Risk-Aversion Happens: Hedging Commodity Material Purchases in a Bilateral Supply Chain

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Abstract

This paper explores the merits of hedging stochastic input costs (i.e., reducing the risk of adverse changes in costs) in a decentralized supply chain under price-only contracts. Specifically, we consider a generalized version of the well-known ‘selling to the newsvendor’ model in which both the upstream and the downstream firms face stochastic production costs. The firms’ operations are intertwined – i.e., the downstream buyer depends on the upstream supplier for delivery and the supplier depends on the buyer for purchase. We show that the stochastic costs reverberate through the supply chain and will be, ex ante, impounded into the wholesale price. In some cases, if input costs significantly increase, one of the supply chain members may not be able to fulfill its contractual requirements, causing the entire supply chain to break down. We identify conditions under which the risk of the supply chain breakdown and its impacts on the firms’ operations will cause the supply chain members to hedge their input costs: (i) The downstream buyer’s market power exceeds a critical threshold; or (ii) the downstream firm operates on a small margin, there is a high baseline demand for its final product, and its market power is below a critical threshold. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. As an extension, we consider the case when firms’ operations are financed with borrowing and show that hedging can be profitable even in the absence of breakdown risk. Here, the equilibrium hedging policy is a full hedging policy.
1 Introduction

Much of the extant supply chain and inventory literature takes demand as stochastic and production costs as fixed. The latter assumption, however, is in contrast to many industrial settings in which manufacturing firms are exposed to price changes of commodities that affect the direct costs of raw materials, components, subassemblies, and packaging materials as well as indirect costs from the energy consumed in operations and transportation. According to a survey of large US non-financial firms (e.g., Bodnar et al., 1995), approximately 40% of the responding firms routinely hedge price risks such as those mentioned above. Why do these firms hedge?

In order to provide a rationale for hedging, extant theories consider situations in which a ‘standalone’ firm is exposed to price changes that stem from direct purchases of inputs (or direct sales of outputs) and rely upon the existence of taxes (e.g., Smith and Stulz, 1985), asymmetric information (e.g., DeMarzo and Duffie, 1991), costly external capital (e.g., Froot et al., 1993), financial frictions (Caldentey and Haugh, 2009), and risk aversion (e.g., Gaur and Seshadri, 2005; Chod et al., 2010). One of the consequences of this approach is that the standalone firm can manage essentially any price risk it wants by hedging alone. This, however, is not necessarily descriptive of the situation many firms face if they are embedded in a supply chain. An example from the auto parts industry illustrates this well (Matthews, 2011). In 2011, steelmakers have increased prices six times, for a total increase of 30%. Consistent with the extant theory, many car and appliance manufactures did not necessarily pay attention to steel prices either because they did not directly purchase steel from steelmakers or because they believed that their products were not sufficiently steel-intensive (for example, steel represents about 4% of the cost of a car). The price increases, however, have been significantly felt by suppliers who buy steel directly from steelmakers and stamp, bend, coat and cut that steel for other companies, such as car and appliance manufacturers. Steel represents their biggest cost, and often their ability to pass on higher raw-material costs onto their buyers is limited – either for contractual or competitive reasons. Significant swings in the price of steel, however, may easily compromise their ability to fulfill their supply agreements, potentially leading to a disruption in supply. This, in turn, will affect their buyers (car and appliance manufacturers) whose ability to produce their end products depends on these suppliers.

1Make investments to reduce the risk of adverse price movements.
The supply chain environment is therefore unique in that a firm may be exposed to price risks not only from direct purchases of commodities, but also to price risks that are embedded in parts, components, products, and services purchased from suppliers. For this reason, supply chain firms may find it difficult to manage some commodity price risks merely by hedging unilaterally. Consequently, supply chain firms may care whether their supply chain partners hedge. (In the above example, some suppliers choose to hedge.) Moreover, the hedging behavior of their supply chain partners, will affect their own hedging behavior, as will be seen.

Following the seminal contributions of Lariviere and Porteus (2001) and Cachon (2004) – which point out that there is an intimate relationship between wholesale price, demand uncertainty, and the distribution of market power in the supply chain – this paper explores the merits of hedging stochastic input costs (i.e., reducing the risk of adverse changes in costs) in a decentralized supply chain under price-only contracts. Specifically, we consider a generalized version of the well-known ‘selling to the newsvendor’ model (Lariviere and Porteus, 2001) in which both the upstream and the downstream firms face stochastic production costs. (In contrast to our paper, in Lariviere and Porteus, 2001, production costs are linear with constant coefficients.) The intuition that underlies our model of hedging in a supply chain is as follows. By agreeing to sell their output at (contractually) pre-determined prices before all factors affecting their productivity are known, both the upstream supplier and the downstream buyer subject themselves to default risk. For example, if one of the firm’s ex post commodity input cost turns out to be excessive, then this firm may rationally choose to default on its contractual obligations to either purchase or to deliver output owing to immoderate production costs, which strain its resources. Since the firms’ operations are interdependent – i.e., the buyer depends on the supplier for delivery and the supplier depends on the buyer for purchase – a default of one firm causes a break in the supply chain. The fact that a supply chain member may default in some states of the world will be, ex ante, impounded into the wholesale price as well as into the order quantity. However, if the firms could somehow ‘guarantee’ that they will fulfill the supply contract (no matter to what happens to their inputs costs), their supply chain counterpart may respond with a more favorable price or a more favorable order quantity. Either firm would like to guarantee supply contract performance if the expected profit associated with the guarantee is greater than the expected profit associated with default in some states. In such circumstances, the firm can commit to the guarantee by purchasing futures contracts.
on the underlying input commodity. The futures contracts will pay off precisely at the time when the firm’s resources are strained. Hence, futures contracts have value because they prevent the firm from defaulting on a contractual obligation when not defaulting is (ex ante) important.

We identify the following conditions under which the supply chain firms should be expected to hedge in equilibrium: (i) The downstream buyer’s market power exceeds a critical threshold; or (ii) the downstream firm operates on a small margin, there is a high baseline demand for its final product, and its market power is below a critical threshold. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. As an extension, we consider the case when firms' operations are financed with borrowing and show that hedging can be profitable even in the absence of breakdown risk. Here, the equilibrium hedging policy is a full hedging policy. (A partial hedging policy leaves the hedger exposed to some residual price risk. A full hedging policy eliminates all price risk.)

To summarize, we provide a rationale for hedging, which is based on a non-cooperative behavior in a supply chain. We illustrate this point with an anecdotal example from a diversified global manufacturing company. The company’s Residential Solutions Division produces a collection of home appliances and power tools, which are sold worldwide. Its Commercial Division manufacturers industrial automation equipment and climate control technology. Many of these products are raw-material-intense. When it comes to supply chain management, the firm plays a variety of roles: Sometimes the firm is a supplier; other times, the firm plays a role of an end-product assembler. Recently, the company saw an increased volatility in the prices of metals, which affected its margins. To combat this, the firm hedged its raw material inputs, but only some. When asked when the firm hedges, a manager who presented the case replied that the firm’s hedging decisions partly depend both on the product and on the firm’s supply chain partners. For example, as a supplier, the firm fully hedges the raw material it uses to produce garbage disposals, which are sold through a major U.S. home-improvement store. This is because the firms’ bargaining power against this retailer is small and it is the retailer who dictates both production quantities and pricing. To avoid funding risk, however, as a buyer, the firm tends not to hedge, when its supplier cannot guarantee supply. The existing risk-management literature does not always provide clear guidance on hedging in cases like these. Either because it requires frictions (e.g., risk-aversion), which do not
apply to large multi-nationals (Carter et al., 2006; Jin and Jorion, 2006); or because it does not allow hedgers to vary their hedging decisions both across products and supply chain counterparts. While our model is a simple selling-to-the-newsvendor setting, this example illustrates that the equilibrium that we advance seem to mirror the facts of these cases (see Part i of Proposition 2).

In what follows, we review related literature, present the model (§3) and derive the firms’ equilibrium behavior under the unhedged and hedged contracts (§4.1 and 4.2). Then, by comparing the expected payoffs both firms can achieve under the unhedged and hedged contracts we make predictions as to when offers of hedged wholesale contract should be expected in equilibrium (§4.3). In §5, we extend our results to the case of leveraged firms. §6 concludes.

2 Related Literature

Recent research in the finance literature has greatly improved our understanding of why and how individual firms should hedge. For instance, it has established that for hedging to be beneficial, the firm’s payoff must be a concave function of the stochastic hedge-able exposure metric. In spirit, this result follows from basic convex-analysis theory: It means that the firm’s expected payoff, a weighted average of a concave function, is lower when the level of variability of the exposure metric is higher. Thus, since hedging reduces variability, then it increases the firm’s expected payoff.

Focusing on a single-firm setting, Smith and Stulz (1985) demonstrate that managerial risk-aversion or market frictions such as corporate taxes cause a firm’s payoff to become a concave function of its hedge-able exposure. One of the implications of this paper’s result is that firms should hedge fully or not at all. Froot et al. (1993) expand the discussion to settings in which firms are exposed to multiple sources of uncertainty that are statistically correlated. They show that in such type of situations, it may become optimal for a firm to utilize partial hedging. Building upon this observation, a large body of the corporate finance literature has studied hedging by taking a firm as the basic ‘unit of analysis’ and by exogenously imposing the afore-mentioned concavity property. This property has been commonly justified by assuming that the hedger’s preferences on the set of its payoffs are consistent with some form of concave utility function. This literature provides insights into the mechanics of hedging using financial instruments such as futures and options. For instance, Neuberger (1999) show how a risk-averse supplier may use short-maturity commodity
futures contracts to hedge a long-term commodity supply commitment. A comprehensive summary of this research is given in Triantis (1999).

While the finance literature focuses on mitigating pricing risk, the operations management literature predominantly considers demand and supply risks. An excellent survey of the early operations literature can be found in Van Mieghem (2003). More recent and influential papers in this area include Gaur and Seshadri (2005) and Caldentey and Haugh (2009). Gaur and Seshadri (2005) address the problem of using market instruments to hedge stochastic demand. The authors consider a newsvendor who faces demand that is a function of the price of an underlying financial asset. They show how firms can use price information in financial assets to set optimal inventory levels, and demonstrate that financial hedging of demand risk can lead to higher return on inventory investment. Caldentey and Haugh (2009) show that financial hedging of stochastic demand can increase supply chain output. Note, however, that there are several important differences between the supply chain considered in Caldentey and Haugh, 2009 and the supply chain in this paper. Their supply chain is centralized, the hedge-able risk in their chain is stochastic demand, and the risk is faced by only one of the firms in the supply chain. In order to provide a rationale for the hedging behavior, Gaur and Seshadri (2005) rely upon risk-aversion; Caldentey and Haugh (2009) require market frictions, which lead to credit rationing.

Other papers in the operations management literature discuss and analyze hedging strategies for managing risky supply. Emergency inventory (e.g., Schmitt et al., 2010); emergency sourcing (e.g., Yang et al., 2009); and dual sourcing (e.g., Babich et al., 2007; Gümüş et al., 2012) are examples of operational hedging strategies capable of mitigating adverse supply shocks. Supplier subsidy (e.g., Swinney and Netessine, 2009; Babich, 2010) is an example of a financial strategy designed to reduce the likelihood of supply contract abandonment. Haksöz and Seshadri (2007), in turn, put financial value on the supplier’s ability to abandon a supply contract. Finally, while intuition may suggest that operational and financial hedging may be substitutes, surprisingly, Chod et al. (2010) show that they can be complements.

We differ from the aforementioned literature by focusing on the management of input cost risk (rather than demand risk) and by extending the discussion of why and how risk-neutral firms hedge from the single-firm setting into a decentralized supply chain. Specifically, we explore the potential benefits of financial hedging for risk-neutral supply chain members who are exposed to both direct
and *indirect* commodity price risks. The direct price risks stem from purchases of production inputs; the indirect price risks are embedded in products and services that move up and down the supply chain. In contrast, the extant theories of corporate hedging (e.g., Smith and Stulz, 1985; Gaur and Seshadri, 2005) only consider direct commodity price risks and cannot necessarily predict hedging behavior in a decentralized supply chain, where firms hedge for reasons that do not arise in the single-firm setting – e.g., the risk of supply disruption.

3 The Model

We study a generalized ‘selling to the newsvendor’ model (see Lariviere and Porteus, 2001) with stochastic input costs. There are two firms: A component supplier (S) selling to a final product assembler (M) who must choose a production quantity, q, before it observes a single realization of stochastic demand, D, for its final product. Stochastic demand for the final product, D, has a distribution $F$ and density $f$. $F$ has support on $[\underline{d}, \overline{d}]$ and an increasing failure rate (see Lariviere and Porteus, 2001, p.295). The retail price of the final product is normalized to 1.

To assemble $q$ units of the final product, the assembler requires $q$ components from the supplier and $q$ units of commodity 2. To produce $q$ components for the assembler, the supplier requires $q$ units of commodity 1. (Figure 1 summarizes the production inputs required to assemble 1 unit of the final product.) For simplicity, unmet demand is lost, unsold stock is worthless, and per unit production and holding costs are zero. Both firms are risk-neutral and are protected by limited liability. Each firm has a reservation payoff $R_j \geq 0$ and $c_j \geq 0$, $j \in \{M, S\}$ dollars in cash-on-hand. (We begin by assuming that there is no borrowing; later, in §5, we discuss what happens if this assumption is relaxed. As explained in Cachon, 2003, p.234, exogenous reservation payoffs, $R_j$, are a standard way to model a firm’s market power.)

One can interpret the firms as playing a game over times $t_0 < t_1 \leq t_2 \leq t_3$. The supplier offers the assembler a wholesale contract at time $t_0$ when only the distribution function of future commodity input costs is known. Due to production lead times, the supplier commences production of components at time $t_1$ and the assembler commences final assembly at time $t_2$. An outcome of the supplier’s commodity input cost, $S_1 \geq 0$, is revealed at time $t_1$; an outcome of an assembler’s commodity input cost, $S_2 \geq 0$, is revealed at time $t_2$. ($S_i$, $i = 1, 2$ represents a time $t_i$ spot price
of commodity \(i\). Assembler’s sales revenue is realized at time \(t_3\) when demand is revealed. For simplicity, the discount rate between dates \(t_0\) and \(t_3\) is taken to be zero. (From CAPM, this is equivalent to assuming that the risk-free rate is zero and that both input commodities are zero beta assets for which there is an empirical support in So, 1988; Pindyck, 1994; Deaves and Krinsky, 1995.) Commodity spot prices, \(S_1\) and \(S_2\), are modeled in a complete probability space \((\Omega, \mathcal{F}, \mathbb{P})\) equipped with filtration \(\{\mathcal{F}_t\}, t \in [t_0, t_3]\). \(S_1\) and \(S_2\) have a joint distribution function, \(H\), and joint density function \(h\).

**Figure 1: Inputs Required to Produce One Unit of Assembler’s Final Product.**

It is rather important to emphasize that either the supplier or the assembler may have an incentive to default on the supply contract. This could happen if the time \(t_i\) commodity input cost turns out be too high in relation to the contractually pre-determined selling price. (Recall that the supplier’s wholesale price, \(w\), is contractually pre-determined at time \(t_0\) and the assembler’s selling price is normalized to 1.) We assume that if a firm defaults on the supply contract, then it incurs a contractually pre-determined default penalty \(L_j \geq 0, j \in \{M, S\}\). Throughout the paper, we assume that the penalty, \(L_j\), is payable to firm \(j\)’s supply chain partner. (It is, however, rather straightforward to change this to a model, in which \(L_j\) represents an indirect reputation cost. For a discussion on reputation costs, see Diamond, 1991.) If both firms choose to produce, then \(q\) is the highest payoff either firm can achieve, as will be seen. *To prevent a situation in which one firm experiences a financial windfall when its supply chain partner defaults, throughout the analysis we therefore assume that \(0 \leq L_S \leq q\) and \(0 \leq L_M \leq q\).* (As an example of a supply contract penalty, see the Plymouth Rubber Company contract in Appendix A – there, \(L_j\) is set as a percentage of the wholesale price. In the Davita, Inc. contract, the penalty is set as 100% of the wholesale price.)

One of the goals of this paper is to investigate whether firms should be expected to *hedge* their commodity input costs in order to avoid default on the underlying supply contract. For this reason we suppose that for each commodity \(i = 1, 2\) there exists a futures contract to purchase a specific amount of commodity \(i\) at time \(t_i\) for a pre-specified (futures) price, \(F_i\). As is convention, the
futures price is set so that the value of the futures contract at inception is zero. The payoff to a futures contract on commodity \( i = 1, 2 \) is realized at time \( t_i \), where the payoff is the difference between the futures price, \( F_i \), and the time \( t_i \) spot price, \( S_i \). We use the variable \( n_j, j \in \{M, S\} \) to represent firm \( j \)'s position in the futures market: \( n_j > 0 \) indicates that firm \( j \) has a 'long position' of \( n_j \) futures contracts and \( n_j < 0 \) indicates that it has 'short position.' If the firm \( j \in \{M, S\} \) buys – or is long in – \( n_j \) futures contracts, its time \( t_i \) payoff is \( n_j (S_i - F_i) \), which will be positive when input prices are high and negative when input prices are low. In our setting, both firms will be hedging by taking long positions. Therefore, hereafter, \( n_j \geq 0, j \in \{M, S\} \).

The futures contracts are negotiated at a futures exchange, which acts as an intermediary. One of the key roles of the exchange is to organize trading so that futures contract defaults are completely avoided (for additional discussion, see §2–3 in Hull, 2009). For this reason, the exchange will limit the number of contracts a firm \( j \in \{M, S\} \) can buy if it knows that firm \( j \) will not have sufficient ex post resources to pay off its long futures contract position when the realized input prices at time \( t_i, i = 1, 2 \) are low.

In summary, there are three key assumptions about penalties and futures contracts that underlie our model of supply chain hedging:

**Assumption 1.** Any default on the underlying supply contract is costly; neither firm, however, stands to unusually profit from a default of its supply chain partner.

**Assumption 2.** To avoid pathological cases, the futures prices \( F_i, i = 1, 2 \) are low enough so that by hedging, neither firm effectively locks in a negative (expected) profit.

**Assumption 3.** The futures market is credible, meaning that neither firm \( j \in \{M, S\} \) defaults on its futures contract position in all possible futures states of the world (Hull, 2009, §2–3).

Later in the paper, we’ll translate Assumptions 1 – 3 into mathematical conditions. It is worth mentioning that all three assumptions are quite standard in the hedging literature, which includes Smith and Stulz, 1985; Froot et al., 1993.
3.1 Game Sequence

What follows is a sequence of stages in the game between the supplier and the assembler. At time $t_0$:

- Firms choose to take positions $N_j \geq 0$, $j \in \{M, S\}$ in the futures contracts.
- The futures exchange accepts $0 \leq n_j \leq N_j$, $j \in \{M, S\}$.
- Supplier, $S$, sets a wholesale price, $w$.
- Assembler, $M$, sets an order quantity, $q$.

This ends time $t_0$. Before time $t_1$ begins the future spot price of commodity 1 price is revealed. At time $t_1$:

- The supplier produces $q$ units for the assembler. (If the supplier chooses not to produce, then it incurs a penalty $L_S$.)

This ends time $t_1$. Before time $t_2$ begins the future spot price of commodity 2 and the supplier’s production decision at time $t_1$ are revealed. If the supplier produced at time $t_1$, then at time $t_2$:

- The assembler produces $q$ units of the final product. (If the assembler chooses not to produce, then it incurs a penalty $L_M$.)

This ends time $t_2$. Before time $t_3$ begins the value of demand $D$ is revealed. If the assembler produced at time $t_2$, then at time $t_3$ the sales revenue $\min\{q, D\}$ is realized.

In terms of the information structure, we assume that all market participants can observe the actions taken by all players and can observe all market outcomes, i.e., information is complete. This assumption can best justified with emerging empirical research in finance, which reveals that companies commonly incorporate covenant restrictions into supply contracts (e.g., see Smith and Stulz, 1985; Gilley and Rasheed, 2000; Roberts and Sufi, 2009; Costello, 2011) that help them reduce or eliminate asymmetric information and moral hazard.

4 Analysis

As mentioned earlier, one of the objectives of this paper is to identify conditions under which the supply chain firms should be expected to hedge their commodity input costs. In the base
case analysis, we evaluate the firms’ preferences over the hedged and unhedged contracts \textit{without} borrowing. Then, as an extension, we allow that both firms borrow from outside investors.

We will proceed with the analysis by first assuming that neither firm hedges its input costs and show that in this case both firms subject themselves to default risk since they agree to sell their output at prices that are fixed before all factors affecting their productivity are known. We’ll refer to this situation as the ‘unhedged wholesale contract.’ Where useful, the subscript ‘U’ will be used to indicate association with this subgame.

We then analyze the case in which the supplier avoids defaulting on the supply contract by entering into \( n_S > 0 \) futures contracts to buy commodity 1 at time \( t_1 \) and the assembler avoids defaulting on the supply contract by entering into \( n_M > 0 \) futures contracts to buy commodity 2 at time \( t_2 \). We’ll refer to this subgame as the ‘hedged wholesale contract’ and use the subscript ‘H’ to represent it. In analyzing this subgame, we determine the number of futures contracts, \( n_j, j \in \{M, S\} \), both firms will be purchasing in equilibrium and consider what happens if \( N_j > 0 \) and \( N_k = 0, j, k \in \{M, S\}, j \neq k \), which is a case in which one of the firms attempts to hedge while its supply chain partner does not hedge.

As is standard, the model is solved in two stages using backward induction: First stage characterizes the equilibrium behavior of the assembler; second stage characterizes the equilibrium behavior of the supplier and identifies the equilibrium in each subgame. The final equilibrium in which both firms decide whether to hedge can be determined by simply comparing the expected profits the supply chain firms generate with and without hedging.

\textbf{Cash Flows.} Before we begin the analysis of the \( U \) and \( H \) subgames, it is convenient to construct cash flows that are due to each firm if the supplier, \( S \), were to enter into an \textit{arbitrary} futures contract position to purchase \( N_S \geq 0 \) units of commodity 1 at time \( t_1 \) and the assembler, \( M \), were to enter into an \textit{arbitrary} futures contract position to purchase \( N_M \geq 0 \) units of commodity 2 at time \( t_2 \). From these, the cash flows that are due to each firm in the subgame \( U \) will be recovered by taking \( N_S = N_M = 0 \); the \( H \)-subgame cash flows will be recovered by setting \( N_M = n_M \) and \( N_S = n_S \), where \( n_S \) and \( n_M \) are values that satisfy the constraints given in Proposition 1 to be presented later in the paper.

The cash flows reveal that, in general, each firm \( j \in \{M, S\} \) will choose not to produce if the
firm $j$’s future spot price, $S_i$, $i = 1, 2$, is either too high in relation to firm $j$’s selling price (retail or wholesale price) or too low in relation to firm $j$’s futures price, $F_i$, $i = 1, 2$. We evaluate the issue of default in detail in Lemma 1 and present the results graphically in Figure 2. Given arbitrary futures contract positions, $N_M \geq 0$ and $N_S \geq 0$, cash flows to both firms are:

$$
\Pi^M = \begin{cases} 
N_M (S_2 - F_2) + \min \{ q, D \} - q S_2 - w q & \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, u_L^M < S_2 \leq u_H^M, t_2 < t_3 \\
N_M (S_2 - F_2) + \min \{ q, D \} (1 - S_2) - w q & \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, u_L^M < S_2 \leq u_H^M, t_2 = t_3 \\
\max \{ N_M (S_2 - F_2) - L_M, -c_M \} & \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, S_2 > u_H^M, t_2 \leq t_3 \\
\max \{ N_M (S_2 - F_2) - L_M, -c_M \} & \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, S_2 \leq u_L^M, t_2 \leq t_3 \\
\max \{ N_M (S_2 - F_2) + \ell_S, -c_M \} & \text{otherwise,}
\end{cases} 
$$

(1a)

$$
\Pi^S = \begin{cases} 
N_S (S_1 - F_1) + q (w - S_1) & \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, u_L^M < S_2 \leq u_H^M, t_1 \leq t_2 \\
N_S (S_1 - F_1) + \ell_M - q S_1 & \text{if } u_{L2}^S < S_1 \leq u_{H2}^S, S_2 > u_H^M, t_1 \leq t_2 \\
N_S (S_1 - F_1) + \ell_M - q S_1 & \text{if } u_{L1}^S < S_1 \leq u_{H1}^S, S_2 \leq u_L^M, t_1 \leq t_2 \\
\max \{ N_S (S_1 - F_1) - L_S, -c_S \} & \text{otherwise},
\end{cases} 
$$

(1b)

where

$$
\ell_S \equiv \min \left\{ L_S, (c_S + N_S (S_1 - F_1))^+ \right\} \quad \text{and} \quad \ell_M \equiv \min \left\{ L_M, (c_M + N_M (S_2 - F_2))^+ \right\}.
$$

(1c)

Above, $\ell_j$, $j \in \{M, S\}$ are expressions for penalties that each firm $j$ winds up paying if it fails to produce. In (1c), $N_j (S_i - F_i)$, $j \in \{M, S\}$, $i = 1, 2$ is firm $j$’s payoff from the futures contract position and $c_j + N_j (S_i - F_i)$ is firm $j$’s total capital at time $t_i$. Taken together, the expressions for $\ell_j$ reflect the fact both firms $j$ have limited liability. The $u$’s in Equations (1a)–(1b) are spot prices of commodities 1 and 2 at which the supplier and the assembler will break the underlying supply contract at times $t_1$ and $t_2$ respectively. Their values are as follows.

**Lemma 1.** If $t_0 < t_1 \leq t_2 \leq t_3$, $0 \leq c_j, L_j$, and $0 \leq N_j \leq q$, $j \in \{M, S\}$, then in (1): $u_{H1}^S = u_{L2}^S = 0$, $u_L^M = 0$, and

$$
u_{H1}^S = \begin{cases} 
\min \left\{ \frac{w q + c_S - N_S F_1}{q - N_S}, w + \frac{L_S}{q} \right\} & \text{if } t_1 = t_2, \\
\min \left\{ \frac{c_S - N_S F_1}{q - N_S}, \frac{q w p \{ S_2 \leq u_H^M \} + \ell_M p \{ S_2 > u_H^M \} + L_S}{q} \right\} & \text{if } t_1 < t_2,
\end{cases}$$

(2a)
If $t_0 < t_1 < t_2 < t_3$, $0 \leq c_S, L_S$, and $0 < q < N_S$, then in (1):

\[
\begin{align*}
    u^{S}_{H2} &= \begin{cases} 
        \min \left\{ \frac{L_M + L_S}{q}, \frac{L_M + c_S - N_S F_1}{q - N_S} \right\} & \text{if } t_1 = t_2, \\
        u^{S}_{H1} & \text{if } t_1 < t_2,
    \end{cases} \\
    u^{M}_{H2} &= \begin{cases} 
        \min \left\{ \frac{\min(q,D) + c_M - N_M F_2 - w q}{\min(q,D) - N_M}, \frac{\min(q,D) + L_M - w q}{\min(q,D) - 1} \right\} & \text{if } t_2 = t_3, \\
        \min \left\{ \frac{c_M - N_M F_2 - w q}{q - N_M}, \frac{\min_{t} (q,D) + L_M - w q}{q - \min_{t} (q,D)} \right\} & \text{if } t_2 < t_3.
    \end{cases}
\end{align*}
\]

Discussion. A careful inspection of the functions $u$ specified in the above lemma reveals that each firm $j \in \{M, S\}$ will choose not to produce if at least one of the following holds:

(1) Firm $j$'s expected profit from production is strictly less than the penalty $L_j$, $j \in \{M, S\}$ firm $j$ agreed to pay for breaking the underlying supply contract. That is, if input prices at time $t_i$ turn out to be high, and the cost of default, $L_j$, is low, then firm $j$ will choose not to produce owing to insufficient expected payoff.

(2) Firm $j$'s lacks the necessary capital to produce. To illustrate, suppose $t_1 < t_2$. Then, the supplier, $S$, will inevitably default at time $t_1$ if $c_j + N_j (S_1 - F_1) < q S_1$ because, in such a case,
the time $t_1$ cost of production inputs, $q S_1$, exceeds the time $t_1$ available capital, $c_j + N_j (S_1 - F_1)$. This situation will arise if either the time $t_1$ spot price of commodity 1 turns out to be high and the supplier’s futures contract position, $N_S$, is too small, or if the time $t_1$ spot price of commodity 1 turns out to be low and the supplier’s futures contract position, $N_j$, is too large. In both cases, the supplier will fail to produce owing to insufficient resources.

Figure 2, which illustrates the functions $u$ graphically confirms that the supplier, $S$, will rationally choose to produce if it knows that the assembler will produce and the time $t_1$ spot price of commodity 1 is between $u_{L1}^S$ and $u_{H1}^S$. In this case the supplier will earn a payoff of $q(w - S_1) + N_S(S_1 - F_1)$. However, the figure also reveals that the supplier will choose to produce if it knows that the assembler will not produce and if its time $t_1$ spot price of commodity 1 is between $u_{L2}^S$ and $u_{H2}^S$. In this case, the supplier will receive a payoff of $N_S(S_1 - F_1) + \ell_M - qS_1$, which is in excess of the penalty payment $\ell_S$. The assembler, on the other hand, will only produce if its commodity input cost is between $u_{L}^M$ and $u_{H}^M$ and if the supplier produces. (This is because the assembler’s production process requires an input from the supplier.)
A closer examination at the functions $u$ given in the lemma also reveals that the exogenously
given capital endowment, $c_j \geq 0$, $j \in \{M, S\}$, causes ‘parallel shifts’ to the functions $u$, but it does
not qualitatively change how each firm $j$ behaves as its endogenously chosen position in the futures
market, $N_j$, and the spot price of commodity $i = 1, 2$ change. (Except when $c_j$ is very large; then
the role of the firm $j$’s futures contract position, $N_j$, diminishes.) It is for this reason that hereafter we analyze the model in the analytically simplest way by taking $c_j = 0$. Low cash reserves are also
consistent with some empirical evidence, e.g., see de Blasio (2005). Moreover, to allow firms to
produce without having to borrow, we take $0 < t_1 = t_2 = t_3$. Then both firms incur revenues and
costs simultaneously and production is feasible without cash on hand and without borrowing.

4.1 Analysis of the Unhedged Contract

Suppose now that neither firm hedges its commodity input cost. The assembler’s and supplier’s time
$t_0$ payoffs, $\Pi_U^M$ and $\Pi_U^S$, can be recovered from Equations (1) and (2) and by setting $N_M = N_S = 0.$
Since $c_M = c_S = 0$, then Equations (1c) imply $\ell_M = \ell_S = 0$ for all $L_S \geq 0$, $L_M \geq 0$, implying that
default penalties in the unhedged case are irrelevant.

**Equilibrium Behavior for the Assembler.** By risk-neutrality, the assembler’s best response
order quantity, $q$, maximizes its time $t_0$ expected profit, $E_{t_0} \Pi_U^M (q)$. (The subscript on the expectation
operator indicates filtration.) Using (1), the assembler solves the following problem:

$$
\max_{q \leq p \leq d} \int_0^{u_H^S} \int_q^{u_H^M(q,p)} \int_0^{w_U^M(x,q)} q (1 - y + w_U) f(x) h(y,z) \, dy \, dx \, dz
+ \int_0^{u_H^S} \int_q^{u_H^M(x,p)} (x - x y - w_U \, q) f(x) h(y,z) \, dy \, dx \, dz. \tag{4}
$$

Using the necessary optimality conditions with respect to $q$, (e.g., Bertsekas, 2003, Proposition 1.1.1) and the Leibniz’s rule we can obtain the implicit inverse demand curve, $w_U(q)$, that the supplier faces:

$$
w_U(q) = \frac{\int_0^{u_H^S} \int_q^{u_H^M(q,p)} (1 - y) f(x) h(y,z) \, dy \, dx \, dz}{\mathbb{P} \left\{ S_1 \leq u_H^S, S_2 \leq u_H^M \right\}}. \tag{5}
$$
By substituting the right side of (5) for the wholesale price, \( w_U \), into the maximand in (4), we can obtain an expression for the assembler’s expected payoff (in terms of \( q \)):

\[
\mathbb{E}_{t_0} \Pi_U^M(q) = \int_0^{u_H^S} \int_q \int_0^{u_H^S(x,q)} x (1 - y) f(x) h(y,z) \, dy \, dx \, dz. \tag{6}
\]

Since the distribution of demand, \( F \), has support on \([d, \bar{d}]\), then \( \mathbb{E}_{t_0} \Pi_U^M(q) : [d, \bar{d}] \to \mathbb{R} \), given by (6), is a continuous real-valued mapping, where \([d, \bar{d}]\) is non-empty subset of \( \mathbb{R} \). There exists a quantity \( q_M^* \) such that \( \mathbb{E}_{t_0} \Pi_U^M(q) \leq \mathbb{E}_{t_0} \Pi_U^M[q_M^*] \) for all \( q \in [d, \bar{d}] \) (Weierstrass proposition). It follows that if \( \mathbb{E}_{t_0} \Pi_U^M[q_M^*] \leq R_M \), then the assembler will place no order.

**Equilibrium Behavior for the Supplier.** Let \( w_U(q) \) be implicitly given by Equation (5). The supplier’s expected profit is:

\[
\mathbb{E}_{t_0} \Pi_U^S(q) = q \left( w_U(q) - \mathbb{E}_{t_0} \left( S_1 \mid S_1 \leq u_H^S, S_2 \leq u_H^M \right) \right) \mathbb{P} \left( S_1 \leq u_H^S, S_2 \leq u_H^M \right),
\]

or, equivalently,

\[
\mathbb{E}_{t_0} \Pi_U^S(q) = q \int_0^{u_H^S(q,q)} \int_q \int_0^{u_H^S(x,q)} (1 - y) f(x) h(y,z) \, dz \, dx \, dy - q \int_0^{u_H^S(q,q)} \int_0^{u_H^S} z h(y,z) \, dz \, dy. \tag{7}
\]

Since \( F \) has support on \([d, \bar{d}]\), then \( \mathbb{E}_{t_0} \Pi_U^S(q) : [d, \bar{d}] \to \mathbb{R} \) is a continuous real-valued mapping where \([d, \bar{d}]\) is a non-empty subset of \( \mathbb{R} \). There exists a quantity \( q_U^* \) such that \( \mathbb{E}_{t_0} \Pi_U^S(q) \leq \mathbb{E}_{t_0} \Pi_U^S[q_U^*] \) for all \( q \in [d, \bar{d}] \) (Weierstrass proposition). The supplier’s optimal order quantity \( q_U^* \) is given by \( \arg \max_{q \geq d} \mathbb{E}_{t_0} \Pi_U^S(q) \) s.t. \( \mathbb{E}_{t_0} \Pi_U^M(q) \geq R_M \) and \( \mathbb{E}_{t_0} \Pi_U^S(q) \geq R_S \), where \( R_S, R_M \) are the firms’ reservation payoffs (note that if \( R_M \) and \( R_S \) are excessively large, then \( q_U^* \) may not exist). It follows that if the supplier were able to choose any wholesale price, it would choose \( w_U(q_U^*) \), where \( w_U \) solves Equation (5).

### 4.2 Analysis of the Hedged Contract

Suppose now that both firms hedge by entering into \( n_j \geq 0 \), \( j \in \{M,S\} \) futures contracts to purchase commodity \( i = 1, 2 \) at time \( t_i \). Each firm \( j \)'s time \( t_i \) payoff from the futures contract position will be \( n_j (S_i - F_i) \), which is positive when the realized future spot price, \( S_i \), is high (i.e.,
when \( 0 \leq F_i < S_i \leq \infty \) and negative when it is low (i.e., when \( 0 \leq S_i < F_i \leq \infty \)). Before we begin the Stage 1 analysis of the supply chain contract with hedging, we present a preliminary result that deals with firms’ equilibrium long futures contract positions.

**Proposition 1.** Suppose Assumption 3 holds and \( c_j = 0 \), \( j \in \{M,S\} \). The number of futures contracts, \( n_j \geq 0 \), that can be supported as a subgame perfect equilibrium (SPE) must satisfy the following constraints:

\[
\frac{q L_S}{L_S + q(w_H - F_1)} \leq n_S \leq \min \left\{ \frac{q w_H}{F_1}, \frac{L_M}{F_1}, q \right\}, \tag{8a}
\]

\[
\frac{d L_M}{d(1 - F_2) + L_M - w_H q} \leq n_M \leq \min \left\{ \frac{d - q w_H}{F_2}, \frac{L_S}{F_2}, q \right\}. \tag{8b}
\]

**Discussion.** As specified in Assumption 3, for the futures market to be credible, the futures exchange must organize trading in futures contracts so that defaults are completely avoided. In practice, the exchange may require parties to post margin accounts so that performance can be credibly guaranteed. \( c_j = 0 \), however, implies that neither firm \( j \in \{M,S\} \) is able to post a margin. Therefore the exchange guarantees performance by accepting futures contract positions \( n_j \geq 0 \), that it knows each firm \( j \) can sustain without default for all \( 0 \leq S_1 < \infty, 0 \leq S_2 < \infty \), and \( d \leq D \leq \bar{d} \). It turns out that such values of \( n_j \) must satisfy the bounds in given (8).

Reading from the left, the first upper bound on \( n_j \) given in (8) is in force when both supply chain members choose to produce at time \( t_i, i = 1, 2 \) and firm \( j \)’s realized input price, \( S_i \), at time \( t_i \) is less than the futures price, \( F_i \). The bound ensures that firm \( j \)’s operating profit is sufficient to offset any loss from firm \( j \)’s futures contract position, \( n_j (S_i - F_i) < 0, j \in \{M,S\}, i = 1, 2 \). For otherwise firm \( j \) would be at risk of defaulting on its futures contract position due to insufficient resources.

The second upper bound on \( n_j \) given in (8) is in force when firm \( j \)’s realized time \( t_i \) input price is again low (i.e., \( S_i < F_i \)) and its supply chain counterpart, firm \( k \in \{M,S\}, j \neq k \), chooses not to produce at time \( t_l, l = 1, 2, l \neq i \). This will occur exactly when firm \( k \)’s realized input cost, \( S_i \), at time \( t_i \) is high (infinite). In this situation firm \( k \) will be contractually obligated to use the payoff from its futures contract position, \( n_k (S_i - F_i) > 0 \), to compensate firm \( j \) with a default penalty, \( L_k \geq 0 \). (Note that if firm \( k \) breaks the supply contract, the penalty payment, \( L_k \), is firm \( j \)’s only revenue.) As before, the upper bound on \( n_j \) ensures that \( L_k \) is sufficient to offset any loss from
firm $j$’s futures contract position, $n_j (S_i - F_i) < 0$. At the same time, the lower bound on $n_k$ given in (8) ensures that firm $k$’s payoff from its futures contract position, $n_k (S_i - F_i) > 0$, is enough to pay at least $L_k$ to firm $j$.

In summary, Proposition 1 can be viewed a set of necessary and sufficient conditions under which neither supply chain member defaults on its futures contract position in all possible futures states of the world – as specified in our Assumption 3.

Since the lower bounds on $n_M$ and $n_S$ are strictly positive, then Proposition 1 also reveals that, in equilibrium, either both firms $j \in \{M, S\}$ simultaneously enter into futures contract positions $n_j > 0$ or they do not hedge at all. This reflects the fact that a situation in which one firm hedges while its supply chain partner does not hedge, makes the hedger susceptible to default on the futures contract. This default occurs exactly when the realized future spot price of the firm that did not hedge is high (causing it to default on the underlying supply contract) and the realized future spot price of the firm that did hedge is low (causing it to default on the futures contract).

Corollary 1 summarizes the result.

**Corollary 1.** If $c_j = 0$, $j \in \{M, S\}$, then in equilibrium, either both firms $j$ simultaneously enter into futures contract positions $n_j > 0$ or they don’t hedge at all.

Finally, to ensure that the ranges for $n_j$, $j \in \{M, S\}$ given in Proposition 1 are non-empty we require the following two conditions:

**Condition 1.** $w_H q \leq L_M \leq q$ and $\min\{q, D\}(1 - S_2)^+ \leq L_S \leq q$.

**Condition 2.** $F_1 \leq w_H$ and $F_2 \leq 1 - \frac{q}{d}$.

Condition 1 corresponds to Assumption 1, which requires that a default on the underlying supply contract be costly; neither firm, however, should unusually profit from a default of its supply chain partner. Interestingly, the penalties that satisfy the bounds given in Condition 1 not only ensure that neither firm $j \in \{M, S\}$ defaults on its futures contract position, but also that each firm $j$ is indifferent as to whether its supply chain partner chooses to produce or not. To see this, consider the case when $w_H q \leq L_M$, and $\min\{q, D\}(1 - S_2)^+ \leq L_S$ and when both firms hedge. Then the supplier is guaranteed to receive a revenue of $w_H q$ and the assembler is guaranteed to receive a revenue of $\min\{q, D\}(1 - S_2)^+$, whatever action its supply chain partner takes. It is
worth mentioning that penalties that satisfy Condition 1 are not inconsistent with what one can observe in empirical practice. For example, in the Davita, Inc. contract included in Appendix A, the penalty is set as 100% of the wholesale price.

Condition 2 corresponds to Assumption 2, which requires upper bounds on the futures prices of commodities 1 and 2. Without these upper bounds, firms cannot make money in expectation and hedging is essentially infeasible.

**Lemma 2.** If Conditions 1–2 hold, then the range for \( n_j, j \in \{M,S\} \) given in (8) is non-empty for all \( 0 \leq S_1 < \infty, 0 \leq S_2 < \infty, d \leq D \leq \overline{d}, \) and \( c_j = 0. \)

We illustrate Proposition 1 with the following simple example.

**Example 1.** Suppose \( 0 \leq S_1 < \infty \) and \( 0 \leq S_2 < \infty \) are log-normally distributed with a mean \( F_1 = F_2 = $0.25. \) Demand is uniform on [100,150] and the order quantity \( q = 110. \) With these parameters, the highest payoff either firm can achieve is $110. If the equilibrium wholesale price, \( w_H \) is $0.5, then Condition 1 stipulates \( 55 \leq L_M \leq 110 \) and \( \min\{110,D\}(1-S_2) \leq L_S \leq 110. \) It is easy to check that \( F_1 \) and \( F_2 \) satisfy Condition 2. Finally, the number of futures contracts, \( n_M \) and \( n_S, \) that can be supported as an SPE satisfies: \( 83.33 \leq n_M \leq 110 \) and \( 86.27 \leq n_S \leq 110. \)

**Partial vs. Complete Hedging.** Following SFAS 52 and 80, which are generally accepted accounting principles (GAAP) rules for the accounting treatment of hedged transactions (see Mian, 1996, p.425), in an earlier version of this paper\(^2\), we adopted the assumption that a futures trade is not considered to be a hedge unless the underlying transaction is a firm commitment. Some empirical support for this assumption can be found in the airline industry: Carter et al. (2006, p.26), for example, report that most airlines hedge their fuel costs, but do not necessarily cancel flights when selling jet fuel on the open market is more profitable than using the fuel to fly planes. Under this assumption, it is straightforward to show that if firms hedge, then without loss of optimality, they can hedge fully by taking futures contract positions \( n_j = q, j \in \{M,S\}, \) i.e., they can completely insulate their market values from hedge-able risks.

In this version of the paper, we allow that firms adopt the ‘take-the-money-and-run’ strategy under which firms default on the underlying supply contract and sell their commodity inputs on

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\(^2\)The previous version of the paper is available from the authors upon request.
the open market – whenever it is profitable to do so.\textsuperscript{3} There is some empirical evidence for this conduct as well: Hillier et al. (2008, p.768), for example, describe a case where in 2001 an aluminum producer Alcoa temporarily shut down its smelters and sold its electricity futures contracts on the open market. (Since it could make more money by selling electricity than by selling aluminum.)

Allowing firms to adopt the take-the-money-and-run strategy implies that the hedged supply contract is no longer a firm commitment and the results in Proposition 1 reveal that in equilibrium firms may only hedge partially (see above). (By partial hedging, it is meant that $0 < n_j < q$ or $0 < q < n_j$, $j \in \{M, S\}$.)

It is worth mentioning that previous models of hedging in the single-firm setting have also shown that firms may only hedge partially (see Froot et al., 1993). There is, however, an important difference between our result and the findings in Froot et al. (1993): Partial hedging in Froot et al. (1993) requires multiple correlated sources of uncertainty; here, such assumption plays no role.

**Equilibrium Behavior for the Assembler and for the Supplier.** Let Conditions 1 – 2 hold and let $n_j \geq 0$, $j \in \{M, S\}$ satisfy the bounds given in Proposition 1. The equilibrium behavior for both firms can now obtained by adapting results from the existing literature (for example, Lariviere and Porteus, 2001). Using Equations (1), the assembler’s *expected* payoff is given by:

$$
E_{t0} \Pi_M^H(q, w_H) = \int_0^\infty \int_q^1 \int_0^1 q (1 - y) f(x) h(y, z) dy \ dx \ dz + \int_0^\infty \int_0^q \int_0^1 (1 - y) x f(x) h(y, z) dy \ dx \ dz - q w_H + n_M (E_{t0} S_2 - F_2). \tag{9}
$$

Using the fact that $F_2 = E_{t0} S_2$, the assembler’s optimal order quantity implicitly defined by $\int_0^\infty \int_q^1 \int_0^1 (1 - y) f(x) h(y, z) dy \ dx \ dz - w_H = 0$, from which we can solve for the inverse demand, say $w_H(q)$, the supplier faces:

$$
w_H(q) = \min \left\{ \int_0^\infty \int_q^1 \int_0^1 (1 - y) f(x) h(y, z) dy \ dx \ dz, \frac{d(1 - F_2)}{q} \right\}. \tag{10}
$$

We can then use Equations (9) and (10) to yield

$$
E_{t0} \Pi_M^H(q) = \max \left\{ \int_0^\infty \int_q^1 \int_0^1 x (1 - y) f(x) h(y, z) dy \ dx \ dz, \int_0^\infty \int_q^1 \int_0^1 q (1 - y) f(x) h(y, z) dy \ dx \ dz \right\}
$$

\textsuperscript{4}We thank an anonymous reviewer whose comments encouraged this extension.
\[
+ \int_0^\infty \int_a^1 \int_0^1 (1 - y) x f(y, z) dy dx dz - d(1 - F_2)
\] 
(11)

which is assembler’s expected payoff in terms of \(q\). Differentiation reveals that the assembler’s payoff is increasing in \(q\). To see this, the derivatives of the first and the second expression on the right side of (11) are:

\[
\int_0^\infty \int_0^1 q (1 - y) f(q(y, z) dy dz \geq 0 \quad \text{and} \quad \int_0^\infty \int_a^1 (1 - y) f(y, z) dy dx dz \geq 0.
\] 
(12)

The supplier’s expected profit is:

\[
E_{t_0} \Pi_H^S(q) = q w_H(q) - E_{t_0} \min \{q S_1, \min\{q, D\} (1 - S_2)^+\} + n_S (E_{t_0} S_1 - F_1),
\] 
(13)

The supplier’s most preferred order quantity, \(q_H^*\), is given by \(\arg \max_{q \geq \overline{q}} E_{t_0} \Pi_H^S(q)\); the supplier’s optimal order quantity \(q_H^{**}\) is given by \(\arg \max_{q \geq \overline{q}} E_{t_0} \Pi_H^M(q)\) s.t. \(E_{t_0} \Pi_H^M(q) \geq R_M\) and \(E_{t_0} \Pi_H^S(q) \geq R_S\), where \(R_S, R_M\) are the firms’ reservation payoffs. The results in Lariviere and Porteus (2001) imply that \(E_{t_0} \Pi_H^S(q)\), given by (13), is unimodal in \(q\).

Since the supplier’s expected payoff, \(E_{t_0} \Pi_H^S(q)\), is unimodal and the assembler’s expected payoff, \(E_{t_0} \Pi_H^M(q)\), is strictly increasing in the order quantity, \(q\), it follows that \(q_H^{**} \in H\), where \(H := \{q \mid q_H^* \leq q \leq \overline{q}\}\). To see this, suppose the supplier selects a wholesale contract \((q', w_H(q'))\) (see Figure 3a). Such a contract must be Pareto dominated by the wholesale contract \((q'', w_H(q''))\) because \(E_{t_0} \Pi_H^S(q') = E_{t_0} \Pi_H^S(q'')\) and \(E_{t_0} \Pi_H^M(q') < E_{t_0} \Pi_H^M(q'')\). Hence all contracts associated with order quantities in \([\overline{q}, q_H^*]\) are Pareto dominated by contracts with order quantities selected from the set \(H\). Now, for example, why would the supplier select some order quantity in \((q'', \overline{q})\) (rather than its most preferred order quantity \(q_H^*\))? Suppose the assembler’s reservation profit \(R_M = E_{t_0} \Pi_H^M(q'')\). Then it is readily seen in Figure 3a that only contracts in \((q'', \overline{q})\) are feasible. (For visual comparison, Figure 3d combines the payoffs under both unhedged and hedged contracts into a single diagram.)

### 4.3 Equilibrium Among Unhedged and Hedged Contracts

We now allow the firms to enter into the supply chain contract by choosing whether they wish to hedge. Depending on the model parameters, it is possible to have equilibria where: (a) Neither
firm hedges; or (b) both firms hedge. Due to Proposition 1, case (a) will occur if one of the firms in the model decides not to hedge at time $t_0$. If, on the other hand, there exists an SPE in which

$$\mathbb{E}_{t_0} \Pi^S_H(q^{**}_H) \geq \mathbb{E}_{t_0} \Pi^S_U(q^{**}_U),$$  \hspace{1cm} (14a)

then the supplier, $S$, will hedge, and as will be seen in Lemma 3, Part (i), the assembler will hedge as well. Similarly, the supplier will hedge, in cases where the unhedged wholesale contract may be infeasible. This occurs when:

$$\mathbb{E}_{t_0} \Pi^M_U(q^*_M) \leq R_M, \text{ and there exists } q \in H \text{ for which } R_S \leq \mathbb{E}_{t_0} \Pi^S_H(q) \text{ and } R_M \leq \mathbb{E}_{t_0} \Pi^M_H(q).$$ \hspace{1cm} (14b)

The following Lemma 3 is needed to establish that Conditions (14a) and (14b) can hold in equilibrium. Parts (i) and (ii) describe how the risk of supply contract default affects both the assembler’s expected payoff and the wholesale price. Parts (iii) and (iv) give sufficient conditions under which (for both the supplier and the assembler) there exist expected payoff levels that may only be achieved via the hedged wholesale contract.

**Lemma 3.** Let $w_U(q), \mathbb{E}_{t_0} \Pi^M_U(q), \mathbb{E}_{t_0} \Pi^S_U(q), w_H(q), \mathbb{E}_{t_0} \Pi^M_H(q),$ and $\mathbb{E}_{t_0} \Pi^S_H(q)$ respectively be given by (5), (6), (7), (10), (11), and (13). If $c_j = 0$ and Conditions 1 – 2 hold then:

(i) $0 \leq \mathbb{E}_{t_0} \Pi^M_U(q) \leq \mathbb{E}_{t_0} \Pi^M_H(q)$.

(ii) $w_H(q) \leq w_U(q)$ (if $S_1$ and $S_2$ are independent).

(iii) The set $H_M := \{ q \in H \mid \mathbb{E}_{t_0} \Pi^M_U(q^*_M) \leq \mathbb{E}_{t_0} \Pi^M_H(q) \}$ is non-empty. (An example of the set $H_M$ can be seen graphically in Figure 3b.)

(iv) $H_S := \{ q \in H \mid \mathbb{E}_{t_0} \Pi^S_U(q^*_U) \leq \mathbb{E}_{t_0} \Pi^S_H(q) \}$ is non-empty \left( \frac{\int_0^{\mathbb{E}} (1 - F(q))}{1 - F_2} \right) \int_0^1 (1 - y) h(y, z) dy dz \leq \frac{q \int_0^{\mathbb{E}} (1 - F(q))}{1 - F_2} \int_0^1 (1 - y) h(y, z) dy dz. \text{ An example of the set } H_S \text{ can be seen graphically in Figure 3c.}\right)

**Discussion.** Part (i) asserts that for any order quantity, $q$, the assembler prefers the hedged wholesale contract to the unhedged one. When compared to the no hedge case, a hedged supplier can guarantee supply on a greater range of $S_1$ while at the same time charge a lower wholesale price, $w_H$, as will be seen in Part (ii). Together these imply that the hedged contract is preferable to the assembler because it allows it to produce on a much greater range of input prices. For the case when $S_1$ and $S_2$ are independent, Part (ii) of the lemma formally establishes that the equilibrium
wholesale price is lower when the downstream assembler hedges. This result reflects the fact that, by hedging, the assembler guarantees the supply contract performance and the rational supplier responds to this guarantee by lowering the unit wholesale price.

Parts (iii) and (iv) give sufficient conditions under which there exist expected payoff levels that can only be achieved via the hedged wholesale contract. As a consequence of Part (i) of the lemma, the set $H_M$ will always be non-empty. The set $H_S$ will be non-empty if $d$ is sufficiently large and $\mathbb{E}_{t_0} \min\{q, D\} (1 - S_2)^+ + \text{is sufficiently small. The former is consistent with a situation in which

\begin{itemize}
\item \textbf{Note.} (i) For the order quantities $\hat{q}_S$ and $\hat{q}_M$ respectively we have $\mathbb{E}_{t_0} \Pi_H^S(\hat{q}_S) = \mathbb{E}_{t_0} \Pi_U^S(q^* U)$ and $\mathbb{E}_{t_0} \Pi_H^M(\hat{q}_M) = \mathbb{E}_{t_0} \Pi_U^M(q^* M)$. \end{itemize}

\begin{itemize}
\item \textbf{Note.} (ii) For the order quantity $q = \hat{q}_S$ we have $\mathbb{E}_{t_0} \Pi_H^S(\hat{q}_S) = \mathbb{E}_{t_0} \Pi_U^S(q^* U)$; for the quantity $q = \hat{q}_M$ we have $\mathbb{E}_{t_0} \Pi_H^M(\hat{q}_M) = \mathbb{E}_{t_0} \Pi_U^M(q^* M)$. $D \sim \mathcal{U}(10, 75)$; $S_1$ and $S_2$ are independent and log-normally distributed; time $t_0$ prices of commodities 1 and 2 are $0.3$; lead-time $= t_1 - t_0 = 6$ weeks; $t_2 = t_1$; annual volatility of commodity 1 and 2 spot prices is $60\%$.
\end{itemize}
there is a high baseline demand for the assembler’s final product. The latter is consistent with a situation in which the assembler operates on a small margin, which implies that, without hedging, the assembler prone to defaulting on the underlying supply contract. Under these conditions, not defaulting is ex-ante important to the supplier who wants to capitalize on the high baseline demand without risking default. The supplier therefore prefers that the assembler hedge. The primary use of Lemma 3 is in establishing the next Proposition 2.

**Proposition 2.** If \( c_j = 0 \) and Conditions 1 – 2 hold then there exist SPEs in which both firms will hedge. Both firms will hedge if (i) \( R_M \geq \mathbb{E}_{t_0} \Pi_M^H(q_M^*) \); or (ii) if \( R_M = 0 \) and the set \( H_S \) is non-empty (see Part iv of Lemma 3).

**Discussion.** Proposition 2 points out a strategic interaction between supply chain hedging and the market power of the downstream firm. In particular, Part (i) of the Proposition 2 suggests that hedged wholesale contract offers with order quantities drawn from the set \( H_M \) (seen graphically in Figure 3b) should be expected when the assembler has a sufficiently high reservation payoff. Recall that \( \mathbb{E}_{t_0} \Pi^M_U(q^*_M) \) is the highest payoff the assembler can achieve under the unhedged contract. If \( R_M \geq \mathbb{E}_{t_0} \Pi^M_U(q^*_M) \) then the supplier must choose a hedged contract with \( q \in (\hat{q}_M, \bar{q}] \), where \( \mathbb{E}_{t_0} \Pi^H_M(\hat{q}_M) = \mathbb{E}_{t_0} \Pi^M_U(q^*_M) \). This follows because \( \mathbb{E}_{t_0} \Pi^M_H(q) \), given by (12), is strictly increasing in \( q \). (In the proof of Part i, we show that such contracts satisfy the condition 14b.) Based on Part (i), we can therefore make an empirical prediction:

**Prediction 1.** Offers of the hedged wholesale contract should be expected if the downstream firm’s market power exceeds a critical threshold.

Prediction 1 essentially identifies situations when hedging is mainly important to the downstream assembler. A second empirical prediction regarding the use of hedged contract can be made using Part (ii) of Proposition 2. Prediction 2 identifies situations when hedging is mainly important to the upstream supplier.

**Prediction 2.** Offers of the hedged wholesale contract should be expected if the downstream firm operates on a small margin, there is a high baseline demand for its final product, and its market power is below a critical threshold.
Table 1 presents some data on how much better the assembler can do with the hedged contract: If the unhedged equilibrium order quantity is $q$, then column (L) of Table 1 shows the percentage profit increase the assembler experiences if it is offered an alternative hedged contract that leaves the supplier no worse off than the original unhedged contract. The data shows that the assembler’s expected payoff can increase 1% to 75%. Column G of the same table presents comparable data for the supplier.

As might be expected, the value derived from using the hedged contract will depend on lead-times, spot price volatilities, and spot price correlation. In computing the results in Table 1, we assumed that the spot prices of commodities 1 and 2 were independent and chose annual volatilities ranging from 30%–60% (a range comparable to equities; e.g., see Hull, 2009, p.238). Interestingly, in practice, annual volatility of commodity prices can easily exceed 60%: To illustrate, silver, an industrial metal used in electrical contacts and in catalysis of chemical reactions, was trading at around $18 per ounce in April and May of 2010. Less than a year later, silver almost tripled in value to $49 per ounce during the final week of April 2011 – see Christian (2011). Our lead-times, namely the difference between $t_1$ and $t_0$, were 4 to 12 weeks (e.g., see Fisher, 1997). For simplicity, we took $t_2 = t_1$. All remaining assumptions are stated at the bottom of Table 1.

We can also circle back some of our predictions both to results found in the research existing literature and to the existing industry practices. Because of the aforementioned interaction between the distribution of market power and the incentive to hedge, one may conjecture that the presence of a powerful supply chain member may render hedging unprofitable for the less powerful supply chain members. Figure 3d confirms this intuition: Suppose that the assembler’s market power is sufficiently high so that the equilibrium order quantity is in $H_M$. Then both firms will hedge their commodity inputs. However, as Figure 3d reveals, for some order quantities the supplier’s expected payoff under the unhedged contract may be lower than under the hedged contract. This observation is not necessarily consistent with the existing theories of hedging, which imply that hedging should increase firms’ market values.

It does, however, appear to be consistent with the empirical findings in Jin and Jorion (2006) who report that while many firms hedge, hedging does not necessarily increase their market values. In addition, it appears to be consistent with several anecdotal examples from the popular business press, which stipulate that commodity price movements routinely affect delivery of products and
Table 1: The Impact of Switching to the Supplier’s (Assembler’s) Best Hedged Contract That Leaves the Assembler (Supplier) No Worse Off.

<table>
<thead>
<tr>
<th>Lead-time (weeks)</th>
<th>Supplier’s Best Hedged Contract (q′, w_H(q′)) subject to E_t_t_a^M(q′) ≥ E_t_t_a^M(q)</th>
<th>Assembler’s Best Hedged Contract (q″, w_H(q″)) subject to E_t_t_a^N(q″) ≥ E_t_t_a^N(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
<td>(B)</td>
</tr>
<tr>
<td>4 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>24.</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>27.2</td>
<td>0.428</td>
</tr>
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<td></td>
<td>30.4</td>
<td>0.396</td>
</tr>
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<td></td>
<td>33.6</td>
<td>0.364</td>
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<td></td>
<td>36.8</td>
<td>0.332</td>
</tr>
<tr>
<td></td>
<td>40.0</td>
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</tr>
<tr>
<td>8 45</td>
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<tr>
<td></td>
<td>24.</td>
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<td></td>
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<tr>
<td>12 60</td>
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<tr>
<td></td>
<td>24.</td>
<td>0.464</td>
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<td></td>
<td>27.2</td>
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<td></td>
<td>30.4</td>
<td>0.397</td>
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<td>33.6</td>
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<td>36.8</td>
<td>0.332</td>
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<tr>
<td></td>
<td>40.0</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Note. \( D \sim \mathcal{U}(10, 75) \); \( S_1 \) and \( S_2 \) are independent and log-normally distributed; time \( t_0 \) prices of commodities 1 and 2 are \( \$0.3 \).

Col(A): Lead-time = \( t_1 - t_0 \). The example assumes \( t_0 < t_1 = t_2 < t_3 \) (see ‘Timing of Events’ in Section 3).

Col(B): Measures standard deviations of price returns.

Cols(C) and (D): Equilibrium unhedged wholesale contract, \((q, w_H(q))\).

Cols(E) and (F): Supplier’s best hedged contract \((q', w_H(q'))\) subject to \( E_{t_0} \Pi_H^M(q') \geq E_{t_0} \Pi_H^M(q) \).

Cols(J) and (K): Assembler’s best hedged contract \((q''', w_H(q'''))\) subject to \( E_{t_0} \Pi_H^N(q''') \geq E_{t_0} \Pi_H^N(q) \).
services in auto parts supply chains (e.g., Hakim, 2003; Matthews, 2011), food-processing supply chains (e.g., Bjerga, 2013; Eckblad, 2012), and energy supply chains (e.g., Thakkar, 2013).

5 Extensions: Production with Outside Financing

So far we identified some conditions under which firms that do not borrow from outside investors find it profitable to hedge their production inputs. Since firms commonly borrow, one important issue to consider is how the results presented in Section 4 change if the supply chain participants can borrow. Borrowing may undoubtedly introduce some new effects.

In order to tease out the role borrowing plays in our model, we will deliberately impose conditions under which the supply chain would not be operational in the absence of external financing.\(^4\) Suppose \(t_1 < t_2 < t_3\) and \(c_S = c_M = 0\). Then, in order to produce, the supplier, \(S\), must borrow \(b_S = qS_1\) at time \(t_1\) and the assembler, \(M\), must borrow \(b_M = q(w_U + S_2)\) at time \(t_2\). To ensure that borrowing is always feasible, we assume that \(S_1\) and \(S_2\) have a joint distribution function, \(H\), which has a support on \([0, w_U] \times [0, 1 - w_U]\). Finally, for simplicity, we also set \(d = 0\). Interestingly, these assumptions also imply that, in the absence of financing, neither firm would hedge.

As before, operational revenues are determined by the realization of random variables reflecting demand and commodity input cost, together with the output decision made by the supplier. Both firms must pay debt claims, \(b_j, j \in \{M, S\}\), out of these operating revenues if possible. If the firm’s operating revenue cannot cover its debt obligation, the firm goes into bankruptcy. In practice, the meaning of bankruptcy is that a firm enters into a legal process as a result of declaring its inability to pay its debts as they come due. The firm may continue operations while in bankruptcy, or it may be liquidated. In either case, the shareholders lose their immediate claim over the earnings of the firm and the lenders become residual claimants. The advent of bankruptcy imposes extra costs on the operations of the firm (e.g., legal costs, bargaining costs, and a loss of revenues if the firm’s operations are inhibited). Following Dotan and Ravid (1985, p.505), we model these costs as a fixed number, \(B_j \geq 0, j \in \{M, S\}\). Who bears the bankruptcy cost? Under the absolute priority rule, most of the firm’s value in the event of bankruptcy is transferred to its lenders. Since the cost of bankruptcy diminishes the value of the firm, most bankruptcy costs are ultimately borne by

\(^4\)A derivation under more general assumptions is presented in an earlier version of this paper, which is available from the authors.
the firm’s lenders. However, knowing that they would have to bear a potential loss, lenders adjust the interest rate and, therefore, increase the firm’s borrowing costs. (For an additional discussion on bankruptcy costs, see Hillier et al., 2008, §16.) With $c_S = c_M = 0$, $\Pi^M_U$ and $\Pi^S_U$, the unhedged assembler’s and supplier’s time $t_0$ payoffs, are as follows:

\[
\Pi^M_U = \begin{cases} 
\min \{q, D\} - q (w_U + S_2) - \rho_M (b_M) & \text{if } D > k^M = b_M + \rho_M (b_M), \\
0 & \text{otherwise}, 
\end{cases} 
\]  

(15a)

\[
\Pi^S_U = q (w_U - S_1) - \rho_S (b_S). 
\]  

(15b)

where $k^M$ is the value of demand, $d$, for which the end-of-period operating income is just sufficient to cover the promised payment to the lenders (if $d < k^M$, the assembler defaults due to insufficient demand). $\rho_M$ and $\rho_S$ respectively represent the total interest payable to $M$’s and $S$’s lenders who receive:

\[
\Pi^L_M = \begin{cases} 
b_M + \rho_M (b_M) & \text{if } D > k^M \\
\left( \min \{q, D\} - B_M \right)^+ & \text{if } D \leq k^M, \\
0 & \text{otherwise} 
\end{cases} 
\]

and

\[
\Pi^L_S = b_S + \rho_S (b_S). 
\]  

(16)

As explained in Tirole (2006, §3.2.1), the total interest paid to the lenders, $\rho_M$ and $\rho_S$ respectively, must guarantee that expected payments, $\mathbb{E}_{t_0} \Pi^M_U$ and $\mathbb{E}_{t_0} \Pi^S_U$, equal the return obtained at the risk-free rate (assumed to be zero). It follows, then, that we must have $\mathbb{E}_{t_0} \Pi^L_M = b_M$ and $\mathbb{E}_{t_0} \Pi^L_S = b_S$, which are used to determine the equilibrium values for $\rho_M$ and $\rho_S$. (It follows that $\rho_S = 0$.)

By risk-neutrality, for a given wholesale price, $w_U$, the time $t_0$ assembler’s optimal order quantity, $q$, maximizes its time $t_0$ expected profit given by:

\[
\mathbb{E}_{t_0} \Pi^M_U (q) = \mathbb{E}_{t_0} \min \{q, D\} - q (w_U + \mathbb{E}_{t_0} S_2) - \mathbb{E}_{t_0} C_M (b_M), \quad \text{where} \quad C_M (b_M) := \rho_M (b_M) F \left[ k^M (b_M) \right] + \int_0^{k^M (b_M)} (x - b_M) f(x) dx \geq 0. 
\]  

(17a)

(17b)

The derived total cost of financing, $C_M$, consists of two components: The first component, $\rho_M (b_M) F \left[ k^M (b_M) \right]$, is cost as it represents the interest payment in the case that $M$ does not go bankrupt; the second component is a benefit as it represents the total amount of loan proceeds that $M$ does not
pay back to the lenders in the case of bankruptcy. (In the absence of bank financing $C_M(b_M) = 0$.)

The supplier maximizes $E_{t_0} \Pi^S_U(q) = q(w_U(q) - E_{t_0}S_2)$, where $w_U(q)$ is inverse demand. The next Lemma 4 asserts that if the distribution of demand has an increasing failure rate (see Lariviere and Porteus, 2001, p. 295), then $C_M$, is convex increasing.

**Lemma 4.** If demand, $D$, is an IFR r.v., then $C_M(\cdot)$ given by (17b) is convex increasing.

Using Lemma 4, if $D$ is IFR, then the assembler’s payoff, $\Pi^M_U(q)$, is concave in $S_2$. The supplier’s payoff, $\Pi^S_U(q)$ is linear in $S_1$. For the assembler’s expected payoff, Equation (17a) and Jensen’s inequality (see Shreve, 2004, proposition 1.3.4) imply $E_{t_0} \Pi^M_U(q, S_2) \leq E_{t_0} \Pi^M_U(q, E_{t_0}S_2)$. For the supplier’s payoff, Jensen’s inequality implies $E_{t_0} \Pi^S_U(q, S_1) = E_{t_0} \Pi^S_U(q, E_{t_0}S_1)$. Taken together, these readily imply the following result.

**Proposition 3.** The leveraged assembler, $M$, will hedge by going long $q$ futures contracts at time $t_0$. The leveraged supplier, $S$, will be indifferent towards hedging.

### 6 Conclusion

The production processes of many firms depend on raw materials whose prices can be highly volatile (e.g., Carter et al., 2006; Ziobro, Oct 24, 2011). Moreover, many firms who purchase raw materials also depend on suppliers who purchase raw materials for their own production. Of course, suppliers experience price volatility, too, and they may request price increases and surcharges. In some cases, if commodity prices significantly increase, the suppliers may not be able to fulfill contractual requirements, causing a breakdown in the supply chain. It is possible to find examples of this phenomenon from various industries, including auto parts, e.g., Hakim, 2003; Matthews, 2011; food-processing, e.g., Bjerga, 2013; Eckblad, 2012; energy and utilities, e.g., Thakkar, 2013; and heavy manufacturing, e.g., Matthews, 2011. It is therefore important to study how firms can manage this volatility. Specifically:

1. When should firms hedge their stochastic input costs?
2. Should they hedge fully or partially?
3. Do the answers depend on whether the firms’ supply chain partners hedge?
To answer these questions, we consider a simple supply chain model – the ‘selling to the newsvendor’ model (Lariviere and Porteus, 2001) – and generalize it by assuming that both the upstream and the downstream firms face stochastic production costs. (In contrast to our paper, in Lariviere and Porteus, 2001, production costs are linear with constant coefficients.) The stochastic costs could represent the raw material costs. We show that the stochastic costs reverberate through the supply chain and will be, ex ante, impounded into the wholesale price. In some cases, if input costs significantly increase, one of the supply chain members may not be able to fulfill its contractual requirements, causing the entire supply chain to break down. We identify conditions under which the risk of the supply chain breakdown and its impacts on the firms’ operations will cause the supply chain members to hedge their input costs: (i) The downstream buyer’s market power exceeds a critical threshold; or (ii) the downstream firm operates on a small margin, there is a high baseline demand for its final product, and its market power is below a critical threshold. To sustain hedging in equilibrium, both firms must hedge and supply chain breakdown must be costly. The equilibrium hedging policy will (in general) be a partial hedging policy. As an extension, we consider the case when firms’ operations are financed with borrowing and show that hedging can be profitable even in the absence of breakdown risk. Here, the equilibrium hedging policy is a full hedging policy.

Example of the condition (i) already given in the §1, where describe a situation in which an appliance manufacturer sells garbage disposals through a large home improvement store. As an example of the condition (ii), we may use Eastman Kodak Co., which manufacturers digital cameras, a product for which there has been a considerable demand as consumers are moving away from traditional film cameras. However, there is also considerable competition in this market and we can conjecture that margins are small. Silver is an important raw material used to make film, screens, batteries, and digital cameras. Moreover, the price of silver can be highly volatile. Compatible with our results (see Part ii of Proposition 2) Eastman Kodak Co. hedges its silver purchases (see Cui and Mattioli, 2011).

The analysis presented in this paper can be generalized in several ways. First, we assumed the selling to the newsvendor, in which the downstream firm has no pricing power. One may examine how the paper’s results would change if the downstream firm were to compete for demand by setting prices. Second, future research may include supply competition in the model and examine if competition can induce hedging. Finally, future research may consider hedging strategies, in
which firms purchase options rather than futures contracts.

References


Dotan, A., S. A. Ravid. 1985. On the interaction of real and financial decisions of the firm under


Appendices

A Sample Penalty Clauses

Taken from the Contract between Plymouth Rubber Company, Inc. (the Buyer) and Kleinewefers Kunststoffanlagen GmbH (the Supplier) (Source: Plymouth’s July 17, 1997, 10-K filing).

§10.1: If due to the responsibility of the Supplier components have not been delivered at the relevant dates according to Article 4.1, the Supplier shall be obliged to pay the Buyer penalty that shall not exceed 5% of the contract price.

Taken from the Contract between Davita, Inc. (the Buyer) and Rockwell Medical Technologies, Inc. (the Supplier) (Source: Rockwell’s March 28, 2003, 10-K filing).

Failure to Perform Supply Obligation. In the event ROCKWELL is unable to fulfill DAVITA’S orders at any time during the Term of this Agreement, DAVITA may, as its sole and exclusive remedy, upon prior notice to ROCKWELL, seek other suppliers
to fill purchase orders for some or all Products. If DAVITA is required to purchase Products from a third party under this Section 8, ROCKWELL will provide DAVITA with a purchase credit equal to the difference, if any, in the then current purchase prices for the Products which ROCKWELL was unable to deliver and the purchase prices DAVITA is reasonably required to pay, including legitimate freight charges incurred, in order to obtain similar or equivalent products from a different supplier(s).

B Proofs

Proof of Lemma 1. For brevity, we’ll only show the derivation of the expression for $u_{H1}^S$ and $u_{H2}^S$ for the case when $t_1 = t_2$ and $N_j \leq q$, $j \in \{M,S\}$. The other cases follow in a similar manner. Suppose that the supplier and the assembler respectively are long $N_S$ and $N_M$ futures contracts to purchase commodity $i$ at time $t_i$, $i = 1, 2$. In order for the supplier to produce the following must hold:

1. The hedged supplier has sufficient cash to procure the required production inputs. That is:

$$q w + c_S + N_S (S_1 - F_1) \geq q S_1 \quad \text{if} \quad S_2 \leq u_{H1}^M,$$ (B.1a)

$$\min \left\{ L_M, w q, (c_M + N_M (S_2 - F_2))^+ \right\} + c_S + N_S (S_1 - F_1) \geq q S_1 \quad \text{if} \quad S_2 > u_{H1}^M.$$ (B.1b)

2. The hedged supplier stands to earn a higher expected payoff by producing than by defaulting on the supply contract, selling commodity 1 on the open market and incurring the default penalty $\max \{-L_S, -N_S (S_1 - F_1) - c_S\}$. That is:

$$N_S (S_1 - F_1) + q w - q S_1 \geq \max \{N_S (S_1 - F_1) - L_S, -c_S\} \quad \text{if} \quad S_2 \leq u_{H1}^M,$$ (B.2a)

$$N_S (S_1 - F_1) + \min \left\{ L_M, w q, (c_M + N_M (S_2 - F_2))^+ \right\} - q S_1 \geq \max \{N_S (S_1 - F_1) - L_S, -c_S\} \quad \text{if} \quad S_2 > u_{H1}^M.$$ (B.2b)

Together, Conditions (B.1a) and (B.2a) imply that the supplier will commit to production if and only if:

$$S_1 \leq \min \left\{ \frac{q w + c_S - N_S F_1}{q - N_S}, w + \frac{L_S}{q} \right\} \iff S_1 \leq u_{H1}^S.$$
Similarly, Conditions (B.1b) and (B.2b) imply that the supplier will commit to production if and only if:

$$S_1 \leq \min \left\{ \frac{\min \left\{ L_M, wq, (c_M + N_M (S_2 - F_2))^+ \right\}}{q} + L_S, \frac{\min \left\{ L_M, wq, (c_M + N_M (S_2 - F_2))^+ \right\} + c_S - N_SF_1}{q - N_S} \right\} \iff S_1 \leq u^s_{H2}.$$

The proof is analogous for the assembler.

*Proof of Proposition 1.* As we explain in Section 3, as a guarantor, the futures exchange must organize trading in futures contracts so that defaults are completely avoided. This is accomplished by only accepting futures contract positions $0 \leq n_j \leq q$, that the exchange knows each firm $j$ can sustain without default for all for all $0 \leq S_1 < \infty$, $0 \leq S_2 < \infty$, and $d \leq D \leq \tilde{d}$. To establish the bounds on $n_j$ given in (8), we begin by defining $\bar{s}_1$ and $\bar{s}_2$ respectively as values of $S_1$ and $S_2$ for which:

$$q (w_H - S_1) = -L_S \quad \text{and} \quad d (1 - S_2)^+ - q w_H = -L_M.$$

The proof now proceeds in cases.

**Case 1:** $\bar{s}_1 < S_1 < \infty$, $0 \leq S_2 < \infty$. To avoid the assembler’s default on the futures contract, the following must hold for all $(S_1, S_2)$: $L_S + n_M (S_2 - F_2) \geq 0$ and $q (w_H - \bar{s}_1) + n_S (\bar{s}_1 - F_1) \geq 0$. This gives $\frac{q L_S}{L_S + q (w_H - F_1)} \leq n_S$ and $n_M \leq \frac{L_S}{F_2}$.

**Case 2:** $0 \leq S_1 \leq \bar{s}_1$, $0 \leq S_2 \leq \bar{s}_2$. To eliminate the risk that either the assembler or supplier default, the following must hold for all $(S_1, S_2)$: $q (w_H - S_1) + n_S (S_1 - F_1) \geq 0$ and $\min \{q, D\} (1 - S_2)^+ - q w_H + n_M (S_2 - F_2) \geq 0$. This gives $n_S \leq \frac{q w_H}{F_1}$ and $n_M \leq \frac{d - q w_H}{F_2}$.

**Case 3:** $0 \leq S_1 \leq \bar{s}_1$, $\bar{s}_2 < S_2 < \infty$. To avoid the supplier’s default on the futures contract, the following must hold for all $(S_1, S_2)$: $L_M + n_S (S_1 - F_1) \geq 0$ and $\min \{q, D\} (1 - \bar{s}_2)^+ - q w_H + n_M (\bar{s}_2 - F_2) \geq 0$. This gives $n_S \leq \frac{L_M}{F_1}$ and $\frac{d L_M}{d (1 - F_2) + L_M - w_H q} \leq n_M$.

Together, Cases 1–3 give (8).
Proof of Lemma 2. In Lemma 1, Condition 1 ensures that at \( S_1 = 0 \) and \( S_2 = 0 \) (see Cases 1 and 3 in the proof of Proposition 1):

\[
\frac{q \cdot w_H}{F_1} \leq \frac{L_M}{F_1} \quad \text{and} \quad \frac{d - q \cdot w_H}{F_2} \leq \frac{L_S}{F_2}.
\] (B.3)

Condition 2 ensures that:

\[
\frac{q \cdot L_S}{L_S + q \cdot (w_H - F_1)} \leq \frac{q \cdot w_H}{F_1} \quad \text{and} \quad \frac{d \cdot L_M}{d \cdot (1 - F_2) + L_M - w_H q} \leq \frac{d - q \cdot w_H}{F_2}.
\] (B.4)

Together (B.3) and (B.4) imply that the range for \( n_j, j \in \{M, S\} \) given in (8) is non-empty. \( \square \)

Proof of Lemma 3. (i) Let \( h_Y(\cdot) \) and \( h_Z(\cdot) \) be the marginal densities of \( S_2 \) and \( S_1 \). Equations (5) and (10) give:

\[
w_H(q) = \min \left\{ \frac{(1 - F(q)) \int_0^q \int_0^1 (1 - y) \cdot h_Y(y) \cdot h_Z(z) \, dy \, dz}{\mathbb{P}\{S_1 \leq \infty\} \cdot \mathbb{P}\{S_2 \leq \infty\}}, \frac{d \cdot (1 - F_2)}{q} \right\} = \min \left\{ \frac{(1 - F(q)) \int_0^q \int_0^1 (1 - y) \cdot h_Y(y) \cdot h_Z(z) \, dy \, dz}{\mathbb{P}\{S_2 \leq 1\}}, \frac{d \cdot (1 - F_2)}{q} \right\} \leq \frac{(1 - F(q)) \int_0^{u_H^M(q,q)} \int_0^1 (1 - y) \cdot h_Y(y) \, dy \, dz}{\mathbb{P}\{S_2 \leq u_H^M\}} = w_U(q).
\]

(ii) Equations (6) and (12) give:

\[
\mathbb{E}_{t_0} \Pi_U^M (q) = \int_0^{u_H^M(q)} \int_0^q \int_0^{u_H^M(x,q)} x \cdot (1 - y) \cdot f(x) \cdot h_Y(y) \cdot h_Z(z) \, dy \, dx \, dz \\
\leq \max \left\{ \int_0^\infty \int_0^q \int_0^{u_H^M(x,q)} x \cdot (1 - y) \cdot f(x) \cdot h(y,z) \, dy \, dx \, dz, \int_0^\infty \int_0^q \int_0^{u_H^M(x,q)} q \cdot (1 - y) \cdot f(x) \cdot h(y,z) \, dy \, dx \, dz \\
+ \int_0^\infty \int_0^q \int_0^{u_H^M(x,q)} (1 - y) \cdot f(x) \cdot h(y,z) \, dy \, dx \, dz - \frac{d \cdot (1 - F_2)}{q} \right\} = \mathbb{E}_{t_0} \Pi_H^M (q).
\]

(iii) Part (ii) asserts \( \mathbb{E}_{t_0} \Pi_U^M (q) \leq \mathbb{E}_{t_0} \Pi_H^M (q) \) for all \( q \in H \). Moreover, it follows that there exist an order quantity \( q_M \) for which \( \mathbb{E}_{t_0} \Pi_H^M (q_M) = \mathbb{E}_{t_0} \Pi_U^M (q_M) \) and \( \mathbb{E}_{t_0} \Pi_U^M (q) \leq \mathbb{E}_{t_0} \Pi_H^M (q) \) for all \( \hat{q}_M \leq q \in H \).

(iv) \( \frac{q \cdot (1 - F(q))}{(1 - F_2)} \cdot \int_0^\infty \int_0^1 (1 - y) \cdot h(y,z) \, dy \, dz \leq \frac{d}{q} \) implies \( \int_0^\infty \int_q \int_0^1 (1 - y) \cdot f(x) \cdot h(y,z) \, dy \, dx \, dz \leq \frac{d \cdot (1 - F_2)}{q} \).
\[ q \int_0^{u_M} (1-y) f(x) h(y, z) dy dy - q \int_0^{u_M} z h(y, z) dz dy \]

Equations (12), (13), and (7) give:

\[ E_{t_0} \Pi_U^S(q) = q \int_0^{u_M} (1-y) f(x) h(y, z) dx dy + \int_0^{u_M} z h(y, z) dz dy \]

Proof of Proposition 2. (i) Since the set \( H_M \) is non-empty, then there exists \( q \in H \) such that \( E_{t_0} \Pi_U^M(q_M^*) < E_{t_0} \Pi_H^M(q) \) and \( R_S = 0 \leq E_{t_0} \Pi_H^S(q) \). Because \( E_{t_0} \Pi_U^M(q_M^*) \) is the highest payoff the assembler can achieve under the unhedged contract and \( R_M \geq E_{t_0} \Pi_U^M(q_M^*) \), then the supplier must choose a hedged contract with \( q \in (\hat{q}_M, \bar{q}] \subset H_M \), where \( E_{t_0} \Pi_H^M(q_M^*) = E_{t_0} \Pi_U^M(q_M^*) \), which is condition (14b). (The last claim follows because \( E_{t_0} \Pi_H^M(q) \), given by (12), is strictly increasing in \( q \).)

(ii) \( H_S \) is non-empty implies that there exists some \( q \in H \) such that \( E_{t_0} \Pi_U^S(q^*_U) < E_{t_0} \Pi_H^S(q) \).

Since \( q_H^* \in H \) and \( q_H^* = \arg \max_{q \geq 2} E_{t_0} \Pi_H^S(q) \), then \( E_{t_0} \Pi_U^S(q_U^*) < E_{t_0} \Pi_H^S(q_H^*) \). Now, \( R_M = 0 \)
\[ \Rightarrow E_{t_0} \Pi_U^S(q_U^*) = E_{t_0} \Pi_U^S(q_U^*) \text{ and } E_{t_0} \Pi_H^S(q_H^*) = E_{t_0} \Pi_H^S(q_H^*) \].

Therefore \( E_{t_0} \Pi_H^S(q_H^*) \) is condition (14a).

Proof of Lemma 4. To show that \( C_M \) is increasing, differentiate (17b) once with respect to \( b_M \) to obtain

\[ (C_M)'(b_M) = -F \left| k_M(b_M) \right| + \int_{k_M(b_M)}^{a} (\rho_M)'(b_M) f(x) dx \]  \hspace{1cm} (B.5)

where we have used the fact that \( (b_M - k_M(b_M) + \rho_M(b_M)) = 0 \), which is (15a). The expression for \( (\rho_M)'(b_M) \) is obtained by implicit differentiation of the lender’s condition (see Equation (9) in Dotan and Ravid, 1985):

\[ E_{t_0} \left( b_M + \rho(b_M), \epsilon(D) \right) = b_M, \quad \text{where } \epsilon(D) = D - B. \]  \hspace{1cm} (B.6)

Differentiate (B.6) with respect to \( b_M \) and solve for \( (\rho_M)'(b_M) \). Next, substitute for \( (\rho_M)'(b_M) \) in
(B.5) and manipulate to yield

\[(C_M)'(b_M) = (b_M - \epsilon [k_M (b_M)] + \rho_M (b_M)) f [k_M (b_M)] (k_M)'(b_M). \]  

The above expression is positive because \(\epsilon [k_M (b_M)] < (b_M + \rho_M (b_M))\) and \((k_M)'(b_M) > 0\) (see Lemma B.1 below). To show that \(C_M\) is convex, differentiate (17b) twice with respect to \(b_M\) to obtain:

\[
(C_M)''(b_M) = \left( (b_M - \epsilon [k_M (b_M)] + \rho_M (b_M)) f [k_M (b_M)] \right)^2 
+ (1 - F [k_M (b_M)]) f [k_M (b_M)] (1 - \epsilon [k_M (b_M)] 
+ (1 - F [k_M (b_M)]) (b_M - \epsilon [k_M (b_M)] + \rho_M (b_M)) f' [k_M (b_M)] 
\left/ \left( (1 - F [k_M (b_M)]) - (b_M - \epsilon [k_M (b_M)] + \rho_M (b_M)) f [k_M (b_M)] \right)^3 \right). 
\]  

(B.8)

Now, the denominator of (B.9) is positive due to the following lemma.

**Lemma B.1.**

\[
(1 - F [k_M (b_M)]) - (b_M - \epsilon [k_M (b_M)] + \rho_M (b_M)) f [k_M (b_M)] > 0. \]  

(B.10)

**Proof of Lemma B.1.** To ensure that a loan is granted, we must have \((\rho_M)'(b_M) > -1\), which implies

\[
(k_M)'(b_M) = 1 + (\rho_M)'(b_M) > 0. \]

Therefore \((k_M)'(b_M) > 0\). Differentiating once both sides of (B.6) with respect to \(b_M\), substituting
\((-1 + (k_M)'(b_M))\) for \((\rho_M)'(b_M)\), and solving for \((k_M)'(b_M)\) yields:

\[
(k_M)'(b_M) = \frac{1}{(1 - F[k_M(b_M)]) - (b_M - \epsilon [k_M(b_M)] + \rho_M(b_M)) f[k_M(b_M)]}.
\]

Since \((k_M)'(b_M) > 0\), then right side of the above equation implies (B.10).

To show that the numerator of (B.9) is positive, note that IFR demand assumption implies
\[
f'[k_M(b_M)] > \frac{f[k_M(b_M)]^2}{1+F[k_M(b_M)]}.
\]
Using this assumption we have:

\[
\left( (b_M - \epsilon [k_M(b_M)] + \rho_M(b_M)) f[k_M(b_M)]^2 + (1 - F[k_M(b_M)]) f[k_M(b_M)] (1 - \epsilon' [k_M(b_M)]) \\
+ (1 - F[k_M(b_M)]) (b_M - \epsilon [k_M(b_M)] + \rho_M(b_M)) f'[k_M(b_M)] \right)
\geq \left( (b_M - \epsilon [k_M(b_M)] + \rho_M(b_M)) f[k_M(b_M)]^2 + (1 - F[k_M(b_M)]) f[k_M(b_M)] (1 - \epsilon' [k_M(b_M)]) \\
- (1 - F[k_M(b_M)]) (b_M - \epsilon [k_M(b_M)] + \rho_M(b_M)) \frac{f[k_M(b_M)]^2}{1 - F[k_M(b_M)]} \right)
= (1 - F[k_M(b_M)]) f[k_M(b_M)] (1 - \epsilon' [k_M(b_M)]) \geq 0,
\]
where last inequality follows due to \(\epsilon' [k_M(b_M)] \leq 1\). This implies that the numerator and the entire right side of (B.9) are positive. This establishes that \(C_M\) is convex.