-c_f-v_f}{c_s-c_f} \leq \frac{p-c_s-v_s}{p-c_s} \). Since \( c_f + v_f < c_s + v_s \) and \( c_s > c_f \), \( \frac{c_s+v_s-c_f-v_f}{c_s-c_f} > 0 \). Thus, for \( K_s = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \) and \( K_f = F^{-1} \left( \frac{c_s+v_s-c_f-v_f}{c_s-c_f} \right) \), \( \partial \Pi_{cs}/\partial K_s = 0 \) and \( \partial \Pi_{cs}/\partial K_f = 0 \). Since \( \Pi_{cs} \) is jointly concave, this solution is optimal to (1).

Case \( \frac{p-c_s}{v_s} < \frac{p-c_f}{v_f} \): (10) implies that the optimal capacity decisions exist in either of the following two intervals:

Interval 1: \( K_s + K_f = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) , K_s \geq 0 \) and \( K_f \geq 0 \),

Interval 2: \( K_s = 0 \) and \( K_f \geq F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \).

For Interval 1, substituting \( K_s = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) - K_f \) in (1) and taking the derivative with respect to (w.r.t.) \( K_f \), we obtain \( \partial \Pi_{cs}/\partial K_f = \frac{c_s+v_s-c_f-v_f}{c_s-c_f} - (c_s - c_f) F (K_f) \).
For Interval 2, substituting $K_s = 0$ in (1) and taking the first and second derivatives w.r.t. $K_f$, we obtain

\[ \frac{\partial \Pi_{cs}}{\partial K_f} = p - c_f - v_f - (p - c_f) F(K_f) \]

\[ \frac{\partial^2 \Pi_{cs}}{\partial K_f^2} = -(p - c_f) f(K_f) \quad (11) \]

Since $p > c_f$, $\frac{\partial^2 \Pi_{cs}}{\partial K_f^2} < 0$, and thus $\Pi_{cs}$ is concave in $K_f$.

Since $\frac{p - c_s}{v_s} < \frac{p - c_f}{v_f}$ and $\frac{p - c_s - v_s}{p - c_s} < \frac{c_s + v_s - c_f - v_f}{c_s - c_f}$, therefore, (11) implies $\frac{\partial \Pi_{cs}}{\partial K_f} > 0$ in Interval 1, and thus $K_f = F^{1-} \left( \frac{p - c_f - v_f}{p - c_f} \right)$ and $K_s = 0$ is optimal in Interval 1. Since $\frac{p - c_s}{v_s} < \frac{p - c_f}{v_f}$ is also equivalent to $\frac{p - c_s - v_s}{p - c_s} < \frac{p - c_f - v_f}{p - c_f}$, then $K_f = F^{1-} \left( \frac{p - c_f - v_f}{p - c_f} \right)$ is optimal in Interval 2. The optimal solution in Interval 1 is also in Interval 2 and is dominated by that in Interval 2. Hence, under the condition of $\frac{p - c_s}{v_s} < \frac{p - c_f}{v_f}$, $K_c = \left( 0, F^{1-} \left( \frac{p - c_f - v_f}{p - c_f} \right) \right)$ is optimal.

Second, we consider the case of $c_s < c_f$. Taking the first and second derivatives of (2) and the corresponding Hessian, $H$, w.r.t. $K_s$ and $K_f$, we obtain

\[ \frac{\partial \Pi_{cs}}{\partial K_s} = p - c_s - v_s - (c_f - c_s) F(K_s) - (p - c_f) F(K_s + K_f) \]

\[ \frac{\partial \Pi_{cs}}{\partial K_f} = p - c_f - v_f - (p - c_f) F(K_s + K_f) \]

\[ \frac{\partial^2 \Pi_{cs}}{\partial K_s^2} = -(c_f - c_s) f(K_s) - (p - c_f) f(K_s + K_f) \]

\[ \frac{\partial^2 \Pi_{cs}}{\partial K_f^2} = -(p - c_f) f(K_s + K_f) \]

\[ \frac{\partial^2 \Pi_{cs}}{\partial K_s \partial K_f} = \frac{\partial^2 \Pi_{cs}}{\partial K_f \partial K_s} = -(p - c_f) f(K_s + K_f) \]

\[ \det (H) = (p - c_f)(c_f - c_s) f(K_s) f(K_s + K_f) \]

Since $c_f > c_s$ and $p - c_f$, $\frac{\partial^2 \Pi_{cs}}{\partial K_s^2} < 0$, $\frac{\partial^2 \Pi_{cs}}{\partial K_f^2} < 0$, and $\det (H) > 0$.

Therefore, $\Pi_{cs}$ is jointly concave.

Case $\frac{p - c_s}{v_s} < \frac{p - c_f}{v_f}$: (12) implies that the optimal solution exists in either of the following two intervals:

Interval 1: $K_s + K_f = F^{-1} \left( \frac{p - c_f - v_f}{p - c_f} \right), K_s \geq 0$ and $K_f \geq 0$,

Interval 2: $K_s \geq F^{-1} \left( \frac{p - c_f - v_f}{p - c_f} \right)$ and $K_f = 0$.

Substituting $K_f = F^{-1} \left( \frac{p - c_f - v_f}{p - c_f} \right) - K_s$ in (2) and taking the derivative w.r.t. $K_s$, we obtain

\[ \frac{\partial \Pi_{cs}}{\partial K_s} = -(c_s + v_s - c_f - v_f) - (c_f - c_s) F(K_s) \]  

(13)

Since $c_f + v_f < c_s + v_s$ and $c_f > c_s$, $\frac{\partial \Pi_{cs}}{\partial K_s} < 0$. Therefore, $K_s = 0$ and $K_f = F^{-1} \left( \frac{p - c_f - v_f}{p - c_f} \right)$ are optimal in (2) in Interval 1.
For Interval 2, substituting $K_f = 0$ in (2) and taking the first and second order derivatives w.r.t. $K_s$, we obtain $\partial \Pi_c s / \partial K_s = p - c_s - v_s - (p - c_s) F(K_s)$ and $\partial^2 \Pi_c s / \partial K_s^2 = -(p - c_s) f(K_s)$. Since $p > c_s$, $\partial^2 \Pi_c s / \partial K_s^2 < 0$, and thus $\Pi_c s$ is concave in $K_s$. $\frac{p-c_s}{v_s} < \frac{p-c_f}{v_f}$ is equivalent to $\frac{p-c_s-v_s}{p-c_s} < \frac{p-c_f-v_f}{p-c_f}$. Hence, $\partial \Pi_c s / \partial K_s < 0$ in Interval 2, and thus $K_s = F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$ and $K_f = 0$ are optimal in Interval 2. Since this solution is also in Interval 1, it is dominated by the optimal solution in Interval 1. Hence, under the conditions of $c_f + v_f < c_s + v_s$ and $\frac{p-c_s}{v_s} < \frac{p-c_f}{v_f}$, $K_c = \left(0, F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)\right)$ is optimal.

Case $\frac{p-c_f}{v_f} \leq \frac{p-c_s}{v_s}$: $c_f + v_f < c_s + v_s$ and $c_s < c_f$ imply this case is impossible, as explained as follows: $\frac{p-c_f}{v_f} \leq \frac{p-c_s}{v_s}$ is equivalent to $\frac{p-c_f-v_f}{v_f} \leq \frac{p-c_s-v_s}{v_s}$, and $c_f + v_f < c_s + v_s$ implies $p - c_f - v_f > p - c_s - v_s$. In order for $\frac{p-c_f-v_f}{v_f} \leq \frac{p-c_s-v_s}{v_s}$ to hold, $v_f > v_s$ must hold. However, $c_f + v_f < c_s + v_s$ and $c_s < c_f$ imply $v_s > v_f$, and thus $\frac{p-c_f-v_f}{v_f} \leq \frac{p-c_s-v_s}{v_s}$ cannot hold.

Third, we consider the case of $c_s = c_f$. This implies $\frac{p-c_f}{v_f} > \frac{p-c_s}{v_s}$ by the assumption of $c_f + v_f < c_s + v_s$. Taking the first and second order derivatives of (3) w.r.t. $K_s$, we obtain $\partial \Pi_c s / \partial K_s = p - c_s - v_s - (p - c_s) F(K_s + K_f)$ and $\partial^2 \Pi_c s / \partial K_s^2 = -(p - c_s) f(K_s + K_f). p > c_f$ implies $\partial^2 \Pi_c s / \partial K_s^2 < 0$. That is, $\Pi_c s$ is concave in $K_s$. Hence, the optimal solution exists in either of the following two intervals:

\[
\begin{align*}
\text{Interval 1: } & K_s + K_f = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right), K_s \geq 0 \text{ and } K_f \geq 0 \\
\text{Interval 2: } & K_s = 0 \text{ and } K_f \geq F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)
\end{align*}
\]

Substituting $K_s + K_f = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)$ in (3), it is easy to see that $K_s = 0$ and $K_f = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)$ are optimal in Interval 1. For Interval 2, substituting $K_s = 0$ in (3) and noting $c_f = c_s$, we obtain $\partial \Pi_c s / \partial K_f = p - c_f - v_f - (p - c_f) F(K_s + K_f)$ and $\partial^2 \Pi_c s / \partial K_f^2 = -(p - c_f) f(K_s + K_f)$. Since $p > c_f$, $\Pi_c s$ is concave in $K_f$. $c_f = c_s$ and $v_f < v_s$ imply $\frac{p-c_s-v_s}{p-c_s} < \frac{p-c_f-v_f}{p-c_f}$. Hence, in Interval 2, $K_s = 0$ and $K_f = F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$ is optimal. Since the optimal solution in Interval 1 is also in Interval 2, $K_s = 0$ and $K_f = F^{-1}\left(\frac{p-c_f-v_f}{p-c_f}\right)$ is optimal for this case.

**Proof of Lemma 2.** The ZRCs are degenerate cases of capacity reservation contracts.

When $w \leq c_s$, the foundry’s and IDM’s expected profits are derived by setting $R = 0$ in Equations 4 and 5 in Lemma 3(a). When $w \geq c_s$, $Z_f$ and $Z_s$ are derived by substituting $R = 0$ in Lemma 3(b). Substituting $R = 0$ in Lemma 3(c) leads to the same result.

**Proof of Proposition 2.** (a) Assume that $w \leq c_s$ holds. By Lemma 2, $\Pi_f = (w - c_f) f^K_f (1 - F(x)) dx - v_f K_f$. Taking the first and second derivatives w.r.t. $K_f$, we obtain
\[ \frac{d\Pi_f}{dK_f} = w - c_f - v_f - (w - c_f) F(K_f) \] and \[ \frac{d^2\Pi_f}{dK_f^2} = -(w - c_f)f(K_f). \] Since \( w > c_f + v_f > c_f \), \( \frac{d^2\Pi_f}{dK_f^2} < 0 \), and thus \( \Pi_f \) is concave in \( K_f \). Therefore, the optimal solution \( K_f^* = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \).

In Step 1, knowing \( K_f^* \), the IDM maximizes

\[ \Pi_s = (p - w) \int_0^{K_f^*} (1 - F(x)) \, dx + (p - c_s) \int_{K_f^*}^{K_s + K_f^*} (1 - F(x)) \, dx - v_s K_s. \]

Taking the first and second order derivatives w.r.t. \( K_s \), we obtain \( \frac{d\Pi_s}{dK_s} = p - c_s - v_s - (p - c_s) F(K_s + K_f^*) \) and \( \frac{d^2\Pi_s}{dK_s^2} = -(p - c_s) f(K_s + K_f^*) \). Since \( p > c_s + v_s > c_s \), \( \frac{d^2\Pi_s}{dK_s^2} < 0 \) and thus \( \Pi_s \) is concave in \( K_s \). Note that \( \frac{w-c_f}{v_f} \leq \frac{p-c_s}{v_s} \) is equivalent to \( \frac{w-c_f}{v_f} \leq \frac{p-c_s}{v_s} \). Hence, if \( \frac{w-c_f}{v_f} \leq \frac{p-c_s}{v_s} \), \( K_s^* = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - K_f^* = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \). If \( \frac{w-c_f}{v_f} > \frac{p-c_s}{v_s} \), then for \( K_s \geq 0 \), \( \frac{d\Pi_s}{dK_s} = p - c_s - v_s - (p - c_s) F(K_s + K_f^*) \leq p - c_s - v_s - (p - c_s) F(K_f^*) = p - c_s - v_s - (p - c_s) \left(\frac{w-c_f}{v_f} - \frac{w-c_f-v_f}{w-c_f}\right) < 0 \), and thus \( K_s^* = 0 \). These along with \( K_f^* \) obtained above complete the proof for the case of \( w \leq c_s \).

(b) Assume that \( c_s < w \leq c_s + v_s \) holds. \( \Pi_f \) and \( \Pi_s \) to be maximized can be expressed as those in Lemma 3(b) with \( R = 0 \). In Step 2, the foundry maximizes \( \Pi_f = (w - c_f) \int_{K_s}^{K_s + K_f^*} (1 - F(x)) \, dx - v_f K_f^* \). Taking the first and second order derivatives with respect to \( K_f^* \), we obtain \( \frac{d\Pi_f}{dK_f^*} = w - c_f - v_f - (w - c_f)f(K_s + K_f^*) \) and \( \frac{d^2\Pi_f}{dK_f^*} = -(w - c_f)f(K_s + K_f^*) \). Since \( w > c_f + v_f > c_f \), \( \frac{d^2\Pi_f}{dK_f^*} < 0 \), and thus \( \Pi_f \) is concave in \( K_f^* \). Therefore, the optimal solution

\[ K_f^* = \begin{cases} 0, & \text{for } K_s \leq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s, \\ F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s, & \text{for } K_s \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right). \end{cases} \]

In Step 1, knowing \( K_f^* \), the IDM maximizes

\[ \Pi_s = (p - w) \int_{K_s}^{K_s + K_f^*} (1 - F(x)) \, dx + (p - c_s) \int_0^{K_s} (1 - F(x)) \, dx - v_s K_s. \quad (14) \]

Case b1. \( 0 \leq K_s \leq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \): Then, substituting \( K_f^* = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s \) into \( \Pi_s \) and taking the first order derivative of \( \Pi_s \) with respect to \( K_s \), we obtain \( \frac{d\Pi_s}{dK_s} = -\left( p - c_s + v_s - w \right) - (w - c_s) F(K_s) \). Since \( w > c_s \) and \( w \leq c_s + v_s \), then \( \frac{d\Pi_s}{dK_s} \leq 0 \). Hence, \( K_s = 0 \) is optimal for \( 0 \leq K_s \leq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \).

Case b2. \( K_s \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \): Substituting \( K_f^* = 0 \) into \( \Pi_s \) and taking the first and second order derivatives of \( \Pi_s \) with respect to \( K_s \), we obtain \( \frac{d\Pi_s}{dK_s} = p - c_s - v_s - (p - c_s) F(K_s) \)
and \( \frac{d^2 \Pi_s}{dK_s^2} = -(p - c_s) f(K_s) \). Since \( p > c_s + v_s > c_s \), \( \Pi_s \) is concave in \( K_s \). We consider the following two cases.

Case b2.1. \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \): Note that \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \) is equivalent to \( \frac{w-c_f-v_f}{w-c_f} < \frac{p-c_s-v_s}{p-c_s} \). Hence, if \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \), then by the expression of \( \frac{d \Pi_s}{dK_s} = K_s = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \) is optimal for \( K_s \geq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \).

Case b2.2. \( \frac{w-c_f}{v_f} \geq \frac{p-c_s}{v_s} \): Since \( \frac{w-c_f}{v_f} \geq \frac{p-c_s}{v_s} \), \( \frac{d \Pi_s}{dK_s} \leq 0 \) for \( K_s \geq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \). Hence, if \( \frac{w-c_f}{v_f} \geq \frac{p-c_s}{v_s} \), \( K_s = F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \) is optimal for \( K_s \geq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \).

Case \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \): Combining Cases b1 and b2.1, \( K_s = 0 \) is the local optimum over interval \([0, F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right)] \) and \( K_s = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \) is the local optimum over \([F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right), \infty] \), and hence either \( K^* = \left( 0, F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \) or \( K^* = \left( F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right) \) is optimal in this case.

Case \( \frac{w-c_f}{v_f} \geq \frac{p-c_s}{v_s} \): Combining Cases b1 and b2.2, \( K_s = 0 \) is the local optimum over the first interval \([0, F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right)] \) and \( K_s = F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \) is the local optimum over the second interval \([F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right), \infty] \), which implies the second interval is dominated by the first one and \( K_s = 0 \) is the global optimum for this case. That is, \( K^* = \left( 0, F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \right) \) is optimal in this case.

(c) Assume that \( w > c_s + v_s \) holds. The proof for this case is the same as that for the case of \( c_s < w \leq c_s + v_s \) up to the derivation of (14).

Case c1. \( 0 \leq K_s \leq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \): Similar to the proof for the case of \( c_s < w \leq c_s + v_s \), \( \frac{d \Pi_s}{dK_s} = \frac{d^2 \Pi_s}{dK_s^2} = F(K_s) \). Thus, \( \Pi_s \) is concave in \( K_s \).

Case c1.1. \( \frac{w-c_f}{v_f} \leq \frac{w-c_f-v_f}{w-c_f} \): \( \frac{w-c_f-v_f}{w-c_f} \) is implied. For \( K_s \leq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \), \( \frac{d \Pi_s}{dK_s} \geq 0 \). Hence, if \( \frac{w-c_f}{v_f} \leq \frac{w-c_f-v_f}{w-c_f} \), \( K_s = F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \) is optimal for \( 0 \leq K_s \leq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \).

Case c1.2. \( \frac{w-c_f}{v_f} > \frac{w-c_f-v_f}{w-c_f} \): \( K_s = F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \) is optimal for \( 0 \leq K_s \leq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \).

Case c2. \( K_s \geq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \): Note that Cases b2.1 and b2.2 for \( c_s < w \leq c_s + v_s \) also hold for \( w > c_s + v_s \). We consider the following three mutually exclusive and collectively exhaustive cases:

Case \( \frac{w-c_f}{v_f} < \frac{w-c_f-v_f}{w-c_f} \): Since \( p > w \), \( \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \) also holds. Therefore, Case c1.1 and Case b2.1 apply, i.e., \( K_s = F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \) is the local optimum over the first interval \([0, F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right)] \) and \( K_s = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \) is the local optimum over the second interval \([F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right), \infty] \), which implies the first interval is dominated by the second interval and hence \( K^* = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \). That is, \( K^* = \left( F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right), 0 \right) \).
Case $\frac{w-c_s}{v_s} \leq \frac{w-c_f}{v_f}$: Since $p > w$, \( \frac{w-c_s}{v_s} < \frac{w-c_f}{v_f} \) also holds. Therefore, Case c1.2 and Case b2.2 apply, i.e., \( K_s = F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right) \) is the local optimum over the first interval \([0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)]\) and \( K_s = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \) is the local optimum over the second interval \([F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \infty)\], which implies the second interval is dominated by the first interval and hence \( K^*_s = F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right) \). That is, \( K^* = (F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)) \).

Case $\frac{w-c_s}{v_s} < \frac{w-c_f}{v_f} < \frac{p-c_s}{v_s}$: Case c1.2 and Case b2.1 apply. Then, \( K_s = F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right) \) is the local optimum over \([0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)]\) and \( K_s = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) \) is the local optimum over \([F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), \infty)\]. Thus, either \( K^* = (F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right), 0) \) or \( K^* = (F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right), F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - F^{-1}\left(\frac{w-c_s-v_s}{w-c_s}\right)) \).

**Proof of Lemma 3(a).** As the demand $x$ is realized in Step 3 of Table 1, the IDM determines the production quantities $y$ and $z$ to minimize his total cost in Step 4. The IDM pays $wy + r (R - y)^+$ to the foundry including the capacity reservation cost. Hence, the ex-post profits for the IDM and foundry for given $y$ and $z$ can be represented respectively by $\pi_s = (p - w)y + (p - c_s)z - v_sK_s - r (R - y)^+$, and $\pi_f = (w - c_f)y - v_fK_f + r (R - y)^+$.

That is, the IDM determines $y$ and $z$ by solving the following maximization problem:

$$
\max_{y+z\leq x, 0\leq y\leq K_f, 0\leq z\leq K_s} \pi_s.
$$

For case $w \leq c_s$, $p - w \geq p - c_s$ holds and, to maximize the objective function, $y$ must be set as large as possible subject to the constraints. That is, the foundry’s capacity has to be used first. Hence,

$$
(y^*, z^*) = \begin{cases} 
(x, 0), & \text{for } 0 \leq x \leq K_f, \\
(K_f, x - K_f), & \text{for } K_f \leq x \leq K_s + K_f, \\
(K_f, K_s), & \text{for } x \geq K_s + K_f.
\end{cases}
(3a.1)
$$

In Step 2, the foundry determines the capacity $K_f$ to maximize her expected profit $\Pi_f$, constrained by $K_f \geq R$, anticipating that the IDM will determine $y$ and $z$ as in (3a.1). That is

$$
\max_{K_f \geq R} \Pi_f = \max_{K_f \geq R} (w - c_f)E[y^*] - v_fK_f + rE[(R - y^*)^+].
(3a.2)
$$

Using (3a.1), it is easy to verify the following:

$$
E[y^*] = \int_0^{K_f} xf(x)dx + \int_{K_f}^{\infty} K_f f(x)dx = \int_0^{K_f} (1 - F(x)) dx,
$$
\[ E[z^*] = \int_{K_f}^{K_s+K_f} (x - K_f) f(x) \, dx + \int_{K_s+K_f}^{\infty} K_s f(x) \, dx = \int_{K_f}^{K_s+K_f} (1 - F(x)) \, dx, \]  

\[ (3a.3) \]

\[ E[(R - y^*)^+] = \int_{R}^{\infty} (R - x) f(x) \, dx = \int_{R}^{\infty} F(x) \, dx. \]

Hence, the objective function in (3(a).2) can be expressed as

\[ \Pi_f = (w - c_f) \int_{0}^{K_f} (1 - F(x)) \, dx - v_f K_f + r \int_{0}^{R} F(x) \, dx. \]  

\[ (3a.4) \]

The first and second order derivatives of \( \Pi_f \) are

\[ \frac{d\Pi_f}{dK_f} = (w - c_f) (1 - F(K_f)) - v_f \]

and

\[ \frac{d^2\Pi_f}{dK_f^2} = -(w - c_f) f(K_f). \]

\( w > c_f \) implies \( \frac{d^2\Pi_f}{dK_f^2} < 0 \). With the constraint that \( K_f \geq R \), the optimal solution can be expressed as follows:

\[ K_f^* = \begin{cases} 
F^{-1}\left(\frac{w - c_f - v_f}{w - c_f}\right), & \text{for } 0 \leq R \leq F^{-1}\left(\frac{w - c_f - v_f}{w - c_f}\right), \\
R, & \text{for } R \geq F^{-1}\left(\frac{w - c_f - v_f}{w - c_f}\right).
\end{cases} \]  

\[ (3a.5) \]

In Step 1, the IDM determines \( K_s \) and \( R \) to maximize his expected profit \( \Pi_s \) with \( y = y^* \), \( z = z^* \) and \( K_f = K_f^* \). That is,

\[ \max_{K_s, R \geq 0} \Pi_s = (p - w) E[y^*] + (p - c_s) E[z^*] - v_s K_s - r E[(R - y^*)^+]. \]  

\[ (3a.6) \]

Substituting (3(a).3) into (3(a).6) yields

\[ \Pi_s = (p - w) \int_{0}^{K_f} (1 - F(x)) \, dx + (p - c_s) \int_{K_f}^{K_s+K_f} (1 - F(x)) \, dx - v_s K_s - r \int_{0}^{R} F(x) \, dx. \]  

\[ (3a.7) \]

Hence, (3(a.4), (3(a.5) and (3(a.7) complete the proof of Lemma 3(a).

Lemma 3(b)(c) can be proved similarly.

**Proof of Proposition 3.** (a) Assume \( MOF_s \geq MOF_f \). Under this condition, \( PRCs(w, r) \) in (a2) and (c2) of Lemma 4 are candidate \( PRCs \) to replicate the capacity investment and production decisions in Proposition 1. We first show that \( PRCs(w, r) \) in (c2) of Lemma 4 can replicate, and then show that \( PRCs(w, r) \) in (a2) of Lemma 4 fail.

**PRCs(w, r) in (c2) of Lemma 4:** To match the capacities, we set

\[ \frac{c_s + v_s - c_f - v_f}{c_s - c_f} = \frac{c_s + v_s - w}{c_s + r - w}. \]  

\[ (3a.1) \]

Since \( w > c_f + v_f \), we denote

\[ w = c_f + v_f + \alpha. \]  

\[ (3a.2) \]
where $\alpha > 0$. Since

$$w < p, \alpha < p - c_f - v_f,$$  \hspace{1cm} (3a.3)  

(3a.1) and (3a.2) lead to

$$r = \frac{(c_s - c_f) (c_s + v_s - w)}{c_s + v_s - c_f - v_f} - (c_s - w)$$  \hspace{1cm} (3a.4)  

$$= \frac{v_f c_s - v_s c_f + w (v_s - v_f)}{c_s + v_s - c_f - v_f} = \frac{v_f + \alpha (v_s - v_f)}{c_s + v_s - c_f - v_f}.$$  

The conditions in (c2) of Lemma 4 are as follows:

$$w > c_s$$  \hspace{1cm} (3a.5)  

$$r > w - c_s$$  \hspace{1cm} (3a.6)  

$$w \leq c_s + v_s$$  \hspace{1cm} (3a.7)  

$$\frac{w - c_f}{v_f} < \frac{p - c_s}{v_s}$$  \hspace{1cm} (3a.8)  

$$\frac{r}{v_s} \geq \frac{v_s (p - w)}{p - c_s - v_s}$$  \hspace{1cm} (3a.9)  

We shall show that there exists $\alpha > 0$ that satisfies (3a.5)-(3a.9). Substituting (3a.2) and (3a.4) in (3a.5)-(3a.9), we obtain the following: (3a.5) is equivalent to

$$\alpha > c_s - c_f - v_f$$  \hspace{1cm} (3a.10)  

(3a.6) is equivalent to $v_f + \alpha \frac{v_s - v_f}{c_s + v_s - c_f - v_f} > c_f + v_f + \alpha - c_s$, which is $c_s - c_f > \alpha \frac{c_s - c_f}{c_s + v_s - c_f - v_f}$. Since $c_f + v_f < c_s + v_s$ and $MOF_s \geq MOF_f$ imply $c_s - c_f > 0$, (3a.6) is equivalent to

$$\alpha < c_s + v_s - c_f - v_f.$$  \hspace{1cm} (3a.11)  

(3a.7) is easily verified to be

$$\alpha \leq c_s + v_s - c_f - v_f.$$  \hspace{1cm} (3a.12)  

If (3a.3) holds, then $p > w$ and thus

$$\frac{p - c_s}{v_s} \geq \frac{p - c_f}{v_f} > \frac{w - c_f}{v_f}.$$  \hspace{1cm} (3a.13)  

That is, if (3a.3) holds, (3a.8) holds. Substituting the expression of $r$ in (3a.4), (3a.9) can be expressed as $v_f + \alpha \frac{v_s - v_f}{c_s + v_s - c_f - v_f} \geq v_s \frac{p - c_f - v_f - \alpha}{p - c_s - v_s}$. By rearranging the terms, we obtain
Similarly, we have
\[ \frac{v_s(p - cf)}{c_s + v_s - cf - v_f} \leq v_s(p - cf) - v_f(p - c_s), \]
which is equivalent to \( \alpha \leq c_s + v_s - cf - v_f \)
because \( \frac{p - cf}{v_f} \leq \frac{p - c_s}{v_s} \) implies \( v_s(p - cf) - v_f(p - c_s) < 0 \). Thus, (3a.9) is equivalent to
\[ \alpha \leq c_s + v_s - cf - v_f. \tag{3a.14} \]

Since \( c_s + v_s - cf - v_f < p - cf - v_f \), by (3a.10)–(3a.14), \( \alpha \) in \( (c_s - cf - v_f)^+ < \alpha < c_s + v_s - cf - v_f \) satisfies (3a.5)–(3a.9). Thus, PRCs\((w, r)\) in (c2) of Lemma 4 can coordinate capacity investment and production decisions.

Thus, the PRCs with \( w = cf + v_f + \alpha \) and \( r = v_f + \frac{\alpha(v_s - v_f)}{c_s + v_s - cf - v_f} \), where \( (c_s - cf - v_f)^+ < \alpha < c_s + v_s - cf - v_f \), can coordinate the capacity investment and production decisions.

The total expected profit in the centralized system is expressed as
\[ \Pi_{cs} = (p - cf) \int_0^{K_{cf}} (1 - F(x)) \, dx + (p - c_s) \int_{K_{cf}}^{K_{cs} + K_{cf}} (1 - F(x)) \, dx - v_f K_{cf} - v_s K_{cs}. \]

Note that the corresponding expected profits for the PRC can be derived by Lemma 3 with \( K_s = K_{cs} \), \( R = K_{cf} \) and \( K_f = K_{cf} \):

\[
\Pi_f = (w - cf) \int_0^{K_{cf}} (1 - F(x)) \, dx - v_f K_{cf} + r \int_0^{K_{cf}} F(x) \, dx,
\]
\[
\Pi_s = (p - w) \int_0^{K_{cf}} (1 - F(x)) \, dx + (p - c_s) \int_{K_{cf}}^{K_{cs} + K_{cf}} (1 - F(x)) \, dx - v_s K_{cs} - r \int_{K_{cf}}^{K_{cf}} F(x) \, dx.
\]

Substituting \( w = cf + v_f + \alpha \) and \( r = v_f + \frac{\alpha(v_s - v_f)}{c_s + v_s - cf - v_f} \) in these equations, we obtain
\[
\Pi_f = (v_f + \alpha) \int_0^{K_{cf}} (1 - F(x)) \, dx - v_f K_{cf} + \left(v_f - \frac{\alpha(v_f - v_s)}{c_s + v_s - cf - v_f}\right) \int_0^{K_{cf}} F(x) \, dx
\]
\[
= \frac{\alpha}{c_s + v_s - cf - v_f} \left((c_s - cf) \int_0^{K_{cf}} (1 - F(x)) \, dx - (v_f - v_s) K_{cf}\right),
\]
\[
\Pi_s = \Pi_{cs} - \Pi_f.
\]

The capacities are as shown in Proposition 1.

**PRC\((w, r)\) in (a2) of Lemma 4:** To match capacities, we set \( \frac{c_s + v_s - cf - v_f}{c_s - cf} = \frac{c_s + v_s - w}{c_s + r - w} \).

Similarly, we have \( w = cf + v_f + \alpha \), and \( r = v_f + \frac{\alpha(v_s - v_f)}{c_s + v_s - cf - v_f} \). We restrict \( \alpha \) to satisfy the conditions in (a2) of Lemma 4, i.e.,
\[
w \leq c_s \Rightarrow \alpha \leq (c_s - cf - v_f)^+,
\]
\[
\frac{w - cf}{v_f} < \frac{p - c_s}{v_s} \Rightarrow \alpha < \frac{v_f(p - c_s - v_s)}{v_s},
\]
\[
r \geq \frac{v_s(p - w)}{p - c_s - v_s} \Rightarrow \alpha \leq c_s + v_s - cf - v_f,
\]
\[ r < \frac{v_f c_s - v_s c_f + w(v_s - v_f)}{w - c_f - v_f} \Rightarrow \]
\[- (v_f - v_s) \alpha^2 + (2v_f - v_s)(c_s + v_s - c_f - v_f) \alpha - v_f (c_f + v_f - c_s - v_s)^2 > 0. \quad (3a.16)\]

Since \( \alpha > 0 \), by (3a.15), \( c_s > c_f + v_f \). This together with \( MOF_s \geq MOF_f \) implies \( v_s < v_f \).

Define \( \Psi(\alpha) \equiv -(v_f - v_s) \alpha^2 + (2v_f - v_s)(c_s + v_s - c_f - v_f) \alpha - v_f (c_f + v_f - c_s - v_s)^2 \). Then the maximum value of \( \Psi(\alpha) \) is achieved at \( \alpha = \frac{(2v_f - v_s)(c_s + v_s - c_f - v_f)}{2(v_f - v_s)} \) and the maximum \( \Psi(\alpha) \) is equal to \(-\frac{(2c_s - v_s)(c_f + v_f - c_s - v_s)^2}{4(c_s - v_f)^2} \). By \( c_s > c_f + v_f \) and \( v_s < v_f \), we have \( 2c_s - v_s > 0 \) and \( 2c_s v_f - 3v_s v_f + v_s^2 > 0 \). This implies the maximum of \( \Psi(\alpha) \) is less than 0, so (3a.16) and (3a.15) cannot hold at the same time, that is, \( PRCs(w, r) \) in (a2) of Lemma 4 cannot replicate the centralized capacity investment decision.

This completes the proof for the case of \( MOF_s \geq MOF_f \) and \( c_s > c_f \).

(b) Assume that \( MOF_s < MOF_f \) holds. We shall prove that under these conditions, the expected profit and capacities in Proposition 1 can be realized with \( w \) and \( r \) in (a1), (a3), (b1), (c1), and (c3) of Lemma 4. To match the capacities, we set

\[
\frac{p - c_f - v_f}{p - c_f} = \frac{p - w}{p - w + r}. \quad (3b.1)
\]

We also denote \( w = c_f + v_f + \alpha \); then (3b.1) can be rewritten as

\[
r = v_f \frac{p - w}{p - c_f - v_f} = v_f - \alpha \frac{v_f}{p - c_f - v_f}. \quad (3b.2)
\]

First, we use (a1) and (c1) in Lemma 4 to coordinate the capacity investments. By combining the two cases, we have

\[
w \leq c_s + v_s \quad (3b.3)
\]
\[
w < c_s + r \quad (3b.4)
\]
\[
r < v_s (p - w) / (p - c_s - v_s) \quad (3b.5)
\]
\[
\frac{w - c_f}{v_f} < \frac{p - c_s}{v_s} \quad (3b.6)
\]

We shall show that there exists \( \alpha > 0 \) that satisfies (3b.3)-(3b.6). Substituting (3a.2) and (3b.2) in (3b.3)-(3b.6), we find (3b.3)-(3b.6) are equivalent to the following four conditions respectively:

\[
\alpha \leq c_s + v_s - c_f - v_f,
\]
\[
\alpha < \frac{(c_s - c_f)(p - c_f - v_f)}{p - c_f},
\]
\[ \alpha < p - c_f - v_f, \]
\[ \alpha < \frac{v_f}{v_s} (p - c_s) - v_f. \]

It can be shown as follows that \( MOF_f > MOF_s \) is equivalent to \( c_s + v_s - c_f - v_f > \left( \frac{c_s - c_f}{p - c_f} \right) \left( \frac{p - c_f - v_f}{p - c_f} \right) \) \( \Leftrightarrow \) \( (p - c_f)(v_s - v_f) > -v_f(c_s - c_f) \Leftrightarrow (p - c_f)v_s > v_f(p - c_s) \Leftrightarrow \frac{p - c_s}{v_s} > \frac{p - c_f}{v_f} \). Hence, \( w = c_f + v_f + \alpha \) and \( r = v_f - \alpha \frac{v_f}{p - c_f - v_f} \) with
\[
\alpha < \frac{(c_s - c_f)(p - c_f - v_f)}{p - c_f} \quad \text{and} \quad \alpha < \frac{v_f}{v_s} (p - c_s) - v_f \quad (3b.7)
\]
can coordinate the capacity investment decisions.

Second, we use (a3) and (c3) in Lemma 4 to coordinate. By combining the conditions in these two cases, we have
\[
w < c_s + r \quad (3b.8) \\
r < v_f (p - w) / (w - c_f - v_f) \quad (3b.9) \\
\frac{p - c_s}{v_s} < \frac{w - c_f}{v_f} \quad (3b.10)
\]

We shall show that there exists \( \alpha > 0 \) that satisfies (3b.8)–(3b.10). Substituting (3a.2) and (3b.2) into (3b.8)–(3b.10), we find that (3b.8) is equivalent to \( \alpha < \left( \frac{c_s - c_f}{p - c_f} \right) \left( \frac{p - c_f - v_f}{p - c_f} \right) \); (3b.9) always holds; (3b.10) is equivalent to \( \alpha \geq \frac{v_f}{v_s} (p - c_s) - v_f \). Thus, \( w = c_f + v_f + \alpha \) and \( r = v_f - \alpha \frac{v_f}{p - c_f - v_f} \) with
\[
\alpha < \frac{(c_s - c_f)(p - c_f - v_f)}{p - c_f} \quad \text{and} \quad \alpha \geq \frac{v_f}{v_s} (p - c_s) - v_f \quad (3b.11)
\]
can coordinate the capacity investment decisions.

Combining \( \alpha \) in (3b.7) and (3b.11), we have that \( w = c_f + v_f + \alpha \) and \( r = v_f - \alpha \frac{v_f}{p - c_f - v_f} \) with
\[
\alpha < \frac{(c_s - c_f)(p - c_f - v_f)}{p - c_f} \quad (3b.12)
\]
can coordinate the capacity investment decisions.

Third, we use (b1) in Lemma 4 to coordinate. The conditions in (b1) are
\[
w \leq c_s + v_s, \quad (3b.13) \\
w \geq c_s + r, \quad (3b.14) \\
r < \frac{v_f (p - w)}{w - c_f - v_f}. \quad (3b.15)
\]
After substituting (3a.2) and (3b.2) into (3b.13)–(3b.15), we find that (3b.15) always holds, and (3b.13) and (3b.14) correspond to \( \alpha \leq c_s + v_s - c_f - v_f \) and \( \alpha \geq \frac{(c_s - c_f)(p - c_f - v_f)}{p - c_f} \) respectively. Thus, \( w = c_f + v_f + \alpha \) and \( r = v_f - \alpha \frac{v_f}{p - c_f - v_f} \) with

\[
\frac{(c_s - c_f)(p - c_f - v_f)}{p - c_f} \leq \alpha \leq c_s + v_s - c_f - v_f
\]

(3b.16)
can coordinate the capacity investment decisions.

Combining \( \alpha \) in (3b.12) and (3b.16), we have that \( w = c_f + v_f + \alpha \) and \( r = v_f - \alpha \frac{v_f}{p - c_f - v_f} \) with

\[
0 < \alpha \leq c_s + v_s - c_f - v_f
\]
can coordinate the capacity investment decisions.

The total profit in the centralized system is expressed as

\[
\Pi_{cs} = (p - c_f) \int_0^{K_{cf}} (1 - F(x)) \, dx - v_f K_{cf}.
\]

Note that the corresponding expected profits for the PRC can be derived by Lemmas 3(a), 3(b) or 3(c) with \( K_s = 0, R = K_{cf} \) and \( K_f = K_{cf} \).

\[
\Pi_f = (w - c_f) \int_0^{K_{cf}} (1 - F(x)) \, dx - v_f K_{cf} + r \int_0^{K_{cf}} F(x) \, dx,
\]

\[
\Pi_s = (p - w) \int_0^{K_{cf}} (1 - F(x)) \, dx - r \int_0^{K_{cf}} F(x) \, dx.
\]

Substituting \( w = c_f + v_f + \alpha \) and \( r = v_f - \frac{\alpha v_f}{p - c_f - v_f} \) in these equations, we obtain

\[
\Pi_f = (v_f + \alpha) \int_0^{K_{cf}} (1 - F(x)) \, dx - v_f K_{cf} + \left(v_f - \frac{\alpha v_f}{p - c_f - v_f}\right) \int_0^{K_{cf}} F(x) \, dx
\]

\[
= \frac{\alpha}{p - c_f - v_f} \left(p - c_f\right) \int_0^{K_{cf}} (1 - F(x)) \, dx - v_f K_{cf}
\]

\[
= \frac{\alpha}{p - c_f - v_f} \Pi_{cs},
\]

\[
\Pi_s = \left(1 - \frac{\alpha}{p - c_f - v_f}\right) \Pi_{cs}.
\]

**Proof of Proposition 4.** (i) It is straightforward to observe that \( \Pi_f \) increases in \( \alpha \). As \( w = \alpha + c_f + v_f \), \( \Pi_f \) also increases in \( w \). Since \( \Pi_s = \Pi_{cs} - \Pi_f \) and \( \Pi_{cs} \) is independent of \( w \), \( \Pi_s \) decreases in \( w \). By \( w \) and \( r \) specified in Proposition 3(a), as \( w \) converges to \( c_s + v_s \), \( r \) converges to \( v_s \). We argue in the following that under the limit PRC \((w = c_s + v_s, r = v_s)\), \( \Pi_s = \Pi_f \). Note that the total capacity under PRCs in Proposition 3(a) is \( F^{-1}\left(\frac{p - c_s - v_s}{p - c_s}\right)\),
which is the same as the capacity built under the IDM acting independently case. We consider the IDM’s expected profit as composed of ex-ante capacity investment cost and ex-post gain from satisfying the demand. Ex ante, to secure the same amount of capacity, the IDM incurs the same cost by either reserving from the foundry or building by himself, because \( r = v_s \). Ex post, for a unit of realized demand, the IDM collects the same revenue by using either his own capacity (gain \( p - c_s \)) or reserved capacity from the foundry (gain \( p - w + r = p - (c_s + v_s) + v_s = p - c_s \)). As a result, as long as the total capacity under the \( \text{PRC} (w = c_s + v_s, r = v_s) \) is the same as that under the “IDM independent” case, \( \Pi_s^e = \Pi_s^i \). This result, along with the property that \( \Pi_s^e \) decreases in \( w \), completes the proof of (i).

(ii) Under the \( \text{PRC}s \) specified in Proposition 3(b), \( \Pi_s^i = \left(1 - \frac{\alpha}{p - c_f - v_f}\right) \Pi_{cs} \), which decreases in \( \alpha \). Since \( w = c_f + v_f + \alpha \), \( w \) increases in \( \alpha \). Thus, \( \Pi_s^i \) decreases in \( w \). The smallest \( \Pi_s^i \) is achieved when \( w = c_s + v_s \). Next, we show that under the \( \text{PRC}s \) specified in Proposition 3(b), when only the foundry builds capacity, \( \Pi_s \) is \( \frac{p - w}{p - c_f - v_f} \) portion of the system profit, regardless of the amount of the foundry’s capacity and the demand distribution. The expected profits are equal to the ex-post gain from satisfying the demand net ex-ante capacity investment cost. Since only the foundry builds capacity, the IDM’s proportion of capacity investment cost is \( \frac{c}{v_f} = 1 - \frac{\alpha}{p - c_f - v_f} = 1 - \frac{w - c_f - v_f}{p - c_f - v_f} = \frac{p - w}{p - c_f - v_f} \). The IDM’s proportion of ex-post gain is \( \frac{p - w + r}{p - c_f} = \frac{p - w + v_f - \alpha v_f}{p - c_f} \). By substituting \( w = c_f + v_f \) into the right hand side of the equation, we get \( \frac{p - w + r}{p - c_f} = \frac{p - w}{p - c_f - v_f} \). Since the IDM’s both ex-ante cost proportion and ex-post gain proportion are \( \frac{p - w}{p - c_f - v_f} \), his expected profit proportion is \( \frac{p - w}{p - c_f - v_f} \), independent of the amount of the foundry’s capacity and the demand distribution.

Note that as we have argued in (i), under the \( \text{PRC} \) with \( r = v_s \) and \( w = c_s + v_s \), if the total capacity under this \( \text{PRC} \) is the same as that under the “IDM independent” case, then \( \Pi_s \) under this \( \text{PRC} \) equals \( \Pi_s^i \). Let \( K_3 = (0, F^{-1}\left(\frac{w - c_s}{p - c_s}\right)) \). The total capacity of \( K_3 \) equals that under the “IDM independent” case. For the \( \text{PRC} \) with \( r < v_s \) and \( w = c_s + v_s \) and the capacity investments \( K_3 \), the IDM’s expected profit is greater than \( \Pi_s \). Since \( r \) specified in Proposition 3(b) can be shown to be strictly less than \( v_s \),

\[
[\Pi_s]_{K_3}^{(w, r)} > \Pi_s^i \text{ when } w = c_s + v_s.
\]

Denote the total and the IDM’s expected profits under the \( (w, r) \) specified in Proposition 3(b) and the capacity setting \( K \) as \( [\Pi]_{K}^{(w, r)} \) and \( [\Pi]_{K_s}^{(w, r)} \), respectively. We have shown that \( [\Pi]_{K_3}^{(w, r)} = \frac{p - w}{p - c_f - v_f} [\Pi]_{K_3}^{(w, r)} \). As \( K_c = (K_{cs}, K_{cf}) = (0, F^{-1}\left(\frac{w - c_f - v_f}{p - c_f}\right)) \) is the centralized capacity investments, \( [\Pi]_{K_c}^{(w, r)} \geq [\Pi]_{K_3}^{(w, r)} \). \( [\Pi]_{K_c}^{(w, r)} = \frac{p - w}{p - c_f - v_f} [\Pi]_{K_c}^{(w, r)} \), which together
with \([\Pi^i_s]_{K_3}^{(w,r)} = \frac{p-w}{p-c_f-v_f} [\Pi^j_f]_{K_3}^{(w,r)}\) and \([\Pi^j_f]_{K_3}^{(w,r)} \geq [\Pi^j_e]_{K_3}^{(w,r)}\), implies \([\Pi^i_s]_{K_3}^{(w,r)} \geq [\Pi^i_s]_{K_3}^{(w,r)}\). This along with (15) implies \([\Pi^j_f]_{K_3}^{(w,r)} > \Pi^i_s\) when \(w = c_s + v_s\). Because \([\Pi^i_s]_{K_3}^{(w,r)}\) decreases in \(w\) and \(w \leq c_s + v_s\) in Proposition 3, \(\Pi^i_s\) under any \(PRC\) specified in Proposition 3(b) is strictly greater than \(\Pi^i_s\).

**Proof of Proposition 5.** We prove the result for the \(CCPRCs\) in Proposition 3(a) and 3(b) in Steps A and B, respectively.

**Step A:** For \((w,r)\) specified in Proposition 3(a), we show \(v_s < r < v_f\) as follows: In Proposition 3(a), \(MOF_s \geq MOF_f\) implies \(c_s > c_f, c_s > c_f\) and \(\frac{p-c_s}{v_s} \geq \frac{p-c_f}{v_f}\) imply \(v_s < v_f\), \(r - v_s = (v_f - v_s) \frac{c_s + v_s - c_f - v_f - \alpha}{c_s + v_s - c_f - v_f} > 0\) by \(v_s < v_f\) and \(\alpha < c_s + v_s - c_f - v_f; r - v_f = -\frac{\alpha(v_f - v_s)}{c_s + v_s - c_f - v_f} < 0\), \(v_s < r < v_f\) and \(c_s > c_f\) imply \(w - r = \frac{p-r}{v_f - v_s} c_f + \frac{v_f - r}{v_f - v_s} c_f < \frac{p-v_s}{v_f - v_s} c_s + \frac{v_f - r}{v_f - v_s} c_s = c_s\). By \(w = c_f + v_f + \alpha\) and \(\alpha > (c_s - c_f - v_f)^+\), we have \(w > c_s\). Therefore, \(c_s < w < c_s + r\) holds, and the IDM will allocate the demand to the foundry first up to \(R\). This implies that if the foundry builds a capacity level that is less than or equal to \(R\), then the foundry’s expected profit is not influenced by the IDM’s capacity. Hence we obtain the foundry’s expected profit by setting \(K_s = 0\) without loss of generality. Let \([\Pi^j_f]_{K_f}^{(w,r)}\) denote the foundry’s expected profit under the \(CCPRC(w,r)\) when she builds a capacity of \(K_f\), and \([\Pi^j_f]_{K_f}^{(w,r)}\) denote her expected profit under the \(CCPRC(w,r)\) when she builds the capacity level \(K_f\) desired in the centralized system. Under the \(CCPRCs\) in Proposition 3(a), \(K_f = R\). Since \(c_s < w < c_s + r\), the expected profit expression in Lemma 3(c) is used. By setting \(K_s = 0\) and \(R = K_f\), we have

\[
[\Pi^j_f]_{K_f}^{(w,r)} = (w - c_f) \int_0^{K_f} (1 - F(x)) dx - v_f K_f + r \int_0^{K_f} F(x) dx.
\]

Note that \([\Pi^j_f]_{K_f}^{(w,r)}\) increases in \(K_f\) for \(K_f \leq F^{-1}(\frac{w - c_f - v_f}{w - c_f})\).

By substituting \(w\) and \(r\) specified in Proposition 3(a), we have \(\frac{w - c_f - v_f}{w - c_f - r} = c_s + v_s - c_f - v_f\). As \(\frac{w - c_f - v_f}{w - c_f} < \frac{w - c_f - v_f}{w - c_f - r} = \frac{c_s + v_s - c_f - v_f}{c_s - c_f}\), we have

\[
[\Pi^j_f]_{K_f = F^{-1}(\frac{w - c_f - v_f}{w - c_f})}^{(w,r)} < [\Pi^j_f]_{K_f = F^{-1}(\frac{c_s + v_s - c_f - v_f}{c_s - c_f})}^{(w,r)} = [\Pi^j_f]_{K_f = F^{-1}(\frac{w - c_f - v_f}{w - c_f})}^{(w,r)} = [\Pi^j_f]_{K_f}^{(w,r)}.
\]

Next we compare \([\Pi^j_f]_{K_f = F^{-1}(\frac{w - c_f - v_f}{w - c_f})}^{(w,r)}\) under the \(CCPRC(w,r)\) and the foundry’s expected profit under the \(ZRC\) for the same \(w\), denoted as \(\Pi^z_f(w)\). Under \(ZRCs\) with \(c_s < w < c_s + v_s\), Proposition 2 is applied. As the relationship between \(\frac{w - c_f - v_f}{w - c_f}\) and \(\frac{p-c_s}{p-c_s}\) in Proposition 2 cannot be determined, we consider the IDM’s problem as finding the optimal capacity pair \(K^*_s\) out of \(K^*_1 = 0, F^{-1}(\frac{w - c_f - v_f}{w - c_f})\) and \(K^*_2 = F^{-1}(\frac{p-c_s - v_s}{p-c_s})\). If \(K^* = K^*_2\), then it is actually
“IDM independent case”, and $K_f = 0$ and $\Pi_f = 0$, which is less than $[\Pi_f]^{(w,r)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)}$.

If $K^* = K^*_1$, it is indeed the ZRC$^f$ case, and we need to compare $[\Pi_f]^{(w,r)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)}$ under the ZRC$^f$ with $[\Pi_f]^{(w,r)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)}$ under the CCPRC$(w,r)$. Under the CCPRCs in Proposition 3(a), foundry’s capacity is used first. In the ZRC$^f$s, $K_s = 0$, and thus only the foundry’s capacity is used. Given the foundry establishing the same capacity, the foundry’s expected profit under the CCPRC is greater than that under the ZRC$^f$. This is because ex ante, the foundry is paid the reservation fee under the CCPRC. The foundry’s ex-post profits are the same if realized demand is higher than $F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. If realized demand is less than $F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, the foundry’s ex-post profit under the CCPRC is higher than that under the ZRC$^f$ because of the reservation fee. Therefore, ex ante, if $K^* = \left(0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$, we have $[\Pi_f]^{(w,r)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)} > [\Pi_f]^{(w)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)}$. This result, together with (16), implies $\Pi_f^{(w,r)} = [\Pi_f]^{(w,r)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)} > [\Pi_f]^{(w)}_{K_f = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)}$.

Thus, for either $K^* = K^*_2$ or $K^* = K^*_1$, we can conclude $[\Pi_f]^{(w,r)} > [\Pi_f]^{(w)}$.

Step B: For CCPRCs as specified in Proposition 3 (b), $\Pi_f = \frac{\alpha}{p-c_f-v_f}\Pi_{cs} = \frac{w-c_f-v_f}{p-c_f-v_f}\Pi_{cs}$.

Under ZRCs with $w \leq c_s + v_s$, similar argument to Step A follows: The IDM’s problem boils down to finding the optimal capacity pair $K^*$ out of $K^*_1 = \left(0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$ and $K^*_2 = \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right), 0\right)$. If $K^* = K^*_2$, the foundry’s expected profit is 0, less than $\Pi_f$. If $K^* = K^*_1$, then when the demand is greater than or equal to $F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, only $F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$ of the demand can be satisfied, and the foundry’s ex-post profit achieves the highest proportion, which is $\frac{w-c_f-v_f}{p-c_f-v_f}$ of the total ex-post profit (sum of the IDM’s and foundry’s profits). However, ex ante, the foundry’s capacity cost proportion is 1, and with positive probability, the foundry will have leftover capacity. As a result, $\Pi_f^{(w)}$ is less than $\frac{w-c_f-v_f}{p-c_f-v_f}$ of the total expected profit. However, under the CCPRC, $\Pi_f^{(w,r)} = \frac{w-c_f-v_f}{p-c_f-v_f}\Pi_{cs}$, where $\Pi_{cs}$ is the centralized system profit. Therefore, $\Pi_f^{(w,r)} > \Pi_f^{(w)}$.

Proof of Proposition 6. (i) If $MOF_s \geq MOF_f$ and $c_s \leq c_f + v_f$, by the observation from Proposition 3, $w$ can be arbitrarily close to $c_f + v_f$; by firms’ profit expressions, if $w$ converges to $c_f + v_f$, then the foundry’s profit converges to zero while the IDM’s profit $\Pi^c$ converges to the maximum system profit, which must be greater than $\Pi^s_f$. Since $\Pi^c$ decreases in $w$, as $w$ increases from $c_f + v_f$, $\Pi^c_f$ decreases from the maximum system profit. Then there exists a $w$ value, say $\bar{w}$ ($\bar{w} < c_s + v_s$), so that the CCPRC with $w = \bar{w}$ leads to
Thus, $\Pi^c = \Pi^{c*}$, and the $CCPRCs$ with $w < \bar{w}$ are strictly preferred by the IDM to his optimal $ZRC$. On the other hand, as $w$ converges to the upper bound of $c_s + v_s$, $\Pi^c$ converges to $\Pi^i$, and thus $\Pi^{c*} + \Pi^f < \Pi^i$ implies $\Pi^{c*} + \Pi^f < \Pi^* + \Pi^f$ for $w$ converging to $c_s + v_s$. Since $\Pi^{c*} \geq \Pi^f$, the foundry’s profit under the $ZRC$, $\Pi^f$ must be less than that under the $CCPRC$ with $w$ converging to $c_s + v_s$. As $\Pi^f$ under the $CCPRCs$ increases in $w$, there exists a lower bound $w$ so that $\Pi^f = \Pi^{f*}$ at $w$, and all the $CCPRCs$ with $w > w$ are preferred by the foundry to the IDM’s optimal $ZRC$.

Next, we show that $\bar{w} > w$. At $\bar{w}$, $\Pi^c = \Pi^{c*}$, so $\Pi^f > \Pi^{f*}$ considering that the IDM’s optimal $ZRC$ cannot achieve the maximum system profit (Section 5). At $w$, $\Pi^c = \Pi^{c*}$. Thus, $\Pi^f$ at $\bar{w}$ is strictly greater than $\Pi^f$ at $w$. Because $\Pi^f$ increases in $w$, $\bar{w} > w$. Since the $CCPRCs$ with $w < \bar{w}$ satisfy the IDM’s individual rationality condition, and the $CCPRCs$ with $w > w$ satisfy the foundry’s individual rationality condition, we conclude that under $MOF_s \geq MOF_f$ and $c_s \leq c_f + v_f$, there is a continuous range of coordinating $PRCs$ ($CPRCs$) with $w \in (w, \bar{w})$.

(ii) If $MOF_f > MOF_s$, $w$ can also be arbitrarily close to $c_f + v_f$ by Proposition 3. Following the same argument as in (i), there exists a $\bar{w}$ so that $CCPRCs$ with $w < \bar{w}$ satisfy the IDM’s individual rationality condition. As for the foundry’s individual rationality condition, because the lower bound of $\Pi^c$ is strictly greater than $\Pi^i$, the above argument for the case of $MOF_s \geq MOF_f$ does not apply and an alternative argument follows.

Proposition 2 shows that for $w \leq c_s + v_s$, the foundry always builds a capacity of $F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. It is straightforward to show that the foundry’s expected profit under $ZRCs$ increases in $w$ for $w \leq c_s + v_s$. This result, together with Proposition 5, implies that $\Pi^f$ under the $CCPRC$ with $w = c_s + v_s$ is greater than the foundry’s expected profit under any $ZRC$ with $w \leq c_s + v_s$. If the IDM’s optimal $ZRC$ has $w \leq c_s + v_s$, then $\Pi^f$ at $w = c_s + v_s$ is strictly greater than $\Pi^{f*}$. As $\Pi^f$ increases in $w$, there exists a lower bound $w$ so that $\Pi^f = \Pi^{f*}$ at $w$, and all the $CCPRCs$ with $w > w$ are preferred by the foundry to the $ZRC$. As shown in (i), $\bar{w} > w$. Therefore, we conclude that under $MOF_f > MOF_s$, if the IDM’s optimal $ZRC$ has $w \leq c_s + v_s$, then there is a continuous range of $CPRCs$ with $w \in (w, \bar{w})$.

**Proof of Lemma 4(a).** Assume that $w \leq c_s$ holds. In Step 1, the IDM determines $K_s$ and $R$ to maximize $\Pi_s$ in Lemma 3(a) subject to $K_s$, $R \geq 0$:

$$
\max_{K_s, R \geq 0} \Pi_s = (p-w) \int_0^{K_f} (1 - F(x)) \, dx + (p-c_s) \int_{K_f}^{K_s+K_f} (1 - F(x)) \, dx - v_s K_s - r \int_0^R F(x) \, dx.
$$

(4a.1)
If \( 0 \leq R \leq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \), then Lemma 3(a) implies that \( K_f^* = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \); \( \frac{\partial \Pi_s}{\partial R} = -r F(R) \leq 0 \), and thus \( \Pi_s \) is maximized at \( R = 0 \) for \( 0 \leq R \leq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \). That is, in this case, the positive capacity reservation contract does not increase the IDM’s expected profit. Hence, we consider in the rest of this proof the optimization problem (4a.1) with \( K_f^* = R \) subject to \( K_s \geq 0 \) and

\[
R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right).
\]  

(4a.2)

The first and second order derivatives of \( \Pi_s \) w.r.t. \( K_s \) are as follows.

\[
\frac{\partial \Pi_s}{\partial K_s} = p - c_s - v_s - (p - c_s) F(K_s + R),
\]

(4a.3)

\[
\frac{\partial^2 \Pi_s}{\partial K_s^2} = - (p - c_s) f(K_s + R).
\]

Since \( p > c_s \), then \( \frac{\partial^2 \Pi_s}{\partial K_s^2} < 0 \). Therefore, \( \Pi_s \) is concave in \( K_s \). We now consider the combinations of the following cases noting their equivalence:

\[
\frac{w-c_f}{v_f} < \frac{p-c_s}{v_s} \iff \frac{w-c_f-v_f}{w-c_f} < \frac{p-c_s-v_s}{p-c_s},
\]

(A1)

\[
\frac{p-c_s}{v_s} \leq \frac{w-c_f}{v_f} \iff \frac{p-c_s-v_s}{w-c_f} \leq \frac{w-c_f-v_f}{w-c_f},
\]

(A2)

\[
r < \frac{v_s(p-w)}{p-c_s-v_s} \iff \frac{p-c_s-v_s}{p-c_s} < \frac{c_s+v_s-w}{c_s+r-w} \iff \frac{p-c_s-v_s}{p-c_s} < \frac{p-w}{p-w+r},
\]

(B1)

\[
r \geq \frac{v_s(p-w)}{p-c_s-v_s} \iff \frac{c_s+v_s-w}{c_s+r-w} \leq \frac{p-c_s-v_s}{p-c_s} \iff \frac{p-w}{p-w+r} \leq \frac{p-c_s-v_s}{p-c_s},
\]

(B2)

\[
r < \frac{v_f c_s-v_s c_f+w(v_s-v_f)}{w-c_f-v_f} \iff \frac{w-c_f-v_f}{w-c_f} < \frac{c_s+v_s-w}{c_s+r-w},
\]

(C1)

\[
r \geq \frac{v_f c_s-v_s c_f+w(v_s-v_f)}{w-c_f-v_f} \iff \frac{c_s+v_s-w}{c_s+r-w} \leq \frac{w-c_f-v_f}{w-c_f},
\]

(C2)

\[
r < \frac{v_f (p-w)}{w-c_f-v_f} \iff \frac{w-c_f-v_f}{w-c_f} < \frac{p-w}{p-w+r},
\]

(D1)

\[
r \geq \frac{v_f (p-w)}{w-c_f-v_f} \iff \frac{p-w}{p-w+r} \leq \frac{w-c_f-v_f}{w-c_f}.
\]

(D2)

Case Lemma 4(a1) [(A1) and (B1)]: Because of the concavity of \( \Pi_s \) w.r.t. \( K_s \), (4a.2), (4a.3) and (A1) imply that the optimal solution is in either of the following two intervals:

Interval 1: \( K_s + R = F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) \), \( K_s \geq 0 \), and \( R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) \),
Interval 2: $K_s = 0$ and $R \geq F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right)$

For Interval 1, $\frac{d\Pi_s}{dK_s} = 0$. Substituting $K_s = F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right)$ and then taking the first and second order derivatives w.r.t. $R$, we have

\[
\frac{d\Pi_s}{dR} = (p - w) (1 - F(R)) - (p - c_s) (1 - F(R)) + v_s - rF(R) = c_s + v_s - w - (c_s + r - w) F(R)
\]

(4a.4)

\[
\frac{d^2\Pi_s}{dR^2} = - (c_s + r - w) f(R).
\]

$w \leq c_s$ and $r > 0$ imply $\frac{d^2\Pi_s}{dR^2} < 0$. That is, $\Pi_s$ is concave in $R$. (B1) and $R \leq F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right)$ in Interval 1 imply $\frac{d\Pi_s}{dR} > 0$. Therefore, $K = \left( 0, F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right) \right)$ maximizes $\Pi_s$ in Interval 1.

For Interval 2, $K_s = 0$. Substituting this in (4a.1), and taking the first and second order derivatives w.r.t. $R$, we have

\[
\frac{d\Pi_s}{dR} = (p - w) (1 - F(R)) - rF(R) = p - w - (p - w + r) F(R)
\]

(4a.5)

\[
\frac{d^2\Pi_s}{dR^2} = - (p - w + r) f(R).
\]

Since $p > w$ and $r > 0$, $\frac{d^2\Pi_s}{dR^2} < 0$. That is, $\Pi_s$ is concave in $R$. Since (B1) holds, $\frac{p - c_s - v_s}{p - c_s} < \frac{p - w}{p - w + r}$. Thus, there exists $R = F^{-1} \left( \frac{p - w}{p - w + r} \right) > F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right)$ that satisfies $\frac{d\Pi_s}{dR} = 0$; that is, $K = \left( 0, F^{-1} \left( \frac{p - w}{p - w + r} \right) \right)$ maximizes $\Pi_s$ in Interval 2.

Since the optimal $K = \left( 0, F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right) \right)$ in Interval 1 is also in Interval 2, $K = \left( 0, F^{-1} \left( \frac{p - w}{p - w + r} \right) \right)$ is optimal given that positive capacity is reserved. This completes the proof of Lemma 4(a1).

Case Lemma 4(a2) [(A1), (B2) and (C1)]: Still we consider the two intervals defined above separately.

For Interval 1: (B2) and (C1) together lead to $\frac{w - c_f - v_f}{w - c_f} < \frac{c_s - v_s - w}{c_s + r - w} \leq \frac{p - c_s - v_s}{p - c_s}$. Hence, in Interval 1, there exists $R$ that satisfies $\frac{d\Pi_s}{dR} = 0$ in (4a.4). Thus, in Interval 1, $\Pi_s$ is maximized at $K = \left( F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right) , F^{-1} \left( \frac{c_s + v_s - w}{c_s + r - w} \right) \right)$.

For Interval 2: (B2) is equivalent to $\frac{p - w}{p - w + r} \leq \frac{p - c_s - v_s}{p - c_s}$. This along with (4a.5) implies that for any $R \geq F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right)$, $\frac{d\Pi_s}{dR} \leq 0$. Thus, $K = \left( 0, F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right) \right)$ maximizes $\Pi_s$ in Interval 2.

Combining the two intervals, $K = \left( F^{-1} \left( \frac{p - c_s - v_s}{p - c_s} \right) , F^{-1} \left( \frac{c_s + v_s - w}{c_s + r - w} \right) \right)$ is optimal given that positive capacity is reserved, since the optimal $K$ in Interval 2 is also in Interval 1. This completes the proof of Lemma 4(a2).
Case Lemma 4(a3) [(A2) and (D1)]: (A2) and (4a.3) imply that $\frac{d\Pi}{dK_s} \leq 0$ for any $K_s \geq 0$ and $R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. Therefore, the optimal solution exists in the interval of $K_s = 0$ and $R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. Substituting $K_s = 0$ in (4a.1), and taking the first order derivative w.r.t. $R$, we have (4a.5). By (D1), $\frac{w-c_f-v_f}{w-c_f} < \frac{p-w}{p-w+r}$. Hence, $K = \left(0, F^{-1}\left(\frac{p-w}{p-w+r}\right)\right)$ maximizes $\Pi_s$ and may increase the IDM’s expected profit from that of the corresponding zero reservation contract. This completes the proof of Lemma 4(a3).

Next, we show for the rest two cases, there is no PRC that is superior to the ZRC.

Case (A1), (B2) and (C2): Still we consider two intervals defined above separately.

For Interval 1: (C2) and (4a.4) imply that $\frac{d\Pi}{dR} \leq 0$ holds in Interval 1. Therefore, $\Pi_s$ is maximized at $K = \left(F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right) - F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right), F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$. However, the case of $w \leq c_s$ in Proposition 2 indicates that this $K$ can be realized without PRC. Hence, for this case PRC cannot be superior to the corresponding ZRC.

For Interval 2: As discussed in Case Lemma 4(a2) above, $K = \left(0, F^{-1}\left(\frac{p-c_s-v_s}{p-c_s}\right)\right)$ maximizes $\Pi_s$ in this interval. However, this $K$ is also in Interval 1, and thus it is dominated by the optimal $K$ in Interval 1.

Hence, under this case, there is no PRC that is superior to the ZRC.

Case (A2) and (D2): (A2) $\frac{p-c_s-v_s}{p-c_s} \leq \frac{w-c_f-v_f}{w-c_f}$ implies for $R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, $\frac{d\Pi}{dK_s} \leq 0$ in (4a.3); thus, $K_s = 0$ and $\frac{d\Pi}{dR}$ is given in (4a.5). (D2) $\frac{p-w}{p-w+r} \leq \frac{w-c_f-v_f}{w-c_f}$ implies $\frac{d\Pi}{dR} \leq 0$ for $R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. Thus, $R = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$ and $K = \left(0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$ maximizes $\Pi_s$. However, this $K$ is the same as that for the case of $c_s < w \leq c_s + v_s$ under ZRCs in Proposition 2. That is, for this case, PRC cannot be superior to ZRC.

Proof of Lemma 4(b). Assume that $w \geq c_s + r$ holds. In Step 1, the IDM determines $K_s$ and $R$ to maximize $\Pi_s$ in Lemma 3(b) subject to $K_s$, $R \geq 0$:

$$
\max_{K_s, R \geq 0} \Pi_s = (p-w) \int_{K_s}^{K_s+K_f} (1-F(x))dx + (p-c_s) \int_0^{K_s} (1-F(x))dx - v_sK_s - r \int_{K_s}^{K_s+R} F(x)dx.
$$

(4b.1)

If $K_s + R \leq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, then Lemma 3(b) shows that $K_f^* = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s$, which is independent of $R$. In this case, it is optimal for the IDM to set $R = 0$. Hence, we consider in the rest of this proof the optimization problem (4b.1) with $K_f^* = R$ subject to $K_s \geq 0$ and

$$
R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right) - K_s.
$$

(4b.2)

The first and second order derivatives of $\Pi_s$ in (4b.1) w.r.t. $R$ are as follows

$$
\frac{d\Pi_s}{dR} = p-w - (p-w+r) F(K_s+R)
$$

(4b.3)
\[ \frac{d^2 \Pi_s}{dR^2} = -(p - w + r) f(K_s + R) \]

\( p > w \) and \( r > 0 \) imply \( \frac{d\Pi_s}{dR} < 0 \). Therefore, \( \Pi_s \) is concave in \( R \). Below we consider Cases Lemma 4(b1), (b2), and the rest cases.

Case (D1): (D1) is equivalent to \( \frac{w - c_s - v_s}{w - c_s} < \frac{p - w}{p - w + r} \). Because of the concavity of \( \Pi_s \) with respect to \( R \), (4b.2) and (4b.3) imply that the optimal solution exists in either of the following two intervals.

Interval 1: \( K_s + R = F^{-1}\left(\frac{p - w}{p - w + r}\right) \), \( K_s \geq 0 \) and \( R > 0 \)

Interval 2: \( R = 0 \) and \( K_s \geq F^{-1}\left(\frac{p - w}{p - w + r}\right) \).

Since \( R = 0 \) in Interval 2, the optimal solution in Interval 2 is dominated by that of the corresponding ZRC. Hence, we need to consider Interval 1 to identify an optimal solution that may be superior to the corresponding ZRC. We assume \( K_s + R = F^{-1}\left(\frac{p - w}{p - w + r}\right) \).

Substituting this in (4b.1) and taking the first order derivative w.r.t. \( K_s \), we obtain

\[ \frac{d\Pi_s}{dK_s} = w - c_s - v_s - (w - c_s - r) F(K_s). \]  

(4b.4)

Case Lemma 4(b1) \( [w \leq c_s + v_s, \text{ and } (D1)] \):

\( w \leq c_s + v_s \) and \( w \geq c_s + r \) imply \( \frac{d\Pi_s}{dK_s} \leq 0 \) in Interval 1. Therefore, \( K = \left(0, F^{-1}\left(\frac{p - w}{p - w + r}\right)\right) \)

maximizes \( \Pi_s \) and may increase the IDM’s expected profit from that of the corresponding ZRC. This completes the proof of Lemma 4(b1).

Case Lemma 4(b2) \( [w > c_s + v_s, (D1), (B1) \text{ and } w > c_s + r] \): (B1) is equivalent to \( \frac{p - w}{p - w + r} > \frac{w - c_s - v_s}{w - c_s} \). Therefore, there exists \( K_s \) such that \( 0 < K_s < F^{-1}\left(\frac{p - w}{p - w + r}\right) \) and \( \frac{d\Pi_s}{dK_s} = 0 \). Hence, by (4b.4), \( K = \left(F^{-1}\left(\frac{w - c_s - v_s}{w - c_s - r}\right), F^{-1}\left(\frac{p - w}{p - w + r}\right) - F^{-1}\left(\frac{w - c_s - v_s}{w - c_s - r}\right)\right) \)

maximizes \( \Pi_s \) and may increase the IDM’s expected profit from that of the corresponding ZRC. This proves Lemma 4(b2).

Next, we prove for the rest cases, there does not exist a PRC that is superior to the ZRC for the IDM.

Case \( w > c_s + v_s, (D1), (B2) \) and \( w > c_s + r \): (B2) is equivalent to \( \frac{p - w}{p - w + r} \leq \frac{w - c_s - v_s}{w - c_s - r} \). For any \( K_s \) in \( 0 \leq K_s \leq F^{-1}\left(\frac{p - w}{p - w + r}\right) \), \( \frac{d\Pi_s}{dK_s} \geq 0 \) in Interval 1. Therefore, \( K = \left(F^{-1}\left(\frac{p - w}{p - w + r}\right), 0\right) \)

maximizes \( \Pi_s \) in this interval. Since \( R = 0 \), this \( K \) is dominated by the ZRC.

Case \( w > c_s + v_s, (D1) \), and \( w = c_s + r \): \( \frac{d\Pi_s}{dK_s} > 0 \) by (4b.4), and thus \( K = \left(F^{-1}\left(\frac{p - w}{p - w + r}\right), 0\right) \)

maximizes \( \Pi_s \) in Interval 1. Since \( R = 0 \), this \( K \) is dominated by the ZRC.
Case (D2): (D2) is equivalent to \( \frac{p-w}{p-w+c_f} \leq \frac{w-c_f-v_f}{w-c_f} \). Since we consider the optimization problem (4b.1) subject to \( R \geq F^{-1}(\frac{w-c_f-v_f}{w-c_f}) - K_s \), (4b.3), (D2), and the concavity of \( \Pi_s \) imply that the optimal solution exists in either of the following two intervals:

Interval 3: \( K_s + R = F^{-1}(\frac{w-c_f-v_f}{w-c_f}) \), \( K_s \geq 0 \), and \( R \geq 0 \)

Interval 4: \( R = 0 \) and \( K_s \geq F^{-1}(\frac{w-c_f-v_f}{w-c_f}) \).

Since \( R = 0 \) in Interval 4, the optimal solution in Interval 4 is dominated by the ZRC.

For Interval 3, substituting \( R = F^{-1}(\frac{w-c_f-v_f}{w-c_f}) - K_s \) in (4b.1) and taking the first order derivative w.r.t. \( K_s \), we obtain \( \frac{d\Pi_s}{dK_s} \) as in (4b.4).

Case \( w \leq c_s + v_s \) and (D2): Since \( w \leq c_s + v_s \) and \( r \leq w - c_s \), \( \frac{d\Pi_s}{dK_s} \leq 0 \) in Interval 3. Therefore, \( K = (0, F^{-1}(\frac{w-c_f-v_f}{w-c_f})) \) maximizes \( \Pi_s \). The case of \( c_s < w \leq c_s + v_s \) in Proposition 2 implies that this \( K \) is dominated by the corresponding ZRC.

Case \( w > c_s + v_s \) and (D2): There are three subcases.

Case \( w > c_s + v_s \), (D2), (C1) and \( r < w - c_s \) : (C1) is equivalent to \( \frac{w-c_s-v_s}{w-c_s-r} > \frac{w-c_f-v_f}{w-c_f} \). For any \( K_s \) in \( 0 \leq K_s \leq F^{-1}(\frac{w-c_f-v_f}{w-c_f}) \), \( \frac{d\Pi_s}{dK_s} \geq 0 \) in Interval 3. Therefore, \( K = (F^{-1}(\frac{w-c_f-v_f}{w-c_f}), 0) \) maximizes \( \Pi_s \) in this interval. Since \( R = 0 \), this \( K \) is dominated by the corresponding ZRC.

Case \( w > c_s + v_s \), (D2), (C2) and \( r < w - c_s \) : (C2) is equivalent to \( \frac{w-c_f-v_f}{w-c_f} \geq \frac{w-c_s-v_s}{w-c_s-r} \). Then, there exists \( K_s \) such that \( 0 < K_s < F^{-1}(\frac{w-c_f-v_f}{w-c_f}) \) and \( \frac{d\Pi_s}{dK_s} = 0 \) in (4b.4). Hence, under this case, \( K = (F^{-1}(\frac{w-c_f-v_f}{w-c_f}), F^{-1}(\frac{w-c_f-v_f}{w-c_f}) - F^{-1}(\frac{w-c_s-v_s}{w-c_s-r})) \) maximizes \( \Pi_s \). Then \( K_s + R = F^{-1}(\frac{w-c_f-v_f}{w-c_f}) \); however, as mentioned at the beginning of this proof, for \( K_s + R = F^{-1}(\frac{w-c_f-v_f}{w-c_f}) \), \( K_s^* = F^{-1}(\frac{w-c_f-v_f}{w-c_f}) - K_s \) is independent of \( R \), and thus the optimal \( R = 0 \). Therefore, in this case PRC cannot be superior to ZRC.

Case \( w > c_s + v_s \), (D2) and \( r = w - c_s \) : \( w > c_s + v_s \) and \( r = w - c_s \) in (4b.4) imply that \( \frac{d\Pi_s}{dK_s} > 0 \) in Interval 3. Therefore, \( K = (F^{-1}(\frac{w-c_f-v_f}{w-c_f}), 0) \) maximizes \( \Pi_s \) in this interval. Since \( R = 0 \), this \( K \) is dominated by the corresponding ZRC.

**Proof of Lemma 4(c).** Assume that \( c_s < w < c_s + r \) holds. In Step 1, the IDM determines \( K_s \) and \( R \) to maximize \( \Pi_s \) in Lemma 3(c) subject to \( K_s, R \geq 0 \).

\[
\begin{align*}
\max_{K_s, R \geq 0} & \quad (p-w) \left( \int_{0}^{R} (1 - F(x)) \, dx + \int_{K_s+R}^{K_s+K_f} (1 - F(x)) \, dx \right) \\
& + (p-c_s) \int_{R}^{K_s+R} (1 - F(x)) \, dt - v_s K_s - r \int_{0}^{R} F(x) \, dx.
\end{align*}
\]
If \( K_s + R \leq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) \), then Lemma 3(c) shows that \( K^*_f = F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) - K_s \) in (4c.1). Then, by \( w > c_s \) and \( r > w - c_s \), \( \frac{\partial}{\partial R} = -(c_s + r - w) F(R) - (w - c_s) F(K_s + R) < 0 \), and thus \( \Pi_s \) takes on the maximum value at \( R = 0 \) for \( 0 \leq R \leq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) - K_s \), \( K_s \geq 0 \) and \( R \geq 0 \). In this region, the PRC does not increase the IDM’s expected profit. Hence, we consider, in the rest of this proof, the optimization problem (4c.1) with \( K^*_f = R \) subject to \( K_s \geq 0 \) and

\[
R \geq F^{-1} \left( \frac{w-c_f-v_f}{w-c_f} \right) - K_s. \tag{4c.2}
\]

The first and second order derivatives of \( \Pi_s \) w.r.t. \( K_s \) are as follows,

\[
\frac{\partial \Pi_s}{\partial K_s} = (p - c_s)(1 - F(K_s + R)) - v_s \tag{4c.3}
\]

\[
\frac{\partial^2 \Pi_s}{\partial K_s^2} = -(p - c_s) f(K_s + R). \tag{4c.4}
\]

\( p > c_s \) implies \( \frac{\partial^2 \Pi_s}{\partial K_s^2} < 0 \). That is, \( \Pi_s \) is concave in \( K_s \).

Case (A1): By (A1), \( \frac{w-c_f-v_f}{w-c_f} < \frac{p-c_s-v_s}{p-c_s} \). Because of the concavity of \( \Pi_s \) with respect to \( K_s \), (4c.2), (4c.3) and (A1) imply that the optimal solution exists in either of the following two intervals:

Interval 1: \( K_s + R = F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \), \( K_s \geq 0 \) and \( R \geq 0 \),

\[
\frac{d\Pi_s}{dR} = c_s + v_s - w - (c_s + r - w) F(R). \tag{4c.4}
\]

\[
\frac{d^2 \Pi_s}{dR^2} = -(c_s + r - w) F(R). \tag{4c.5}
\]

\( r > w - c_s \) implies \( \frac{d^2 \Pi_s}{dR^2} < 0 \). That is, \( \Pi_s \) is concave in \( R \). For \( w < c_s + v_s \), \( \frac{p-c_s-v_s}{p-c_s} < \frac{c_s+v_s-w}{c_s+r-w} \). Then, \( \frac{d\Pi_s}{dR} \geq 0 \) in Interval 1 for both \( w < c_s + v_s \) and \( w = c_s + v_s \). Therefore, \( K = \left( 0, F^{-1} \left( \frac{p-c_s-v_s}{p-c_s} \right) \right) \) maximizes \( \Pi_s \) in this case.

For Interval 2, \( K_s = 0 \). Substituting this into (4c.1) and taking the derivatives w.r.t. \( R \), we obtain

\[
\frac{d\Pi_s}{dR} = p - w - (p - w + r) F(R). \tag{4c.5}
\]
\[
\frac{d^2 \Pi_s}{dR^2} = -(p - w + r) f(R).
\]

\(p > w\) and \(r > 0\) imply \(\frac{d^2 \Pi_s}{dR^2} < 0\). That is, \(\Pi_s\) is concave in \(R\). By (B1), \(\frac{w - c_s - v_s}{p - c_s} < \frac{w - w + r}{p - w + r}\). Therefore, there exists \(K\) in Interval 2 such that \(\frac{d \Pi_s}{dR} = 0\) in (4c.4), and \(K = \left(0, F^{-1} \left(\frac{p - w}{p - w + r}\right)\right)\) maximizes \(\Pi_s\) in this case. Note that this \(K\) dominates the optimal solution in Interval 1 because \(K_s = 0\) is in both cases.

Hence, under this case, \(K = \left(0, F^{-1} \left(\frac{p - w}{p - w + r}\right)\right)\) is optimal given that positive capacity is reserved. This completes the proof of Lemma 4(c1).

Case Lemma 4(c2) \([w \leq c_s + v_s, (A1)\) and (B2)\]: We first consider Interval 1. By \(w \leq c_s + v_s\) and (B2), \(\frac{c_s + v_s - w}{c_s + r - w} \leq \frac{p - c_s - v_s}{p - c_s}\), for (4c.4) there exists \(R\) that satisfies \(\frac{d \Pi_s}{dR} = 0\). Hence, \(K = \left(F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right) - F^{-1} \left(\frac{c_s + v_s - w}{c_s + r - w}\right)\right)\) maximizes \(\Pi_s\) in this case.

For Interval 2: By (B2), \(\frac{p - w}{p - w + r} \leq \frac{c_s + v_s - w}{p - c_s}\), for (4c.5) \(\frac{d \Pi_s}{dR} \leq 0\) for any \(R\) in \(R \geq F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right)\). Therefore, \(K = \left(0, F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right)\right)\) maximizes \(\Pi_s\) in this case. Since this \(K\) is also in Interval 1, it is dominated by the optimal solution in Interval 1.

Hence, under this case, \(K = \left(F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right) - F^{-1} \left(\frac{c_s + v_s - w}{c_s + r - w}\right)\right)\) is optimal given that the positive capacity is reserved. This completes the proof of Lemma 4(c2).

Case \(w > c_s + v_s\) and (A1): We first consider Interval 1. \(w > c_s + v_s\) and (4c.4) imply \(\frac{d \Pi_s}{dR} < 0\) in Interval 1. Therefore, \(K = \left(F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right), 0\right)\) maximizes \(\Pi_s\) in this interval. Since \(R = 0\), this \(K\) is dominated by the corresponding ZRC.

For Interval 2: Assume that \(r > w - c_s\) and (B1) hold. Then, \(w - c_s < r < \frac{v_s(p - w)}{p - c_s - v_s}\). This leads to \((p - c_s)(c_s + v_s - w) > 0\) which contradicts with \(w > c_s + v_s\) and \(p > c_s\). Hence, (B2) holds. Then, \(\frac{p - w}{p - w + r} \leq \frac{c_s + v_s - w}{p - c_s}\). Therefore, \(\frac{d \Pi_s}{dR} \leq 0\) for any \(R\) in \(R \geq F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right)\), and thus \(K = \left(0, F^{-1} \left(\frac{p - c_s - v_s}{p - c_s}\right)\right)\) maximizes \(\Pi_s\) in this case. Since this \(K\) is also in Interval 1, it is dominated by the optimal solution in Interval 1. By the optimal solution in Interval 1, we conclude that PRC cannot improve the IDM’s expected profit over ZRC in this case.

Case (A2): By (A2), \(\frac{w - c_s - v_s}{p - c_s} \leq \frac{w - c_f - v_f}{w - c_f}\). Because of the concavity of \(\Pi_s\) in \(K_s\), (4c.2), (4c.3) and (A2) imply that the optimal solution exists in either of the following two intervals:

\begin{itemize}
  \item Interval 3: \(K_s + R = F^{-1} \left(\frac{w - c_f - v_f}{w - c_f}\right)\), \(K_s \geq 0\), and \(R \geq 0\).
  \item Interval 4: \(K_s = 0\) and \(R \geq F^{-1} \left(\frac{w - c_f - v_f}{w - c_f}\right)\).
\end{itemize}

As mentioned before, if \(K_s + R = F^{-1} \left(\frac{w - c_f - v_f}{w - c_f}\right)\), then \(K_s = F^{-1} \left(\frac{w - c_f - v_f}{w - c_f}\right) - K_s\), which is independent of \(R\), and thus it is optimal to set \(R = 0\). Thus, in Interval 3, PRC does
not increase the IDM’s expected profit over ZRC. Hence, we consider only Interval 4 in this case.

Case Lemma 4(c3) [(A2) and (D1)]: We obtain $\Pi_s = (p-w) \int_0^R (1-F(x)) \, dx - r \int_0^R F(x) \, dx$ in Interval 4 by substituting $K_s = 0$ in (4c.1). $\frac{d\Pi_s}{dR} = p-w-(p-w+r)F(R)$. By (D1) $\frac{w-c_f-v_f}{w-c_f} < \frac{p-w}{p-w+r}$ and the concavity of $\Pi_s$ in $R$, $K = \left(0, F^{-1}\left(\frac{p-w}{p-w+r}\right)\right)$ maximizes $\Pi_s$ in Interval 4, and may increase the IDM’s expected profit from that of the corresponding ZRC. This completes the proof of Lemma 4(c3).

Case (A2) and (D2): By (D2), $\frac{p-w}{p-w+r} \leq \frac{w-c_f-v_f}{w-c_f}$. Then, in Interval 4, $\frac{d\Pi_s}{dR} \leq 0$ for $R \geq F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$. Then $R = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, and $K = \left(0, F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)\right)$ maximizes $\Pi_s$ in Interval 4. Since in this $K$, $K_s + R = F^{-1}\left(\frac{w-c_f-v_f}{w-c_f}\right)$, the optimal $R$ should be 0 as mentioned before. Thus, in this case, PRC cannot improve the IDM’s expected profit over ZRC.