

Information and Inventory in Distribution Channels*

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Abstract

We examine the trade-offs between demand information and inventory in a distribution channel. While better demand information has a positive direct effect for the manufacturer in improving the efficiency of holding inventory in a channel, it can also have the strategic effect of increasing retail prices and limiting the extraction of retail profits. Having inventory in the channel can help the manufacturer to manage retail pricing behavior while better extracting retail surplus. Thus even if the information system is perfectly reliable, the manufacturer might not always want to institute an information enabled channel over a channel with inventory.

We show this first in a channel with a single retailer, where the channel with perfect information is preferred over the channel with inventory only if the marginal cost of production is sufficiently high. We also analyze a channel with an imperfectly reliable information system and find that if the manufacturer were to choose the precision of the demand information system, it might not prefer perfect information, even if such information was costless to acquire. In a channel with competing retailers, the channel with perfect information is preferred when retail competition is sufficiently intense. Thus, the presence of inventory can play a role in managing competition among retailers and in helping the manufacturers to appropriate surplus especially when retailers are sufficiently differentiated.

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1 Introduction

A basic aspect of retailing is that retailers must carry inventory of products to meet potential demand. While the need for holding inventory can simply arise due to the time required for physical delivery of the good at the retail location, in many channels uncertainty about demand at the time of the contract can also be an important factor. If demand is uncertain, then the inability to forecast demand accurately can result in a mismatch between the inventory that is held by the retailer and the realized demand. This leads to inefficient production and to either understocking or overstocking of the good as compared to the realized demand. An important response to the demand uncertainty in channels is the institution of better information systems to help the channel better align inventory holding with demand. Aligning the inventory holding through better demand information is one characteristic of many initiatives in retailing such as quick response, collaborative planning and forecasting and other such practices. Information-enabled retailing has been adopted in industries such as apparel, consumer electronics and automobile retailing (see Hammond et. al. 1991 for examples).

Interestingly, despite the obvious advantages of reduced inventory, the experience of manufacturers adopting improved information systems has been somewhat mixed. The fashion goods industry highlights this issue. The literature suggests that manufacturers of fashion products should benefit substantially from the adoption of these practices due to the industry characteristics of high demand uncertainty and short selling cycles. But manufacturers in the industry seem to be less than enthusiastic about investing in channel information systems (Hammond et al 1991). Fisher (1997) presents evidence from several other industries (that are contrary to popular belief) which suggests that initiatives that reduce the inventory in a channel need not always be attractive for manufacturers.

We investigate the trade-off between information and inventory in a decentralized channel. To begin the argument consider a world in which complete contracting is possible which is equivalent to either the manufacturer being able to sell directly to consumers or being able to achieve a first-best vertically integrated channel. In such a world, we find that having better information is always more efficient than having inventory. Thus it is only in a decentralized channel with limited contracting (where the manufacturer cannot contractually achieve the vertical integration) that there exists a trade-off between inventory and information.

Imperfect demand information will result in the mis-alignment of inventory with realized demand. One possibility for the manufacturer then is to consider selling the good to the retailer using a wholesale price contract, but with a provision to take back returns of the unsold good. As the

quality of the information improves there is less need for the returns contract, and at the extreme with perfect information there need not be any returns and the contract will endogenously consist of only the wholesale price. The trade-off that we show is the following: Better information about demand has a positive direct effect on manufacturer profits due to improved efficiency of inventory holding in the channel. But there is also an indirect or strategic effect of information that negatively affects manufacturer profits as it leads to greater double marginalization and too high retail prices.¹

We find that the manufacturer would not always want to institute a channel with information over a channel with inventory even if the information system is perfect. This will be the case when the strategic effect of information on retailer behavior dominates the direct benefit of information in terms of efficient inventory holding. While in a channel with inventory the manufacturer faces the disadvantage that there might be underproduction or overproduction of inventory, it also endogenously facilitates the offer of returns of unsold goods. As a result, the surplus of retailer is better extracted while limiting the double marginalization. This presents a rationale for why having costly retail inventory might be “good” for the manufacturer from a strategic point of view. We find that the channel with perfect information dominates the channel with inventory if the marginal costs of production are sufficiently high or (in a model with retail competition) if retail competition is sufficiently intense. Higher marginal costs of production lead to less double marginalization (i.e., lower strategic effect) and higher cost of having excess inventory in the channel (i.e., greater direct effect). This makes the channel with information more attractive for the manufacturer. Similarly, more intense retail competition counteracts the double marginalization problem also lowering the disadvantage of the strategic effect and making the channel with information more attractive.

We also investigate the effect of an imperfect information system which can predict the true state of demand with less than perfect reliability. In this case too, there can be possible understocking or overstocking of the good compared to the actual demand. We find that the manufacturer’s profit is in general maximized at some intermediate value of the reliability of the information system. In other words, if the manufacturer were to choose the precision of the demand information system, the manufacturer would not prefer perfect information, even if such information was costless to acquire. We also find that when the reliability of the information system is a choice variable, the manufacturer profits are maximized at higher values of the reliability when the production costs or the upside

¹This idea of limited contracting as a driver of organizational or contractual choice has also been investigated in other contexts such as in customer relationship management (Anderson 2002) or bargaining in a vertical channel (Iyer and Villas-Boas 2003).

demand potential is higher.

1.1 Related Research

Milgrom and Roberts (1988) examine the relationship between information and inventory in the organization of production for a multi-product firm and show that inventory and information can be strict substitutes in production. Dudley and Lasserre (1989) provide empirical evidence for this idea. In contrast, our paper examines the information versus inventory question in the context of a distribution channel in which the manufacturer has to sell to the market through a retail channel. In a distribution channel the trade-off between information and inventory is not only governed by the direct effect of information in aligning the production to actual demand realizations, but also by the strategic effect that information and inventory have on retailer behavior. Thus our analysis shows that in a distribution channel information need not always substitute for inventory.

Our work is related to the literature on channel coordination (e.g., Jeuland and Shugan 1983, Moorthy 1987, Lal 1990, Villas-Boas 1998 and Iyer 1998). In this paper we examine the role of better demand information and retail inventory in enabling the manufacturer to control retail pricing behavior. Our paper also endogenizes the return of unsold merchandise to the manufacturer and points out a different strategic role for the practice than what was shown in the previous literature (Padmanabhan and Png 1997). The literature in operations management and marketing has examined the role of contractual practices in the face of demand uncertainty. For example, Cachon and Fisher (2000) study sharing of demand information between an upstream and a downstream partner. More recently, Bialogorsky and Koenigsberg (2005) have analyzed how lead times affect the allocation of ownership of channel inventory between the manufacturer and the retailer. As in these papers, our work also analyzes a channel with demand uncertainty, yet the focus here is on highlighting the strategic trade-off between inventory and information in a distribution channel relationship.

The rest of this paper is organized as follows. In the next section, we develop a model of a distribution channel and introduce the elements of demand, uncertainty and the information system that the manufacturer and retailer can use. Section 3 starts with the analysis of the two polar cases of information reliability, i.e., when the information is either completely reliable or completely unreliable. Later in the section, we analyze the case of imperfect reliability. Section 4 presents a model with downstream competition. Section 5 concludes and discusses some limitations and directions for future research.

2 The Model

We develop a simple model where all parties in the channel are uncertain about demand and explore the role of information and inventory in this distribution channel. The channel consists of a manufacturer who sells through a retailer who decides on the retail price that affects consumer demand. The manufacturer has a constant marginal cost of production given by c . We also assume that the manufacturer has no salvage value for the product but may endogenously decide to allow returns of the unsold goods from the retailer at a price specified in the contract. Note that in this way the marginal cost is also the cost of excess production in the channel which is borne by the manufacturer if he accepts returns and by the retailer if no returns are accepted.²

The retail demand is uncertain and is given by $q = \theta_s - p$, where p is the retail price and subscript s represents the uncertainty in the market demand potential. We assume that there are two states of demand ($s = h, l$) that occur with equal probability. We normalize the low state of demand $\theta_l = 1$ and denote the high state of demand as $\theta_h = z > 1$. Thus z is a measure of the spread in the distribution representing the degree of demand uncertainty or the upside potential in demand.

Firms can use the demand information generated by the information system to respond to demand uncertainty. The information system may not be perfect in predicting the state of the world *ex-ante*, but it does predict the demand better than what is implied by the prior distribution on θ_s . Thus the demand information system can be interpreted as a knowledge system which provides better information to decision maker than his prior belief in the sense of the match of the belief (after getting the information) to actual ex-post state that will be realized (see for example Chen et. al 2001). The output of the information system is a common signal (to both parties) which indicates whether $\theta_s = 1$ or z , which in general may not be fully reliable. The reliability of the information system is modeled as follows: Let h and l refer to the true state of demand being high or low, and let \tilde{h} and \tilde{l} denote the signal indicating that the demand will be h or l respectively. As in Chen et. al (2001), we define the reliability of the information system by a measure $\rho \in [0, 1]$. Our formulation has the following properties: i) When $\rho = 1$ the system is perfectly reliable in the sense that $\Pr(\tilde{h}|h) = \Pr(\tilde{l}|l) = 1$. At the other extreme, when the system is completely unreliable and $\rho = 0$, the signal is no improvement over the prior. ii) The information system is unbiased in that the unconditional probability of a signal indicating demand to be high (low) is equal to the true prior probability of demand being high (low): i.e., $Pr(h) = Pr(\tilde{h})$ and $Pr(l) = Pr(\tilde{l})$. iii) A consistency

²If there is a positive salvage value, then the cost of excess production will be $c - f$. The qualitative results of the paper would hold for any positive salvage value $f < c$.

property $Pr(\tilde{h}) = Pr(h)Pr(\tilde{h}|h) + Pr(l)Pr(\tilde{h}|l)$ and analogously for $Pr(\tilde{l})$. This simply implies that the expected likelihood (over the true distribution) of each signal is the equal to the probability of the signal. Given these properties we can derive $Pr(\tilde{h}|h) = Pr(\tilde{l}|l) = \frac{1}{2} + \frac{\rho}{2}$ and $Pr(\tilde{h}|l) = Pr(\tilde{l}|h) = \frac{1}{2} - \frac{\rho}{2}$.

A decision maker in a firm observes the signal from the information system and uses it to revise the conditional probability (belief) of the demand being high or low given the signal. In other words, the decision makers at the firms would compute the probabilities such as $Pr(h|\tilde{h})$ and $Pr(l|\tilde{h})$. For example, to compute $Pr(h|\tilde{h})$, note that $Pr(h \cap \tilde{h}) = Pr(\tilde{h}|h)Pr(h) = Pr(h|\tilde{h})Pr(\tilde{h})$. Now given that $Pr(h) = Pr(\tilde{h})$, we have that $Pr(h|\tilde{h}) = \frac{1}{2} + \frac{\rho}{2}$. Note that for any $\rho > 0$ the signal is always meaningful in that $Pr(h|\tilde{h}) = Pr(l|\tilde{l}) > Pr(h) = Pr(l)$. This implies that having obtained the signal, a firm will not ignore the signal as it improves the probability of being correct. As ρ increases towards 1, the reliability of signal improves and the probability of wrong prediction of the state decreases. However, as long as ρ is less than 1, there is a positive probability that the channel ends up with a mismatch between the inventory and the realized demand.

Note that the contract and actions of both the manufacturer and the retailer will be signal dependent whenever $\rho > 0$. The timing of the game of this model is as shown in Figure 1. Based on the common information signal, the manufacturer offers a contract ($w_{\tilde{k}}$ and $R_{\tilde{k}}$). Because the signal (of \tilde{h} or \tilde{l}) may be imperfect, the retailer faces the possibility of ordering a quantity $Q_{\tilde{k}}$, which may be too high or too low for the realized state of demand. After the quantity stocking decision the retailer chooses a retail price $p_{\tilde{k}}$. At this point, the true state of demand is revealed and the retailer sells according to the demand but only up to the quantity stocked. If there is excess stock after meeting the demand and if the manufacturer accepts returns in the contract (i.e., $R_{\tilde{k}} > 0$), then the retailer also gets revenue from returning the unsold stock.

The resolution of uncertainty after retail decisions on pricing and stocking represents a variety of situations in which the marketing actions of the retailer have to be deployed before the exogenous uncertainty about consumer demand is revealed. For example, it can represent the case in which retailers have to commit to posted prices before consumers buy and where the cost of changing prices is substantial.³ Large retailers like Safeway have organizational and operational constraints which restrict the number of price changes to a frequency of at best once a week or less. Furthermore, it is a reality in retailing that promotional programs have to be planned in advance and then be advertised to consumers before they make purchasing decisions and the demand uncertainty is resolved.

³Levy et. al (1997) report menu costs at about 52 cents per price change on an average. The costs of price changes amount to around 35 percent of net margins in US supermarket industry

3 Analysis of the Model

Consider first that complete contracts are possible that allow the manufacturer to achieve the first best vertically integrated channel solution in which it is as if the manufacturer takes the downstream retail actions. We present the general solution for the vertically integrated channel in the Appendix and show that the expected channel profits are always (weakly) increasing in the reliability of the information signal ρ . This implies that when contracts are complete or when the manufacturer can implement a vertically integrated channel, it always pays the manufacturer to have a channel with information.

The natural question to ask now is whether there will be a trade-off between information and inventory in the channel if the manufacturer cannot implement a vertically integrated solution. Can it be the case that the manufacturer will prefer to have inventory in a decentralized channel? This question can be addressed by comparing the two polar cases of information reliability: the case when $\rho = 0$ in which the information signal does not provide the players information that is any better than the prior, and the case of perfect information when $\rho = 1$ in which the manufacturer and the retailer have the knowledge to adjust their actions to the exact demand realizations.

3.1 Channel With Perfect Information ($\rho = 1$)

If the information system is perfect, then the manufacturer will be able to align the production of inventory with the actual realization of demand and there is no need for excess inventory. Thus this format implies a demand information system that enables the channel to line up inventory according to the demand realization.⁴ In this manner, perfect information works as a substitute for inventory. The sequence of moves is a special case of that in Figure 1 since the manufacturer and the retailer are able to choose actions after receiving perfect information about the demand state.

We denote this perfect information case by the subscript x and provide the solution below. After the information system provides a (perfect) signal of the state of demand $\theta_s = z$ or 1 that will be realized, the manufacturer chooses a wholesale price w_s and the retailer then chooses a retail price p_s . Notice that because there is no possibility of excess demand there is no need for a returns price, R_s in any state of demand. The retail profit for each state is $\pi_{rxs} = (p_s - w_s)(\theta_s - p_s)$ and so the optimal retail price for each state s is $p_s(w_s) = \frac{\theta_s + w_s}{2}$. The manufacturer's profit function for each state is

⁴For example, retailers such as Wal-Mart are equipped with information technology (e.g., scanner data, electronic warehouse links, and in-store audits) and ongoing marketing research information that helps to predict demand.

$\pi_{mxs} = (w_s - c)(\theta_s - p_s(w_s))$ and the optimal wholesale price is $w_{xs} = \frac{\theta_s + c}{2}$. The expected manufacturer profit for this channel with information can be calculated as $E_\theta(\pi_{mx}) = \frac{1}{16}[(z - c)^2 + (1 - c)^2]$. Similarly, the *ex-ante* expected retailer profit is $E_\theta(\pi_{rx}) = \frac{1}{32}[(z - c)^2 + (1 - c)^2]$.⁵

Perfect information for the retailer about the demand realization implies that there will be no inefficiency arising from the mis-alignment of the inventory ordered to the demand realization. The manufacturer and the retailer are both able to use the demand information to respond accurately to each state of demand. Thus the wholesale price and the retail price both respond to the actual demand realizations. This ability of the manufacturer (retailer) in the channel with perfect information to adjust the price to the actual state of demand makes the manufacturer's (and retailer's) profit increasing in the variance of the distribution.

3.2 Channel With Inventory / No Information ($\rho = 0$)

When $\rho = 0$ the channel members will choose actions based only on the prior distribution. This means that the manufacturer produces the good and sells to the retailer the inventory of the good for the wholesale price. The retailer then chooses the retail price based on which the demand is realized. This captures the idea that inventory at the retail level is necessary for doing business and that the realized demand can be met if the retailer has the product in stock while setting the price. In the first stage the manufacturer produces the goods and chooses the contract which consists of a wholesale price w and possibly an offer to accept returns of unsold goods at a non-negative returns price R .⁶ The retailer then chooses how much of the good to stock and the retail price. The retailer can return any unsold good if the manufacturer accepts returns for a positive returns price. Note that there is potential inefficiency in the channel here arising from excess production of inventory that might not be sold irrespective of who actually bears its cost.

In general, the manufacturer's contract will consist of a wholesale price w and a return price R for any unsold inventory. Let us denote the quantity of inventory that the retailer orders by Q . Given the specification of demand uncertainty there are two types of equilibria that are feasible given the inventory and pricing choices of the retailer: The first is the case in which the retailer possibly "understocks" in equilibrium. In this case, given the contract, the quantity bought by the retailer is less than the demand in the high state but equal to the demand in low state ($(1 - p) = Q < (z - p)$).

⁵It is interesting to note that with perfect information, both the manufacturer and the retailer profits are increasing in the variance of the demand.

⁶Return of unsold merchandise are offered in many industries such as books and music publishing, diamonds and jewelry which are characterized by high degree of uncertainty of demand (Padmanabhan and Png 1997).

The second case is one in which there is possible “overstocking” by the retailer. In this case the quantity bought by the retailer is equal to the demand in the high state, but higher than the demand if the low state were to occur ($(1 - p) < Q = (z - p)$). As we show in the appendix there is no other case. In particular, the “in-between” case in which the retailer understocks if the high state were to be realized, but overstocks if the low state were to be realized does not occur in equilibrium.

Understocking Equilibrium: In this case, denoted by u , the retailer’s realized demand is higher than the quantity Q_u ordered if the state of demand turns out to be high, but is exactly equal to the quantity in the low state of demand. Therefore $Q_u = 1 - p_u$. The retailer’s profit function is given by $\pi_{ru} = (1 - p_u)(p_u - w_u)$ from which the optimal retail price will be $p_u(w_u) = \frac{1+w_u}{2}$. For understocking to be an equilibrium, it must be the case that the retailer has no incentive to deviate to any other choice of inventory and price given the contract. Thus the manufacturer’s problem is to choose $w_u > 0$ to maximize its profits $\pi_{mu} = (w_u - c)(1 - p_u(w_u))$ subject to the retailer getting at least as much profit as is available from the best possible deviation, which is a deviation to the overstocking of inventory, i.e., $\pi_{rd}(w_u) = \frac{1}{2}(z - p_d)p_d + \frac{1}{2}\min\{0, (1 - p_d)p_d\} - (z - p_d)w_u$. Solving the Lagrangian for this constrained optimization problem, the equilibrium retail price, quantity and profits are respectively $p_u^* = \frac{3+c}{4}$, $Q_u^* = \frac{1-c}{4}$ and $\pi_{ru}^* = \frac{(1-c)^2}{16}$, and $\pi_{mu}^* = \frac{(1-c)^2}{8}$.

Overstocking Equilibrium: In this case, denoted by o , the retailer’s ordered quantity is $Q_o = z - p_o$. We continue to analyze the case in which there is positive demand in equilibrium in both states.⁷ Given this the retailer’s profit function is,

$$\pi_{ro} = p_o\left(\frac{1}{2}(z - p_o) + \frac{1}{2}(1 - p_o)\right) + \frac{1}{2}R_o(z - 1) - w_o(z - p_o) \quad (1)$$

In the retailer’s profit function, there are two possible sources of revenue, either from the consumer demand that is realized (the first term), or from returning unsold units to the manufacturer for the return price (the second term). The last term represents the wholesale price payment to the manufacturer for the quantity Q_o . Note that the returns revenue in the second term is independent of the retail price. From this the retail price function can be derived as $p_o(w_o) = \frac{1+z+2w_o}{4}$ and $Q_o(w_o) = \frac{3z-1-2w_o}{4}$. The manufacturer’s problem is to maximize $\pi_{mo} = (w_o - c)(z - p_o(w_o)) - \frac{1}{2}R_o(z - 1)$ subject to $w_o > 0$ and $R_o \geq 0$ with the retailer getting at least as much profit as the best possible deviation. Note that the only possible deviation for the retailer is to the case of understocking of inventory, i.e., $\pi_{rd}(w_o) = (1 - p_d)(p_d - w_o)$. We again set up and solve a Lagrangian for this constrained

⁷This implies that z be not too large (specifically $z < 2 - c$ for this section). There can also be a possible case in which the equilibrium retail demand is not positive if the low state of demand were to be realized. But this case is dominated by the channel with perfect information and therefore it does not affect the results in this section.

optimization problem and find that an overstocking equilibrium exists if the degree of uncertainty is not high and $z < (2 - c)$ which is the condition for positive demand in all states. The returns price $R_o^* = \frac{3+6c-z}{8}$ is strictly positive and it is independent of the retail price chosen, and the manufacturer can use it along with the wholesale price to extract retail surplus. Consequently, in equilibrium, the retailer's equilibrium profit is exactly the deviation profits and the manufacturer is able to extract all the excess profit over and above π_{rd} for any wholesale price that it charges. The equilibrium wholesale and retail prices are $w_o^* = \frac{1+c}{2}$ and $p_o^* = \frac{z+c+2}{4}$ respectively. Finally the equilibrium manufacturer and retailer profits are $\pi_{mo}^* = \frac{(z^2+2z-12zc+2c^2+8c-1)}{16}$ and $\pi_{ro}^* = \frac{(1-c)^2}{16}$.

Comparing the equilibrium manufacturer profits π_{mo}^* and π_{mu}^* , we present the lemma below with conditions under which the channel equilibrium will involve over- versus under stocking.

Lemma 1 *i. If $c < \frac{1}{3}$, then manufacturer profits are higher in the overstocking equilibrium than in the understocking equilibrium.*

ii. If $c > \frac{1}{3}$, then manufacturer profits are higher in the overstocking equilibrium if $z > (12c - 3)$, and they are higher in the understocking equilibrium when the reverse is true.

Figure 2 depicts the conditions in the lemma for the overstocking and understocking cases. The manufacturer prefers overstocking by the retailer in zone 1 and zone 2 while in zone 3 represented by the hatched area to the right of the line $z = (12c - 3)$ understocking is preferred. In the understocking equilibrium, there is no inefficiency in terms of unsold stock being returned to manufacturer which has no salvage value. However, this fails to capitalize on the upside potential of high demand. Furthermore, the equilibrium contract in the understocking case consists of only the wholesale price. In contrast, in the overstocking equilibrium, the manufacturer offers a wholesale price and a returns contract. With overstocking, there is an inefficiency cost for the manufacturer in that there is excess stock which the retailer may return and for which there is no value. However, the manufacturer has now two instruments, the wholesale price and the returns price, to manage the retailer actions. This allows the manufacturer to reduce double marginalization while appropriating greater surplus. In addition, with overstocking the manufacturer is able to take advantage of the upside potential if the high state of the demand is realized.

When the marginal cost of production is small enough, the overstocking equilibrium always dominates for the manufacturer. This is because manufacturer is able to better extract retail surplus while at the same time facing little cost due to the excess unsold stock that is returned and which has no value. When marginal cost of production is large, the overstocking equilibrium is only preferred

by the manufacturer if the demand in the high state is large enough so as to balance the inefficiency of the unsold stock.

3.3 Comparing the Two Cases

The following proposition highlights the trade-off between information and inventory in this bilateral monopoly channel.

Proposition 1 *If $c > \frac{1}{5}$, the channel with perfect information is preferred to the channel with inventory for the manufacturer. If $c < \frac{1}{5}$, the channel with inventory is preferred to the channel with perfect information. In this case channel with inventory involves overstocking. For the manufacturer, the channel with perfect information always dominates the channel with understocking of inventory.*

Proof. Follows directly from comparing the equilibrium profits of the two cases. ■

This proposition highlights the value of demand information in a distribution channel. The point that we are interested in examining is whether the channel with inventory may be optimal for the manufacturer, even if perfect information about the demand is available. The availability of demand information allows the parties in the channel to optimally adjust their actions to the actual demand realization. However, there is greater double marginalization and the manufacturer is unable to fully extract retailer profits.

In Figure 2, in the area to left of the dotted line $c = \frac{1}{5}$, the manufacturer prefers the channel with inventory over the channel with perfect information. In this range, the equilibrium for the channel with inventory involves overstocking and the contract involves two instruments w and R . This allows the manufacturer to better extract retailer profits while controlling the extent of double marginalization. The disadvantage of the overstocking equilibrium is that the manufacturer takes back returns which have no value and this disadvantage is small enough if $c < \frac{1}{5}$. In contrast, the strategic effect of perfect information which leads to greater double marginalization is higher when the production costs are low. Thus when $c < \frac{1}{5}$ holding inventory in the channel is more attractive for the manufacturer than a channel with perfect demand information. When $c > \frac{1}{5}$ the strategic effect of information in creating greater double marginalization is mitigated. At the same time it becomes more costly for the manufacturer to hold excess inventory in the channel. Therefore the manufacturer prefers the channel with information.⁸

⁸We have assumed in the model that the two states are equiprobable which implies maximum variance for the prior demand distribution. We also analyzed the model for general values of the probability of high demand to be some

Note that the channel with perfect information always dominates the channel with understocking of inventory for the manufacturer. In the understocking equilibrium the manufacturer is able to use only the wholesale price and is unable to extract retailer profits. At the same time in the understocking equilibrium the manufacturer suffers from the inability to respond to the actual realization of demand.

Thus we find that the information enabled format is not preferred by the manufacturer even when the information system provides perfect information about the demand. The trade-off that our analysis highlights is that while better demand information allows the channel to adjust exactly to the realization, it also has the strategic effect of resulting in higher retail prices because the manufacturer cannot fully extract retailer profits. Holding inventory in the channel mitigates this strategic effect and helps the manufacturer to better extract retail profits.

3.4 The General Case of Imperfect Information System

Consider now the general case with an imperfect demand information system, i.e., $0 < \rho < 1$. In Figure 1, for each signal node denoted \tilde{k} , (where $\tilde{k} = \tilde{h}$ or \tilde{l}), the manufacturer announces a signal-dependent contract, which is then followed by the quantity to be ordered and the setting of the retail price. After that the true state of demand is revealed and there may be possible returns depending on the state of the world and the specification of the contract.

For each \tilde{k} , the retailer's expected profit function after the manufacturer announces the contract is given by:

$$\begin{aligned} \pi_{r\tilde{k}} = & \Pr(h|\tilde{k})\{p_{\tilde{k}}[\min((z - p_{\tilde{k}}), Q_{\tilde{k}})] + R_{\tilde{k}}[\max((Q_{\tilde{k}} - (z - p_{\tilde{k}})), 0)]\} \\ & + \Pr(l|\tilde{k})\{p_{\tilde{k}}[\min((1 - p_{\tilde{k}}), Q_{\tilde{k}})] + R_{\tilde{k}}[\max((Q_{\tilde{k}} - (1 - p_{\tilde{k}})), 0)]\} - w_{\tilde{k}}Q_{\tilde{k}} \end{aligned} \quad (2)$$

Given \tilde{k} , the first term denotes the revenue if the high demand were to be realized. This includes revenue from either selling the goods to consumers or returning the unsold goods to manufacturer. The second term is the corresponding revenue if the low demand were to be realized and the final term is the cost of buying the stock Q_k . As in the previous case of $\rho = 0$ there will be two possible types of equilibrium involving either overstocking or understocking. However, now the strategies of the firms also depend on the received signal resulting in four possible types of outcomes (high signal/ low signal $\lambda \in (0, 1)$ and this does not change any of the main insights. Basically, with high enough λ , a channel with overstocking of inventory becomes more attractive compared to the channel with perfect information, while the channel with perfect information be preferred for lower values of λ . We thank an anonymous reviewer for raising this issue.

× overstocking / understocking). We present the derivation of these outcomes in the Appendix and discuss the main results starting with the high signal case.

Result 1 *Given a high signal,*

- i. when $z \leq \min(\frac{(3-\rho)(1-c)}{4(1-\rho)}, \frac{7-2c+\rho}{5+\rho})$, there is an overstocking equilibrium in which the manufacturer contract includes $w_{\tilde{h}1} = \frac{1}{4}(3z - 1 + 2c - (z - 1)\rho)$ and no returns, i.e., $R_{\tilde{h}1} = 0$, which leads to a retail price of $p_{\tilde{h}1} = \frac{1}{8}(5z + 1 + 2c + \rho(z - 1))$ and retailer buying quantity $q_{\tilde{h}1} = z - p_{\tilde{h}1}$.
- ii. when $z \leq \min(\frac{2-c+\rho}{1+\rho}, \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2})$, there is another type of overstocking equilibrium where the manufacturer contract involves a strictly positive returns price $R_{\tilde{h}2} = \frac{1}{8(1-\rho)}[3(1+2c) - z - \rho^2(z - 1) - 2\rho(z + c + 2)]$ and $w_{\tilde{h}2} = \frac{1+c}{2}$, which leads to a retail price of $p_{\tilde{h}2} = \frac{1}{4}(z + 2 + c + \rho(z - 1))$ and retailer buying quantity $q_{\tilde{h}2} = z - p_{\tilde{h}2}$.
- iii. when $z \leq \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2}$, there is an understocking equilibrium where the manufacturer contract includes $w_{\tilde{h}3} = \frac{1+c}{2}$ and no return, i.e., $R_{\tilde{h}3} = 0$, which leads to a retail price of $p_{\tilde{h}3} = \frac{3+c}{4}$ and retailer buying quantity $q_{\tilde{h}3} = \frac{1-c}{4}$.
- iv. when $z > \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2}$, there is an understocking equilibrium where the manufacturer contract includes $w_{\tilde{h}4} = \frac{3+z(1+\rho)^2+\rho(2-\rho)}{4(3-\rho)}$ and no return, i.e., $R_{\tilde{h}4} = 0$, which leads to a retail price of $p_{\tilde{h}4} = \frac{15-\rho(2+\rho)+z(1+\rho)^2}{8(3-\rho)}$ and retailer buying quantity $q_{\tilde{h}4} = 1 - p_{\tilde{h}4}$.

When the signal indicates that the demand is likely to be high, one might expect the channel equilibrium to involve overstocking by the retailer in order to take advantage of the high state of demand being realized. Furthermore, as in the case of $\rho = 0$ we should expect that the overstocking equilibrium is likely as long as z is not too high (so that the inefficiency of returning unsold stock is mitigated). This is precisely what we find as shown in parts (i) and (ii) of the result above.

Parts (iii) and (iv) of the Proposition characterize the possible understocking equilibria. As can be noticed an understocking equilibrium of one or the other type will always be feasible. However, given a high signal, the overstocking equilibrium when it is feasible will always imply greater manufacturer profits than the understocking equilibrium. We next move to the possible equilibria under the low information signal ($\tilde{k} = \tilde{l}$).

Result 2 *Given a low signal*

- i. Under the low signal, an overstocking equilibrium exists only if $\rho < \rho^l = \frac{z+1+2c-[2(1-c)^2+8cz]^{\frac{1}{2}}}{z-1}$ and $z \leq \min(\frac{2-c-\rho}{1-\rho}, \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2})$. The manufacturer contract includes a strictly positive

$R_{\tilde{l}1} = \frac{1}{8(1+\rho)}[3(1+2c) - z + \rho^2(z-1) + 2\rho(z+c+2)]$ and $w_{\tilde{l}1} = \frac{1+c}{2}$. This leads to a retail price of $p_{\tilde{l}1} = \frac{1}{4}(z+2+c-\rho(z-1))$ and retailer buying quantity $q_{\tilde{l}1} = z - p_{\tilde{l}1}$. If $\rho > \rho^l$, an overstocking equilibrium does not exist for the low signal.

ii. when $z \leq \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2}$, there is an understocking equilibrium where the manufacturer contract includes $w_{\tilde{l}u1} = \frac{1+c}{2}$ and no return, i.e., $R_{\tilde{l}u1} = 0$, which leads to a retail price of $p_{\tilde{l}u1} = \frac{3+c}{4}$ and retailer buying quantity $q_{\tilde{l}u1} = \frac{1-c}{4}$.

iii. when $z > \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2}$, there is another understocking equilibrium where the manufacturer contract includes $w_{\tilde{l}u2} = \frac{3+z(1-\rho)^2-\rho(2+\rho)}{4(3+\rho)}$ and still no return, i.e., $R_{\tilde{l}u2} = 0$. This leads to a retail price of $p_{\tilde{l}u2} = \frac{15+\rho(2-\rho)+z(1-\rho)^2}{8(3+\rho)}$ and retailer buying quantity $q_{\tilde{l}u2} = 1 - p_{\tilde{l}u2}$.⁹

If the low signal is obtained and it comes from a very reliable information system, then inducing overstocking is inefficient as the chances of wasteful production are high and the overstocking equilibrium will not exist. Only when the information system is relatively unreliable will overstocking occur if a low signal is realized. This is because the probability of a high state of demand is high enough in this case and the manufacturer also finds it optimal to offer returns. On the other hand, given a low signal, an understocking equilibrium is always feasible.

The results above characterize all the possible equilibria and the particular equilibrium that prevails will depend on which of the feasible equilibria yields the highest profits for the manufacturer. In order to characterize the equilibrium for different parameter values, we should ensure feasibility and compare profits values given in Results 1 and 2 above. We demonstrate the comparisons and the results for a particular value of $c = \frac{1}{5}$ and $z = 1.1$ for all $\rho \in (0, 1)$.¹⁰ Figure 3 shows the manufacturer's ex-ante expected profit on the vertical axis and the reliability of the information system (ρ) on the horizontal axis. The cutoff values of ρ in the figure indicates where the equilibrium contract for one or the other signal changes. For low values of ρ ($\rho < \rho_1$) and a high signal, the equilibrium strategy is to induce an overstocking equilibrium by offering a contract $(w_{\tilde{h}2}, R_{\tilde{h}2})$ with a strictly positive returns price. This is because with lower reliability the probability of actual state being low is high enough and therefore offering returns to the retailer is necessary to optimally control retailer pricing. Also, in this range an overstocking equilibrium is also optimal for low signal and the manufacturer offers the

⁹The understocking equilibria might also be subject to the conditions shown in the Appendix which rule out deviations by the retailer to a case where off the equilibrium there is demand only in the high state. The analysis that follows pertains to the case where those conditions are subsumed in the conditions shown in the result.

¹⁰The qualitative behavior of the manufacturer profit for different values of z is similar to that in Figure 3.

contract (w_{l1}^*, R_{l1}^*) because the probability of the high state is high enough making it worth the cost of excess stocking.

As the reliability increases and goes beyond ρ_1 but is less than ρ_2 , the high signal equilibrium does not change, but for low signal the low state is now likely enough that stocking for the high state is not worth the cost of excess stocking. Therefore, it is best for the manufacturer to offer $(w_{lu1}^*, R_{lu1}^* = 0)$ and induce an understocking equilibrium. For still higher reliability levels between ρ_2 and ρ_3 , we observe a discontinuity in the profit function. In this range, given a high signal, neither of the two contracts that would induce an overstocking contract are feasible because there is profitable deviation for the retailer from either one. Thus, in this range only an understocking equilibrium is induced by offering (w_{h4}^*, R_{h4}^*) . Clearly, understocking ignores the upside potential of the high state and results in a lower manufacturer profit. For low signal, the equilibrium continues to be understocking and involves $(w_{lu1}^*, R_{lu1}^* = 0)$. Finally, for still higher levels of reliability $\rho > \rho_3$, the reliability is high enough that the manufacturer does not need to provide a protection to the retailer against low state happening with a positive returns price if a high signal is received. Thus for high signal, an overstocking equilibrium is induced by offering the contract $(w_{h1}^*, R_{h1}^* = 0)$. For low signal, the equilibrium still continues to be understocking and involves (w_{lu1}^*, R_{lu1}^*) .

The main take-away here is that the manufacturer's profit is not monotonic in the reliability parameter ρ . In fact, for high enough ρ , an increase in ρ may have the strategic effect of making the retail prices too high in the channel. To see this notice that in the range $\rho > \rho_3$ the equilibrium contract involves no returns implying greater double marginalization of the retail price and the expected profits fall with higher reliability as a result. Thus even if a manufacturer could choose the reliability of the information system and even if higher reliability came at no extra cost, the manufacturer would not want to institute a perfect information system. As expected, since $c = \frac{1}{5}$, in Figure 3, we recover the result of Proposition 1, that the manufacturer profits at the two extremes of ρ are equal. We also want to point out, that the total *ex-ante* channel profit, i.e., sum of manufacturer's and retailer's profit, shows a monotonic increase with respect to ρ even as the manufacturer's profit alone falls. This once again highlights the point that given availability of more complete contracts, a higher degree of reliability becomes more attractive for the manufacturer.

4 Retail Competition

In this section we analyze how retail competition affects the manufacturer's choice of the channel format in the basic model involving the comparison of the channel with inventory ($\rho = 0$) with the channel with perfect information ($\rho = 1$). Suppose there are two competing retailers (i and j) and the retail market demand for retailer i it is given by $q_{is} = \theta_s - p_i + \gamma(p_j - p_i)$ where p_i and p_j are the retail prices. In this setup, $\gamma > 0$ is proportional to the cross-price elasticity of retail demand and higher γ results in more intense retail competition.

Channel with Perfect Information ($\rho=1$): After the information system truly reveals the state of demand $\theta_s = z$ or 1 , the manufacturer chooses a wholesale price w_s and the two retailers simultaneously choose the retail prices. The expected equilibrium profit (over the states) for the manufacturer can be calculated to be $E_\theta(\pi_{mzs}) = \frac{(\gamma+1)[(z-c)^2+(1-c)^2]}{4(\gamma+2)}$. Similarly, the ex-ante expected profit of each retailer is obtained as $E_\theta(\pi_{rs}) = \frac{(\gamma+1)[(z-c)^2+(1-c)^2]}{8(\gamma+2)^2}$. Note that perfect information for the retailer about the demand realization implies that there will be no inefficiency arising from the mis-alignment of the inventory ordered to the demand realization. The manufacturer and the retailer are both able to use the demand information to respond accurately to each state of demand. Consequently, as in the case of the single retailer, we find that the expected manufacturer and retailer profits are increasing in the variance of the demand distribution. Also, as expected, we find that the manufacturer profits increase when retail competition increases (i.e., γ increases), whereas the retailer profits decrease with retail competition.

*Channel with Inventory ($\rho = 0$):*As in the single retailer case, we have the possibility of equilibrium with overstocking and understocking of inventory which is derived in the appendix for both these cases. At the contracting stage the manufacturer will choose whether to induce either overstocking or understocking of the inventory. In all cases we look for the symmetric equilibrium in retail strategies.

Understocking Equilibrium: In the understocking case, denoted by subscript u , the retailers' order quantity $Q_{iu} = 1 - p_{iu} + \gamma(p_{ju} - p_{iu})$ ($i, j = 1, 2; i \neq j$). Given this $\pi_{riu} = (p_{iu} - w_u)(1 - p_{iu} + \gamma(p_{ju} - p_{iu}))$ and the optimal retail price will be $p_{iu}(w_u) = p_{ju}(w_u) = \frac{1+w_u(1+\gamma)}{2+\gamma}$. Given this response function and solving the manufacturer problem, we find the equilibrium wholesale price to be once again $w_u^* = \frac{1+c}{2}$. Thus the equilibrium retail price, quantity and profits are respectively $p_{1u}^* = p_{2u}^* = \frac{3+c(1+\gamma)+\gamma}{2(2+\gamma)}$, $Q_{1u}^* = Q_{2u}^* = \frac{(1-c)(1+\gamma)}{2(2+\gamma)}$, $\pi_{r1u}^* = \pi_{r2u}^* = \frac{(1-c)^2(1+\gamma)}{4(2+\gamma)^2}$, and $\pi_{mu}^* = \frac{(1-c)^2(1+\gamma)}{2(2+\gamma)}$.

Overstocking Equilibrium: In this case, denoted by subscript o , retailer i 's order quantity is $Q_{io} =$

$z - p_{io} + \gamma(p_{jo} - p_{io})(i, j = 1, 2; i \neq j)$. For $i, j = 1, 2; i \neq j$, retailer i 's profit function is,

$$\begin{aligned} \pi_{rio} &= p_o \left(\frac{1}{2}(z - p_{io} + \gamma(p_{jo} - p_{io})) + \frac{1}{2}(1 - p_{io} + \gamma(p_{jo} - p_{io})) \right) \\ &\quad + \frac{1}{2}R(z - 1) - w_o(z - p_{io} + \gamma(p_{jo} - p_{io})). \end{aligned} \quad (3)$$

By backward induction we get $w_o^* = \frac{2(1+c)+\gamma(1+z+2c)}{4(1+\gamma)}$ and $R_o^* = \frac{2\gamma^2(1+z+2c)+z(3\gamma-2)+4c(4\gamma+3)+5\gamma+6}{8(1+\gamma)(2+\gamma)}$. Given w_o^* and R_o^* , both the retailers find it optimal to order $Q_{1o}^* = Q_{2o}^* = \frac{3z(2+\gamma)-2c(1+\gamma)-4-\gamma}{4(2+\gamma)}$ and price $p_{1o}^* = p_{2o}^* = \frac{2(c+z+2)+\gamma(1+z+2c)}{4(2+\gamma)}$. Finally the expected profits for the manufacturer and retailer, respectively are $\pi_{mo}^* = \frac{(2+\gamma)(1+z)^2+4c(c+c\gamma+\gamma-3z\gamma+4+6z)-4}{8(2+\gamma)}$ and $\pi_{r1o}^* = \pi_{r2o}^* = \frac{(1-c)^2(1+\gamma)}{4(2+\gamma)^2}$.

Comparison of the overstocking and the understocking equilibrium profits for the manufacturer under retail competition provides analogous results as in Lemma 1 for the single retailer case. Getting to the question of the choice of channel format, we first observe that the manufacturer profit for a channel with demand information is strictly larger compared to that in the channel with inventory, whenever understocking is a preferred equilibrium. Comparing the profits between the overstocking equilibrium and the channel with information we get the following proposition:

Proposition 2 *When $c \geq \frac{1}{5}$ the channel with perfect information dominates for the manufacturer compared to the channel with inventory. When $c < \frac{1}{5}$, then the channel with perfect information dominates for the manufacturer only when retail competition is sufficiently intense $\gamma > 4(1 - 5c)$. Otherwise, the manufacturer prefers to have a channel with overstocking of inventory.*

This proposition highlights the effect of retail competition on the choice of the channel format by the manufacturer. As in the case of the single retailer, when the marginal cost of production is sufficiently large, the manufacturer always prefers the channel with information. However, if $c < \frac{1}{5}$, then having inventory becomes attractive. Thus when the loss from excess production is low enough, the manufacturer favors a channel with inventory depending upon the degree of retail competition. In particular in markets with greater retail differentiation (less intense competition) the manufacturer prefers to have the channel with inventory. Whereas the channel with information becomes optimal when the intensity of retail competition is higher. Recall, that while the (perfect) information system has the advantage of eliminating excess production, it also results in higher retail prices. More intense retail competition places a downward pressure on retail prices which counteracts upward pressure on the retail prices caused by the information system. Therefore, with greater retail competition the channel with perfect information becomes more attractive.

5 Conclusion and Limitations

In this paper we highlight a trade-off between information and inventory in a distribution channel. While better demand information has a positive direct effect for the manufacturer in improving the efficiency of inventory holding, it also has the strategic effect of increasing double marginalization in the channel and limiting the manufacturer's ability to extract retail surplus. Despite the inefficiency that excess inventory brings through returns of the goods which remain unsold, having inventory in the channel helps the manufacturer to counter double marginalization while better extracting retail surplus. Thus even if the information system is perfectly reliable, the manufacturer might not always want to institute an information enabled channel over a channel with inventory. We find that the channel with information is more attractive and preferred over the channel with inventory if the marginal cost of production is sufficiently high and when retail competition is sufficiently intense. Thus we show that the presence of inventory might be "good" for the manufacturer from a strategic point of view. We also considered the case of an imperfectly reliable information system to show that the manufacturer profits are maximized at an intermediate level of the reliability of the information system. In other words, if the manufacturer were to choose the precision of the demand information system, the manufacturer would not prefer perfect information, even if such information was costless to acquire.

There are some limitations of this research that can be usefully pursued in future research. We have considered the case in which the channel members have joint access to the information system resulting in symmetric information about demand uncertainty for all the channel members. The case of asymmetric information leading to signaling (or screening) incentives can be an interesting extension (Desai and Srinivasan 1995). It can be the case that the signaling distortions in the channel are greater when the information system is more reliable. Next we have only considered the case in which the manufacturer and the retailer choose their actions before the actual state of demand is revealed. It might also be the case that the retailer buys the stock under uncertainty, but may be able to adjust the prices after the actual demand is realized. For example, this could pertain to cases where the retailers have to buy the stock well in advance, but can sell to consumers over a relatively long selling cycle. Finally, the information system modeled in this paper is a knowledge system which provides the decision maker with better information about the actual state of the world than the prior. It will be also be interesting to examine the alternative characterization of the information system as an estimator of the true state of the world.

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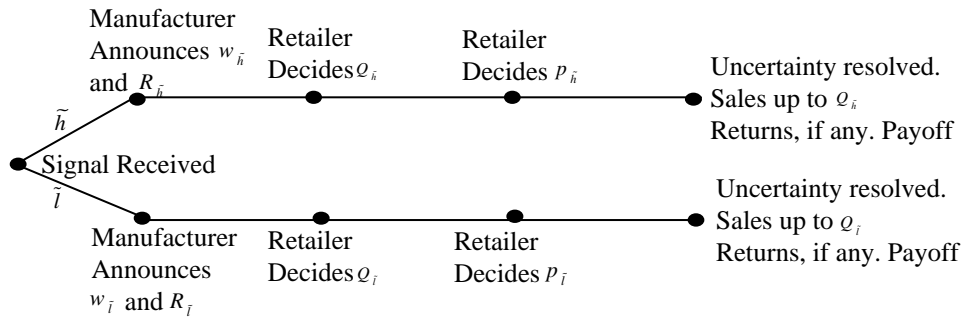


Figure 1: Sequence of Moves

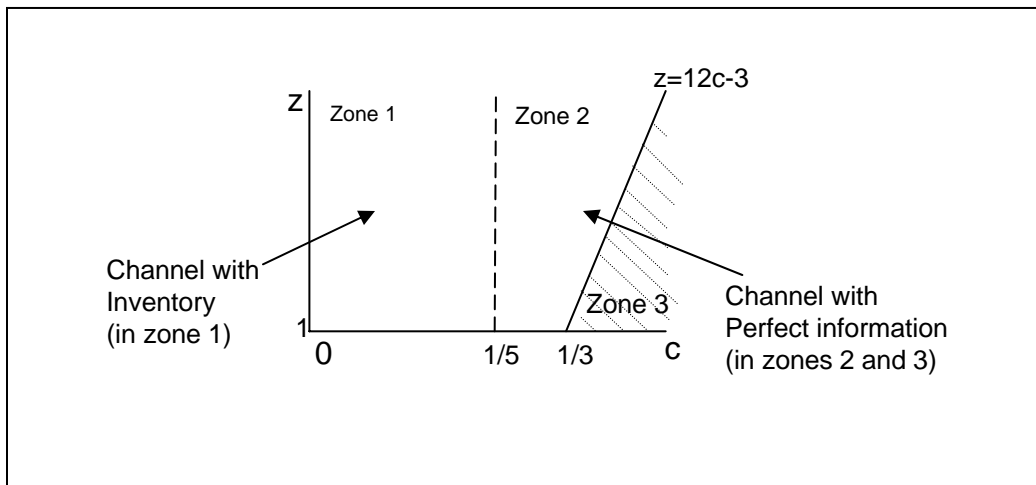


Figure 2: Illustration of Lemma 1 and Proposition 1

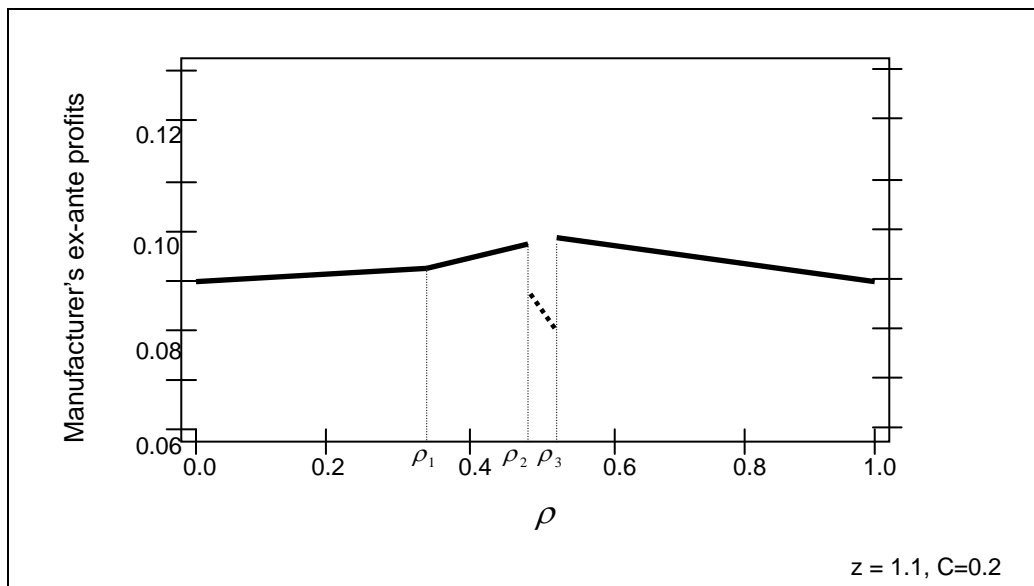


Figure 3: Types of Equilibria in $0 < \rho < 1$ case

Appendix for “Information and Inventory in Distribution Channels”

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First-best Vertically Integrated Channel:

Consider a vertically integrated manufacturer who first produces a quantity and then sets up the price for the product in an uncertain demand environment. Like in the model with decentralized channel, the demand is given by $q = \theta_s - p$, with $s = h$ or l with equal probability. This firm may have access to an information system that gives out signals $\tilde{k} = \tilde{h}, \tilde{l}$; with possibly imperfect reliability, i.e., ρ is free parameter between 0 and 1. The firm’s profit function is:

$\Pi_{\tilde{k}} = \Pr(h|\tilde{k})[p_{\tilde{k}}(\min(z - p_{\tilde{k}}), Q_{\tilde{k}})] + \Pr(l|\tilde{k})[p_{\tilde{k}}(\min(1 - p_{\tilde{k}}), Q_{\tilde{k}})] - cQ_{\tilde{k}}$ where $Q_{\tilde{k}}$ and $p_{\tilde{k}}$ are signal dependent amount of production and retail price, respectively. Firm can either (1) produce and price for selling out exactly in high state but this amounts to overstocking in low state or (2) produce and price for selling out exactly in low state, in which case it obviously understocks for the high state in which it cannot meet all the demand or (3) produce and price so that it has more than enough if low state is realized but cannot meet all the demand if high state is realized. We start by ruling out (3) first. For the high signal, the profit function in the pricing stage becomes $\Pi_{\tilde{h}} = (\frac{1+\rho}{2})[p_{\tilde{h}}Q_{\tilde{h}}] + \frac{1-\rho}{2}[p_{\tilde{h}}(1 - p_{\tilde{h}})]$. For a given $Q_{\tilde{h}}$ it leads to an optimal price of $\frac{z(1+\rho)+Q_{\tilde{h}}(1-\rho)}{2(1+\rho)}$. Going back to the quantity stage, however, we see that it leads to a profit function which is quadratic function in $Q_{\tilde{h}}$ with a strictly positive second derivative in $Q_{\tilde{h}}$. This means the function has an interior minimum and the profit maximizing quantity can be found only at one (or both) the corners of this regime which correspond to (1) or (2) above. We call these Overstocking and Understocking respectively. A similar conclusion is reached for low signal also. Thus the optimal production quantity and price for the vertically integrated firm can be found possibly in analyzing the Overstocking and Understocking cases.

The profit function for an understocking equilibrium is simply $(p_u - c)((1 - p_u))$, since the firm always produces and prices for low state which it can always sell. This is irrespective of the received signal. It is easy to see that it leads to an optimal price of $\frac{1+c}{2}$ and an optimal profit of $\Pi_u^* = \frac{1}{4}(1 - c)^2$.

The profit function for Overstocking case is signal-dependent, and denoting the signal by \tilde{k} , it can be expressed as $\Pi_{ok}^{\tilde{k}} = \Pr(h|\tilde{k})(z - p_{ok}^{\tilde{k}})(p_{ok}^{\tilde{k}}) + \Pr(l|\tilde{k})(1 - p_{ok}^{\tilde{k}})(p_{ok}^{\tilde{k}}) - (z - p_{ok}^{\tilde{k}})c$.

Conditional on the signal, the firm fixes the price and it can be easily seen that the profit maximizing price and resulting profit for the high signal is $p_{oh}^{\tilde{k}} = \frac{1}{4}((z + 1) + \rho(z - 1) + 2c)$ leading to $\Pi_{oh}^* = \frac{1}{16}[(z + 1)^2 + 4c(1 + c + \rho(z - 1) - 3z) + \rho^2(z - 1)^2 + 2\rho(z^2 - 1)]$. Similarly, for low signal $p_{ol}^{\tilde{k}} = \frac{1}{4}((z + 1) - \rho(z - 1) + 2c)$ leading to $\Pi_{ol}^* = \frac{1}{16}[(z + 1)^2 + 4c(1 + c - \rho(z - 1) - 3z) + \rho^2(z - 1)^2 - 2\rho(z^2 - 1)]$.

What the firm will do for any given signal is then dependent on whether understocking or overstocking is better strategy to follow for any given ρ . After a little bit of algebra, we find three types of strategies emerge in equilibrium for different values of ρ . These are (i) Overstocking if \tilde{h} and Overstocking if \tilde{l} , (ii) Overstocking if \tilde{h} and Under-stocking if \tilde{l} , and (iii) Under-stocking if \tilde{h} and Under-stocking if \tilde{l} .

The expected profits from the three types of strategies can simply be calculated by giving equal-probability weights to the two signals. Finally, we can calculate the first derivatives of the three types of equilibrium profits as below:

$$\begin{aligned}\frac{\partial(\frac{1}{2}\Pi_{oh}^* + \frac{1}{2}\Pi_{ol}^*)}{\partial\rho} &= \frac{1}{8}\rho(z - 1)^2 > 0 \\ \frac{\partial(\frac{1}{2}\Pi_{oh}^* + \frac{1}{2}\Pi_u^*)}{\partial\rho} &= \frac{1}{16}[(z^2 + 1) + \rho(z - 1)^2 + 2c(z - 1)] > 0 \text{ and} \\ \frac{\partial(\Pi_u^*)}{\partial\rho} &= 0.\end{aligned}$$

Therefore, for the vertically integrated firm, expected profits are (weakly) increasing in the signal reliability parameter ρ .

The Single Retailer Channel

Channel with Inventory ($\rho = 0$)

Denote the inventory bought by the retailer as Q and the retail price as p . There are three possible types of channel equilibria given the inventory and pricing choices of the retailer:

- i A case of “understocking” in which the retailer chooses inventory that is exactly equal to the demand in the low state but less than the realized demand in the high state. Thus $(1 - p) = Q < (z - p)$.
- ii A case of “overstocking” in which the retailer chooses inventory that is exactly equal to the realized

demand if the high state of demand were to be realized, but has excess inventory if the low state were to be realized. Thus $(1 - p) < Q = (z - p)$.

- iii An “in-between” case in which the retailer the retailer understocks if the high state were to be realized, but overstocks if the low state were to be realized. Thus in this case $(1 - p) < Q < (z - p)$.

We begin the analysis by showing that the in-between case (iii) described above can never be part of an equilibrium. Given a contract (w, R) , the retailer’s objective function is $\pi_{rb} = \frac{1}{2}pQ + \frac{1}{2}p(1 - p) + \frac{1}{2}R(Q - (1 - p)) - wQ$. From this we can solve $p = \frac{1+R+Q}{2}$ and substituting this into the objective function we get a quadratic in Q with a minimum ($\frac{\partial^2 \pi_{rb}}{\partial Q^2} = \frac{1}{4} > 0$). Therefore there is no interior maximum for Q and the inventory choice by the retailer will be at the boundary characterized either by the understocking or the overstocking choice analyzed below. It can also be noted that it will never the case in equilibrium that the retailer chooses an inventory greater than the maximum possible demand ($Q > z - p$) because in that case the retailer will always be better off reducing the inventory or the retail price charged. Similarly, the retailer will never choose an inventory less than the minimum possible demand ($Q < 1 - p$), because in that case the retailer will always be better off increasing the inventory or the price.

Understocking Equilibrium

Based on the w offered by the manufacturer, the retailer chooses the inventory and the retail price. The retailer’s profit function is $\pi_{ru} = (1 - p_u)(p_u - w_u)$. From this the optimal retail price is $p_u(w_u) = \frac{1+w_u}{2}$ and the retailer profit becomes $\pi_{ru}(w_u) = \frac{(1-w_u)^2}{4}$. While choosing the contract w_u the manufacturer also has to account for the possibility that the retailer might deviate to any choice of inventory and price other than the equilibrium one. Specifically, the retailer might deviate to overstocking of inventory (because it is the case that the deviation to the case of in-between inventory is not optimal). Denoting the deviation by d , and given w_u , suppose the retailer deviates to choosing $(1 - p_d) < Q = (z - p_d)$.

Let us begin with the case where the retailer chooses the deviation price p_d so that there is demand in the off-equilibrium path irrespective of the state. The deviation profit will be $\pi_{rd} = \frac{1}{2}(z - p_d)p_d + \frac{1}{2}(1 - p_d)p_d - (z - p_d)w_u$. Thus for given w_u we will have the optimal deviation price to be $p_d(w_u) = \frac{(1+z+2w_u)}{4}$. Thus if the channel equilibrium were to involve understocking, then this understocking equilibrium must provide the retailer with at least as much profit as $\pi_{rd}(p_d(w_u))$.

The manufacturer’s problem is to choose w_u to maximize $\pi_{mu} = (1 - p_u)(w_u - c)$ subject to $w_u > 0$ and the retailer getting atleast the profit available from the best possible deviation. The corresponding Lagrangian

will be,

$$L_{mu} = (1 - p_u)(w_u - c) + \mu_{wu}w_u + \mu_d(\pi_{ru}(w_u) - \pi_{rd}(w_u))$$

where the μ 's are the corresponding Lagrangian multipliers. Note that we are looking for a solution with $w_u > 0$ and therefore $\frac{\partial L_{mu}}{\partial w_u} = 0$ and $\mu_{wu} = 0$. Let us first consider the case for which $\mu_d > 0$ which means that π_{rd} is binding for the retailer. Then $\frac{\partial L_{mu}}{\partial \mu_d} = 0$ from which we get that $w_u = \frac{z+3}{12}$. Next from $\frac{\partial L_{mu}}{\partial w_u} = 0$ we can derive $\mu_d = \frac{z-3-6c}{z-1}$. And $\mu_d > 0 \implies z > (3 + 6c)$. Substituting the wholesale price the equilibrium manufacturer profit for this case turns out to be $\pi_{mu} = \frac{(9-z)(z+3-12c)}{288}$. The retailer profit is $\pi_{ru} = \frac{(9-z)^2}{576}$. Next note that this equilibrium is supported by the off-equilibrium retailer price p_d and the requirement that there is positive demand in both states in the off-equilibrium path. This requires that $(1 - p_d) > 0$ which implies $\frac{1}{8}(5 - \frac{7z}{3}) > 0 \implies z < \frac{15}{7}$. However, comparing this with the earlier condition $z > 3 + 6c$ we can see that there is no feasible value of z for which this case exists.

Next consider the case when $\mu_d = 0$. This implies that $\frac{\partial L_{mu}}{\partial \mu_d} > 0$, from which we get that $z < 3 + 6c$. Also given that $w_u > 0$ which implies $\frac{\partial L_{mu}}{\partial w_u} = 0$ and $w_u = \frac{1+c}{2}$. We require $(1 - p_d) > 0$ and this implies $z < (2 - c)$. Thus this case exists for $z < (2 - c)$. The equilibrium manufacturer and retailer profits are respectively $\pi_{mu}^* = \frac{(1-c)^2}{8}$ and $\pi_{ru}^* = \frac{(1-c)^2}{16}$.

Now we consider the case in which the possible deviation by the retailer involves demand only in the high state. The deviation profit will be $\pi_{rd} = \frac{1}{2}(z - p_d)p_d - (z - p_d)w_u$. This means $p_d(w_u) = \frac{z}{2} + w_u$ and $\pi_{rd} = \frac{1}{8}(z - 2w_u)^2$. Therefore, the Lagrangian for the manufacturer's optimization for this case will be,

$$L_{mu} = (1 - p)(w_u - c) + \mu_w w_u + \mu_d \left(\frac{(1 - w_u)^2}{4} - \frac{(z - 2w_u)^2}{8} \right)$$

First consider when $\mu_d > 0$, then $(\frac{(1-w)^2}{4} - \frac{(z-2w)^2}{8}) = 0$. This is a quadratic in w_u which can be solved for $w_{u1} = (z - 1) + \frac{1}{\sqrt{2}}(z - 2)$ and $w_{u2} = (z - 1) - \frac{1}{\sqrt{2}}(z - 2)$. We can calculate μ_d from the condition $\frac{\partial L_{mu}}{\partial w_u} = 0$ to be $\mu_d = \frac{1-2w_u+c}{1+w_u-z}$. Substituting w_{u1} for w_u in the equation for μ_d we get that $\mu_d < 0$ which is impossible. Therefore w_{u1} cannot be a feasible solution. Consider the second root w_{u2} . Substituting it into the retailer price response we get $p(w_{u2}) = (\frac{3z}{2} - 1) + \sqrt{2}(1 - \frac{z}{2})$. Now for the equilibrium understocking demand to be positive $(1 - p(w_{u2})) > 0$ implies that $z < 0.7387$ which is impossible by assumption. Therefore, there cannot be a deviation by the retailer which involving positive demand only in the high state and with $\mu_d > 0$.

Finally, consider the case in which $\mu_d = 0$ and $\frac{\partial L_{mu}}{\partial \mu_d} > 0$. As before $w_u > 0$ and therefore $\frac{\partial L_{mu}}{\partial w_u} = 0$ and $\mu_{wu} = 0$. From this we get $w_u = \frac{(1+c)}{2}$. From $\frac{\partial L_{mu}}{\partial \mu_d} > 0$ we get the condition $z < (1 + c) + \frac{(1-c)}{\sqrt{2}}$. The equilibrium profits are once again $\pi_{mu}^* = \frac{(1-c)^2}{8}$ and $\pi_{ru}^* = \frac{(1-c)^2}{16}$. To summarize, in the understocking equilibrium $w_u^* = \frac{(1+c)}{2}$, $p_u^* = \frac{3+c}{4}$, $\pi_{mu}^* = \frac{(1-c)^2}{8}$, and $\pi_{ru}^* = \frac{(1-c)^2}{16}$.

Overstocking Equilibrium

The retailer's profit function in the overstocking case is $\pi_{ro} = p_o(\frac{z-p_o}{2} + \frac{1-p_o}{2}) + \frac{R_o}{2}(z-1) - w_o(z-p_o)$. From this the optimal retail price $p_o(w_o) = \frac{z+1+2w_o}{4}$. The manufacturer problem is to maximize the profit function $\pi_{mo} = (w_o - c)(z - p_o) - \frac{R_o}{2}(z - 1)$ subject to $w_o > 0$, $R_o \geq 0$ and the retailer getting atleast the profit available from the best possible deviation. Note again that the only feasible deviation for the retailer will be to an understocking case. The retailer's off-equilibrium deviation profit will be $\pi_{rd} = (1 - p_d)(p_d - w_o)$. Finding the optimal $p_d(w_o)$ and substituting back we get $\pi_{rd}(w_o) = \frac{(1-w_o)^2}{4}$. The Lagrangian for the manufacturer's optimization will be,

$$L_{mo} = (w_o - c)(z - p_o) - \frac{R_o}{2}(z - 1) + \mu_{w_o}w_o + \mu_R R_o + \mu_d(\pi_{ro}(w_o, R_o) - \pi_{rd}(w_o))$$

Consider first the case in which $R_o > 0$ and $\mu_d > 0$ which implies that the retailer profit condition is binding. Because $R_o > 0$, $\mu_R = 0$ and $\frac{\partial L_{mo}}{\partial R_o} = 0$ which implies that $\mu_d = 1$. Because $w_o > 0$, $\frac{\partial L_{mo}}{\partial w_o} = 0$ and $\mu_{w_o} = 0$ and from this we can calculate $w_o = \frac{1+c}{2}$. The corresponding retail price is $p_o^* = \frac{z+c+2}{4}$. Because $\mu_d > 0$ we will have $\frac{\partial L_{mo}}{\partial \mu_d} = 0$ from which we can calculate $R_o = \frac{3+6c-z}{8}$. Since we need $R_o > 0$, this results in the condition $z < (3 + 6c)$. The equilibrium manufacturer profit can be calculated to be $\pi_{mo}^* = \frac{(z^2+2z-12zc+2c^2+8c-1)}{16}$ and the equilibrium retailer profit is $\pi_{ro}^* = \frac{(1-c)^2}{16}$. Finally, for the demand in the low state to be positive we need $(1 - p_o^*) > 0 \implies z < (2 - c)$. Thus the binding condition for existence of this overstocking case is $z < (2 - c)$.

Finally, suppose that $\mu_d = 0$. Note as usual because $w_o > 0$ then $\mu_{w_o} = 0$. Now suppose $R_o > 0$, then $\frac{\partial L_{mo}}{\partial R_o} = 0$ and $\mu_{R_o} = 0$. But that will imply that $\mu_d = 1$ which contradicts our assumption. Therefore, $R_o = 0$ if $\mu_d = 0$. Of course, if $\mu_d = 0$, then we must have that $\pi_{ro}(w_o, R_o) - \pi_{rd}(w_o) > 0$. But $\pi_{ro}(w_o, R_o) - \pi_{rd}(w_o) = -\frac{(z-1)(4z-3(1-c))}{8} < 0$ always. Therefore this case never exists.

General Case ($0 < \rho < 1$)

In this analysis, we proceed like in the channel with inventory case above, however we need to analyze both types of equilibrium (overstocking and understocking), for both types of possible signal (\tilde{h} or \tilde{l}). We start with the \tilde{h} signal followed by the \tilde{l} signal.

Understocking given \tilde{h} : Based on $w_{\tilde{h}}$ offered by the manufacturer, the retailer chooses the inventory and the retail price. The retailer's profit function is $\pi_{r\tilde{h}u} = (1 - p_{\tilde{h}})(p_{\tilde{h}} - w_{\tilde{h}})$. From this the optimal retail price is $p_{\tilde{h}}(w_{\tilde{h}}) = \frac{1+w_{\tilde{h}}}{2}$ and the retailer profit becomes $\pi_{r\tilde{h}u}(w_{\tilde{h}}) = \frac{(1-w_{\tilde{h}})^2}{4}$. In choosing the contract $w_{\tilde{h}}$ the manufacturer must guard against two possible deviations from the equilibrium. Either the retailer can deviate to overstocking with positive demand in both states or the retailer can deviate to overstocking with positive demand only in the high state. We derive the equilibrium for the case in which deviation that is relevant for

the manufacturer to guard against in supporting the equilibrium wholesale price is the one in which the retailer may deviate to overstocking with positive demand in both states.

In deviating to overstocking with positive demand in both states (denoted by subscript d), and given $w_{\tilde{h}}$, if the deviation price is $p_{\tilde{h}d}$, then the deviation profit will be $\pi_{r\tilde{h}d} = \Pr(h|\tilde{h})(z - p_{\tilde{h}d})p_{\tilde{h}d} + \Pr(l|\tilde{h})(1 - p_{\tilde{h}d})p_{\tilde{h}d} - (z - p_{\tilde{h}d})w_{\tilde{h}}$. This leads to an optimal deviation price to be $p_{\tilde{h}d}(w_{\tilde{h}}) = \frac{1+z+2w_{\tilde{h}}+(z-1)\rho}{4}$. Thus if the channel equilibrium were to involve understocking, then this understocking equilibrium must provide the retailer with atleast as much profit as $\pi_{r\tilde{h}d}(p_{\tilde{h}d}(w_{\tilde{h}}))$.

The manufacturer's problem is to choose $w_{\tilde{h}}$ to maximize $\pi_{m\tilde{h}u} = (1-p_{\tilde{h}})(w_{\tilde{h}}-c)$ subject to $w_{\tilde{h}} > 0$ and the retailer getting atleast the profit available from the best possible deviation. The corresponding Lagrangian is:

$$L_{m\tilde{h}u} = (1-p_{\tilde{h}})(w_{\tilde{h}}-c) + \mu_{w_{\tilde{h}}}w_{\tilde{h}} + \mu_{\tilde{h}d}(\pi_{r\tilde{h}u}(w_{\tilde{h}}) - \pi_{r\tilde{h}d}(p_{\tilde{h}d}(w_{\tilde{h}})))$$

We consider only $w_{\tilde{h}} > 0$ and therefore $\frac{\partial L_{m\tilde{h}u}}{\partial w_{\tilde{h}}} = 0$ and $\mu_{w_{\tilde{h}}} = 0$. Let us first consider the case for which $\mu_{\tilde{h}d} = 0$, which means that the deviation constraint is not binding for the retailer. Solving $\frac{\partial L_{m\tilde{h}u}}{\partial w_{\tilde{h}}} = 0$ gives us $w_{\tilde{h}u1} = \frac{1+c}{2}$. In order for $(\pi_{r\tilde{h}u}(w_{\tilde{h}}) > \pi_{r\tilde{h}d}(p_{\tilde{h}d}(w_{\tilde{h}})))$, we must have $(1 < z < \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2})$. This is one possible solution that leads to a manufacturer profit of $\pi_{m\tilde{h}u1}^* = \frac{(1-c)^2}{8}$. Next we consider the case when the deviation constraint is binding, i.e., $(\pi_{r\tilde{h}u}(w_{\tilde{h}}) = \pi_{r\tilde{h}d}(p_{\tilde{h}d}(w_{\tilde{h}})))$, and that implies $\mu_{\tilde{h}d} \geq 0$. Solving this along with $\frac{\partial L_{m\tilde{h}u}}{\partial w_{\tilde{h}}} = 0$ gives us $w_{\tilde{h}u2} = \frac{3+z(1+\rho)^2+\rho(2-\rho)}{4(3-\rho)}$ and $\mu_{\tilde{h}d} = \frac{z(\rho+1)^2-3(1+2c)+\rho(4+2c-\rho)}{(3(3-2\rho)+\rho^2)(z-1)}$. Note that the Lagrange multiplier must be positive, which is satisfied for $z > \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2}$. This solution gives the manufacturer profit as $\pi_{m\tilde{h}u2}^* = \frac{(\rho^2-2z\rho-z\rho-z-3-2\rho+12c-4c\rho)(6\rho-9+2z\rho+z\rho^2+z-\rho^2)}{32(3-\rho)^2}$. Thus, subject to the regularity condition, an understocking equilibrium given the signal \tilde{h} may involves $w_{\tilde{h}u}^* = w_{\tilde{h}u1}^*, \pi_{m\tilde{h}u}^* = \pi_{m\tilde{h}u1}^*$, or, $w_{\tilde{h}u}^* = w_{\tilde{h}u2}^*$ and $\pi_{m\tilde{h}u}^* = \pi_{m\tilde{h}u2}^*$ subject to their respective regularity conditions for existence.

Next, we need to find conditions for which the equilibrium outcomes identified above is immune to the retailer deviating to a case where he sells only in the h state. Notice that the deviation profit for the retailer in this case is $\pi_{rd} = \Pr(h|\tilde{h})(z - p_d)p_d - (z - p_d)w_{\tilde{h}}$. This means $p_d(w_{\tilde{h}}) = \frac{z(\rho+1)+2w_{\tilde{h}}}{2(1+\rho)}$ and $\pi_{rd}(w_{\tilde{h}}) = \frac{[z(1+\rho)-2w_{\tilde{h}}]^2}{8(1+\rho)}$. As long as this deviation profit remains lower than $\pi_{r\tilde{h}u1}^* = \frac{(1-c)^2}{16}$ for $w_{\tilde{h}} = w_{\tilde{h}u1}^* = \frac{(1+c)}{2}$, the first equilibrium identified above remains robust to a retailer deviation of this kind. The condition simplifies to $z < \frac{2(1+c)+\sqrt{2(1+\rho)(1-c)^2}}{2(1+\rho)}$. Following similar approach, the second equilibrium identified above remains robust to a retailer deviation for $z < \frac{(3-\rho)(2\rho+2+4\sqrt{2\rho+2})}{2(7+6\rho-\rho^2)}$.

Overstocking given \tilde{h} : The retailer's profit function in the overstocking case is $\pi_{r\tilde{h}o} = \Pr(h|\tilde{h})(p_{\tilde{h}})(z - p_{\tilde{h}}) + \Pr(l|\tilde{h})[(p_{\tilde{h}})(1 - p_{\tilde{h}}) + R_{\tilde{h}}(z - 1)] - w_{\tilde{h}}(z - p_{\tilde{h}})$. From this the optimal retail price is $p_{\tilde{h}}(w_{\tilde{h}}) = \frac{z+1+2w_{\tilde{h}}+(z-1)\rho}{4}$. The manufacturer problem is to maximize the profit function $\pi_{mo\tilde{h}} = (w_{\tilde{h}} - c)(z - p_{\tilde{h}}) - \Pr(l|\tilde{h})R_{\tilde{h}}(z - 1)$

subject to $w_{\tilde{h}} > 0, R_{\tilde{h}} \geq 0$ and the retailer getting at least the profit available from the best possible deviation to an understocking case. The retailer's deviation profit will be $\pi_{r\tilde{h}d} = (1 - p_{\tilde{h}d})(p_{\tilde{h}d} - w_{\tilde{h}})$. Finding the optimal $p_{\tilde{h}d}(w_{\tilde{h}})$ and substituting back we get $\pi_{r\tilde{h}d}(w_{\tilde{h}}) = \frac{(1-w_{\tilde{h}})^2}{4}$. The Lagrangian for the manufacturer's optimization will be,

$$L_{m\tilde{h}o} = (w_{\tilde{h}} - c)(z - p_{\tilde{h}}) - \Pr(l|\tilde{h})R_{\tilde{h}}(z - 1) + \mu_{w_{\tilde{h}}}w_{\tilde{h}} + \mu_{R_{\tilde{h}}}R_{\tilde{h}} + \mu_{\tilde{h}d}(\pi_{r\tilde{h}o}(w_{\tilde{h}}, R_{\tilde{h}}) - \pi_{r\tilde{h}d}(w_{\tilde{h}}))$$

Starting with the case of $\mu_{\tilde{h}d} = 0$ and the deviation constraint not binding: since $w_{\tilde{h}} > 0$ then we must have $\mu_{w_{\tilde{h}}} = 0$. Now suppose $\frac{\partial L_{m\tilde{h}o}}{\partial R_{\tilde{h}}} = 0$, which gives us $\mu_{R_{\tilde{h}}} = \frac{(1-\rho)(z-1)}{2} > 0$. This implies that $R_{\tilde{h}} = 0$. Now substituting these into $\mu_{w_{\tilde{h}}} = 0$, we get $w_{\tilde{h}1} = \frac{1}{4}(3z - 1 + 2c - (z - 1)\rho)$. Substituting $R_{\tilde{h}} = 0$ and $w_{\tilde{h}} = w_{\tilde{h}1}$, we get the deviation constraint $(\pi_{r\tilde{h}o}(w_{\tilde{h}}, R_{\tilde{h}}) - \pi_{r\tilde{h}d}(w_{\tilde{h}}))$. For this solution to be valid, this must be non-negative. This and the condition for demand to be positive in both states gives us a regularity conditions in z as $z \leq \min(\frac{(3-\rho)(1-c)}{4(1-\rho)}, \frac{7-2c+\rho}{5+\rho})$. Under this solution, the manufacturer's expected profit is $\pi_{m\tilde{h}o1}^* = \frac{(1-3z+z\rho-\rho+2c)^2}{32}$.

Consider next the case in which $R_{\tilde{h}} > 0$ and $\mu_{\tilde{h}d} > 0$, which implies that the retailer profit condition is binding. Solving $\frac{\partial L_{m\tilde{h}o}}{\partial R_{\tilde{h}}} = 0$, $\frac{\partial L_{m\tilde{h}o}}{\partial w_{\tilde{h}}} = 0$ and the binding profit condition together, we get $w_{\tilde{h}} = \frac{1+c}{2}$, $\mu_{\tilde{h}d} = 1$ and $R_{\tilde{h}} = \frac{3+6c-z-\rho^2(z-1)-2\rho(z+c+2)}{8(1-\rho)}$. The corresponding retail price is $p_{\tilde{h}}^* = \frac{z+c+2+\rho(z-1)}{4}$. From the positive demand condition in off-equilibrium path, we get a condition: $z < \frac{2-c+\rho}{1+\rho}$. In addition, the requirement for positive $R_{\tilde{h}}$ gives us another condition, $z \leq \frac{3+6c-\rho(4+2c-\rho)}{(1+\rho)^2}$. Under these regularity conditions, the equilibrium manufacturer profit can be calculated to be $\pi_{m\tilde{h}o2}^* = \frac{(2z-1+8c-12cz+2c^2+z^2)+(z-1)^2\rho^2+2\rho(z^2+2c(z-1)-1)}{16}$ and the equilibrium retailer profit is $\pi_{r\tilde{h}}^* = \frac{(1-c)^2}{16}$. Depending upon the parameter values, one or the other solution obtained above may not exist. Whenever both solutions exist, we can compare $\pi_{m\tilde{h}o1}^*$ and $\pi_{m\tilde{h}o2}^*$ and show that for $\rho < \rho^{\tilde{h}} = \frac{2[8z(c+z)+(1-c)^2]^{\frac{1}{2}} - (5z+1+2c)}{(z-1)}$, $\pi_{m\tilde{h}o1}^*$ is higher for the manufacturer and hence for that, $w_{\tilde{h}} = \frac{1}{4}(3z - 1 + 2c - (z - 1)\rho)$ and $R_{\tilde{h}} = 0$. Otherwise, $w_{\tilde{h}} = \frac{1+c}{2}$ and $R_{\tilde{h}} = \frac{3+6c-z-\rho^2(z-1)-2\rho(z+c+2)}{8(1-\rho)}$.

Understocking given \tilde{l} : Given $w_{\tilde{l}}$ from the manufacturer, the retailer chooses the inventory and the retail price. The retailer's profit function is $\pi_{r\tilde{l}} = (1 - p_{\tilde{l}})(p_{\tilde{l}} - w_{\tilde{l}})$. From this the optimal retail price is $p_{\tilde{l}}(w_{\tilde{l}}) = \frac{1+w_{\tilde{l}}}{2}$ and the retailer profit becomes $\pi_{r\tilde{l}u}(w_{\tilde{l}}) = \frac{(1-w_{\tilde{l}})^2}{4}$. Once again, all deviations from the understocking equilibrium need to be considered. Just as we did for the \tilde{h} signal, we again derive the equilibrium considering the case where the relevant off-equilibrium deviation is that of the retailer moving to overstocking when there is positive demand in both states of demand. Given $w_{\tilde{l}}$, let the retailer choose the deviation price $p_{\tilde{l}d}$ so that there is demand in the off-equilibrium path irrespective of the state. The deviation profit will be $\pi_{r\tilde{l}d} = \Pr(h|\tilde{l})(z - p_{\tilde{l}d})p_{\tilde{l}d} + \Pr(l|\tilde{l})(1 - p_{\tilde{l}d})p_{\tilde{l}d} - (z - p_{\tilde{l}d})w_{\tilde{l}}$. This leads to an optimal deviation price of $p_{\tilde{l}d}(w_{\tilde{l}}) = \frac{1+z+2w_{\tilde{l}}-(z-1)\rho}{4}$. Thus if the channel equilibrium were to involve understocking, then this understocking equilibrium must provide

the retailer with at least as much profit as $\pi_{r\tilde{d}}(p_{\tilde{d}}(w_{\tilde{t}}))$.

The manufacturer's problem is to choose $w_{\tilde{t}}$ to maximize $\pi_{m\tilde{u}} = (1 - p_{\tilde{t}})(w_{\tilde{t}} - c)$ subject to $w_{\tilde{t}} > 0$ and the retailer getting at least the profit available from the best possible deviation. The corresponding Lagrangian will be:

$$L_{m\tilde{u}} = (1 - p_{\tilde{t}})(w_{\tilde{t}} - c) + \mu_{w_{\tilde{t}}}w_{\tilde{t}} + \mu_{\tilde{d}}(\pi_{ru}(w_{\tilde{t}}) - \pi_{rd}(p_{\tilde{d}}(w_{\tilde{t}})))$$

Let us first consider the case for which $\mu_{\tilde{d}} = 0$ which means that the deviation constraint is not binding for the retailer. Solving $\frac{\partial L_{m\tilde{u}}}{\partial w_{\tilde{t}}} = 0$ gives us $w_{\tilde{t}u1} = \frac{1+c}{2}$. In order for $\pi_{rd} \geq 0$, we must have $(1 < z < \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2})$. This is one possible solution that leads to a manufacturer profit of $\pi_{m\tilde{u}1}^* = \frac{(1-c)^2}{8}$ and $\pi_{r\tilde{u}1}^* = \frac{(1-c)^2}{16}$. Next we consider the case when the deviation profit constraint is binding. This implies $\mu_{\tilde{d}} \geq 0$. Solving this along with $\frac{\partial L_{m\tilde{u}}}{\partial w_{\tilde{t}}} = 0$ gives us $w_{\tilde{t}u2} = \frac{3+z(1-\rho)^2-\rho(2+\rho)}{4(3+\rho)}$ and $\mu_{\tilde{d}} = \frac{z(1-\rho)^2-3(1+2c)-\rho(4+2c+\rho)}{(3(3+2\rho)+\rho^2)(z-1)}$. Note that for $\mu_{\tilde{d}} \geq 0$, we must have $z > \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2}$, which is the regularity condition for existence of this solution. Thus given the signal \tilde{l} , an understocking equilibrium may involve $w_{\tilde{t}u1}^* = \frac{(1+c)}{2}$ and no returns price, or $w_{\tilde{t}u2} = \frac{3+z(1-\rho)^2-\rho(2+\rho)}{4(3+\rho)}$ and no returns price.

Finally, to identify condition when the above outcome is immune to a retailer deviation involving demand only in high state, we start by identifying that the deviation profit will be $\pi_{rd} = \Pr(h|\tilde{l})(z - p_d)p_d - (z - p_d)w_{\tilde{t}}$. This means $p_d^*(w_{\tilde{t}}) = \frac{z(1-\rho)+2w_{\tilde{t}}}{2(1-\rho)}$ and $\pi_{rd}(w_{\tilde{t}}) = \frac{[z(1-\rho)-2w_{\tilde{t}}]^2}{8(1-\rho)}$. Notice that this is valid only if $p_d^*(w_{\tilde{t}}) < z$ as otherwise demand in High state is also negative which is meaningless. Subject to this regularity condition, comparing the equilibrium profit with the deviation profit, we get that whenever $\frac{2(1+c)-[2(1-c)^2(1-\rho)]^{\frac{1}{2}}}{2(1-\rho)} \leq z \leq \frac{2(1+c)+[2(1-c)^2(1-\rho)]^{\frac{1}{2}}}{2(1-\rho)}$, the identified equilibrium is immune to this deviation.

Overstocking given \tilde{l} : The retailer's profit function in the overstocking case is $\pi_{r\tilde{o}} = \Pr(h|\tilde{l})(p_{\tilde{t}})(z - p_{\tilde{t}}) + \Pr(l|\tilde{l})[(p_{\tilde{t}})(1 - p_{\tilde{t}}) + R_{\tilde{t}}(z - 1)] - w_{\tilde{t}}(z - p_{\tilde{t}})$. From this the optimal retail price $p_{\tilde{t}}(w_{\tilde{t}}) = \frac{z+1+2w_{\tilde{t}}-(z-1)\rho}{4}$. The manufacturer problem is to maximize the profit function $\pi_{m\tilde{o}} = (w_{\tilde{t}} - c)(z - p_{\tilde{t}}) - \Pr(l|\tilde{l})R_{\tilde{t}}(z - 1)$ subject to $w_{\tilde{t}} > 0, R_{\tilde{t}} \geq 0$ and the retailer getting at least the profit available from the best possible deviation to an understocking case. The retailer's deviation profit will be $\pi_{r\tilde{d}} = (1 - p_{\tilde{d}})(p_{\tilde{d}} - w_{\tilde{t}})$. Finding the optimal $p_{\tilde{d}}(w_{\tilde{t}})$ and substituting back we get $\pi_{r\tilde{d}}(w_{\tilde{t}}) = \frac{(1-w_{\tilde{t}})^2}{4}$. The Lagrangian for the manufacturer's optimization will be,

$$L_{m\tilde{o}} = \Pr(h|\tilde{l})(w_{\tilde{t}} - c)(z - p_{\tilde{t}}) - \Pr(l|\tilde{l})R_{\tilde{t}}(z - 1) + \mu_{w_{\tilde{t}}}w_{\tilde{t}} + \mu_{R_{\tilde{t}}}R_{\tilde{t}} + \mu_{\tilde{d}}(\pi_{r\tilde{o}}(w_{\tilde{t}}, R_{\tilde{t}}) - \pi_{r\tilde{d}}(w_{\tilde{t}}))$$

Starting with the case of deviation constraint not binding, since $w_{\tilde{t}} > 0$ then we must have $\mu_{w_{\tilde{t}}} = 0$. Now suppose $\frac{\partial L_{m\tilde{o}}}{\partial R_{\tilde{t}}} = 0$, which gives us $\mu_{R_{\tilde{t}}} = \frac{(1-\rho)(z-1)}{2} > 0$. This implies that $R_{\tilde{t}} = 0$. Now substituting these into $\mu_{w_{\tilde{t}}} = 0$, we get $w_{\tilde{t}1}^* = \frac{1}{4}(3z - 1 + 2c + (z - 1)\rho)$. Substituting $R_{\tilde{t}}^* = 0$ and $w_{\tilde{t}} = w_{\tilde{t}1}^*$ we can calculate the deviation constraint, which must be non-negative to be valid. However, that requires $(1 \geq z \geq \frac{(3+\rho)(1-c)}{4(1-\rho)})$, which is not possible so this solution is ruled out.

Consider next the case in which $R_{\tilde{t}} > 0$ and $\mu_{\tilde{d}} > 0$, which implies that the retailer profit condition is binding. Solving $\frac{\partial L_{m\tilde{o}}}{\partial R_{\tilde{t}}} = 0$, and $\frac{\partial L_{m\tilde{o}}}{\partial w_{\tilde{t}}} = 0$ and the binding profit condition together, we get $w_{\tilde{t}} = \frac{1+c}{2}$,

$\mu_{id} = 1$ and $R_i = \frac{3+6c-z+\rho^2(z-1)+2\rho(z+c+2)}{8(1+\rho)}$. Since R_i must be positive, we must have $z \leq \frac{3+6c+\rho(4+2c+\rho)}{(1-\rho)^2}$. The corresponding retail price is $p_i^* = \frac{z+c+2-\rho(z-1)}{4}$. From the positive demand condition in both states, we get another regularity condition: $z < \frac{2-c-\rho}{1-\rho}$. Under these conditions, the equilibrium manufacturer profit can be calculated to be $\pi_{mlo2}^* = \frac{(2z-1+8c-12cz+2c^2+z^2)+(z-1)^2\rho^2-2\rho(z^2+2c(z-1)-1)}{16}$ and the equilibrium retailer profit is $\pi_{rlo}^* = \frac{(1-c)^2}{16}$. For $\pi_{mlo2}^* \geq 0$, we must have low enough reliability i.e., $\rho < \rho^l = \frac{z+1+2c-[2(1-c)^2+8cz]^{\frac{1}{2}}}{z-1}$. This is the only surviving solution subject to the regularity conditions already mentioned above.

Downstream Retail Competition

Channel with Inventory ($\rho = 0$)

In the competitive case also, we need to consider two types of equilibrium as before, as the in-between case can be ruled out for equilibrium (and deviations) for reasons similar to the single retailer channel. We solve for the symmetric equilibrium.

Understocking Equilibrium

Based on the w offered by the manufacturer, the retailers choose the inventory and the retail price. Retailer i 's profit function is $\pi_{riu} = [1-p_i + \gamma(p_j - p_i)](p_i - w)$. From this the optimal retail price is $p_1(w) = p_2(w) = \frac{1+w(1+\gamma)}{2+\gamma}$ and retailer i 's profit can be expressed as $\pi_{riu}(w)$. Given the contract no retailer's should have the incentive to unilaterally deviate to any choice of inventory and price other than understocking. Like in the single retailer case, once again the deviation possibilities involve overstocking, either with positive demand in both states or only in the high state. We will consider the equilibrium where the relevant off-equilibrium deviation supporting the equilibrium involves the case where there is demand in both states. Later on we will identify conditions when this equilibrium is immune to the deviation to the other type of overstocking. Without loss of generality, let the deviating retailer be retailer 1.

If retailer 1 deviates to overstocking, it chooses $Q = (z - p_1 + \gamma(p_2 - p_1))$. Let retailer 1's choice of the deviation price be called p_{1d} . The deviation profit function will be $\pi_{r1d} = \frac{1}{2}(z - p_{1d} + \gamma(p_2(w) - p_{1d}))p_{1d} + \frac{1}{2}(1 - p_{1d} + \gamma(p_2(w) - p_{1d}))p_{1d} - (z - p_{1d} + \gamma(p_2(w) - p_{1d}))w$. Thus for a given w , we will have the optimal deviation price to be $p_{1d}(w) = \frac{4w(\gamma+1)^2+z(\gamma+2)+3\gamma+2}{4(\gamma^2+3\gamma+2)}$. The equilibrium contract must provide the retailer with atleast as much profit as $\pi_{r1d}(p_{1d}(w))$.

The manufacturer's problem is to choose w to maximize $\pi_{mu} = [(1 - p_1 + \gamma(p_2 - p_1)) + (1 - p_2 + \gamma(p_1 - p_2))](w - c)$ subject to $w > 0$ and the retailer getting atleast the profit available from the deviation identified above. The corresponding Lagrangian will be,

$$L_{mu} = [(1 - p_1 + \gamma(p_2 - p_1)) + (1 - p_2 + \gamma(p_1 - p_2))](w - c) + \mu_w w + \mu_d (\pi_{riu}(w) - \pi_{rid}(p_{1d}(w)))$$

where the μ 's are the corresponding Lagrangian multipliers. Parallel to the analysis in the single retailer case, we

look at the solution candidates one by one based on the relevant first order conditions. One solution candidate is $w_1^* = \frac{1+c}{2}$, which exists only if $z \in (1, \frac{4c(\gamma^2+4\gamma+3)+(4\gamma^2+9\gamma+6)}{2+\gamma})$ and the deviation constraint is not binding. Whenever it exists, it gives $\pi_{mu} = \pi_{mu1}^* = \frac{(1+\gamma)(1-c)^2}{2(2+\gamma)}$ as the manufacturer profit. Another solution candidate is $w_2^* = \frac{z(\gamma+2)+7\gamma+6}{8(\gamma^2+4\gamma+3)}$, which exists only if $[\frac{4c\gamma^2+4\gamma^2-9\gamma-16c\gamma-12c-6}{2+\gamma} < z < (9+8\gamma)]$ and the deviation constraint is binding. Whenever this exists, it gives $\pi_{mu} = \pi_{mu2}^* = \frac{(8\gamma+9-z)(z\gamma+2z+7\gamma+6-8c\gamma^2-32c\gamma-24c)}{32(3+\gamma)(\gamma^2+4\gamma+3)}$. It can be shown that $\pi_{mu2}^* \leq \pi_{mu1}^*$. Thus the maximum equilibrium manufacturer profits are $\pi_{mu}^* = \frac{(1+\gamma)(1-c)^2}{2(2+\gamma)}$ and $w^* = \frac{1+c}{2}$.

To see if the equilibrium with w_1^* is immune against one retailer unilaterally deviating to the overstocking case of the type involving only high state demand, we start with calculating that the deviating retailer's profit function will be $\pi_{rid2}(w) = \frac{[z(\gamma+2)+\gamma-w(5\gamma+\gamma^2+4)]^2}{(2+\gamma)(3\gamma+\gamma^2+2)}$. For the retailer deviating from the equilibrium path to this deviation, however, it is sufficient that $(\pi_{riu}(w) - \pi_{rid2}(w))$ be negative for $w^* = \frac{1+c}{2}$. The condition for that turns out to be that $z \in (\frac{c(8+14\gamma+\gamma^3+7\gamma^2)+(5\gamma^2+10\gamma+\gamma^3+8)\pm 2[2((1-c)^2(4+6\gamma^3+13\gamma^2+12\gamma+\gamma^4)-12(1-\gamma))]}{2(\gamma^2+4\gamma+4)})^{\frac{1}{2}}$. Thus as long as z is between the two roots, the equilibrium identified above is valid. Finally for the manufacturer w_1^* is always the best strategy and it can be shown that no other strategy (that makes the retailer deviation involving the demand in the high state only relevant) is profit improving.

Overstocking Equilibrium

The retailer's profit function in the overstocking case is $\pi_{rio} = \frac{1}{2}[(z - p_i + \gamma(p_j - p_i))(p_i) + (1 - p_i + \gamma(p_j - p_i))(p_i) + R(z - 1)] - w(z - p_i + \gamma(p_j - p_i))$. From this the optimal retail price $p_i(w) = \frac{2w(1+\gamma)+z+1}{2(\gamma+2)}$. Given this, the manufacturer problem is to maximize the profits given by $\pi_{mo} = (w - c)[(z - p_1 + \gamma(p_2 - p_1)) + (z - p_2 + \gamma(p_1 - p_2))] - R(z - 1)$, subject to $w > 0$, $R \geq 0$ and the retailer getting at least the profit available from the best possible deviation. We need to consider deviation to the understocking case. Again, without loss of generality, let retailer 1 deviate and charge a price of p_{1d} . We can show that $p_{1d}(w) = \frac{4w(1+\gamma)^2+(3+z)\gamma+4}{4(2+3\gamma+\gamma^2)}$ and $\pi_{r1d}(p_{1d}(w)) = \frac{[3\gamma+4+z\gamma-4w(1+\gamma)]^2}{16(\gamma+2)(3\gamma+2+\gamma^2)}$. To solve the manufacturer's problem, we can set up the following Lagrangian:

$$\begin{aligned} L_{mo} &= (w - c)[(z - p_1 + \gamma(p_2 - p_1)) + (z - p_2 + \gamma(p_1 - p_2))] - R(z - 1) \\ &\quad + \mu_w w + \mu_R R + \mu_d (\pi_{rio}(w, R) - \pi_{rid}(w)) \end{aligned}$$

Proceeding in the usual manner to consider all solution candidates, we can show that the only solution candidate that can survive involves retailer's deviation constraint to be binding and it involves a positive returns price R . Whenever $z < \frac{3\gamma+4-2c(1+\gamma)}{2+\gamma}$ the manufacturer may offer a wholesale price $w^* = \frac{2c(1+\gamma)+2+(\gamma(z+1))}{4(1+\gamma)}$ and the returns price $R^* = \frac{2\gamma^2 z + 3z\gamma - 2z + 2(2c+1)\gamma^2 + 16c\gamma + 5\gamma + 12c + 6}{3\gamma + 2 + \gamma^2}$. This results in a manufacturer profit of $\pi_{mo}^* = \frac{(\gamma+2)(z+1)^2 + 4c(c+c\gamma+\gamma-3z\gamma+4+6z)-4}{8(\gamma+2)}$.