

**DECOMPOSING PURCHASE ELASTICITY WITH A DYNAMIC STRUCTURAL  
MODEL OF FLEXIBLE CONSUMPTION**

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## ABSTRACT

It is well known that in package goods categories a temporary price cut of a brand leads to increase in the sales of that brand in the current period. Under the assumption of stable consumption rate, previous literature has identified that the sources for the increase in sales are brand switching, purchase acceleration, and increase in quantity. However, there are very few studies that have formally modeled the impact of price promotions on consumption when consumption rate in a category is not constant. In this paper we offer a methodology to decompose the effects of price promotions into brand switching, stockpiling and change in consumption and explicitly allow for consumer heterogeneity in brand preferences and consumption needs. A dynamic structural model of a household that decides when, what, how much to buy as well as how much to consume to maximize its expected utility over an infinite horizon is developed. By making certain simplifying assumptions we are able to reduce the dimensionality of the problem. We estimate the proposed model using a scanner panel data of 1000 households on canned tuna purchases for 12 product alternatives over a two-year period. The results from the model shed insights on the decomposition of the price elasticity into its components. This could help managers make inferences about which brands sales are most responsive to household stockpiling and consumption expansion as well as understand how temporary price cuts affect future sales. Contrary to previous literature, we find that brand switching is not the dominant force for the increase in sales. We also find that a household's brand preference has significant impact on its stockpiling and flexible consumption. More specifically, we show that brand loyalists respond to a price promotion mainly with stockpiling for future consumption while brand switchers do not stockpile at all. We also find that heavy users

stockpile more but their consumption rate does not increase as much as for the light users when there is a price promotion.

*Key words:* flexible consumption, decomposition of price elasticity, consumer heterogeneity, dynamic structural model.

## 1. INTRODUCTION

Modeling and understanding the demand for a product is at the core of applied marketing research. In the applied literature, following Guadagni and Little's (1983) seminal work, researchers have emphasized the need for and advantages of using individual level data to model demand. Subsequently research has evolved in the direction of breaking down the demand into its sub components of purchase incidence, brand choice and purchase quantity (Gupta (1988), Chiang (1991), Chintagunta (1993)), modeling observed and unobserved heterogeneity in the demand parameters (Chintagunta et. al. (1991), Gonul and Srinivasan (1993)), incorporating consideration sets in choice (Andrews and Srinivasan (1995), Chiang et. al. (1999)), and modeling state dependence on choice (Seetharaman et. al. (1998), Roy et. al. (1999)). In these streams of research the quantity modeled is the quantity purchased. In the case of packaged goods most of the within household variance in quantity bought is due to price promotions. If households anticipate such promotions to be temporary, the quantity bought would be a complex function of its consumption, inventory on hand, inventory carrying cost, current marketing mix, and its expectations about future. Therefore it seems natural to parcel out the effect of current and future marketing mix on consumption and quantity purchased due to the reasons we explore below. Moreover, whether and how a household responds to price promotions would also interact with the household brand preferences and consumption needs. A household that is particular about its brand or consumes more in the category is more likely to stockpile than a similar household that is indifferent across many brands or consumes little. Our focus in this paper is to develop a structural model of utility maximizing households, allowing observed and unobserved heterogeneity, that simultaneously decides on consumption and purchase decisions taking into account not only the current but also future utility derived from consumption.

Underlying the economic model is the notion that households maximize utility subject to a budget constraint. The standard static model does not distinguish between consumption and purchase quantity since whatever is purchased is assumed to be consumed by that household within the period. But, for many frequently purchased packaged goods, the household enters the market periodically and past literature has documented that current prices or promotions affect not only which brand and how much the household purchases today but also the amount consumed (Ailawadi and Neslin (1998), Bell et. al. (2002), Sun (2003)). It is this effect that needs to be properly understood from a manufacturer or a retailer perspective and is the major focus of this paper. For example, does a promotion lead to an increase in consumption, which we define as an increase in consumption due to either the income or the substitution (away from other categories) effect? Or does a promotion simply lead to households' stockpiling the product for future use without any increase in their consumption? The manufacturer of a product wants to increase the expected (current and discounted future) profits of his product while the retailer wants to increase the expected profits from the category in his store. How promotions affect sales clearly have different implications for the manufacturer and the retailer in terms of managing the pricing and promotion policies. An increase in consumption benefits both manufacturers and retailers. A pure increase in stockpiling hurts the long term profits of both manufacturers and retailers. Manufacturers benefit from brand switching from other brands, while whether retailers can benefit from brand switching or not will depend on the profit margins of different brands they stock.

Researchers and managers interested in evaluating the competitive position of a brand relative to others in a category have relied on the concepts of clout and vulnerability (Kamakura and Russell (1989)). To compute these measures we need estimates of own and cross price

elasticities. Since elasticity estimates for most packaged goods are based on temporary price promotions, our discussion above points to another reason why understanding elasticities and their decomposition due to flexible consumption are important. For example, a brand's clout and vulnerability indexes need to be reevaluated if the increase in elasticity is primarily through stockpiling vs. through increase in consumption.

Another reason why we believe that this problem is important is the implication on competitive strategies when households are heterogeneous in their response to price promotions. Marketing has a rich tradition of analytical models that explain the existence of consumer and trade promotions that occur in frequently purchased package goods (Jeuland and Narasimhan (1985), Narasimhan (1988), Lal (1990), Rao (1991), Lal et. al. (1996)). In these models households are segmented into brand loyalists and brand switchers (non-loyalists) but their purchasing behavior in terms of quantity purchased is assumed to be the same. However, if this is not true, the difference in quantity purchased then needs to be considered in the profit implications of competitive strategies. The analyses from this paper should therefore be of interest to analytical modelers who would want to enrich their models to provide deeper understanding of the strategic games among manufacturers and retailers.

The findings on household heterogeneity respect to price promotion responses can further help managers to design better pricing and promotion strategies. Households' brand preferences and consumption needs, inventory cost, and expectations about future determine whether they will respond to price promotions with switching brands, stockpiling or increasing consumption as well as determine the relative magnitude of these responses. A general concern that managers have when they design a pricing/promotion strategy will be making sure that the strategy will discourage the group(s) that will respond in a way that does not lead to improvement in profits

from responding while encourage the other group(s) to do so. Managers can also determine whether a pricing/promotion strategy should be used given the composition of households on the market. Our goal is to address these issues and provide some specific guidelines.

In summary, there are three issues that we attempt to address in this paper:

1. What is the composition of sales increase induced by price promotions? Further, is the increase mainly due to brand switching, stockpiling or due to an increase in consumption? Is the composition different across brands?
2. Is there heterogeneity across households in their response to price promotions? Specifically, do brand loyals stockpile more since they want to save for future consumption when the price of their favorite brand could be high? Do brand switchers increase consumption and don't stockpile since they can always buy the least expensive brand. How do consumption needs affect households' consumption flexibility and stockpiling behavior?
3. Given the results from the above two issues, what are the managerial implications? Specifically, how can manufacturers or retailers use the information to formulate their pricing strategies? How can manufacturers or retailers price discriminate among different household types to maximize their long term profits?

To address these issues we develop a dynamic structural model of a household that maximizes discounted utility from its consumption in a category. Consumption in each period is endogenous and is allowed to vary based on current price, inventory levels, and consumption preferences. In deciding to buy today, the household trades off the inventory cost with the opportunity cost of buying the product in the future at a higher price. We allow a household's inventory cost and price responsiveness to be functions of its demographics, and also allow

brand and consumption preferences to vary across households. We apply our model to scanner panel data on canned tuna category with 12 product alternatives. By imposing simplifying behavioral assumptions the infinite planning horizon dynamic programming problem is reduced to a finite horizon problem. We propose a solution to overcome the dimensionality problem in model estimation.

We run simulations (policy experiments) using the estimates from the proposed model as input. We are able to parcel out the effect of increase in sales due to a price promotion into its components of consumption increase, brand switching occurring in both current and future periods, and stockpiling. Our analysis reveals several interesting insights: (1) contrary to what is shown in most of relevant previous literature (Gupta (1988), Chiang (1991), Sun (2003) etc.)<sup>2</sup>, brand switching is not the dominant effect; (2) majority of promotion induced sales increase from larger share brands is attributed to stockpiling; (3) the consumption effect is greater for smaller share brands. We further show that a household's brand preference has significant impact on its stockpiling and flexible consumption behavior. More specifically, brand loyalists mainly respond to a price promotion with stockpiling while brand switchers do not stockpile at all. We also find that systematic differences exist between heavy and light users in the category on their responses to a promotion. Heavy users stockpile more for future consumption and increase less consumption than light users do. These findings provide managerial implications for pricing and promotion strategies of both manufacturers and retailers.

The rest of the paper is organized as follows. In the next section we describe related research and color our contribution with respect to the literature. Following this we describe our econometric model. The section following that describes the data we use to estimate the proposed model, discusses the estimation as well as the simulation results. Also in that section

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<sup>2</sup> Our finding is consistent with that in Erdem et al. (2003), Van Heerde et al. (2003).

we provide specific suggestions to managers based on our findings. Finally, we conclude the paper with some extensions and suggestions for future research.

## **2. BACKGROUND**

In this section we describe prior literature to position our paper and its contributions. We argued earlier that it is important to model flexible consumption in order to understand the spike in sales more fully. Moreover, we argued that there may be systematic differences across consumers<sup>3</sup> based on brand preferences and their consumption needs (brand loyals vs. switchers and light vs. heavy users) in their responses to price promotions. There are three but not necessarily separate streams of literature that are relevant to which we potentially contribute. These are (1) research that focuses on breaking down the purchase elasticity into its components, (2) research that focuses on modeling consumption behavior with consumer future expectations, and (3) dynamic structural models accounting for flexible consumption. At the end we will also briefly discuss our contributions from a methodology perspective.

### **2.1 Breaking down total elasticity into its components**

The sales increase of a brand under temporary price promotions is due to several factors: (1) consumers who would normally buy some other brand might switch to buy the promoted brand (brand switching); (2) consumers might purchase more in current period for future consumption (stockpiling); consumers might also consume more in current or future periods (consumption increase). Gupta (1988) was the earliest to decompose the promotional response (using promotional elasticity as the metric) into brand switching, purchase (incidence)

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<sup>3</sup> In our paper, “consumer” and “household” are used interchangeably.

acceleration and stockpiling effects using coffee data. He found that the dominant force was brand switching accounting for 84% of the change in response while purchase (incidence) acceleration accounted for 14% and stockpiling accounted for 2%. Similar results were reported by Chiang (1991). Bell et. al (1999) pursue this further using a much broader dataset involving 13 categories and more than 170 brands. While their results in general confirm Gupta's findings they also report substantial variances in the brand switching effect (69% to 81%) across categories. The above papers assume that consumption rate is fixed. Sun (2003) relaxes this assumption and allows consumers to decide on consumption rate each period based on their inventories and new purchases. She finds that the brand switching effect of a promotion is overestimated when consumption is fixed but it is still the dominant effect. Van Heerde et. al (2003) proposes a different decomposition measure based on sales units. Using the same dataset as Gupta (1988) did, they found that only 33% of unit sales increase is due to losses of other brands. Taking off from this stream, we breakdown the promotional effect into the brand switching, stockpiling and consumption effects. We measure the impact of a price promotion not just in the promotion period but over a long term horizon. We find that most sales increase due to a price promotion can be attributed to consumption increase and stockpiling rather than brand switching.

## **2.2 Modeling Consumers' Consumption Behavior under Price Uncertainty**

When consumers perceive costs of purchasing in future different from purchasing today, they are likely to change their behavior today. For example, based on their expectation about future prices, consumers might change the amount they buy today and the amount to stockpile (Golabi (1985), Meyer and Assuncao (1990), Krishna (1992, 1994)). Following the framework

of Golabi (1985), Assuncao and Meyer (1993) develop a normative model of a consumer who endogenously chooses the optimal level of consumption by assuming a particular price process for the future. They model a consumer as maximizing an inter-temporal utility function and optimally choosing a consumption and purchase stream. Their model predicts that consumption is an increasing function of inventory and this has subsequently found empirical support in the works of Wansink (1996), Ailawadi and Neslin (1998), Erdem et al (2003), and Sun (2003) etc. Bell et. al (2002) develop an analytical model that tries to capture the effect of flexible consumption on the part of the consumers and explores the pricing strategies of identical retailers. Like these papers we also model forward looking consumers and allow them to rationally choose not only the quantity to be bought but also their consumption stream. We take this stream further by empirically exploring differences across customer groups (brand loyals vs. brand switchers and light vs. heavy users) in the impact of promotions on their consumption and stockpiling behavior.

### **2.3 Dynamic Structural Models with Flexible Consumption**

It has been demonstrated in the marketing literature that a dynamic structural model has its advantages when modeling a consumer's purchase, stockpiling and consumption behavior under price uncertainty in package goods categories (e.g. Gunul and Srinivasan (1996), Erdem et al (2003), Hendel and Nevo (2003), Sun et al (2003) and Sun (2003)). There is a stream of literature that accounts for flexible consumption under price uncertainty using dynamic structural models. Erdem et al (2003) assume that consumers have an exogenous usage requirement each period which is revealed to consumers after their purchases. Consumers are allowed to stock out but there is a stock-out cost. In their model there are a fixed number of segments in terms of

usage rate and consumers can move from one segment to another following a Markov process. Hendel and Nevo (2003) and Sun (2003) relax the assumption of exogenous consumption rates and model consumption as a decision variable of consumers. In Hendel and Nevo (2003), the assumption that utility of a brand is derived at the point of purchase enables them to model consumers' optimization of purchase quantity in a dynamic programming setting and then separately model consumers' brand choice decisions in a static framework. This results in significant computational simplification but also comes at a cost (see the discussions in Erdem et al (2003)). Moreover, consumers are assumed to choose package sizes rather than units, limiting the applications of the model to markets where package sizes rather than multiple purchases are considerations of consumers<sup>4</sup>. Sun (2003) has shown that a dynamic structural model with endogenous consumption rate improves the goodness of fit to data as well as model prediction. The focus of her paper is on the impact of promotion on consumption in general. She confirms that consumers' consumption rate on canned tuna is not constant and that short term consumption increases when there is a price promotion. We follow this stream of research on endogenous consumption, but we propose a hedonic approach to model consumer utility. The focus of our study is the decomposition of price elasticity to its components of brand switching, stockpiling and consumption increase over the planning horizon<sup>5</sup>. Another distinct feature in our model is that we also explicitly allow consumer heterogeneity in terms of brand preferences, consumption needs, and consumption flexibility. We are interested in how these affect their responses to price promotions. For example, which types of consumers (brand loyals vs. brand switchers, light users vs. heavy users) will mainly increase consumption or mainly stockpile when facing temporary price promotions?

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<sup>4</sup> For example, it is not suitable to model the purchases in categories like canned tuna and yogurt.

<sup>5</sup> As will become clear later on, we contrast our results to the "short term perspective" where the decomposition is done for the period of promotion only. Our "long term perspective" is defined over the planning horizon of consumers.

From methodology perspective, we demonstrate that the infinite horizon dynamic problem can be reduced into a finite one, helping reducing the computational burden. We impose further behavioral assumptions<sup>6</sup> under which the problem is further simplified. Simulated Method of Moments (SMM) is used to estimate our model. Although our model has its limitations due to the simplifying assumptions, it offers an alternative solution to overcome the problem of “curse of dimensionality” in dynamic optimization problem. Hence, it is useful to researchers when they are facing high-dimension problems or dealing with large datasets.

### 3. MODEL

#### 3.1 THE HOUSEHOLD’S PROBLEM

Assume that there are  $J$  products or brands, and  $H$  households in the market. Let  $y_{ht}$  be a  $J \times 1$  vector of household  $h$ ’s continuous consumption quantity,  $x_{ht}$  a  $J \times 1$  vector of continuous quantity purchased, in week  $t$ , and  $u_h(\cdot)$  be the utility function of consumption<sup>7</sup>.

In week  $t$ , household  $h$  has to decide which product to purchase, how much to purchase, as well as how much to consume. Since quantity bought at time  $t$  can be held over and consumed in future periods, the infinite horizon planning problem can be stated as follows:

$$\begin{aligned} & \sup_{\{x_{hs}, y_{hs}\}} E_t \left\{ \sum_{s=t}^{\infty} \gamma^{s-t} [u_h(y_{hs}) - \lambda_h \cdot p_s' x_{hs} - c_h \left( \sum_{j=1}^J I_{hsj} \right)] \mid \sigma_{ht} \right\} \\ & s.t. \quad I_{hs} = x_{hs} + I_{hs-1} - y_{hs} \\ & \quad \quad x_{hs}, y_{hs}, I_{hs} \geq 0 \end{aligned} \tag{1}$$

where  $p_s$  is a  $J \times 1$  vector of prices, and  $I_{hs}$  a  $J \times 1$  vector of the inventory level of products in week  $s$ . We use  $\lambda_h$  to denote the marginal utility of income,  $c_h$  the inventory cost of one

<sup>6</sup> We will discuss these assumptions in detail in section 3.

<sup>7</sup> Its functional form will be specified in detail in equation (2) in section 3.2.

standardized unit for household  $h$ , and  $\gamma$  the discount rate (assumed to be equal across households in a standard way).  $E_t\{\cdot|\sigma_{ht}\}$  is the expectation operator conditional on the information set at  $t$ ,  $\sigma_{ht}$ . The information set includes the inventory inherited from previous period,  $I_{ht-1}$ , past marketing variables such as prices, features and displays, and household demographic variables. The endogenous variables in (1) include  $\{x_{ht}, x_{ht+1}, \dots; y_{ht}, y_{ht+1}, \dots\}$ .

### 3.2 DETAILS OF THE INDIRECT UTILITY FUNCTION

We assume, for  $s \geq t$ , a utility function of consumption as follows:

$$E_t u_h(y_{hs}) = (\Psi_h' A' y_{hs} + \phi_h)^{\alpha_h} \quad (2)$$

where,  $A$  is a  $J \times (C + J)$  characteristic matrix. The first  $C$  columns represent observable product attributes including brand names, flavor etc. The last  $J$  columns of the matrix  $A$  is an identity matrix of dimension  $J$ .  $\Psi_h$  is a vector of preferences and tastes coefficients consisting of a  $C \times 1$  vector of time-invariant household  $h$ 's preferences for the observed attributes,  $\psi_h$ , and a  $J \times 1$  vector of time-invariant household specific taste preferences for products,  $\xi_h$ .<sup>8</sup> Hence,  $\Psi_h = (\psi_h | \xi_h)$  in (2). We assume that  $\xi_h$  in (1) are *i.i.d.* continuously distributed over households. Specifically, we assume  $\xi_h \sim normal(0, \sigma_\xi^2)$ . Finally, we use a random coefficient approach by assuming that

$$\Psi_h = \bar{\Psi} + \eta_h \quad (3)$$

where  $\eta_h$  is a vector of random variables that are continuously distributed. In the model estimation, we assume  $\eta_h \sim normal(0, \sigma_\eta^2)$ , and they are *i.i.d.* over households.

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<sup>8</sup> We note that state-dependence like variety seeking behavior is not modeled here.

The parameters  $\alpha_h$  and  $\varphi_h$  control the rate of diminishing marginal returns and translation in the utility function<sup>9</sup>, where  $\alpha_h$  is restricted to be in the unit interval. To allow for heterogeneity across households in consumption, we assume that  $\alpha_h$  is a function of household demographics. In the estimation, we assume that there exist  $K$  types of households; for type  $k$ ,

$$\alpha_h = \frac{\exp(\alpha_{k,0} + \alpha_{k,1} \cdot FMY_h)}{1 + \exp(\alpha_{k,0} + \alpha_{k,1} \cdot FMY_h)} \quad (4)$$

where  $FMY_h$  represents the family size of household  $h$ , and  $\alpha_{k,0}$  and  $\alpha_{k,1}$  are parameters to be estimated. Households with larger  $\alpha_h$  are more likely to purchase the product category and consume larger quantity compared to those households with smaller  $\alpha_h$ .

To make sure that marginal utility does not go to infinity when the consumption level is zero,  $\varphi_h$  is restricted to be positive. In this case corner solutions are allowed. Following Kim et. al (2002), we fix  $\varphi_h = 1$  in the equation because it is difficult to identify  $\alpha_h$  and  $\varphi_h$  separately.

To allow for heterogeneity in price sensitivity, we further assume that

$$\lambda_h = 1 + \lambda_1 \cdot (INCOME_h - AvgINCOME) + \lambda_2 \cdot EMPLOY_h \quad (5)$$

where  $INCOME_h$  is the income of household  $h$ ,  $AvgINCOME$  is the average of household income, and  $EMPLOY_h$  is the employment status of the female household head.

To allow for heterogeneity in inventory costs, we assume that

$$c_h = c_0 + c_1 \cdot (INCOME_h - AvgINCOME) + c_2 \cdot RESIDENCE_h \quad (6)$$

where  $RESIDENCE_h$  is the residence type of the household  $h$  such as single family house or others.

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<sup>9</sup> For more detailed explanation see Kim, Allenby, and Rossi (2002).

We do not observe the initial inventory that household  $h$  has at the time when we observe its very first purchase. Intuitively, if household  $h$  makes a purchase when the price in that period is higher than its regular price, it could indicate that this household's inventory is low at the beginning of that first period. In the estimation model we assume that

$$I_{h,0} = \exp(v_h), \text{ and } v_h \sim \text{normal}(\rho \cdot (p_{h,1,j} - p_j^0), 1) \quad (7)$$

where  $I_{h,0}$  is the initial inventory of  $h$ ,  $p_{h,1,j}$  is the observed price of product  $j$  in the period when household  $h$  made the first purchase in data, and  $p_j^0$  is the perceived cost of purchasing of product  $j$ .<sup>10</sup>  $\rho$  is a parameter to be estimated. If  $\rho$  is negative, a higher first purchase price will indicate a lower initial inventory.

### 3.3 SOME SIMPLIFYING ASSUMPTIONS AND PROPOSED SOLUTION

Using Bellman's equation, equation (1) can be re-written as<sup>11</sup>:

$$V(\sigma_t) = \sup_{\{x_t, y_t\}} \left\{ E_t[u(y_t) - \lambda p_t' x_t - c \cdot \left( \sum_{j=1}^J I_{ij} \right) | \sigma_t] + \gamma \cdot \int_{\sigma} V(\sigma') dP(\sigma' | \sigma_t, x_t, y_t) \right\} \quad (8)$$

where  $P(\cdot | \cdot, x, y)$  is the Markov transition kernel for  $\{\sigma_t\}$  conditional on actions  $\{x, y\}$ .

Optimal controls  $\{x, y\}$  are solved from (8) under the constraints of inventory and non-negativity.

In practice, equation (8) is difficult, if not infeasible, to solve because of the "curse of dimensionality". There are  $J \times 2$  continuous actions (i.e.,  $x_t, y_t$ ), hence, the dimension of controls are infinite. Moreover, we do not observe inventory or consumption rate.

<sup>10</sup> We will discuss how to derive the perceived cost of purchasing later in this section.

<sup>11</sup> For simplicity we omit the subscript  $h$  hereafter.

We propose a solution to overcome the problems mentioned above. Suppose  $c_h > 0$  for all  $h$ ,  $p_{ij} > 0$  for all  $t$  and  $j$ . Let  $\bar{p}_j$  be the highest future price that product  $j$  will possibly charge. We assume  $\bar{p}_j - p_{ij} < b < \infty$  for all  $t$  and  $j$ . In Appendix A we show that there exists a finite time period,  $T$ , such that, households will not purchase at  $t$  and stockpile for the consumption of periods beyond  $t+T$  no matter how much inventory they are holding. Thus, we can rewrite the problem in (1) as a finite horizon problem,

$$\begin{aligned} & \sup_{\{x_s, y_s\}} E_t \left\{ \sum_{s=t}^{t+T} \gamma^{s-t} [u(y_s) - \lambda \cdot p'_s x_s - c \left( \sum_{j=1}^J I_{sj} \right)] \mid \sigma_t \right\} \\ & \text{s.t. } I_s = x_s + I_{s-1} - y_s \\ & \quad x_s, y_s, I_s \geq 0 \end{aligned} \tag{9}$$

The optimal solutions  $\{x_t^*, y_t^*\}$  for the week  $t$  in (9), which are the focal decision variables of the problem, are equivalent to the optimal solutions we would obtain from solving the infinite horizon problem in (1). This implies that empirical researchers may start from a reasonably large number of  $T$  and solve this finite horizon dynamic programming problem. This substantially reduces the computational burden compared to the traditional method of successive approximations in solving the Bellman equation in (8).<sup>12</sup>

Although the problem is reduced to a finite horizon dynamic programming problem, solving for the optimal decisions in (9) is still not an easy task, particularly, since  $\{x_s, y_s; s = t, t+1, \dots\}$  are  $J$ -dimensional vectors. We impose two more assumptions to simplify the problem. We first adopt an assumption used by Erdem et. al (2003).

### 1 Assumption 1:

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<sup>12</sup> For example, see a detailed explanation of different solution methods in Rust (1994).

Households use each product in their inventory proportionately. That is, given their inventory after the purchase,  $I_{t-1} + x_t$ , households will consume a proportion  $\delta_s$  for each product  $j$ , where  $\delta_s \geq 0$  and  $\sum_{s=t}^{t+T} \delta_s = 1$ , for each future period  $s$ .

Under the above assumption, households' consumption from their inventory at  $s$  is  $y_s = \delta_s (I_{t-1} + x_t)$ . Here  $\delta_s$  is a scalar. We then only need to solve for a  $T \times 1$  vector of  $\delta$ 's instead of a  $T \times J$  matrix of  $y$ 's in (9).

Second, we assume that households use a simple yet, intuitive updating rule regarding what they perceive as the future cost of purchasing.

1 *Assumption 2:*

In any given week  $t < s$ , households form a “perceived cost of purchasing” for buying brand  $j$  in week  $s$ ,  $p_{t \rightarrow s, j}^0$ , using current and past prices. These prices are expected to remain constant for all weeks  $l > t$ , i.e.,  $p_{t \rightarrow l, j}^0 = p_{t, j}^0$ .<sup>13</sup>

The above assumption implies that, facing a trade-off between buying now and buying later for future consumption, households would compare the perceived cost of purchasing with the current observed prices. Such a decision rule is a simplification from the traditional optimization algorithm, where households consider all possible future price paths and maximize the expected discounted utility over them. Since the focuses of our paper are on flexible consumption and stockpiling, this simplifying assumption will help us capture the main effects while the implied decision rules are still consistent with intuitive behavior. A flexible specification for the perceived cost of purchasing should provide an approximation to the

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<sup>13</sup> This assumption still allows the flexibility of incorporating different updating rules for future prices. For example, in our empirical analysis we assume households update their perceived future cost of purchasing using:  $p_{t, j}^0 = p_j^0 + \omega \cdot (p_j^0 - p_{t, j})$ , where  $p_j^0$  is the regular price of product  $j$ ,  $p_{t, j}$  the observed price at time  $t$ , and  $\omega$  a parameter to be estimated.

solution of a dynamic planning problem under price uncertainty. Note that the perceived cost of purchasing should not be confused with the expected future prices. In particular, if households are risk-averse and prefer not to commit to consuming any product in the future by holding inventory,  $p_t^0$  can be lower than the expected future prices in model estimation.

Under the assumptions of stationary perceived cost of purchasing and positive inventory cost, it is not optimal for households to make purchases in a future period  $s$  and hold inventory for the consumption in a further future period  $u$ , where  $u > s$ . Based on this reasoning and along with Assumptions 1 and 2, equation (9) can be re-written as

$$\begin{aligned} & \sup_{\{x_t, \dots, x_{t+T}, \delta_t, \dots, \delta_{t+T}\}} \{E_t u(\delta_t \cdot (I_{t-1} + x_t)) - \lambda p_t' x_t - c \cdot (1 - \delta_t) \cdot \sum_{j=1}^J (I_{t-1,j} + x_{t,j}) \\ & + \sum_{s=t+1}^{t+T} E_t u(\delta_s \cdot (I_{t-1} + x_t) + x_s) - \lambda p_t^0' x_s - c \cdot (1 - \delta_t - \dots - \delta_s) \cdot \sum_{j=1}^J (I_{t-1,j} + x_{t,j}) \quad (10) \\ \text{s.t.} \quad & x_t, \dots, x_{t+T}, \delta_t, \dots, \delta_{t+T} \geq 0, \quad \sum_{s=t}^{t+T} \delta_s = 1 \end{aligned}$$

Note that  $x_s$  in (10) only enters the expected utility function in period  $s$  and does not affect the expected inventory in any future period. The dynamic programming problem in a finite horizon of  $T$  is then vastly simplified.

Next, let us examine how a household decides whether to buy, which product to buy, how much to consume and how much to buy. To simplify the analysis, first suppose that the household does not hold inventory in week  $t$ . The utility function in (2) implies that, after observing  $p_t$ , a household will choose at most one product  $j^*$  such that, for all  $k = 1, \dots, J$ ,

$$\begin{aligned} \frac{MU_{j^*}(0)}{\lambda p_{t,j^*}} &= \frac{\alpha \cdot \psi' A_{j^*}}{\lambda p_{t,j^*}} \geq \frac{\alpha \cdot \psi' A_k}{\lambda p_{t,k}} = \frac{MU_k(0)}{\lambda p_{t,k}} \\ \Rightarrow \frac{\psi' A_{j^*}}{p_{t,j^*}} &\geq \frac{\psi' A_k}{p_{t,k}} \end{aligned} \quad (11)$$

where  $MU_{j^*}(0)$  is the marginal utility level with respect to  $j^*$  at  $y_k = 0, \forall k = 1, \dots, J$ . Since we assume that a household's preference is stationary over time,  $j^*$  will also be the product chosen if the household purchases now and holds it for future consumption. Corner solution exists when  $\max_{\{k\}} \left\{ \frac{\alpha \cdot \Psi' A_k}{\lambda p_{t,k}^0} \right\} < 1$ , for all  $k$ . In this case we have  $x_t = 0$ . This occurs when the household finds current prices too high to purchase the product category at week  $t$

With the perceived cost of purchasing equal to  $p_t^0$  and the utility function as specified in (2), the household expects to choose at most one product  $j^0$  such that, for all  $k$ ,

$$\begin{aligned} \frac{MU_{j^0}(0)}{\lambda p_{t,j}^0} &= \frac{\alpha \cdot \Psi' A_{j^0}}{\lambda p_{t,j}^0} \geq \frac{\alpha \cdot \Psi' A_k}{\lambda p_{t,k}^0} = \frac{MU_k(0)}{\lambda p_{t,k}^0} \\ \Rightarrow \frac{\Psi' A_{j^0}}{p_{t,j}^0} &\geq \frac{\Psi' A_k}{p_{t,k}^0} \end{aligned} \quad (12)$$

Again, when  $\max_{\{k\}} \left\{ \frac{\alpha \cdot \Psi' A_k}{\lambda p_{t,k}^0} \right\} < 1$ , for all  $k$ , the expected purchase at the perceived cost of purchasing will be zero. This occurs when the household does not normally purchase or consume the category, and will only purchase when there is a big price promotion.

As implied by (10), the household will purchase in week  $t$  and hold it for consumption in week  $s$ , where  $s > t$ , only if the following two conditions are satisfied:

$$(i) \quad \gamma^{s-t} \cdot MU_{j^*}(0) - \lambda p_{t,j^*} - c \cdot \sum_{u=0}^{s-t} \gamma^u \geq 0$$

and

$$(ii) \quad \gamma^{s-t} \cdot MU_{j^*}(0) - \lambda p_{t,j^*} - c \cdot \sum_{u=0}^{s-t} \gamma^u \geq \gamma^{s-t} \cdot [MU_{j^0}(0) - \lambda p_{t,j^0}^0]$$

Condition (i) ensures that discounted consumption utility in week  $s$  net of purchasing costs in week  $t$  and discounted inventory cost is non-negative. Condition (ii) ensures that it is worthwhile to buy now and hold inventory until week  $s$ .<sup>14</sup>

Suppose the above two conditions are satisfied, *i.e.*, the household purchases in week  $t$  and holds it for consumption in week  $s$ . The optimal purchase quantity then satisfies the third condition that is derived from the first-order condition:

$$(iii) \quad \gamma^{s-t} \cdot MU(y_{t,s,j^*}) - \lambda p_{t,j^*} - c \cdot \sum_{u=0}^{s-t} \gamma^u = 0$$

where  $y_{t,s,j^*}$  is the optimal quantity purchased in week  $t$  for the consumption in week  $s$ . The optimal level of  $x_t$  in (10) is equal to the sum of  $y_{t,s,j^*}$ ,  $s=t, \dots, T$ , in the  $j^*$ -th row and zero elsewhere. Thus, the optimal level of  $\delta_s^*$  is  $\frac{y_{t,s,j^*}}{x_{t,j^*}}$ .

Note that our model should not be treated as a substitute for the dynamic models with rational expectations because of the simplifying behavioral assumptions we make, but it offers an alternative solution to capture the main effects while overcoming the problem of the “curse of dimensionality” in dynamic optimization problem.

When the household holds positive inventory in week  $t$ , solution  $y_{t,s,j^*}$  for all  $s$  cannot be solved separately as in conditions (i) to (iii), since the household has to additionally consider the

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<sup>14</sup> This decision rule is similar to those in reference price literature (Winer (1986), Mayhew and Winer (1992), Mazumdar and Papatla (2000), etc.). But in our setting households are comparing current prices with the perceived cost of purchasing in the future. Indeed we may treat  $\min\{\frac{1}{\lambda}\{\gamma^{s-t} \cdot MU_{j^*}(0) - c \cdot \sum_{u=0}^{s-t} \gamma^u\}, \frac{1}{\lambda}\{\gamma^{s-t} \cdot [MU_{j^*}(0) - MU_{j^0}(0)] - c \cdot \sum_{u=0}^{s-t} \gamma^u + \gamma^{s-t} \cdot \lambda p_{t,j^0}^0\}\}$  as an “inter-temporal” reference price of buying  $j^*$  at week  $t$  for consumption at future period  $s$ . In this case the “inter-temporal” reference price is a function of brand and consumption preferences, inventory cost, and perceived cost of purchasing.

benefit and cost of consuming the inventory *vs.* buying in current week. However, basic principles of the solution concept discussed above still apply. In our estimation algorithm, we directly solve the non-linear constrained optimization problem in (10), given parameters  $\theta$  and inventory level  $I_{t-1}$ . Our estimator is a non-linear least square estimator using the Method of Moments. Details of the estimation algorithm will be provided in Appendix B.<sup>15</sup>

### 3.4 IDENTIFICATION OF DIFFERENT TYPES OF CONSUMPTION BEHAVIOR

Depending on (1) preferences for the attributes of different products, (2) inventory cost, and (3) flexibility in consumption as determined by  $\alpha$ , a price promotion will have different effect on different households. One major identification issue for model estimation is that we, as researchers, do not observe consumption and inventory of households in the data. We only observe whether a household makes a purchase and if so which product it buys and the quantity it purchases in each period. Still, we can identify the parameters of a household's utility function from the variations in its purchasing pattern over time: Brand switching patterns of households over time help identify the differences in product preferences of households, and variations in purchase quantity and time-intervals between purchases help identify the inventory cost and consumption rate changes. For example, suppose that there are two households, *A* and *B*, who buy one unit of the product in each period at the same price. Suppose the price is cut by 10 percent in the current period, and both *A* and *B* increase their purchase from 1 to 2 units. If household *A* comes back to purchase 1 unit again in the next period, but household *B* does not make a purchase until period 3, we can infer that *A* increases its consumption and does not stockpile in the current period, while household *B* does the opposite. In this case household *A* has

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<sup>15</sup> In this application we fix the number of finite periods to  $T=12$ , and the number of simulation draws to just one. Since we have a large number of observations, estimator efficiency is not our major concern.

a flexible consumption rate but a higher inventory cost than household *B*. Suppose there is another household, *C*, which buys 4 units of the product during promotion, and only comes back to market in period 3. Then we infer that household *C* may have a more flexible consumption rate than household *B* but a lower inventory cost than household *A*. Hence, the parameters are identified if there are enough variations in responses to prices, even though we just observe the quantity purchased. Given these parameter values, effects of stockpiling, brand-switching and consumption increase due to temporary price promotions can also be identified. For example, when there is a price promotion household *A* will show a larger consumption effect but a smaller stockpiling effect than household *B* does, while household *C* will show a larger consumption effect as well as a stockpiling effect.

## **4. EMPIRICAL ANALYSIS**

### **4.1 DESCRIPTION OF DATA**

We estimate the proposed model using the A. C. Nielsen scanner panel data on canned tuna. The reason we choose this category is that canned tuna is easily storable and potentially a good candidate for stockpiling and flexible consumption. The dataset on canned tuna contains weekly data on prices, feature advertising, and displays from 19 stores in Sioux Falls, South Dakota for a period of 123 weeks from January 1985 to May 1987. It also provides purchase history information of 4308 households. There are totally 10 package sizes in the data, among which the one size, 6.5 oz., has the largest share, accounting for 94.2% of the total quantity sold, and 93.7% of the total purchase occasions. Also, among the 4308 households, 3250 or 75.4% of total households, purchases canned tuna of this size only. Given the obvious dominance of this package size, in order to avoid dealing with the issue of

quantity discount with the existence of multiple package sizes, we focus our analysis on the 6.5 oz size and purchases from these 3250 households.

There are total 33 SKUs of 6.5 oz size in the data. We use product attributes to group the SKUs. We identify three main product attributes: brand, water/oil based, and light/regular. There are total 10 brands among which four brands, Star-Kist, Chicken of the Sea (CKN), 3 Diamond, and CTL (store controlled brand) account for 99.5% of all purchases. The grouping of SKUs by product attributes generates 12 product alternatives (brands\*attributes), of which 11 product alternatives represent the SKUs that belongs to one of the four brands, water or oil, and light or regular. We combine the rest of the SKUs that belong to a brand other than the four brands into the 12th product alternative<sup>16</sup>. Table 1 provides the summary statistics for these 12 products. To keep the size of the dataset manageable, we randomly chose 1000 households from the 3250 households. These households made 13394 purchases during the sample periods. The average number of units per purchase occasion was 2.15 units and the average inter-purchase time was 9.84 weeks. Figure 1 gives the histogram of inter-purchase times of the 1000 households over the 123 weeks.

The price, feature, and display of each product assumed to be observed by a household in each week is constructed as follows:

- 1 For a product that a household purchases in a week, price, feature and display are constructed as the weighted average over the SKUs that belong to the product alternative.

The weight used is the quantity of purchased;

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<sup>16</sup> In each purchase occasion, a household purchases one of the 12 product alternatives, so there is no underestimate of household inventory. Henceforth we will use the term product to refer to one of these 12 alternatives.

- 1 For a product that a household does not purchase in a week, the price, feature and display are constructed as the numerical average over all the SKUs that belong to the product alternative in the household's preferred store.

The dataset also contains the demographic characteristics of the households such as family size, income, the employment status of the female head of the household, and type of residency, etc. We incorporate these variables in the estimation of our proposed model.

[Insert Table 1 and Figure 1 here]

## 4.2 EMPIRICAL RESULTS

We estimate two model specifications: In model A, the perceived costs of purchasing are assumed to be the regular prices<sup>17</sup> in the data. In model B, a household updates its perceived cost of purchasing after observing current price. Specifically, we assume that:

$$p_{t,j}^0 = p_j^0 + \omega \cdot (p_j^0 - p_{t,j})$$

where  $p_j^0$  is the regular price of product  $j$ ,  $p_{t,j}$  the observed price at time  $t$ , and  $\omega$  a parameter to be estimated. If  $\omega$  is negative, it suggests that a household lowers its perceived cost of purchasing product  $j$  because of the price promotion at time  $t$ .

Since model B yields a lower value of the objective function than model A, we report the results for model B in Table 2. From the table, we can see that the estimate of mean preference for all households for the category of canned tuna is -0.12. However, the preference is heterogeneous among households since the variance of the normally distributed preference parameter is 0.50, which is statically significant. The most preferred brand is Star-Kist while the least preferred brand is 3 Diamond. Further, households' preferences for Star-Kist and CKN are

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<sup>17</sup> For each household, the regular price of each product is the average price over the 123 weeks.

heterogeneous (i.e.,  $\sigma_2^2, \sigma_3^2$  are statistically significant) while those for 3 Diamond and CTL are not. The estimates for households' mean preferences for water based and light tuna are positive (1.83 and 0.33 respectively), implying that households like water-based more than oil-based tuna, and like light more than regular tuna. Consistent with our expectations, the households also have positive responses to feature advertising and store display (1.53 and 2.47 respectively). The household specific taste preferences for products are assumed to be i.i.d normal distribution with zero mean, over households and product alternatives. The estimation shows that this preference is heterogeneous among households (i.e.,  $\sigma^2$  is 1.89 and statistically significant).

The coefficient of price is normalized to one for the model parameters to be identified. The coefficients of income and employment status of the female head of the household are not statistically significant. The estimate for inventory cost of canned tuna is \$0.03 per can per week. It is within a reasonable range and consistent with our expectation. The income and the residence type a household has in do not seem to have significant impact on its inventory cost. One explanation for this could be that because canned tuna is relatively easy to store households do not run into space constraints caused by income or residency types.

The consumption level in our model is determined by  $\alpha = \frac{\exp(\alpha_0 + \tau \cdot FMYSIZE)}{1 + \exp(\alpha_0 + \tau \cdot FMYSIZE)}$ . Our estimation indicates that there are two segments with respect to  $\alpha_0$  and  $\tau$ . The first segment accounts for 76.43% of all the households. Interestingly, the negative sign for  $\tau$  implies that smaller families have a larger consumption rate for this category. The estimate of  $\rho$  is negative (-2.22), implying that if a household buys at a higher price during its first observed purchase, it indicates that the household has a lower inventory level. This is consistent with our intuition.

### 4.3 SIMULATION AND DISCUSSIONS

In order to answer the questions that we raised earlier regarding the effects of price promotions on brand switching, stockpiling and consumption, we conduct simulations using the parameter estimates from our dynamic structural model. One advantage of structural models is that the underlying behavioral process of a rational household is explicitly specified, thus the estimated parameters are invariant to marketing activities and are suitable for policy experiments.

In the simulations, we used the 1000 households observed in data, and for each household we take 10 random draws out of the distribution. Therefore, we have 10,000 observations. We first simulate the purchases and consumption of these households when they face the regular prices<sup>18</sup> for all products for 12 continuous weeks. Then, we simulate the purchases and consumption for the same households for the 12 weeks when all the product alternatives that belong to a brand (the focal brand) offer an equal amount of price discount in the first week. The comparisons of the sales before and after the price cut allow us to decompose the effects of price promotions into those of consumption increase, brand switching and stockpiling. We define the total effects of the price cut as the sales increases in week one. That is,

$$TE_b = \sum_{h=1}^{10000} x_{hb1}(p_b^1) - x_{hb1}(p^0)$$

where  $p^0$  is the price vector when all brands charge their regular prices, and  $p_b^1$  is the price vector when brand  $b$  has a price cut while other brands remain their regular prices, and  $x_{hb1}(\cdot)$  the quantity of brand  $b$  that household  $h$  purchases in week one under the corresponding price vector.

The consumption effect is defined as the difference in the total sales in all 12 weeks before and after the price cut. That is:

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<sup>18</sup> We use average prices of each product alternative from the data.

$$CE_b = \sum_{t=1}^{12} \sum_{h=1}^{10000} x_{hbt}(p_b^1) - x_{hbt}(p^0)$$

Note that increase in consumption comes from those households that purchase only on promotions and households that buy during non-promoted weeks as well. Since we assume that the planning horizon of households are 12 weeks, i.e., households will not purchase some quantity in week one and hold them as inventory for future consumption beyond the 12<sup>th</sup> week, given that prices in all weeks after week one are the same regular prices<sup>19</sup>, it is valid to use the difference in total purchases for all 12 weeks as the measure for changes in total consumption<sup>20</sup>.

The brand switching effect is measured by the decrease in total sales of other brands (whose prices remain the same) in all 12 weeks as the focal brand cuts its price in week one. We further decompose the brand switching effect into two components: brand-switching that occurs in the first week ( $BS_{b1}$ ) and in later weeks ( $BS_{bL}$ ). That is:

$$BS_{b1} = \sum_{h \in H_{-b1}} x_{h1}(p^0) - \sum_{h \in H_{-b1}} x_{h1}(p_b^1)$$

$$BS_{bL} = \sum_{t=2}^{12} \sum_{h \in H_{-bt}} x_{ht}(p^0) - \sum_{t=2}^{12} \sum_{h \in H_{-bt}} x_{ht}(p_b^1)$$

where  $H_{-bt}$  are those households who purchase brands other than  $b$  in week  $t$ . Notice that since these households are paying the same prices for the other brands before and after the price cut of brand  $b$ , there is no consumption or stockpiling effect in  $x_{ht}(p_b^1)$ ; thus the differences must be pure brand switching effects.

The residual is then the stockpiling effect. That is:

$$SP_b = TE_b - CE_b - BS_{b1} - BS_{bL}$$

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<sup>19</sup> So from week 2 to 11, households will purchase tuna only for consumption of that week without stockpiling for future weeks.

<sup>20</sup> We can also measure the consumption effect by comparing the total quantity consumed during the 12 weeks before and after the price cut. However, it is simpler to use the method we mentioned here, and the results are equivalent.

this is derived from the fact that total sales increase of a brand due to a reduction in price in week 1 has to come from increase in consumption of tuna, households' brand switching from other brands, and households' stockpiling for future consumption.

#### **4.3.1. Decomposition of Price Elasticities**

We decompose demand elasticities<sup>21</sup> into the above components. We first examine the case with a price cut of 10 cents. The decomposition for all 1,000 households is given in Table 3.

[Insert Table 3 here]

First, we notice that the brand switching effect for all brands is much smaller than what is identified in most previous relevant literature (e.g., Gupta (1988), Chiang (1991)), and is not the dominant force. The discrepancy is due to two differences between our model and those of Gupta (1988) and Chiang (1991):

1. In our model, a household is assumed to maximize its consumption utility over a planning horizon (e.g., in our estimation of canned tuna category, we let it be 12 weeks) while in the models of Gupta (1988) and Chiang (1991), a household is assumed to maximize its purchase utility only in current period.
2. In our model, consumption is a decision variable for the households while the consumption rate is fixed in Gupta (1988) and Chiang (1991).

Moreover, the brand switching effect in our model is computed as the lost sales of other brands in the current period as well as in later periods while in Chiang (1991) and Gupta (1988) it is calculated as the increased choice probability in current period for choosing focal brand

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<sup>21</sup> In each table, we report the decomposition of elasticities both in terms of numerical values as well as in percentages relative to the total elasticities. The latter is the number in parentheses.

(conditional on purchase incidence<sup>22</sup>). Since the consumption is restricted to be fixed, the brand switching effect in their models in fact includes the increased quantity purchased by the household for consuming more or and stockpiling for future consumption, while those two effects are separately identified in our model. Therefore, the brand switching effect in their models is inflated.<sup>23</sup>

Second, we notice the different responses exhibited between larger share brands (i.e., Star-Kist and CKN) and smaller share brands (i.e., 3 Diamond and CTL):

1. Smaller share brands have higher price elasticity than larger share brands. This is consistent with the findings in Chintagunta (1993).
2. Majority of sales increase from the two large brands is due to stockpiling effects (62% and 67% respectively) while the stockpiling effects for smaller share brands are smaller (37% for 3 Diamond and 39% for CTL).
3. Brand switching effect is relatively small for larger share brands (8% for both brands) but substantially higher for smaller share brands (26% for 3 Diamond and 25% for CKN)<sup>24</sup>.
4. Consumption effect is significant for all brands. But this effect is higher for smaller share brands than for larger share brands.

The comparisons imply that, from a manufacturer's perspective, the strategy of temporarily cutting prices to steal sales from other brands might not be very effective for large share brands.

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<sup>22</sup> This is the condition in the model of Chiang (1991). Gupta (1988) assumes three decisions to be independent.

<sup>23</sup> Notice that in Sun (2003) a household is also assumed to decide on consumption each period to maximize total consumption utility over a long period of time, but she still finds the brand switching effect is the dominant force. We suspect the discrepancy is due to the fact that in her model the price elasticity is calculated base on the sales increase (caused by the price cut) in the current period (i.e., the period when a price cut occurs) only, while in our model the price elasticity is calculated based on the sales increase occurring during the planning horizon (i.e., 12 weeks)

<sup>24</sup> Since the calculation of price elasticity is based on the sales at regular prices (for smaller share brands, they have much fewer sales than those of larger share brands), one should be cautious to draw the conclusion that smaller share brands can draw more sales by price promotions than larger share brands by simply comparing the brand switching elasticity (or the corresponding percentage measure).

Unlike in a one period game where large share brand might benefit if the small brand's price is higher, in this case its profits could be hurt in the long term since a greater portion of the sales increase in current period comes at the cost of future sales. Relatively, a price promotion seems to be more attractive for a smaller brand since majority of its sales increase comes from consumption increase and brand switching.

Since the demand curve of households in our model is highly non-linear, in order to understand households' responses when facing different levels of price discounts, we did two sets of experiments: one for a price cut of 10 cents for each brand, and the other for a price cut of 20 cents. We report the decomposition of price elasticity for all households given the price cut of 20 cents in Table 4.

[Insert Table 4 here]

Comparing the elasticity decomposition in Table 3 and Table 4, we can see consistent patterns across all brands:

1. The price elasticity is higher when the price cut is 20 cents than when the price cut is 10 cents.
2. Compared with a price cut of 10 cents, the price cut of 20 cents results in a larger increase in consumption but a smaller increase in stockpiling<sup>25</sup>.

The above results suggest that, households are more responsive to a larger price discount and they also respond by increasing their consumption more than stockpiling for future use. To understand this note that when a household faces a price cut it increases its expected utility by either increasing its current consumption or stockpiling for future or both. As can be seen from table 3 for a 10 cents price cut a household increases its consumption and also buys more for

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<sup>25</sup> With the price cut of 20 cents, the decomposition for stockpiling effect is smaller in terms of percentage values but still greater in terms of absolute values.

future use as well but the latter effect dominates. For a deeper price cut of 20 cents, the increase in stockpiling relative to the 10 cents case is small because of the convexity of the inventory cost function and therefore the household increases utility by increasing current consumption much more. Thus, a larger price discount seems to have relatively more positive impact on long term sales from the manufacturer's point of view.

#### **4.3.2. Comparisons of the Elasticity Decomposition across Households**

First, we investigate the impact of brand preferences on households' responses to price promotions. For each brand, we group all the households into two segments. The households in one segment purchase the brand at its regular price, while the households in the other segment do not do so. When there is a price cut for brand  $b$ , we can expect the households in brand  $b$ 's first segment will continue purchasing this brand and some of households in its second segment may also be induced to purchase brand  $b$ . We call the households in the first segment "brand loyals" and those in the second segment "brand switchers"<sup>26</sup> of brand  $b$ <sup>27</sup>. We focus on the differences in the responses of brand loyals and brand switchers for a price reduction of 10 cents.<sup>28</sup> The results are shown in Table 5 and Table 6 respectively.

[Insert Table 5 and Table 6 here]

Comparing the figures in Tables 5 and 6 yields the following insights:

1. Brand loyals and brand switchers demonstrate very different stockpiling behaviors.

For brand loyals, majority of the increase in purchases from price promotions can be

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<sup>26</sup> Brand switchers include households that buy other brands when those brands are priced at their regular prices as well as households that make no purchase when all brands are priced at their regular prices.

<sup>27</sup> According to this definition, the brand loyals/switchers for one brand can be the brand switchers/loyals for another brand.

<sup>28</sup> We have also done the analyses for the price cut of 20 cents and the results are consistent with what we find for the price cut of 10 cents.

attributed to stockpiling (69%, 73%, 61% and 63% respectively for Star-Kist, CKN, 3 Diamond and CTL) while for brand switchers the stockpiling effect is almost zero.

2. Brand loyals also increase their consumption more than brand switchers do.
3. Brand loyals are more price elastic than brand switchers as consumption effect is explicitly modeled.

These findings suggest that when a price promotion is used in a market with a large number of brand loyals, the brand will lose profits from its brand loyals not only in the current period (the foregone profit since loyals would have bought the product anyway), but also in future periods because these brand loyals are stockpiling at a lower price for future consumption. The gain from brand loyals' consumption increase that is also induced by the price cut will not be sufficient to compensate the loss. On the other hand, in a market with high proportion of brand switchers (a very competitive market), temporary price cuts of a brand have the benefit of stealing sales from other brands as well as an increase in consumption, and furthermore it does not have the cost of inducing brand switchers to stockpile for future consumption. Therefore, knowing the market composition in terms of households' brand preferences will help managers decide whether it is appropriate to use price promotion strategies.

Next, we investigate the impact of consumption levels on households' responses to price promotions. We divide all households into two groups: heavy users and light users. A heavy user is the one whose total quantity purchased in all 12 weeks is above the average. Otherwise he is a light user. Table 7 and Table 8 show the decomposition of price elasticity for heavy users and light users respectively, given a price cut of 10 cents.

[Insert Table 7 and Table 8 here]

Comparisons of Table 7 and Table 8 show that:

1. Lighter users show a larger consumption effect than heavy users.
2. Heavy users show a larger stockpiling effect than light users.

The first finding is intuitive and consistent with the commonly known satiation effect. Compared with heavy users, light users are more likely to increase consumption due to a lower price. The second finding suggests that heavy users are more strategic in terms of planning for future consumptions. This is consistent with the findings in Zhang, Seetharaman and Narasimhan (2003) and can be attributed to the different importance of the tuna category to the two groups of households. By using past purchase history of households, the firm can target heavy users and light users with different incentives (vs. give temporary price cuts to all households). For example, the firm can use targeting coupons to induce light users to start purchasing the product or purchase more quantity.

The two important findings that we have identified: when facing a price promotion (1) brand loyals mainly respond with stockpiling and also some consumption increase, while brand switchers do not stockpile but switch brands and increase consumption; (2) heavy users stockpile more and do not increase their consumption as much as light users, can help managers design better pricing and promotion strategies. Though a rigorous analysis of the optimal pricing strategy a firm can use under these settings is out of the scope of this paper, we will provide some general guidance here: In general a firm can increase its long term profits by discouraging brand loyals and heavy users from responding to price promotions while encouraging brand switchers and light users to do so. For example, a firm can give price discounts on small package sizes to induce brand switchers and light users to switch. To more efficiently price discriminate and discourage brand loyals and heavy users from taking advantage of these price discounts the per unit price of large package sizes should not be higher than the discounted per unit price of

smaller package size. This shows that the presence of loyal and heavy users constrains a firm's strategy in competing for light users and brand switchers. Meanwhile, since brand switchers are not likely to increase much consumption at lower prices, it is optimal for the firm to charge prices just low enough to induce them from switching.

## **5. CONCLUSIONS AND FUTURE RESEARCH**

We proposed a dynamic structural model to study the impact of temporary price promotions on households' behavior and to identify the relative influence of brand switching, stockpiling and consumption increase on the total impact. Using a hedonic approach to model household utility and imposing some simplifying behavioral assumptions enabled us to reduce the dimensionality of the problem and apply our model to a market in the presence of large number of households and product alternatives. Contrary to most previous research, our decomposition of price elasticity shows that the major effect of a price promotion is not inducing brand switching, but increasing consumption and stockpiling. We also find that larger share brands will be hurt by price promotions since majority of promotion induced sales increase from these brands is attributed to stockpiling for future consumption. We then investigate the household heterogeneity in terms of brand preferences and consumption levels, and find that with price promotions brand loyalists mainly stockpile for future consumption while brand switchers almost do not stockpile. We also documented that when there is a price promotion light users consume more but heavy users stockpile more for future consumption. These findings can help managers better design their pricing and promotion strategies.

In this paper, we did not consider state dependence, i.e., households might seek variety from one purchase to another or be inertia. It might be of interest to explore the impact of price

promotions when both variety seeking (or inertia) and flexible consumption behavior are present. It will also be of interest to explore cross-category generalization of our empirical findings and see whether there exists heterogeneity across categories in terms of households' responses to price promotions that could imply different strategies in different categories. By comparing two depths of price discounts from regular prices we note that a 20 cent price discount seems to have a more positive long term effects on the sales of a product. An optimization model might be useful to further investigate the optimal level of price discounts.

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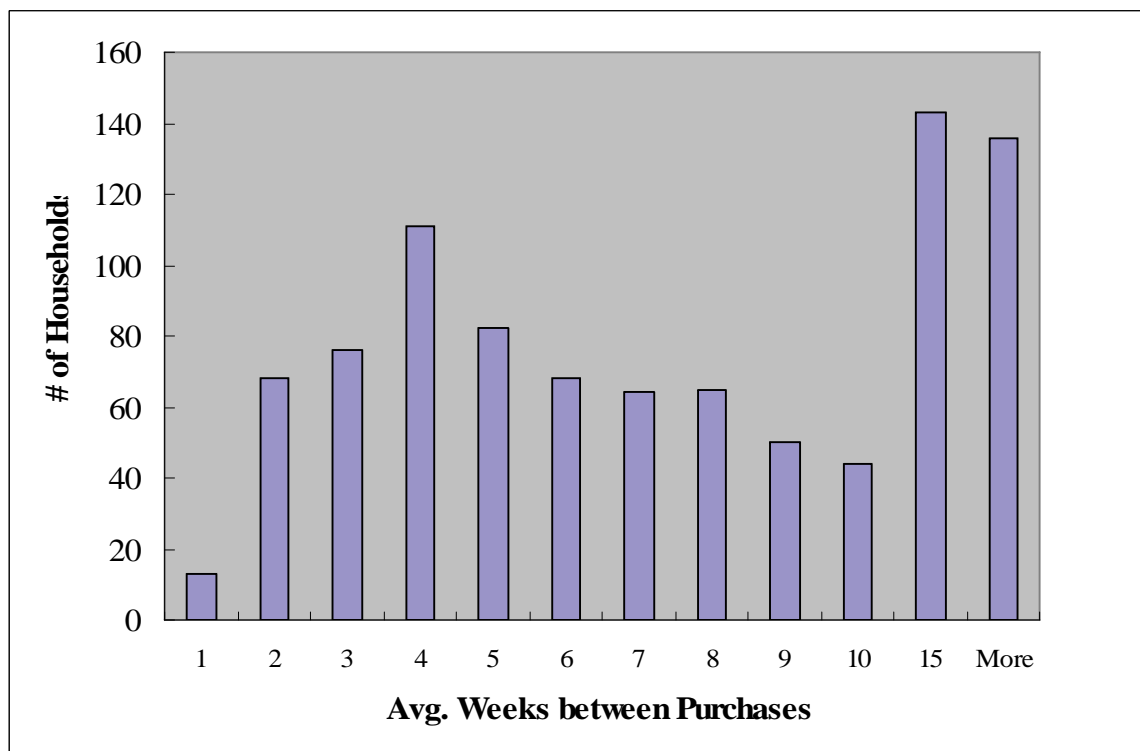
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**Table 1. Market Share of Product alternatives**

Product alternative	Market Share (%)	Avg. Price (\$/Unit)	Avg. Feature	Avg. Display
Star-Kist, Water, Light	36.32	0.78	0.14	0.04
CKN, Water, Light	32.56	0.81	0.13	0.05
Star-Kist, Oil, Light	13.00	0.70	0.16	0.04
CKN, Oil, Light	10.05	0.82	0.12	0.05
CTL, Water, Light	3.44	0.65	0.02	0.04
3 Diamond, Water, Light	1.66	0.62	0.01	0.08
CTL, Oil, Light	1.24	0.65	0.03	0.04
3 Diamond, Oil, Light	0.71	0.62	0.01	0.08
Other Brands	0.54	1.14	0.00	0.00
CTL, Oil, White	0.27	0.64	0.02	0.11
Star, Water, White	0.19	1.54	0.00	0.00
CTL, Water, White	0.01	1.22	0.00	0.00

**Figure 1. Histogram of Avg. Inter-Purchase Time of Households**



**Table 2. Estimation Results of the Proposed Model**

<b>Parameters</b>	<b>Est.</b>	<b>Sta. Err.</b>
Tuna	-0.12	0.04
$\sigma_1^2$ (Tuna)	0.50	0.01
Star-Kist	3.06	0.01
CKN	1.43	0.02
3 Diamond	-1.53	0.03
CTL	-1.27	0.06
$\sigma_2^2$ (Star-Kist)	1.32	0.01
$\sigma_3^2$ (CKN)	5.50	0.01
$\sigma_4^2$ (3 Diamond)	0.05	0.07
$\sigma_5^2$ (CTL)	0.06	0.05
Water	1.83	0.01
Light	0.33	0.04
$\sigma_6^2$ (Water)	0.86	0.01
$\sigma_7^2$ (Light)	0.28	0.01
Feature	1.53	0.02
Display	2.47	0.02
$\sigma_8^2$ (Feature)	0.23	0.01
$\sigma_9^2$ (Display)	0.14	0.02
$\sigma^2$ (Product Specific Household Preferences)	1.89	0.01
Price*Income	0.00	0.00
Price*Employ	-0.01	0.01
$C_0$ (Inventory)	0.03	0.01
$C_1$ (Invent*Income)	0.00	0.00
$C_2$ (Invent*RedType)	0.00	0.00
$\alpha_{01}$	-0.56	0.03
$\alpha_{02}$	0.00	0.01
$\tau_1$ (FMYSIZE)	-0.60	0.02
$\tau_2$ (FMYSIZE)	-0.14	0.01
PROB(Segment 1)	76.43%	0.17
$\rho$	-2.22	0.14
$\omega$	-0.01	0.01
Function Value	673.45	

**Table 3. Decomposition of Price Elasticity for All Households (10 cents cut)**

	Total Price Elasticity in 1st Week	Total Consumption Elasticity	Brand Switching Elasticity in 1st Week	Brand Switching Elasticity in later Weeks	Stockpiling Elasticity
Star-Kist	-0.97 (100%)	-0.29 (30%)	-0.07 (7%)	-0.01 (1%)	-0.60 (62%)
CKN	-0.94 (100%)	-0.24 (25%)	-0.06 (7%)	-0.01 (1%)	-0.62 (67%)
3 Diamond	-1.39 (100%)	-0.53 (38%)	-0.33 (24%)	-0.02 (2%)	-0.51 (37%)
CTL	-1.34 (100%)	-0.48 (36%)	-0.31 (23%)	-0.03 (2%)	-0.53 (39%)

**Table 4. Decomposition of Price Elasticity for All Households (20 cents cut)**

	Total Price Elasticity in 1st Week	Total Consumption Elasticity	Brand Switching Elasticity in 1st Week	Brand Switching Elasticity in later Weeks	Stockpiling Elasticity
Star-Kist	-1.38 (100%)	-0.57 (42%)	-0.08 (6%)	-0.07 (5%)	-0.66 (48%)
CKN	-1.25 (100%)	-0.46 (37%)	-0.06 (5%)	-0.05 (4%)	-0.68 (54%)
3 Diamond	-3.16 (100%)	-1.52 (48%)	-0.50 (16%)	-0.36 (11%)	-0.77 (24%)
CTL	-2.97 (100%)	-1.33 (45%)	-0.47 (16%)	-0.36 (12%)	-0.80 (27%)

**Table 5. Decomposition of Price Elasticity for Brand Loyals (10 cents cut)**

	Total Price Elasticity in 1st Week	Total Consumption Elasticity	Stockpiling Elasticity
Star-Kist	-0.86 (100%)	-0.26 (31%)	-0.60 (69%)
CKN	-0.85 (100%)	-0.23 (27%)	-0.62 (73%)
3 Diamond	-0.83 (100%)	-0.32 (39%)	-0.51 (61%)
CTL	-0.84 (100%)	-0.31 (37%)	-0.53 (63%)

**Table 6. Decomposition of Price Elasticity for Brand Switchers (10 cents cut)**

	Total Price Elasticity in 1st Week	Total Consumption Elasticity	Brand Switching Elasticity in 1st Week	Brand Switching Elasticity in later Weeks	Stockpiling Elasticity
Star-Kist	-0.10 (100%)	-0.02 (21%)	-0.07 (69%)	-0.01 (10%)	0.00 (0%)
CKN	-0.08 (100%)	-0.01 (10%)	-0.06 (78%)	-0.01 (12%)	0.00 (0%)
3 Diamond	-0.56 (100%)	-0.21 (38%)	-0.33 (58%)	-0.02 (4%)	0.00 (0%)
CTL	-0.50 (100%)	-0.17 (33%)	-0.31 (62%)	-0.03 (5%)	0.00 (0%)

**Table 7. Decomposition of Price Elasticity for Heavy Users (10 cents cut)**

	Total Price Elasticity in 1st Week	Total Consumption Elasticity	Brand Switching Elasticity in 1st Week	Brand Switching Elasticity in later Weeks	Stockpiling Elasticity
Star-Kist	-0.99 (100%)	-0.27 (28%)	-0.07 (7%)	-0.01 (1%)	-0.63 (64%)
CKN	-0.97 (100%)	-0.23 (24%)	-0.06 (7%)	-0.01 (1%)	-0.67 (68%)
3 Diamond	-1.33 (100%)	-0.48 (36%)	-0.31 (24%)	-0.02 (1%)	-0.52 (39%)
CTL	-1.32 (100%)	-0.45 (34%)	-0.30 (23%)	-0.03 (2%)	-0.54 (41%)

**Table 8. Decomposition of Price Elasticity for Light Users (10 cents cut)**

	Total Price Elasticity in 1st Week	Total Consumption Elasticity	Brand Switching Elasticity in 1st Week	Brand Switching Elasticity in later Weeks	Stockpiling Elasticity
Star-Kist	-0.82 (100%)	-0.38 (46%)	-0.07 (9%)	0.00 (0%)	-0.37 (45%)
CKN	-0.59 (100%)	-0.27 (45%)	-0.06 (11%)	0.00 (0%)	-0.27 (45%)
3 Diamond	-1.86 (100%)	-0.91 (49%)	-0.44 (24%)	-0.05 (2%)	-0.46 (25%)
CTL	-1.51 (100%)	-0.72 (48%)	-0.36 (24%)	0.00 (0%)	-0.43 (29%)

## Appendix A. Reduce a Infinite-Horizon Problem to a Finite-Horizon Problem

Our objective is to reduce the infinite-horizon problem in (1) to a finite-horizon problem. First, we can replace the purchase and the inventory in period  $t$ ,  $x_t, I_{t-1}$ , with the consumption at current and all future periods, i.e.,  $x_t = \sum_{s=t}^{\infty} y_{t,s}$  and  $I_t = \sum_{s=t}^{\infty} y_{t-1,s}$ . The subscript “ $t$ ” in  $y_{t,s}$  denotes the time of purchase, and “ $s$ ” the time of consumption, and “ $t-1$ ” in  $y_{t-1,s}$  indicates that it is quantity inherited from inventory at period  $t-1$ . Then we can rewrite (1) into

$$\begin{aligned} & \sup_{\{\}} E_t u(y_{t-1,t} + y_{t,t}) - \lambda \cdot p_t' \left( \sum_{s=t}^{\infty} y_{t,s} \right) - c \left\{ \sum_{j=1}^J \left[ \sum_{s=t+1}^{\infty} (y_{t-1,s,j} + y_{t,s,j}) \right] \right\} \\ & + \sum_{s=t+1}^{\infty} \gamma^{s-t} E_t \left\{ u(y_{t-1,s} + y_{t,s} + \dots + y_{s,s}) - \lambda \cdot p_s^{o'} \left( \sum_{u=s}^{\infty} y_{s,u} \right) - c \left[ \sum_{j=1}^J \sum_{u=s+1}^{\infty} (y_{t-1,u,j} + y_{t,u,j} + \dots + y_{s,u,j}) \right] \mid \sigma_t \right\} \\ & s.t. \quad I_{t-1} = \sum_{s=t}^{\infty} y_{t-1,s} \\ & \quad y_{s,u,k} \geq 0, u \geq s, k = 1, \dots, J \end{aligned} \tag{A.1}$$

Since  $\bar{p}_j - p_{ij} < b < \infty$ , for all  $t$  and  $j$ , that is, the difference between the highest price of and the observed price of product  $j$  is finite, there exists a finite time period,  $T_1$ , such that households will not purchase at time  $t$  for the consumption of time  $u$ , for all  $u > T_1$ , under the condition that the cost from purchasing the product today and stockpiling for consumption at period  $T_1$  is greater than the cost of having to buying the product at period  $T_1$  and consume it at the same period (even with the highest price), i.e.,

$$\begin{aligned} & \lambda \cdot p_{ij} + c \left( \sum_{u=t}^{T_1} \gamma^{u-t} \right) > \lambda \cdot (\gamma^{T_1-t} \bar{p}_j) \\ & \Rightarrow T_1 - t < \frac{\log \left( \left( \frac{c}{\lambda} + p_{ij} \right) / \bar{p}_j \right)}{\log(\gamma)} \end{aligned} \tag{A.2}$$

In this case,  $y_{t,u,j} = 0$ , for all  $u > T_1$ . That is, no household will purchase product  $j$  at  $t$  and hold it as inventory for the consumption after period  $T_1$ .

Now let us consider the endogenous variables  $\{y_{t-1,b}, y_{t-1,t+1}, \dots\}$ , that is, the inventory inherited from period  $t-1$  which will be used for future consumption. We want to show that there exists a finite period  $T_2$  such that  $y_{t-1,u} = 0$  for all  $u > T_2$ .

By assumption, marginal utility (i.e.,  $\frac{\partial E_t u_h(y_{hs})}{\partial y_j} \Big|_{y_{hs}=0} = \alpha_h \cdot \Psi_{ht} \cdot A_j$ ) is finite at zero consumption level. Then there exists a  $T_2$  such that

$$\begin{aligned} \alpha \cdot \Psi \cdot A_j \cdot \gamma^{T_2-t} &< c \cdot \sum_{u=t}^{T_2} \gamma^{u-t} \\ \Rightarrow T_2 - t &< \frac{\log(c) - \log(\alpha \cdot \Psi \cdot A_j \cdot (1-\gamma) + c)}{\log \gamma} \end{aligned} \quad (\text{A.3})$$

This implies that a household will not stockpile current inventory,  $I_t$ , for the consumption after period  $T_2$  due to the existence of its inventory cost and the discount rate.

Let  $T = \max\{T_1, T_2\}$ . We can write a finite horizon problem equivalent to the infinite horizon problem in (A.1):

$$\begin{aligned} \sup_{\{y\}} & E_t u(y_{t-1,t} + y_{t,t}) - \lambda \cdot p_t' \left( \sum_{s=t}^T y_{t,s} \right) - c \left\{ \sum_{j=1}^J \left[ \sum_{s=t+1}^T (y_{t-1,s,j} + y_{t,s,j}) \right] \right\} \\ & + \sum_{s=t+1}^T \gamma^{s-t} E_t \left\{ u(y_{t-1,s} + y_{t,s} + \dots + y_{s,s}) - \lambda \cdot p_s^{0'} \left( \sum_{u=s}^T y_{s,u} \right) - c \left[ \sum_{j=1}^J \sum_{u=s+1}^T (y_{t-1,u,j} + y_{t,u,j} + \dots + y_{s,u,j}) \right] \mid \sigma_t \right\} \\ \text{s.t. } & I_{t-1} = \sum_{s=t}^T y_{t-1,s} \\ & y_{s,u,k} \geq 0, u \geq s, k = 1, \dots, J \end{aligned} \quad (\text{A.4})$$

Solutions of  $y_{t-1,s}$  and  $y_{t,s}$ , for all  $s \geq t$ , in (A.4) are equivalent to solutions in (A.1). Therefore, our infinite horizon problem has been reduced to a finite horizon planning problem.

## Appendix B. Model Estimation (Simulated Method of Moments)

Based on the discussion that how solutions from a finite-horizon problem are equivalent to an infinite one in the setting of our model, a natural way in model estimation is to use the Method of Moments by matching the expected quantity purchased from solving the problem with the observed purchases. The estimation procedure involves a nested algorithm for estimating parameters  $\theta$ : an “inner” algorithm computes a simulated quantity purchased which solves the problem in (10) for each trial value of  $\theta$ , and an “outer” algorithm searches for the value of  $\theta$  that minimize a criterion function. The inner algorithm is repeated every time when  $\theta$  is updated by the outer algorithm.

Procedures of the inner algorithm are the following: Given the conditional distribution function of random variables  $\epsilon$ ,<sup>29</sup>  $F(\cdot | X_t)$ , where  $X_t$  represents all explanatory variables that we used in the model including marketing variables such as prices, product features and displays, as well as demographic variables like household size, income level, residence type, female employment status, we can solve the expected values of  $\{x_t, \dots, x_{t+T}; \delta_t, \dots, \delta_{t+T}\}$  from (10). That is,

$$\{x_t^*, \dots, x_{t+T}^*; \delta_t^*, \dots, \delta_{t+T}^*\} = \int \arg \max_{\{x_t, \dots, x_{t+T}; \delta_t, \dots, \delta_{t+T}\}} \{\text{problem in (10)}\} F(d\epsilon | X_t) \quad (\text{B.1})$$

However,  $x_t^*$  in (B.1) is non-tractable because of the non-negativity constraints. Instead, we use simulation method: for each household  $h$  and period  $t$ , we draw  $\tilde{\epsilon}_{ht,1}, \dots, \tilde{\epsilon}_{ht,ns}$  from the distribution function  $F(\cdot | X_{ht})$ , where  $ns$  is the number of simulation draws. Conditional on each simulation draw  $\tilde{\epsilon}_{ht,s}$ , we solve a non-linear constrained optimization problem in (10) for the optimal quantity purchased levels at time  $t$ ,  $\tilde{x}(X_{ht}; \theta; \tilde{\epsilon}_{ht,s})$ , using a derivative search procedure called the Sequential Quadratic Programming. In this method variables are updated in a series of iterations beginning with a starting value that satisfies the constraints in (10). Let  $\tilde{x}_n$  be the current value at iteration  $n$ , the succeeding value is  $\tilde{x}_{n+1} = \tilde{x}_n + \rho \delta$ , where  $\delta$  is a direction vector, and  $\rho$  a scalar

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<sup>29</sup> Here  $\epsilon$  includes (i) all stochastic components in the utility function such as individual product preferences  $\xi_h$  and  $\eta_h$ , (ii) a standard normal variable that relates to the initial inventory level  $I_{h,0}$ , and (iii) discrete distributed stochastic variables relates to the types of  $\alpha_h$  a household has in the utility function.

step length. The iterated  $\tilde{x}_n$  are converging under some nice properties such as concavity of the utility function. The procedure is repeated for every simulation draw and hence generate a simulated data  $\tilde{x}(X_{ht};\theta) = (1/ns) \cdot \sum_{s=1}^{ns} \tilde{x}(X_{ht};\theta;\tilde{\epsilon}_{ht,s})$ . When the utility function is concave and its Jacobian and Hessian matrices can be written down in analytical forms, convergence of  $\tilde{x}$  is fast.

The outer algorithm searches for the estimator  $\theta$ . We make a major identifying assumption that there are no unobserved characteristics in the model.<sup>30</sup> Therefore, there is no endogeneity issue for marketing variables such as prices. Although this assumption can be challenged, it solves the data problem as good instruments for weekly changing prices are not available. Under the identifying assumption, this yields a moment condition:

$$E[x_{ht} - \tilde{x}(X_{ht};\theta_0)] = 0 \quad (\text{B.2})$$

where  $\theta_0$  is the true parameters. The estimator  $\theta_n$  is obtained by the following non-linear least-square estimator

$$\theta_n = \arg \min_{\theta \in \Theta} Q_n(\theta) = \arg \min_{\theta \in \Theta} \frac{1}{H \times T_h} \sum_{h=1}^H \sum_{t=1}^{T_h} [x_{ht} - \tilde{x}(X_{ht};\theta)]^2 \quad (\text{B.3})$$

We use the Nelder-Meade (1965) nonderivative simplex method to search for  $\theta_n$ . Estimators based on this moment condition are called the Simulated Method of Moment (SMM) estimators (Pakes (1984), Pakes and Pollard (1989), McFadden (1989)). One major advantage of using the SMM is that  $ns$  can be finite (even when  $ns = 1$ ) and we still obtain consistent estimators. This helps to reduce the computational burden in model estimation. Our methodology here is very similar to Chan (2003). However, his model is to estimate multiple-product, multiple-unit purchase decisions, while our model is to estimate the purchase and consumption decisions over multiple periods.

The following are the summary of our estimation procedures. They also include more details of searching for  $\tilde{x}(X_{ht};\theta;\tilde{\epsilon}_{ht,s})$ .

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<sup>30</sup> More precisely, we assume that all product characteristics are either observed ones that we use in the model, or included in the household specific product preferences that we will estimate. Also, we assume that there are no promotional activities unobserved in data are correlated with pricing decisions. We note that this is a typical implicit assumption used in all dynamic optimization empirical research so our estimation method is no more restrictive than the others.

- (1) Draw  $\varepsilon_{ht,1}, \dots, \varepsilon_{ht,ns}$  from the distribution function  $F(\cdot | X_{ht})$  for each  $h$  and  $t$ . These are saved to be used for the whole process of parameters searching. We also fix a large number for  $T$ .<sup>31</sup>
- (2) Starting from the inner algorithm. Given parameters  $\theta$  and random draws  $\varepsilon_{ht,1}, \dots, \varepsilon_{ht,ns}$ , we solve the product choice  $j^*$  under observed prices  $p_t$  and  $j^0$  under perceived costs of purchasing  $p_t^0$  using conditions (11) and (12).<sup>32</sup> Conditional on  $p_t^0$ ,  $\delta_{hs}$ ,  $x_{ht}$  and  $I_{ht}$ , we can then solve for simulated solutions,  $\tilde{x}_{hs,j^0}$ ,  $t < s \leq T$ , and restrict the  $j^0$ -th row of  $\tilde{x}_{hs}$  being  $\tilde{x}_{hs,j^0}$ , and 0 otherwise.
- (3) Now we are only left with endogenous variables  $\{\tilde{x}_{ht}; \tilde{\delta}_{ht}, \dots, \tilde{\delta}_{hT}\}$  in the problem (10). We use numerical method to search for the optimal level of  $\{\tilde{x}_{ht}; \tilde{\delta}_{ht}, \dots, \tilde{\delta}_{hT}\}$ .
- (4) The observed purchased quantity,  $x_{ht}$ , in the data is likely to be different from the simulated quantity  $\tilde{x}_{ht}$ . To update the inventory level at  $t$ ,  $I_{ht}$ , we use the observed data  $x_{ht}$ , conditional on the estimated  $\tilde{\delta}_{ht}$ . The updated inventory level is then used to compute the optimal level of  $\{\tilde{x}_{h,t+1}; \tilde{\delta}_{h,t+1}, \dots, \tilde{\delta}_{h,T+1}\}$  for period  $t+1$ . This methodology is similar to the dynamic conditional moment in Gourieroux and Monfort (1996) since the observed  $x_t$  is used. They showed that this estimator is asymptotically consistent.
- (5) For the outer algorithm we search for estimators  $\theta_n$  using the moment condition in (B.3). For this outer level of parameters search, we use the Nelder-Meade simplex method. We repeat procedures (2) to (4) for each  $\theta$ .

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<sup>31</sup> Theoretically  $T$  can be estimated. In practice we fix  $T$  to be a large number so that the chance that households will hold the inventory for consumption after  $T$  is very small. For example, in our empirical analysis for canned tuna category  $T$  is fixed as  $t+12$ . This implies that households will not buy tuna at period  $t$  and plan to consume it at periods later than three months. We believe that this is a reasonable assumption.

<sup>32</sup> Note that  $j^*$  and  $j^0$  can be null sets if  $\max_{\{k\}} \left\{ \frac{\alpha \cdot \Psi' A_k}{\lambda p_{t,k}} \right\} < 1$  or  $\max_{\{k\}} \left\{ \frac{\alpha \cdot \Psi' A_k}{\lambda p_k^0} \right\} < 1$ .