Parameter Stability and the Valuation of Mortgages

And Mortgage-backed Securities

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Abstract

We examine the effect of parameter instability on the valuation of mortgages and mortgage-backed securities. In particular, we price 1997 issue mortgages subject to the 1998 bond market rally events assuming an empirically derived prepayment model constructed on data reflecting the 1993 experience and compare results to those that would have been obtained had actual parameter drift been captured. Using carefully constructed micro-data, we find that the refinancing propensity was greater in 1998 for a 1997 issue given the same incentives, compared to the 1993 performance of a 1992 issue. The associated change in cash flow patterns produces a 1.7% change in asset values. Results are consistent with technological improvement and greater efficiency in the mortgage market and may help explain the large losses often sustained by mortgage market participants during bond market rallies.

Introduction

Economic analysis of portfolio choice and equilibrium asset pricing typically relies on the assumption that investors are perfectly informed as the value of parameters that affect asset returns. For example, investors need to know both expected returns and the covariance matrix of returns to construct an optimally diversified portfolio. But in the real world investors must estimate parameters from noisy empirical data. As a result, those parameter estimates are imperfect at any particular point in time. To make matters worse, economic conditions and technological change may alter the fundamental underlying relationships so that parameter values are non-stationary over time. The effect of this parameter instability on asset pricing has been a topic of wide interest in finance; however, most existing research focuses on equity markets. In this paper, we extend consideration to the mortgage market, today the largest component of the overall
bond market, and evaluate the effect of estimation, or model, risk on pricing mortgages and mortgage-backed securities. Model risk is particularly acute in the valuation of mortgages and mortgage-related assets since cash flows are to a considerable extent interest rate dependent due to the call option embedded in the typical contract1.

Many commentators have noted the size and growing importance of the mortgage market in the overall capital market. Indeed, thanks to recent government surpluses and repayments on the national debt, by some calculations the mortgage market is now larger than the government bond market2. Given the embedded call option in mortgage contracts, correctly modeling borrower refinancing behavior is a central problem in mortgage valuation. Econometric models of mortgage prepayment typically use data from prior periods to estimate the probability of refinancing conditional on interest rate levels, loan seasoning, and other factors. These models are then used to predict future prepayment rates under simulated interest rate conditions. But if market conditions are evolving over time such that model parameters are unstable, predictions may diverge widely from actual results. In the current paper, we examine this problem and price mortgages using standard option-adjusted spread (OAS) techniques incorporating prepayment models based on disaggregated loan level event histories. Results suggest an economically significant difference in asset values associated with parameter instability

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1 In addition, where credit risk is insured, such as in agency MBS, any defaults in the underlying pool translate into prepayments to the investor.

2 According to the Bond Market Association, as of Q3 2000, the total mortgage-backed securities market was $2.4 trillion. After adding $0.8 trillion in asset-backed securities, which includes non-shelter credit card receivables, as well as home equity, and manufactured housing assets, the total market comes to $3.2 trillion. The Treasury bond market (excluding agency issues) has been declining in total size since 1996 and now stands at approximately $3.0 trillion.
over time. This model risk may help explain the large losses often incurred by mortgage market participants during bond market rallies.

**Literature Review**

Early theoretical research focusing on portfolio choice under parameter uncertainty includes Barry [1974], Klein and Bawa [1976], Klein and Bawa [1977], Brown [1979] and Bawa and Brown [1979], Barry and Brown [1985], Coles and Lowenstein [1988], and Coles, Lowenstein, and Suay [1995]. These papers focus on equity markets, for example, construction of portfolios when mean return vectors are known but covariance matrices are not.

The literature is much smaller when we turn to the bond market. It has become widely recognized that term structure volatility is not constant over time and, accordingly, the pricing of fixed income instruments optimally incorporates stochastic volatility. Bliss and Smith [1997] argue that choice of model and parameter stability is closely connected. They re-examine the often cited Chan, Karolyi, Longstaff, and Sanders [1992] analysis of term structure volatility and the ability of various single factor term structure diffusion processes to model rate movements over the period 1964-1989. Bliss and Smith argue that a structural break occurred during the early 1980s due to Fed policy, i.e. that parameters were not stable over the study period. A related analysis focusing on the effect of alternative interest rate processes on the value of mortgages, in particular,
appears in Archer and Ling [1995]. We have not been able to find published research that specifically quantifies model risk in mortgages, however.

Turning now to the mortgage research literature, we note initially that the volume of work on termination risk in general, and prepayments in particular, is vast, so our survey here is necessarily abbreviated. A number of themes are, nevertheless, evident. Among the earliest to examine the topic were Green and Shoven (1986) and Quigley (1988), both of who identified the role of interest rates as well as borrower mobility on rates of mortgage prepayment, and Schwartz and Torous (1989) who related prepayment rates to the valuation of mortgage-backed securities. Academic interest in the topic accelerated during the early 1990s with the mainly theoretical papers by Brueckner (1992), Follain, Scott, and Yang (1992), Kau et al (1992a, 1992b), Brueckner (1994), and Stanton (1994). These papers dealt with optimal exercise of the borrower’s call option given stochastic interest rates and the implications for mortgage contract design and pricing, both of mortgages and mortgage-backed securities. Concurrently, Wall Street firms were developing proprietary prepayment models for use in valuation routines that supported their trading strategies (Richard and Roll [1989], Patruno [1994], and Hayre and Rajan [1995], Hayre, Chaudhary, and Young [2000]).

The refinancing wave of 1993 followed by the sharp increase in rates during 1994 produced large losses for many market participants, reinforcing the business imperative to develop better models3. During the latter half of the 1990s, many researchers turned

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3 The latest example of major losses involving the valuation of mortgages is Homescape Lending, Inc. This top ten U.S. mortgage lender was acquired by National Bank of Australia in 1997 for approximately $1.0
their attention to institutional constraints that might reduce prepayments, even when call options appeared to be deep in the money (Peristiani et al [1996], Archer et al [1996, 1997], Caplin et al [1997], Green and LaCour-Little [1999]). These studies identified declines in collateral value, credit status, and other macroeconomic forces, such as unemployment, as significant factors inhibiting prepayment. Research in the late 1990s and into the new century re-focused on the role of borrower mobility (Clapp, Harding, and LaCour-Little [2000]), Pavlov [2001]), and a more complex specification of mortgage termination risk using a competing risk framework (Deng, Quigley, and Van Order [2000], Ambrose and LaCour-Little [2001]).

Relatively less has been written linking institutional and technological change in the mortgage industry with borrower behavior. Arora, Heike, and Mattu (2000) review historical returns in the mortgage-backed securities sector during the 1990s. They find a “…steady improvement in refinancing efficiency over the past decade, which has caused prepayment models to consistently understate mortgage callability”4 LaCour-Little (2001) reviews the effect of technology on the mortgage industry over the last decade, noting clear evidence of declining mortgage servicing costs but limited evidence of declining mortgage origination costs. Peristiani, Bennett, and Peach (1998) argue that the mortgage market has evolved considerably since the 1980s as a result of technological, institutional, and competitive market forces causing greater rates of prepayment during the 1990s than would otherwise have been expected. In their empirical analysis, Peristiani et al use loan level data, including credit scores, to compare a cohort of loans

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4 billion; the parent announced in September 2001 a write-off of $1.2 billion to cover unanticipated losses in the valuation of mortgages and mortgage servicing rights.
originated during 1984-1990 to a cohort originated during 1991-1994, using hazard model techniques. They report an increased propensity to refinance among the latter group.

Our contribution here is to re-focus attention on the temporal stability of the parameters of the prepayment function, using carefully constructed micro-level data from the two most recent refinancing waves, and the pricing impact of this instability. Specifically, we estimate hazard models for each of the two cohorts and find the response to the refinancing incentive to be much greater in the 1997 cohort, compared to the 1992 cohort, when measured over comparable duration. An interesting additional effect appears to be a decline in the constraining effect of contemporaneous LTV, apparent in the 1992 cohort but significantly diminished in the 1997 cohort.

**The Theory of Mortgage Prepayment**

Following Archer, Ling, and McGill [1996], one may represent the total probability of mortgage termination, \( \lambda_{Tt} \), for all reasons, as:

\[
\lambda_{Tt} = \lambda_{Dt} + (1 - \lambda_{Dt}) [\lambda_{Mt} + (1-\lambda_{Mt}) \lambda_{PNMt}] \tag{1}
\]

where \( \lambda_{Dt} \) is the probability of termination due to default at time \( t \), \( \lambda_{Mt} \) is the probability of terminating due to moving at time \( t \), and \( \lambda_{PNMt} \) is the probability of terminating due refinancing in place (prepaying and not moving) at time \( t \). Since we employ a very short

\[\text{Arora, Heike, and Mattu (page 6).}\]
time horizon in our data (essentially the first 36 months of loan life), we make the simplifying assumption that $\lambda_{D_t} = \lambda_{M_t} = 0$. Clapp, Harding, and LaCour-Little (2000), using data in which movers could be separated from refinancers, found that the cumulative probability of mobility for FRM borrowers was approximately .045 over a three and one-half year period during the mid-1990s, so our assumption regarding mobility seems reasonable.

The potential savings that can be realized over the remaining life of the loan drive refinancing decisions. Consider a mortgage with book value $BV(r_c,t)$, where $r_c$ is the contract rate of interest. The value of $BV(r_c,t)$ declines over time according to its amortization schedule, with a loan term of $n$. At any point in time, $t$, household wealth is given by

$$W_t = FA_t + (H_t - MV_t) \quad (2)$$

where $FA_t$ are other net financial assets at time $t$, $H_t$ is house value at time $t$, and $MV_t$ is the market value of the mortgage at time $t$. $MV_t$ is a function of contractual loan payments, remaining loan term, $n-t$, and the market rate of interest, $r_m$.

Household wealth maximization implies a pure refinancing strategy, namely, minimizing the value of $MV_t$. This is accomplished by following the rule:

$$\text{Prepay when } (MV(r_m,t) / BV(r_c,t)) > 1 \quad (3)$$
In other words, the borrower should prepay the loan as soon as the option goes into-the-money, that is, as soon as the market rate of interest declines below the contract rate on \( BV(r_c,t) \). In the presence of transaction costs, \( TC \), however, the rule becomes:\textsuperscript{5}

\[
\text{Prepay when } \frac{MV(r_m,t)}{(BV(r_c,t) + TC)} > 1
\]

(4)

If transaction costs were declining over time, due to institutional and technological change, we would expect to see rates of prepayment increasing over time for a given level of refinancing incentive, i.e. parameters of the prepayment function would be not be stable over time. In the econometric specification that follows, we use \( MV/BV \) as the measure of refinancing incentive.

Given the computational burden of evaluating partial differential equations, pricing of mortgages and mortgage-backed securities given a term structure diffusion and prepayment and default models is typically done via Monte Carlo methods. Multiple interest rate paths are generated using the chosen term structure model and along each path the prepayment model generates cash flows as a function of simulated interest levels and other factors.\textsuperscript{6} For a given price, the option adjusted spread (OAS) is calculated as the spread over simulated treasury rates that equates price to the average present value of expected cash flows over all simulated paths. Alternatively, for a given OAS, we may solve for price by averaging the present value of expected cash flows discounted at

\textsuperscript{5} Transactions costs will also sometimes lead borrowers to postpone prepayment even when the option is in the money, because of the hysteresis effect described in Dixit and Pindyck (1998).

\textsuperscript{6} Typical factors included in Wall Street and proprietary prepayment models include loan age, loan balance, contract type, coupon, some measure of path-dependent coupon burnout and, possibly, loan-to-value ratio, geography, origination channel, and credit score.
treasury plus OAS over all simulated paths. In the analysis that follows, we use the commercially available WINOAS software to generate simulated interest rate paths and an empirically derived prepayment model to generate cash flows given simulated interest rates. We then discount the cash flows generated at typical OAS values to compute asset prices.

**Data**

Data is drawn from the servicing records of a major mortgage market participant that prefers anonymity. Samples were drawn from two origination years: 1992 and 1997, henceforth referred to as the 92 cohort and the 97 cohort. The mortgage market changed in a fairly significant way in between these two years, since credit scoring and then automated underwriting was widely adopted during 1995. Figure 1 shows the path of mortgage interest rates between 1990-2000. As can be seen from this graph in both cohort years, mortgage interest rates were predominately in the 7.50-8.50% range, followed by market rallies in 1993 and 1998 that produced rates about 100 basis points lower, in the 6.50-7.50% range. We select 30 year fixed rate mortgage loans from both cohorts with note rates ranging from 8.00-9.00%, so as to have loans that clearly would experience incentives to refinance within 24 months of loan origination. Since, with this data set, we cannot distinguish mobility from refinancing behavior, we are effectively assuming that mobility within 24 months of home purchase is negligible. All of the loans used are 30-year conventional fixed rate mortgages that were subsequently sold to Fannie Mae. All were originated for home purchase, as opposed to refinancing an existing loan.
This is an important distinction, since with refinancing loans, mortgage age is not equal to time at current residence\(^7\). Many prepayment analyses pool these distinct loan types together applying a single seasoning factor, a questionable procedure given the measurement error in the age variable. Table 1 presents descriptive statistics for the two cohorts.

**Empirical Methodology for Prepayment Model**

Hazard models have been widely used in the mortgage termination literature, so we provide only a brief overview here, preferring to focus on our innovation, namely, construction of wider confidence intervals to control for temporal instability in the refinancing parameter.

The set-up for the Cox proportional hazard model begins by defining the time to prepayment, \(T\), as a random variable, which has a continuous probability distribution, \(f(t)\), with \(t\) a realization of \(T\). The cumulative probability is defined as

\[
F(t) = \int_0^t f(s)ds = \Pr(T \leq t)
\]

from which the survival function follows as

\[
S(t) = 1 - F(t) = \exp\left(-\int_0^t f(s)ds\right)
\]

\(^7\) In econometric terms, the borrower’s tenure in the house, when measured by loan age, is a truncated variable, whereas for purchase money loans, housing tenure is equal to loan age. For refinancing loans, we
\[ S(t) = 1 - F(t) = \Pr(T \geq t). \tag{6} \]

The survival function describes the probability that the time to prepayment will be of length at least \( t \). The probability \( l(t) \) that a prepayment will occur in the next short interval of time, \( \Delta t \), given that the borrower has not prepaid prior to time \( t \) is defined as:

\[ l(t, \Delta t) = \Pr(t \leq T \leq t + \Delta t \mid T \geq t). \tag{7} \]

From this probability, the hazard rate is defined as

\[ h(t) = \lim_{\Delta \to 0^+} \frac{\Pr(t \leq T < t + \Delta t \mid T \geq t)}{\Delta} = \frac{f(t)}{S(t)}. \tag{8} \]

Adding \( k \) covariates, some of which may be time-varying, we can write the model as:

\[ H_i(t) = \lambda_0(t) \exp (\beta_1 X_{i1} + \beta_2 X_{i2}(t) + \ldots + \beta_k X_{ik}) \tag{9} \]

In (5), \( \lambda_0(t) \) is the baseline hazard rate, \( \mathbf{X} \) is a vector of covariates, and \( \boldsymbol{\beta} \) is the vector of parameters to be estimated. Finally, taking logs of both sides, we have the model to be estimated by the Cox method of partial likelihood:

\[ \log H_i(t) = \alpha(t) + \beta_1 X_{i1} + \beta_2 X_{i2}(t) + \ldots + \beta_k X_{ik} \tag{10} \]

know only that the homeowner has been in the house for at least the length of the current loan.
We have available as covariates both loan characteristics and a limited set of borrower demographic variables. Loan characteristics include original loan size (ORIBAL), original and time-varying loan-to-value ratio\(^8\) (ORIGLTV, CLTV), and discount points paid at loan origination (DISCPT), and loan origination date. The note rate on the mortgage is captured in the time-varying ratio of the market value of the loan to the book value of the loan (VALUEOFE), a proxy for the value of the borrower’s prepayment option. Borrower characteristics at time of loan origination include age (BRWAGE) and income (INCOME), measures often used to proxy for household wealth and level of financial sophistication. We tried several specifications but, in the interest of brevity, only report the best results, which do not include either of the borrower characteristics. Our final specification includes only original loan size, original and contemporaneous LTV, and refinancing incentive. We represent contemporaneous LTV by a dummy variable equal to one, if the CLTV is greater than 80% and zero otherwise, since such mortgages may be relatively more difficult to refinance. Consistent with much of the previous literature, the refinancing incentive is defined as the present value of the mortgage at market rates divided by current book value.

Obviously, the 1992 cohort has had a much longer history than the 1997 cohort. In order to control for this difference, we only examine prepayment performance over the first 38 months of loan life, i.e. 1992-1995 (roughly) for the 92 cohort and 1997-2000 for the 97 cohort. We chose 38 months, since this was the longest duration available from the 97

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\(^8\) Loan balances were amortized based on stated term and contract rate and house values updated using the publicly available OFHEO state level house price indices. In future versions of this paper, we hope to use zip code level CSW house price indices for collateral valuation updating.
cohort. In any event, each 38-month period incorporates the refinancing waves of 1993 and 1998.

Results

Table 2 presents results of three hazard models with identical specifications: first the 92 cohort, then the 97 cohort, and finally both cohorts combined. Combining the two cohorts effectively averages the rather different baseline hazard rates over the two cohorts. We note that coefficients on original loan size and original LTV are essentially identical across the three sets of models. In contrast, the coefficient on refinancing incentive is approximately twice as large using the 97 cohort rather than the 92 cohort. In other words, the 97 cohort was considerably more sensitive to refinancing opportunities, relative to the 92 cohort. In addition, the coefficient on CLTV over 80% is much smaller using the 97 cohort, compared to the 92 cohort. We interpret this to mean that high LTV loans became less difficult to refinance in the late 1990s, relative to the early 1990s, a story consistent with the better risk metrics associated with credit scoring and automated underwriting adopted mid-decade.

Model Error and the Pricing of Mortgages

Given that models are estimated historically but used prospectively, how does model error affect mortgage pricing? We use simulation to address this issue. In particular, we simulate 100 interest rate paths and price a hypothetical 8.00% mortgage using the two
different prepayment hazard models generated from the 92 and the 97 cohorts. The difference is approximately 1.7% in value. To summarize briefly, pricing the 1997 issue (as of June 1997) using the 1992 prepayment model parameters and an OAS of 75 basis points\(^9\) produces a value of 101.08, compared to a value of 102.79, using the parameters of the 1997 model. In what follows, we describe our method in more detail.

We use Mortgage Industry Advisory Corporation’s WINOAS software\(^{10}\) to generate 100 random interest rate paths according to a standard lognormal diffusion process, with a long term mean rate of 8%, a mean reversion rate of 10% and spot rate volatility rate of 18%. We assume a constant mortgage-treasury spread and default rates of zero. We then price a normalized $1 mortgage loan subject to (partial) prepayment risk described by the hazard models based on either the 92 cohort or the 97 cohort of data, by taking the present value of the cash flows at various OAS values over simulated treasury rates.

Figures 1 shows the distribution of prices at various OAS levels for the two sets of prepayment model parameters (the 92 Model and the 97 Model). For comparison purposes, we also valued the same pool using identical term structure model settings in WINOAS and one of its current industry standard prepayment models\(^{11}\). Since this is same collateral and the same set of interest rate scenarios, so the entirety of the value differences is attributable to the prepayment model used.


\(^{10}\) This WINDOWS compatible software is used by a number of major mortgage firms to value mortgages and mortgage servicing rights and has the flexibility of incorporating alternative term structure and prepayment models into the valuation process.
Conclusions

We have analyzed the prepayment response rates of loans originated in the early 1990s compared to the late 1990s, a period during which technological change and increased efficiency, together with generally robust macroeconomic conditions, made mortgage refinancing an increasingly available option to most households. We conclude that interest rate sensitivity has changed significantly over the decade and that the constraints of high LTV have diminished. Moreover, the model risk associated with pricing using one response function versus another can produce economically significant differences in asset values. Accordingly, practitioners would be well advised to incorporate parameter drift into models used to price mortgages and mortgage-backed securities. In future research, we intend to examine the robustness of the results reported here to changes in the underlying term structure model used.

11 WINOAS supports an array of industry standard prepayment models. The model used, of course, has benefited from having experienced the 1998 refinancing wave, so its values are much closer to those produced by the 97 model than the 92 model.
References


Archer, W., D. Ling, and G. McGill. 1993 or TBD. The Effect of Term Structure Model on the Value of Mortgages. Journal of Housing Research TBD.


### Table 1
Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>1992</th>
<th>1997</th>
<th>Both years combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage Balance at Origination (00000)</td>
<td>115,957 (45,863)</td>
<td>115,043 (50,000)</td>
<td>115,531 (47,827)</td>
</tr>
<tr>
<td>Loan-to-value at Origination</td>
<td>.718 (.144)</td>
<td>.793 (.166)</td>
<td>.753 (.154)</td>
</tr>
<tr>
<td>Present value of payments to balance ratio</td>
<td>1.08 (0.07)</td>
<td>1.05 (0.07)</td>
<td>1.07 (0.07)</td>
</tr>
<tr>
<td>Contemporaneous LTV &gt; .80</td>
<td>0.07 (0.26)</td>
<td>0.23 (0.42)</td>
<td>0.15 (0.35)</td>
</tr>
<tr>
<td>N</td>
<td>1,215</td>
<td>1,061</td>
<td>2,276</td>
</tr>
<tr>
<td>Survivors to right-censoring or t=40</td>
<td>736</td>
<td>532</td>
<td>1,268</td>
</tr>
</tbody>
</table>

Note: Top number in cell is mean; number in parenthesis is standard deviation.

### Table 2
Coefficients on Cox Proportional Hazard Prepayment Models

<table>
<thead>
<tr>
<th></th>
<th>1992</th>
<th>1997</th>
<th>Both years combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage Balance at Origination (00000)</td>
<td>0.53 (0.07)</td>
<td>0.65 (0.09)</td>
<td>0.55 (0.06)</td>
</tr>
<tr>
<td>Loan-to-value at Origination</td>
<td>-0.14 (0.002)</td>
<td>-0.02 (0.003)</td>
<td>-0.14 (.002)</td>
</tr>
<tr>
<td>Present value of payments to balance ratio</td>
<td>12.7 (0.59)</td>
<td>23.0 (0.88)</td>
<td>13.5 (0.44)</td>
</tr>
<tr>
<td>Contemporaneous LTV &gt; .80</td>
<td>1.23 (0.12)</td>
<td>0.44 (0.12)</td>
<td>1.13 (0.08)</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-5,678</td>
<td>-2,990</td>
<td>-9,558</td>
</tr>
</tbody>
</table>

Chi-squared statistic that both years combined is the correct model is 890.

Note: to make baselines comparable, means were removed from the mortgage balance and loan-to-value variables.
Figure 1: Value of 8% Mortgage Pool as of June 1997
(Alternative Prepayment Models)

Price

OAS

- 92 Model
- WINOAS (2001 Commercial Model)
- 97 Model