A Theory of B2B Exchange Formation

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Abstract

The recent explosion of attempts to form B2B exchanges and the large failure rate of these attempts raise questions about when and why B2B exchange formation succeeds. Our model provides a theory of B2B exchange formation by investigating conditions under which B2B exchanges attract enough buyers and suppliers to form. Our most important result is that, when the number of potential suppliers is large enough, successful formation of a B2B exchange hinges on its ability to subsidize suppliers selectively. Since there are externalities among participation decisions, charging the marginal cost of connection does not lead to the efficient outcome. Offering a subsidy to a selective group of suppliers is to “divide and conquer” them to induce full participation. Selective subsidy is also necessary to insure that the optimal number of suppliers join the exchange. When such subsidy is feasible, the full participation equilibrium becomes the unique subgame-perfect Nash equilibrium. The theory also yields implications for the ownership structure needed to support B2B exchange formation.

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1 Introduction

In the early summer of 2001, the Financial Times reported widespread suspicions about the suitability of B2B exchanges for the airline industry. In the article, one consultant commented that suppliers “continue to be reluctant to sign up to portals and other e-mechanisms created by the prime contractors. The key reason for this is that the primary objective of e-procurement is perceived to be a reduction in the purchase price, therefore forcing pressures on [supplier] margins.” The article recommended that the industry forget about the promise of large savings from B2B exchanges and instead use the portals to manage collaboration on existing programs (see Odell [12]).

This outcome, should it obtain, may not be unique to the airline industry. Announcements of B2B exchange formation are widespread across many industries. Indeed, over 1,600 B2B exchanges had been launched or announced by the beginning of 2001 (e.g., Harrington [10]). However, the number of successful formations are few and announcements terminating formation efforts are rising in number. By May of 2001, for instance, over 400 B2B exchanges had shut down and countless more exchanges never materialized according to Harrington [10]. Moreover, industry observers report that in 2001, only about 100 B2B exchanges handled any genuine transactions (see May 19 issue of the Economist [7]) and predict that perhaps as few as a handful of public exchanges may survive in the long-run (see Cronin [5]).

Such a large and rapid failure rate raises questions about when B2B-based exchanges create value and who captures it, questions that have been the source of hot debate in the management and economics literatures (for surveys see Borenstein and Saloner [3] and Kauffman and Walden [9]). Simple application of basic auction theory, the principal methodology used to inform the debate, typically assumes supplier participation and implies that B2B exchanges create value by reducing search costs and increasing the average number of bidders, which facilitates buyers capturing more value than is created by the B2B exchange because it reduces supplier profit margins. However, the aforementioned consultant’s comment about airline industry suppliers and, more generally, the widespread failure of B2B exchange formation efforts indicate that supplier participation is not assured and thus should not be assumed. Indeed, without sufficient supplier incentives to participate, attempts to form a B2B exchange will fail. Thus, supplier participation must be central to a theory of B2B exchange formation.
Our study is one of the few that attempts to present an analytic framework to analyze auction-based B2B exchange formation. One such study that analyzes B2B exchange participation incentives is Fath and Sarvary [8].\footnote{There are three critical differences between our model and that of Fath and Sarvary [8]. First, they abstract away from asymmetric information problems by assuming that suppliers set the same price for every buyer, which is not true in most procurement processes. We assume a typical auction setting in which suppliers have private information about their costs. Second, we allow buyers and suppliers to make coordinated decisions. Third, we assume that a supplier’s entry into each auction is endogenous whereas Fath and Sarvary assume exogenous random matching between buyers and suppliers.} They use a dynamic model in which entry and exit depends on the relative surplus enjoyed by both buyers and suppliers. With no switching cost, they identify two stable equilibria: low participation and high participation; and a set of unstable equilibria in which any deviation from the set evolves to one of the two stable equilibria.\footnote{With a positive switching cost, the set of unstable equilibria, which is a curve in the number of suppliers-number of buyers space, turns into a band in which interior points are more stable in the sense that small deviations lead to neither low or high participation equilibria.}

Our theory of B2B exchange formation is quite general in that it endogenize both the participation in the B2B exchange and the participation in individual auctions within and outside the exchange. Hence, our model informs not only when a B2B exchange can be successfully introduced but also what features are needed for the exchange to successfully attract participants. We especially clarify the role of subsidies in achieving the sufficient number of participants for formation. The theory delivers useful implications about the optimal ownership form of exchanges.

The paper considers two potential benefits of B2B exchanges, although our primary focus is on the first one. First, we assume that a B2B exchange reduces pre-contract transaction costs such as search, communication, and formal bid preparation and then analyze suppliers’ decision to participate in a B2B exchange. We do so by building on a model of endogenous entry introduced by Seshadri et al. [14]. For each procurement opportunity, each supplier draws an entry cost from a certain distribution and participates in the auction only if the expected profit exceeds the entry cost. A B2B exchange reduces the entry cost and thus increases the number of bidders in equilibrium. Second, although in the form of extension, we later discuss the possibility that a B2B exchange reduces post-contract transaction costs by
facilitating collaboration and coordination in product design, production, and delivery, leading to higher quality and performance. We use this assumption to evaluate the impact of the adoption of a B2B exchange on suppliers’ profit using a result from Che [4] on the analysis of an auction with quality competition.

In our formal procurement model that analyzes the first of the above two benefits, we use subgame-perfect Nash equilibrium as the solution concept. Our first result is that when connection fees to join the B2B exchange are uniform among suppliers there are at most two subgame-perfect Nash equilibria: no participation equilibrium and full participation equilibrium. In the former, no firms participate, and in the latter, all buyers and a subset of suppliers participate in the B2B exchange and non-participating suppliers exit the market. We also show that when there are two equilibria, the no participation equilibrium is the unique perfectly coalition-proof Nash equilibrium. Coalition-proofness is an appropriate equilibrium concept in many settings when buyers and suppliers in many instances can freely discuss their interests and potentially can make collective decisions about entry. In a perfectly coalition-proof Nash equilibrium, any collective decision among coalition participants must be self-enforcing: no subcoalition of firms must be able to arrange plausible, mutually beneficial deviations from the collective decision.

Our most important result is that, when the number of potential suppliers is large enough, successful formation of a B2B exchange hinges on its ability to subsidize suppliers selectively, possibly by providing hardware, software, or training for personnel, or even by offering direct payments. There is a negative externality among supplier participation decisions and suppliers could avoid an unprofitable full participation equilibrium by adopting a trigger strategy: I will join if others do but will not otherwise. Offering a subsidy to a selective group of suppliers is to “divide and conquer” them to induce full participation. Selective subsidy is also necessary to insure that the optimal number of suppliers join the exchange. When such subsidy is feasible, the full participation equilibrium becomes the unique subgame-perfect Nash equilibrium. B2B exchanges do not need to discriminate among buyers because no negative externality is generated by their participation. Buyers are always profitably induced to participate without selective subsidy whenever the B2B exchange is socially efficient.

We can summarize our model’s main findings from a social welfare point of view. We show
that a B2B exchange chooses the efficient number of participants when the B2B exchange can control the number of participants through subsidies. Nonetheless, the model identifies two kinds of inefficiencies that could arise in the adoption of a B2B exchange. A socially efficient B2B exchange may fail to form when there are enough potential suppliers and the exchange cannot subsidize suppliers selectively. Alternatively, a socially undesirable B2B exchange may be formed because it can always capture more rent than is created by its formation. More specifically, the B2B exchange does not take into consideration any social efficiency loss created by the exit of suppliers from the market when it designs its optimal subsidy policy.

A number of factors can affect the optimal connection fees. The marginal cost of connection is one of them, but charging the marginal cost to participants is not efficient, in general, because of the externality among participation decisions. Other factors that affect the optimal connection fees include the number of potential buyers and suppliers, the distribution of production costs and the distribution of entry costs.

Finally, as an extension, we consider the case when a B2B exchange reduces post-contract transactions costs such as the cost of communication or information exchange in product design, production, and delivery. Although this additional role of a B2B exchange does not affect the likelihood of successful B2B formation, it could affect the optimal connection fees significantly. Suppose a B2B exchange reduces its marginal cost of providing quality by facilitating post-contractual collaboration and coordination. It is easily shown that the B2B exchange can charge higher fees to both buyers and suppliers if the B2B exchange does not affect the distribution of production costs significantly. The optimal supplier (buyer) connection fee is further increased (decreased) when the B2B exchange augments the dispersion of production costs among suppliers. For example, augmentation may occur when suppliers with higher flexibility and delivery performance are in a better position than others to leverage collaboration and coordination enabled by a B2B exchange.

The paper proceeds by first introducing a procurement model sans a B2B exchange. We then expand this model to consider B2B exchange formation focusing on benefits arising from pre-contract transaction cost reductions. In the analyses section, we show that a socially efficient B2B exchange can always achieve full participation as the unique subgame-perfect Nash equilibrium if selective subsidy is allowed. Welfare implications are also discussed in
the section. We conclude our analysis by modeling procurement with quality competition. Section 4 illuminates our propositions by discussing when different ownership structures may be needed for exchange formation. We conclude by discussing opportunities for and impediments to empirical examination of our propositions.

2 Analyses

2.1 Base Procurement Model

We first consider a procurement environment consisting of one downstream firm (“buyer”) and many potential upstream firms (“suppliers”) to derive buyer and supplier profits from each procurement before formally defining a B2B exchange. The buyer procures one unit of intermediate product using an auction. Let \( N_S \) be the number of potential suppliers. \( N_S \) is common knowledge for all firms and all firms are risk neutral. The value of the intermediate product for the buyer is fixed at \( v \).

The procurement process we consider is similar to the one introduced by Seshadri et al. [14]. The buyer announces a request for procurement (RFP) and potential suppliers decide whether to join the auction or not. In order to participate in the auction, each potential supplier has to search for such opportunities, communicate with the buyer, develop the required product, and prepare and transmit a formal bid. We call the sum of these the auction entry cost, denoted by \( \tau_i \) where \( i \) indicates supplier \( i \). We assume that each potential supplier privately observes \( \tau_i \), which is drawn from the same continuously differentiable distribution \( G(\tau) \) on \([0, +\infty)\), with a density function of \( g(\tau) \). \( \tau_i \) is independent over \( i \). Once supplier \( i \) decides to enter the auction, it privately observes its production cost \( c_i \), which is independently and identically distributed by \( F(c) \) over \([c, \bar{c}]\) with density function \( f(c) \). We assume that \( \tau_i \) and \( c_i \) are independent. Since suppliers are symmetric and products are homogeneous, the subscription \( i \) is dropped in the remainder of the analysis. After entry into the auction, the number of bidders, \( n \), becomes public knowledge and the buyer conducts a standard auction. For the moment, we maintain the general form of an auction by allowing \textit{ex ante} admission fee \( e \), and ceiling price \( \bar{P} \). We also consider a subsidy to bidders by allowing \( e \) to be negative. When no supplier bids, the buyer can procure the product elsewhere (in-house production for instance) at cost \( c_0 \). Therefore,
the buyer only can commit to $\bar{p} \leq c_0$

We make the following assumption in order to simplify the analysis by ruling out the case in which the buyer needs to set the ceiling price lower than $\bar{v}$.

**Assumption 1.** $v > c_0 \geq \bar{v}$

We do not specify an auction mechanism in the paper because the revenue equivalence theorem extended to auctions with entry by Levin and Smith [11] also applies to our model. According to the theorem, any auction mechanism in which (i) the order always goes to the supplier with the lowest cost, and (ii) any bidder with the highest-possible cost expects zero surplus, yields the same expected procurement cost.

We now start with a standard result of auction theory. Let $\pi(c, n)$ be the expected profit of a supplier with cost $c$ when it enters the auction. Then,

$$
\pi(c, n) = \int_c^{\min(\pi, p)} (1 - F(t))^{n-1}dt \quad \text{for } c \leq p.
$$

for $n \geq 2$ and $\pi(c, 1) = p - c$ for $c \leq p$ and 0 otherwise. Let $\pi(n)$ be the expected profit of a supplier in an auction when there are $n$ participants. Then,

$$
\pi(n) = \int_{c_0}^{\min(\pi, p)} \pi(c, n)f(c)dc
$$

$$
= \int_{c_0}^{\min(\pi, p)} F(c)(1 - F(c))^{n-1}dc.
$$

for $n \geq 2$ and $\pi(1) = F(\min(\pi, p))(p - E[c | c \leq \bar{p}])$

Normally, when a supplier makes a decision on whether to enter an auction, it does not know how many of the potential suppliers will bid because entry decisions are endogenous and made almost simultaneously. Nonetheless, each supplier can evaluate the possible number of entrants and develop its *ex ante* expected profits, and hence make its entry decision. Let $\pi^*$ be the *ex ante* expected profit from participating in an auction.

$$
\pi^* = E_n[\pi(n)] - e = \sum_{n=1}^{N} \pi(n) \Pr\{\# = n | i \text{ joins}\} - e
$$

Firm $j$ should enter if $\pi^* > \tau_j$. Therefore,

$$
\Pr\{\# = n | i \text{ joins}\} = \binom{N-1}{n-1} G(\pi^*)^{n-1}(1 - G(\pi^*))^{N-n}
$$
Plugging this into (2) yields
\[
\pi^* = \sum_{n=1}^{N} \binom{N-1}{n-1} \pi(n) G(\pi^*)^{n-1} (1 - G(\pi^*))^{N-n} - e
\]  
(3)

Since one bidder can view another potential supplier as entering the auction randomly with the probability of \( G(\pi^*) \), the number of other bidders, \( n-1 \), follows a binomial distribution with \( N-1 \) independent Bernoulli trials and the probability of “success”, \( G(\pi^*) \), as is indicated by (3). Hence, the average number of participants in an auction is determined by \( E[n] = NG(\pi^*) \). Note that \( n-1 \) is increasing in \( G(\pi^*) \) in the sense of first-order stochastic dominance. Then, as is suggested by Lemma 2.1, when a function \( \pi(n) \) is a decreasing function, \( E_n[\pi(n)] \) is decreasing in \( \pi^* \). Therefore, the right-hand side of (3) is a decreasing and continuous function of \( \pi^* \) and its intersection with the 45° line represents the unique equilibrium.

**Lemma 1** Let \( n \) be drawn from a binomial distribution. Consider the following expected value of \( g \), a function of \( n \).

\[
E_n[g(n); p] = \sum_{n=0}^{N} \binom{N}{n} g(n)p^n(1-p)^{N-n}
\]  
(4)

When \( g \) is decreasing (increasing) in \( n \), \( E_n[g(n); p] \) is decreasing (increasing) in \( p \).

Proof is in the appendix.

We characterize the equilibrium in this base model in our first proposition. The first half of the proof of the proposition is due to Levin and Smith [11] who produced the result for a slightly different model where bidders randomize entry decision in the symmetric equilibrium.

**Proposition 1** The entry level is socially efficient when \( \overline{p} = c_0 \) and \( e = 0 \). However, the buyer will choose \( e > 0 \) or \( e < 0 \) and induce a socially inefficient level of entries in general.

Proof is in the appendix.

To see this result, note that the expected social value added by including one more supplier to an auction is the probability that it wins the auction times the difference between its cost and that of the next lowest cost supplier, minus the entry cost. But this is exactly the expected profit of the supplier from joining the auction. Therefore, free entry with no distortion induces the socially efficient outcome. But, this is not necessarily optimal for the buyers in general. Although the entry fee has a direct positive impact on the buyer’s revenue, it also decreases
the number of bidders in the auction leading to a higher price. The entry fee that is optimal for the buyers depends on the distributions of the entry cost and production cost and could be positive or negative.

2.2 On-line Procurement

One of the important benefits of a B2B exchange is a reduction of pre-contract transaction costs such as search and communication costs and formal bid preparation. We interpret this benefit as a reduction of entry cost defined in our model. We consider the following game in order to investigate the return to participation in a B2B exchange. Now, we assume that there are $N_B$ buyers and $N_S$ potential suppliers in the market. Each of the buyers conducts auctions $T$ times in each of which it procures one unit of homogeneous intermediate product from one of the suppliers. Entry costs and production costs are identically and independently distributed across suppliers and auctions and we use the same notation for cost distribution as in the previous section.

Consider the possibility of forming one B2B exchange in the industry. Competition among multiple B2B exchanges is beyond the scope of this paper. Participation in the B2B exchange reduces the entry cost for suppliers from $\tau_i$ to $\alpha \tau_i$ where $0 < \alpha < 1$ for all auctions conducted by buyers in the B2B exchange. For simplicity, the B2B exchange is assumed to bear any marginal costs from connecting one supplier to the exchange. The marginal cost $k$ includes the one-time cost of necessary changes in the B2B database, installation of computer hardware and software to the supplier system, and training personnel for the new technology. Let $t_S$ be the one-time connection fee charged on suppliers. Suppliers in the B2B exchange can enter both on-line and traditional auctions conducted by buyers inside and outside the B2B exchange while suppliers not in the B2B exchange only can enter traditional procurement process requested by the buyers outside the B2B exchange.

Similarly, the buyers in the B2B exchange can use both on-line and traditional auctions to attract suppliers. However, once a buyer adopts one procurement technology for an auction, it cannot redo the auction in the other form just because the first auction has not generated the expected result. In other words, a delay in procurement is so costly that in each auction a buyer has to accept the result regardless of the outcome. The marginal cost of connecting one
buyer to the B2B exchange is $K$, which again is borne by the exchange. The exchange charges $t_B$ on every participating buyer. Both $k$ and $K$ are exogenously given and independent of the number of participants. Let $M_B$ and $M_S$ be the number of buyers and suppliers, respectively, in the B2B exchange.

In the following analyses, we assume that the ceiling fee is set at the alternative production cost (i.e. $\tilde{p} = c_0$) and the entrance fee for each individual auction is set to zero (i.e. $e = 0$) regardless of whether it is an on-line auction or traditional auction. This assumption makes sense for on-line auctions because, as we show later, B2B exchanges can exploit any efficiency gain and thus should require all auctions not to charge entrance fees. Although this assumption is not appropriate for traditional procurement, we believe that this simplification only makes traditional procurement more favorable for suppliers than in reality and therefore makes the conditions for successful formation of B2B exchange more stringent without changing the qualitative nature of our results.

How will the introduction of the B2B exchange affect the equilibrium number of bidders and their expected profits? Let $\pi^{B2B}_{M_S}$ and $\pi^O$ be the expected gross profit for a supplier from a B2B on-line auction and a traditional auction, respectively, when there are $M_S$ number of participating suppliers in the B2B exchange. Participant $i$ enters an on-line auction if $\pi^{B2B}_{M_S} > \alpha \tau_i$ while supplier $j$ enters a traditional auction if $\pi^O > \tau_j$. Since one bidder in the B2B exchange views another potential supplier in the B2B exchange as entering the auction randomly with the probability of $G(\frac{\pi^{B2B}_{M_S}}{\alpha})$, the expected profit from entering an on-line auction is

$$\pi^{B2B}_{M_S} = \sum_{n=0}^{M_S-1} \binom{M_S-1}{n} \pi(n+1)G\left(\frac{\pi^{B2B}_{M_S}}{\alpha}\right)^n(1 - G\left(\frac{\pi^{B2B}_{M_S}}{\alpha}\right))^{M_S-1-n}$$  \hspace{1cm} (5)

Since a buyer also can attract any potential suppliers through a traditional auction, the expected profit from entering such an auction does not depend on the number of suppliers in the B2B exchange.

$$\pi^O = \sum_{n=0}^{N_S-1} \binom{N_S-1}{n} \pi(n+1)G(\pi^O)^n(1 - G(\pi^O))^{N_S-1-n}$$  \hspace{1cm} (6)

As is shown in the previous section, the pair $(\pi^{B2B}_{M_S}, \pi^O)$ that satisfies both (5) and (6) embodies the equilibrium. Let $\Pi^{B2B}_S(M_B, M_S)$ and $\Pi^O_S(M_B)$ be the expected total profit net
of entry costs for a supplier inside and outside a B2B exchange, respectively, when the numbers of buyers and suppliers in the B2B exchange are $M_B$ and $M_S$ and all the participating buyers conduct on-line auctions.

\[
\Pi^{B_{2B}}_S(M_B, M_S) = TM_B(\pi^{B_{2B}}_{M_S} - E[\alpha|\pi^{B_{2B}}_{M_S} > \alpha])G(\pi^{B_{2B}}_{M_S} / \alpha) + T(N_B - M_B)(\pi^O - E[\tau|\pi^O > \tau])G(\pi^O)
\]

\[
= T\alpha M_B \int_0^{\pi^{B_{2B}}_{M_S}} G(\tau)d\tau + T(N_B - M_B) \int_{\pi^O}^{\infty} G(\tau)d\tau
\]

\[
\Pi^O_S(M_B) = T(N_B - M_B)(\pi^O - E[\tau|\pi^O > \tau])G(\pi^O) = T(N_B - M_B) \int_{\pi^O}^{\infty} G(\tau)d\tau
\]

Let $\Pi^{B_{2B}}_B(M_S)$ and $\Pi^O_B$ be the expected total profit for the buyers inside and outside the B2B exchange, respectively, when there are $M_S$ suppliers in the B2B exchange and all the participating buyers conduct on-line auctions. Then,

\[
\Pi^{B_{2B}}_B(M_S) = T \sum_{n=0}^{M_S} \binom{M_S}{n} \{v - c_n - n\pi(n)\}G(\pi^{B_{2B}}_{M_S} / \alpha)^n(1 - G(\pi^{B_{2B}}_{M_S} / \alpha))^{M_S-n}
\]

\[
\Pi^O_B = T \sum_{n=0}^{N_S} \binom{N_S}{n} \{v - c_n - n\pi(n)\}G(\pi^O)^n(1 - G(\pi^O))^{N_S-n}
\]

where $c_n$ is the lowest among the production costs of $n$ suppliers (i.e. $c_n \equiv \int_0^{\pi}(1 - F(c))^n dc$).

The next proposition characterizes the profit functions.

**Proposition 2** $\pi^{B_{2B}}_{M_S}$ and $\Pi^{B_{2B}}_S(M_B, M_S)$ are decreasing in $M_S$ while $\Pi^{B_{2B}}_B(M_S)$ is increasing in $M_S$. $\Pi^{B_{2B}}_S(M_B, M_S)$ is increasing in $M_B$ while $\Pi^O_S(M_B)$ is decreasing in $M_B$.

Proof is in the appendix.

One major characteristic of the market that affects the competitive pressure created by a B2B exchange is entry cost heterogeneity. Note that when the distribution of $G$ is narrow, the reduction of entry cost could substantially increase the probability of entering each auction and hence raise the number of bidders, which leads to a lower profit margin. We confirm
this intuition in the following result by developing parameterized results for three specific, but commonly used, distributions. There are no comparative statics results that hold for any distributions or any parameter range. However, by demonstrating the results for a reasonably wide range of parameters for commonly used distributions, we attempt to show the typical impact of entry cost heterogeneity on the benefit of participation in a B2B exchange.

**Proposition 3** Assume \( \tau \) follows a uniform distribution (\( G(\tau) = \frac{\tau}{\sigma} \) for \( 0 \leq \tau \leq \sigma \)), censored normal distribution (\( G(\tau) = \Phi\left(\frac{\tau - \mu}{\sigma}\right) \) for \( \tau \geq 0 \) and 0 for \( \tau < 0 \)) or log-normal distribution (\( G(\tau) = \Phi\left(\frac{\log(\tau - \mu)}{\sigma}\right) \)) where \( \Phi \) is the cumulative standard normal distribution function. Consider the parameter range for which \( G(\pi^O) \leq 0.5 \) and \( 0.5 \leq G\left(\frac{\pi^O_B}{\alpha}\right) \leq 1 - G(0) \). Then, \( \frac{d\pi^B_S}{d\sigma} > 0 \) and \( \frac{d\pi^B_O}{d\sigma} > 0 \), and for a fixed \( MS \), \( \frac{d\pi^B_S(M_S)}{d\sigma} < 0 \), \( \frac{d\pi^O_S(0)}{d\sigma} < 0 \) and \( \frac{d\pi^B_B(N_B, M_S)}{d\sigma} > 0 \).

Proof is in the Appendix.

Proposition 3 implies that more heterogeneous entry costs make it more profitable for suppliers to join a B2B exchange while they make it less profitable for the buyers to do so given a fixed connection fee. In the next section, we provide equilibrium analysis of participation decisions.

### 2.3 Reduced-Form Participation Game

Now that profit functions are defined we turn our attention to identifying supplier and buyer participation decisions. We consider a reduced-form game where the participation decisions generate the outcome \( \{\pi^B_S(M_B, M_S), \pi^O_S(M_B), \pi^B_B(M_S), \pi^O_B\} \) for participating and non-participating suppliers and buyers, respectively. We consider the following extensive-form game: (1) firms are arbitrarily ordered to make a decision whether to participate or not sequentially; (2) a firm cannot cancel its decision once it decides to join the B2B exchange; (3) when a firm decides not to participate, it will be given a chance to reconsider its decision if there are any new entries after the decision; (4) the game ends when no additional firms have decided to join after a round of decisions by all non-participating firms.

\[3\] The inequality \( G\left(\frac{\pi^O_S}{\alpha}\right) \leq 1 - G(0) \) constrains the range only for censored normal distributions because \( G(0) = 0 \) for the other two.
Consider a buyer’s incentive to join the B2B exchange. Let \( m(\geq 2)\) be the minimum number that satisfies \( \Pi_B^{B2B}(M_S) \geq \Pi_B^O \). Even if some buyers are connected to the B2B exchange, they will likely use the traditional procurement process when the B2B exchange fails to attract enough suppliers, namely \( \Pi_B^{B2B}(M_S) < \Pi_B^O \). Suppose \( M_S \) is the expected number of suppliers in the B2B exchange. Then, buyers will participate in the exchange if

\[
\Pi_B^{B2B}(M_S) - t_B \geq \Pi_B^O. \tag{9}
\]

Let \( M_S(t_B) \) be the minimum number that satisfies this inequality and non-decreasing in \( t_B \). The threshold \( M_S(t_B) \) represents the “critical mass” often discussed among the practitioners. If the B2B exchange succeeds in signing up enough suppliers, the B2B exchange will expand toward the full participation equilibrium on its own. Obviously, \( M_S(t_B) \geq m \) whenever \( t_B \geq 0 \). Since \( \Pi_B^{B2B}(M_S) \) is increasing in \( M_S \), \( \Pi_B^{B2B}(M_S) - t_B > \Pi_B^O \) for all \( M_S > M_S(t_B) \).

Next, we consider the supplier incentive to join the B2B exchange. Suppose a supplier has seen all buyers and \( M_S - 1 \) other suppliers join the B2B exchange. When \( M_S < m \), then no buyer in the B2B exchange actually conducts on-line auctions even if the supplier participates in the B2B exchange. Then, the supplier has no incentive to join unless additional suppliers join or are expected to join. Suppose \( M_S \geq m \). A supplier would join the B2B exchange if \( \Pi_S^{B2B}(N_B, M_S) - t_s > \Pi_S^O(0) = 0 \). Let \( \overline{M}_S(M_B, t_S) \) be the maximum number such that \( \Pi_S^{B2B}(M_B, \overline{M}_S(t_S)) - t_s > \Pi_S^O(M_B) \) and \( \overline{M}_S(t_S) \geq m \). Then,

\[
\Pi_S^{B2B}(M_B, \overline{M}_S(t_S)) - t_s > \Pi_S^O(M_B) \iff \alpha TM_B \int_0^{\frac{t_S}{\overline{M}_S}} G(\tau)d\tau > t_s \tag{10}
\]

Let \( \overline{M}_S(M_B, t_S) = 0 \) if there is no such number: i.e., \( \Pi_S^{B2B}(M_B, M_S) - t_s \leq \Pi_S^O(M_B) \) for all \( M_S \geq m \). When \( M_B = N_B \), we denote \( \overline{M}_S(N_B, t_S) = \overline{M}_S(t_S) \). \( \overline{M}_S(t_S) \) is the number of suppliers in the B2B exchange should all buyers join. Since \( \alpha TM_B \int_0^{\frac{t_S}{\overline{M}_S}} G(\tau)d\tau \) is increasing in \( M_B \) and decreasing in \( M_S \), \( \overline{M}_S(M_B, t_S) \) is non-decreasing in \( M_B \) and non-increasing in \( t_S \). To simplify our discussion in the rest of the paper, we add one regularity condition.

**Assumption 1**  \( \overline{M}_S(M_B, t_S) \) is strictly increasing in \( M_B \).

We can now identify two types of pure strategy Nash equilibria.

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\(^4\)When \( M_S = 1 \), the supplier’s bidding price is always equal to the ceiling price \( \bar{p} = c_0 \). Thus, there is no gain by holding on-line auctions.
Proposition 4  There are at most two subgame-perfect Nash equilibria. When $\Pi^B_S(N_B, M_S(t_S)) - t_S \leq \Pi^Q_S(0)$, there exists an equilibrium in which no buyers or suppliers participate. When $M_S(t_B)$ exists and $M_S(t_S) \geq M_S(t_B)$, there is another equilibrium in which all buyers and $M_S(t_S)$ suppliers participate in the B2B exchange. No participation is the unique subgame-perfect Nash equilibrium outcome when $M_S(t_B)$ does not exist or $M_S(t_S) < M_S(t_B)$, while full buyer participation is the unique subgame-perfect Nash equilibrium outcome when $\Pi^B_S(N_B, M_S(t_S)) - t_S > \Pi^Q_S(0)$.

Proof. We prove the Proposition in three steps.

Step 1: When $\Pi^B_S(N_B, M_S(t_S)) - t_S \leq \Pi^Q_S(0)$, there exists an equilibrium in which no buyers or suppliers participate. This is the unique subgame-perfect Nash equilibrium outcome when $M_S(t_B)$ does not exist or $M_S(t_S) < M_S(t_B)$.

It is easily verified that the second part is true, namely, no participation is the unique subgame-perfect Nash equilibrium outcome. So suppose $M_S(t_B)$ exists and $M_S(t_S) \geq M_S(t_B)$ and consider the following strategy profile: (S1B) a buyer joins if and only if there are no less than $m$ suppliers in the exchange; and (S1S) a supplier joins if and only if there are no less than $m$ but less than $M_S(t_S)$ suppliers in the exchange. We show that this strategy profile constitutes a subgame-perfect Nash equilibrium and induces no participation. Consider the subgame in which $M_B$ buyers and $M_S$ suppliers are already in the B2B exchange.

We prove that (S1B) is the best response for a buyer to (S1B) taken by the rest of the buyers and (S1S) taken by the suppliers. First suppose $M_S \geq m$. Then, (S1B) and (S1S) implies that all other buyers and $M_S(t_S)$ suppliers would eventually join regardless of the buyer’s decision. Since $M_S(t_S) \geq M_S(t_B)$, joining is the best response. Next suppose $M_S < m$. Since the buyer’s participation does not trigger participation by any other buyers or suppliers and participating buyers would not profitably conduct on-line auctions, not joining strictly dominates joining. Therefore, (S1B) is the best response for a buyer.

Next, we show that (S1S) is the best response to (S1B) taken by all buyers and (S1S) taken by the rest of the suppliers. Suppose $m \leq M_S < M_S(t_S)$. (S1B) and (S1S) imply that, regardless of the next supplier’s decision, all buyers and $M_S(t_S)$ suppliers join eventually. Thus the supplier should get $\Pi^B_S(N_B, M_S(t_S)) - t_S(> 0)$ by joining and get nothing by not joining at the time. Hence, joining is the best response. Next, when $M_S \geq M_S(t_S)$, not joining is
obviously the unique best response. Finally, suppose $M_S < m$. If $M_S < m - 1$, the next supplier will be worse off by joining because no other buyers or suppliers would join and participating buyers would not conduct on-line auctions. If $M_S = m - 1$, (S1B) and (S1S) imply that the supplier’s participation would induce all buyers and $\overline{M}_S(t_S)$ suppliers to join. But, this final outcome is not strictly preferred by the supplier because $\Pi^{B2B}_S(N_B, \overline{M}_S(t_S)) - t_S \leq \Pi^{O}_S(0)$. Hence, not joining is still a best response. We have shown that (S1S) is the best response.

Therefore, (S1B) and (S1S) constitutes a subgame-perfect Nash equilibrium and obviously no firm participates in the equilibrium.

**Step 2:** When $\underline{M}_S(t_B)$ exists and $\overline{M}_S(t_S) \geq \underline{M}_S(t_B)$, there is a subgame-perfect Nash equilibrium in which all buyers and $\overline{M}_S(t_S)$ suppliers participate in the B2B exchange. This is the unique subgame-perfect Nash equilibrium outcome when $\Pi^{B2B}_S(N_B, \overline{M}_S(t_S)) - t_S > \Pi^{O}_S(0)$.

Consider the following strategy profile: (S2B) a buyer always decides to participate; and (S2S) a supplier participates if and only if there are no more than $\overline{M}_S(t_S)$ suppliers in the exchange. It is easily verified that this strategy profile is a subgame-perfect Nash equilibrium. We only need to show that full buyer participation is the unique subgame-perfect Nash equilibrium outcome. Consider a subgame in which there are already $M_B$ buyers and $\underline{M}_S(t_B)$ suppliers in the exchange at the beginning. In this subgame, if $M_B < N_B$, the rest of the buyers should participate in the equilibrium and additional $\overline{M}_S(t_S) - \underline{M}_S(t_B)$ suppliers should join, too. Thus, each supplier earns $\Pi^{B2B}_S(N_B, \overline{M}_S(t_S)) - t_S$. Consider a subgame in which there are already $M_B$ buyers and $\underline{M}_S(t_B) - 1$ suppliers in the exchange where $M_B$ is arbitrary. Every supplier which still remains outside the B2B exchange should participate as long as less than $\overline{M}_S(t_S)$ suppliers are in the exchange because participation by one more supplier induces all outside buyers to participate and the supplier earns $\Pi^{B2B}_S(N_B, \overline{M}_S(t_S)) - t_S > \Pi^{O}_S(0) \geq \Pi^{O}_S(M_B)$. Note that it gets $\Pi^{O}_S(M_B)$ at most by not joining and $\Pi^{B2B}_S(N_B, \overline{M}_S(t_S)) - t_S > \Pi^{O}_S(0) \geq \Pi^{O}_S(M_B)$. The supplier strictly prefers participating to not participating. By the usual backward induction, you can show that all remaining buyers and additional $(l + \overline{M}_S(t_S) - \underline{M}_S(t_B))$ suppliers always choose to participate in the B2B exchange in a subgame in which there are already $M_B$ buyers and $\underline{M}_S(t_B) - l$ suppliers where $M_B$ and $0 < l \leq \overline{M}_S(t_B)$ are arbitrary. Therefore, all buyers and $\overline{M}_S(t_S)$ suppliers participate in any subgame-perfect Nash equilibrium. This concludes the proof.
Step 3: There is no other type of equilibria, namely, \((M^e_B, M^e_S) \neq (0, 0), (N_B, M_S(t_S))\), where \(M^e_B\) and \(M^e_S\) are the equilibrium numbers of buyers and suppliers in the B2B exchange, respectively.

Suppose \((M^e_B, M^e_S) \neq (0, 0)\). If there is no buyer in the exchange, no supplier has an incentive to join. Thus, \(M^e_B > 0\). When \(M^e_B = N_B\), it is obvious that \(M_S(t_S)\) suppliers should join the exchange in the equilibrium leading to the second type of subgame-perfect Nash equilibria.

Hence, let’s start by assuming that \(0 < M^e_B < N_B\). Because some buyers participate but not all do, \(\Pi_B^{B2B}(M^e_S) - t_B = \Pi_B^O\). This implies that \(M^e_S = M_S(t_B)\). Also, \(M^e_S \leq m(M^e_B, t_S)\) because \(M^e_S\) suppliers are better off by joining. We will show that this outcome cannot happen in a subgame-perfect Nash equilibrium. Consider the subgame immediately after \((M^e_B + M^e_S)\) participants joined the B2B exchange. In this subgame, outside buyers lose nothing by deviating to participate because \(\Pi_B(M^e_S) - t_B = \Pi_B^O\). Consider the subgame of the subgame in which \(N_B\) buyers are in the B2B exchange. The game should end after \(M_S(t_S)\) suppliers join the exchange and all buyers receive the payoff \(\Pi_B^{B2B}(M_S(t_S)) - t_B\). The strict inequality in the parenthesis is induced by Assumption 1. In the subgame in which \(N_B - 1\) buyers are in the exchange, the last buyer should enter because it should get \(\Pi_B^{B2B}(M_S(t_S)) - t_B\) eventually and be better off by doing so. The usual backward induction takes us to the conclusion that all buyers should participate once we reach the subgame in which \(M^e_B\) buyers and \(M^e_S\) suppliers are in the B2B exchange, contradicting our assumption that \(M^e_B\) and \(M^e_S\) are the equilibrium numbers of buyers and suppliers in the B2B exchange.

We call the first type the no participation equilibrium and the second type the full participation equilibrium, respectively. Note that buyers are always better off in the full participation equilibrium when it exists.

Note that participation of a supplier in a B2B exchange has a negative externality on other suppliers’ profit in the B2B exchange: \(i.e.,\) an increase in \(M_S\) reduces \(\Pi_S^{B2B}(M_B, M_S)\). Thus, it’s very likely that every supplier prefers the no participation equilibrium to the full participation equilibrium. Namely, \(\Pi_S^{B2B}(N_B, m(N_B)) - t_S < \Pi_S^O(0)\), which can make achieving a “critical mass” difficult. The next proposition formalizes this intuition.
**Proposition 5** When $\Pi_{S}^{B2B}(N_{B}, M_{S}(t_{S})) - t_{S} < \Pi_{S}^{Q}(0)$, no participation is the unique perfectly coalition-proof Nash equilibrium outcome\(^5\).

Proof is in the Appendix.

Full participation may appear as the unique subgame-perfect Nash equilibrium and the unique perfectly coalition-proof Nash equilibrium when $N_{S}$ is small enough so that $\Pi_{S}^{B2B}(N_{B}, N_{S}) - t_{S} > \Pi_{S}^{Q}(0)$. In this case, suppliers retain a part of the rent created by the reduction in transaction costs and have a strong incentive to participate in the B2B exchange. This is more likely so, by Proposition 3, when suppliers are more heterogeneous in entry costs.

For a reasonable range of connection fees, namely, $M_{S}(t_{S}) \geq M_{S}(t_{B})$ and $\Pi_{S}^{B2B}(N_{B}, M_{S}(t_{S})) - t_{S} \leq \Pi_{S}^{Q}(0)$, both the no participation and full participation equilibria are subgame-perfect.

For almost all choices of connection fees (i.e., except when $\Pi_{S}^{B2B}(N_{B}, M_{S}(t_{S})) - t_{S} = \Pi_{S}^{Q}(0)$), however, no participation is the more likely outcome whenever both types of equilibria are subgame-perfect according to Proposition 5. Therefore, ensuring successful formation of a B2B exchange may be difficult as long as the exchange charges a uniform connection fee on all suppliers. In the next section, we show that B2B exchanges can induce full participation as the unique subgame-perfect Nash equilibrium outcome by adopting a “divide and conquer” strategy, namely, by discriminating among suppliers.

### 2.4 Optimization with Selective Subsidy

In the previous section, we have assumed that the B2B exchange charges the same connection fee on all buyers or all suppliers. However, it is more realistic to assume that some B2B exchanges can discriminate among potential participants by offering different connection fees. We argue that price discrimination among suppliers allows a B2B exchange to exploit the rent from suppliers while keeping the full participation equilibrium as the unique perfectly coalition-proof Nash equilibrium and also to control the number of suppliers at the optimal

\[^5\text{A perfectly coalition-proof Nash equilibrium is a subgame-perfect Nash equilibrium and requires that no coalition should be able to make a mutually advantageous deviation from the equilibrium strategy profile in any subgame in a dynamically self-enforcing way (the deviation is subgame-perfect in the game imposed on the subgame and the coalition by fixing the strategies for the complement of the coalition unchanged, and there are no such deviations from the deviation). See Bernheim, Peleg and Whinston [2] for the formal definition.}\]
level. We assume that the B2B exchange can divide buyers or suppliers into two groups and offer differentiated connection fees to them. Let $t_{kl}$ ($k = B, S, l = 1, 2$) be the connection fee for the $l$-th group of buyers ($k = B$) and suppliers ($k = S$). The fee profile is called \emph{feasible} only when the combination of connection fees balances its budget. Namely, $M_{B1}(t_{B1} - K) + M_{B2}(t_{B2} - K) + M_{S1}(t_{S1} - k) + M_{S2}(t_{S2} - k) \geq 0$ where $M_{kl}$ is the number of buyers and suppliers in the $l$-th group which participated in the B2B exchange. When $t_{B1} < t_{B2}$ or $t_{S1} < t_{S2}$, one group of buyers or suppliers can be perceived as being subsidized. Therefore, we call this scheme \emph{selective subsidy}. For simplicity, we assume that offered subsidies become common knowledge at the beginning of the game.

When selective subsidy is allowed, a B2B exchange can fully control the number of participants in the exchange. Suppose the B2B exchange plans to induce $M_B$ buyers and $M_S$ suppliers to participate in the exchange. When $M_S < m$, no on-line auctions will be conducted and thus the exchange cannot expect to make any profit. Hence, assume $M_S \geq m$. The next proposition maintains that the exchange can achieve the participation by $M_B$ buyers and $M_S$ suppliers as the unique subgame-perfect Nash equilibrium outcome if it offers $t_{B1}$, $t_{B2}$, $t_{S1}$ and $t_{S2}$ that satisfy the following incentive compatibility constraints to $M_B$ buyers, $m$ suppliers and $N_S - m$ suppliers, respectively.\footnote{The conditions also make such outcome the unique perfectly coalition-proof Nash equilibrium outcome.}

\begin{align}
\Pi^{B2B}_{B}(M_S) - t_{B1} & > \Pi^{0}_{B} \quad \text{(ICB 1)} \\
\Pi^{B2B}_{B}(M_S) - t_{B2} & < \Pi^{0}_{B} \quad \text{(ICB 2)} \\
\Pi^{S2B}_{S}(M_B, M_S) - t_{S1} & > \Pi^{0}_{S}(0) \quad \text{(ICS 1)} \\
\Pi^{S2B}_{S}(M_B, M_S) - t_{S2} & > \Pi^{0}_{S}(M_B) \quad \text{(ICS 2)} \\
\Pi^{B2B}_{S}(M_B, M_S + 1) - t_{S2} & < \Pi^{0}_{S}(M_B) \quad \text{(ICS 3)}
\end{align}

(ICS 1) ensure that $M_B$ buyers and $m$ suppliers strictly prefers the full participation equilibrium to the no participation equilibrium. (ICB 2), (ICS 2) and (ICS 3) induce exact $M_B$ buyers and $M_S$ suppliers participate as the unique subgame-perfect Nash equilibrium outcome.
Proposition 6 Suppose the B2B exchange offers $t_{B1}$, $t_{B2}$, $t_{S1}$ and $t_{S2}$ that satisfy (11) where $M_S \geq m$ to $M_B$ buyers, $N_B - M_B$ buyers, $m$ suppliers and $N_S - m$ suppliers, respectively. Then, $M_B$ buyers and $M_S$ suppliers participate in the B2B exchange as the unique subgame perfectly Nash equilibrium.

Proof. Consider the subgame in which there are already $M_B$ buyers and $m$ suppliers in the exchange. The next $M_S - m$ suppliers should participate because they get at least $\Pi_S^{B2B}(M_B, M_S) - t_{S2}$ by joining and $\Pi_S^O(M_B)$ by not joining. (ICS2) implies that joining strictly dominates not joining. Consider the subgame in which $M_B$ buyers and $m - 1$ suppliers are in the exchange. Knowing that the participation by one more supplier would induce other $M_S - m$ suppliers to join, at least one more supplier which is offered $t_{S1}$ should have an incentive to participate immediately. The same is true for the subgame in which $M_B - 1$ buyers and $m$ suppliers are in the exchange. The usual backward induction leads to the conclusion that $M_B$ buyers and $M_S$ suppliers participate immediately as the unique subgame-perfect Nash equilibrium outcome.

Let $\Pi_E^{B2B}(M_B, M_S)$ be the supremum of profits for an exchange that attract $M_B$ buyers and $M_S$ suppliers when $M_S \geq m$. Then,

$$\Pi_E^{B2B}(M_B, M_S) = \sup_{t_{B1}, t_{S1}, t_{S2}} M_B(t_{B1} - K) + m(t_{S1} - k) + (M_S - m)(t_{S2} - k)$$

s.t. (11)

$$= M_B(\Pi_B^{B2B}(M_S) - \Pi_B^O - K) + m(\Pi_S^{B2B}(M_B, M_S) - \Pi_S^O(0) - k)$$

$$+ (M_S - m)(\Pi_S^{B2B}(M_B, M_S) - \Pi_S^O(M_B) - k)$$

(12)

Note that $t_{B2}$ does not affect the profit because no buyers that are offered $t_{B2}$ join the exchange.

Let us define $\Delta W(M_B, M_S)$, which is the gain in social surplus created by causing $M_B$ buyers and $M_S$ suppliers to join a B2B exchange.

$$\Delta W(M_B, M_S) = M_B(\Pi_B^{B2B}(M_S) - \Pi_B^O - K) + M_S(\Pi_S^{B2B}(M_B, M_S) - \Pi_S^O(0) - k)$$

$$+ (N_S - M_S)(\Pi_S^O(M_B) - \Pi_S^O(0))$$

(13)
The next proposition shows that a B2B exchange whose formation is socially desirable can always make a positive profit.

**Proposition 7** Suppose $N_S > m$ and selective subsidy is feasible. When the formation of a B2B exchange with $M_B$ buyers and $M_S$ suppliers is socially desirable, the B2B exchange can always profitably attract $M_B$ buyers and $M_S$ suppliers. i.e. $\Pi_E^{B2B}(N_B, M_S) > \Delta W(N_B, M_S)$

**Proof.**

\[
\Pi_E^{B2B}(M_B, M_S) = (N_S - m)(\Pi_S^Q(0) - \Pi_S^Q(M_B)) + \Delta W(M_B, M_S) > \Delta W(M_B, M_S)
\]

Here, the exchange’s ability to discriminate among participants is crucial. Without being able to differentiate prices, the B2B exchange cannot control the number of suppliers in it. For instance, imagine that the exchange charges a uniform connection fee $t_S$ to all suppliers. In order to achieve the critical mass of suppliers, it has to be set below $\Pi_S^{B2B}(M_B, M_S) - \Pi_S^Q(0)$ from (11). However, this connection fee should cause more than $M_S$ suppliers to participate if $\Pi_S^{B2B}(M_B, M_S + 1) - t_S > \Pi_S^Q(M_B)$ (i.e. $\Pi_S^{B2B}(M_B, M_S + 1) - \Pi_S^Q(M_B) > \Pi_S^{B2B}(M_B, M_S) - \Pi_S^Q(0)$). If it is the case and $M_S (> M_S)$ suppliers have joined the exchange, it is quite likely that $\Pi_S^{B2B}(M_B, M_S) - t_S < \Pi_S^Q(0)$ because $\Pi_S^{B2B}(M_B, M_S + 1) - t_S < \Pi_S^Q(M_B)$. Then, the fee is not low enough to bring in a large enough number of suppliers. Therefore, the fee needs to be further reduced, which would lead to participation by more suppliers. While this process continues, the profit for the B2B exchange continues to decline and, eventually, it may not be able to profitably attract enough suppliers to form.

Let $(M_B^*, M_S^*)$ be the optimal numbers of participating buyers and suppliers for the B2B exchange and $(M_B^e, M_S^e)$ be the socially efficient numbers of participating buyers and suppliers. Namely,

\[
(M_B^*, M_S^*) = \arg\max \Pi_E^{B2B}(M_B, M_S) \quad \text{when} \quad \max_{M_B \geq 0, M_S \geq m} \Pi_E^{B2B}(M_B, M_S) > 0
\]
\[
= (0, 0) \quad \text{otherwise.}
\]

\[
(M_B^e, M_S^e) = \arg\max \Delta W(M_B, M_S) \quad \text{when} \quad \max_{M_B \geq 0, M_S \geq m} \Delta W(M_B, M_S) > 0
\]
\[
= (0, 0) \quad \text{otherwise.}
\]
Proposition 8 \( M_B^* = N_B \) whenever \( \max_{M_B \geq 0, M_S \geq m} \Pi_{E}^{2B}(M_B, M_S) > 0 \) and \( M_B^e = N_B \) whenever \( \max_{M_B \geq 0, M_S \geq m} \Delta W(M_B, M_S) > 0 \).

Proof. Because \( \Pi_{S}^{2B}(M_B, M_S) = T\alpha M_B \int_{0}^{\frac{M_B}{M_S}} G(\tau)d\tau + T(N_B - M_B) \int_{0}^{\frac{M_B}{M_S}} G(\tau)d\tau \) and \( \Pi_{S}^{2B}(M_B, M_S) - \Pi_{S}^{O}(M_B) = T\alpha M_B \int_{0}^{\frac{M_B}{M_S}} G(\tau)d\tau \), by definition in (12), \( \Pi_{E}^{2B}(M_B, M_S) \) is linear in \( M_B \). Since \( \Pi_{S}^{2B}(0, M_S) = -M_S k < 0 \), \( \max_{M_B \geq 0, M_S \geq m} \Pi_{E}^{2B}(M_B, M_S) > 0 \) implies \( \Pi_{E}^{2B}(M_B, M_S) \) is strictly increasing in \( M_B \) for any \( M_S \geq m \). Therefore, \( M_B^* = N_B \). Similar for \( M_B^e = N_B \).

Therefore, the exchanges always choose \( M_B^* = N_B \) and it is the efficient choice when \( \max_{M_B \geq 0, M_S \geq m} \Delta W(M_B, M_S) > 0 \). Proposition 8 has two important implications. First, returns to scale are increasing for B2B exchanges, which is due to the one-time connection fee\(^7\) and the efficiency created by the increase in the numbers of bidders in individual auctions. Secondly, B2B exchanges do not need to discriminate among buyers. The “divide and conquer” strategy works only for suppliers because there is a negative externality only among supplier participation decisions.

2.5 Efficiency

Proposition 7 does not rule out the possibility that a socially undesirable B2B exchange is formed. \( i.e. \Pi_{E}^{2B}(N_B, M_S^*) > 0 > \Delta W(N_B, M_S^*) \) where \( M_S^* \) is the optimal number of on-line suppliers for the exchange. Note that the difference between \( \Pi_{E}^{2B}(N_B, M_S^*) \) and \( \Delta W(N_B, M_S^*) \) includes the efficiency loss caused by the exit of \( N_S - M_S \) suppliers, \( (N_S - M_S)\Pi_{S}^{O}(0) \). When \( N_S \) is large enough, inefficient B2B exchange may be introduced because the B2B exchange can exploit more rent than is created by its formation and, more specifically, it does not consider the loss to exiting suppliers.

Next, we evaluate whether B2B exchanges always choose efficient \( M_S^* \). The answer is yes because a B2B exchange can capture the entire rent created by the addition of a supplier to the B2B exchange.

Proposition 9 \( M_S^* = M_S^e \)

\( ^7\)The setup cost for a B2B exchange which is neglected in our model also helps to create scale economy.
Proof. Note that the following difference between $\Pi_{E}^{B2B}(N_B, M_S)$ and $\Delta W(N_B, M_S)$ does not depend on $M_S$:

$$\Pi_{E}^{B2B}(N_B, M_S) - \Delta W(N_B, M_S) = (N_S - m)\Pi_{S}^{O}(0).$$

This concludes the proof.

2.6 Extension: Procurement Model With Quality Competition

In previous sections, we focused only on one role of B2B exchanges to reduce pre-contract transaction costs. Another important benefit of a B2B exchange is to reduce post-contract transactions costs such as the costs of communication or information exchange in product design, production, and delivery. As an extension of the model, we discuss the possibility that a B2B exchange may reduce the marginal cost of improving quality by facilitating collaboration and coordination. We introduce a separate model of products with quality competition for which collaboration and coordination after winning an auction is important and investigate the information rent captured by suppliers. The implication for the optimal connection fee profile for the B2B exchange is straight-forward. Here, quality characteristics include technical product characteristics, design ascetics, production flexibility, delivery performance, etc.

Suppose that the value of a product is not fixed any more and, instead, is a supplier choice variable. We analyze how the participation in a B2B exchange affects the information rent captured by suppliers. Let $q$ be the value, which also represents quality. The cost of producing a product with quality level $q$ is $c(q, \theta_i, \lambda_i)$, where $\theta_i$ is the cost parameter for supplier $i$ and $\lambda_i$ is the choice of a procurement system. $c$ is increasing in $(q, \theta_i, \lambda_i)$. Here, we assume that $\lambda_i$ is a discrete choice from $\{0, 1\}$: $\lambda_i = 0$ indicates traditional procurement and $\lambda_i = 1$ indicates on-line procurement. On-line procurement requires connection to the B2B exchange and additional cost, $c(q, \theta_i, 1) - c(q, \theta_i, 0) > 0$. $\theta_i$ is independently and identically distributed over $[\theta, \bar{\theta}]$ according to a distribution function $\Phi$ for which there exists a continuous density function $\phi$. We assume $c_{qq} \geq 0$, $c_{q\theta} > 0$, $c_{qq\theta} \geq 0$ and $c_q(q, \theta_i, 1) - c_q(q, \theta_i, 0) < 0$. The following regularity assumption is also needed to guarantee strict monotonicity of the quality choice with respect to $\theta_i$. Since suppliers are symmetric, the subscription $i$ is dropped in the remainder of the analysis.

Assumption 3 $c_q + \frac{\partial}{\partial \theta}c_{q\theta}$ is nondecreasing in $\theta$. 

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We consider two-dimensional auctions, in which each supplier bids on both quality and price and bids are evaluated in accordance to the score defined by \( q - p \), which is surplus given to the buyer. We can assume what Che [4] called either a “first-score” auction or a “second-score” auction. Each supplier in a first-score auction submits a sealed bid and, upon winning, produces the offered quality at the offered price. In a “second-score” auction, the winner is required to match the highest rejected score but is not required to match the exact quality-price combination. Thanks to the revenue equivalence result proved by Che [4], we do not have to specify which type of auction is used because the supplier’s expected profit is the same between the two types of auctions. Let \( \pi(n, \lambda) \) be the expected profit for a supplier given that there are \( n \) bidders when the supplier chooses the procurement system \( \lambda \). Then, we can state the following proposition, which is due to Che [4]. The proof is omitted.

**Proposition 10** (Che 1993) All suppliers choose the quality level \( q^*(\theta, \lambda) = \arg \max q - c(q, \theta, \lambda) \) and

\[
\pi(n, \lambda) = \int_0^\infty \int_0^\infty c_\theta(q^*(\theta', 1), \theta', \lambda)(1 - F(\theta'))^{n-1} d\theta' \phi(\theta) d\theta.
\]

We now can analyze how \( \pi(n, \lambda) \) changes with a change in \( \lambda \).

\[
\pi(n, 1) - \pi(n, 0) = \int_0^\infty \int_0^\infty \{c_\theta(q^*(\theta', 1, 0), 1, 0) - c_\theta(q^*(\theta', 1, 1, 0))\}(1 - \Phi(\theta'))^{n-1} d\theta' \phi(\theta) d\theta + \int_0^\infty \int_0^\infty \{c_\theta(q^*(\theta', 1, 0), 0, 0) - c_\theta(q^*(\theta', 0), 1, 0))\}(1 - \Phi(\theta'))^{n-1} d\theta' \phi(\theta) d\theta
\]

The first term measures how a new on-line procurement system affects the production cost distribution. \( c_\theta(q^*(\theta', 1, 0) > c_\theta(q^*(\theta', 1, 1, 0))) \) indicates that a new system widens the cost differentials while \( c_\theta(q^*(\theta', 0, 1) < c_\theta(q^*(\theta', 0, 0))) \) implies that the cost differentials are narrowed. Since the cost distribution determines the information rent captured by the suppliers, the first term provides the direct impact of the new system on the information rent received by the suppliers. When the new system increases production cost heterogeneity among suppliers, the first term is positive and thus the adoption of a B2B exchange is more likely to lead to a higher supplier profit given a fixed \( n \), which is the case when high performers (those with low \( \theta \)) are in a better position to leverage collaboration and coordination enabled by a
B2B exchange. On the other hand, if the information exchange facilitated by a B2B exchange reduces production cost heterogeneity among the suppliers, on-line auctions might put more pressure on their profit margin. Thus, the model is ambiguous about the sign of the first term.

The sign of the second term can be determined without ambiguity. Note that we assume that the marginal cost of quality (i.e., \( c_{q\theta} > 0 \)) increases with the cost parameter. Since Proposition 10 implies that the introduction of a B2B exchange improves the equilibrium quality choices for all suppliers by lowering the marginal cost of quality and thus the cost differentials among suppliers, the second term is positive.

If the combined impact of these two terms is positive, suppliers can expect higher profits in on-line auctions than in traditional auctions and vice versa for a fixed \( n \). By replacing \( \pi(n) \) and \( v - c_n \) in (5) and (7) with \( \pi(n, 1) \) and \( E[q^*(\theta, 1) - c(q^*(\theta, 1), \theta, 1)|\theta = \min_i \theta_i] \), and \( \pi(n) \) and \( v - c_n \) in (6) and (8) with \( \pi(n, 0) \) and \( E[q^*(\theta, 0) - c(q^*(\theta, 0), \theta, 0)|\theta = \min_i \theta_i] \), we obtain the same results that we got for products without quality competition. However, this additional features of B2B exchanges could further increase efficiency and enable the B2B owners to exploit more rents from their formation. Whether a B2B exchange enlarges or narrows production cost differentials among suppliers determines the relative size of the optimal connection fees between buyers and suppliers.

3 Discussion

We illuminate our propositions by relating ability to subsidize optimally to B2B exchange ownership. We have shown that the likelihood of a B2B exchange largely depends on the ability to subsidize suppliers selectively. Such ability is likely to depend on the ownership of the B2B exchange: private or consortium (owned by buyers) or public (owned by a third party).²

²Two types of non-private B2B exchanges have been described in the literature: public exchanges and consortium exchanges. Public exchanges are owned and operated by a third-party whereas consortium exchanges are owned by a group, typically buyers, within a single industry. A third type exchange is a private one owned by a single buyer. A recent development is that public exchanges now provide IT and hosting services for individual buyers to create an out-sourced version of a private exchange. Such outsourced private exchanges rely on the buyer using the communication standards set by the third-party, which appears to yield an economy of scale (e.g., Jones [6], Wilson [15]). Nonetheless, these public-hosted exchanges are private in the sense that information in the exchange is restricted to the buyer and participating suppliers.
If the B2B exchange is managed by a third party, selecting suppliers in a discriminating way may not be feasible because of asymmetric information. The B2B exchange would face higher cost of assessing product values and supplier costs than individual buyers that have amassed knowledge from past dealings. In addition, since participating buyers have an incentive to distort their private information in order to increase the number of bidders at other parties’ cost, choosing the optimal price and the optimal number of “subsidized” suppliers should be difficult for such B2B exchanges. On the other hand, consortium exchanges and, even more so, private exchanges, should be able to implement price discrimination at a lower cost because information is less asymmetric and information distortion incentives are diminished for consortium and eliminated for private B2B exchanges.

In contrast, assuming there exists some economies scale to the information technology infrastructure of a B2B exchange, public ownership may offer lower cost for buyers than consortium or private ownership. Thus, there is a trade-off between the ability to subsidize selectively and economies of scale. As the potential for information asymmetry between buyer and exchange increases, the formation of a B2B exchange that requires subsidy is more likely through consortia and, even more so, through a private ownership. This logic implies that we should observe more public exchanges in the industries where products are mostly standardized while consortium or private exchanges are likely to be formed in the industries where product innovation is rapid and products have to be customized.

The result that the exchange always chooses the efficient number of suppliers in Proposition 9 is not affected by the ownership structure of the B2B exchange except that the number of buyers is constrained to be one in private exchanges or the number of founders and their allies in consortium exchanges. Buyer-owned exchanges still choose the efficient number of suppliers because the exchange can always capture the rent that buyers can possibly obtain from participating in the exchange. Hence, maximizing the profit of the B2B exchange is equivalent to maximizing the joint profit of the exchange and the buyers.

A number of factors including the number of potential buyers and suppliers and supplier heterogeneity in production costs and entry costs influence the optimal connection fees. Entry

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9Economies of scale for B2B exchanges have not been established empirically. Nonetheless, industry observers assert that economies of scale are substantial when common standards are adopted (e.g., Jones [6]).
cost heterogeneity, as well as production cost heterogeneity, is likely to be related to technological conditions among the suppliers like the rate of technological advance or the diversity of alternative technologies because technology may vary in the time and cost needed to evaluate the manufacturability of procured products. Heterogeneity also may be related to the age of the supplier industry because a wide range of alternative but competing technologies is usually present in young industries. Yet another source of heterogeneity in suppliers’ entry costs is geographic dispersion. Broad geographic dispersion implies that some suppliers may have high communications costs and hence high entry costs. Moreover, the more dispersed are suppliers the more likely that some have high entry costs because of language difficulties. Proposition 3 and the set of inequalities in (11) imply that the more homogeneous are suppliers the lower should be their connection fee.

Finally, when there are numerous number of buyers and suppliers, auction-based B2B exchanges are less likely to form at all. A large number of potential suppliers should require selective subsidy, but a large number of buyers implies that asymmetric information among buyers may be quite large because their existing supplier bases are less likely to overlap. Without shared information about the value of products to the buyers and about the supplier cost distribution, an exchange won’t be able to determine the optimal subsidy and thus is doomed to failure.

Our propositions are empirically operationalizable. Empirical analysis of our propositions could proceed by investigating the formation of B2B exchanges and compare successful v. failed formation by the industry conditions and ownership structures described above; however, the fruit from empirical analysis may not yet be ripe. The industry still is in the throws of rapid growth, entry, and exit. Additional time may be needed for empirical regularities to take shape.
Appendices

A Proof of Lemma 1

First, the following result of the first-order stochastic dominance only requires standard techniques in statistics and the proof is omitted. For $p_1 > p_2$ and arbitrary $m$,

$$
\sum_{n=0}^{m} \binom{N}{n} p_1^n (1 - p_1)^{N-n} < \sum_{n=0}^{m} \binom{N}{n} p_2^n (1 - p_2)^{N-n}
$$

As the next step, we prove the following lemma. Lemma 2.1 follows immediately.

**Lemma 1a** Suppose the distribution $F(n; p) = \sum_{k=0}^{n} f(k; p)$ depends on the parameter $p$. For any $p_1 > p_2$, $F(n; p_1)$ strictly dominates $F(n; p_2)$ in the sense of the first-order stochastic dominance i.e. $F(n; p_1) < F(n; p_2)$. Then, if $g$ is decreasing (increasing),

$$
\sum_{n=0}^{N} g(n)f(n; p_1) < (>) \sum_{n=0}^{N} g(n)f(n; p_2)
$$

Suppose $g$ is decreasing. Then, $g(n) - g(n+1) > 0$ for all $n$. By rearranging the summation in (14) and defining $g(N + 1) = 0$,

$$
\sum_{n=0}^{N} g(n)f(n; p_1) = \sum_{m=0}^{N} \sum_{n=0}^{m} \{g(m) - g(m + 1)\} f(n; p_1)
\quad = \sum_{m=0}^{N} \{g(m) - g(m + 1)\} F(m; p_1)
\quad < \sum_{m=0}^{N} \{g(m) - g(m + 1)\} F(m; p_2)
\quad = \sum_{n=0}^{N} g(n)f(n; p_2)
$$

where the inequality is by assumption. Similar for the case when $g$ is increasing.

B Proof of Proposition 1

Let $c_n(\overline{p})$ be the expected lowest cost among $n$ suppliers when $c_i < \overline{p}$ for at least one supplier.

Suppose $\overline{p} < \overline{q}$. The cumulative distribution function for the actual lowest cost given that
$c_i < \bar{p}$ for at least one supplier is

$$F_{n, \bar{p}}(c) = \frac{1 - (1 - F(c))^n}{1 - (1 - F(\bar{p}))^n}. $$

Therefore,

$$c_n(\bar{p}) = \frac{\int_\bar{p}^- \text{ncf}(c)(1 - F(c))^{n-1} dc}{1 - (1 - F(\bar{p}))^n}.$$  \hspace{1cm} (15)

For $\bar{p} > \bar{c}$, $c_n(\bar{p}) = c_n \equiv \int_\bar{p}^- (1 - F(c))^n dc$, the $n$-th order statistics of the cost distribution.

Let $W$ be the social surplus. We’ll show that $W$ is maximized when $\bar{p} = c_0$ and $e = 0$.

$$W = \sum_{n=1}^{N} \left( \begin{array}{c} N \\ n \end{array} \right) B(n, \bar{p}) G(\pi^*)^n (1 - G(\pi^*))^{N-n}$$

$$+ (1 - G(\pi^*))^N (g - c_0) - NG(\pi^*) E[\tau | \tau < \pi^*]$$

where

$$B(n, \bar{p}) = \{1 - (1 - F(\min(\bar{c}, \bar{p}))))^n \} (g - c_n(\bar{p})) + (1 - F(\min(\bar{c}, \bar{p}))))^n (g - c_0)$$

$$= g - (1 - F(\min(\bar{c}, \bar{p}))))^n (g - c_0) - \int_{\bar{p}^-} \text{ncf}(c)(1 - F(c))^{n-1} dc$$

If there is an interior maximum, the first-order conditions are

$$\frac{dW}{de} = \frac{\partial W}{\partial \pi^*} \frac{d\pi^*}{de} = 0 \hspace{1cm} (16)$$

and

$$\frac{dW}{d\bar{p}} = \frac{\partial W}{\partial \pi^*} \frac{d\pi^*}{d\bar{p}} + 1_{\bar{p} < \bar{c}} \sum_{n=1}^{N} \left( \begin{array}{c} N \\ n \end{array} \right) \frac{dB(n, \bar{p})}{d\bar{p}} G(\pi^*)^n (1 - G(\pi^*))^{N-n} = 0 \hspace{1cm} (17)$$

where $1_{\bar{p} < \bar{c}}$ is the indicator function for the choice $\bar{p} < \bar{c}$ and $\frac{dB(n, \bar{p})}{d\bar{p}} = n f(\bar{p})(1 - F(\bar{p}))^{n-1}(c_0 - \bar{p})$. From the way $\pi^*$ is determined, it is easily shown that $\frac{d\pi^*}{d\bar{p}} > 0$ and $\frac{d\pi^*}{de} < 0$. Therefore, $\bar{p} \geq \bar{c}$ and $\frac{dW}{d\pi^*} = 0$ determine the solution.
\[
\frac{\partial W}{\partial \pi} = N g(\pi^*) \left[ \sum_{n=1}^{N} \binom{N-1}{n-1} B(n, \bar{p}) G(\pi^*)^{n-1}(1 - G(\pi^*))^{N-n} \right. \\
- \sum_{n=1}^{N-1} \binom{N-1}{n-1} B(n, \bar{p}) G(\pi^*)^{n}(1 - G(\pi^*))^{N-n-1} \\
- (1 - G(\pi^*))^{N-1}(g - c_0) - \pi^* \right] \\
= N g(\pi^*) \left[ \sum_{n=1}^{N} \binom{N-1}{n-1} (B(n, \bar{p}) - B(n-1, \bar{p})) G(\pi^*)^{n-1}(1 - G(\pi^*))^{N-n} - \pi^* \right]. 
\]

Use \( \bar{p} \geq \pi \) and (1), and we obtain

\[
B(n, \bar{p}) - B(n-1, \bar{p}) = \int_0^{\bar{p}} F(c)(1 - F(c))^{n-1} dc = \pi(n) \text{ for } n \geq 2 \\
= c_0 - c_1 = \pi(1) + (c_0 - \bar{p}) \text{ for } n = 1.
\]

By substituting them into (18) and using the equation (3), we get

\[
\frac{\partial W}{\partial \pi} = N g(\pi^*) \{ e + (1 - G(\pi^*))^{N-1}(c_0 - \bar{p}) \} = 0
\]

Therefore, the first-order conditions hold for any \( \bar{p} \leq \pi \leq c_0 \) and \( e \leq 0 \) that satisfy \( e + (1 - G(\pi^*))^{N-1}(c_0 - \bar{p}) = 0 \). Along this line, \( \pi^* \) and \( W \) are constant. We need to show that they are global maxima. Take as arbitrary \((\hat{\pi}, \hat{e})\), which satisfy (19). It is easily checked that if \( e \leq \hat{e} \), \( \frac{dW}{de} \geq 0 \) and if \( \bar{p} \leq \hat{\pi} \), \( \frac{dW}{d\bar{p}} \geq 0 \). It is also clear that there is no corner solution in the range \( \bar{p} < \bar{p} \), because given \( \frac{d\pi^*}{de} = 0 \), \( \frac{dW}{d\bar{p}} > 0 \) for any \( \bar{p} < \bar{p} \). Therefore, \((\hat{\pi}, \hat{e})\) is a global maximum.

Next, we prove the second half of the proposition. The first step is to show that the buyer chooses \( \bar{p} \geq \pi \). Suppose not, that is \( \bar{p} < \pi \). Then, the ceiling price will prevent \textit{ex post} efficient trades as you can see in (17). The buyer can raise its profit by raising \( \bar{p} \) while reducing \( e \) so that \( \pi^* \) does not change. In other words, the money transfer enables the buyer to capture the entire efficiency gain from choosing the efficient ceiling price. Let \( \bar{p} = c_0 \) so that the socially efficient \( e = 0 \). The other choice of \( \bar{p} \) does not change the level of the buyer’s profit if \( e \) is chosen optimally. Let \( p_n \) be the expected winning price when there are \( n \) suppliers. Then,
\[ p_n = c_n + n\pi(n). \] The buyer’s expected profit \( \Pi \) is
\[ \Pi = \sum_{n=0}^{N} \binom{N}{n} (g + ne - p_n)G(\pi^*)^n(1 - G(\pi^*))^{N-n} \]
where we define \( p_0 = c_0 \). Note that the first-order condition for the optimal \( e \) is\( \frac{d\Pi}{de} = \frac{\partial \Pi}{\partial e} \frac{de}{de} + \frac{\partial \Pi}{\partial e} = 0 \) where \( \frac{de}{de} < 0 \), \( \frac{\partial \Pi}{\partial e} = NG(\pi^*) > 0 \) and
\[
\frac{\partial \Pi}{\partial \pi^*} = N\frac{\partial g(\pi^*)}{\partial \pi^*} \left\{ \sum_{k=1}^{N} \binom{N-1}{n-1} (p_{n-1} - p_n)G(\pi^*)^{n-1}(1 - G(\pi^*))^{N-n} + \varepsilon \right\}
\]
\[ = N g(\pi^*) \left\{ \sum_{k=2}^{N} \binom{N-1}{n-1} (n-1)(\pi(n-1) - \pi(n))G(\pi^*)^{n-1}(1 - G(\pi^*))^{N-n} + \varepsilon \right\} \]

where \( c_{n-1} - c_n = \pi(n) \) for \( n \geq 2 \) is used to produce the second line. The sign of \( \frac{d\Pi}{de} \) evaluated at \( e = 0 \) is indeterminate.

This concludes the proof. \( \blacksquare \)

\section*{C Proof of Proposition 2}

Define the distribution function \( F(n; M) = \sum_{k=0}^{n} f(k; M) = \sum_{k=0}^{n} (\binom{M}{k})p^k(1-p)^{M-k} \). We first prove the following lemma.

\textbf{Lemma 2a} For all \( M \), \( F(n; M + 1) \) strictly dominates \( F(n; M) \).

\textbf{Proof.} Suppose not. Then, for some \( n \), \( \sum_{k=0}^{n} (\binom{M+1}{k})p^k(1-p)^{M+1-k} > \sum_{k=0}^{n} (\binom{M}{k})p^k(1-p)^{M-k} \). Then, for some \( k' \leq n \), \( (\binom{M+1}{k'})p^{k'}(1-p)^{M+1-k'} > (\binom{M}{k'})p^{k'}(1-p)^{M-k'} \) which is equivalent to \( \frac{M+1}{M+1-k'}(1-p) > 1 \). Since the left-hand side is increasing in \( k' \), \( (\binom{M+1}{k'})p^{k'}(1-p)^{M+1-k'} > (\binom{M}{k'})p^{k'}(1-p)^{M+1-k'} \) holds for all \( k \geq k' \). Therefore, \( \sum_{k=0}^{M+1} (\binom{M+1}{k})p^k(1-p)^{M+1-k} > \sum_{k=0}^{M} (\binom{M}{k})p^k(1-p)^{M-k} \). But, this contradicts with the fact that \( F(M; M) = 1 \) for all \( M \). this concludes the proof. \( \blacksquare \)

Now, define \( H(\pi, M) = \sum_{n=0}^{M} (\binom{M}{n})\pi(n+1)G(\frac{\pi}{M})^n(1 - G(\frac{\pi}{M}))^{M-n} \). By applying Lemma 1a to Lemma 2a (simply, replace \( p \) with \( M \)), we get that \( H(\pi, M) \) is decreasing in \( M \).

We can derive the monotonicity as follows: Note that \( \pi_{B22}^M \) is the solution for \( \pi_{B22}^M = \)
\(H(\pi_{M_s}^{B2B}, M_s - 1)\). Suppose \(\pi_{M_{s+1}}^{B2B} \geq \pi_{M_s}^{B2B}\), then

\[
\pi_{M_{s+1}}^{B2B} - \pi_{M_s}^{B2B} = H(\pi_{M_{s+1}}^{B2B}, M_s) - H(\pi_{M_s}^{B2B}, M_s) + H(\pi_{M_s}^{B2B}, M_s) - H(\pi_{M_s}^{B2B}, M_s - 1)
\]

\[
= \int_{\pi_{M_s}^{B2B}}^{\pi_{M_{s+1}}^{B2B}} \frac{\partial H(\pi, M_s)}{\partial \pi} d\pi + H(\pi_{M_s}^{B2B}, M_s) - H(\pi_{M_s}^{B2B}, M_s - 1) < 0.
\]

The last inequality is derived from \(\frac{\partial H(\pi, M_s)}{\partial \pi} < 0\) (by Lemma 1a) and \(H(\pi_{M_s}^{B2B}, M_s) - H(\pi_{M_s}^{B2B}, M_s - 1) < 0\) \((H(\pi, M)\) is decreasing in \(M)\).

By contradiction, \(\pi_{M_{s+1}}^{B2B} < \pi_{M_s}^{B2B}\). Therefore, \(\pi_{M_s}^{B2B}\) is decreasing in \(M_s\). The fact that \(\Pi_{S}^{B2B}(M_B, M_S)\) is decreasing in \(M_S\) directly follows.

Similarly, define \(I(\pi_{M_s}^{B2B}, M_s) = \sum_{n=0}^{M_s} (M_s)\{v -cn - n\pi(n)\}G(\pi_{M_s}^{B2B}/\alpha)^n(1 - G(\pi_{M_s}^{B2B}/\alpha))^{M_s-n}\). Note that \(v-cn-n\pi(n)\) is increasing in \(n\). By applying Lemma 1a to Lemma 2a (simply, replace \(p\) with \(M_S\)), we get that \(I(\pi_{M_s}^{B2B}, M_s)\) is increasing in \(M_s\), and by Lemma 1a \(\frac{\partial I(\pi, M_s)}{\partial \pi} > 0\).

Then,

\[
\Pi_{B}^{B2B}(M_s + 1) - \Pi_{B}^{B2B}(M_s)
\]

\[
= I(\pi_{M_{s+1}}^{B2B}, M_s + 1) - I(\pi_{M_s}^{B2B}, M_s + 1) + I(\pi_{M_s}^{B2B}, M_s + 1) - I(\pi_{M_s}^{B2B}, M_s)
\]

\[
= \int_{\pi_{M_s}^{B2B}}^{\pi_{M_{s+1}}^{B2B}} \frac{\partial I(\pi, M_s)}{\partial \pi} d\pi + I(\pi_{M_s}^{B2B}, M_s + 1) - I(\pi_{M_s}^{B2B}, M_s) > 0.
\]

In order to prove that \(\Pi_{S}^{B2B}(M_B, M_S)\) is increasing in \(M_B\), you only need to show that \(\alpha\int_0^\alpha G(\tau)d\tau - \int_0^C G(\tau)d\tau > 0\). Define \(J(\alpha) = \alpha\int_0^\alpha G(\tau)d\tau - \int_0^C G(\tau)d\tau\). Since \(\frac{dJ(\alpha)}{d\alpha} = \int_0^\alpha \frac{G(\tau)}{\alpha}\tau - \frac{\pi_{M_s}^{B2B}}{\alpha}G(\frac{\pi_{M_s}^{B2B}}{\alpha}) < 0\) and \(\lim_{\alpha\to1} J(\alpha) = 0, J(\alpha) > 0\). Hence \(\Pi_{S}^{B2B}(M_B, M_S)\) is increasing in \(M_B\). It is trivial from the definition that \(\Pi_{S}^{B2B}(M_B)\) is decreasing in \(M_B\).

**D  Proof of Proposition 3**

We prove the result only for a censored normal distribution. The proof for a log-normal distribution is similar.

When \(G(\frac{\pi_{M_s}^{B2B}}{\alpha}) > 0.5\), \(\frac{dG(\frac{\pi_{M_s}^{B2B}}{\alpha})}{d\alpha} = -\frac{\pi_{M_s}^{B2B}}{\sigma^2}G(\frac{\pi_{M_s}^{B2B}}{\alpha}) < 0\) where \(\phi\) is standard normal.
density function. Then,

\[
\frac{d\pi^{B2B}_{MS}}{d\sigma} = (M_S - 1) \left( \frac{dG\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right)}{d\sigma} + \frac{g\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right)}{\alpha} \frac{d\pi^{B2B}_{MS}}{d\sigma} \right) \\
\times \left\{ \sum_{n=0}^{M_S-2} \left( \begin{array}{c} M_S - 2 \\ n \end{array} \right) (\pi(n+2) - \pi(n+1))G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right)^n(1 - G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right))^{M_S-2-n} \right\}
\]

We obtain

\[
\frac{d\pi^{B2B}_{MS}}{d\sigma} = (M_S - 1) \frac{\pi^{B2B}_{MS}}{\sigma} \frac{\alpha}{\sigma} \phi\left(\frac{\pi^{B2B}_{MS}}{\alpha} - \mu\right)K(M_S) > 0 \text{ for } G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right) > 0.5
\]

where \( K(M_S) = \sum_{n=0}^{M_S-2} \left( \begin{array}{c} M_S - 2 \\ n \end{array} \right) (\pi(n+1) - \pi(n+2))G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right)^n(1 - G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right))^{M_S-2-n} > 0. \)

Hence, an increase in variance improves the supplier profit in each auction when the B2B exchange enables each participating supplier to enter more than half of the auctions conducted on-line.

However, the buyer profit will be squeezed by entry cost heterogeneity.

\[
\frac{d\Pi^{B2B}_B(M_S)}{d\sigma} = M_S\left( - \frac{\pi^{B2B}_{MS}}{\alpha} - \frac{\mu}{\sigma} + \frac{1}{\alpha} \frac{d\pi^{B2B}_{MS}}{d\sigma} g\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right) \right) \\
\times \left\{ \sum_{n=0}^{M_S-1} \left( \begin{array}{c} M_S - 1 \\ n \end{array} \right) (c_n - c_{n+1} + n\pi(n) - (n+1)\pi(n+1))G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right)^n(1 - G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right))^{M_S-1-n} \right\}
\]

\[
= - \frac{M_S}{\sigma} \left( \frac{\pi^{B2B}_{MS}}{\alpha} - \frac{\mu}{\sigma} \right) \frac{\alpha}{\sigma} \phi\left(\frac{\pi^{B2B}_{MS}}{\alpha} - \mu\right)K(M_S) \\
\times \left\{ \sum_{n=0}^{M_S-1} \left( \begin{array}{c} M_S - 1 \\ n \end{array} \right) (c_n - c_{n+1} + n\pi(n) - (n+1)\pi(n+1))G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right)^n(1 - G\left(\frac{\pi^{B2B}_{MS}}{\alpha}\right))^{M_S-1-n} \right\}
\]

\[
< 0
\]

Next, we show that the suppliers’ overall profit net of entry costs is also better for a larger
where the last inequality is derived from the assumption \( G(\frac{\pi_{B2B}}{\alpha}) < 1 - G(0) \), which leads to \( 2\mu - \frac{\pi_{B2B}}{\alpha} > 0 \).

The comparative statics with respect to \( \pi^O \) is similar. First,

\[
\frac{d\pi^O}{d\sigma} = \frac{(N_S - 1)\frac{\pi^O - \mu}{\sigma} \phi(\frac{\pi^O - \mu}{\sigma}) K(N_S)}{1 + (N_S - 1)\frac{1}{\alpha} \phi(\frac{\pi^O - \mu}{\sigma}) K(N_S)} < 0 \text{ for } G(\pi^O) < 0.5.
\]
\[
\frac{d\Pi_S^Q(0)}{d\sigma} = N_B G(\pi^O) \frac{d\pi^O}{d\sigma} - N_B \frac{\pi^O - \mu}{\sigma} G(\pi^O) - \frac{N_B}{\sigma} \mu G(0) + \frac{N_B}{\sigma} \int_0^{\pi^O} G(\tau) d\tau
\]

< \frac{N_B}{\sigma} (\mu - \pi^O) G(\pi^O) - \frac{N_B}{\sigma} \mu G(0) + \frac{N_B}{\sigma} \int_0^{\pi^O} G(\tau) d\tau

= \frac{N_B}{\sigma} \int_{\pi^O}^{\mu} (G(\pi^O) - G(0)) d\tau

E Proof of Proposition 5

We first need to show that the no participation equilibrium is perfectly coalition-proof. Since we have shown that it is subgame-perfect, we only need to show that there is no coalition such that the strategy profile that requires all firms in the coalition to join a B2B exchange is a perfectly coalition-proof Nash equilibrium, in the game induced on the coalition by fixing the strategies for the complement of the coalition as those in the no participation equilibrium. Suppose there is such a coalition and denote it by \( J \). Let \( J_B \) and \( J_S \) be the number of buyers and suppliers in \( J \). Obviously \( J_B > 0 \). If \( J_B = N_B \), the deviation by the coalition should trigger the participation of \( \overline{M_S}(t_S) - J_S \) other suppliers because their equilibrium strategy is subgame-perfect. Then, this deviation should not be self-enforcing because \( \Pi_S^{B2B}(N_B, \overline{M_S}(t_S)) - t_S < \Pi_S^Q(0) \). Therefore, \( J_B < N_B \) and \( J_S < \overline{M_S}(t_S) \). However, once \( J_B \) buyers and \( J_S \) suppliers participate in the B2B exchange, it is in the best interest of the remaining \( N_B - J_B \) buyers to participate because it would trigger \( \overline{M_S}(t_S) - J_S \) other suppliers to join as we have argued earlier and these buyers earn more profits than not participating, i.e. \( \Pi_S^{B2B}(\overline{M_S}(t_S)) - t_B > \Pi_B^{B2B}(J_S) - t_B \geq \Pi_B^O \). Therefore, this deviation by \( J \) cannot be self-enforcing.

Next, we prove that the full participation is not perfectly coalition-proof. Consider the deviation by all suppliers such that all of them adopt the strategy (S1S) described in the proof of Proposition 4. It is easily verified that this strategy profile is a perfectly coalition-proof Nash equilibrium in the game induced on the supplier coalition by fixing the buyer’s strategy. This concludes the proof.
References


