

Persuasive Advertising and Product Line Design

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ABSTRACT

Firms design product lines to satisfy heterogeneous consumers. Apart from the core product attributes consumers gain utility through other marketing mix variables such as advertising that appeals to their emotional/social needs. An interesting question is whether a firm should use such advertising on all products it offers or exclusively target on a subset. Moreover how does such a choice affect the product design decision?

To explore this, a product line design model is constructed with persuasive advertising effect entering consumer's utility function. We first derive a monopolist's optimal choice of quality and prices without advertising and with advertising on the entire line or targeted exclusively on one product. Profit comparison indicates that it is better for the monopolist to use persuasive advertising on the entire product line. We then study a duopolist's decisions in managing both the cannibalization and inter firm competition. Under moderate competitive intensity, we find that firms tend to pursue a targeted advertising strategy. Our analysis illustrates how firms should use multiple marketing mix variables (product, advertising and pricing) to handle multiple strategic issues (cannibalization, competition) simultaneously and how choice in one dimension constrains or enhances another. The study also generates hypotheses that can be empirically tested with data on advertising spending, advertising content, pricing and sales from durable product categories such as automobiles.

Key words: persuasive advertising, product line design, multiple marketing strategies, competitive strategies, game theory

INTRODUCTION

Quality-based segmentation is common in many durable product categories such as automobile, appliances, consumer electronics, clothing and even internet services. We use the term quality in this paper to connote a combination of attributes exhibiting “more-is-better” property. Under this strategy, firms provide a line of related products or services to cater to consumers with heterogeneous preferences and incomes that often translate in differences in the marginal valuation for quality. In the automobile industry, for example, Toyota markets luxury cars under the Lexus brand name and economy and mid priced cars under the Toyota name and recently added Scion to attract younger segment of consumers. Similarly in the clothing industry Gap Inc uses the parent brand to sell casual clothes to shoppers in their 20s and 30s, Old Navy for targeting teens and moms shopping for families, Banana Republic as the luxury brand and more recently using Forth & Towne for baby-boomer women. Higher-price, higher-quality products are targeted at consumers who care more about quality and are willing to pay incrementally higher prices for the added quality. Lower-price, lower-quality products are meant for consumers who value quality less. That in the absence of perfect discrimination offering such a menu of product quality-price pairs leads to second degree discrimination is well known and is preferred in general (yields higher profits) to mass marketing one single product to all consumers. This strategy requires the firms to set prices taking into account potential cannibalization resulting from the high end consumers’ incentive to trade down to lower-quality products. There are a variety of instruments and strategies firm can use to mitigate such an opportunistic behavior by the high end consumers. These include model changes, channel selection, branding, etc. In this paper we examine yet another strategy, viz., the use of persuasive appeals through advertising to enhance the charm of one or more products by directly increasing consumers’ preference towards the advertised product.

Advertising is regarded as “second to product development as the most important area to allocate incremental \$”.¹ Every year industries such as automobiles invest billions of dollars in various forms of advertising, trying to influence consumers to buy their product. Our focus is on those advertising strategies that affect consumers’ preference towards the advertised product as opposed to advertising that provides information about the existence of or describing what a product has. One such example could be the TV commercial “Eyes wide shut” for Lexus GS featuring a husband who pretends to be a philanderer to mislead his wife from the fact that he was driving the vehicle all night. The advertising hardly contains any information about product attributes, pricing or financing, instead, it aims to “persuade” potential buyers about the passion of owning this vehicle, the desirability of driving, etc. A

¹ “A Day in the Life of C-Level Executives”, Forbes.com.

typical advertisement for Toyota Corolla, on the other hand, usually provides detailed information about the vehicle's physical attributes, promotions and financing terms in effect at that time. This pattern seems to exist among other automobile companies as well.² We think these examples illustrate a general pattern of advertising strategies incorporated by automobile firms and firms in other durable product categories where firms provide products to satisfy consumers' needs while using advertising to affect their valuation towards the product.

Our objective in this paper is to explore the precise nature of the linkage among a firm's strategies along these dimensions. More specifically, if enhancing consumers' valuation towards a product through persuasive advertising is beneficial to a firm, should this activity always be conducted on its entire product line? If the implementation is exclusive or targeted on only one product is it related to demand effects and/or due to competitive effects? How does the choice of such advertising strategy interact with the design of product qualities? To explore this we first develop a benchmark model of a monopolist that designs products and engages in persuasive advertising that is either confined to one single product (exclusive or targeted) or is extended to the whole product line (uniform). Following this we consider a duopoly to explore the strategic rationale behind such choices.

We follow the existing literature and consider a market consisting of two consumer segments that differ in their willingness to pay for quality. By implementing persuasive appeals on a product (Lexus GS in the above example) a firm affects consumers' preferences that leads to a greater willingness to pay for this product. We derive the optimal prices, qualities and advertising strategies for a monopolist. We find that a monopolist has greater incentive to provide a product of higher quality with advertising than without. Further a monopolist prefers to use this advertising strategy for all products along the product line (uniform) rather than targeting the high end product (exclusive) or to no advertising at all.

We then consider a duopoly with each firm marketing two products. While between-segment heterogeneity is driven by the difference in marginal willingness to pay for quality, within-segment heterogeneity exists in the relative preference of these consumers to the two competing firms. Each firm has to manage not only the intra firm cannibalization but also inter firm competition. Under moderate competitive intensity, we find that firms tend to pursue a targeted advertising strategy. This is because a firm's advertising on the lower end of the product line while being costly also has a negative strategic effect on its high end product that arises due to inter brand competition. The loss, combined

² According to Austin Business Journal's study on Super Bowl ads, the TV commercial of Cadillac at 2003 Super Bowl resulted in a large gain in both preference and recognition by featuring a man admiring the upscale automobiles while riding on a subway train.

with the advertising cost, cannot be compensated by the increased value created at the low end which the firm is unable to capture fully because of the inter brand competition at the low end. Our analyses thus demonstrate that the selective use of advertising acts as an additional strategic tool in controlling product line cannibalization, together with other marketing mix variables when there is inter firm competition. We further extend our model to incorporate additional consumer heterogeneity in advertising receptiveness as well as brand sensitivity. The qualitative results of the base model are shown to be robust to these extensions. Finally we compare product specific persuasive appeals with brand specific advertising and provide managerial guidance for choosing the proper form of advertising in handling multiple strategic issues simultaneously.

The remainder of this paper is organized as follows. Next section discusses the relevant literature and positions our contributions. It is followed by the analytical results of monopoly and duopoly models. We conclude the paper discussing some extensions of the basic model and suggestions for future research.

BACKGROUND

This paper is mainly in the nexus of two streams of literature: product line design and pricing, and incorporating the effect of persuasive advertising in a demand model. Since we build on the extant work in these two streams, we discuss the important ways we borrow from and add to these streams.

Product Line Design and Pricing

Mussa and Rosen (1978) and Moorthy (1984) provide the theoretical framework and analyses of how product quality and pricing interact in a model of a monopolist facing a set of heterogeneous demanders that differ in their marginal willingness to pay for quality. They analyze the optimal choices of qualities and prices a monopolist would choose. An important result from these papers is that to mitigate the cannibalization problem (i.e., the incentive of high end consumers to trade down), except for the highest type consumers others may not get their efficient quality and some consumers may be priced out of the market. That is, there is a downward distortion in the quality provision to all but the consumers with the highest marginal willingness to pay. Katz (1984) and Moorthy (1988) extend these models to duopoly. Desai (2001) examines firm's price and quality decisions in a model with two dimensions of consumer heterogeneity, viz., quality valuation and taste preference. He also considers the cases of incomplete and complete market coverage. He finds that, unlike earlier papers, depending on consumer and firm characteristics, both efficient and inefficient quality provisions are possible in both monopoly and duopoly settings. Villas-Boas (1998) shows that in the presence of a retailer, the

distortion in the quality at the lower end will be even more due to the double marginalization in a bilateral monopoly. In Villas-Boas (2004), advertising affects the aggregate demand as consumers only consider buying the product upon receiving the advertisement. He finds that advertising cost will affect the optimal number as well as the quality levels of vertically differentiated products. We build on this literature by using similar framework as in these papers to model consumers' and firms' choices. However, we depart from the past literature by incorporating the effect of persuasive advertising and the choice of targeted vs. uniform use of advertising strategy in our model extending this stream in important ways. Thus, our contribution to this stream is in the introduction of other marketing mix variables and in particular examining the interaction between advertising and choice of quality-price levels.

Persuasive Advertising

It is commonly recognized that advertising can influence consumer behavior in different ways. There has been much debate about the informative role of advertising (e.g., Stigler [1961], Telser [1964], Nelson [1970, 1974], Stigler and Becker [1977]) and the persuasive role of advertising (e.g., Bain [1956], Comanor and Wilson [1974], Dixit and Norman [1978], Kotowitz and Mathewson [1979], Shapiro [1980], Schmalensee [1986], Becker and Murphy [1993]). The key factor that distinguishes persuasive effect from its informative counterpart is whether consumers' consumption utility could change through a change in the preference weight upon receipt of advertisement. Depending on which effect is hypothesized to be in effect, the impact of advertising on price elasticity can be different.³ A recent study investigating the dual role of advertising is Banerjee and Bandyopadhyay (2003). von der Fehr and Stevik (1998) further elaborate on the ideas of Becker and Murphy (1993) by classifying three different ways of modeling the effect of persuasive advertising on consumer preferences. They assume that a representative consumer's gross consumption utility can be expressed as $U_x^A = s - t(x, d)$, $U_x^B = s - t(1 - x, d)$ where $d \geq 0$ is a measure of the degree of product differentiation between the two competing firms (A, B), x is the consumer's location on a Hotelling line representing her ideal product and s is her reservation price. Persuasive advertising can either raise the degree of product differentiation (d), or move the consumer's ideal point (x) closer to the advertised product, or increase consumer valuation (s). We use the findings in this stream to incorporate the role of advertising in our model. Similar to von der Fehr and Stevik (1998) we hypothesize that advertising affects consumers' preference for quality positively. Thus, we assume that quality perceptions are unambiguous and same across consumers and that advertising enhances the value customers place on

³ Interested readers are referred to Bagwell (2005) for a comprehensive survey.

the quality attribute. We add to this stream by exploring whether it is optimal for a firm to adopt this strategy exclusively, i.e., for only part of its product line or for the entire product line and how such a choice might interact with the design of the product quality and the pricing decision. Henceforth we will use the terms “uniform” when the firm uses persuasive appeal on *all* its products and “exclusive” or “targeted” to denote the case when the firm uses it on only one product. Note that the use of the term targeting is product specific in our paper which is different from the conventional use of this term.

The strategic choice between uniform and targeted advertising campaign studied in this paper could also be linked to the “umbrella branding” stream of literature. Umbrella branding refers to a firm using an established brand name to position a new product through advertising and packaging. Wernerfelt (1988) using a signaling framework suggests that a false signal sent out via umbrella branding may damage the existing product and while there is a direct saving in communication cost there could be a significant indirect cost to umbrella branding. Wernerfelt (1990) argues that brand advertising creates value for consumers, since it enables them to signal through brand choice their types to each other. Our model captures some of the features in this literature when studying the question whether firm should use persuasive appeals covering different product qualities of the product line. In his model setting, Wernerfelt (1988) points out that it will be more costly to put an existing name on a new product. This helps us justify our cost model of advertising. We thus add to this literature by considering a product line and show that even in the absence of signaling arguments firms have incentives to treat products asymmetrically with image oriented persuasive appeals.

MODEL

We first develop a benchmark monopoly model to illustrate non competitive trade offs when a firm decides whether to adopt persuasive advertising and its interaction with product line design and pricing decisions. We then consider the competitive case.

Consumers

Our assumptions about the demand side are similar to what has been assumed in the literature (see Moorthy [1984], von der Fehr and Stevik [1998] for example)

- A randomly chosen consumer derives a net utility of $U = \theta q - P(q)$ where q is the index of quality, $P(q)$ is the price for product of quality q and θ is the consumer’s marginal willingness to pay for quality.⁴

⁴ Note in addition to physical attributes, the quality index q could also capture other benefits that may be conveyed through packaging, brand name, etc.

- Total mass of consumers in the market is normalized to one without loss of generality. Consumers belong to either one of two segments that differ in their valuation for quality – a fraction λ with marginal willingness to pay θ_H and the remaining $(1 - \lambda)$ with marginal willingness to pay θ_L . $\theta_H > \theta_L > 0$.
- A consumer will buy at most one unit as long as the net utility is non-negative.

The Monopolist

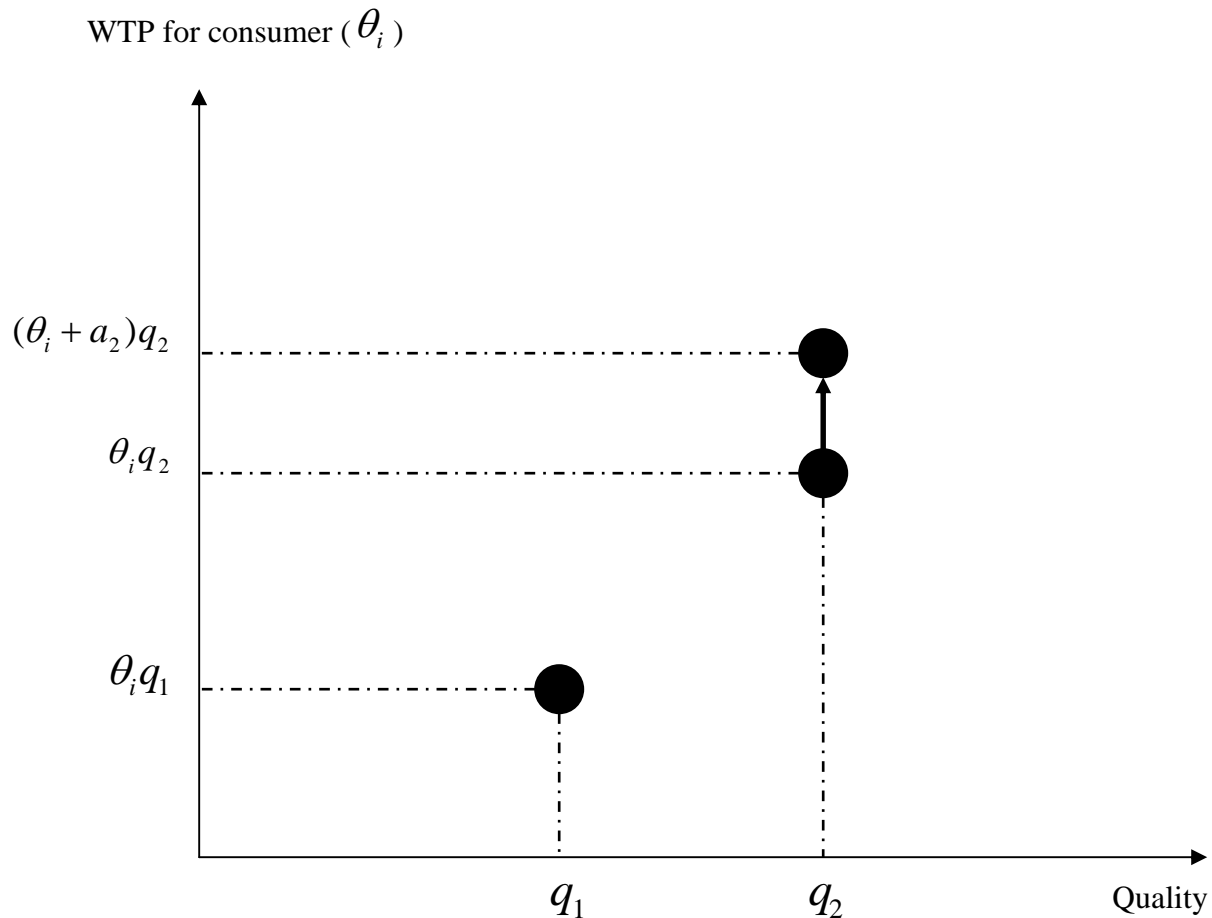
The firm incurs production cost and advertising cost when one is undertaken. Given that there are two segments of consumers, from standard arguments (see Moorthy [1988] for example) it follows that the firm would market and sell at most two products. Since we are interested in examining the conditions when the firm sells two products and the potential asymmetry in the choice on advertising, we assume that the parameters (λ, θ_H , and θ_L) are such that the firm's product line consists of a low and a high quality product denoted by (q_1, q_2) with $(0 <) q_1 < q_2$. The production cost of a good of quality q is assumed to be increasing and convex and given by $C(q) = q^2$. Facing the two consumer segments in the market, firm's product line design problem is to determine quality levels (q_1, q_2) and set prices (p_1, p_2) . We assume that firm knows the distribution of consumer types but cannot identify the type of a randomly chosen consumer. It is well known that if the monopolist can perfectly discriminate he will choose for segment i ($i=H, L$) the efficient quality $q_e = \frac{\theta_i}{2}$. This is referred to as the efficient quality level since this is what a social planner would choose equating the marginal willingness to pay to the marginal cost.

Advertising

The firm has technology to implement an advertising campaign for any product in its product line. To simplify matters, we assume that the firm conducts some base level of advertising informing consumers about the existence of the product and establishing the quality levels (q_1, q_2) in the minds of the consumers (as in Villas-Boas [2004]). We assume that while informative advertising does not affect preferences, persuasive advertising affects consumers' preference or taste for quality, viz., θ . As shown in figure 1, if a firm implements persuasive advertising for product of quality q_j ($j = 1, 2$) which has the effect of a_j , gross utility for consumer i ($i=H, L$) from this product becomes $(\theta_i + a_j)q_j$ where

a_j is the impact of advertising.⁵ We assume that the cost of inducing this affect a_j is $c(a_j) = a_j^2, j = 1, 2$, which implies that it costs more to generate greater advertising impact, and the incremental return of advertising spending is diminishing. Since the product line consists of a low and a high quality product, the firm should decide whether to use persuasive appeals uniformly or exclusively and the level of each ad campaign (a_j).⁶

Figure 1: Persuasive Advertising Affect Marginal WTP for q_2



Analysis

Depending on the spread between the quality valuations of the two segments and their relative size, the monopolist may or may not find it optimal to produce different quality levels and serve all the

⁵ We can think of the firm choosing an ad expenditure of \$x which through a concave function $f(x)$ translates to impact. The concavity of $f(x)$ implies diminishing return of ad spending. We start with the inverse of $f(x)$ which results in a variable instead of function entering consumer utility. The convexity of our advertising cost function ensures that our setting is equivalent to this conventional setup with significant reduction in the computational burden. Therefore, to say that the firm decides on the impact of ad campaign is the same as saying firm decides the corresponding advertising expenditure.

⁶ The persuasive advertising is product specific, and has no spill over effect across products. This is shown in figure 1 where persuasive advertising for product quality q_2 will not enhance consumer's valuation towards product quality q_1 .

consumers in the market. It is well known that in the standard adverse-selection model without advertising, it is optimal for monopolist to serve both consumer segments if $0 < \lambda < \frac{\theta_L}{\theta_H}$. As we will see

later this constraint may have to be modified when the firm uses advertising to enhance the appeal of its product. The monopolist chooses qualities and prices to maximize his profits and as can be seen from table 1, the high end segment gets its efficient quality while the quality for the low end segment is distorted downward from its efficient quality level.

We now consider the case where the monopolist uses persuasive advertising uniformly along the product line. Since we assume persuasive advertising is product specific, we consider this as two different campaigns, one generating affect a_2 for the high end product (e.g. Lexus GS) and the other generating affect a_1 for the low end product (e.g. Toyota Corolla) respectively.⁷

Monopoly Product Line Design with Uniform Persuasive Ad

Per our discussion above, we modify the utility function for a randomly chosen consumer who buys quality level q_j as

$$U_{ij} = (\theta_i + a_j)q_j - p_j, \quad j = 1, 2, \quad i = H, L \quad (1)$$

The monopoly's profit function now becomes

$$\Pi = \lambda(p_2 - q_2^2) + (1 - \lambda)(p_1 - q_1^2) - (a_2^2 + a_1^2) \quad (2)^8$$

The individual rationality and incentive compatibility constraints need to be modified to solve this problem. As in the standard case we proceed to solve by identifying the binding IC and IR constraints. Using the superscript U to connote the optimal levels when persuasive advertising is used for both high and low end products, we obtain the condition that the high end segment will be provided a higher quality when the marginal cost of advertising $a_2 > a_1$ is satisfied.⁹ Thus the advertising for the higher end product is more costly than for the lower end product. The optimal quality and profit levels (see appendix for the derivations and table 1 for the summary of monopoly results) are

$$q_2^U = \frac{2\theta_H}{4 - \lambda}, \quad q_1^U = \frac{2(\theta_L - \lambda\theta_H)}{(1 - \lambda)(3 + \lambda)} \quad (3)$$

⁷ Note we put no restriction on the relative magnitudes of a_1 and a_2 . In other words, they are not required to be the same or different.

⁸ Throughout this study, we focus on the case where market is completely covered.

⁹ This is the monotone likelihood ratio property in the standard adverse selection problem.

$$\Pi^U = \frac{\lambda\theta_H^2}{4-\lambda} + \frac{(\theta_L - \lambda\theta_H)^2}{(3+\lambda)(1-\lambda)} \quad (4)$$

A few observations can be made from the optimal quality levels. First it can be easily verified that $q_1^U > q_1^S$ and $q_2^U > q_2^S$,¹⁰ which implies that with persuasive ad campaigns for all products on the product line, monopolist will increase the quality levels he provides to both consumer segments. This leads to a rise in the average quality level of the product line, i.e. $\left[\frac{q_1^U + q_2^U}{2}\right] > \left[\frac{q_1^S + q_2^S}{2}\right]$. While the cannibalization problem still exists, it can be shown that quality gap between high and low end products gets enlarged compared to the case of no advertising, i.e., $(q_2^U - q_1^U) > (q_2^S - q_1^S)$. The interesting insight here is that due to persuasive advertising, the average quality increases and the breadth of the line also increases.¹¹

We next solve for the case when the firm uses targeted advertising on the high end product.¹² We call this advertising strategy as “targeting” on the high end product. Recall that when a firm uses persuasive advertising on a product a consumer’s willingness to pay for this product goes up while his willingness to pay for other product remains the same (see figure 1). This would seem to suggest that the monopolist may be better off using persuasive advertising exclusively for the high end product. To explore this we analyze the design of product line with targeted advertising on high end product alone.

Monopoly Product Line Design with Exclusive or Targeted Advertising

The utilities for a θ_H consumer from buying the high and low end products are:

$$U_H(q_2) = (\theta_H + a_2)q_2 - p_2, \quad U_H(q_1) = \theta_H q_1 - p_1 \quad (5)$$

Similarly we can write the corresponding utilities for a θ_L consumer as:

$$U_L(q_2) = (\theta_L + a_2)q_2 - p_2, \quad U_L(q_1) = \theta_L q_1 - p_1 \quad (6)$$

Monopolist’s profit function is:

¹⁰ Similar to the superscript U , we use the superscript S to connote the optimal quality and price levels when no persuasive advertising is implemented for either product, i.e. the no-ad case.

¹¹ An added observation is that the efficient quality levels for the two segments are now $q_{2e}^U = \frac{\theta_H + a_2}{2}$, $q_{1e}^U = \frac{\theta_L + a_1}{2}$ due to the increase of consumers’ marginal willingness to pay (θ) under advertising. Thus to measure the degree of quality distortion needed to manage the adverse selection in this model, the correct comparison ought to be $q_{je}^U - q_j^U$, $j = 1, 2$, rather than using the efficient quality levels in the previous case when there is no persuasive advertising.

¹² It is straightforward to show that advertising only the low end product is a dominated strategy as it will make the cannibalization problem more severe.

$$\Pi = \lambda (p_2 - q_2^2) + (1 - \lambda)(p_1 - q_1^2) - a_2^2 \quad (7)$$

We have to modify the earlier individual rationality and incentive compatibility constraints to take into account the targeting. The resulting optimal quality and profit levels are

$$q_2^T = \frac{2\theta_H}{4-\lambda}, \quad q_1^T = \frac{(\theta_L - \lambda\theta_H)}{2(1-\lambda)} \quad (8)$$

$$\Pi^T = \frac{\lambda\theta_H^2}{4-\lambda} + \frac{(\theta_L - \lambda\theta_H)^2}{4(1-\lambda)} \quad (9)$$

The derivations of the optimal prices and advertising levels are reported in the appendix and summarized in table 1 as well. Here we use superscript “ T ” to indicate optimal values under the targeted advertising case.

Table 1: Optimal Values from the Monopoly Model

	q_2	q_1	a_2	a_1
N	$\frac{\theta_H}{2}^\dagger$	$\frac{\theta_L - \lambda\theta_H}{2(1-\lambda)}$		
T	$\frac{2\theta_H}{4-\lambda}$	$\frac{\theta_L - \lambda\theta_H}{2(1-\lambda)}$	$\frac{\lambda\theta_H}{4-\lambda}$	
U	$\frac{2\theta_H}{4-\lambda}$	$\frac{2(\theta_L - \lambda\theta_H)}{(1-\lambda)(3+\lambda)}$	$\frac{\lambda\theta_H}{4-\lambda}$	$\frac{\theta_L - \lambda\theta_H}{3+\lambda}$

(\dagger This is the efficient quality for the θ_H segment)

	$p(q_2)$	$p(q_1)$	π
N	$\frac{\theta_H^2 - (\theta_H - \theta_L)(\theta_L - \lambda\theta_H)}{2(1-\lambda)}$	$\frac{\theta_L(\theta_L - \lambda\theta_H)}{2(1-\lambda)}$	$\frac{\lambda\theta_H^2 - 2\lambda\theta_H\theta_L + \theta_L^2}{4(1-\lambda)}$
T	$\frac{8\theta_H^2}{(4-\lambda)^2} - \frac{(\theta_L - \lambda\theta_H)(\theta_H - \theta_L)}{2(1-\lambda)}$	$\frac{\theta_L(\theta_L - \lambda\theta_H)}{2(1-\lambda)}$	$\frac{\lambda\theta_H^2}{4-\lambda} + \frac{(\theta_L - \lambda\theta_H)^2}{4(1-\lambda)}$
U	$\frac{8\theta_H^2}{(4-\lambda)^2} - \frac{2(\theta_L - \lambda\theta_H)(\theta_H - \theta_L)}{(1-\lambda)(3+\lambda)}$	$\frac{2(\theta_L - \lambda\theta_H)(4\theta_L - \lambda(\theta_H - \theta_L))}{(1-\lambda)(3+\lambda)}$	$\frac{\lambda\theta_H^2}{4-\lambda} + \frac{(\theta_L - \lambda\theta_H)^2}{(3+\lambda)(1-\lambda)}$

It is straightforward to see that $q_2^T = q_2^U > q_2^S$, $q_1^U > q_1^T = q_1^S$, which when combined with the result in uniform case, implies $(q_2^T - q_1^T) > (q_2^U - q_1^U) > (q_2^S - q_1^S)$. Using persuasive advertising only on the high end product leads to a greater quality gap along the monopoly product line making the low

quality product even less attractive for the high valuation consumers to deviate and buy. However, the following proposition shows that the monopolist will not prefer targeted advertising the high end product only.

Proposition 1: *It is optimal for the monopolist to implement persuasive advertisings for both high and low end products.*

Proof: see Appendix.

Figure 2: No Ad vs. Target vs. Uniform Ad (*Monopoly Model*)

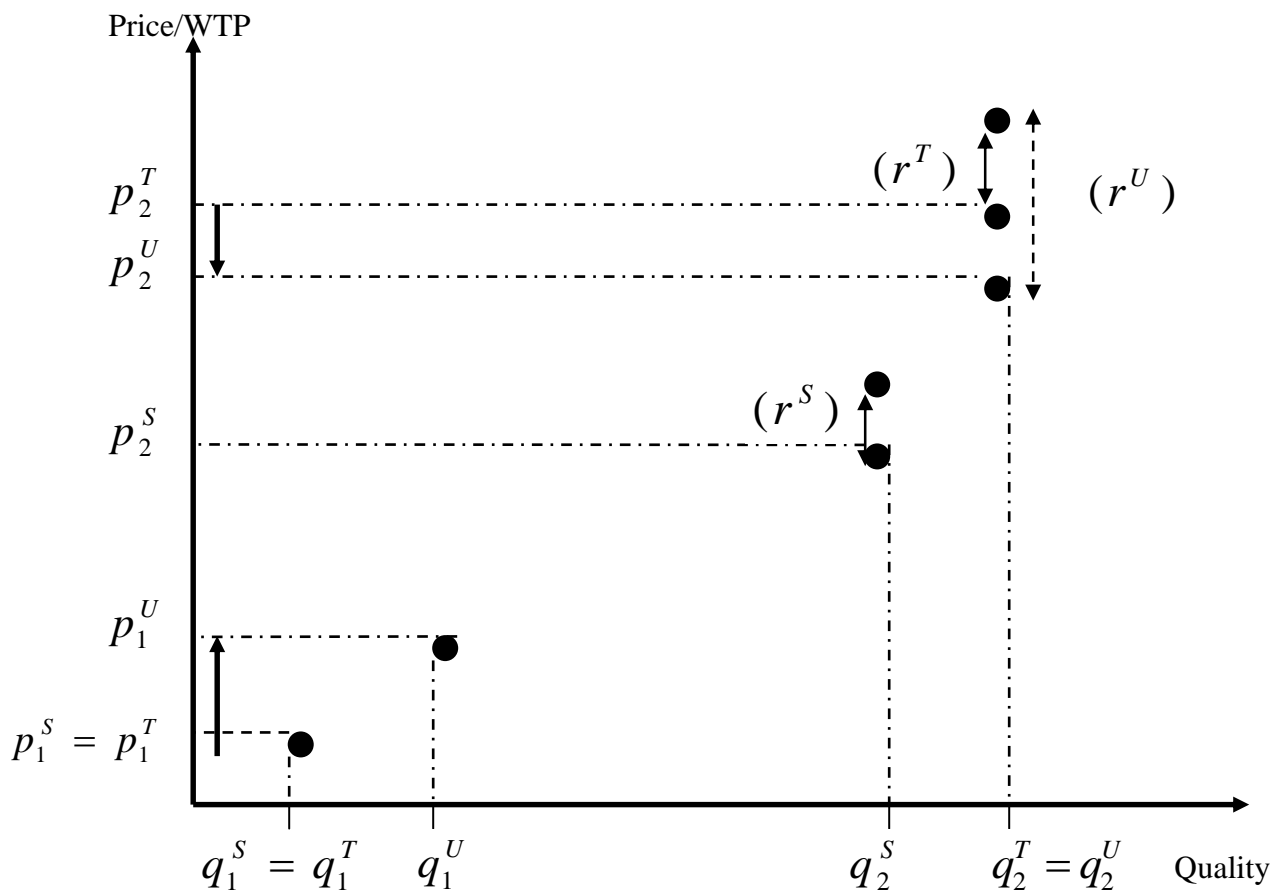


Figure 2 illustrates the trade offs the monopolist makes between targeted advertising on the high end product and uniform advertising on all products. In figure 2, r^U, r^T stand for the information rent given up by the monopolist to each θ_H consumer in the case of uniform and targeted advertising respectively. Note that while $q_2^T = q_2^U$, $p_2^T > p_2^U$ implying that in the targeting case the monopolist gives up less rent to θ_H consumers. Denote $p_2^{SM}, p_2^{TM}, p_2^{UM}$ as the perfect discrimination price that can be charged by monopolist to the high quality product in the cases of no advertising, target advertising and

uniform advertising respectively. Simple algebra shows that $p_2^{UM} - p_2^U > p_2^{TM} - p_2^T = p_2^{SM} - p_2^S$. Further $p_2^T - p_2^U = r^U - r^T$. In other words, the firm gives up same amount of surplus to the θ_H consumers in the standard (no ad) case and in targeting case which is less than what the firm gives up in the uniform case. What then, leads to the higher monopoly profit when implementing persuasive advertising for both high and low end products?

To answer this we need to realize that there are two sources of profits for the monopolist, one from each consumer segment. By creating more value at the lower end the monopolist is able to extract this surplus fully. This can be seen from figure 2. Note that $q_1^T < q_1^U$. Persuasive advertising on the low end product enhances consumers' willingness to pay for quality resulting in a choice of higher quality level provided for θ_L segment and greater profits from these customers. Comparing the prices across the three cases we see that $p_1^U > p_1^T = p_1^S$, which implies that the firm extracts highest surplus from each θ_L consumer in the uniform advertising case. The key here is to recognize that the return for a \$1 increase in the valuation of the θ_L consumers is greater than the return the monopolist obtains by increasing the valuation at the upper end segment. With sufficient number of θ_L consumers $(1-\lambda)$,¹³ the opportunity of completely capturing the surplus from the low end outweighs the relatively more information rent given up to the θ_H consumers plus the additional advertising cost. The combined profits from both market segments thus drive the monopolist to use persuasive advertising for both high and low end products. It is both the opportunity of complete value appropriation from the θ_L segment and the sufficient heterogeneity across the segments that make it optimal for the monopolist to use persuasive advertising along the entire product line. To summarize under the optimal strategy, the width of the product line increases relative to the no advertising case and the uniform advertising conveys a benevolent advantage on the high end consumers.

Table 2 reports the comparative static analysis for the monopoly results. Of particular interest is the effect of high end consumer segment size on monopoly's optimal product line design and advertising decisions. This is represented by parameter λ . As the size of high end consumer segment goes up, the complete value extraction from the low end segment becomes less and less significant. Firm will then focus more on the high valuation segment by providing better quality with greater advertising effort to

¹³ The discussion again emphasizes that sufficient heterogeneity among consumers (in terms of product valuation as well as segment size) is the driving factor for a firm to offer products of different qualities in the first place.

prevent the consumers from trading down. Meanwhile, given that advertising is costly and affects both segments, the firm will decide to cut the level at the low end.

Table 2: Monopoly Comparative Statics¹⁴

No Ad	q_1	p_1	q_2	p_2	a_1	a_2
θ_H	-	-	+	+		
θ_L	+	+	0	-		
λ	-	-	0	+		
r						

Target

θ_H	-	-	+	+		+
θ_L	+	+	0	-		0
λ	-	-	+	+		+
r	0	0	+	+		+

Uniform

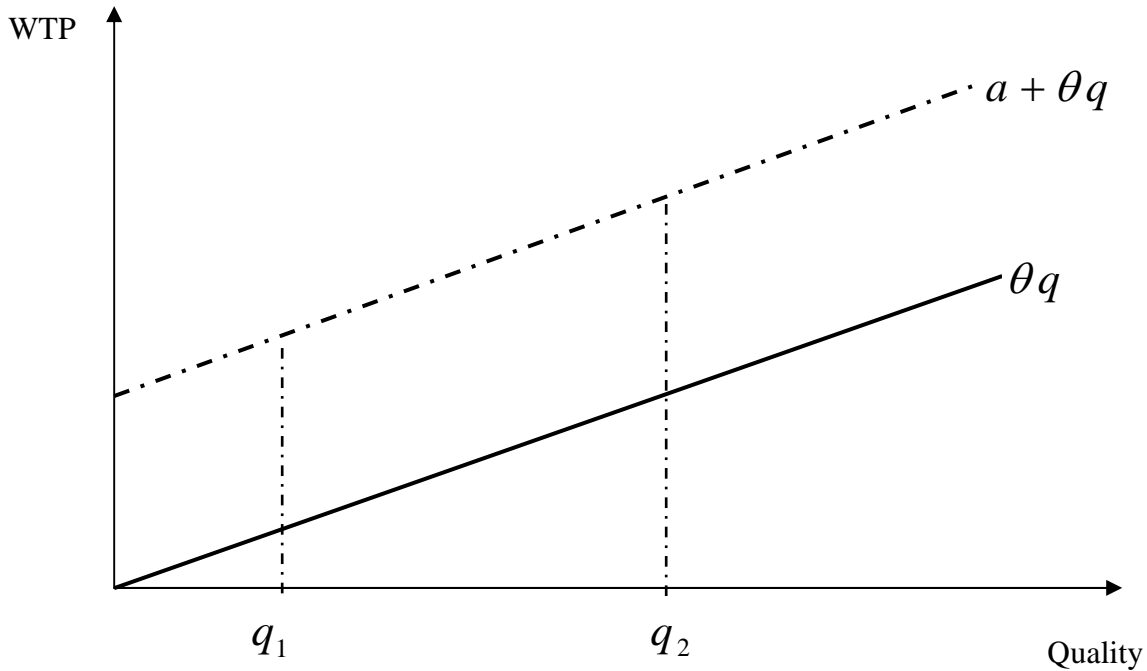
θ_H	-	-	+	+	-	+
θ_L	+	+	0	-	+	0
λ	-	-	+	+	-	+
r	-	-	+	+	-	+

Note that in our framework the high end segment's valuation of the low end product also increases in the uniform advertising case. In other words, we allow complete leakage of advertising messages across segments. Going through the similar solution procedure, it can be shown that if firm prevents the message leakage such that advertising becomes both product *and* segment specific, the optimal results would tilt more to the implementation of uniform advertising. A comment on the way we have modeled persuasive advertising is also worth making here. In our framework advertising affects how much

¹⁴ Note results in table 2 are based on the extended model where advertising has greater impact ($r > 1$) for high end consumer segments.

consumers are willing to pay for additional product quality. Another way of modeling is to assume that advertising affects the overall evaluation of the product through an intercept in consumer utility function. Referring to figure 3, this would lead to parallel shifting, instead of rotation of the willingness to pay function. The analysis still indicates the monopolist would prefer uniform advertising. However, the optimal quality levels are now the same across all three cases.¹⁵

Figure 3: Persuasive Advertising Affect Overall Valuation



Moving to the competitive environment, both intra firm cannibalization and inter firm competition now come into play. Will the firms still use persuasive advertising on the entire line? How does inter firm competition affect the advertising strategy we saw as optimal for a monopolist? How does consumer heterogeneity in brand preference and price sensitivity affect firms' strategic decisions in equilibrium? We examine these questions in a model of symmetric duopolists.

Duopoly

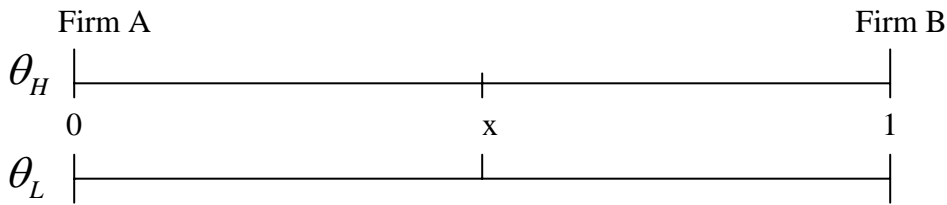
In this section we consider a model of two firms each offering a product line. We assume that each product line consists of only two products, a high quality and a low quality product. The firms have similar technology in production and advertising: as in the monopoly model marginal cost of production and advertising is product specific.

¹⁵ Derivations are available upon request from the authors.

As in the monopoly case, consumers are segmented in terms of marginal willingness to pay ($\theta_i, i = H, L$) for quality. Given that there are two firms we need to capture the consumers' preferences for these two firms in addition to the heterogeneity on product valuation. We therefore modify a representative consumer's utility function to account for her relative brand preference. We use the standard Hotelling (1929) line to model consumers' relative preference for two firms. As shown in figure 4, consumers within each segment are uniformly located on a $[0, 1]$ interval. We assume that the two firms (A, B) are located at the two ends of this line, representing the perceptual position of the two firms. A consumer's location x on the Hotelling line describes the consumer's ideal brand. A consumer from segment i located at x incurs a "psychic" cost or disutility of k per unit length traveled buying from either firm. A firm's choice variables are the two product qualities, two advertising levels (if the firm decides to advertise either product), and the two prices for the products. We denote the set of these strategic variables as $\{q_j^f, a_j^f, p_j^f\}$, $j = 1, 2$, $f = A, B$ where q stands for quality, a for advertising, p for price, f is the indicator variable for the firm and j is the indicator variable for the product. With these assumptions, the net utility for a consumer in segment i located at x if she were to purchase a product of quality q_j from firm A when it is advertised is:

$$U_i^A(q_j) = (\theta_i + a_j^A)q_j - kx - p_j^A, \quad i = H, L \quad (10)$$

Figure 4: Schematic Representation of the Competitive Market



The Game Structure and Sequence

We consider a three-stage game with the following sequence of moves by the duopolists:

Stage 1: the two firms simultaneously design quality levels for their product lines.

Stage 2: firms simultaneously decide on the advertising strategy, viz., whether to adopt persuasive advertising on the entire line or target on the high end only, and the corresponding levels of advertising.

Stage 3: firms set prices of all the products simultaneously.

Finally consumers decide which product to buy and from which firm after observing the quality-advertising-price choices made by both firms. This sequence captures the stickiness and the long term

implications of quality and advertising choices a firm makes. Reputations on these dimensions are enduring and even though we employ a static model, the game sequence captures the spirit of this.

The Advertising-Price Competition

We start the analyses by exploring the equilibrium advertising and pricing strategy in the game described above. Following quality choice in the first stage which we shall assume to be symmetric, in stage 2 each firm has three options in terms of advertising strategy, viz., no persuasive advertising (N), target persuasive appeal on high end product only (T), or use persuasive advertising for both and low end products (U). This generates a total number of nine possible strategic combinations in the advertising game (see table 3). Focusing on advertising equilibrium in pure strategies, we are particularly interested to see if the (U, U) constitutes Nash equilibrium, in other words, each firm advertises exactly the same way as if in the monopoly world. Or under certain conditions (T, T) will constitute Nash equilibrium, i.e., each firm targets persuasive appeal on the high end product only.¹⁶

Table 3: Normal Form of Duopoly Advertising Game

		B		
		N	T	U
A	N	$(\pi_A(N, N), \pi_B(N, N))$	$(\pi_A(N, T), \pi_B(N, T))$	$(\pi_A(N, U), \pi_B(N, U))$
	T	$(\pi_A(T, N), \pi_B(T, N))$	$(\pi_A(T, T), \pi_B(T, T))$	$(\pi_A(T, U), \pi_B(T, U))$
	U	$(\pi_A(U, N), \pi_B(U, N))$	$(\pi_A(U, T), \pi_B(U, T))$	$(\pi_A(U, U), \pi_B(U, U))$

We solve the game by the standard method of backward induction. We first derive the pricing strategies that would result in the third stage. To do this, we need to describe a randomly chosen consumer's choice given qualities, advertising and prices. For the purpose of illustration assume that both firms have decided to use targeted advertising in stage 2 and have decided on the level of advertising effect. Thus we are in the (T, T) cell of table 3.

Consider a consumer from the high valuation (θ_H) segment located at x_H . Suppose this consumer were to purchase firm A's high quality product, her net utility then needs to satisfy three constraints. The first one is the same as in the monopoly case, viz., the consumer has no incentive to buy the low quality product from firm A implying that her net utility from purchasing firm A's high end product

¹⁶ We assume complete coverage of the market where firms compete for both consumer segments and have positive demand for all the products he offers. .

should be greater than or equal to that from purchasing firm A's low end product. We denote this vertical incentive compatibility constraint due to firm A's own product line cannibalization as IC_A^{H-L} .

$$(\theta_H + a_2^A)q_2 - kx_H - p_2^A \geq \theta_H q_1 - kx_H - p_1^A \quad (11)$$

The second constraint is that the net utility from purchasing firm A's high end product should be greater than or equal to that from purchasing firm B's high end product. We denote this horizontal incentive compatibility constraint resulting from competition as IC_{A-B}^H .

$$(\theta_H + a_2^A)q_2 - kx_H - p_2^A \geq (\theta_H + a_2^B)q_2 - k(1-x_H) - p_2^B \quad (12)^{17}$$

Finally the third constraint arises by recognizing that the net utility from purchasing firm A's high end product should be greater than or equal to that from purchasing firm B's low end product. We denote this "cross firm" vertical incentive compatibility constraint as IC_{A-B}^{H-L} .

$$(\theta_H + a_2^A)q_2 - kx_H - p_2^A \geq \theta_H q_1 - k(1-x_H) - p_1^B \quad (13)$$

In a similar manner, we can write down the three incentive compatibility constraints for a consumer from the low valuation (θ_L) segment located at x_L along the Hotelling line if she were to purchase firm A's low quality product. Derivation of the equilibrium prices in the third stage thus involves simultaneously solving the two firms' profit maximization problems subject to the three types of incentive constraints specified above. The resulting expressions will be a function of the qualities as well as advertising decisions made by the firms in earlier stages. We can then move backward to the second stage of the game to simultaneously solve for the level of persuasive advertising in the (T, T) case, viz., when both firms decide to target persuasive appeal on the high end products only.

The analysis in the appendix indicates that there are only symmetric advertising equilibria following firms' symmetric quality decision. In table 4 we report the equilibrium advertising, price and profit levels for the three symmetric, viz., (N, N), (T, T) and (U, U) equilibria. Proposition 2 provides condition under which these equilibria would survive.

Proposition 2: *(N, N) is the unique advertising equilibrium when $k < k^*$; (T,T) is the unique advertising equilibrium when $k \in (k^*, k^{**})$; (U,U) is the unique advertising equilibrium when $k > k^{**}$ and*

$$k^* = \max\left\{\frac{6(\lambda\theta_H(q_2 - q_1) - \lambda q_2^2 - (1-\lambda)q_1^2) + \lambda^2 q_2^2}{6}, \frac{\lambda^2 q_2^2}{18}\right\},$$

¹⁷ Given the assumption of symmetric quality choice, there is no firm indicator in the quality variables.

$$k^{**} = \min\left\{\frac{6(\lambda\theta_H(q_2 - q_1) - \lambda q_2^2 - (1-\lambda)q_1^2 + \theta_L q_1) + \lambda^2 q_2^2}{9}, \frac{\lambda^2 q_2^2 + (1-\lambda)^2 q_1^2}{18}\right\}.$$

Proof: see Appendix.

Table 4: Equilibrium Results for the Duopoly Advertising-Price Game

	$p(q_2)$	$p(q_1)$
(N, N)	$\frac{6(k + (1-\lambda)\theta_H(q_2 - q_1) + \lambda q_2^2 + (1-\lambda)q_1^2)}{6}$	$\frac{6(k - \lambda\theta_H(q_2 - q_1) + \lambda q_2^2 + (1-\lambda)q_1^2)}{6}$
(T, T)	$\frac{6(k + (1-\lambda)\theta_H(q_2 - q_1) + \lambda q_2^2 + (1-\lambda)q_1^2) + \lambda(1-\lambda)q_2^2}{6}$	$\frac{6(k - \lambda\theta_H(q_2 - q_1) + \lambda q_2^2 + (1-\lambda)q_1^2) - \lambda^2 q_2^2}{6}$
(U, U)	$\frac{6(k + (1-\lambda)\theta_H(q_2 - q_1) + \lambda q_2^2 + (1-\lambda)q_1^2) + \lambda(1-\lambda)q_2^2 - (1-\lambda)^2 q_1^2}{6}$	$\frac{6(k - \lambda\theta_H(q_2 - q_1) + \lambda q_2^2 + (1-\lambda)q_1^2) - \lambda^2 q_2^2 + \lambda(1-\lambda)q_1^2}{6}$

	a_2	a_1	π
(N, N)			$\frac{k}{2}$
(T, T)	$\frac{\lambda q_2}{6}$		$\frac{18k - \lambda^2 q_2^2}{36}$
(U, U)	$\frac{\lambda q_2}{6}$	$\frac{(1-\lambda)q_1}{6}$	$\frac{18k - \lambda^2 q_2^2 - (1-\lambda)^2 q_1^2}{36}$

To understand the intuition behind proposition 2, it is important to note that as we move from the monopoly to the duopoly world, the biggest change is the inability of the duopolists to capture the entire surplus from the lower end of the market and a further reduced ability to capture what is available from the higher end market. The reason for the above is due to the horizontal competition at both ends of the product line. Under the duopoly setting, the per unit travel cost k measures the initial level of differentiation in the market. Intuitively, as k becomes sufficiently large a consumer incurs a much larger penalty for patronizing a product farther away. The two firms act like monopolists, which

leads to using persuasive advertising for both high and low end products.¹⁸ On the other hand, as k gets sufficiently small, consumers consider the products as undifferentiated. In such an extremely competitive market, any potential surplus due to persuasive advertising will be quickly competed away due to intense price competition, which gives neither firm an incentive to implement persuasive advertising.¹⁹ Between the two extreme conditions, proposition 2 indicates that advertising equilibrium will have each firm targeting the persuasive appeal on its high end product only.

As shown in the appendix, it is straightforward to see that given that the competitor is only using targeted appeal on the high end product, it does not pay for a firm to withdraw all advertising effort, i.e., moving from (T, T) to (N, T) or (T, N) (see table 3). This is because the competitive disadvantage on the high end product due to lack of advertising results in reduction in profit that is more than the advertising cost. What is left is to make sure that the firm has no incentive to use persuasive appeals for its entire product line given that the competitor is doing targeted advertising. And it turns out such a potential deviation will not pay off either because of the coexistence of intra firm cannibalization and inter firm competition as we explain below.

Suppose that firm A decides to use persuasive appeals for both high and low end products, given that B is only targeting on the high end product (i.e., from [T, T] to [U, T] in table 3), consumers' valuation towards firm A's low end product will be enhanced, which forces firm B to price more aggressively to overcome the disadvantaged position in the low end due to lack of advertising. Such an action would lead to loss in sales of its high end product due to cannibalization and firm B therefore has to lower its price on the high end product as well.²⁰ This in turn forces firm A to lower its price of its high end product. In this sense, the price competition at the low end has a strategically negative effect on the high end. Unlike in the monopoly case the increased consumer valuation on its low end product due to persuasive advertising cannot be fully captured by firm A. The loss from high end market plus the additional advertising cost cannot be covered by the incomplete surplus extraction in the low end market, which removes firm A's incentive to deviate from (T, T) and use persuasive advertising for both of his high and low end products.

¹⁸ Note that if k is too large, complete coverage of the market cannot be achieved as consumers located in the middle of the line (e.g. $x=1/2$) will have negative utility from purchasing either firm's product. But firms would still want to adopt persuasive advertising to the entire product line and sell to their monopoly territories. The intuition is the same as in monopoly model.

¹⁹ There is also a lower bound for k below which firms get negative profit margin from the low end product.

²⁰ It needs to be emphasized that according to the game structure when thinking about changing advertising strategy, e.g., from targeting to uniform, firm cannot adjust the product quality to control for cannibalization as the quality level is a decision made at an earlier stage of the game.

Full Equilibrium of the Duopoly Game

Proposition 2 characterizes the Nash equilibrium in the advertising-price subgame, while the subgame perfect Nash equilibrium (SPE) for the entire duopoly game involves the two competing firms' strategic decisions in each of the three stages, viz., quality, advertising and price. Unfortunately, the mathematical expressions become too complicated to track when we allow the quality choice in the first stage to be continuous and there is no closed form solution.

To obtain intuition on the full equilibrium, we model the duopoly game by restricting the quality choices in the first stage to one of two discrete values. Retaining the assumption of symmetric quality choice by the two firms, we further assume that at the high end, quality levels could only be either $\bar{q}_h = \alpha$ or $\bar{q}_h = \beta$. Similarly on the low end, quality levels could only be either $\bar{q}_l = \phi$ or $\bar{q}_l = \omega$. And $\alpha > \beta > \phi > \omega$. Now continuing the backward induction to stage 1, we are able to write down full equilibrium for the entire duopoly game in the following proposition.

Proposition 3: *When $k \in (k^*, k^{**})$, the unique symmetric subgame perfect Nash equilibrium (SPE) in pure strategies for the three-stage duopoly game is:*

Stage1: each firm produces $\bar{q}_h = \beta$ on the high end and $\bar{q}_l = \omega$ on the low end;

Stage2: Each firm targets persuasive appeal to the high end product with affect \bar{a}_h^ ;²¹*

Stage3: Each firm charges \bar{p}_h^ for the high end product and \bar{p}_l^* for the low end product.*

Proof: see Appendix.

Since firms are unable to capture the incremental surplus fully the correlation between persuasive advertising and quality offered is weaker in the duopoly case than in the monopoly case. This is established in proposition 3 which shows that although persuasive appeal is targeted on the high end product, firms do not choose to produce the highest feasible level of quality α .

To summarize, in the duopoly case each firm faces two complex tasks: managing own product line cannibalization while at the same time trying to mitigate inter firm competition. Price competition at the lower end has a strategic effect (apart from the direct cannibalization from the high end) that arises due to competitor managing its cannibalization problem. Based on the above analysis, the *selective use* of persuasive advertising is in fact a useful tool, together with other strategic variables (quality, price), for firms to fight the two enemies, viz., cannibalization and competition at the same time.

²¹ The expressions of critical k values, \bar{a}_j^* and \bar{p}_j^* can be obtained from table 4 in the appendix by substituting in the corresponding values of \bar{q}_1, \bar{q}_2 in the discrete quality case.

EXTENSIONS

Additional Heterogeneity

In the foregoing analysis between segment heterogeneity arises from the difference in the marginal willingness to pay (θ) for quality. We now check the robustness of our results to additional assumptions on heterogeneity in consumers' advertising receptiveness and brand preference.

Since persuasive advertising tends to emphasize imagery cues and often convey the higher status of owning the product, such kind of advertising could have a higher impact on consumers with higher income level.²² To account for this we incorporate a segment specific parameter r for the advertising effect in consumer's utility function:

$$U_{ij} = (\theta_i + r_i a_j) q_j - p_j, \quad j = 1, 2, \quad i = H, L \quad (14)$$

Based on the above rationale, we normalize $r_L = 1$ while $r_H > 1$.

Higher valuation consumers could also have stronger taste preference when choosing between the product lines of two competing firms. In other words, consumers from θ_H segment could be more brand sensitive. Following earlier papers (Iyer [1998], Deasi [2001] for example), we model this possibility by having the per unit "psychic" cost k positively correlated with consumer's marginal willingness to pay (θ). Under these additional assumptions, we can write down the net utility for a consumer in segment i located at x if she were to buy a product of quality q_j from A under persuasive appeal:

$$U_i^A(q_j) = (\theta_i + r_i a_j^A) q_j - k \theta_i x - p_j^A, \quad i = H, L \quad (15)$$

Our analysis indicates that the qualitative results of the basic model are preserved with the above two additional dimensions of consumer heterogeneity, with only some modification in the condition on the parameters that is required to ensure that the equilibria exists.

Brand versus Product Advertising

The persuasive advertising in our model affects consumers' marginal valuation on the advertised quality. Hence it is product specific as modeled in a representative consumer's utility function (see equation 1 for example). We can also compare persuasive advertising with another common type of advertising that builds up the firm's image in consumers' mind, i.e., the brand advertising. If a firm

²² The behavior literatures have well documented the heterogeneity in consumers' response to emotional appeals in advertising (e.g. Moore, Harris and Chen [1995]).

implements brand advertising for its product line which has the effect of a_f ²³, gross utility for consumer i ($i=H, L$) from buying a certain product of this firm becomes $a_f + \theta_i q_j$. Our interest here is the driving factor for firms' choice between implementing brand advertising across the product line and product-specific persuasive advertising.

Our analysis indicates that in addition to different cost structure and competitive intensity, firm's choice between brand and product advertising also depends on the extent of persuasive advertising spillover. As is shown in equation 10, our model assumes the extreme case of no spillover of advertising effect. In other words, firm B's advertising on its high end product will not enhance consumers' valuation towards firm A's high end product. This could be restrictive given the symmetric assumption on firms' quality choice at the first stage. The general case can be expressed in the following mathematical form:

$$U_i^A(q_j) = (\theta_i + a_j^A + \delta a_j^B)q_j - kx - p_j^A, \quad i = H, L \quad (16)$$

And the current model is in fact the polar case where $\delta = 0$ or sufficiently small. Intuitively, as the advertising spillover gets to larger extent, firms may hesitate to use persuasive advertising even when the market is under the relevant condition specified in proposition 2 or 3. As a result, they will resort more heavily to the brand specific advertising. This analysis thus highlights another important factor in managerial consideration of using proper form of advertising, along with other strategic variables to handle multiple strategic issues (cannibalization, competition) simultaneously.

CONCLUSIONS AND FUTURE WORK

In this paper, we model the influence of multiple marketing mix variables and focus on the interaction of persuasive advertising with product line design and pricing decisions. In a monopoly we find that the ability to completely appropriate increased surplus due to advertising effect from one market segment drives the monopolist to use persuasive advertising on both high and low end products even though that results in giving up a bit more surplus to the other market segment. In the duopoly case, we illustrate the link between cannibalization and competition through negative strategic effect on one market segment that arises due to the competition for the other market segment and transformed through product line cannibalization.

Our model's predictions can be tested by examining data from the automobile industry using leading manufacturers' advertising spending, pricing and sales figures on new luxury versus economy cars.

²³ Notice the brand advertising effect is firm specific which affects all products on the product line.

Based on the differences in advertising content, we can identify firm's choices on persuasive advertising. The following are some testable hypotheses:

- Firm tends to use targeted advertising in more competitive markets.
- In markets where a single firm is dominant (monopoly) advertising leads to widening of product line.
- Under targeted advertising, the price difference between high and low quality products is bigger than under uniform advertising case.
- If advertising spillover effects exist, firms are more likely to use brand image building than trying to enhance customers' valuation of one of its product relative to others in its product line.

Several directions are worth pursuing to continue the exploration began in this paper. We assume that the market size is fixed. Suppose the market is not fully covered or there is potential for market expansion, will it pay to use persuasive advertising to enhance existing consumers' valuation towards the product, or the input should be shifted on informative advertising to increase awareness of the product and enlarge the market base? Second, the duopoly results we derived are based on two symmetric firms each offering a product line. Interesting results may arise once we allow for asymmetries in technology, brand position or product specialization between the two competing firms. The research question then becomes, for example, what the advertising-price equilibrium would be if firm A, decides to be a niche of luxury/economy specialized manufacturer, while competing with firm B who offers a complete product line? Third, the model could further be extended to the situation where a particular end of a firm's product line also consists of multiple quality levels.²⁴ We believe that the study of firms' strategic decisions on multiple dimensions and the interaction between one and another is a rich and promising area for modelers.

²⁴ For example, within Toyota line, there are cars ranging from small Tercel to full size Avalon.

APPENDIX

Proof of Proposition 1

Monopolist's constrained profit maximization problems under the three cases, viz., no ad, targeted and uniform ad are constructed respectively in the following.

No-Ad case:

$$\max_{\{q_1, q_2\}, \{p_1, p_2\}} \Pi = \lambda \left(p_2 - (q_2)^2 \right) + (1 - \lambda) \left(p_1 - (q_1)^2 \right)$$

$$s.t. \quad \theta_H q_2 - p_2 \geq 0 \quad (IR_H)$$

$$\theta_H q_2 - p_2 \geq \theta_H q_1 - p_1 \quad (IC_H)$$

$$\theta_L q_1 - p_1 \geq 0 \quad (IR_L)$$

$$\theta_L q_1 - p_1 \geq \theta_L q_2 - p_2 \quad (IC_L)$$

Target case:

$$\max_{\{q_1, q_2\}, \{p_1, p_2\}, \{a_2\}} \Pi = \lambda \left(p_2 - (q_2)^2 \right) + (1 - \lambda) \left(p_1 - (q_1)^2 \right) - (a_2)^2$$

$$s.t. \quad (\theta_H + a_2)q_2 - p_2 \geq 0 \quad (IR_H)$$

$$(\theta_H + a_2)q_2 - p_2 \geq \theta_H q_1 - p_1 \quad (IC_H)$$

$$\theta_L q_1 - p_1 \geq 0 \quad (IR_L)$$

$$\theta_L q_1 - p_1 \geq (\theta_L + a_2)q_2 - p_2 \quad (IC_L)$$

Uniform case:

$$\max_{\{q_1, q_2\}, \{p_1, p_2\}, \{a_1, a_2\}} \Pi = \lambda \left(p_2 - (q_2)^2 \right) + (1 - \lambda) \left(p_1 - (q_1)^2 \right) - (a_1)^2 - (a_2)^2$$

$$s.t. \quad (\theta_H + a_2)q_2 - p_2 \geq 0 \quad (IR_H)$$

$$(\theta_H + a_2)q_2 - p_2 \geq (\theta_H + a_1)q_1 - p_1 \quad (IC_H)$$

$$(\theta_L + a_1)q_1 - p_1 \geq 0 \quad (IR_L)$$

$$(\theta_L + a_1)q_1 - p_1 \geq (\theta_L + a_2)q_2 - p_2 \quad (IC_L)$$

Note the individual rationality (IR) and incentive compatibility (IC) constraints are adjusted corresponding to different use of persuasive advertising.

Following the standard procedure in consumer adverse selection literature, we start the solution algorithm with the low valuation consumer's IR constraint binding, so is the high valuation consumer's

IC constraint, viz., IR_L and IC_H . This indicates the monopolist wants to extract full surplus from the low valuation segment, in the mean time, would not want to give more rents to the high valuation consumers than the minimum necessary for them to be indifferent between buying the high quality product designed for them and trading downwards for the low quality product. These two binding constraints yield the prices for the two products as functions of qualities, advertising (if in uniform or target cases) and other model parameters. In the uniform case, these are²⁵:

$$p_1^U = (\theta_L + a_1^U)q_1^U, \text{ and } p_2^U = (\theta_H + a_2^U)q_2 - (\theta_H - \theta_L)q_1^U \quad (\text{A1})$$

Substituting the above price expressions back to monopoly profit, we have

$$\begin{aligned} \Pi = & \lambda \left((\theta_H + a_2^U)q_2^U - (\theta_H - \theta_L)q_1^U - (q_2^U)^2 \right) + (1 - \lambda) \left((\theta_L + a_1^U)q_1^U - (q_1^U)^2 \right) \\ & - (a_1^U)^2 - (a_2^U)^2 \end{aligned} \quad (\text{A2})$$

Optimal qualities and advertising effects are obtained from solving the first-order conditions of the above profit function and check the negative semi-definite of the second order Hessian matrix. The optimal prices can then be derived by substituting the optimal quality and advertising back to the above equation A1. We need to check the satisfaction of the remaining two constraints, viz., IR_H and IC_L . In the standard adverse selection problem without advertising, these two constraints are non-binding under the monotone likelihood condition, viz., the optimal quality level $q_2 > q_1$. In the case of uniform advertising, there is an additional condition regarding the marginal cost of advertising different products. In particular, $a_2 > a_1$. All results are reported in table 1.

The optimal results in the other two cases (no ad, target) are derived via the similar procedure. Proposition 1 is proved through direct comparison of Π^S, Π^T and Π^U .

Proof of Proposition 2

Following the symmetric quality decision in the first stage, there are nine proper subgames corresponding to each possible action chosen by the firm in the second stage (see table 3). In the following we prove that (T, T) is the unique equilibrium in the advertising subgame under the specified condition. Proofs for the remaining part of proposition 2 follow similar procedure.

We start by looking at the two constraints resulting from horizontal competition, viz., IC_{A-B}^H and IC_{A-B}^L . From these we can identify a marginal consumer (x_H) in the high valuation segment who is indifferent between buying high quality product from firm A and from firm B. That is,

²⁵ Given the solution procedures are similar across the three cases, we use the case of uniform advertising to illustrate the solution procedure.

$$\begin{aligned}
(\theta_H + a_2^A)q_2 - kx_H - p_2^A &= (\theta_H + a_2^B)q_2 - k(1 - x_H) - p_2^B \\
\rightarrow x_H &= \frac{q_2(a_2^A - a_2^B) + (p_2^B - p_2^A) + k}{2k}
\end{aligned} \tag{A3}$$

Similarly from the low end market, we have $x_L = \frac{p_1^B - p_1^A + k}{2k}$. Clearly IC_{A-B}^H will be non binding for the high valuation consumers located to the left of x_H . Similarly, IC_{A-B}^L will be non binding for the low valuation consumers located to the left of x_L . Now consider the multi-product firm's own cannibalization constraint which is still in presence under competitive environment, viz., IC_A^{H-L}, IC_B^{H-L} .

These restrict the prices of high end product offered by the duopolists:

$$\begin{aligned}
(\theta_H + a_2^A)q_2 - kx_H - p_2^A &= \theta_H q_1 - kx_H - p_1^A \rightarrow p_2^A = \theta_H (q_2 - q_1) + a_2^A q_2 + p_1^A \\
(\theta_H + a_2^B)q_2 - k(1 - x_H) - p_2^B &= \theta_H q_1 - k(1 - x_H) - p_1^B \rightarrow p_2^B = \theta_H (q_2 - q_1) + a_2^B q_2 + p_1^B
\end{aligned} \tag{A4}$$

Substituting the above price expressions into the x_H yields firms' high end market shares, similarly for x_L . Also simple algebra can show that under the symmetric equilibrium, high valuation consumers' cross firm cannibalization constraints $IC_{A-B}^{H-L}, IC_{B-A}^{H-L}$ are not binding. Thus the profit functions for the duopolists can be rewritten as:

$$\begin{aligned}
\pi_A^T &= \lambda(\theta_H (q_2 - q_1) + a_2^A q_2 + p_1^A - q_2^2)x_H + (1 - \lambda)(p_1^A - q_1^2)x_L - (a_2^A)^2 \\
\pi_B^T &= \lambda(\theta_H (q_2 - q_1) + a_2^B q_2 + p_1^B - q_2^2)(1 - x_H) + (1 - \lambda)(p_1^B - q_1^2)(1 - x_L) - (a_2^B)^2
\end{aligned} \tag{A5}$$

Simultaneously solve for the above two profit maximization problems will give the price expressions for the low end products in (T, T) advertising subgame, which are functions of firms' advertising and quality levels. Substitute the price expressions back to the above (A5), we can then solve for the ad effect in (T, T), which is symmetric between two firms. Substituting back to price equation, we obtain the equilibrium price and profit levels under (T, T) reported in table 4. Notice the feasibility condition (e.g., marginal θ_L consumer's IR condition) yields the following parameter condition:

$$\frac{6(\lambda\theta_H (q_2 - q_1) - \lambda q_2^2 - (1 - \lambda)q_1^2) + \lambda^2 q_2^2}{6} < k < \frac{6(\lambda\theta_H (q_2 - q_1) - \lambda q_2^2 - (1 - \lambda)q_1^2 + \theta_L q_1) + \lambda^2 q_2^2}{9} \tag{A6}$$

The above derivation provides a candidate of ad equilibria: (T, T). For it to be an equilibrium, we need to ensure neither firm has incentive to deviate given the competitor adopts targeted advertising. For the purpose of illustration, we assume firm A is the one considering deviation. The potential deviations include not advertising at all (N) as well as advertising both high and low end products (U). Here we show the derivation of conditions under which firm A has no incentive to deviate from T to U given firm B is targeting. Ruling out the other option follows similar procedure.

Given firm B chooses to target persuasive advertising on the high end product at the level of a_2^T ,²⁶ if firm A uses persuasive appeals on the entire product line, the horizontal competition on the low end market results in:

$$(\theta_L + a_1^A)q_1 - kx_L - p_1^A = \theta_L q_1 - k(1 - x_L) - p_1^B \rightarrow x_L = \frac{a_1^A q_1 + p_1^B - p_1^A + k}{2k} \quad (\text{A7})$$

Solving for the price competition in stage 3 and then move back to the advertising stage, we get firm A's optimal advertising level on the low end product:

$$a_1^A(\text{Given } B[T]) = \frac{(1-\lambda)q_1}{6} \frac{18k - \lambda^2 q_2^2}{18k - \lambda^2 q_2^2 - (1-\lambda)^2 q_1^2} \quad (\text{A8})$$

Notice the RHS of (A8) is negative as long as

$$\frac{\lambda^2 q_2^2}{18} < k < \frac{\lambda^2 q_2^2 + (1-\lambda)^2 q_1^2}{18} \quad (\text{A9})$$

Under this condition, firm A will have no unilateral incentive to deviate from T to U. Combining (A6) and A(9) yields the range (k^*, k^{**}) shown in proposition 2 within which (T, T) is an advertising equilibrium.

What is left is the uniqueness proof which requires we check the other eight strategic profiles (see table 3) and make sure none of them can be sustained as advertising equilibrium under the range of (k^*, k^{**}) . We illustrate in the following the failure of (U, U) to be advertising equilibrium when $k \in (k^*, k^{**})$. The examination of other symmetric as well as asymmetric profiles follows similar path.

Under (U, U), both firms use persuasive advertising for the entire product line. Backward induction gives the levels of prices and advertising as reported in table 4. If firm A considers unilaterally deviating to T, profit comparison indicates

$$\Pi_A(\text{T given } B[\text{U}]) - \Pi_A(\text{U, U}) = \frac{q_1^2 (1-\lambda)^2 (18k - \lambda^2 q_2^2 - (1-\lambda)^2 q_1^2)}{(18k - \lambda^2 q_2^2)} \quad (\text{A10})$$

Note RHS of A(10) is positive as long as $\frac{\lambda^2 q_2^2}{18} < k < \frac{\lambda^2 q_2^2 + (1-\lambda)^2 q_1^2}{18}$ which falls into the range of (k^*, k^{**}) . Therefore, firm A will have incentive to deviate from U to T, which implies that (U, U) is not an equilibrium when $k \in (k^*, k^{**})$.

²⁶ It needs to be emphasized that firm B cannot adjust the advertising affect level as we are checking firm A's unilateral incentive to deviate from (T, T) in the second stage. But firm B can adjust its prices according to A's new advertising decisions because price is the choice made in the third stage *after* the advertising decision.

Proof of Proposition 3

This is done by continuing the backward induction to the first stage of quality decisions. Note under the symmetric assumption, there are only four quality equilibria candidates under the symmetry constraint, viz., $(q_1^f = \phi, q_2^f = \alpha)$, $(q_1^f = \omega, q_2^f = \alpha)$, $(q_1^f = \phi, q_2^f = \beta)$ and $(q_1^f = \omega, q_2^f = \beta)$, $f=A, B$. Details of derivation are similar to that of proposition 2.

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