Supply Chain Coordination with Financial Constraints and Bankruptcy Costs

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We study the coordination of a supply chain in which both the supplier and retailer are working capital constrained and need short-term financing. Competitively priced loans for relevant risks are available from a bank to finance the supplier’s production and retailer’s procurement requirements, and failure of loan repayment leads to a costly bankruptcy. In this work, we focus on the case that the centralized supply chain has bankruptcy risks and costs. We first study a general “demand-independent (DI)” contract, i.e., basic contract parameters not depending on the demand realization. This type of contract includes and can be reduced to the traditional buyback, quantity discount, quantity flexibility, and revenue sharing contracts by giving the appropriate values to contract parameters. We show that DI contracts cannot coordinate the supply chain in general under conditions where either the retailer or both the supplier and retailer need to borrow to finance their working capital needs and thus incur bankruptcy risks. We show that a coordinating contract should have basic contract parameters contingent on the demand realization, i.e., be a “demand-dependent (DP)” contract. We provide a revenue sharing DP contract that can coordinate the capital constrained supply chain through voluntary compliance and arbitrarily allocate supply chain profits.

Key words: Newsvendor, Supply Contract, Coordination, Bankruptcy, and Bankruptcy Costs

History:

1. Introduction

In this paper, we consider a linear supply chain of one supplier and one retailer facing uncertain demand in an imperfect capital market with bankruptcy costs. Both the supplier and retailer are financially constrained, and they might need to borrow short-term loans from a bank in order to finance the supplier’s production and retailer’s procurement decisions. Failure to repay the loan leads to a costly bankruptcy, with bankruptcy costs reflecting the fixed administrative costs and variable costs proportional to realized sales. These costs are discussed in details in the corporate finance literature (see Ang, Chua, and McConnell, 1982, and Tirole, 2006), and in the recent supply
chain management literature (see Kouvelis and Zhao, 2009). In the event of a bankruptcy, the bank has the right to seize all sales receipts, but only after covering the fixed administrative costs for agents engaging in such bankruptcy proceedings.

We assume that banks operate in highly competitive financial markets and therefore all bank loans are competitively (“fairly”) pricing risks. For ease of exposition, we assume that the supplier and retailer are both long-term financed by equity solely, and the bank is the only short-term creditor of the two firms. If the realized retail sales are low (i.e., the sales receipts are less than the owed amounts) the supplier and retailer might not be able to repay the loans and declare bankruptcy. To account for the relevant bankruptcy risks and related costs, the bank charges an interest premium above the risk free rate.

We consider the supply chain coordination issue in such a supply chain setting. When there are no bankruptcy costs, Kouvelis and Zhao (2008) results imply that all supply contracts that coordinate the supply chain without financial constraints can still coordinate the chain with financial constraints, even if bankruptcy risks exist, as long as the needed financing is through competitively priced bank loans. In this paper, we focus on supply chain coordination in the presence of capital market imperfections such as the existence of bankruptcy costs.

Kouvelis and Zhao (2009) show that in the presence of bankruptcy risks, both a retailer’s profit and order quantity increase in his working capital. In general, the retailer’s working capital is less than the channel’s working capital (unless the supplier has zero working capital), which implies that the order quantity of the decentralized supply chain (DSC) is smaller than the corresponding centralized supply chain (CSC). As a result, in order to design a coordinating contract, we need to go besides coordinating the retailer’s order quantity through the wholesale price and sales revenue allocation, as in the traditional coordinating contracts, and into coordinating the use of the supply chain’s working capital for effective management of the channel’s bankruptcy risks and costs.

Our study contributes to the literature in multiple ways. To the best of our knowledge, our paper is the first to address supply chain coordination issues in the presence of bankruptcy costs. We first study the coordinating efficiency of “Demand-Independent (DI)” contracts, i.e., the ones
whose basic contract parameters are not depending on the realized demand. Specifically, we study a general contract that includes and can be reduced to the traditional buyback contract, quantity flexibility contract, all-unit quantity discount contract, and revenue sharing contract. We show that DI contracts cannot coordinate the supply chain when the supplier is adequately self-financed and does not need to borrow to produce the retailer’s order.

For situations that both the supplier and retailer need to borrow to finance their production and ordering decisions respectively, we establish that DI contracts can coordinate the supply chain only if there are no fixed administrative bankruptcy costs. However, when the fixed administrative bankruptcy costs exist, the DI contracts cannot coordinate the supply chain, unless the bankruptcy risk of the CSC is small.

Then, to design supply chain coordination contracts, we study the class of “Demand-Dependent (DP)” contracts, i.e., the ones whose basic contract parameters depend on the realized demand. We propose one such DP contract that can coordinate the supply chain through voluntary compliance and arbitrarily allocate the supply chain profits between the supplier and retailer.

The organization of the paper is the following. In Section 2, we briefly review the related literature. Then, in Section 3, we present our basic model, introduce relevant notations and assumptions, and describe important background results on the retailer’s (newsvendor like) order quantity and profits. In Section 4, we establish that a general DI contract cannot coordinate the supply chain when the CSC faces bankruptcy risk and the supplier is adequately self-financed for her production needs. In Section 5, we further establish that the general DI contracts are not supply chain coordinating even when both supplier and retailer need to borrow for financing their working capital needs and both face bankruptcy risks. In Section 6, we propose a DP contract that can coordinate the supply chain through voluntary compliance and arbitrarily allocate the supply chain profits between supplier and retailer. Finally, Section 7 presents conclusions and future research.

2. Literature Review

Our work fits in the broad area of interfaces of operations and financial decisions, which has recently received substantial interest (representative references are Babich and Sobel, 2004, and
Ding et al., 2007). We are particularly interested on the impact of capital liquidity constraints on inventory decisions and supply chain coordinations. Until the last few years, most of the supply chain management and coordination literature either ignored the impact of short-term financing issues on operational decisions (an observation reaffirmed in Dada and Hu, 2008) or considered the financing issues in an environment without bankruptcy costs (see Zhang and Xu, 2008, Zhou and Groenevelt, 2007, and Kouvelis and Zhao, 2009 for more references).

Zhang and Xu (2008) consider the selection of supply chain coordinating contracts for chains with capital constrained retailers. In the paper, the supply chain never borrows money from external sources (e.g., a bank). This results in the supplier having to offer strong incentives to induce the retailer to order the coordinating quantity, and thus the retailer ends up appropriating most of the supply chain profit. Since the chain does not need to borrow from external financial institutions, supply chain coordination issues in the presence of bankruptcy risks and costs, which is the focus of our work, are not addressed.

In Dada and Hu (2008), non-linear coordinating bank loan schedules are offered by banks to retailers. The banks in Dada and Hu (2008) are profit maximizers, as opposed to purely pricing risk in a perfectly competitive environment, and supply chain coordination in the presence of bankruptcy risks and costs of both supplier and retailer are not discussed. Supply chain coordinating contracts in the presence of supplier financing schemes, such as trade credit contracts, are briefly discussed in Zhou and Groenevelt (2007), but without bankruptcy costs.

Kouvelis and Zhao (2009) study the retailer’s newsvendor ordering problem under a wholesale price-only contract and in the presence of bankruptcy costs (general form that includes such costs proportional to sales revenue, proportional to the pledged to the loan collateral value, and fixed administrative costs). They show that the retailer’s order quantity and profits increase in the retailer’s wealth. They also obtain the Stackelberg equilibrium in wholesale price and order quantity for the linear supplier and retailer supply chain (solve the selling to the financially constrained newsvendor with a wholesale price contract). However, there is no discussion of supply chain coordination issues under bankruptcy costs. Our current paper focuses on supply chain coordination...
issues in the presence of bankruptcy risks and bankruptcy costs (both fixed and proportional to sales) for both the retailer and supplier.

3. Model Description, Notations and Assumptions, and Background Results

3.1. Model Description, Notations and Assumptions

The retailer (he) orders a single product from the supplier (she) now (time 0) to satisfy the future uncertain demand. The sequence of events is as follows:

1. At time 0, the supplier presents a take-it-or-leave-it contract to the retailer. If the retailer accepts the contract, he places an order and transfers the required payment to the supplier. Then, the supplier produces the order for the retailer. If necessary, the supplier and retailer borrow competitively priced bank loans independently to finance their working capital needs in executing their production and order decisions, respectively.

2. At the time of demand realization, the retailer transfers the agreed upon share of sales revenue to the supplier and the supplier buys back at an agreed price leftover inventory, if such provisions are part of the agreed upon contract. Then, the supplier and retailer repay their debt to the bank. Any party who defaults has to declare bankruptcy, and the bank gains control of the defaulting party’s remaining wealth after bankruptcy proceedings and covering all the relevant bankruptcy costs. (Sales revenue and transfer payments from the other party in the supply chain as contractually agreed, e.g., buyback payments to the retailer and share of sales revenue to the supplier, occur prior to bankruptcy proceedings.)

As common in practice, bankruptcy proceedings involve fixed administrative costs (lawyers, accountants, and other bureaucratic proceedings), let us say $B$. Any bankruptcy proceedings costs proportional to the size of business are accounted for by proportional to sales revenue bankruptcy costs. Let $0 \leq \alpha \leq 1$ of sales revenue be the proportional to sales bankruptcy costs for the supplier or retailer. We refer to these two parameters, $\alpha$ and $B$, as bankruptcy cost coefficients, and without loss of generality, we assume them being the same for supplier and retailer.
If necessary, we use superscript $r$, $s$, $c$, and $d$ to denote the retailer, supplier, centralized supply chain (CSC) and decentralized supply chain (DSC), respectively. For example, if we use $y$ to represent the working capital of a generic firm, then $y^r$, $y^s$, and $y^c$ are the working capital of the retailer, supplier, and CSC, respectively. Notice that $y^c = y^r + y^s$. Let the retailer’s order quantity be $q$, the supplier’s wholesale price be $w$, and the bank’s interest rate be $r$.

Exogenous parameters are $r_f$, the risk-free interest rate in a period from time 0 to demand realization, $p$, the retail price at demand realization, and $c$, the unit production cost at time 0. We ignore the salvage value of the unsold items, and assume no goodwill loss for unmet demand. To avoid trivial cases, assume that $p > c(1 + r_f)$.

Let random variable $D$ be the demand on the support $[0, \infty)$. Let $\xi$ be its realization. The probability density function (PDF) of $D$ is $f(\cdot)$, cumulative distribution function (CDF) is $F(\cdot)$, and complementary CDF is $\bar{F}(\cdot) = 1 - F(\cdot)$. We assume that $F$ is differentiable, strictly increasing and $F(0) = 0$. The failure rate is defined as $z(\xi) = f(\xi)/\bar{F}(\xi)$ for $\xi \in [0, \infty)$. In our paper, we restrict our attention to demand distributions of an increasing failure rate (IFR). Also, we require that $z(0) > 0$. Then, $z(\xi_2) \geq z(\xi_1) > 0$ for $0 \leq \xi_1 < \xi_2 < \infty$. Major notations used in our paper are summarized in Table 1.

Our other modeling assumptions are summarized in the following Table 2.

### 3.2. The Newsvendor Profit Function with Bankruptcy Costs

Let a generic retail firm with working capital $y$ order $q$ product units at time 0 and get $1 - \theta$, $0 \leq \theta \leq 1$, share of sales revenue at the time of demand realization. The wholesale price is $w$ where $w(1 + r_f) < (1 - \theta)p$. The loan he needs to borrow at time 0 is $Q = \max\{wq - y, 0\}$. The repayment amount at demand realization is $Q(1 + r)$. Let $k(q)$ be the firm’s bankruptcy threshold, the minimal realized demand level $\xi$ for which the firm is able to fully repay its loan obligation. Then,

$$Q(1 + r) = (1 - \theta)pk(q). \quad (1)$$

If $Q > 0$, i.e., the firm borrows to finance its working capital needs, then $k(q) > 0$ and the firm faces bankruptcy risks. We refer to this case as “in the presence of bankruptcy risks”.
Table 1  Notations (If necessary, we use superscript $r$, $s$, $c$, and $d$ to denote the retailer, supplier, centralized supply chain (CSC) and decentralized supply chain (DSC), respectively)

- $p$: Retail price at demand realization,
- $c$: Supplier’s production cost at time 0, $c(1+r_f) < p$,
- $r_f$: Risk-free interest rate in a period from time 0 to demand realization,
- $D$: Random variable, the future uncertain demand,
- $\xi$: The realization of the future uncertain demand,
- $\alpha$: Bankruptcy costs proportional to sales revenue,
- $B$: Fixed administrative bankruptcy costs,
- $b$: Fixed bankruptcy cost threshold,
- $y$: Working capital at time 0,
- $q$: Retailer’s order quantity at time 0,
- $Q$: The principal of the bank loan at time 0,
- $r$: Bank’s interest rate on its loan determined at time 0 after competitively pricing the risks associated with the loan,
- $k$: The bankruptcy threshold, i.e., the minimal realized demand for a firm to fully repay its loan obligation,
- $w$: Contract parameter, supplier’s wholesale price,
- $\gamma$: Contract parameter, the unit buyback price for the retailer’s leftover inventory,
- $\theta$: Contract parameter, the supplier’s revenue share from the sales,
- $\delta$: Contract parameter, the portion of the retailer’s order quantity up to which the supplier buys back,
- $\pi$: The expected profit of a firm,
- $\Pi$: The expected bankruptcy cost of a firm,
- $\Gamma$: The bank’s expected repayment from a firm.

Table 2  Assumptions

- $A1$: The bank, retailer, and supplier are risk neutral,
- $A2$: All bank loans are competitively priced, and the bank is assumed to face no bankruptcy risk,
- $A3$: Both the supplier and retailer have long-term capital structure that is solely equity financed (without loss of generality and only for exposition convenience),
- $A4$: The supplier, retailer, and CSC have the same bankruptcy cost coefficients $\alpha$ and $B$ (without loss of generality and only for exposition convenience).

The firm’s expected sales revenue are

$$R(q) = (1 - \theta)pE[\min(\xi, q)].$$

The working capital $y$ paid to the supplier at time 0 is equal to $y(1 + r_f)$ at demand realization.

When $Q > 0$, if the realized demand $\xi$ is less than or equal to the bankruptcy threshold $k(q)$, then the firm declares bankruptcy (used to model “limited liability” to loan obligations rather than going out of business situations), and loses $(1 - \theta)p\xi$. However, if $\xi \geq k(q)$, the firm can fully repay the loan, and the cost is $Q(1 + r) = (1 - \theta)pk(q)$. Then, the expected total cost of the retail firm is
\[
C(q) = \mathbb{E}[\min((1 - \theta)p\xi, Q(1 + r))] + y(1 + r_f) = (1 - \theta)p\mathbb{E}[\min(\xi, k(q))] + y(1 + r_f). \quad (3)
\]

When \( Q = 0 \) so that \( k(q) = 0 \), we have \( r = r_f \). In this case, \( C(q) = wq(1 + r_f) \). As a result, the firm’s profit, \( \pi^g(q) = R(q) - C(q) \) where for clarity, superscript \( g \) represents the generic firm (later \( g \) will be either \( s \), for the supplier, \( r \), for the retailer, or \( c \), for the centralized supply chain (CSC)), is

\[
\pi^g(q) = \begin{cases} 
(1 - \theta)p\{\mathbb{E}[\min(\xi, q)] - \mathbb{E}[\min(\xi, k(q))]\} - y(1 + r_f), & \text{if } Q > 0, \\
(1 - \theta)p\mathbb{E}[\min(\xi, q)] - wq(1 + r_f), & \text{if } Q = 0.
\end{cases} \quad (4a)
\]

Lemmma 1. Let \( y > 0 \) and \( \theta < 1 \). Let \( q^* \) be the optimal order quantity. In the presence of the firm’s bankruptcy risks, i.e., \( Q > 0 \), we have that \( k(q^*) < q^* \) and \( \pi^g(q^*) \geq 0 \).

The proof is presented in the Appendix. Lemma 1 establishes that even in the presence of bankruptcy risks and costs, when the firm orders the optimal quantity, the firm has a bankruptcy threshold smaller than the order quantity, and gets non-negative profits. Therefore, we only focus on the case that \( k(q) < q \) in this paper.

### 3.3. The Bank’s Expected Repayment Functions

The expected loan repayment the bank gets from the firm includes two parts: the repayment when the firm gets into bankruptcy, \( \Theta(\xi) \), which is the total remaining wealth of the firm minus bankruptcy costs, and the repayment when the firm fully repays the loan, \( Q(1 + r) = (1 - \theta)p k(q) \).

Then, the expected total repayment the bank gets, \( \Gamma^g(k(q), 1 - \theta, B) \) where superscript \( g \) represents that the repayments are from the generic firm (\( g \in \{s, r, c\} \)), is

\[
\Gamma^g(k(q), 1 - \theta, B) = \mathbb{E}_{\xi \leq k(q)}[\Theta(\xi)] + \mathbb{E}_{\xi \geq k(q)}[(1 - \theta)p k(q)]. \quad (5)
\]

According to our assumption, the bank loan is competitively priced. Then,

\[
Q(1 + r_f) \equiv \Gamma^g(k(q), 1 - \theta, B), \quad (6)
\]

which we refer to as the “competitively priced loan equation”.

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1 Kouvelis and Zhao (2009) prove that the newsvendor retailer has a finite debt capacity, and thus \( q \) and \( k(q) \) are finite.
As previous explained, there are fixed and proportional to sales bankruptcy costs. Thus,

$$\Theta(\xi) = [(1-\alpha)(1-\theta)p\xi - B]^+ = \max\{(1-\alpha)(1-\theta)p\xi - B, 0\}, \tag{7}$$

where $\xi$ is the realized demand and $\xi < k(q)$. Note that only if the realized demand $\xi$ is higher than a threshold level $b$, where

$$b = \frac{B}{(1-\alpha)(1-\theta)p} \geq 0,$$

is the bank able to recover the fixed bankruptcy cost $B$ and thus justify engaging in the bankruptcy proceedings. We refer to $b$ as the “fixed bankruptcy cost threshold”. Then, we have Lemma 2.

**Lemma 2.** Consider the case that $k(q) > 0$, i.e., in the presence of bankruptcy risks. Then,

1) The bank’s expected repayment from the generic retail firm is

$$\Gamma^g(k(q), 1-\theta, B) = \begin{cases} (1-\theta)pk(q)\bar{F}(k(q)), & \text{if } 0 < k(q) \leq b, \tag{8a} \\ (1-\alpha)(1-\theta)p\{E[\min(\xi, k(q))] - E[\min(\xi, b)]\} \\
+ [\alpha(1-\theta)pk(q) + B]\bar{F}(k(q)), & \text{if } k(q) > b. \tag{8b} \end{cases}$$

2) The first order derivative $\frac{\partial \Gamma^g(k(q), 1-\theta, B)}{\partial k} = (1-\theta)p\bar{F}(k(q))G(k(q))$ where

$$G(k(q)) = \begin{cases} 1 - k(q)z(k(q)), & \text{if } 0 < k(q) \leq b, \tag{9a} \\ 1 - [\alpha k(q) + (1-\alpha)b]z(k(q)), & \text{if } k(q) > b. \tag{9b} \end{cases}$$

Bank’s repayment model (8a) tells us that if the firm’s bankruptcy threshold is less than or equal to the fixed bankruptcy cost threshold, then when the firm declares bankruptcy since the realized demand is less than the bankruptcy threshold, all the remaining sales revenue of the firm will be used to pay the fixed bankruptcy costs $B$, and the bank will get nothing. In other words, the bank has no incentive to get into the firm’s bankruptcy process in this case (but the bank will take this into account when determine the interest rate). On the other hand, in (8b), since $k(q) > b$, the bank still gets some remaining sales even if the retailer runs into bankruptcy.

Although $G(k(q))$ is also a function of $1-\theta$ and $B$, we omit them from the notation for convenience of exposition. Without bankruptcy costs, $\alpha = B = 0$ and $G(k(q)) \equiv 1$. Since larger loans lead to larger bankruptcy thresholds and larger expected repayments to the bank (thus $\frac{\partial \Gamma^g(k(q), 1-\theta, B)}{\partial k} =$
(1 − θ)p\bar{F}(k(q))G(k(q)) \geq 0), we note that G(k(q)) \geq 0 (for more discussion, please refer to Kouvelis and Zhao, 2009, and please refer to the proof of Lemma 5 in the Appendix for a similar but more complicated case). Finally, for IFR demand distributions where z(·) = f(·)/\bar{F}(·) is an increasing function, G(k(q)) decreases in the bankruptcy threshold k(q). Moreover, since we require z(0) > 0, we have G(k(q)) < 1 for any k(q) > 0.

3.4. The Expected Bankruptcy Cost

The existence of bankruptcy costs in the case of capital constrained supplier and retailer substantially complicates supply chain coordination and allocation of the supply chain risks among the supply chain firms. Therefore, we need to better understand the expected bankruptcy costs of the generic firm.

From (3), the total expected cost of the firm at demand realization is (1 − θ)pE[min(ξ, k(q))]. However, the total expected loan repayment to the bank is Γ^g(k(q), 1 − θ, B), which equals Q(1 + r_f) from the competitively priced loan equation (6). The difference is the expected bankruptcy cost,

\[ \Pi^g(Q, 1 - \theta, B) = (1 - \theta)pE[min(\xi, k(q))] - \Gamma^g(k(q), 1 - \theta, B) = (1 - \theta)pE[min(\xi, k(q))] - Q(1 + r_f), \]

(10)

where k(q) can be solved from the competitively priced loan equation (6). Thus, we do not treat k(q) as an independent variable in Π^g. In (10), still, the superscript g represents the generic firm.

Here we use the loan amount Q as one of the independent variables of Π^g. We study how changes in Q, \theta, and B affect the expected bankruptcy costs.

**Lemma 3.** Consider a given order quantity q. For k(q) > 0, we have that

1) \[ \frac{\partial \Pi^g(Q, 1 - \theta, B)}{\partial Q(1 + r_f)} = \frac{1}{G(k(q))} - 1 > 0, \text{ and } \frac{\partial \Pi^g(Q, 1 - \theta, B)}{\partial Q(1 + r_f)} \text{ increases in } Q. \]

For bank’s payment models (8a) and (8b),

2) \[ \frac{\partial \Pi^g(Q, 1 - \theta, B)}{\partial \theta} > 0, \text{ i.e., the expected bankruptcy cost increases in the firm’s revenue share.} \]

3) \[ \frac{\partial \Pi^g(Q, 1 - \theta, B)}{\partial B} \geq 0, \text{ where equalities hold only when } 0 < k(q) \leq b. \text{ That is, the expected bankruptcy cost increases in the fixed bankruptcy costs } B. \]
From Lemma 3, the expected bankruptcy cost $\Pi^g$ is both strictly increasing and convex in the loan amount $Q$, due to the increase of the interest rate charged by the bank. Also, for our discussed bank’s repayment models, the expected bankruptcy cost $\Pi^g$ strictly increases in $\theta$, and thus decreases in the retail firm’s revenue share $1 - \theta$. This can be thought equivalently as the retail price decreases from $p$ to $(1 - \theta)p$. Therefore, in this case, the expected bankruptcy costs increase as compared to the corresponding wholesale price contract with price $p$, if the firm orders the same quantity under the two contracts. Finally, Lemma 3 shows that the expected bankruptcy cost increases in the fixed bankruptcy costs $B$.

### 3.5. Supply Chain Coordination with Bankruptcy Costs

Let us express the firm’s expected profits in terms of the expected bankruptcy costs. From (10),

$$(1 - \theta)pE[\min(\xi, k(q))] = \Pi^g(Q, 1 - \theta, B) + wq(1 + r_j) - y(1 + r_f).$$

Then, (4a) can be rewritten as

$$\pi^g(q) = (1 - \theta)pE[\min(\xi, q)] - wq(1 + r_j) - \Pi^g(Q, 1 - \theta, B).$$

Then, for the retailer, supplier, and CSC, after replacing the superscript $g$ with $r$ and $s$, the expected profits are

$$\pi^r(q) = (1 - \theta)pE[\min(\xi, q)] - wq(1 + r_j) - \Pi^r(Q^r, 1 - \theta, B),$$

$$\pi^s(q) = \theta pE[\min(\xi, q)] + wq(1 + r_j) - cq(1 + r_f) - \Pi^s(Q^s, \theta, B),$$

$$\pi^c(q) = pE[\min(\xi, q)] - cq(1 + r_f) - \Pi^c(Q^c, 1, B).$$

Let

$$\Omega(q) = pE[\min(\xi, q)] - cq(1 + r_f)$$

be the profit of the centralized supply chain profit without capital constraints. Therefore, the expected profits of the centralized supply chain (CSC) and the corresponding decentralized one (DSC) are given respectively in (11) and (12):

$$\pi^c(q) = \Omega(q) - \Pi^c(Q^c, 1, B),$$

(11)
\[ \pi^d(q) = \Omega(q) - [\Pi^c(Q_r^*, 1, B) + \Pi^r(Q_r^*, 1 - \theta, B)] \]  

Note that \( \Omega(q) \) is only a function of \( q \). Therefore, in order to coordinate a supply chain without financial constraints, the necessary and sufficient condition is that the DSC orders the optimal order quantity of the CSC (see Cachon, 2003). However, in the presence of bankruptcy costs, the presence of the two bankruptcy terms, \( \Pi^c(Q_c^*, 1, B) \) and \( \Pi^r(Q_r^*, 1 - \theta, B) + \Pi^s(Q_s^*, \theta, B) \), complicates the condition for achieving supply chain coordination. Even if the DSC orders the same order quantity as the CSC, the expected bankruptcy cost of the DSC might be larger than that of the CSC. Furthermore, it might be possible that the DSC can achieve the CSC’s profits by ordering an optimal order quantity different from that of the CSC, due to the complex form of the expected bankruptcy costs. In this case, the DSC is still coordinated to the CSC.

As a result, if we want to show that a supply contract cannot coordinate the DSC, then we need to show that \( \pi^c(q) > \pi^d(q) \) for any given \( q > 0 \). In other words, \( \Pi^c(Q_c^*, 1, B) < \Pi^r(Q_r^*, 1 - \theta, B) + \Pi^s(Q_s^*, \theta, B) \) for any given \( q > 0 \). This guarantees that the maximal profit of the DSC is smaller than that of the CSC.

In this paper, we concentrate on the challenging supply chain coordination problem when the CSC is capital constrained, and thus needs to borrow a bank loan and faces bankruptcy risks.

4. Demand-Independent (DI) Contracts and Coordination of Capital Constrained Supply Chains

Let us consider contracts whose basic contract parameters are not functions of the realized demand, \( \xi \), and therefore, not functions of the bankruptcy threshold, \( k(q) \). However, note that the basic contract parameters can be functions of the order quantity \( q \), or of the parameters of the demand distributions (for example, mean and standard deviation) if needed. We refer to such contracts as “Demand-Independent (DI)” contracts. As an example, an all-unit quantity discount is such a contract, where the wholesale price \( w(q) \) is a function of \( q \), but the quantity discount is independent of the realized demand.

Also, note that in DI contracts, only the basic contract parameters are not functions of the realized demand. It is possible that some derived contract parameters depend on the realized
demand. For example, consider a revenue sharing contract \((w, \theta)\) where \(w = c(1-\theta)\) is the wholesale price and \(\theta\) is the supplier’s share of the sales revenue. If \(\theta\) is not a function of the realized demand, i.e., the supplier requires a fixed share of the sales revenue regardless of the demand realization, then this is a DI contract. Notice that the total revenue the supplier receives is \(\theta p \min(\xi, q)\), which is a function of the realized demand.

4.1. General DI Contracts with Four Parameters

In Sections 4 and 5, we study a general DI contract \((w, \gamma, \delta, \theta)\) where \(w\) is the unit wholesale price, \(\gamma\) is the unit buyback price for the leftover inventory up to \(\delta q\), where \(0 \leq \delta \leq 1\), and \(0 \leq \theta \leq 1\) is the share of sales revenue that the supplier gets (the retailer gets \((1-\theta)\)). According to our discussion, the four parameters can be functions of \(q\), but do not rely on the demand realization. Furthermore, note that if the retailer fixes his order quantity at a given \(q\), the contract parameters being functions of \(q\) are all fixed at the levels corresponding to \(q\). For ease of exposition, we suppress the possible dependence of those parameters on \(q\), and denote them as single parameters.

If \(\gamma = \delta = \theta = 0\), then this contract reduces to an all-unit quantity discount contract \((w(q)\) should be an appropriate function in order to coordinate); if \(\delta = 1\) and \(\theta = 0\), then this contract reduces to a buyback contract; if \(\gamma = w\) and \(\theta = 0\), then this contract reduces to a quantity flexibility contract; and if \(\gamma = \delta = 0\) and \(w = c(1-\theta)\), then this contract reduces to a revenue sharing contract. Note that most of the coordinating contracts discussed in Cachon (2003), except the sales rebate contracts, become special cases of the above described general contract.

4.2. DI Contracts and Coordination When the Supplier does not Need to Borrow

In this section, we discuss the case that the supplier has adequate working capital to implement her production decision without borrowing. Let us fix the order quantity at any given \(q > 0\) where the CSC faces bankruptcy risks for this \(q\), i.e., \(cq - y^c > 0\). At time 0, the supplier receives \(wq\) from the retailer, has \(y^c\) working capital, and produces the items at the cost \(cq\). Under our assumption of the supplier not being capital constrained to implement his production decision, \(wq - cq + y^c \geq 0\). At demand realization, she uses all or some of her working capital, \((wq - cq + y^c)(1+r_f) \geq 0\), to
compensate the retailer. Therefore, in order to honor the contract, she has to choose the contract parameters $\gamma$ and $\delta$ carefully so that she can compensate the retailer $\gamma \delta q$ even if the realized demand $\xi = 0$. That is,

$$\gamma \delta q \leq (wq - cq + y^r)(1 + r_f). \quad (13)$$

At time 0, after transferring $wq$ to the supplier, the retailer’s working capital is $y^r - wq \leq y^r - cq + y^s = -(cq - y^c) < 0$. Therefore, he needs to borrow a bank loan $Q_r = wq - y^r \geq Q^c = cq - y^c > 0$.

As a result, the retailer faces bankruptcy risks and costs. For the CSC and DSC, we have that

$$\pi^c(q) = \Omega(q) - \Pi^c(Q^c, 1, B),$$
$$\pi^d(q) = \Omega(q) - \Pi^r(Q^r, 1 - \theta, B).$$

Next, we examine how each contract parameter influences the expected bankruptcy cost of the retailer, $\Pi^r(Q^r, 1 - \theta, B)$. For the revenue share parameter $\theta$, Property 2) of Lemma 3 establishes that from a supply chain perspective the retailer who faces bankruptcy risks takes all the share of sales revenue. Any share taken by the supplier who does not face bankruptcy risks, strictly decrease the supply chain profits by increasing the expected bankruptcy costs. As a result, in considering the expected bankruptcy costs for the DSC, we work with the best possible case for it with $\theta = 0$.

Next, we examine the buyback parameters $\gamma$ and $\delta$ of the contract in the following lemma.

**Lemma 4.** If $\gamma \delta = 0$, i.e., the supplier does not compensate the retailer at demand realization, then $\Pi^r(Q^r, 1, B) \geq \Pi^c(Q^c, 1, B)$ where equality holds only when $wq = cq - y^s$, i.e., the transfer payment plus the supplier’s working capital is barely enough for the supplier to produce.

From Lemma 4, if $\gamma \delta = 0$ and the supplier does not compensate the retailer at demand realization for the leftover inventory, then any transfer payment higher than the amount she needs to produce (i.e., $cq - y^s$) will make the retailer’s loan larger than that of the CSC. Therefore, the supply chain expected bankruptcy cost is larger and the DI contract cannot coordinate the supply chain. The only way to coordinate the supply chain when $\gamma \delta = 0$ will require the supplier to not only have
zero profit but also to forego her working capital. This is definitely not acceptable for the supplier. As a result, we focus the remaining of our discussion in this section to the case that \( \gamma \delta > 0 \).

If \( \gamma \delta > 0 \), the supplier compensates the retailer for \( \gamma(\delta q - \min\{\xi, \delta q\}) \) at demand realization. From the retailer’s perspective, we can divide this cash flow at demand realization into two components: the positive cash flow, \( \gamma \delta q \), which is a fixed value independent of the demand realization, and the negative cash flow, \( -\gamma \min\{\xi, \delta q\} \), which can be interpreted as the lost buyback price for each unit sold. Another way to interpret the negative cash flow \( -\gamma \min\{\xi, \delta q\} \) is as share of sales revenue the supplier takes. However, this revenue share is not “explicit” in the contract, as in the case of a revenue sharing contract, but is “implicit” in the realized cash flow at the sales time induced by the buyback amount.

To establish that the expected bankruptcy cost of the DSC is larger than that of the CSC, we construct a “new” decentralized supply chain (we will refer to it as the “Fixed Payback/Revenue Share (FP/RS)” chain) where

1) At time 0, the retailer transfers the same amount \( wq \) to the supplier as in the DSC, and

2) At demand realization, the supplier guarantees a fixed cash flow \( \gamma \delta q \) to the retailer but wants a revenue share \( \frac{\delta}{\delta q} \) up to \( \delta q \) sales and the retailer gets all the remaining sales revenue.

Note that as long as (13) holds, the supplier guarantees to give the fixed cash flow \( \gamma \delta q \) to the retailer at demand realization. If we can establish that this FP/RS chain has an expected bankruptcy cost smaller than that of the DSC, but larger than that of the CSC, then we can conclude that the expected bankruptcy cost of the DSC is larger than that of the CSC. Lemma 5 establishes that the expected bankruptcy cost of the FP/RS chain is smaller than that of the DSC.

**Lemma 5.** When \( \gamma \delta > 0 \), the expected bankruptcy cost of the FP/RS chain is smaller than or equal to that of the DSC.

Note that in the DSC, the retailer nominally takes all the sales revenue at demand realization (as we explained above, we are interested in the case \( \theta = 0 \)), and for each demand realization \( \xi \) below the bankruptcy threshold, his bankruptcy cost proportional to sales is \( \alpha p \xi \). However, in the
FP/RS chain, the retailer takes $\frac{\gamma}{p}$ share of the sales revenue, after he guarantees the $\frac{\gamma}{p}$ shares of sales revenue to the supplier according to the contract. Thus, the retailer’s bankruptcy cost proportional to sales is $\alpha(p - \gamma)\xi \leq \alpha p \xi$, where equality holds only if $\gamma = 0$ or $\alpha = 0$. As a result, the expected bankruptcy cost of the FP/RS chain is smaller than or equal to that of the DSC.

Now, if we can show that the expected bankruptcy cost of the FP/RS chain is larger than that of the CSC, then we can conclude that the expected bankruptcy cost of the DSC is larger than that of the CSC for any given $q$ satisfying $cq - y^c > 0$, and as a result, the DI contract cannot coordinate the supply chain. In order to obtain the minimal expected bankruptcy cost of the FP/RS chain, we assume that constraint (13) is binding, i.e., $\gamma \delta q = (wq - cq + y^c)(1 + r_f)$.

Note that in the FP/RS chain, the loan principal $Q^r$ of the retailer is $Q^r = wq - y^c = cq - y^c + \gamma \delta q \frac{1}{1 + r_f}$, which is larger than the loan principal of the CSC by $\gamma \delta q \frac{1}{1 + r_f}$. On the other hand, in the FP/RS chain, the retailer gets the buyback transfer payment $\gamma \delta q$ from the supplier at demand realization. The following Lemma 6 shows that the expected bankruptcy cost of the FP/RS chain is larger than or equal to that of the CSC, and strictly larger if $B > 0$.

**Lemma 6.** Let $x$ be a pre-determined fixed cash flow at demand realization that the newsvendor firm gets. Also, the firm has to borrow an additional amount $\frac{\gamma}{1 + r_f}$ at time 0. Then, the firm’s expected bankruptcy cost is the same for any $x$ if $B = 0$, but strictly increases in $x$ if $B > 0$.

From Lemma 6, if $B = 0$, the buyback transfer payment of $\gamma \delta q$ at demand realization fully balances the effect of the retailer’s larger loan size at time 0, and the expected bankruptcy cost stays the same. An intuitive explanation is that if $B = 0$, then the bank can get the full value of $\gamma \delta q$, regardless of the time it is obtained, since it is guaranteed by the supplier, and thus risk free.

However, if $B > 0$ and if the realized demand is low, the supplier’s buyback transfer payment will be used first to cover the fixed administrative bankruptcy cost $B$, and thus in these cases, the bank is not guaranteed to receive the full amount of $\gamma \delta q$. As this is not risk free for the bank, it will result into higher bank charges and an increase in the expected bankruptcy cost. That is, the expected bankruptcy cost of the FP/RS chain is higher than that of a newsvendor with a loan size
Finally, note that $\frac{\gamma}{p}$ is the supplier’s explicit share of sales revenue in the FP/RS chain. Property 2) of Lemma 3 implies that this revenue share itself decreases the supply chain efficiency by increasing the retailer’s expected bankruptcy costs. Therefore, the expected bankruptcy cost of the FP/RS chain is strictly smaller than that the CSC establishes, even for $B = 0$. Consequently, DI contract cannot coordinate the supply chain.

**Proposition 1.** When $\gamma \delta > 0$, the DI contract cannot coordinate the supply chain.

From Lemma 4 and Proposition 1, we have the following theorem.

**Theorem 1.** In the presence of bankruptcy risks and costs, the DI contract cannot coordinate the supply chain for any set of parameters $(w, \gamma, \delta, \theta)$, if the supplier has adequate working capital to produce without borrowing.

Theorem 1 establishes that for a capital constrained retailer and overall supply chain, but with a supplier having adequate working capital to produce without borrowing, most of the traditional contracts, such as buyback, all-unit quantity discount, and quantity flexibility, do not coordinate the chain in general. The logic is, if the supplier does not borrow, then the best thing she can do is to use all her working capital to help the retailer avoid bankruptcy by transferring more to him at demand realization, which, however, is not effective when $B > 0$. Moreover, whenever she does not transfer all her working capital to the retailer at demand realization, or wants to share some of the retailer’s sales revenue (either explicitly through the revenue sharing parameter $\theta$ or implicitly through the buyback amount), the retailer incurs larger expected bankruptcy costs and the DSC has even worse performance. Finally, our discussion in this section can be easily extended to the DI contracts where the retailer does not need to borrow.

Theorem 1 implies two possible directions to design supply chain coordinating contracts in the presence of bankruptcy costs: (a) having both the supplier and retailer borrow competitively priced bank loans under DI contracts, or (b) changing DI contracts to contracts whose basic parameters depend on the realized demand (we refer to them as “Demand-Dependent (DP)” contracts). In
Section 5, we discuss DI contracts and supply chain coordination when both the supplier and retailer borrow, and in Section 6, we study DP contracts and their ability to coordinate the chain.

5. DI Contracts and Coordination When both the Supplier and Retailer Borrow and Incur Bankruptcy Risks

We now consider a supply chain facing capital constraints and bankruptcy risks with both supplier and retailer in need to borrow to produce and order, respectively. We first examine a DI revenue sharing contract in Section 5.1, and then extend our discussion to general DI contracts in Section 5.2.

5.1. A DI-Revenue Sharing Contract

A DI-revenue sharing contract is our general DI contract \((w, \gamma, \delta, \theta)\) with \(\gamma = \delta = 0\). We know that in the absence of capital constraints and bankruptcy costs, revenue sharing contracts coordinate the supply chain. However, we will argue that this is no longer the case when there exist fixed administrative bankruptcy costs, i.e., \(B > 0\).

Now, we study the expected bankruptcy cost of the retailer and supplier in the DSC. Recall that \(\theta\) is the share of the sales revenue to the supplier. We have the following lemma.

**Lemma 7.** We have \(\Pi^r(k^r, 1 - \theta, B) = (1 - \theta)\Pi^r(k^r, 1, \frac{B}{1 - \theta})\) and \(\Pi^s(k^s, \theta, B) = \theta\Pi^s(k^s, 1, \frac{B}{\theta})\).

Let \(B^r = \frac{B}{1 - \theta} \geq B\) and \(b^r = \frac{B}{(1 - \alpha)(1 - \theta)p} = \frac{B^r}{(1 - \alpha)p}\). We refer to \(B^r\) as the “revenue share adjusted fixed bankruptcy costs” (or thereon as “adjusted fixed bankruptcy costs”) to the retailer as it reflects the situation where the retailer receives all revenue share. Similarly, we can define \(B^s = \frac{B}{\theta}\) as the “adjusted fixed bankruptcy costs” to the supplier.

We want to know that under the revenue sharing contract, if we adjust the retailer’s loan size \(Q^r\) (with the supplier’s loan size changing accordingly), how the expected bankruptcy costs of the supply chain will change. Due to the bankruptcy costs, the supplier and retailer do not want to borrow loans larger than necessary. As a result, we consider the case that \(Q^r + Q^s = Q^c\). Because of this relationship, \(Q^s\) is not an independent variable. We use hat, i.e., “\(\hat{\ }\)”, to denote the optimal value of a variable. Then, we have the following lemma.
Lemma 8. Let $\theta$, the supplier’s share of sales revenue, be fixed. There exists an optimal loan size $\hat{Q}^r$ for the DSC. Let the corresponding bankruptcy thresholds of the retailer and supplier be $\hat{k}^r$ and $\hat{k}^s$. Then, we have that $G(\hat{k}^r) = G(\hat{k}^s)$, where function $G(\cdot)$ is defined by (9a) or (9b).

From Lemma 8, the way that the total loan is split between the supplier and retailer does influence the total expected bankruptcy costs of the DSC. Also, Lemma 8 establishes the optimality condition $G(\hat{k}^r) = G(\hat{k}^s)$ to calculate the optimal loan sizes $\hat{Q}^r$ and $\hat{Q}^s$.

Lemma 9. If $B = 0$ or $\alpha = 1$, then $\hat{k}^r = \hat{k}^s = k^c$ and $\hat{Q}^r = (1 - \theta)Q^c$ and $\hat{Q}^s = \theta Q^c$. The DI-revenue sharing contract can fully coordinate the supply chain.

If $B = 0$, i.e., no fixed bankruptcy costs, then Lemma 9 implies that the DI-revenue sharing contract can coordinate the supply chain. The reason is that the bankruptcy cost proportional to the sales revenue can be split perfectly between the supplier and retailer by the revenue sharing contract. As a result, the loan sizes of the supplier and retailer are proportional to the revenue share each obtains, and the supplier and retailer have the same bankruptcy thresholds as the CSC. Similarly, when $\alpha = 1$, all the remaining sales revenue after a party declares bankruptcy is lost as the bankruptcy costs. Therefore, the fixed bankruptcy cost $B$ is meaningless to the bank, because he will get nothing and thus never engage in the bankruptcy process. Instead, he will charge the interest rate to reflect the situation. Note that this also applies to the CSC. As a result, the DI-revenue sharing contract can still coordinate the supply chain.

However, for $B > 0$ and $\alpha < 1$, $B^r = \frac{B}{1-\theta} \neq \frac{B}{\theta} = B^s$ in general, $G(\hat{k}^r) = G(\hat{k}^s)$ cannot be reduced to $\hat{k}^r = \hat{k}^s$ and $\hat{Q}^r \neq (1 - \theta)Q^c$ and $\hat{Q}^s \neq \theta Q^c$. In other words, surprisingly, due to the existence of the fixed administrative bankruptcy cost $B$, the optimal loan size assignment between the supplier and retailer is not any more proportional to the revenue share that each receives.

Next, we show that the expected bankruptcy cost of the DSC is larger than that of the CSC. Without loss of generality, let $0 \leq \theta \leq \frac{1}{2}$. For this case, $B^r = \frac{B}{1-\theta} \leq \frac{B}{\theta} = B^s$. That is, the adjusted fixed bankruptcy cost of the retailer is smaller than that of the supplier, and both are larger than that of the CSC. We construct a virtual supply chain (VSC) whose variables are denoted by the
corresponding variables with a prime, i.e., “′”. Let the VSC be identical to the DSC except that 
\[ B^{′s} \equiv \frac{B}{1-\theta} \leq B^{s} = \frac{B}{\theta}. \] Then, 
\[ B^{′s} = B^{′r} \geq B \] and 
\[ b^{′r} = b^{′s} = \frac{B}{(1-\alpha)(1-\theta)}. \] In other words, the supplier of the VSC faces an adjusted fixed bankruptcy cost smaller than that of the DSC. Note that the adjusted fixed bankruptcy cost of either the supplier or the retailer, when each receives all revenue shares, is larger than that of the CSC. The following Lemma 10 establishes that this virtual supply chain has total expected bankruptcy costs smaller than or equal to those of the DSC.

**Lemma 10.** For the VSC, 
\[ \hat{k}^{′r} = \hat{k}^{′s}, \quad \hat{Q}^{′r} = (1-\theta)Q^c \quad \text{and} \quad \hat{Q}^{′s} = \theta Q^c. \] That is, the optimal loan sizes of the supplier and retailer are proportional to their revenue share, and the bankruptcy thresholds of the supplier and retailer are the same. Furthermore, the expected bankruptcy cost of the VSC is smaller than or equal to that of the DSC.

It is not surprising that the expected bankruptcy cost of the VSC is smaller than or equal to that of the DSC, since the VSC is obtained by reducing the administrative bankruptcy cost that the supplier faces. Then, in the VSC, when the supplier or retailer receives all the revenue shares, each has the same administrative bankruptcy cost. As a result, the supplier and retailer’s loan sizes are proportional to their revenue shares, and they have the identical bankruptcy thresholds.

Note that 
\[ \frac{B}{1-\theta}, \] the adjusted fixed bankruptcy cost of the supplier and retailer, is larger than or equal to 
\[ B, \] the fixed bankruptcy cost of the CSC, where equal to holds only when \( \theta = 0 \). Therefore, we expect that the expected bankruptcy cost of the VSC is larger than or equal to that of the CSC. The result is formally established in the following lemma.

**Lemma 11.** For the VSC, we have 
\[ \hat{k}^{r} = k^{s} \geq k^c, \] i.e., the bankruptcy thresholds of the supplier and retailer are larger than or equal to that of the CSC. Furthermore, the expected bankruptcy cost is larger than or equal to that of the CSC, where equal to holds when 
\[ k^c \leq b^c. \]

Lemmas 11 and 10 imply that the expected bankruptcy cost of the DSC is larger than or equal to that of the CSC, where equal to holds when 
\[ k^c \leq b^c, \] i.e., the bankruptcy risk of the CSC is low. In this case, since the CSC loses all remaining sales after declaring bankruptcy so that the bankruptcy threshold does not increase if we increase the fixed bankruptcy cost 
\( B \) of the CSC.
toward the adjusted fixed bankruptcy costs of the VSC. The following lemma establishes that we can guarantee the revenue sharing contract fully coordinates the supply chain, for $0 < k^c \leq b^c$.

**Lemma 12.** In the presence of fixed bankruptcy costs (i.e., $B > 0$) and under low bankruptcy risk (i.e., $0 < k^c \leq b^c$), the DI-revenue sharing contract can fully coordinate the supply chain, and can allocate the supply chain profits arbitrarily between the supplier and retailer.

The reason why the revenue sharing contract can coordinate the supply chain is that when $k^c \leq b^c$, the contract splits the supplier and retailer’s loan sizes proportional to the revenue shares each claims. After splitting, the supplier and retailer have the same bankruptcy thresholds, which are equal to the bankruptcy threshold $k^c$ but smaller than or equal to their thresholds $b^s$ and $b^r$ (note that $b^r \geq b^c$ and $b^s \geq b^c$). Hence, when the supplier, retailer, and the CSC run into bankruptcy, the bank cannot get anything from the bankruptcy processes. In other words, the supplier and retailer are the perfect duplication of the CSC, with coefficients $\theta$ and $1 - \theta$, respectively. Thus, the revenue sharing contract can coordinate the supply chain for any $0 \leq \theta \leq 1$.

However, when $k^c > b^c$, the following theorem shows that the revenue sharing contract cannot coordinate the supply chain.

**Theorem 2.** In the presence of fixed bankruptcy costs (i.e., $B > 0$) and under moderate-to-high bankruptcy risk (i.e., $k^c > b^c$), the DI-revenue sharing contract does not coordinate the supply chain.

An intuitive explanation to Theorem 2 is that under the revenue sharing contract and for a CSC that is capital constrained, both the supplier and retailer need to borrow, and consequently face bankruptcy risks. That is, for low demand realizations, both go bankrupt. A bankrupt CSC gets assessed only one fixed administrative bankruptcy cost $B$. However, in the DSC, the bank will incur $B$ for processing two bankruptcies, those of the supplier and retailer, respectively. Depending on the relative values of $k^r$ and $b^r$, $k^s$ and $b^s$, and $k^c$ and $b^c$, the bank gets different residue values of the three parties, and thus sets the interest rates on each party differently. As a result, the revenue sharing contract does not coordinate the supply chain in general.
5.2. General DI Contracts

In Section 4.2, we limit our discussion of DI contracts to the case that the supplier has adequate working capital to implement her production decision without borrowing. Therefore, the supplier’s buyback ability at demand realization is constrained by (13). Now, we remove this constraint by assuming that the supplier will also borrow at time 0 if necessary. The supplier can borrow either for the production requirement, or for the buyback requirement.

We now consider a general DI contract \((w, \gamma, \delta, \theta)\) where \(\gamma \delta > 0\). We assume that

\[
\gamma \delta q > (wq - cq + y^s)(1 + r_f).
\]

Then, the supplier’s loan size is

\[
Q_s = \frac{\gamma \delta q}{1 + r_f} - (wq - cq + y^s)
\]

(she does not need to borrow more than necessary). Since \(Q_r = wq - y^r\), the total loan size of the DSC is

\[
Q_r + Q_s = cq - y^c + \frac{\gamma \delta q}{1 + r_f} = Q_r^c + \frac{\gamma \delta q}{1 + r_f} > Q_r^c. \tag{14}
\]

For convenience of exposition and without loss of generality, we assume that \(\theta = 0\). At demand realization, the supplier has \(\gamma \delta q\) working capital. After the supplier buybacks the leftover inventory of the retailer based on the contract, her remaining working capital is

\[
\gamma \min\{\xi, \delta q\},
\]

which is subject to the bankruptcy costs. Then, for \(\xi \leq k^s\), the amount that the bank gets from the supplier is

\[
\Theta^s(\xi) = \max\{(1 - \alpha)\gamma \min\{\xi, \delta q\} - B, 0\}.
\]

For \(\xi \leq k^r\), the amount that the bank gets from the retailer is

\[
\Theta^r(\xi) = \max\{(1 - \alpha)p\xi + (\gamma \delta q - \gamma \min\{\xi, \delta q\}) - B, 0\}
\]

\[
= \max\{(1 - \alpha)p\xi - \gamma \min\{\xi, \delta q\} + \gamma \delta q - B, 0\}.
\]

We still consider the FP/RS chain defined in Section 4.2, in which the supplier takes the \(\frac{\gamma}{p}\) share of sales revenue up to \(\delta q\) explicitly but guarantees the buyback amount \(\gamma \delta q\) at demand realization. Denote the corresponding variables of this FP/RS chain with a prime, i.e., “\(^\prime\)”. For the supplier in
this chain, $\Theta^s(\xi)$, the amount that the bank obtains when the supplier bankrupts, equals $\Theta^s(\xi)$. However, the amount that the bank gets from the retailer in the case of bankruptcy, is
\[
\Theta'^r(\xi) = \max\{(1 - \alpha)p\xi - (1 - \alpha)\gamma \min\{\xi, \delta q\} + \gamma \delta q - B, 0\} \leq \Theta^r(\xi),
\]
where equality holds only when $\alpha = 0$. By using the same argument as in Lemma 5, we obtain the following lemma, which confirms that from the supply chain perspective, it is better for the supplier to share the sales revenue explicitly.

**Lemma 13.** When $\gamma \delta > 0$, the expected bankruptcy cost of the FP/RS chain is smaller than or equal to that of the DSC, when both the supplier and retailer borrow bank loans.

Now, we compare the expected bankruptcy costs of the FP/RS chain and the CSC. From (14), the total loan size of the FP/RS chain is $\frac{\gamma \delta q}{1 + \gamma f}$ larger than the loan size of the CSC. Note that $\gamma \delta q$ is the amount available to the retailer at demand realization. Very similar to Lemma 6, we can show that if $B > 0$ and the retailer is subject to bankruptcy risks, the amount $\gamma \delta q$ is not risk free to the bank. Thus, the expected bankruptcy cost of the FP/RS chain is larger than or equal to that of the CSC, and strictly larger if $B > 0$. Therefore, together with Lemma 13, we can conclude that when $\gamma \delta > 0$, DI contracts cannot coordinate the supply chain.

So far, when $\gamma \delta > 0$, even without considering how the loan is split between the supplier and retailer (recall that we assume $\theta = 0$), we can conclude that the DI contracts with buyback terms do not coordinate the supply chain. Two factors increase the expected bankruptcy costs of the DSC: the first one is the implicit revenue share by the supplier, due to the buyback amount (as in our discussion in Section 4.2), and the second one is due to the buyback cash flow losing value when the retailer bankrupts. Note that such observations continue to hold for even $\theta > 0$, in which case the contract can be viewed as a superposition of a revenue sharing contract and a buyback contract. The imposition of any buyback terms decrease the efficiency (and corresponding total expected profits of the DSC) of any revenue sharing type DI contracts they are added on. We summarize our results in the following theorem.
Theorem 3. In the presence of fixed bankruptcy costs (i.e., \( B > 0 \)) and under moderate-to-high bankruptcy risk (i.e., \( k^c > b_c \)), DI contracts do not coordinate the supply chain, when both the supplier and retailer borrow and incur bankruptcy risks. Moreover, the imposition of buyback terms in DI contracts decreases the coordinating efficiency by incurring larger expected bankruptcy costs.

From Theorem 3, we conclude that for \( B > 0 \) and \( k^c > b_c \), our general DI contracts cannot coordinate the supply chain, no matter whether the supplier borrows or not. Therefore, to coordinate the supply chain, we discuss demand-dependent (DP) contract in Section 6.

6. Demand-Dependent Supply Contracts

6.1. Demand-Dependent Revenue Sharing Contracts

In this section, we introduce a demand-dependent supply contract that can coordinate the supply chain. We design the contract terms and contract protocol as follows:

1) At time 0, the retailer transfers payment \( cq - y_s \) to the supplier by borrowing a loan \( Q^r = cq - y_s - y_r = cq - y_c \) from a bank that competitively assesses the retailer’s demand risks with the working capital of the whole supply chain pledged towards covering the loan. After receiving the transfer payment, the supplier has \( cq \) total working capital and produces the products for the retailer. (We would like to remark that in the above protocol the retailer is the one borrowing for the supply chain using all its working capital. However, it is immaterial who borrows for the chain, as long as the loan borrowed uses all working capital in the chain.)

2) At demand realization, if the realized demand is lower than the bankruptcy threshold, i.e., \( \xi \leq k^r(q) \), then the supplier does not claim any sales revenue. However, if \( \xi > k^r(q) \), then the supplier requests a \( \lambda \) share of the net sale profits, which is \( \lambda p[\min\{\xi, q\} - k^r(q)] \), where \( \lambda \) is a constant value, the desired share of the net supply chain profit that she would like to enjoy. Thus, the retailer’s revenue share is a function of the realized demand:

\[
1 - \theta(\xi) = \begin{cases} 
1, & \text{if } \xi \leq k^r(q), \\
1 - \lambda, & \text{if } \xi > k^r(q). 
\end{cases}
\]
Under the above contract, the expected sales revenue that the supplier gets is
\[ \lambda \{ pE[\min(\xi, q)] - pE[\min(\xi, k^*(q))] \} \]
so that the supplier’s expected net profit is
\[ \pi^s(q) = \lambda \{ pE[\min(\xi, q)] - pE[\min(\xi, k^*(q))] \} - y^s(1 + r_f). \]
The expected sales revenue that the retailer gets is
\[ (1 - \lambda) \{ pE[\min(\xi, q)] - pE[\min(\xi, k^*(q))] \} \]
so that the retailer’s expected net profit is
\[ \pi^r(q) = (1 - \lambda) \{ pE[\min(\xi, q)] - pE[\min(\xi, k^*(q))] \} - y^r(1 + r_f). \]
Comparing with (4a), the expected profit of a generic firm,
\[ \pi^g(q) = pE[\min(\xi, q)] - pE[\min(\xi, k(q))] - y(1 + r_f), \]
we can conclude that for any given \( \lambda \), to give out \( \lambda \) portion of sales revenue to the supplier if the retailer does not bankrupt does not change the retailer’s optimal \( q \) and \( k^*(q) \) and the bank’s assessed interest rate \( r^r \) to the retailer.
For ease of exposition, let \( \Omega(q) = \{ pE[\min(\xi, q)] - pE[\min(\xi, k^*(q))] \} \). Let \( q^* \) be the retailer’s optimal order quantity under the contract (note that \( q^* \) is also the optimal order quantity of the CSC). Then, the share of the net supply chain profit of the retailer is
\[ \beta = \frac{\pi^r(q^*)}{\pi^r(q^*) + \pi^s(q^*)} = \frac{(1 - \lambda)\Omega(q^*) - y^r(1 + r_f)}{\Omega(q^*) - y^r(1 + r_f)}. \]
When the CSC orders the optimal quantity, \( \Omega(q^*) - y^r(1 + r_f) > 0 \) (otherwise, the CSC does not want to do the business). Then,
\[ \lambda = 1 - \beta \left( 1 - \frac{y^c(1 + r_f)}{\Omega(q^*)} \right) - \frac{y^r(1 + r_f)}{\Omega(q^*)}. \]
Notice that \( \lambda \) and \( \beta \) have a linear relationship for fixed and known values of the \( q^* \) and \( k^*(q^*) \) (these appear in \( \Omega(q^*) \)). Therefore, the supply chain profit can be arbitrarily allocated for any \( 0 \leq \beta \leq 1 \). The range of \( \lambda \) corresponding to the [0,1] range of \( \beta \) is
\[ 0 \leq \frac{y^r(1 + r_f)}{\Omega(q^*)} \leq \lambda \leq 1 - \frac{y^r(1 + r_f)}{\Omega(q^*)} \leq 1. \]
In summary, the proposed DP contract \((w, 0, 0, \theta(\xi))\), which can be thought as a realized demand dependent revenue sharing contract (we will refer to it onwards as DP-revenue sharing) coordinates the supply chain.

**Theorem 4.** For capital constrained central supply chain, supplier, and retailer, and with all of them in need of financing and facing bankruptcy risks with costly bankruptcy proceedings, the proposed DP-revenue sharing contract coordinates the supply chain through voluntary compliance and allocates the supply chain profits arbitrarily.

From the above discussion, we can see that the DP-revenue sharing contract we propose includes two parts: when the realized demand is low, the contract uses all the working capital in the chain and any realized sales as part of a limited liability effort to cover obligations against the bank loan, and in this way the supply chain behaves exactly as the CSC. When the realized demand is high, then the contract behaves exactly like a traditional revenue sharing contract and allows the supplier and retailer to share sales revenues. Obviously, for a supply chain without financial constraints, i.e., \(k^r(q) = 0\) and thus \(k^c(q) = 0\), our DP-revenue sharing contract is reduced to the traditional revenue sharing contract.

### 6.2. Further Discussion and Managerial Insights

For the DP-revenue sharing contract discussed in Section 6.1, the supplier and retailer coordinate not only against the demand risks, but also against financial risks. At time 0, each of them will determine the amount of working capital they want to pledge to the business, and borrow the bank loan with the total working capital. If bankruptcy happens when the realized demand is low, each of them knows that they will lose their part of the working capital, due to the limited liability of the bank loan. Moreover, the amount of the working capital contributed, together with each firm’s market power, will be the two important factors affecting the ensuing bargaining process to determine the revenue share each party takes.

In the DP-revenue sharing contract, the supplier financially help the retailer with the loan principal at time 0. In Zhou and Groenevelt (2007), open account financing is discussed, and
Kouvelis and Zhao (2008) imply that open account financing cannot coordinate the supply chain. In a frequently used open account financing practice (see provided example in Zhou and Groenevelt, 2007), the supplier finances the retailer by subsidizing the interest of the retailer’s loan at demand realization. If the supplier does not need to borrow bank loans and subsidizes the retailer with her working capital, then the subsidized interest is similar as a buyback term, and our results in Section 4.2 imply that we cannot achieve supply chain coordination. On the other hand, if the supplier needs to borrow bank loans as well, our Lemma 6 implies that when there exist fixed bankruptcy costs, i.e., \( B > 0 \), the cash flow at demand realization is less efficient than the cash flow at time 0 for the party that borrows the bank loan, since it lowers the bankruptcy risks by reducing the loan principal. Therefore, supplier’s efforts to subsidize the retailer’s interest cannot coordinate the chain, and the best way for the supplier to use her working capital is to reduce the retailer’s loan principal at time 0. We summarize the result in the following theorem.

**Theorem 5.** In the presence of fixed bankruptcy costs (i.e., \( B > 0 \)), any supplier subsidized cash flow to the retailer at demand realization is less efficient than subsidizing the retailer for the loan principal at time 0, and thus cannot coordinate the supply chain.

From Theorem 5, to achieve the supply chain coordination, it is a must that the supplier and retailer financially work together for reducing the loan principal to the extent possible. Then, when the realized demand is high and the retailer does not bankrupt, the supplier and retailer can find ways to share the sales revenues and coordinate the supply chain.

### 7. Conclusions and Future Research

In this paper, we discuss the supply chain coordination issue in a supply chain where both the supplier and retailer are capital constrained. We assume that the supplier and retailer are in an imperfect capital market with bankruptcy costs.

In the presence of bankruptcy risk, to coordinate the supply chain, a contract not only needs to coordinate the retailer’s order quantity but also the working capital and thus the bankruptcy risks of the supply chain. We investigate a general DI contract, in which the basic contract parameters
are not functions of the realized demand. The contract can be reduced to buyback, all unit quantity discount, quantity flexibility, and revenue sharing contracts by setting the contract parameters appropriately. We show that when the CSC has bankruptcy risks, such contracts cannot coordinate the supply chain, if the supplier does not need to borrow under the contracts.

This discussion implies two directions to modify a DI contract to make it supply chain coordinating: The first direction is to make both the supplier and retailer borrow under the contract, and the second one is to change a DI contract into a DP contract. However, we show that, due to the existence of the fixed bankruptcy cost $B$, even if both the supplier and retailer borrow and incur bankruptcy risks, a DI-revenue sharing contract still cannot coordinate the supply chain in general (only coordinates if the bankruptcy risk is small or if there do not exist fixed bankruptcy costs). For DI contracts with buyback options, we also show that they cannot coordinate the supply chain, as long as there exist fixed bankruptcy costs.

We provide a demand-dependent contract that coordinates the supply chain by voluntary compliance and allocates the supply chain profits arbitrarily. In this contract, the supplier only shares the sales revenue when the realized demand is larger than the retailer’s bankruptcy threshold. On the other hand, for low demand realizations, the whole working capital of the supply chain and all realized sales are given to the bank as part of the limited liability contract signed for short-term borrowing purpose. In the latter case, both supplier and retailer lose all their working capital and receive zero revenue.

Finally, in the presence of the fixed bankruptcy costs, our results imply that to achieve supply chain coordination, any efforts of the supplier to subsidize cash flows of the retailer at demand realization are less efficient than subsidizing the retailer’s cash flows at time 0. In other words, the only way to achieve supply chain coordination is that the supplier and retailer financially coordinate to reduce the loan principal with the bank.

Although our general DI contract discussed includes and can be reduced to most of the traditional contracts, sales rebate contracts are not covered by it. Future study of the coordinating efficiency of sales rebate contracts in the presence of bankruptcy cost may be of interest. We are further
interested in understanding other market imperfections, such as credit line limits or information asymmetry, on supply chain coordination in the presence of bankruptcy risks and costs.

References


