

# Information and Inventory in Distribution Channels\*

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April 2005

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\*We thank Erik Durbin, V. Padmanabhan and seminar participants at the Harvard Business School, and University of Texas at Dallas, Yale University and Marketing Science Conference, 2002, for comments. We also thank the Editor, Associate Editor and reviewers of this journal for their comments. Email: giyer@haas.berkeley.edu, narasimhan@mail.olin.wustl.edu, rkni-raj@marshall.usc.edu.

## Abstract

We examine the trade-offs between demand information and inventory in a distribution channel. While better demand information has a positive direct effect for the manufacturer of improving the efficiency of holding inventory in a channel, it can also have the strategic effect of increasing retail prices and limiting the extraction of retail profits. Having inventory in the channel can help the manufacturer to manage retail pricing behavior while better extracting retail surplus. Thus even if the information system is perfectly reliable, the manufacturer might not always want to institute an information enabled channel over a channel with inventory. In a bilateral channel an imperfectly reliable information system, we show that if the manufacturer were to choose the precision of the demand information system, the manufacturer might not prefer perfect information, even if such information was costless to acquire.

In a channel with a single retailer, we show that the channel with information is preferred over the channel with inventory if the marginal cost of production is sufficiently high. In a channel with competing retailers the information enabled channel is preferred when retail competition is sufficiently intense. The presence of inventory can play a role in managing competition among retailers and in helping the manufacturer's to appropriate surplus especially when retailers are sufficiently differentiated.

# 1 Introduction

A basic aspect of retailing is that retailers must carry inventory of products to meet potential demand. While the need for holding inventory can simply arise due to the time required for physical delivery of the good at the retail location, in many channels uncertainty about demand at the time of the contract can also be an important factor. If demand is uncertain, then the imperfect ability to forecast demand can result in a mismatch between the inventory that is held by the retailer and the actual demand realization. In such a case there is inefficient production of the good leading to either understocking or overstocking of the good as compared to the realized demand.

An important response to the demand uncertainty in channels is the institution of better information systems which help the channel to better align the inventory holding to the demand realization. Aligning the inventory holding through better demand information is one characteristic of many initiatives in retailing such as quick response, collaborative planning and forecasting and other such practices. Information-enabled retailing has been adopted in industries such as apparel, consumer electronics and automobile retailing.<sup>1</sup> Interestingly, despite the obvious advantages of reduced inventory, the experience of manufacturers adopting improved information systems has been somewhat mixed. The fashion goods industry highlights this issue. The literature suggests that manufacturers of fashion products should benefit substantially from the adoption of these practices due to the industry characteristics of high demand uncertainty and short selling cycles. But manufacturers in the industry seem to be less than enthusiastic about investing in channel information systems (Hammond et al 1991, Irastorza 1992). Fisher (1997) presents evidence from several other industries (that are contrary to popular belief) which suggests that initiatives that reduce the inventory in a channel need not always be attractive for the manufacturer.

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<sup>1</sup>Adoption of these practices in industries like apparel (Hammond et. al. 1991), and corporations like General Electric and Thomson Electronics of France (Stern et al. 1999) are documented.

We present a model of a decentralized channel that shows the trade-off between inventory and information in a distribution channel. This is most apparent in the extreme case of the channel with perfect demand information versus the channel with inventory without the information system. The trade-off that we model is the following: Better information about demand has a positive direct effect on manufacturer profits due to improved efficiency of inventory holding in the channel. But there is also an indirect or strategic effect of information that negatively affects manufacturer profits as it leads to greater double marginalization.

Specifically, in a channel with inventory (without demand information), we look at the situation in which a manufacturer has to sell the inventory to the retailer before the retailer sets the retail price and the demand is realized. The manufacturer faces the disadvantage that there might be underproduction or overproduction of inventory in equilibrium. When the marginal cost of production is small or when the upside demand potential is sufficiently large, the manufacturer would find it optimal to induce the retailer to overstock (i.e., have unsold stock if the demand were to be low). In this case the manufacturer endogenously offers to take back unsold goods by fixing returns in the contract. This allows for better extraction of retailer profits while limiting double marginalization. In contrast, with a perfectly reliable demand information system, there is no need for returns and the equilibrium contract consists of the manufacturer selling the realized demand for a wholesale price. While the manufacturer can produce to exactly meet the demand and avoid excess inventory, the information enabled system leads to greater double marginalization of retail prices.

We find that the manufacturer would not always want to institute an information enabled channel over a channel with inventory even if the information system is perfect. In a model of a bilateral channel with a single retailer we find that the channel with perfect information dominates the channel with inventory only if the marginal costs of production and hence, cost of excess production, are sufficiently high. We also investigate the effect of an imperfect information system which can

predict the true state of demand with less than perfect reliability. In this case too, there can be possible understocking or overstocking of the good compared to the actual demand. It turns out that the manufacturer's profit is generally maximized at some intermediate value of the reliability of the information system. In other words, if the manufacturer were to choose the precision of the demand information system, the manufacturer would not prefer perfect information, even if such information was costless to acquire. We also make the point that an information enabled channel will become more attractive for the manufacturer when contracts become complete so as to achieve the first best vertically integrated channel outcome or when they become flexible enough to adjust to the realization of the information signal.

With retail competition, the manufacturer prefers the information enabled channel format in markets where the intensity of retail competition is sufficiently high. In this case the double marginalization caused by the information system counterbalances the downward pressure on the retail prices due to the competition between the retailers. In contrast, when the retail market is sufficiently differentiated, it becomes more attractive to have a channel with inventory. Thus we show that the presence of inventory can play a role in managing retail competition and allowing the manufacturer's to better appropriate retail surplus. This presents a rationale for why having costly retail inventory might be "good" for the manufacturer from a strategic point of view.

## **1.1 Related Research**

This work is related to the literature on channel control and coordination. Research in control issues includes Jeuland and Shugan (1983), who examine the role of quantity discounts, Moorthy (1987), who analyzes two-part tariffs, Lal (1990) who examines the coordination of a double moral hazard, Villas-Boas (1998) who analyzes product line decisions in a channel, and Iyer(1998) who examines coordination in a channel with price and non-price competition at the retail level. Our aim is to examine how

manufacturers can use information and the choice of the retail format to manage the price competition between retailers in markets with demand uncertainty. In doing so we are able to study the trade off between better demand information and retail inventory and the manner in which it helps the manufacturer to control retail pricing behavior.

Our paper also endogenizes (for the case where the retailer carries inventory) the return of unsold merchandise to the manufacturer and points out an important strategic role for the practice that determines the tradeoff between inventory and information in the channel. In doing so, we add to the existing literature on manufacturer returns policies (Marvel and Peck 1995, Padmanabhan and Png 1997).

The rest of this paper is organized as follows. In the next section, we develop our general model of a distribution channel with a single retailer. We introduce the basic elements of demand, uncertainty and the information system that firms in the distribution channel may utilize in this section. Section 3 starts with the analysis of the two extreme cases of the information system reliability, i.e., when the system is either completely reliable or completely unreliable. Later in the section, we analyze the case of the imperfect, but strictly positive reliability. In section 4, we present the model of with downstream competition and also the analysis of this channel with only the two extreme cases of information system reliability. Section 5 concludes and provides some directions for future research.

## **2 The Model**

We now develop a simple model with uncertainty of demand to explore the role of information and inventory in a distribution channel. The vertical channel consists of a manufacturer who sells through a retailer who decides on a marketing-mix activity that affects the end-consumer market and the retailer's profits. In our model, the marketing-mix activity is represented by the retail price. The manufacturer has a

constant marginal cost of production given by  $c$ . We also assume that the manufacturer has no salvage value for the product but may endogenously decide to take back the unsold goods from the retailer at a price specified in the contract. Note that in this way the marginal cost is also the cost of excess production in the channel which is borne by the manufacturer if he accepts returns and by the retailer if no returns are accepted.<sup>2</sup>

The retail demand is uncertain and is given by  $q = \theta_s - p$ , where  $p$  is the retail price and subscript  $s = h, l$  represents the uncertainty in the market demand potential. Specifically, we assume that the two states of demand occur with equal probability and normalize the low state of demand  $\theta_l = 1$  and denote the high state of demand as  $\theta_h = z > 1$ . Thus  $z$  is a measure of the spread in the distribution representing the degree of market demand uncertainty. Firms can use the demand information generated by the information system to respond to demand uncertainty. The information system may not be perfect in predicting the state of the world *ex ante*, but it does predict the demand better than the prior distribution given by probabilities  $\frac{1}{2}$  and  $\frac{1}{2}$  for the two possible values of  $\theta_s$ .

We can think of the output of the information system as a signal, which indicates whether  $\theta_s = 1$  or  $z$ , but the signal may not be fully reliable. The reliability of the information system is modeled as follows. Let  $h$  and  $l$  respectively refer to the true state of demand being high ( $\theta_s = z$ ) or low ( $\theta_s = 1$ ), and let  $\tilde{h}$  and  $\tilde{l}$  respectively denote the signal indicating that the demand will be  $h$  ( $\theta_s = z$ ) or  $l$  ( $\theta_s = 1$ ). Similar to Chen et. al (2001), we define the reliability of the information system by a measure  $\rho \in [0, 1]$ . We specify  $\Pr(\tilde{h}|h) = \Pr(\tilde{l}|l) = \frac{1}{2} + \frac{\rho}{2}$  and  $\Pr(\tilde{h}|l) = \Pr(\tilde{l}|h) = \frac{1}{2} - \frac{\rho}{2}$ . The formulation has the following properties: i) When  $\rho = 1$  the system is perfectly reliable in the sense that  $\Pr(\tilde{h}|h) = \Pr(\tilde{l}|l) = 1$ . At the other extreme, when the system is completely unreliable and  $\rho = 0$ , the signal is no improvement over the prior. ii)

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<sup>2</sup>The cost of excess production in general case may need to be adjusted for a positive salvage value. The qualitative results of the paper would hold for any positive salvage value  $f < c$ .

The information system is unbiased in that the unconditional probability of a signal indicating demand to be high (low) is equal to the true unconditional probability of demand being high (low): i.e.,  $Pr(h) = Pr(\tilde{h})$  and  $Pr(l) = Pr(\tilde{l})$ .

A decision maker in a firm observes the signal from the information system and uses it to compute the conditional probability of demand being high or low given the signal. In other words, the decision makers at the firms would compute the probabilities like  $Pr(h|\tilde{h})$  and  $Pr(l|\tilde{h})$ . For example, to compute  $Pr(h|\tilde{h})$ , note that  $Pr(\tilde{h}|h)Pr(h) = Pr(h|\tilde{h})Pr(\tilde{h}) = Pr(h \cap \tilde{h})$ . Now given that  $Pr(h) = Pr(\tilde{h})$ , we have that  $Pr(h|\tilde{h}) = \frac{1}{2} + \frac{\rho}{2}$ . Other probabilities conditioned on signals like  $Pr(l|\tilde{h})$ ,  $Pr(h|\tilde{l})$  and  $Pr(l|\tilde{l})$  can be similarly computed. Note that for any strictly positive values of  $\rho$ , the signal is always meaningful in the sense that  $Pr(h|\tilde{h}) = Pr(l|\tilde{l}) > Pr(h) = Pr(l)$ . This implies that having obtained the signal, a firm will not ignore the signal as it improves the probability of being correct. As  $\rho$  increases towards 1, the reliability of signal improves and the probability of wrong prediction of the state decreases. However, as long as  $\rho$  is less than 1, there is a positive probability that the channel ends up with a mismatch between the inventory and the realized demand.

Note that the contracts and actions of the firms, both for the manufacturer and for the retailer, will be signal dependent, whenever  $\rho > 0$ . The game sequence of this model is as shown in Figure 1.

First the same information signal is received by both the channel members. The manufacturer then offers a contract  $(w_{\tilde{k}} \text{ and } R_{\tilde{k}})$ . Because this signal (of  $\tilde{h}$  or  $\tilde{l}$ ) may be imperfect, the retailer faces the possibility of ordering a quantity  $Q_{\tilde{k}}$ , which is too high or too low for the realized state of demand. After a decision about how much quantity to stock has been made, a retail price  $p_{\tilde{k}}$  is set by the retailer. At this point, the uncertainty is revealed and the retailer sells according to the demand but only up to the quantity stocked. If there is excess stock after meeting the demand and if the manufacturer accepts returns in the contract (i.e.,  $R_{\tilde{k}} > 0$ ), then the retailer also gets revenue from returning unsold stock.

### 3 Analysis of the Model

We begin the analysis with the two extreme cases of information reliability: the case of  $\rho = 0$  in which the information signal does not provide the players information that is any better than the prior, and the case of perfect information  $\rho = 1$  in which both the manufacturer and the retailer are able to adjust their actions to the exact demand realizations. This analysis helps us to highlight the basic trade-offs between information and inventory in the distribution channel.

#### 3.1 Channel With Perfect Information ( $\rho = 1$ )

If the information system is perfect, then based on this information the manufacturer will be able to align the production of inventory with the actual realization of demand and there is no need for excess inventory. Thus this format implies a demand forecasting system that substitutes for the presence of retail inventory and enables the channel to line up inventory according to the demand realization.<sup>3</sup> Thus perfect information works as a substitute for inventory. The sequence of moves is as shown in Figure 2. The manufacturer and the retailer are able to choose actions after receiving perfect information about the demand state. Later in this section, we will analyze the case in which the demand information is imperfect.

We denote this perfect information case by the subscript  $x$  and provide the solution below. After the information system provides a (perfect) signal of the state of demand  $\theta_s = z$  or 1 that will be realized, the manufacturer chooses a wholesale price  $w_s$  and the retailer accordingly chooses the price  $p_s$ . Notice that because there is no possibility of excess demand there is no need for a returns price,  $R_s$  in any state of demand. The retail profit for each state is  $\pi_{rxs} = (p_s - w_s)(\theta_s - p_s)$  and so

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<sup>3</sup>For example, retailers such as Wal-Mart are equipped with information technology (e.g., scanner data, electronic warehouse links, and in-store audits) and ongoing marketing research information that helps to indicate the realization of demand given the price that is chosen.

the optimal retail price for each state  $s$  is  $p_s(w_s) = \frac{\theta_s + w_s}{2}$ . The manufacturer's profit function for each state is  $\pi_{mxs} = (w_s - c)(\theta_s - p_s(w_s))$  and the optimal wholesale price is  $w_{xs} = \frac{\theta_s + c}{2}$ . Defining  $\bar{\theta}$  as the mean of the  $\theta$  distribution and  $\sigma_\theta^2$  as the variance we can write the expected manufacturer profit for this channel with information as  $E_\theta(\pi_{mx}) = \frac{1}{8}[\sigma_\theta^2 + (\bar{\theta} - c)^2] = \frac{1}{16}[(z - c)^2 + (1 - c)^2]$ . Similarly, the *ex-ante* expected retailer profit is  $E_\theta(\pi_{rx}) = \frac{1}{16}[\sigma_\theta^2 + (\bar{\theta} - c)^2] = \frac{1}{32}[(z - c)^2 + (1 - c)^2]$ .

Perfect information for the retailer about the demand realization implies that there will be no inefficiency arising from the mis-alignment of the inventory ordered to the demand realization. The manufacturer and the retailer are both able to use the demand information to respond accurately to each state of demand. Thus the wholesale price and the retail price both respond to the actual demand realizations. This results in the expected manufacturer (and retailer) profits to be increasing in the variance of the demand distribution. The ability of the manufacturer (retailer) in the information enabled channel to adjust the price to the actual state of demand makes the manufacturer's (and retailer's) profit increasing in the variance of the distribution.

### 3.2 Channel With Inventory ( $\rho = 0$ )

When  $\rho = 0$  the channel members will choose actions based only on the prior distribution. This means that the manufacturer produces the good and sells to the retailers the inventory of the good through the contract before they choose the retail price based on which the demand is realized. This captures the idea that inventory at the retail level is necessary for doing business and that the realized demand can be met only if the retailer has the product in stock while setting the price.

The timing of moves is as shown in Figure 3. Note that the two information nodes of Figure 1 essentially collapse into one as the signal is completely uninformative. In the first stage the manufacturer produces the goods and chooses the contract which consists of a wholesale price  $w$  and possibly an offer to accept returns of unsold

goods at a non-negative returns price  $R$ .<sup>4</sup> The retailer then chooses how much of the good to stock and the retail price. The retailer can return any unsold good if the manufacturer accepts returns for a positive returns price. Note that there is potential inefficiency in the channel here arising from excess production of inventory that might not be sold and therefore returned to the manufacturer.

We now provide the solution of the model of the channel format with inventory. In general the manufacturer's contract will consist of a wholesale price  $w$  and a return price ( $R$ ) for any unsold inventory. Let us denote the quantity of inventory that the retailer orders by  $Q$ . Given the specification of demand uncertainty there are two types of equilibria that are feasible given the inventory and pricing choices of the retailer: The first is the case in which the retailer "understocks" in equilibrium. In this case, given the contract, the quantity bought by the retailer is less than the demand in the high state but equal to the demand in low state ( $(1 - p) = Q < (z - p)$ ). The second case is one in which there is possible "overstocking" by the retailer. In this case the quantity bought by the retailer is equal to the demand in the high state, but higher than the demand if the low state were to occur ( $(1 - p) < Q = (z - p)$ ). As we show in the appendix there is no other feasible case. In particular, the "in-between" case in which the retailer understocks if the high state were to be realized, but overstocks if the low state were to be realized does not occur in equilibrium. We now present the detailed analysis of the two cases.

*Understocking Equilibrium:* In the understocking case, denoted by  $u$ , the retailer's realized demand is higher than the quantity  $Q_u$  ordered if the state of demand turns out to be high, but is exactly equal to the quantity in the low state of demand. Therefore  $Q_u = 1 - p_u$ . The retailer's profit function is given by  $\pi_{ru} = (1 - p_u)(p_u - w_u)$  from which the optimal retail price will be  $p_u(w_u) = \frac{1+w_u}{2}$ . For understocking to be

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<sup>4</sup>Return of unsold merchandise are offered in many industries like book and music publishing, diamonds and jewelry which are characterized by high degree of uncertainty of demand (Kandel 1996, Padmanabhan and Png 1997).

an equilibrium, it must be the case that the retailer has no incentive to deviate to any other choice of inventory and price given the contract. Thus the manufacturer's problem is to choose  $w_u > 0$  to maximize its profits  $\pi_{mu} = (w_u - c)(1 - p_u(w_u))$  subject to the retailer getting atleast as much profit as is available from the best possible deviation  $\pi_{rd}(w_u) = \frac{1}{2}(z - p_d)p_d + \frac{1}{2}(1 - p_d)p_d - (z - p_d)w_u$ . The corresponding Lagrangian is,

$$L_{mu} = (1 - p_u)(w_u - c) + \mu_d(\pi_{ru}(w_u) - \pi_{rd}(w_u)) \quad (1)$$

We present the derivation of the equilibrium in the Appendix. The equilibrium wholesale price can be derived to be  $w_u^* = \frac{1+c}{2}$ . Thus the equilibrium retail price, quantity and profits are respectively  $p_u^* = \frac{3+c}{4}$ ,  $Q_u^* = \frac{1-c}{4}$  and  $\pi_{ru}^* = \frac{(1-c)^2}{16}$ , and  $\pi_{mu}^* = \frac{(1-c)^2}{8}$ .

*Overstocking Equilibrium:* In this case, denoted by  $o$ , the retailer's ordered quantity is  $Q_o = z - p_o$ . We continue to analyze the case in which there is positive demand in equilibrium in both states.<sup>5</sup> Given this the retailer's profit function is,

$$\pi_{ro} = p_o\left(\frac{1}{2}(z - p_o) + \frac{1}{2}(1 - p_o)\right) + \frac{1}{2}R_o(z - 1) - w_o(z - p_o) \quad (2)$$

In each retailer's profit function, there are two possible sources of revenue, either from the consumer demand that is realized (the first term), or from returning unsold units to the manufacturer for the return price (the second term). The last term represents the wholesale price payment to the manufacturer for the quantity  $Q_o$ . Note that the returns revenue in the second term is independent of the retail price. From this the retail price function can be derived as  $p_o(w_o) = \frac{1+z+2w_o}{4}$  and  $Q_o(w_o) = \frac{3z-1-2w_o}{4}$ .

The manufacturer's problem is to maximize  $\pi_{mo} = (w_o - c)(z - p_o(w_o)) - \frac{1}{2}R_o(z - 1)$  subject to  $w_o > 0$  and  $R_o \geq 0$  with the retailer getting atleast as much profit as the best possible deviation. Note that the possible deviation for the retailer

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<sup>5</sup>This implies that  $z$  be not too large (specifically  $z < 2 - c$  for the analysis in section 3.2). There can also be a possible case in which the retail demand is not positive in equilibrium if the low state of demand were to be realized. But this case is dominated by the channel with information and therefore it does not affect the results pertaining to the comparison between the channel formats.

is to the case of understocking of inventory  $\pi_{rd} = (1 - p_d)(p_d - w)$ . The corresponding Lagrangian for the manufacturer's optimization is,

$$L_{mo} = (w - c)(z - p) + \frac{R}{2}(z - 1) + \mu_R R + \mu_d(\pi_{ro}(w, R) - \pi_{rd}(w)) \quad (3)$$

This equilibrium exists if the degree of uncertainty is not high and  $z < (2 - c)$  which is the condition for positive demand in all states. We find that the returns price  $R_o^* = \frac{3+6c-z}{8}$  is strictly positive and it is independent of the retail price chosen, and the manufacturer can use it along with the wholesale price to extract retail surplus. Consequently, in equilibrium, the retailer's deviation profit is strictly binding and the manufacturer is able to extract all the excess profit over and above  $\pi_{rd}$  for any wholesale price that it charges. The equilibrium wholesale and retail prices are  $w_o^* = \frac{1+c}{2}$  and  $p_o^* = \frac{z+c+2}{4}$  respectively. Finally the equilibrium manufacturer and retailer profits are  $\pi_{mo}^* = \frac{(z^2+2z-12zc+2c^2+8c-1)}{16}$  and  $\pi_{ro}^* = \frac{(1-c)^2}{16}$ . Comparing the equilibrium manufacturer profits derived in the appendix, we can establish the conditions under which the channel equilibrium will involve over- versus under stocking. The following lemma states the conditions.

**Lemma 1** *i. If  $c < \frac{1}{3}$ , then manufacturer profits are higher in the overstocking equilibrium than in the understocking equilibrium.*

*ii. If  $c > \frac{1}{3}$ , then manufacturer profits are higher in the overstocking equilibrium if  $z > (12c - 3)$ , and they are higher in the understocking equilibrium when the reverse is true.*

In the understocking equilibrium, there is no inefficiency in terms of unsold stock being returned to manufacturer which has no salvage value. The decentralized channel contract consists of just a wholesale price instrument because of which there can be double marginalization of the retail price. In contrast, in the overstocking equilibrium, the manufacturer offers a wholesale price and a returns contract. With overstocking, there is an inefficiency in that there is excess stock which the retailer

may return and for which there is no value. However, the manufacturer has now two instruments, the wholesale price and the returns price, to manage the retailer actions. This allows the manufacturer to reduce double marginalization while appropriating greater surplus.

When the marginal cost of production is small enough the overstocking equilibrium always dominates for the manufacturer. The manufacturer is able to better extract retail surplus while at the same time facing little inefficiency due to the excess unsold stock that is returned and which has no value. When marginal cost of production is large, the overstocking equilibrium is only preferred by the manufacturer if the demand in the high state is large enough so as to balance the inefficiency of the unsold stock.

### 3.3 Comparing the Two Formats

In this section we investigate the manufacturer's choice of the channel format. Doing so helps us to highlight the trade-off between information and inventory in this bilateral monopoly channel.

**Proposition 1** *1. For the manufacturer, the information enabled channel always dominates the channel with understocking of inventory.*

*2. If  $c > \frac{1}{5}$ , the information enabled channel dominates the channel with overstocking of inventory. Otherwise, the channel with overstocking of inventory dominates the information enabled channel.*

**Proof.** It follows from directly comparing the equilibrium profits of the three cases.

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This proposition highlights the value of demand information for a manufacturer in a distribution channel. The point that we are interested in examining is whether the channel with inventory may be optimal for the manufacturer, even if perfect

information about the demand is available. The availability of demand information allows the parties in the channel to optimally adjust their actions to the actual demand realization. However, in this format there is double marginalization and the inability of the manufacturer to fully extract retailer profits.

The information enabled channel always dominates the channel with understocking of inventory for the manufacturer. In the understocking equilibrium the manufacturer is able to use only the wholesale price and is unable to extract retailer profits. At the same time in the understocking equilibrium the manufacturer suffers from the inability to respond to the actual realization of demand.

When the equilibrium involves overstocking of the channel inventory, the channel contract involves two instruments  $w$  and  $R$  and so the manufacturer can better extract retailer profits while controlling the extent of double marginalization. However, the disadvantage of the overstocking equilibrium is that the manufacturer takes back returns which have no value. Therefore as the marginal cost of production becomes large, the information enabled format becomes more attractive. With lower levels of  $c$  the overstocking equilibrium becomes more attractive. Now the ability of the manufacturer to extract retailer profits while controlling the retail price makes holding inventory more attractive for the manufacturer than a channel with demand information.

Thus the information enabled format does not always dominate for the manufacturer even if the information system provides perfect information about the demand. The trade-off that this model highlights is that while better demand information allows the channel to adjust exactly to the realization, it also has the strategic effect of resulting in relatively high retail prices because the manufacturer cannot fully extract retailer profits. Holding inventory in the channel mitigates this strategic effect and helps the manufacturer to better extract retail profits.

### 3.3.1 First-Best and Flexible Contracts

We have identified the trade-off between information and inventory in a simple decentralized channel with a wholesale price the possibility of returns of unsold goods. One may ask what happens when contracts are complete and the manufacturer is able to institute the first best vertically integrated channel solution. The vertically integrated channel is one in which the manufacturer takes all the downstream retail actions. The vertically integrated profits for the manufacturer in the information enabled case is  $\frac{1}{8}((1 - c)^2 + (z - c)^2)$  and this is always greater than the profits for the channel with inventory (both for the understocking and the overstocking cases). This implies that when contracts are complete and the manufacturer can implement a vertically integrated channel, it always pays the manufacturer to have an information enabled channel.

It is also useful to note the effect of contract flexibility based on the signal received. The information enabled channel shown in Figure 2 implies that both the manufacturer and the retailer have the flexibility to adjust their actions to the available demand information. Consider now the case in which only the retailer can use the information and not the manufacturer. This can be because the information is only revealed after the contract is written and therefore the manufacturer cannot offer a wholesale price that is contingent on the information. After the contract  $w$  is offered by the manufacturer, the information system provides a (perfect) signal of the state of demand  $\theta_s = z$  or  $1$  that will be realized and the retailer accordingly chooses the price  $p_x^s$ . The retail profit is simply  $\pi_{rx}^s = (\theta_s - p)(p - w)$  from which the optimal retail price for each state  $s$  is  $p_x^s(w) = \frac{\theta_s + w}{2}$ . The manufacturer's expected profit function is  $E_\theta(\pi_{mx}^s) = (w - c)[\frac{1}{2}(1 - p_x^l) + \frac{1}{2}(1 - p_x^h)]$ . The optimal wholesale price is  $w = \frac{\bar{\theta} + c}{2}$  where  $\bar{\theta} = \frac{1+z}{2}$  is the mean of the distribution. Thus the wholesale price responds to the mean demand realization while the retailer is able to use the information and respond accurately to each state of the demand. Comparing this to the case of flexible contracting by the manufacturer as in the analysis of section

3.1, we can show that with a flexible contract the manufacturer's profits are always higher. Thus allowing for contracts that are flexible to the realization of the signal makes the information enabled channel more valuable for the manufacturer.

### 3.4 The General Case of $0 < \rho < 1$

Consider now the general case with an imperfect demand information system. Following the game sequence in Figure 1, for each signal node denoted  $\tilde{k}$ , (where  $\tilde{k} = \tilde{h}$  or  $\tilde{l}$ ), the manufacturer announces a signal-dependent contract, which is then followed by the quantity and the pricing decision of the retailer. After that the uncertainty is revealed and stock is sold or returned, or both, depending on the state of the world and the specification of the contract. Given that information system is not perfect, the inventory constraint is not fully relaxed and retailer should have inventory in order to make a sale.

For each  $\tilde{k}$ , the retailer's expected profit function after the manufacturer announces the contract is given by:

$$\begin{aligned} \pi_{r\tilde{k}} = & \Pr(h|\tilde{k})\{p_{\tilde{k}}[\min((z - p_{\tilde{k}}), Q_{\tilde{k}})] + R_{\tilde{k}}[\max((Q_{\tilde{k}} - (z - p_{\tilde{k}})), 0)]\} \\ & + \Pr(l|\tilde{k})\{p_{\tilde{k}}[\min((1 - p_{\tilde{k}}), Q_{\tilde{k}})] + R_{\tilde{k}}[\max((Q_{\tilde{k}} - (1 - p_{\tilde{k}})), 0)]\} - w_{\tilde{k}}Q_{\tilde{k}} \end{aligned}$$

Given  $\tilde{k}$ , the first term denotes the revenue if the high demand were to be realized, the second term is the corresponding revenue if the low demand were to be realized and the final term is the cost of buying the stock  $Q_{\tilde{k}}$  at the wholesale price  $w_{\tilde{k}}$ . As in the previous case of  $\rho = 0$  there will be two possible types of equilibrium involving either overstocking or understocking.<sup>6</sup> However, in this case the strategies of the firms also depend upon the signal, and so there are four possible types of outcomes (overstocking / understocking  $\times$  high signal / low signal). We present the derivation of the four outcomes in the Appendix and discuss the main results below.

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<sup>6</sup>Here too the in-between case is never an equilibrium for the same reason as in the case of  $\rho = 0$ .

*Overstocking:* In addition to the received signal, the nature of the overstocking equilibrium depends upon the reliability of the information system. We first present the case of a high information signal ( $\tilde{k} = \tilde{h}$ ).

**Result 1** *Under the high signal,*

*i* There is a manufacturer contract which involves  $w_{\tilde{h}} = \frac{1}{4}(3z - 1 + 2c - (z - 1)\rho)$  and no return, i.e.,  $R_{\tilde{h}} = 0$  if  $z \leq \min(\frac{(3-\rho)(1-c)}{4(1-\rho)}, \frac{7-2c+\rho}{5+\rho})$ ,

*ii* There is a manufacturer contract which involves a strictly positive returns price  $R_{\tilde{h}} = \frac{1}{8(1-\rho)}[3(1 + 2c) - z - \rho^2(z - 1) - 2\rho(z + c + 2)]$  and  $w_{\tilde{h}} = \frac{1+c}{2}$  if  $z \leq \min(\frac{2-c+\rho}{1+\rho}, \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2})$ .

*iii* If both the conditions above on  $z$  are satisfied then for  $\rho < \rho^h = \frac{2[8z(c+z)+(1-c)^2]^{\frac{1}{2}} - (5z+1+2c)}{2z}$ , the manufacturer offers the no returns contract in (i) above, otherwise the manufacturer offers the returns contract in (ii) above.

When the signal indicates that the demand state is likely to be high, one might expect the channel equilibrium to involve overstocking by the retailer in order to take advantage of the high state of demand being realized. Furthermore, as in the case of  $\rho = 0$  we should expect that the overstocking equilibrium is likely as long as  $z$  is not too high (so that the inefficiency of returning unsold stock which have no value is mitigated). This is precisely what we find as shown in parts (i) and (ii) of the result above. If the information system is not very reliable ( $\rho < \rho^h$ ), then the manufacturer chooses not to offer any returns because it is more likely that the true state  $l$  is realized given this signal  $\tilde{h}$ . This means that the probability of returns from overstocking increases, increasing the cost to the manufacturer of offering returns for unsold goods. However, in this case because the manufacturer has only a simple wholesale price available not all the retail profits (over and above the profits from the best possible deviation) can be extracted by the manufacturer.

When ( $\rho > \rho^h$ ) the probability of the low state of demand being realized becomes low enough and the manufacturer now offers returns in the contract. This enables the manufacturer to better extract the retail profits. It may be the case that only one of the regularity condition is met and therefore only one type of contract may be feasible to offer. For high values of  $z$  which are greater than that indicated in parts (i) and (ii), neither of the overstocking contracts are feasible.

We now present the case of the low information signal ( $\tilde{k} = \tilde{l}$ ) and overstocking.

**Result 2** *Under the low signal, an overstocking equilibrium exists if  $\rho < \rho^l = \frac{z+1+2c-[2(1-c)^2+8cz]}{z-1}$  and  $z \leq \min(\frac{2-c-\rho}{1-\rho}, \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2})$ . The manufacturer contract includes a strictly positive  $R_{\tilde{l}} = \frac{1}{8(1+\rho)}[3(1+2c) - z + \rho^2(z-1) + 2\rho(z+c+2)]$  and  $w_{\tilde{l}} = \frac{1+c}{2}$ . If  $\rho > \rho^l$ , an overstocking equilibrium does not exist.*

If the low signal is obtained and it comes from a very reliable information system, then clearly inducing overstocking is inefficient as chances of wasteful production are high and the overstocking equilibrium will not exist. Only when the information system is relatively unreliable will overstocking occur if a low signal is realized. This is because the probability of a high state of demand is high enough in this case and the manufacturer may find it optimal to offer returns as it does not expect goods to be returned with high probability.

*Understocking:* We derive in the appendix the equilibrium for the case of understocking by the retailer. When the understocking equilibrium exists the manufacturer offers a wholesale price  $w_{\tilde{k}} = \frac{1+c}{2}$ , no returns are accepted, (i.e.,  $R_{\tilde{k}} = 0$ ) and the retail price is  $p_{\tilde{k}} = \frac{3+c}{4}$  and these outcomes apply for both the signals  $\tilde{k}$ .<sup>7</sup> This equilibrium exists

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<sup>7</sup>There are two possible types of retailer deviations that the manufacturer has to account for while choosing the contract: the retailer might deviate to overstocking and sell irrespective of the realized state, or the deviation might be to overstocking with truncated demand and the retailer selling only if the high state occurs. We analyze the equilibrium in which the deviation which is relevant and which affects the equilibrium contract is the one in which the retailer has positive demand irrespective of the state.

only when  $z \leq \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2}$  for signal  $\tilde{h}$ , and  $z \leq \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2}$  for signal  $\tilde{l}$ . Notice that since the condition for  $\tilde{l}$ , being strictly greater than 1 for all values of  $c$  and  $\rho$ , is more easy to be satisfied. This is intuitive since one would expect it to be easier to sustain an understocking equilibrium, which is suited for the low state, if a low signal is obtained. For this low signal, only for very large  $z$ , a retailer may be tempted to deviate to overstocking and an equilibrium may not exist.

Under both signals, whenever the equilibrium exists, the manufacturer profits are  $\frac{(1-c)^2}{8}$ . Because the manufacturer uses only the wholesale price it is less effective in extracting retail profits and the retailer makes excess profits than what it would have made from the best possible deviation. All this is exactly analogous to the results in the overstocking equilibrium for  $\rho = 0$  that we analyzed in section 3.

We can now combine the different cases above and illustrate how the equilibrium outcome varies with respect to model parameters of marginal cost ( $c$ ), reliability of the information system ( $\rho$ ) and degree of spread in demand uncertainty ( $z$ ). To do this we need to select parameter ranges for which the results above are valid (i.e., the equilibrium is not affected by the deviation to demand only in high state is not relevant) and then compute the *ex-ante* profits of the manufacturer, which in turn requires us to check the existence of the understocking and overstocking equilibria for the two possible received signals, and also which of the possible type of equilibrium outcome would be optimal to induce through the contract. In Figure 4 (attached at the end of the paper), we show the relationship between the manufacturer's profit the reliability of the information system ( $\rho$ ) on the horizontal axis for different values of  $c$  and a given value of  $z$ .

The point here is that the manufacturer's profit is not monotonic in the reliability parameter  $\rho$ . In fact, for high enough  $\rho$ , an increase in  $\rho$  may exacerbate the double marginalization problem in the channel. Thus even if a manufacturer could choose the reliability of the information system and even if higher reliability comes at no extra cost, the manufacturer would not want to institute a perfect information

system. As expected, for  $c = \frac{1}{5}$ , in Figure 4, we recover the result of Proposition 1, that the manufacturer profits at the two extremes of  $\rho$  are equal. We also want to point out, however, that the total *ex-ante* channel profit, i.e., sum of manufacturer's and retailer's profit, does show a monotonic increase with respect to  $\rho$  even as the manufacturer's profit alone falls due to increased double marginalization. This once again highlights the point that given availability of more complete contracts, a higher degree of reliability of information system will become more attractive for the manufacturer.

## 4 Retail Competition

The objective in this section will be to understand how the intensity of retail competition affects the manufacturer's choice of the channel format. To show the effect of competition, we compare the case of the channel with inventory ( $\rho = 0$ ) with the channel with perfect information ( $\rho = 1$ ). Suppose there are two competing retailers denoted by  $i = 1, 2$ . The retail market demand is uncertain and for retailer  $i$  is given by  $q_{is} = \theta_s - p_i + \gamma(p_j - p_i)$  where  $j = 3 - i$  and where  $p_i$  and  $p_j$  are the retail prices. Note that  $\gamma > 0$  is a measure of the cross-price elasticity of retail demand and therefore measures the intensity of retail competition.

*Channel with Demand Information* ( $\rho = 1$ ): After the information system truly reveals the state of demand  $\theta_s = z$  or 1, the manufacturer chooses a wholesale price  $w_s$  and the retailers accordingly choose the price  $p_{is}$  for  $i = 1, 2$ . The retail profit for retailer  $i$ , ( $i, j = 1, 2; i \neq j$ ) each state is  $\pi_{ris} = (p_{is} - w_s)q_{is}$  and so the optimal retail price for each state in the symmetric equilibrium is  $p_{1s}(w_s) = p_{2s}(w_s) \frac{\theta_s + w(\gamma+1)}{\gamma+2}$ . The manufacturer's profit function for each state is  $\pi_{mxs} = (w_s - c)[q_{1s} + q_{2s}]$  and the optimal wholesale price is  $w_{xs} = \frac{\theta_s + c}{2}$ . Substituting these back in the profit expressions of the manufacturer, we get  $E_\theta(\pi_{mxs}) = \frac{(\gamma+1)[(z-c)^2 + (1-c)^2]}{4(\gamma+2)}$ . Similarly, the ex-ante expected profit of each retailer is obtained as  $E_\theta(\pi_{rs}) = \frac{(\gamma+1)[(z-c)^2 + (1-c)^2]}{8(\gamma+2)^2}$ .

Note that perfect information for the retailer about the demand realization implies that there will be no inefficiency arising from the mis-alignment of the inventory ordered to the demand realization. The manufacturer and the retailer are both able to use the demand information to respond accurately to each state of demand. Consequently, as in the case of the single retailer, we find that the expected manufacturer and retailer profits are increasing in the variance of the demand distribution. Also, as expected, we find that the manufacturer profits increase when retail competition increases (i.e.,  $\gamma$  increases), whereas the retailer profits decrease with retail competition.

*Channel with Inventory* ( $\rho = 0$ ): As in the single retailer case, we have the possibility of equilibrium with overstocking and understocking of inventory. We derive these equilibrium for the channel with inventory in the appendix for both these cases. At the stage of the choice of the contract the manufacturer will choose whether to induce either overstocking or understocking of the inventory. In all cases we look for the symmetric equilibrium in retail strategies.

*Understocking Equilibrium:* In the understocking case, denoted by subscript  $u$ , the retailers' demand is higher than the quantity  $Q_{iu}$  ordered if the state of demand turns out to be high, but is exactly equal to the quantity in the low state of demand. Therefore  $Q_{iu} = 1 - p_{iu} + \gamma(p_{ju} - p_{iu})$  ( $i, j = 1, 2; i \neq j$ ). Given this  $\pi_{riu} = (p_{iu} - w_u)(1 - p_{iu} + \gamma(p_{ju} - p_{iu}))$  from which the optimal retail price will be  $p_{iu}(w_u) = p_{ju}(w_u) = \frac{1+w_u(1+\gamma)}{2+\gamma}$ . Given this response function, we can solve the manufacturer's problem as shown in the appendix. The equilibrium wholesale price can be derived as  $w_u^* = \frac{1+c}{2}$ . Thus the equilibrium retail price, quantity and profits are respectively  $p_{1u}^* = p_{2u}^* = \frac{3+c(1+\gamma)+\gamma}{2(2+\gamma)}$ ,  $Q_{1u}^* = Q_{2u}^* = \frac{(1-c)(1+\gamma)}{2(2+\gamma)}$ ,  $\pi_{r1u}^* = \pi_{r2u}^* = \frac{(1-c)^2(1+\gamma)}{4(2+\gamma)^2}$ , and  $\pi_{mu}^* = \frac{(1-c)^2(1+\gamma)}{2(2+\gamma)}$ .

*Overstocking Equilibrium:* In this case, denoted by subscript  $o$ , the retailer's order quantity is  $Q_{io} = z - p_{io} + \gamma(p_{jo} - p_{io})(i, j = 1, 2; i \neq j)$ . For  $i, j = 1, 2; i \neq j$ , retailer

$i$ 's profit function is,

$$\begin{aligned}\pi_{rio} &= p_o\left(\frac{1}{2}(z - p_{io} + \gamma(p_{jo} - p_{io})) + \frac{1}{2}(1 - p_{io}) + \gamma(p_{jo} - p_{io})\right) \\ &+ \frac{1}{2}R(z - 1) - w_o(z - p_{io} + \gamma(p_{jo} - p_{io})).\end{aligned}\quad (4)$$

In each retailer's profit function, there are two possible sources of revenue, either from the consumer demand that is realized (the first term, low or high state, each with probability =  $\frac{1}{2}$ ) or from returning unsold units to the manufacturer for the return price (the second term). The last term represents the wholesale price payment to the manufacturer for the quantity  $Q_o$ . From this the retail price function can be derived as  $p_{1o}(w_o) = p_{2o}(w_o) = \frac{1+z+2w_o(1+\gamma)}{2(2+\gamma)}$ . The manufacturer equilibrium contract can be then derived as shown in the appendix. The equilibrium contract involves  $w_o^* = \frac{2(1+c)+\gamma(1+z+2c)}{4(1+\gamma)}$  and  $R_o^* = \frac{2\gamma^2(1+z+2c)+z(3\gamma-2)+4c(4\gamma+3)+5\gamma+6}{8(1+\gamma)(2+\gamma)}$ . Given the contract, both the retailers find it optimal to order  $Q_{1o}^* = Q_{2o}^* = \frac{3z(2+\gamma)-2c(1+\gamma)-4-\gamma}{4(2+\gamma)}$  and price  $p_{1o}^* = p_{2o}^* = \frac{2(c+z+2)+\gamma(1+z+2c)}{4(2+\gamma)}$ . Finally the expected profits for the manufacturer and retailer, respectively are  $\pi_{mo}^* = \frac{(2+\gamma)(1+z)^2+4c(c+\gamma+\gamma-3z\gamma+4+6z)-4}{8(2+\gamma)}$  and  $\pi_{r1o}^* = \pi_{r2o}^* = \frac{(1-c)^2(1+\gamma)}{4(2+\gamma)^2}$ .

Comparison of the overstocking and the understocking equilibrium profits for the manufacturer under retail competition provides exactly analogous results as in Lemma 1 for the single retailer case. If  $c < \frac{1}{3}$ , then the manufacturer profit is higher in the overstocking equilibrium than in the understocking equilibrium. If the marginal cost of production is high and  $c > \frac{1}{3}$ , then manufacturer profits are higher in the overstocking equilibrium only if  $z > (12c - 3)$ . For small enough  $c$ , overstocking is always better but when cost of production (and hence cost of inefficient high production) is high enough, then overstocking is preferable only if the upside demand potential, given by  $z$ , is high enough.

Finally, we get to the question of the choice of channel format, we first observe that the manufacturer profit for a channel with demand information is strictly larger compared to that in the channel with inventory, whenever understocking is a preferred

equilibrium. Comparing the profits between the overstocking equilibrium and the channel with information we get the following proposition:

**Proposition 2** *When  $c \geq \frac{1}{5}$  the information enabled channel always dominates for the manufacturer compared to the channel with inventory. When  $c < \frac{1}{5}$ , then the channel with information dominates for the manufacturer only when retail competition is sufficiently intense  $\gamma > 4(1 - 5c)$ . Otherwise, the manufacturer prefers to have a channel with overstocking of inventory.*

This proposition highlights the effect of retail competition on the choice of the channel format by the manufacturer. Note as in the case of the single retailer, when the marginal cost of production is sufficiently large the manufacturer always prefers the channel with information. However, if  $c < \frac{1}{5}$ , then having inventory becomes attractive. Thus when the loss from excess production is low enough, the manufacturer may go in favor of a channel with inventory depending upon the degree of retail competition. In particular in markets with greater retail differentiation (less intense competition) the manufacturer prefers to have the channel with inventory. Whereas, the channel with information becomes optimal when the intensity of retail competition is higher. Recall, that while the (perfect) information system has the advantage of eliminating excess production, it also results in higher retail prices. With more intense retail competition there is already a downward pressure on retail prices and this is counterbalanced by the upward pressure on the retail prices caused by the information system. Therefore, with greater retail competition the information enabled format becomes more attractive.

## 5 Conclusion

In this paper we highlight a trade-off between information and inventory in a distribution channel. In a decentralized channel, while better demand information has

a positive direct effect for the manufacturer of improving the efficiency of inventory holding, it also has the strategic effect of increasing retail prices and limiting the extraction of retail surplus. Under some conditions having inventory in the channel can be beneficial for the manufacturer despite the inefficiency that there might be returns of the good which remains unsold. Having inventory in the channel can help the manufacturer to manage retail pricing behavior while better extracting retail surplus. Thus even if the information system is perfectly reliable, the manufacturer might not always want to institute an information enabled channel over a channel with inventory. We find that the channel with information is more attractive and preferred over the channel with inventory if the marginal cost of production is sufficiently high and when retail competition is sufficiently intense.

We also considered the case of an imperfectly reliable information system to show that the manufacturer profits are maximized at an intermediate level of the reliability of the information system. In other words, if the manufacturer were to choose the precision of the demand information system, the manufacturer would not prefer perfect information, even if such information was costless to acquire.

With retail competition, when the retailers are sufficiently differentiated, it pays to have a channel with inventory. Thus we show that the presence of inventory can play a role in managing competition among retailers and the manufacturer's ability to appropriate surplus and thereby present a rationale for why having costly retail inventory might be a "good" for the manufacturer from a strategic point of view. Finally, we make the general point that an information enabled channel will become more attractive for the manufacturer with more complete contracts or contracts that are flexible to the signal realizations.

There are some interesting issues that could be pursued in future research. In this paper we considered the case in which the channel members have joint access to the information system resulting in symmetric information about demand uncertainty for all the channel members. The case of asymmetric information where only one of

the two parties (say the retailer) have access to the information can be an interesting extension to study. A natural extension is to a model competition at the manufacturer level. Given that the information system can result in higher retail prices, it is possible that an information enabled channel would be more attractive when the manufacturers are less differentiated. Overall, the effect of information in distribution channels presents some interesting opportunities for future investigation.

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## Appendix

### The Single Retailer Channel

#### Channel with Inventory ( $\rho = 0$ )

Denote the inventory bought by the retailer as  $Q$  and the retail price as  $p$ . There are three possible types of channel equilibria given the inventory and pricing choices of the retailer:

- i A case of “understocking” in which the retailer chooses inventory that is exactly equal to the demand in the low state but less than the realized demand in the high state. Thus  $(1 - p) = Q < (z - p)$ .
- ii A case of “overstocking” in which the retailer chooses inventory that is exactly equal to the realized demand if the high state of demand were to be realized, but has excess inventory if the low state were to be realized. Thus  $(1 - p) < Q = (z - p)$ .
- iii An “in-between” case in which the retailer the retailer understocks if the high state were to be realized, but overstocks if the low state were to be realized. Thus in this case  $(1 - p) < Q < (z - p)$ .

We begin the analysis by showing that the in-between case (iii) described above can never be part of an equilibrium. Given a contract  $(w, R)$ , the retailer’s objective function is  $\pi_{rb} = \frac{1}{2}pQ + \frac{1}{2}p(1 - p) + R(Q - (1 - p)) - wQ$ . From this we can solve  $p = \frac{1+R+Q}{2}$  and substituting this into the objective function we get a quadratic in  $Q$  with a minimum ( $\frac{\partial^2 \pi_{rb}}{\partial Q^2} = \frac{1}{4} > 0$ ). Therefore there is no interior maximum for  $Q$  and the inventory choice by the retailer will be at the boundary characterized either by the understocking or the overstocking choice analyzed below. It can also be noted that it will never the case in equilibrium that the retailer chooses an inventory greater than the maximum possible demand ( $Q > z - p$ ) because in that case the retailer will always be better off reducing the inventory or the retail price charged. Similarly, the retailer will never choose an inventory less than the minimum

possible demand ( $Q < 1 - p$ ), because in that case the retailer will always be better off increasing the inventory or the price.

### Understocking Equilibrium

Based on the  $w$  offered by the manufacturer, the retailer chooses the inventory and the retail price. The retailer's profit function is  $\pi_{ru} = (1 - p_u)(p_u - w_u)$ . From this the optimal retail price is  $p_u(w_u) = \frac{1+w_u}{2}$  and the retailer profit becomes  $\pi_{ru}(w_u) = \frac{(1-w_u)^2}{4}$ . While choosing the contract  $w_u$  the manufacturer also has to account for the possibility that the retailer might deviate to any choice of inventory and price other than the equilibrium one. Specifically, the retailer might deviate to overstocking of inventory (because it is the case that the deviation to the case of in-between inventory is not optimal). Denoting the deviation by  $d$ , and given  $w_u$ , suppose the retailer deviates to choosing  $(1 - p_d) < Q = (z - p_d)$ .

Let us begin with the case where the retailer chooses the deviation price  $p_d$  so that there is demand in the off-equilibrium path irrespective of the state. The deviation profit will be  $\pi_{rd} = \frac{1}{2}(z - p_d)p_d + \frac{1}{2}(1 - p_d)p_d - (z - p_d)w_u$ . Thus for given  $w_u$  we will have the optimal deviation price to be  $p_d(w_u) = \frac{(1+z+2w_u)}{4}$ . Thus if the channel equilibrium were to involve understocking, then this understocking equilibrium must provide the retailer with atleast as much profit as  $\pi_{rd}(p_d(w_u))$ .

The manufacturer's problem is to choose  $w_u$  to maximize  $\pi_{mu} = (1 - p_u)(w_u - c)$  subject to  $w_u > 0$  and the retailer getting atleast the profit available from the best possible deviation. The corresponding lagrangian will be,

$$L_{mu} = (1 - p_u)(w_u - c) + \mu_{w_u}w_u + \mu_d(\pi_{ru}(w_u) - \pi_{rd}(w_u))$$

where the  $\mu$ 's are the corresponding lagrangian multipliers. Note that we are looking for a solution with  $w_u > 0$  and therefore  $\frac{\partial L_{mu}}{\partial w_u} = 0$  and  $\mu_{w_u} = 0$ . Let us first consider the case for which  $\mu_d > 0$  which means that  $\pi_{rd}$  is binding for the retailer. Then  $\frac{\partial L_{mu}}{\partial \mu_d} = 0$  from which we get that  $w_u = \frac{z+3}{12}$ . Next from  $\frac{\partial L_{mu}}{\partial w_u} = 0$  we can derive  $\mu_d = \frac{z-3-6c}{z-1}$ . And

$\mu_d > 0 \implies z > (3 + 6c)$ . Substituting the wholesale price the equilibrium manufacturer profit for this case turns out to be  $\pi_{mu} = \frac{(9-z)(z+3-12c)}{288}$ . The retailer profit is  $\pi_{ru} = \frac{(9-z)^2}{576}$ . Next note that this equilibrium is supported by the off-equilibrium retailer price  $p_d$  and the requirement that there is positive demand in both states in the off-equilibrium path. This requires that  $(1 - p_d) > 0$  which implies  $\frac{1}{8}(5 - \frac{7z}{3}) > 0 \implies z < \frac{15}{7}$ . However, comparing this with the earlier condition  $z > 3 + 6c$  we can see that there is no feasible value of  $z$  for which this case exists.

Next consider the case when  $\mu_d = 0$ . This implies that  $\frac{\partial L_{mu}}{\partial \mu_d} > 0$ , from which we get that  $z < 3 + 6c$ . Also given that  $w_u > 0$  which implies  $\frac{\partial L_{mu}}{\partial w_u} = 0$  and  $w_u = \frac{1+c}{2}$ . We require  $(1 - p_d) > 0$  and this implies  $z < (2 - c)$ . Thus this case exists for  $z < (2 - c)$ . The equilibrium manufacturer and retailer profits are respectively  $\pi_{mu}^* = \frac{(1-c)^2}{8}$  and  $\pi_{ru}^* = \frac{(1-c)^2}{16}$ .

Now we consider the case in which the possible deviation by the retailer involves demand only in the high state. The deviation profit will be  $\pi_{rd} = \frac{1}{2}(z - p_d)p_d - (z - p_d)w_u$ . This means  $p_d(w_u) = \frac{z}{2} + w_u$  and  $\pi_{rd} = \frac{1}{8}(z - 2w_u)^2$ . Therefore, the lagrangian for the manufacturer's optimization for this case will be,

$$L_{mu} = (1 - p)(w_u - c) + \mu_w w_u + \mu_d \left( \frac{(1 - w_u)^2}{4} - \frac{(z - 2w_u)^2}{8} \right)$$

First consider when  $\mu_d > 0$ , then  $\left( \frac{(1-w)^2}{4} - \frac{(z-2w)^2}{8} \right) = 0$ . This is a quadratic in  $w_u$  which can be solved for  $w_{u1} = (z - 1) + \frac{1}{\sqrt{2}}(z - 2)$  and  $w_{u2} = (z - 1) - \frac{1}{\sqrt{2}}(z - 2)$ . We can calculate  $\mu_d$  from the condition  $\frac{\partial L_{mu}}{\partial w_u} = 0$  to be  $\mu_d = \frac{1-2w_u+c}{1+w_u-z}$ . Substituting  $w_{u1}$  for  $w_u$  in the equation for  $\mu_d$  we get that  $\mu_d < 0$  which is impossible. Therefore  $w_{u1}$  cannot be a feasible solution. Consider the second root  $w_{u2}$ . Substituting it into the retailer price response we get  $p(w_{u2}) = (\frac{3z}{2} - 1) + \sqrt{\frac{1}{2}}(1 - \frac{z}{2})$ . Now for the equilibrium understocking demand to be positive  $(1 - p(w_{u2})) > 0$  implies that  $z < 0.7387$  which is impossible by assumption. Therefore, there cannot be a deviation by the retailer which involving positive demand only in the high state and with  $\mu_d > 0$ .

Finally, consider the case in which  $\mu_d = 0$  and  $\frac{\partial L_{mu}}{\partial \mu_d} > 0$ . As before  $w_u > 0$  and therefore  $\frac{\partial L_{mu}}{\partial w_u} = 0$  and  $\mu_{w_u} = 0$ . From this we get  $w_u = \frac{(1+c)}{2}$ . From  $\frac{\partial L_{mu}}{\partial \mu_d} > 0$  we get the condition  $z > (1 + c) + \frac{(1-c)}{\sqrt{2}}$ . The equilibrium profits are once again  $\pi_{mu}^* = \frac{(1-c)^2}{8}$

and  $\pi_{ru}^* = \frac{(1-c)^2}{16}$ . To summarize, in the understocking equilibrium  $w_u^* = \frac{(1+c)}{2}$ ,  $p_u^* = \frac{3+c}{4}$ ,  $\pi_{mu}^* = \frac{(1-c)^2}{8}$ , and  $\pi_{ru}^* = \frac{(1-c)^2}{16}$ .

### Overstocking Equilibrium

The retailer's profit function in the overstocking case is  $\pi_{ro} = p_o \left( \frac{z-p_o}{2} + \frac{(1-p_o)}{2} \right) + \frac{R_o}{2} (z-1) - w_o(z-p_o)$ . From this the optimal retail price  $p_o(w_o) = \frac{z+1+2w_o}{4}$ . The manufacturer problem is to maximize the profit function is  $\pi_{mo} = (w_o - c)(z - p_o) + \frac{R_o}{2}(z - 1)$  subject to  $w_o > 0$ ,  $R_o \geq 0$  and the retailer getting atleast the profit available from the best possible deviation. Note again that the only feasible deviation for the retailer will be to an understocking case. The retailer's off-equilibrium deviation profit will be  $\pi_{rd} = (1 - p_d)(p_d - w)$ . Finding the optimal  $p_d(w_o)$  and substituting back we get  $\pi_{rd}(w_o) = \frac{(1-w_o)^2}{4}$ . The lagrangian for the manufacturer's optimization will be,

$$L_{mo} = (w_o - c)(z - p_o) + \frac{R_o}{2}(z - 1) + \mu_{w_o}w_o + \mu_R R_o + \mu_d(\pi_{ro}(w_o, R_o) - \pi_{rd}(w_o))$$

Consider first the case in which  $R_o > 0$  and  $\mu_d > 0$  which implies that the retailer profit condition is binding. Because  $R_o > 0$ ,  $\mu_R = 0$  and  $\frac{\partial L_{mo}}{\partial R_o} = 0$  which implies that  $\mu_d = 1$ . Because  $w_o > 0$ ,  $\frac{\partial L_{mu}}{\partial w_o} = 0$  and  $\mu_{w_o} = 0$  and from this we can calculate  $w_o = \frac{1+c}{2}$ . The corresponding retail price is  $p_o^* = \frac{z+c+2}{4}$ . Because  $\mu_d > 0$  we will have  $\frac{\partial L_{mu}}{\partial \mu_d} = 0$  from which we can calculate  $R_o = \frac{3+6c-z}{8}$ . Since we need  $R_o > 0$ , this results in the condition  $z < (3 + 6c)$ . The equilibrium manufacturer profit can be calculated to be  $\pi_{mo}^* = \frac{(z^2+2z-12zc+2c^2+8c-1)}{16}$  and the equilibrium retailer profit is  $\pi_{ro}^* = \frac{(1-c)^2}{16}$ . Finally, for the demand in the low state to be positive we need  $(1 - p_o^*) > 0 \implies z < (2 - c)$ . Thus the binding condition for existence of this overstocking case is  $z < (2 - c)$ .

Finally, suppose that  $\mu_d = 0$ . Note as usual because  $w_o > 0$  then  $\mu_{w_o} = 0$ . Now suppose  $R_o > 0$ , then  $\frac{\partial L_{mo}}{\partial R_o} = 0$  and  $\mu_{R_o} = 0$ . But that will imply that  $\mu_d = 1$  which contradicts our assumption. Therefore,  $R_o = 0$  if  $\mu_d = 0$ . Of course, if  $\mu_d = 0$ , then we must have that  $\pi_{ro}(w_o, R_o) - \pi_{rd}(w_o) > 0$ . But  $\pi_{ro}(w_o, R_o) - \pi_{rd}(w_o) = -\frac{(z-1)(4z-3(1-c))}{8} < 0$  always. Therefore this case never exists.

**General Case** ( $0 < \rho < 1$ )

In this analysis, we proceed like in the channel with inventory case above, however we need to analyze both types of equilibrium (overstocking and understocking), for both types of possible signal ( $\tilde{h}$  or  $\tilde{l}$ ). We start with the  $\tilde{h}$  signal followed by the  $\tilde{l}$  signal.

*Understocking given  $\tilde{h}$*  : Based on  $w_{\tilde{h}}$  offered by the manufacturer, the retailer chooses the inventory and the retail price. The retailer's profit function is  $\pi_{r\tilde{h}u} = (1 - p_{\tilde{h}})(p_{\tilde{h}} - w_{\tilde{h}})$ . From this the optimal retail price is  $p_{\tilde{h}}(w_{\tilde{h}}) = \frac{1+w_{\tilde{h}}}{2}$  and the retailer profit becomes  $\pi_{r\tilde{h}u}(w_{\tilde{h}}) = \frac{(1-w_{\tilde{h}})^2}{4}$ . In choosing the contract  $w_{\tilde{h}}$  the manufacturer must guard against two possible deviations from the equilibrium. Either the retailer can deviate to overstocking with positive demand in both states or the retailer can deviate to overstocking with positive demand only in the high state. We derive the equilibrium for the case in which deviation that is relevant for the manufacturer to guard against in supporting the equilibrium wholesale price is the one in which the retailer may deviate to overstocking with positive demand in both states.

In deviating to overstocking with positive demand in both states (denoted by subscript  $d$ ), and given  $w_{\tilde{h}}$ , if the deviation price is  $p_{\tilde{h}d}$ , then the deviation profit will be  $\pi_{r\tilde{h}d} = \Pr(h|\tilde{h})(z - p_{\tilde{h}d})p_{\tilde{h}d} + \Pr(l|\tilde{h})(1 - p_{\tilde{h}d})p_{\tilde{h}d} - (z - p_{\tilde{h}d})w_{\tilde{h}}$ . This leads to an optimal deviation price to be  $p_{\tilde{h}d}(w_{\tilde{h}}) = \frac{1+z+2w_{\tilde{h}}+(z-1)\rho}{4}$ . Thus if the channel equilibrium were to involve understocking, then this understocking equilibrium must provide the retailer with atleast as much profit as  $\pi_{r\tilde{h}d}(p_{\tilde{h}d}(w_{\tilde{h}}))$ .

The manufacturer's problem is to choose  $w_{\tilde{h}}$  to maximize  $\pi_{m\tilde{h}u} = (1 - p_{\tilde{h}})(w_{\tilde{h}} - c)$  subject to  $w_{\tilde{h}} > 0$  and the retailer getting atleast the profit available from the best possible deviation. The corresponding lagrangian is:

$$L_{m\tilde{h}u} = (1 - p_{\tilde{h}})(w_{\tilde{h}} - c) + \mu_{w_{\tilde{h}}} w_{\tilde{h}} + \mu_{\tilde{h}d} (\pi_{r\tilde{h}u}(w_{\tilde{h}}) - \pi_{r\tilde{h}d}(p_{\tilde{h}d}(w_{\tilde{h}})))$$

We consider only  $w_{\tilde{h}} > 0$  and therefore  $\frac{\partial L_{m\tilde{h}u}}{\partial w_{\tilde{h}}} = 0$  and  $\mu_{w_{\tilde{h}}} = 0$ . Let us first consider the case for which  $\mu_{\tilde{h}d} = 0$ , which means that  $\pi_{r\tilde{h}d}$  is not binding for the retailer. Solving  $\frac{\partial L_{m\tilde{h}u}}{\partial w_{\tilde{h}}} = 0$  gives us  $w_{\tilde{h}} = \frac{1+c}{2}$ . In order for  $\pi_{r\tilde{h}d} \geq 0$ , we must have  $(1 < z < \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2})$ . This is one possible solution that leads to a manufacturer profit of  $\pi_{m\tilde{h}u1}^* = \frac{(1-c)^2}{8}$ . Next we consider the case when the constraint  $\pi_{r\tilde{h}d} = 0$  that

implies  $\mu_{\tilde{h}d} \geq 0$ . Solving this along with  $\frac{\partial L_{m\tilde{h}u}}{\partial w_{\tilde{h}}} = 0$  gives us  $w_{\tilde{h}} = \frac{3+z(1+\rho)^2+\rho(2-\rho)}{4(3-\rho)}$  and  $\mu_{\tilde{h}d} = \frac{z(\rho+1)^2-3(1+2c)+\rho(4+2c-\rho)}{(3(3-2\rho)+\rho^2)(z-1)}$ . Note that  $\mu_{\tilde{h}d} \geq 0$ , which may not be satisfied if the numerator is non-negative as the denominator is strictly positive. This solution gives the manufacturer profit as  $\pi_{m\tilde{h}u2}^* = \frac{(\rho^2-2z\rho-z\rho-z-3-2\rho+12c-4c\rho)(6\rho-9+2z\rho+z\rho^2+z-\rho^2)}{32(3-\rho)^2}$ . Analytical comparison of the two manufacturer profits is difficult, but a dense grid search of the entire relevant bounded parameter space,  $c$  and  $\rho$  (between 0 and 1) and a range of  $z$  reveals that  $\pi_{m\tilde{h}u2}^* \leq \pi_{m\tilde{h}u1}^*$  with equality holding only at one point. Thus given the signal  $\tilde{h}$ , an understocking equilibrium involves  $w_{\tilde{h}u}^* = \frac{(1+c)}{2}$ ,  $p_{\tilde{h}u}^* = \frac{3+c}{4}$ ,  $\pi_{m\tilde{h}u}^* = \frac{(1-c)^2}{8}$ , and  $\pi_{r\tilde{h}u}^* = \frac{(1-c)^2}{16}$ .

Finally, we need to find conditions for which the equilibrium outcomes identified above is immune to the retailer deviating to a case where he sells only in the  $h$  state. Notice that the deviation profit for the retailer in this case is  $\pi_{rd} = \Pr(h|\tilde{h})(z-p_d)p_d - (z-p_d)w_{\tilde{h}}$ . This means  $p_d(w_{\tilde{h}}) = \frac{z(\rho+1)+2w_{\tilde{h}}}{2(1+\rho)}$  and  $\pi_{rd}(w_{\tilde{h}}) = \frac{[z(1+\rho)-2w_{\tilde{h}}]^2}{8(1+\rho)}$ . As long as this deviation profit remains lower than  $\pi_{r\tilde{h}u}^* = \frac{(1-c)^2}{16}$  for  $w_{\tilde{h}} = w_{\tilde{h}u}^* = \frac{(1+c)}{2}$ , the equilibrium identified above remains robust to a retailer deviation of this kind. The condition simplifies to  $z < \frac{2(1+c)+\sqrt{2(1+\rho)(1+c)^2}}{2(1+\rho)}$ . Finally, for the manufacturer the  $w_{\tilde{h}u}^* = \frac{(1+c)}{2}$ , must be the best strategy and for the parameters chosen for Figure 4 this is indeed the case.

*Overstocking given  $\tilde{h}$* : The retailer's profit function in the overstocking case is  $\pi_{r\tilde{h}o} = \Pr(h|\tilde{h})(p_{\tilde{h}})(z-p_{\tilde{h}}) + \Pr(l|\tilde{h})[(p_{\tilde{h}})(1-p_{\tilde{h}}) + R_{\tilde{h}}(z-1)] - w_{\tilde{h}}(z-p_{\tilde{h}})$ . From this the optimal retail price is  $p_{\tilde{h}}(w_{\tilde{h}}) = \frac{z+1+2w_{\tilde{h}}+(z-1)\rho}{4}$ . The manufacturer problem is to maximize the profit function  $\pi_{mo\tilde{h}} = (w_{\tilde{h}} - c)(z - p_{\tilde{h}}) - \Pr(l|h)R_{\tilde{h}}(z - 1)$  subject to  $w_{\tilde{h}} > 0, R_{\tilde{h}} \geq 0$  and the retailer getting atleast the profit available from the best possible deviation to an understocking case. The retailer's deviation profit will be  $\pi_{r\tilde{h}d} = (1-p_{\tilde{h}d})(p_{\tilde{h}d}-w_{\tilde{h}})$ . Finding the optimal  $p_{\tilde{h}d}(w_{\tilde{h}})$  and substituting back we get  $\pi_{r\tilde{h}d}(w_{\tilde{h}}) = \frac{(1-w_{\tilde{h}})^2}{4}$ . The lagrangian for the manufacturer's optimization will be,

$$L_{m\tilde{h}o} = (w_{\tilde{h}} - c)(z - p_{\tilde{h}}) - \Pr(l|\tilde{h})R_{\tilde{h}}(z - 1) + \mu_{w_{\tilde{h}}}w_{\tilde{h}} + \mu_{R_{\tilde{h}}}R_{\tilde{h}} + \mu_{\tilde{h}d}(\pi_{r\tilde{h}o}(w_{\tilde{h}}, R_{\tilde{h}}) - \pi_{r\tilde{h}d}(w_{\tilde{h}}))$$

Starting with the case of  $\mu_{\tilde{h}d} = 0$  and the deviation constraint not binding: since  $w_{\tilde{h}} > 0$  then we must have  $\mu_{w_{\tilde{h}}} = 0$ . Now suppose  $\frac{\partial L_{m\tilde{h}o}}{\partial R_{\tilde{h}}} = 0$ , which gives us  $\mu_{R_{\tilde{h}}} = \frac{(1-\rho)(z-1)}{2} > 0$ . This implies that  $R_{\tilde{h}} = 0$ . Now substituting these into  $\mu_{w_{\tilde{h}}} = 0$ , we get  $w_{\tilde{h}1} = \frac{1}{4}(3z -$

$1 + 2c - (z - 1)\rho$ . Substituting  $R_{\tilde{h}} = 0$  and  $w_{\tilde{h}} = w_{\tilde{h}}$ , we get the deviation constraint  $(\pi_{r\tilde{h}o}(w_{\tilde{h}}, R_{\tilde{h}}) - \pi_{r\tilde{h}d}(w_{\tilde{h}}))$ . For this solution to be valid, this must be non-negative. We can obtain regularity conditions in  $z$  to ensure this as  $z \leq \min(\frac{(3-\rho)(1-c)}{4(1-\rho)}, \frac{7-2c+\rho}{5+\rho})$ . For positive demand in the off-equilibrium path in both the states, we must have  $z \leq \frac{7-2c+\rho}{5+\rho}$ . Under this solution, the manufacturer's expected profit is  $\pi_{m\tilde{h}o1}^* = \frac{(1-3z+z\rho-\rho+2c)^2}{32}$ .

Consider next the case in which  $R_{\tilde{h}} > 0$  and  $\mu_{\tilde{h}d} > 0$ , which implies that the retailer profit condition is binding. Solving  $\frac{\partial L_{m\tilde{h}o}}{\partial R_{\tilde{h}}} = 0$ ,  $\frac{\partial L_{m\tilde{h}o}}{\partial w_{\tilde{h}}} = 0$  and the binding profit condition together, we get  $w_{\tilde{h}} = \frac{1+c}{2}$ ,  $\mu_{\tilde{h}d} = 1$  and  $R_{\tilde{h}} = \frac{3+6c-z-\rho^2(z-1)-2\rho(z+c+2)}{8(1-\rho)}$ . The corresponding retail price is  $p_{\tilde{h}}^* = \frac{z+c+2+\rho(z-1)}{4}$ . From the positive demand condition in off-equilibrium path, we get a condition:  $z < \frac{2-c+\rho}{1+\rho}$ . In addition, the requirement for positive  $R_{\tilde{h}}$  gives us another condition,  $z \leq \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2}$ . Under these regularity conditions, the equilibrium manufacturer profit can be calculated to be  $\pi_{m\tilde{h}o2}^* = \frac{(2z-1+8c-12cz+2c^2+z^2)+(z-1)^2\rho^2+2\rho(z^2+2c(z-1)-1)}{16}$  and the equilibrium retailer profit is  $\pi_{r\tilde{h}}^* = \frac{(1-c)^2}{16}$ . Depending upon the parameter values, one or the other solution obtained above may not exist. Whenever both solutions exist, we can compare  $\pi_{m\tilde{h}o1}^*$  and  $\pi_{m\tilde{h}o2}^*$  and show that for  $\rho < \rho^{\tilde{h}} = \frac{2[8z(c+z)+(1-c)^2]^{\frac{1}{2}}-(5z+1+2c)}{2z}$ ,  $\pi_{m\tilde{h}o1}^*$  is higher for the manufacturer and hence for that,  $w_{\tilde{h}} = \frac{1}{4}(3z - 1 + 2c - (z - 1)\rho)$  and  $R_{\tilde{h}} = 0$ . Otherwise,  $w_{\tilde{h}} = \frac{1+c}{2}$  and  $R_{\tilde{h}} = \frac{3+6c-z-\rho^2(z-1)-2\rho(z+c+2)}{8(1-\rho)}$ .

*Understocking given  $\tilde{l}$* : Given  $w_{\tilde{l}}$  from the manufacturer, the retailer chooses the inventory and the retail price. The retailer's profit function is  $\pi_{ru} = (1 - p_{\tilde{l}})(p_{\tilde{l}} - w_{\tilde{l}})$ . From this the optimal retail price is  $p_{\tilde{l}}(w_{\tilde{l}}) = \frac{1+w_{\tilde{l}}}{2}$  and the retailer profit becomes  $\pi_{r\tilde{l}u}(w_{\tilde{l}}) = \frac{(1-w_{\tilde{l}})^2}{4}$ . Once again, all deviations from the understocking equilibrium need to be considered. Just as we did for the  $\tilde{h}$  signal, we again derive the equilibrium considering the case where the relevant off-equilibrium deviation is that of the retailer moving to overstocking when there is positive demand in both states of demand. Given  $w_{\tilde{l}}$ , let the retailer choose the deviation price  $p_{\tilde{l}d}$  so that there is demand in the off-equilibrium path irrespective of the state. The deviation profit will be  $\pi_{r\tilde{l}d} = \Pr(h|\tilde{l})(z - p_{\tilde{l}d})p_{\tilde{l}d} + \Pr(l|\tilde{l})(1 - p_{\tilde{l}d})p_{\tilde{l}d} - (z - p_{\tilde{l}d})w_{\tilde{l}}$ . This leads to an optimal deviation price of  $p_{\tilde{l}d}(w_{\tilde{l}}) = \frac{1+z+2w_{\tilde{l}}-(z-1)\rho}{4}$ . Thus if the channel equilibrium were to involve understocking, then this understocking equilibrium must provide the retailer with atleast as much profit as  $\pi_{r\tilde{l}d}(p_{\tilde{l}d}(w_{\tilde{l}}))$ .

The manufacturer's problem is to choose  $w_{\tilde{l}}$  to maximize  $\pi_{m\tilde{l}u} = (1 - p_{\tilde{l}})(w_{\tilde{l}} - c)$  subject to  $w_{\tilde{l}} > 0$  and the retailer getting atleast the profit available from the best possible deviation. The corresponding lagrangian will be:

$$L_{m\tilde{l}u} = (1 - p_{\tilde{l}})(w_{\tilde{l}} - c) + \mu_{w_{\tilde{l}}}w_{\tilde{l}} + \mu_{\tilde{l}d}(\pi_{ru}(w_{\tilde{l}}) - \pi_{rd}(p_{\tilde{l}d}(w_{\tilde{l}})))$$

Let us first consider the case for which  $\mu_{\tilde{l}d} = 0$  which means that  $\pi_{rd}(\cdot)$  is not binding for the retailer. Solving  $\frac{\partial L_{m\tilde{l}u}}{\partial w_{\tilde{l}}} = 0$  gives us  $w_{\tilde{l}} = \frac{1+c}{2}$ . In order for  $\pi_{rd} \geq 0$ , we must have  $(1 < z < \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2})$ . This is one possible solution that leads to a manufacturer profit of  $\pi_{m\tilde{l}u1}^* = \frac{(1-c)^2}{8}$ . Next we consider the case when the deviation profit constraint is binding. This implies  $\mu_{\tilde{l}d} \geq 0$ . Solving this along with  $\frac{\partial L_{m\tilde{l}u}}{\partial w_{\tilde{l}}} = 0$  gives us  $w_{\tilde{l}} = \frac{3+z(1-\rho)^2-\rho(2+\rho)}{4(3+\rho)}$  and  $\mu_{\tilde{l}d} = \frac{z(1-\rho)^2-3(1+2c)-\rho(4+2c+\rho)}{(3(3+2\rho)+\rho^2)(z-1)}$ . Note that  $\mu_{\tilde{l}d} \geq 0$ , which may not be satisfied if the numerator is non-negative as the denominator is strictly positive. This solution gives the manufacturer profit as  $\pi_{m\tilde{l}u2}^*$ , which in the relevant parameter space,  $c$  and  $\rho$  (between 0 and 1) and and a wide range of  $z$  always turns out to be less than or equal to  $\pi_{m\tilde{l}u1}^*$  with equality holding only at one point. Thus given the signal  $\tilde{l}$ , an understocking equilibrium involves  $w_{\tilde{l}u}^* = \frac{(1+c)}{2}$ ,  $p_{\tilde{l}u}^* = \frac{3+c}{4}$ ,  $\pi_{m\tilde{l}u}^* = \frac{(1-c)^2}{8}$ , and  $\pi_{r\tilde{l}u}^* = \frac{(1-c)^2}{16}$ .

To identify condition when the above outcome is immune to a retailer deviation involving demand only in high state, we start by identifying that the deviation profit will be  $\pi_{rd} = \Pr(h|\tilde{l})(z - p_d)p_d - (z - p_d)w_{\tilde{l}}$ . This means  $p_d^*(w_{\tilde{l}}) = \frac{z(1-\rho)+2w_{\tilde{l}}}{2(1-\rho)}$  and  $\pi_{rd}(w_{\tilde{l}}) = \frac{[z(1-\rho)-2w_{\tilde{l}}]^2}{8(1-\rho)}$ . Notice that this is valid only if  $p_d^*(w_{\tilde{l}}) < z$  as otherwise demand in High state is also negative. In addition to this regularity condition, comparing the equilibrium profit with the deviation profit, we get that whenever  $\frac{2(1+c)+[2(1-c)^2(1-\rho)]^{\frac{1}{2}}}{2(1-\rho)} \leq z \leq \frac{2(1+c)-[2(1-c)^2(1-\rho)]^{\frac{1}{2}}}{2(1-\rho)}$ , the identified equilibrium is immune to this deviation. Finally, for the manufacturer the  $w_{\tilde{l}u}^* = \frac{(1+c)}{2}$ , must be the best strategy and for the parameters chosen for Figure 4 this is indeed the case.

*Overstocking given  $\tilde{l}$* : The retailer's profit function in the overstocking case is  $\pi_{r\tilde{l}o} = \Pr(h|\tilde{l})(p_{\tilde{l}})(z - p_{\tilde{l}}) + \Pr(l|\tilde{l})[(p_{\tilde{l}})(1 - p_{\tilde{l}}) + R_{\tilde{l}}(z - 1)] - w_{\tilde{l}}(z - p_{\tilde{l}})$ . From this the optimal retail price  $p_{\tilde{l}}(w_{\tilde{l}}) = \frac{z+1+2w_{\tilde{l}}-(z-1)\rho}{4}$ . The manufacturer problem is to maximize the profit function is  $\pi_{m\tilde{l}o} = (w_{\tilde{l}} - c)(z - p_{\tilde{l}}) - \Pr(l|\tilde{l})R_{\tilde{l}}(z - 1)$  subject to  $w_{\tilde{l}} > 0$ ,  $R_{\tilde{l}} \geq 0$  and the retailer getting atleast the profit available from the best possible deviation to an understocking case. The

retailer's deviation profit will be  $\pi_{r\tilde{d}} = (1 - p_{\tilde{d}})(p_{\tilde{d}} - w_{\tilde{d}})$ . Finding the optimal  $p_{\tilde{d}}(w_{\tilde{d}})$  and substituting back we get  $\pi_{r\tilde{d}}(w_{\tilde{d}}) = \frac{(1-w_{\tilde{d}})^2}{4}$ . The lagrangian for the manufacturer's optimization will be,

$$L_{m\tilde{d}} = \Pr(h|\tilde{l})(w_{\tilde{d}} - c)(z - p_{\tilde{d}}) - \Pr(l|\tilde{l})R_{\tilde{d}}(z - 1) + \mu_{w_{\tilde{d}}}w_{\tilde{d}} + \mu_{R_{\tilde{d}}}R_{\tilde{d}} + \mu_{\tilde{d}}(\pi_{r\tilde{d}}(w_{\tilde{d}}, R_{\tilde{d}}) - \pi_{r\tilde{d}}(w_{\tilde{d}}))$$

Starting with the case of deviation constraint not binding, since  $w_{\tilde{d}} > 0$  then we must have  $\mu_{w_{\tilde{d}}} = 0$ . Now suppose  $\frac{\partial L_{m\tilde{d}}}{\partial R_{\tilde{d}}} = 0$ , which gives us  $\mu_{R_{\tilde{d}}} = \frac{(1-\rho)(z-1)}{2} > 0$ . This implies that  $R_{\tilde{d}} = 0$ . Now substituting these into  $\mu_{w_{\tilde{d}}} = 0$ , we get  $w_{\tilde{d}}^* = \frac{1}{4}(3z - 1 + 2c + (z - 1)\rho)$ . Substituting  $R_{\tilde{d}}^* = 0$  and  $w_{\tilde{d}} = w_{\tilde{d}}^*$  we can calculate the deviation constraint, which must be non-negative to be valid. However, that requires  $(1 \geq z \geq \frac{(3+\rho)(1-c)}{4(1-\rho)})$ , which is not possible so this solution is ruled out.

Consider next the case in which  $R_{\tilde{d}} > 0$  and  $\mu_{\tilde{d}} > 0$ , which implies that the retailer profit condition is binding. Solving  $\frac{\partial L_{m\tilde{d}}}{\partial R_{\tilde{d}}} = 0$ , and  $\frac{\partial L_{m\tilde{d}}}{\partial w_{\tilde{d}}} = 0$  and the binding profit condition together, we get  $w_{\tilde{d}} = \frac{1+c}{2}$ ,  $\mu_{\tilde{d}} = 1$  and  $R_{\tilde{d}} = \frac{3+6c-z+\rho^2(z-1)+2\rho(z+c+2)}{8(1+\rho)}$ . Since  $R_{\tilde{d}}$  must be positive, we must have  $z \leq \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2}$ . The corresponding retail price is  $p_{\tilde{d}}^* = \frac{z+c+2-\rho(z-1)}{4}$ . From the positive demand condition in off-equilibrium path, we get a regularity condition:  $z < \frac{2-c-\rho}{1+\rho}$ . Under these conditions, the equilibrium manufacturer profit can be calculated to be  $\pi_{m\tilde{d}}^* = \frac{(2z-1+8c-12cz+2c^2+z^2)+(z-1)^2\rho^2-2\rho(z^2+2c(z-1)-1)}{16}$  and the equilibrium retailer profit is  $\pi_{r\tilde{d}}^* = \frac{(1-c)^2}{16}$ . For  $\pi_{m\tilde{d}}^* \geq 0$ , we must have low enough reliability i.e.,  $\rho < \rho^{\tilde{l}} = \frac{z+1+2c-[2(1-c)^2+8cz]^{\frac{1}{2}}}{z-1}$ . This is the only surviving solution subject to the regularity conditions already mentioned above.

## Downstream Retail Competition

### Channel with Inventory ( $\rho = 0$ )

In the competitive case also, we need to consider two types of equilibrium as before, as the in-between case can be ruled out for equilibrium (and deviations) for reasons similar to the single retailer channel. We solve for the symmetric equilibrium.

## Understocking Equilibrium

Based on the  $w$  offered by the manufacturer, the retailers choose the inventory and the retail price. Retailer  $i$ 's profit function is  $\pi_{riu} = [1 - p_i + \gamma(p_j - p_i)](p_i - w)$ . From this the optimal retail price is  $p_1(w) = p_2(w) = \frac{1+w(1+\gamma)}{2+\gamma}$  and retailer  $i$ 's profit can be expressed as  $\pi_{riu}(w)$ . Given the contract no retailer's should have the incentive to unilaterally deviate to any choice of inventory and price other than understocking. Like in the single retailer case, once again the deviation possibilities involve overstocking, either with positive demand in both states or only in the high state. We will consider the equilibrium where the relevant off-equilibrium deviation supporting the equilibrium involves the case where there is demand in both states. Later on we will identify conditions when this equilibrium is immune to the deviation to the other type of overstocking. Without loss of generality, let the deviating retailer be retailer 1.

If retailer 1 deviates to overstocking, it chooses  $Q = (z - p_1 + \gamma(p_2 - p_1))$ . Let retailer 1's choice of the deviation price be called  $p_{1d}$ . The deviation profit function will be  $\pi_{r1d} = \frac{1}{2}(z - p_{1d} + \gamma(p_2(w) - p_{1d}))p_{1d} + \frac{1}{2}(1 - p_{1d} + \gamma(p_2(w) - p_{1d}))p_{1d} - (z - p_{1d} + \gamma(p_2(w) - p_{1d}))w$ . Thus for a given  $w$ , we will have the optimal deviation price to be  $p_{1d}(w) = \frac{4w(\gamma+1)^2+z(\gamma+2)+3\gamma+2}{4(\gamma^2+3\gamma+2)}$ . The equilibrium contract must provide the retailer with atleast as much profit as  $\pi_{r1d}(p_{1d}(w))$ .

The manufacturer's problem is to choose  $w$  to maximize  $\pi_{mu} = [(1 - p_1 + \gamma(p_2 - p_1)) + (1 - p_2 + \gamma(p_1 - p_2))](w - c)$  subject to  $w > 0$  and the retailer getting atleast the profit available from the deviation identified above. The corresponding lagrangian will be,

$$L_{mu} = [(1 - p_1 + \gamma(p_2 - p_1)) + (1 - p_2 + \gamma(p_1 - p_2))](w - c) + \mu_w w + \mu_d (\pi_{riu}(w) - \pi_{rid}(p_{1d}(w)))$$

where the  $\mu$ 's are the corresponding lagrangian multipliers. Parallel to the analysis in the single retailer case, we look at the solution candidates one by one based on the relevant first order conditions. One solution candidate is  $w_1^* = \frac{1+c}{2}$ , which exists only if  $z \in (1, \frac{4c(\gamma^2+4\gamma+3)+(4\gamma^2+9\gamma+6)}{2+\gamma})$  and the deviation constraint is not binding. Whenever it exists, it gives  $\pi_{mu} = \pi_{mu1}^* = \frac{(1+\gamma)(1-c)^2}{2(2+\gamma)}$  as the manufacturer profit. Another solution candidate is  $w_2^* = \frac{z(\gamma+2)+7\gamma+6}{8(\gamma^2+4\gamma+3)}$ , which exists only if  $[\frac{4c\gamma^2+4\gamma^2-9\gamma-16c\gamma-12c-6}{2+\gamma} < z < (9 + 8\gamma)]$

and the deviation constraint is binding. Whenever this exists, it gives  $\pi_{mu} = \pi_{mu2}^* = \frac{(8\gamma+9-z)(z\gamma+2z+7\gamma+6-8c\gamma^2-32c\gamma-24c)}{32(3+\gamma)(\gamma^2+4\gamma+3)}$ . It can be shown that  $\pi_{mu2}^* \leq \pi_{mu1}^*$ . Thus the maximum equilibrium manufacturer profits are  $\pi_{mu}^* = \frac{(1+\gamma)(1-c)^2}{2(2+\gamma)}$  and  $w^* = \frac{1+c}{2}$ .

To see if the equilibrium with  $w_1^*$  is immune against one retailer unilaterally deviating to the overstocking case of the type involving only high state demand, we start with calculating that the deviating retailer's profit function will be  $\pi_{rid2}(w) = \frac{[z(\gamma+2)+\gamma-w(5\gamma+\gamma^2+4)]^2}{(2+\gamma)(3\gamma+\gamma^2+2)}$ . For the retailer deviating from the equilibrium path to this deviation, however, it is sufficient that  $(\pi_{riu}(w) - \pi_{rid2}(w))$  be negative for  $w^* = \frac{1+c}{2}$ . The condition for that turns out to be that  $z \in \left( \frac{c(8+14\gamma+\gamma^3+7\gamma^2)+(5\gamma^2+10\gamma+\gamma^3+8)\pm 2[2((1-c)^2(4+6\gamma^3+13\gamma^2+12\gamma+\gamma^4)-12(1-\gamma))]}{2(\gamma^2+4\gamma+4)} \right)^{\frac{1}{2}}$ . Thus as long as  $z$  is between the two roots, the equilibrium identified above is valid. Finally for the manufacturer  $w_1^*$  is always the best strategy and it can be shown that no other strategy (that makes the retailer deviation involving the demand in the high state only relevant) is profit improving.

### Overstocking Equilibrium

The retailer's profit function in the overstocking case is  $\pi_{rio} = \frac{1}{2}[(z - p_i + \gamma(p_j - p_i))(p_i) + (1 - p_i + \gamma(p_j - p_i))(p_i) + R(z - 1)] - w(z - p_i + \gamma(p_j - p_i))$ . From this the optimal retail price  $p_i(w) = \frac{2w(1+\gamma)+z+1}{2(\gamma+2)}$ . Given this, the manufacturer problem is to maximize the profits given by  $\pi_{mo} = (w - c)[(z - p_1 + \gamma(p_2 - p_1)) + (z - p_2 + \gamma(p_1 - p_2))] - R(z - 1)$ , subject to  $w > 0$ ,  $R \geq 0$  and the retailer getting atleast the profit available from the best possible deviation. We need to consider deviation to the understocking case. Again, without loss of generality, let retailer 1 deviate and charge a price of  $p_{1d}$ . We can show that  $p_{1d}(w) = \frac{4w(1+\gamma)^2+(3+z)\gamma+4}{4(2+3\gamma+\gamma^2)}$  and  $\pi_{r1d}(p_{1d}(w)) = \frac{[3\gamma+4+z\gamma-4w(1+\gamma)]^2}{16(\gamma+2)(3\gamma+2+\gamma^2)}$ . To solve the manufacturer's problem, we can set up the following lagrangian:

$$L_{mo} = (w - c)[(z - p_1 + \gamma(p_2 - p_1)) + (z - p_2 + \gamma(p_1 - p_2))] - R(z - 1) \\ + \mu_w w + \mu_R R + \mu_d (\pi_{rio}(w, R) - \pi_{rid}(w))$$

Proceeding in the usual manner to consider all solution candidates, we can show that the only solution candidate that can survive involves retailer's deviation constraint to be binding and it involves a positive returns price  $R$ . Whenever  $z < \frac{3\gamma+4-2c(1+\gamma)}{2+\gamma}$  the

manufacturer may offer a wholesale price  $w^* = \frac{2c(1+\gamma)+2+(\gamma(z+1))}{4(1+\gamma)}$  and the returns price  $R^* = \frac{2\gamma^2z+3z\gamma-2z+2(2c+1)\gamma^2+16c\gamma+5\gamma+12c+6}{3\gamma+2+\gamma^2}$ . This results in a manufacturer profit of  $\pi_{mo}^* = \frac{(\gamma+2)(z+1)^2+4c(c+c\gamma+\gamma-3z\gamma+4+6z)-4}{8(\gamma+2)}$ .

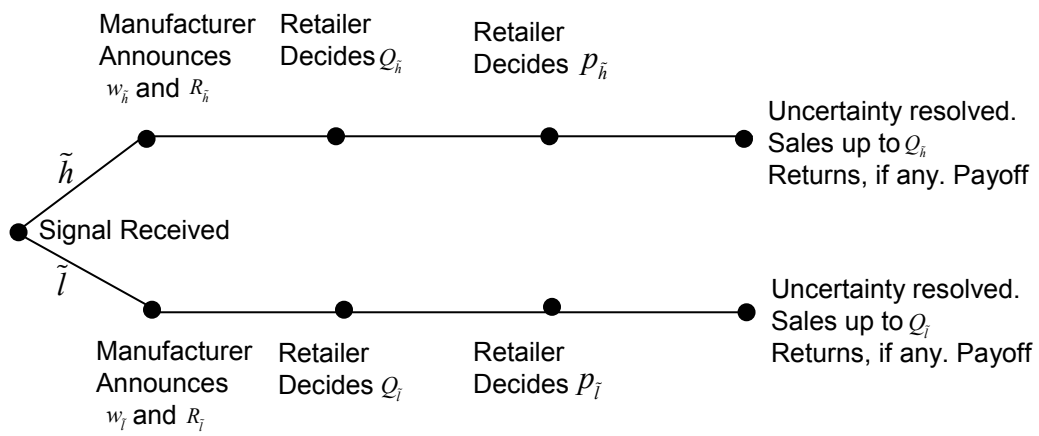


Figure 1

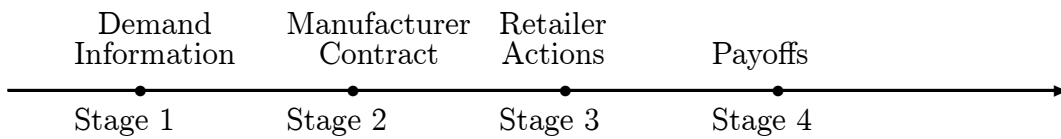


Figure 2: Channel with Perfect Information

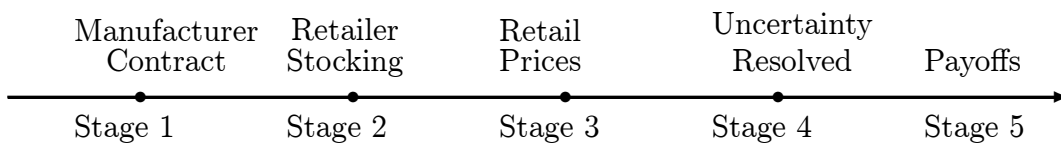


Figure 3: Channel With Inventory

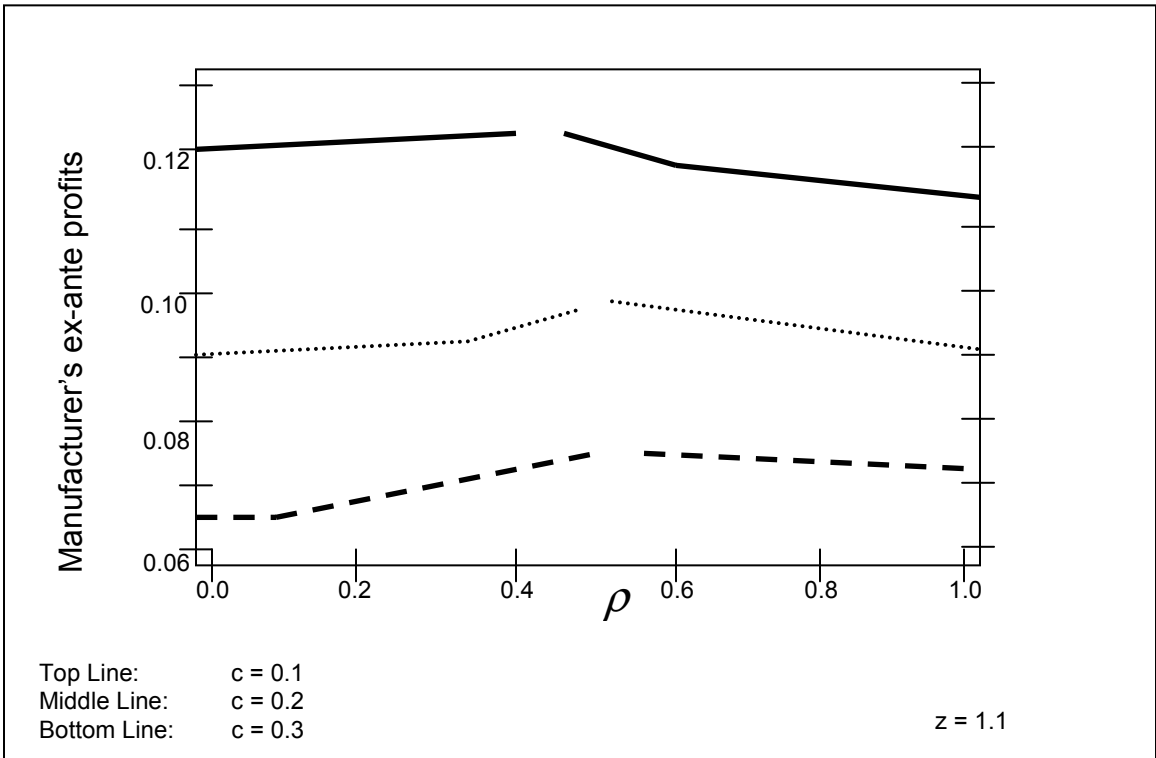


Figure 4