Booms, Busts, and Fraud

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Abstract

We examine firm managers’ incentives to commit fraud in a model where firms seek funding from investors and investors can monitor firms at a cost in order to get more precise information about firm prospects. We show that fraud incentives are highest when business conditions are good, but not too good: in exceptionally good times, even weaker firms can get funded without committing fraud, and in bad times investors are more vigilant and it is harder to commit fraud successfully. As investors’ monitoring costs decrease, the region in which fraud occurs shifts towards better business conditions. It follows that if business conditions are sufficiently strong, a decrease in monitoring costs actually increases the prevalence of fraud. If investors can only observe current business conditions with noise, then the incidence of fraud will be highest when investors begin with positive expectations that are disappointed ex post. Finally, increased disclosure requirements can exacerbate fraud. Our results shed light on the incidence of fraud across the business cycle and across different sectors.

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1 Introduction

“It’s only when the tide goes out that you can see who’s swimming naked.”

Warren Buffett

Booms and busts are a common feature of market economies. Almost as common is the belief that a boom encourages and conceals financial fraud and misrepresentation by firms, which are then revealed by the ensuing bust. Examples in the last century include the 1920s (Galbraith, 1955), the “go-go” market of the 1960s and early 1970s (Labaton, 2002, Schilit, 2002), and the use of junk bonds and LBOs in the 1980s (Kaplan and Stein, 1993). Most recently, the long boom of the 1990s has been followed, first by recession, then by revelations of financial chicanery at many of America’s largest companies.

Some argue that fraud in booms is exacerbated by inadequate rules and regulations. In the 1930s, this view led to the establishment of the SEC and numerous regulations on financial institutions; in the early 1990s, to anti-takeover legislation; and in the crisis just past, to the Sarbanes-Oxley Act. Yet others have argued that the root cause for the fraud lies in investors’ overly optimistic expectations, which make fraudulently positive reports seem more plausible. For example, the Economist (2002) suggests:

The remedy is disclosure, honest accounting, non-executive directors empowered to do their job — and, as always, skeptical shareholders looking out for their own interests.

Without doubt, the last of these is most important of all. Alas, it is beyond the reach of regulators and legislators. ... The most important lesson of this bust, like every bust, is: buyer beware.

In this paper, we examine these arguments in a simple model of financing and investment. Firms require external funding. Firms can be good or bad (i.e., investing in them can be positive or negative net present value), but due to private control benefits, managers want to get funding regardless. Rational investors observe a public but noisy signal of the firm’s true type, after which they can decide whether to investigate the firm’s prospects more closely (“monitor”) at a cost. Managers of bad firms can commit fraud, which increases the chance that the public signal will be high even though the firm is bad. Committing fraud is costly to managers, reflecting effort costs and the chance that they may be caught and penalized.
Finally, although fraud alters the public signal, investors who monitor learn the firm’s true situation.

We show that fraud is most likely to occur if the average firm’s prospects are relatively good. Fraud is less likely in exceptionally good times, however, since investors are rationally willing to fund a firm even if its public signal is low. Fraud is also less likely when the average firm’s prospects are bad to middling; investors are cautious about investing even if a firm’s public signal is high, and so are less easily fooled.

In a dynamic setting, investors’ expectations about the average firm’s prospects will change over time. The incidence of fraud will be highest when the business cycle has turned down but investors are not yet aware of this. Thinking conditions are still reasonably good, investors fund firms with high public signals, and firms with poor prospects commit fraud so as to obtain funding. These very acts of fraud obscure the extent of the downturn. Eventually, reality intrudes, the downturn is revealed, and the incidence of fraud is much greater than anyone anticipated. Yet investors and firms have acted rationally; good times are most likely to lead to widespread fraud when the good times are ending, but the end of good times can only be known ex post. Indeed, it is when a number of firms that had been doing well are seen to be doing badly that investors know that the good times have ended. To reverse Buffett, it’s only when you see a lot of people swimming naked that you know that the tide has gone out.

Thus, a model with rational behavior can reproduce many features of the boom-bust-fraud pattern. Although we do not claim that investors are always perfectly rational, the fact that rationality does not rule out this pattern suggests limits to the “buyer beware” school of policy response. Moreover, our most critical result — that fraud incentives are highest in good times — actually requires a certain amount of “buyer beware” behavior: specifically, investors must be able to monitor and must decide whether to monitor in a rational fashion.

To see why this is true, we discuss our results in more detail. First, consider the case when monitoring is prohibitively expensive, so that investors make financing decisions based on the public signal alone. Fraud is most attractive to a bad firm’s manager when a high public signal definitely leads to financing and a low signal does not. This occurs when prior (pre-signal) uncertainty about the firm’s attractiveness is highest, which is when the prior expected net present value of the firm is close to zero — “so-so” times. By contrast, in good or bad times, the dichotomy between high and low signals is less marked. When good firms are plentiful, a
low signal has a reasonable chance of coming from a good firm that has been unlucky rather than from a bad firm. Thus, investors are more willing to finance low-signal firms; since bad firms have a chance of being funded even if they produce low signals, they have less incentive to commit costly fraud. Similarly, when bad firms are plentiful, a high signal has a good chance of coming from a bad firm that has been lucky, so investors are less willing to finance high-signal firms; since bad firms have less chance of being funded even if they produce a high signal, they have less incentive to commit fraud. Thus, in the absence of monitoring, incentives to commit fraud are highest in “so-so” times rather than good times.

This is not true when monitoring is feasible. Once again, if there are many bad firms in the economy, investors are suspicious of high signals, and managers of bad firms have little incentive to commit fraud: even if fraud produces a high signal, the best they can hope for is being monitored, which will reveal the firm’s true state of affairs and thus preclude funding. Yet the same is likely to be true even in “so-so” times: because initial uncertainty is highest, after the public signal there is enough residual uncertainty about the firm’s prospects that costly monitoring is still attractive. Thus, even after a high signal, investors are likely to monitor, and once again bad firms have little incentive to commit fraud.

As there are more good firms in the economy, eventually investors find it less attractive to monitor high-signal firms. Incentives to commit fraud now increase: fraud increases the odds that a bad firm can generate a high signal and thus get unmonitored funding. Incentives to commit fraud continue to increase with the relative number of good firms until high-signal firms are never monitored. With further increases in the number of good firms, investors begin funding low-signal firms without monitoring, decreasing fraud incentives. If the number of good firms is extremely high, investors may be willing to fund all firms without paying any attention to the free signal, in which case fraud has no benefit at all.¹

Thus, the link between good times and fraud requires that, given their prior beliefs about the distribution of good and bad firms, investors rationally decide whether or not to monitor. As monitoring costs fall, the thresholds for different “regimes” — fund high-signal firms without monitoring, fund low-signal firms without monitoring, etc. — are shifted towards better business conditions. Intuitively, fraud is only attractive when investors do not always

¹ Though we focus our analysis on relative numbers of good and bad firms, similar results obtain if these relative numbers are fixed and it is the relative attractiveness of good and bad firms that varies. We discuss this further at the end of Section 4 below.
monitor high-signal firms. Because lower monitoring costs make monitoring a more attractive option, the prior must be higher before investors cut back on monitoring high-signal firms. Paradoxically, the link between good times and fraud becomes stronger as monitoring costs fall.

Now suppose that investors are not perfectly informed on the relative numbers of good and bad firms: the relative numbers of good firms could be high (“good state of the economy”) or low (“bad state of the economy”), and investors have prior beliefs on the likelihood of these two states. Over time, actual firm successes and failures will reveal more information about the true state of the economy.

Suppose that investors believe there is a low to moderate chance of the good state. As per our previous discussion, investors will only fund high-signal firms after monitoring, if at all, and so bad firms will not commit fraud. Ex post, the economy’s state will prove either to have been bad or good, but either way there will not have been much fraud.

By contrast, suppose that the prior is high enough that high-signal firms are not monitored, though low-signal firms are either monitored or not funded at all. If later events prove that the state of the economy was in fact good, there will not have been much fraud; bad firms did commit fraud, but there were few of them. On the other hand, if later events prove that, despite the prior, the true state was bad, fraud will be prevalent; bad firms did commit fraud, and there were many more of them than expected. In this case, although some may later opine that the problem was that investors were insufficiently skeptical, investors in fact behaved rationally given their prior; the problem was that the true state of the economy was known only noisily and with a lag.²

In fact, the economy evolves over time, so that the relative numbers of good and bad firms are always changing and investors are always updating their beliefs about these numbers. One source of information for such updating is free signals from firms. If these signals can be manipulated, then when bad firms commit fraud, free signals are noisier, and so rational investors are slower to update their beliefs. Supposing that a long stream of positive cash flows does eventually convince investors that times are likely to be good, it will be hard for them to detect when the tide has turned and the number of bad firms has increased – at least, until the

² Again, our main point is that a model with fully rational behavior already exhibits features that are broadly in line with the facts. Nevertheless, if investors are inclined to waves of excessive optimism and pessimism, this will further exacerbate these effects.
projects of the bad firms have come off badly.

In addition to providing a rational explanation for why long booms often seem to end in a wave of failures and fraud, our model yields other predictions, some of them counterintuitive. When times are bad enough that high-signal firms are monitored with some probability, a decrease in the cost of monitoring increases monitoring and decreases fraud, as one would expect. By contrast, when times are good enough that monitoring focuses only on firms that produce low signals, a decrease in the cost of monitoring increases monitoring and increases fraud. This follows because fraud helps bad firms avoid low signals, and in good times, low signals are what triggers monitoring.

Again, this result helps motivate behavior that at first glance seems completely myopic. In bad times (such as the early 1990s or right now), additional financing is hard to come by even for ventures with good ideas and track records. By contrast, in the good times of the late 1990s, shareholders and boards were routinely castigated in the business press for overreacting to bad news, so that the watchword for corporations was to avoid bad news at all costs. Yet even if the ongoing reduction in costs of telecommunication and computing have lowered the cost of analyzing firms, our model suggests that investors may optimally choose to focus their analysis on bad news in good times. If shareholders can only punish managers directly by selling stock (which may then trigger action by the board), then our model is consistent with the behavior that has been seen.

In addition to these “time series” effects, our model has cross-sectional implications for different industries during a given business cycle. For example, if investor priors in a given sector are extremely high, we should see little fraud; if priors in a sector are moderately high, then the potential for fraud increases. This may motivate differences between the “dot-com” and telecom industries during the boom of the 1990s. Investors were so willing to believe in the chances of success of any firm whose name that ended in “dot-com” that fraud per se was largely unnecessary. By contrast, in telecoms, investors, though optimistic, did pay attention to reported revenues and earnings; consistent with our prediction, this sector seems to have experienced far more cases of accounting fraud.

Our results also have policy implications. As we have shown, simply saying “buyer beware” may not do much to prevent fraud. Nevertheless, tougher disclosure standards can actually make fraud more prevalent. If tougher disclosure standards improve the precision of public
signals in the absence of fraud, managers of bad firms have more incentive to commit fraud to “noise things up,” and the probability of fraud increases. To be effective against fraud, disclosure standards must directly make fraud more difficult.3

The plan of the rest of the paper is as follows. We discuss the relevant literature in Section 2. In Section 3 we introduce our model and key assumptions. In Section 4 we analyze the behavior of investors and firms in a setting where all agents know the underlying distribution of good and bad firms in the economy. In Section 5 we show how our results are affected by changes in the underlying parameters and how these can motivate actual behavior by firms and investors. We also show how agents’ beliefs can be grounded in a framework in which the underlying state of the economy is unknown, leading to “surprising” volumes of fraud in certain circumstances. In Section 6 we discuss how our model’s main results are robust to changes in our simple assumptions, and in Section 7 we conclude.

2 Literature Review

Although ours is the first paper that we are aware of that ties fraudulent behavior by firms to changing investor actions over the business cycle, there are a number of papers that are related to the tenor of our analysis. For example, a growing body of work examines “credit cycles” – the idea that banks and other credit suppliers engage in behavior that exacerbates business cycle effects, making credit even tighter in recessions, and looser in expansions, than pure demand-side effects would suggest. Among these, the closest to our paper is Ruckes (1998), who models how competing bank lenders’ incentives to screen potential borrowers exacerbate cyclical variations in credit standards. None of these papers address borrower incentives to commit fraud, which is our key focus.

Another related paper is Persons and Warther’s (1997) model of booms and busts in the adoption of financial innovations. In their model, individual firms decide whether to adopt a new financial technique based on the information that earlier adopters’ experience noisily reveals. They show that such waves of adoption always end on a sour note, in the sense that the most recent adopters always lose money. Ex post, the information that ends the wave is always

3 This is not to say that improved disclosure may not also have beneficial effects. We discuss the impact of improved disclosure in more detail in Section 5.
negative, but the timing of the end is ex ante random, and the latest adopters were behaving rationally based on the information available at the time. Like our model, this suggests that busts are always surprising yet may still be rational. Nevertheless, Persons and Warther do not address the role of fraud, and the mechanism of their model focuses on the evolution of social learning about a static innovation rather than investor-firm conflicts and behavior in the face of private information.

Four recent papers in the finance literature also focus on managerial incentives to commit fraud. Bebchuk and Bar-Gill (2002) present a model in which firms may commit fraud so as to obtain better terms when issuing shares to raise funds for further investments; this incentive to commit fraud increases if managers can sell some of their own shares in the short run or if accounting and legal rules are lax. Goldman and Slezak (2003) present a model where optimal managerial pay-for-performance contracts balance incentives to exert effort against incentives to commit fraud; increased regulatory penalties for fraud can sometimes increase the equilibrium incidence of fraud, and rules that reduce auditor incentives to collude with managers decrease the incidence of fraud but paradoxically reduce firm value. Subrahmanyam (2003) presents a model where more intelligent managers are better both at running firms and at committing successful (undetected) fraud; as a result, investors may prefer more intelligent managers and a higher incidence of fraud in exchange for higher average performance. Unlike our paper, these three papers do not examine how changes in economic conditions affect manager’s incentives to commit fraud and investor’s incentives to monitor managers, which is our primary focus.4

Finally, Noe (2003) analyzes a different type of fraud, in which a firm’s manager “tunnels” value from the firm into her own pocket. He focuses on providing the manager with incentives to perform rather than steal the funds that she has raised.

There are a number of studies in the accounting literature that focus on fraud incentives in the relationships between firms and their auditors. Some of these examine incentives to under-report earnings in order to hide managerial perquisite consumption; see for example Morton (1993). Closer to our focus are papers that examine the incentive to over-report; examples include Newman and Noel (1989), Shibano (1990), and Caplan (1999). Empirical work on SEC enforcement actions aimed at violations of Generally Accepted Accounting Principles (GAAP)

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4 Goldman and Slezak (2003) do show that an influx of naive, overly optimistic investors into the stock market increases the equilibrium incidence of fraud. Again, our model shows that such fluctuations can occur even when all investors are perfectly rational.
suggests that over-reporting aimed at boosting share prices and improving access to additional
capital is in fact the more frequent source of firm-wide financial misrepresentation.\(^5\) Unlike
our paper, these auditing papers on over-reporting focus on the impact of control systems
and auditor incentives; they do not examine how fraud incentives change with overall business
conditions. A further distinction is that auditors are typically penalized for failing to detect
fraud. By focusing on the incentives of investors, we emphasize the fact that investors are not
concerned with finding fraud per se, but rather with finding good investment opportunities.
As already noted, this can lead to counterintuitive results when investors rationally focus their
scrutiny on low signals rather than high ones.

Finally, our work contrasts with the growing literature that examines how bounded rational-
ity can cause market overreactions. The critical difference is that our model relies on rational
behavior throughout. As noted earlier, to the extent that deviations from rationality do lead
investors’ priors to overreact to recent information, they will exacerbate the effects we describe.

3 Basic Model and Assumptions

In this section we lay out the basic model that provides the framework for analyzing the
incidence of fraud in Section 4. The economy consists of equal numbers of firm managers and
investors, each of whom lives for one period. The sequence of events is summarized in Figure
1.

\[
\begin{array}{cccccc}
\text{Firms } j \text{ and} & \text{Commit} & \text{Free signal} & \text{Contract} & \text{Revenue} \\
\text{investors } i & \text{fraud} & s \in \{h, \ell\}; & \text{written} & R \text{ or zero;} \\
\text{matched} & \text{or not} & \text{monitor} & \text{rent } C & \text{or not} \\
\text{randomly} & & \text{or not} & & \\
\end{array}
\]

Figure 1: *Time line*

\(^5\) For example, Peroz et al. (1991) find that fraud usually takes the form of earnings overstatement, and that
news of an SEC enforcement action depresses stock price. Dechow et al. (1996) find that firms that commit
fraud tend to have higher ex ante needs for additional funds.
3.1 Firms and Managers

Each manager controls a firm that requires an investment of $I$ units of cash at the start of the period. At the end of the period, the firm returns a random contractable cash flow that equals $R > I$ with probability $\theta_i$ and zero with probability $1 - \theta_i$, where $i \in \{g, b\}$ is the firm’s type. We assume that $0 \leq \theta_b < \theta_g < 1$. We also assume that

$$N_g = \theta_g R - I > 0 \quad N_b = -(\theta_b R - I) > 0;$$

i.e., $g$ firms are positive net present value investments (“good”), whereas $b$ firms are negative NPV investments (“bad”). Note that $N_b$ is the absolute value of the expected loss from investing in a bad firm.

In addition to generating contractable cash flows, a funded firm generates $C$ in noncontractable control benefits which the manager consumes.\(^6\) This implies that, all else equal, a manager prefers to get her project funded, regardless of her firm’s type.

Managers know their own firm’s type, but outsiders can discover this only by monitoring the firm at a cost, as we discuss below. The prior probability that any given firm is good is given by $\mu$, where $\mu \in (0, 1)$. This prior is common knowledge. For the moment, we take this prior as exogenously given; we discuss how this can be embedded in a multi-period framework in Section 5.

3.2 Investors

Investors are each endowed with $I$ units of the generic good. At the beginning of the period, each investor is randomly matched with a manager and her firm. After being matched, the investor receives a free but noisy signal of the firm’s type, and may then decide whether or not to expend additional effort and learn the firm’s type more precisely. Based on any information that she has, the investor then can make a take-it-or-leave-it investment offer to the manager. The manager does not have time to approach another investor, so if the investor does not make

\(^6\) This could represent nonpecuniary benefits of control or pecuniary benefits that have to be given to the manager in order to elicit reasonable efforts (see for example Diamond (1993)). Moskowitz and Vissing-Jorgensen (2002) give empirical evidence that is consistent with large nonpecuniary benefits; Fee and Hadlock (2004) give evidence on the net pecuniary benefits that CEOs lose if they are dismissed and forced to seek employment elsewhere.
her an offer, the manager cannot get funding for her firm.

Our assumptions of random matching and take-it-or-leave-it offers are made for simplicity; altering them would not change the essentials of our analysis. For simplicity, we also assume that investors cannot pay off bad firms to reveal their type; in practice, doing so is likely to be prohibitively expensive since a large number of incompetent managers would start firms and apply to investors for the sole purpose of receiving that payment. (We return to this issue of entry in Section 6 below.)

Thus, in equilibrium, if the investor does fund the firm, she receives all of the contractable cash flows that it produces. Nevertheless, since the manager receives control benefits $C$ if the firm is funded and nothing if the firm is not funded, she will take any offer that she is given.

### 3.3 Signals, Fraud, and Monitoring

As just mentioned, right after managers and investors are matched, each investor receives a free but noisy signal of the type of the manager’s firm. This signal should be thought of as a financial report or a related public news release by the firm. We assume that this signal takes on one of two values, $h$ (“high”) and $\ell$ (“low”). We also assume that, absent fraud, the signal is positively correlated with the firm’s true type:

$$\Pr\{h|g\} = \gamma > \frac{1}{2} > \beta = \Pr\{h|b, \text{ no fraud}\}.$$  

The free signal is subject to manipulation by the manager (“fraud”). The manager decides whether or not to commit fraud right after she and the investor are matched. Fraud costs the manager an amount $f$, where $f$ reflects both any effort involved in committing fraud and the chance that the manager is later caught and punished. We return to the issue of catching and punishing fraud in Section 6. Fraud increases the probability that a bad firm generates a high signal by $\delta < \gamma - \beta$; that is, $\Pr\{h|b, \text{ fraud}\} = \beta + \delta < \gamma$. Thus, fraud reduces the free signal’s correlation with the firm’s type, but the free signal remains somewhat informative.\(^7\) Fraud is beneficial to the manager to the extent it increases the manager’s chance of collecting control benefits $C$. It follows that fraud will never be attractive unless the cost of fraud $f$ is less than

\(^7\)Allowing $\delta$ to exceed $\gamma - \beta$ would have little effect on our qualitative results; bad firms would never commit fraud with certainty, but comparative statics would be unchanged.
the maximum possible benefit, i.e., \( f < \delta C \). Henceforth, we assume that this condition holds.

In practical terms, fraud should be thought of as deliberate misstatement of the firm’s results, either through altered financial reports or a misleading news release. Such an effort increases the odds that a casual glance at the firm’s results will lead investors to think that the firm is in good shape – in terms of our model, it increases the probability that the public signal is high.

For simplicity, we assume that only bad firms commit fraud. As we discuss in Section 6, allowing good firms to commit fraud leaves most of our results qualitatively unchanged, so long as bad firms have relatively more to gain from fraud.

Suppose that the bad firm commits fraud with probability \( \phi \). Let \( \hat{\mu}_s(\phi) \) be the investor’s posterior probability that the firm is good after she sees the free signal \( s \). Applying Bayes’ Rule, we have

\[
\hat{\mu}_h(\phi) = \Pr [g|h] = \frac{\Pr \{g\} \Pr \{h|g\}}{\Pr \{g\} \Pr \{h|g\} + \Pr \{b\} \Pr \{h|b\}} = \frac{\mu}{\mu + (1-\mu) \frac{\beta + \phi \delta}{\gamma}}
\]

\[
\hat{\mu}_l(\phi) = \Pr [g|\ell] = \frac{\Pr \{g\} \Pr \{\ell|g\}}{\Pr \{g\} \Pr \{\ell|g\} + \Pr \{b\} \Pr \{\ell|b\}} = \frac{\mu}{\mu + (1-\mu) \frac{1 - \beta - \phi \delta}{1 - \gamma}}.
\]

Notice that \( \forall \phi \in (0,1) \),

\[
\hat{\mu}_l(0) < \hat{\mu}_l(\phi) < \hat{\mu}_l(1) < \mu < \hat{\mu}_h(1) < \hat{\mu}_h(\phi) < \hat{\mu}_h(0).
\] (2)

As expected, the posterior probability that the firm is good is higher after observing a high signal than it is after observing a low signal, and fraud makes the signal less precise, i.e. the posterior approaches the prior as either \( \delta \) or \( \phi \) increase.

After receiving the free signal, the investor can choose to investigate the firm further (“monitor”). Monitoring has an effort cost of \( m > 0 \) and perfectly reveals the firm’s type. Once more, the assumption that monitoring is perfect is not essential; the key point is that monitoring gives more precise information about the firm’s type, and that fraud distorts the information from monitoring relatively less than it distorts the free signal.
4 Investor and Firm Behavior

In this section, we analyze the equilibrium actions of the firm’s manager (henceforth, “firm”) and of the investor. As we will see, the incidence of fraud is hump-shaped, first increasing in the prior probability that firms are good, then decreasing. When this prior probability is below the point at which fraud reaches its peak, fraud increases as monitoring decreases; when the prior is above this peak, fraud and monitoring decrease together. Most importantly, whenever monitoring is feasible, the peak in fraud occurs for good priors — those for which the average net present value of a firm’s project is positive — and this peak shifts towards higher priors as monitoring costs decrease. In this sense, fraud is associated with “good times.”

Our analysis proceeds via backwards induction. We begin with the investor’s problem once she has observed the free signal; then we examine the firm’s decision on whether to commit fraud before the free signal is sent. We conclude by characterizing the equilibrium levels of fraud and monitoring as functions of the prior probability that firms are good.

4.1 The Investor’s Ex-Post Problem

After receiving the free signal $s$, the investor has three actions: she can choose not to invest (action “$N$”); she can monitor and then invest if the firm is good (action “$M$”); or she can invest without further monitoring (action “$U$” for unmonitored). Defining $V_A$ as the expected payoff to action $A$, these three actions’ expected payoffs are as follows.

$$V_N = 0$$
$$V_M = \mu N_g - m$$
$$V_U = \mu N_g - (1 - \mu) N_b$$

It is immediate that the investor’s decision depends only on the net present values $N_g$ and $-N_b$ of the two types of firms, the cost of monitoring $m$, and the investor’s posterior belief on the probability $\hat{\mu}$ that the firm is good. For expositional ease, we define the following threshold probabilities: If $\hat{\mu} = \frac{m}{N_g} \equiv \mu_1(m)$ then $V_N = V_M$; if $\hat{\mu} = \frac{N_u}{N_b + N_g} \equiv \mu_2$ then $V_N = V_U$; and if $\hat{\mu} = 1 - \frac{m}{N_b} \equiv \mu_3(m)$ then $V_M = V_U$. The next proposition describes the parameter regions in
which the various actions are optimal.

**Proposition 1** Suppose that, after observing the free signal, the investor believes that the firm is good with probability $\hat{\mu}$. The investor’s optimal action is as follows:

1. Do not invest if $\hat{\mu} < \min (\mu_1(m), \mu_2)$.

2. Invest without monitoring if $\hat{\mu} \geq \max (\mu_2, \mu_3(m))$.

3. Monitor and invest if the firm is good if $\mu_1 < \hat{\mu} \leq \mu_3(m)$ and $m < \frac{N_b N_g}{N_b + N_g} \equiv \overline{m}$.

![Figure 2: Posterior probabilities and optimal investor decisions.](image)

Figure 2 displays key elements of the investor’s decision problem. Given the realization of the free signal, the investor updates her beliefs about the firm’s type. Together, the posterior $\hat{\mu}$ and the cost of monitoring $m$ determine the optimal decision. If the cost of monitoring is above $\overline{m}$, then $\min (\mu_1(m), \mu_2) = \max (\mu_2, \mu_3(m)) = \mu_2$ and monitoring is always dominated either by not investing at all or by unmonitored financing. Here, the investor provides unmonitored finance if and only if the posterior is above the threshold $\mu_2$, which determines where the investor is indifferent between not investing and unmonitored financing.

For monitoring costs below $\overline{m}$, it is possible that the expected benefit from monitoring (avoiding investing in bad firms and losing $N_b$) may exceed the cost of monitoring $m$. If $\hat{\mu}$ is
such that $m = \hat{\mu} N_g$ (the upward sloping line in Figure 2), we have $V_N = V_M$, and the investor is indifferent between monitored finance and not investing. For example, if $m = m'$, the threshold for $\hat{\mu}$ is $\mu_1(m')$. If $\hat{\mu}$ is such that $m = (1 - \hat{\mu}) N_b$ (the downward sloping line), we have $V_M = V_U$, and the investor is indifferent between monitored finance and unmonitored finance. For the example $m = m'$, this defines the threshold $\mu_3(m')$. It follows that monitoring is optimal for intermediate posteriors, and the range of posteriors for which it is optimal increases as the cost of monitoring $m$ decreases.

Note that the investor’s decision depends only on the posterior $\hat{\mu}$, and not on how she forms this posterior; different combinations of the prior $\mu$ and the probability of fraud $\phi$ that lead to the same posterior $\hat{\mu}$ lead to the same action.

### 4.2 The Manager’s Decision to Commit Fraud

Having dealt with the investor’s problem, we now examine the bad firm’s decision on whether to commit fraud. This decision depends on the cost of fraud versus the expected benefit of fraud, which in turn depends on the investor’s response as described in Proposition 1. Since monitoring detects bad firms, the firm only benefits from fraud if fraud increases the firm’s probability of receiving unmonitored funding. This requires two conditions: (i) after a high signal, the investor’s posterior leads her to provide unmonitored funding with positive probability, and (ii) after a low signal, the investor’s posterior leads her to provide unmonitored funding with strictly lower probability than in the high-signal case. On the other hand, as mentioned in the previous section, in equilibrium, fraud makes the signal less precise; this lessens the difference in impact between high and low signals, reducing the gains from fraud.

In equilibrium, the incidence of fraud must be consistent with incentives. Thus, if the manager’s expected benefit strictly exceeds the cost $f$, she undertakes fraud with certainty ($\phi = 1$). If the benefit equals the cost, she is willing to commit fraud with positive probability ($0 < \phi < 1$). Otherwise, she does not commit fraud at all.

We first describe five different ‘regimes’ which characterize the equilibrium; which regime is relevant depends on the prior $\mu$ and on the cost of monitoring $m$. Define

$$\mu_{UF} = \max \{ \mu_3(m), \mu_2 \}.$$
From Proposition 1, $\mu_{UF}$ is the posterior at which the investor is indifferent between investing without monitoring and some other action. As noted above, unmonitored investment is critical to fraud. If the posterior is always above $\mu_{UF}$, there is no point to committing fraud; bad firms always get funding regardless of the signals they send. Similarly, if the posterior is always below $\mu_{UF}$, there is also no point to committing fraud; because firms never get funding without being monitored, bad firms cannot get funding regardless of the signals they send. Thus $\mu_{UF}$ is the key to equilibrium behavior, as we now show.

The regimes are defined as follows (the names are motivated by the results of Proposition 2 below).

1. The Fund-Everything Regime: 
   \[
   \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{1-\gamma}{1-\beta}\right)^{-1} \leq \mu < 1.
   \]

2. The Optimistic Regime: 
   \[
   \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{1-\gamma}{1-\beta-\delta}\right)^{-1} \leq \mu < \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{1-\gamma}{1-\beta}\right)^{-1}.
   \]

3. The Trust-Signals Regime: 
   \[
   \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{\gamma}{\beta+\delta}\right)^{-1} \leq \mu < \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{1-\gamma}{1-\beta-\delta}\right)^{-1}.
   \]

4. The Skeptical Regime: 
   \[
   \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{\gamma}{\beta}\right)^{-1} \leq \mu < \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{\gamma}{\beta+\delta}\right)^{-1}.
   \]

5. The No-Trust Regime: 
   \[
   0 < \mu < \left(1 + \frac{1-\mu_{UF}}{\mu_{UF}} \frac{\gamma}{\beta}\right)^{-1}.
   \]

There are two cases. In one case, monitoring is prohibitively costly, i.e. $m > \bar{m}$; in the other, $m < \bar{m}$, and the investor may monitor in equilibrium. We begin with the case where monitoring is possible.

**Proposition 2** Assume that the monitoring cost $m \leq \bar{m} = \frac{N_b N_g}{N_b + N_g}$. Denote by $\lambda_s$ the probability of monitoring with a signal $s$, by $\kappa_s$ the probability of unmonitored finance with a signal $s$, and by $\phi$ the bad firm’s probability of committing fraud. The equilibrium decisions are as follows:

1. **Fund-Everything Regime.** The investor never monitors ($\lambda_h = \lambda_\ell = 0$), all firms are funded regardless of the signal ($\kappa_h = \kappa_\ell = 1$), and there is no fraud ($\phi = 0$).

2. **Optimistic Regime.** High-signal firms are always funded without monitoring ($\lambda_h = 0$ and $\kappa_h = 1$). Low-signal firms are funded without monitoring with probability $\kappa_\ell = 1 - \frac{f}{\delta C}$ and are monitored otherwise ($\lambda_\ell = \frac{f}{\delta C}$). Bad firms commit fraud with probability $\phi = \frac{1}{\delta} \left(1 - \beta - \frac{\mu}{1-\mu} \frac{m}{N_b-m} (1-\gamma)\right)$. 

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3. **Trust-Signals Regime.** High-signal firms are always funded without monitoring \((\lambda_h = 0\) and \(\kappa_h = 1)\). Low-signal firms are never funded without monitoring \((\kappa_\ell = 0)\). Bad firms always commit fraud \((\phi = 1)\).

4. **Skeptical Regime.** High-signal firms are funded without monitoring with probability \(\kappa_h = \frac{f}{\delta C}\) and are monitored otherwise \((\lambda_h = 1 - \frac{f}{\delta C})\). Low-signal firms are never funded without monitoring \((\kappa_\ell = 0)\). Bad firms commit fraud with probability \(\phi = \frac{1}{\delta} \left( \frac{\mu - m}{1 - \mu N_b - m} \gamma - \beta \right)\).

5. **No-Trust Regime.** Firms are never funded without being monitored \((\kappa_h = \kappa_\ell = 0)\) and there is no fraud \((\phi = 0)\).

**Proof.** See the Appendix.

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**Figure 3: Five Regimes.**

Figure 3 shows which \((\mu, m)\) pairs fall into each regime, both for the case where monitoring is feasible, as described in the preceding proposition, and for the case where monitoring is prohibitively expensive, as described in Proposition 3 below. The darker shaded region consists of all \((\mu, m)\) pairs for which bad firms find it optimal to commit fraud with certainty. In the
lighter shaded regions, bad firms commit fraud with probability strictly between zero and one. In the unshaded regions, there is no fraud at all.

Figure 3 is related to Figure 2, which shows the details of the investor’s ex-post decision problem; the dashed lines in Figure 3 correspond to the solid lines in Figure 2. From the figure, it is clear that fraud takes place in a region centered on $\mu_{UF} = \max\{\mu_3(m), \mu_2\}$, the posterior belief at which the investor is just indifferent to providing unmonitored finance. Intuitively, if the prior is close to this indifference point, the prior uncertainty over whether the firm should receive unmonitored finance or not is greatest. This means that the signal’s outcome has the greatest effect on whether the investor provides unmonitored finance or not: a high signal is most likely to lead to a different outcome from a low signal, which is when incentives for fraud are highest.

Analytically, the results in Proposition 2 follow from the regime definitions that precede the proposition; these are given in terms of the prior $\mu$ and $\mu_{UF}$. (Note that in the case of Proposition 2, $\mu_{UF}$ equals $\mu_3(m)$; i.e., when the investor is indifferent to providing unmonitored finance, her relevant choice is between monitoring and not monitoring.) The expressions for the boundaries of the regimes are derived from critical values of $\hat{\mu}_s(\phi)$, which again is the investor’s posterior belief that the firm is good after seeing the free signal $s$ and assuming that the bad firm commits fraud with probability $\phi$.

As an example, in the Fund-Everything regime, the prior $\mu$ is so high that even a low signal is very likely to have come from a good firm. Specifically, we have $\mu_{UF} = \mu_3(m) \leq \hat{\mu}_\ell(1)$: even after seeing a low signal, and even if bad firms commit fraud with probability one, the investor is willing to extend unmonitored finance to the firm. Using the definition of $\hat{\mu}_\ell(1)$ and rearranging yields the condition given in the definition. Since all firms receive unmonitored finance regardless of the public signal, there is no benefit from committing fraud in this regime.

In the Optimistic regime, either the prior $\mu$ or the cost of monitoring $m$ is somewhat lower, so that $\hat{\mu}_\ell(0) < \mu_3(m) < \hat{\mu}_\ell(1)$. Here, a high signal still leaves the investor choosing to fund the firm without monitoring, but a low signal is bad enough that the investor prefers to monitor with some probability.\(^9\) In this regime, monitoring actually encourages fraud, since bad firms that produce a low signal may be monitored and denied funding.

---

\(^9\) More precisely, if there were no chance of fraud in equilibrium, the investor would strictly prefer to monitor after a low signal; if there were fraud with certainty, the investor would strictly prefer to not monitor; thus, in equilibrium, the investor monitors with probability between 0 and 1.
In the Trust-Signals regime, \( \tilde{\mu}_\ell (1) < \mu_3 (m) < \tilde{\mu}_h (1) \). Here, only high signals receive unmonitored finance; low signals are either monitored or rejected.\(^\text{10}\) Either way, bad firms have no chance of being financed if they produce a low signal, so their incentive to commit fraud is higher than it would be in the Optimistic regime. In this regime, bad firms commit fraud with certainty.

With lower values of \( \mu \) or \( m \), we enter the Skeptical regime, where \( \tilde{\mu}_h (1) < \mu_3 (m) < \tilde{\mu}_h (0) \). The priors in this regime are low enough that the investor finds it optimal to monitor even high signals with positive probability. Because the bad firm may not get financing even if it manages to obtain a high signal, the gains from fraud are lower than those in the Trust-Signals regime. Thus, bad firms commit fraud with probability strictly less than one.

Finally, for very low values of \( \mu \), we have \( \tilde{\mu}_h (0) < \mu_3 (m) \). In this No-Trust regime, the investor’s prior is so low that all firms are either monitored or rejected, regardless of the signal. Since there is no unmonitored finance, there is no gain to committing fraud, and so there is no fraud in equilibrium.

Next, we turn to the case where monitoring costs are so high that monitoring never pays (that is, \( m > \overline{m} \)). The regimes described in Proposition 2 extend to this case in a natural way (see Figure 3):

**Proposition 3** Assume that the monitoring cost \( m > \overline{m} = \frac{N_b N_g}{N_b + N_g} \), so that the investor never monitors. Denote by \( \kappa_s \) the probability of unmonitored finance with a signal \( s \), and by \( \phi \) the bad firm’s probability of committing fraud. The equilibrium decisions are as follows:

1. **Fund-Everything Regime.** All firms are funded regardless of the signal \( (\kappa_h = \kappa_\ell = 1) \), and there is no fraud \( (\phi = 0) \).

2. **Optimistic Regime.** High-signal firms are always funded \( (\kappa_h = 1) \). Low-signal firms are funded with probability \( \kappa_\ell = 1 - \frac{1}{\delta C} \) and denied funding otherwise. Bad firms commit fraud with probability \( \phi = \frac{1}{\delta} \left[ 1 - \beta - \frac{\mu - \mu_3 N_g}{1 - \mu N_b} (1 - \gamma) \right] \).

3. **Trust-Signals Regime.** High-signal firms are always funded \( (\kappa_h = 1) \). Low-signal firms are never funded \( (\kappa_\ell = 0) \). Bad firms always commit fraud \( (\phi = 1) \).

\(^{10}\) The choice depends on whether or not \( \tilde{\mu}_\ell (0) \) exceeds \( \mu_1 (m) \).

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4. Skeptical Regime: High-signal firms are funded without monitoring with probability \( \kappa_h = \frac{1}{\delta C} \) and denied funding otherwise. Low-signal firms are never funded \( (\kappa_\ell = 0) \). Bad firms commit fraud with probability \( \phi = \frac{1}{3} \left( \frac{\mu}{1-\mu} \frac{N_a}{N_b} \gamma - \beta \right) \).

5. No-Trust Regime: firms are never funded \( (\kappa_h = \kappa_\ell = 0) \) and there is no fraud \( (\phi = 0) \).

**Proof.** See the Appendix.

If \( m > \overline{m} \), monitoring is prohibitively expensive, and the investor either rejects the firm or provides unmonitored financing. The five regimes are analogous to those in Proposition 2. One key difference is that if a regime calls for monitoring when \( m \leq \overline{m} \), it calls for denying funding when \( m > \overline{m} \). Another key difference is that when \( m > \overline{m} \), the critical level \( \mu_{UF} \) equals \( \mu_2 \), which does not depend on the monitoring cost \( m \). As a result, the boundaries of the five regimes are constant in \( m \), as can be seen from Figure 3. We will return to the implications of this shortly.

Our next result is a straightforward consequence of Propositions 2 and 3.

**Proposition 4** Both the probability of fraud \( \phi \) conditional on the firm being bad, and the ex-ante probability of fraud \( (1-\mu)\phi \) are hump-shaped in the prior \( \mu \). There is no fraud for the highest and lowest levels of \( \mu \), the Fund-Everything and No-Trust regimes. In the Skeptical regime the probabilities of fraud are increasing in \( \mu \), while in the Optimistic regime they are decreasing. In the Trust-Signals regime, the conditional probability is constant, while the ex-ante probability is decreasing in \( \mu \).

**Proof.** See the Appendix.

Figure 4 shows the conditional and ex-ante probabilities of fraud. The graphs consist of five parts, corresponding to the five regimes described above. In the Skeptical regime, the probabilities increase with \( \mu \). High-signal firms are monitored or denied funding with positive probability, low-signal firms with certainty. Thus the investor is indifferent between monitoring (or denying funding to) high-signal firms and funding them without any further information. All else equal, an increase in the prior \( \mu \) makes the investor strictly unwilling to monitor (or deny funding to) high-signal firms — but then the bad firm would prefer to commit fraud with certainty, worsening the pool of high-signal firms and destroying equilibrium. In equilibrium, the probability of fraud must increase so as to restore balance.
In the Optimistic regime, the probability of fraud decreases with $\mu$. The investor strictly prefers to fund high-signal firms, and is indifferent between monitoring (or denying funding to) low-signal firms and funding them without further information. Here, an increase in the prior makes the investor strictly prefer to fund low-signal firms without monitoring — but then bad firms would have no reason to commit fraud, worsening the pool of low-signal firms and destroying equilibrium. In equilibrium, the probability of fraud decreases so as to restore balance.

The preceding discussion accounts for the results on the bad firms’ conditional probability of fraud $\phi$. The results on the ex ante probability of fraud $(1 - \mu)\phi$ follow immediately.

The last issue we consider in this section has to do with where fraud is most likely — i.e., where the “hump” has its peak. As we discussed following Proposition 2, the region in $(\mu, m)$ space where fraud incentives are highest centers around the line given by $\mu = \mu_{UF}$. When monitoring costs exceed $\overline{m}$, so that monitoring is not feasible, this is a vertical line at $\mu_2$, the prior at which the investor is indifferent between not financing the firm and extending unmonitored financing. But this indifference means that the ex ante expected net present value of a firm is zero. Thus, when monitoring is not feasible, fraud is most prevalent in “so-so” times.

Matters are very different when monitoring costs are low enough that monitoring is sometimes feasible. In this case, $\mu_{UF}$ equals $\mu_3(m)$, which is a downward-sloping line. This means that when monitoring costs fall, the region where fraud is highest shifts towards higher and higher priors. In other words, an association between fraud and “good times” depends on...
investors being able to monitor, and this association is stronger as monitoring costs are lower.

Finally, although our analysis focuses on how fraud changes as the prior probability that a firm is good changes, we obtain similar results if this prior is held fixed and instead the return of a successful firm $R$ changes. It is easy to show that when $R$ is so low that a good firm’s net present value $N_g$ is only slightly positive (and so a bad firm’s negative net present value $N_b$ is large), investors will be cautious even after a high signal. As $R$ increases, eventually investors begin to fund high-signal firms without monitoring, at which point fraud starts to occur; further increases in $R$ lead to the same hump-shaped pattern of fraud that we have already described.\textsuperscript{11} Thus, even if one defines “bad times” and “good times” in terms of the expected return to any given firm rather than the relative numbers of good and bad firms, our predictions still hold.

5 Determinants of Fraud

Having established the properties of equilibria in the various regimes, we now turn to the question of how various parameters affect the incidence of fraud. We show that, while certain results are constant across regimes, others depend heavily on whether the regime is Skeptical or Optimistic. In particular, the Skeptical regime is the more intuitive case; here, monitoring discourages fraud, and other parameter effects are as one would expect. By contrast, the Optimistic regime is counterintuitive; here, monitoring encourages fraud, and several parameter effects are the reverse of what one would expect. We discuss the practical implications of these results. Finally, we discuss how our model’s implications are affected by dynamic considerations.

We begin with the comparative statics of the Skeptical regime.

**Proposition 5** In the Skeptical regime,

(i) The equilibrium probability that bad firms commit fraud ($\phi$) is increasing in the prior $\mu$, weakly increasing in the cost of monitoring $m$, and decreasing in the efficacy of fraud $\delta$.

(ii) If the monitoring cost is low ($m \leq \overline{m}$), then the equilibrium probability that high-signal firms are monitored ($\lambda_h$) is decreasing in the cost of fraud $f$ and increasing in both the efficacy

\textsuperscript{11} Briefly, an increase in $R$ decreases $\mu_2$ and $\mu_3(m)$, and thus $\mu_{UF}$ as well. From the definitions of the five regimes preceding Proposition 2, the boundaries of the regimes are all increasing in $\mu_{UF}$. Thus, an increase in $R$ shifts all regimes to the left in $(\mu, m)$ space, which means that for a fixed prior $\mu$, the regime “improves.” For example, if initially the equilibrium is No-Trust, increasing $R$ leads first to the Skeptical regime, then to the Trust-Signals regime, and so forth.
of fraud $\delta$ and in the level of private benefits $C$. If the monitoring cost is high ($m > \overline{m}$), then the equilibrium probability that high-signal firms are denied funding is decreasing in the cost of fraud $f$ and increasing in both the efficacy of fraud $\delta$ and in the level of private benefits $C$.

The intuition for part $(i)$ of the proposition follows from the effects of parameter changes on the investor’s incentives to monitor the pool of firms that generate high signals. An increase in the prior probability that firms are good improves the pool, lowering the investor’s incentives to monitor or deny funding. This allows the probability that bad firms commit fraud ($\phi$) to increase until equilibrium is restored. An increase in the efficacy of fraud has the opposite effect. Finally, if monitoring costs are sufficiently low ($m \leq \overline{m}$), an increase in the cost of monitoring directly lowers the investor’s monitoring incentives, again allowing the probability of fraud to increase. (If $m > \overline{m}$, the investor never monitors, so changes in $m$ have no effect on the probability of fraud.)

The intuition for part $(ii)$ of the proposition is straightforward. The probability of monitoring or funding denial is determined by the bad firm’s incentive condition – the point at which it is indifferent between committing fraud and not committing fraud. If the cost of fraud increases, then fraud is less attractive, and less intensive monitoring or less frequent funding denial suffices to deter fraud to the point of indifference. Higher private benefits make getting funded more attractive. Because generating a high signal is the only way that a bad firm has a chance of getting funded, fraud is more attractive, and again more intensive monitoring or funding denial is needed. Finally, if fraud is more effective, the pool of high-signal firms worsens, all else equal, and more intensive monitoring or funding denial is needed to restore balance.

As noted above, the Skeptical regime is the intuitive case. The investor’s decision about partial monitoring or funding denial focuses on firms with high signals, and fraud gives a bad firm a higher chance of entering this pool and getting funding. This leads to a direct link between the intensity of monitoring or funding denial and fraud incentives. By contrast, the Optimistic case is less intuitive. Here, partial monitoring or funding denial focuses on firms with low signals, and fraud gives a bad firm a higher chance of exiting this pool by generating a high signal and getting automatic funding. Thus, the link between the intensity of monitoring and fraud incentives is now less direct. This can be seen in the following proposition.
Proposition 6  In the Optimistic regime,

(i) The equilibrium probability that bad firms commit fraud ($\phi$) is decreasing in the prior $\mu$ and the efficacy of fraud $\delta$, and weakly decreasing in the cost of monitoring $m$.

(ii) If the monitoring cost is low ($m \leq \overline{m}$), then the equilibrium probability that low-signal firms are monitored ($\lambda_\ell$) is increasing in the cost of fraud $f$ and decreasing in both the efficacy of fraud $\delta$ and in the level of private benefits $C$. If the monitoring cost is high ($m > \overline{m}$), then the equilibrium probability that low-signal firms are denied funding is increasing in the cost of fraud $f$ and decreasing in both the efficacy of fraud $\delta$ and the level of private benefits $C$.

As before, part (i) of the proposition follows from the effects of parameter changes on the investor’s incentives to tighten funding (i.e., monitor or deny funding, depending on whether or not $m \leq \overline{m}$) for the pool of firms with low signals. An increase in the prior probability that a firm is good increases the fraction of low-signal firms that are good, reducing the investor’s incentives to tighten funding. Since a reduction in monitoring or funding denial makes fraud less attractive (bad firms are more likely to be funded even if they get a low signal), the probability of fraud falls until incentives are restored. An increase in the probability that bad firms can generate high signals through fraud ($\delta$) also increases the fraction of low-signal firms that are good, discouraging fraud. Finally, if monitoring costs are sufficiently low ($m \leq \overline{m}$), an increase in the cost of monitoring directly lowers the investor’s monitoring incentives, discouraging fraud.

Part (ii) follows from the effects of parameter changes on the bad firm’s incentives to commit fraud. The difference is that now, more intensive monitoring or more frequent funding denial decreases the probability that a bad firm with a low signal gets funded, and so tighter funding encourages bad firms to commit fraud so as to improve their odds of generating high signals. When fraud is more costly, fraud is less attractive, so more of the low-signal firms are in fact bad firms, and tighter funding is required to restore equilibrium. Conversely, since more effective fraud or higher private control benefits increase the quality of the pool of low-signal firms, looser funding is required to restore equilibrium.

Thus far, we have not addressed the impact of changes in the base signal’s precision (that is, the signal’s precision in the absence of fraud). The following proposition shows that an increase in this precision always tends to increase the incidence of fraud.
Proposition 7 Suppose that the precision of the base signal improves, so that the probability that good firms send high signals ($\gamma$) increases, or the base probability that bad firms send high signals ($\beta$) decreases, or both.

(i) In both the Skeptical and the Optimistic regimes, the equilibrium probability that bad firms commit fraud ($\phi$) increases.

(ii) The regimes in which fraud occurs expand, encompassing more prior beliefs $\mu$. Specifically, the maximal fraud (Trust-Signals) regime expands, beginning at a lower $\mu$ and ending at a higher $\mu$. The Skeptical regime also begins at a lower prior $\mu$, and the Optimistic regime ends at a higher prior $\mu$.

These results follow from the effect of improved signal precision on investor behavior and thus on bad firms’ incentives to commit fraud. An increase in the probability that good firms generate high signals improves the pool of firms that have high signals and worsens the pool of firms that have low signals. A decrease in the base probability that bad firms generate low signals has similar effects. All else equal, such a change decreases investors’ incentives to monitor or deny funding to high-signal firms and increases their incentives to monitor or deny funding to low-signal firms. This in turn tends to increase bad firms’ incentives to commit fraud. Intuitively, a more precise signal means that the bad firm has more chance of generating a bad signal and then losing funding, but more chance of getting funding if it does generate a high signal; this gives it more incentive to try to commit fraud, “noising up” the signal.

In the Trust-Signals regime, the change in investor incentives does not affect behavior; investors already fund all high-signal firms and never fund low-signal firms without monitoring, so the fraud incentives of bad firms are already maximized (probability of fraud $\phi$ equals 1). By contrast, in the Skeptical regime, the probability that high-signal firms are funded without monitoring increases; this makes generating a high signal through fraud strictly more attractive, and the probability that bad firms commit fraud increases until equilibrium is restored. Similarly, in the Optimistic regime, the probability that low-signal firms are given funding without monitoring decreases; this too makes generating a high signal through fraud strictly more attractive. This explains the results in part (i) of the proposition.

The results in part (ii) follow similar logic. At the outer boundaries of the fraud regimes, improved signal precision introduces a difference in investors’ willingness to extend unmoni-
rored funding to high-signal firms versus low-signal firms, creating an incentive for fraud where none previously existed. Similarly, within the Skeptical and Optimistic regimes, the increase in incentives for fraud expands the set of priors for which these incentives are maximized, expanding the Trust-Signals regime. The upshot is that improved signal precision increases both the set of economic conditions under which fraud occurs and the probability with which it occurs.\footnote{Although our focus is on how increased signal precision increases the incidence of fraud ($\phi$), it is worth noting that, overall, bad firms may be more or less likely to get funded. Within a given regime, an increase in $\gamma$ weakly increases the overall probability that a bad firm is funded, whereas a decrease in $\beta$ weakly decreases this. If the increase in precision causes a regime shift, however, matters are more complex, because the probability $\kappa_s$ that a firm with signal $s$ gets unmonitored funding shifts discontinuously.}

5.1 Implications

We now turn to some direct implications of our model. Perhaps the most striking result is the way that many parameter changes have opposite effects depending on whether the equilibrium is Skeptical or Optimistic. As already suggested, this occurs because of the differing focus of investor scrutiny in these two regimes. In “skeptical” times, investors strictly prefer to be “tough” (monitor or deny funding) with low-signal firms, but they are somewhat “looser” with high-signal firms. As a result, changes in parameters affect investors’ behavior with high-signal firms but not with low-signal firms. The opposite is true in “optimistic” times: now, investors strictly prefer to fund high-signal firms, but they apply somewhat tougher standards to low-signal firms. In this case, changes in parameters affect investors’ behavior with low-signal firms but not with high-signal firms, and so the effects of many parameter changes switch sign.

The results on monitoring in the Optimistic regime seem counterintuitive because we tend to think of monitoring as focusing on detecting fraud. Of course, our model is very stylized, but the underlying point is an important one: monitoring by investors is directed at finding good investment opportunities, not detecting fraud per se. In the Optimistic regime, the chance that a high signal comes from a good firm outweighs the chance that it comes from a bad firm that has committed fraud. As a result, investors begin to loosen funding standards for low-signal firms. Changes that further loosen these standards actually discourage fraud because bad firms see less need for it – why commit fraud when you can get funded without it?

Similarly counterintuitive results arise from changes in the underlying prior that firms are good. In the Skeptical regime, an increase in this prior loosens funding standards and encourages
fraud, which is what we think of as normal behavior. By contrast, in the Optimistic regime, an increase in the prior loosens funding standards and discourages fraud. Again, if investors are sufficiently optimistic, there is less need for fraud to attain funding.

This last result may provide a partial explanation for what happened during the 1990s boom; arguably, as information technology improved, it became easier for analysts and others to “kick the tires” — but during the boom these efforts were concentrated on firms that were known as poor performers. Perversely, this may have increased the prevalence of fraud.

Another implication comes from the result that we discussed at the end of the previous section: namely, as the cost of monitoring falls, the region where fraud occurs shifts towards better prior beliefs. This suggests that as telecommunications and information processing costs have come down, the incidence of fraud may be even more tilted towards better states of the world.

Our model is also consistent with differences in lending behavior across the business cycle. The literature on credit cycles shows that lenders are more willing to make “Type I” errors (rejecting or rationing good credits) in recessions, and more willing to make “Type II” errors (lending to bad credits) in expansions. This is consistent with broad differences between the No-Trust and Skeptical regimes on the one hand and the Optimistic and Fund-Everything regimes on the other. Although our model is not unique in predicting this result, it serves as a useful reality check.

More interestingly, our results also have applications to the prevalence of fraud in different sectors. In the late 1990s, Internet or “dot-com” firms were viewed as “can’t miss” opportunities, because of a widespread conviction that much conventional business would migrate to the Internet in a relatively short period of time. Leaving aside the question of whether so strong a conviction was rational, this view led to the financing of many start-ups that did not even have business plans (see e.g. Schenone, 2003). Yet there have been few accusations of fraud directed at the Internet firms. By contrast, the telecoms sector, though viewed very positively, was not the subject of such strong optimism in the 1990s. Recently, numerous large telecoms firms (including WorldCom, Qwest, Global Crossing, and Lucent) have been accused of fraudulent or misleading accounting. This difference is consistent with our model: Internet firms may have fallen into or close to the Fund-Everything regime, in which case there was no need to commit fraud, whereas the telecoms may have fallen into the lower Optimistic regime, in which case
fraud should have been expected.

Although our analysis suggests that there is an interesting contrast between “optimistic” and “skeptical” regimes, some parameter effects are the same in both. In particular, the probability of fraud increases in the probability $\gamma$ that good firms send high signals and decreases in the probability $\beta$ that bad firms send high signals and in the efficacy of fraud $\delta$. As discussed above, changes in these “signal quality” parameters change the pool of high- and low-signal firms in such a way that they have consistent effects on bad firms’ choice between committing fraud and not committing fraud. An improvement in the precision of the “base” signal (i.e., an increase in $\gamma$ and decrease in $\beta$) increases the prevalence of fraud. By contrast, an increase in the efficacy of fraud $\delta$ makes investors pay less attention to the free signal, and so incentives for fraud decrease.

The signal quality results also have implications for policy makers. Suppose that regulators decide to toughen disclosure standards. If tougher disclosure means releasing more details that give investors a better sense of the firm’s situation, bad firms will be less able to get funding unless they fraudulently alter their results. Something of this sort may have happened in the 1990s. The general trend throughout the decade was for annual reports to release more and more details in the notes to the financial statements, in large part in response to demands for greater revelation from the Financial Accounting Standards Board (FASB). Although many complained that notes were becoming denser, the point is that audited information that was not previously available was now disclosed. In the absence of fraud or misrepresentation, investors could now do a better job of assessing a firm’s situation — and so a number of firms began to game the system, in many cases crossing the line into fraud. Thus, tougher disclosure laws can have the perverse effect of increasing fraud.\(^{13}\) To be effective against fraud, disclosure laws must directly make fraud more difficult.

5.2 Dynamic Considerations

Up until now, we have assumed that investors and firms know the prior distribution of firm types without uncertainty. In practice, such priors are likely to be uncertain, since the “true” state of the economy can only be known ex post, if at all. Moreover, the true state of the

\[^{13}\text{Furthermore, as noted above, increased precision does not necessarily reduce the overall probability with which a bad firm is funded, so the overall effect of tougher disclosure on funding efficiency can be mixed.}\]
economy is dynamic, which can complicate the inference problems of investors and managers. As suggested in the introduction, these considerations can exacerbate the links between fraud, booms, and busts.

To model these issues in a simple way, we assume that there are two possible true states of the economy, one in which there are relatively many good firms (fraction $\mu_u$ of all firms) and one in which there are relatively few good firms (fraction $\mu_d$ of all firms, with $\mu_d < \mu_u$). Furthermore, we assume that $\mu_u$ falls into the Fund-Everything regime, and $\mu_d$ falls into the No-Trust regime. The true state cannot be observed, and all agents share common beliefs: the probability that the state is $\mu_u$ is $p_0$. It follows that the overall prior that any given firm is good is $\mu = p_0\mu_u + (1 - p_0)\mu_d$.

First suppose that $p_0$ is low. In this case, the ex-ante prior $\mu$ is low, corresponding to either the No-Trust or (low) Skeptical regime. Bad firms are unlikely to commit fraud in this case, since even high-signal firms are usually monitored before they are financed. If, ex post, the true state of the economy proves to be $\mu_d$, there will be slightly more bad firms than expected, but the overall incidence of fraud will still be low or nonexistent. If instead the true state proves to be $\mu_u$, there will be even fewer cases of fraud, funded projects will be relatively successful, and investors’ conservatism may seem overblown, as more monitored projects than expected will prove to be good.

Now suppose $p_0$ is high, so that the ex-ante prior $\mu$ falls within the Trust-Signals or Optimistic regime. Although bad firms will be committing fraud, if the true state later proves to be $\mu_u$, there will not be many bad firms, and the actual incidence of fraud will be somewhat lower than expected. By contrast, if the true state proves to be $\mu_d$, the numbers of bad firms and fraud cases will be much higher than expected.

If the prior is higher still, of course, the equilibrium will fall into the upper end of the Optimistic regime or even the Fund-Everything regime. In this case, fraud will be low or nonexistent, even if the state proves to be $\mu_d$, but in this last case many more funded projects than expected will perform poorly.

All of this has taken $p_0$ as given. In reality, $p_0$ will arise from investors getting signals from various firms and from some “actual” realizations (e.g., realized cash flows in our model). Note that the presence of fraud slows down updating in both directions: both high and low signals become noisier. Thus, priors will be slower to shift in the “middle,” where bad firms are likely
to commit fraud. If beliefs begin with a $p_0$ so high that the regime is *Fund-Everything*, and then some bad realizations of the free signal shift $p_0$ and thus $\mu$ into the *Optimistic* or *Trust-Signals* regime, further updating will be slowed.

If there were no change in the underlying state, then over time, investor beliefs would find their way to the true state. A more realistic assumption is that there is always some chance that the underlying state governing the returns on new projects can shift – some chance of transitioning from $\mu_d$ to $\mu_u$, and another chance of transitioning the other way. If by some chance beliefs do find their way close to one or the other extreme, there will always be some chance that the beliefs are “very wrong” due to a transition. Of course, these transition probabilities limit how high or low $p_0$ can go, but there is still a chance that beliefs will be heavily weighted towards one extreme or the other, in which case “surprises” of the sort already discussed will still be possible. In particular, once $p_0$ and thus $\mu$ are in the *Optimistic* regime, a period of slow updating from “free” signals (interim results) could be followed either by a reassuring string of high cash flows or a spate of low cash flows that suddenly reveal that the economy is in recession – followed in the last instance by a wave of revelations of fraud.

In short, the agents in an economy may be “surprised” by changes in the economy’s fundamentals. Although this notion is not especially surprising, it has strong implications for the incidence and prevalence of fraud across the business cycle. As noted, when times are bad — in terms of our model, in the *No-Trust* or *Skeptical* regimes — positive surprises will lead to lower amounts of fraud than expected. The opposite is true when times are good; now surprises lead to higher-than-expected fraud.

It is also important to note that, in the last case, even fraudulent firms are surprised by the extent of fraud. Although they have private information that they are in bad shape, which is a somewhat negative signal for the economy as a whole, this is not the same as knowing that many firms are in bad shape. In a more complex model, this can lead to negative spillovers as firms with weak prospects who see others post high results feel more pressure to do so themselves, precisely because neither they nor investors know whether the others are committing fraud. Something of this sort seems to have happened in the case of WorldCom, whose fraudulent reporting in the 1990s increased the pressure on its rivals (Schiesel, 2002).
6 Robustness and Extensions

In order to streamline our exposition, our analysis has made use of several simplifying assumptions. In this section, we discuss the consequences of loosening three of these: the assumption that the relative numbers of good and bad firms are fixed exogenously, the assumption that the cost of fraud is fixed exogenously, and the assumption that only bad firms commit fraud. As we will see, allowing for endogenous entry or costs of fraud that depend on the probability of getting caught do not change the thrust of our results. Allowing good firms to commit fraud does not change most of our results, but sometimes causes complications that could be resolved in a richer model.

6.1 Allowing Entry and Exit of Firms

Our model has assumed that the distribution of good and bad firms – encapsulated in the prior \( \mu \), which is the proportion of good firms in the economy – is fixed exogenously. Based on the experience of the 1990s boom, however, one might argue that these numbers should be somewhat flexible, as changing beliefs lead to exit and entry by firms. For example, optimistic beliefs on the part of investors and managers should lead to more entry, especially by managers of bad firms. To the extent investors anticipate this, this should limit just how optimistic beliefs about the distribution of firms can be. Conversely, pessimistic beliefs should lead to exit, especially by bad firms; this would limit how pessimistic beliefs can be.

Suppose then that the initial distribution of firms is weighted towards good firms; to be specific, the initial proportion of good firms is \( \mu_0 \), where \( \mu_0 \) is in the Fund-Everything regime. Managers with bad potential projects should then enter the market, bearing any costs of seeking funds (getting matched with an investor) in the hopes of getting control benefits. If investor beliefs did not change, such entry would continue until the supply of potential bad firms is exhausted or the marginal bad firm has a cost of seeking funds equal to the control benefit \( C \). If investors are rational, however, such entry will depress their prior from \( \mu_0 \) to some lower \( \mu_1 \). The prior might fall enough to cross into the Optimistic regime or even lower, where firms are sometimes monitored or denied funding; this would lower the probability that bad firms could get funding, making entry less attractive and lowering the critical cost of seeking funds at which entry is just attractive.
At the other extreme, suppose that the initial distribution is weighted towards bad firms: \( \mu_0 \) falls into the No-Trust regime. In this case, either investors may choose to fund no one, or, if monitoring costs are sufficiently low, firms can only get funded if they are monitored first. In the first case, all firms would exit rather than incur costs of seeking funds, leaving the economy in autarky. In the second case, only bad firms would exit; this would raise the prior, possibly moving the economy into a regime where bad firms have some chance of being funded (and thus lowering the cost of seeking funds at which a bad firm is indifferent to exiting or staying in the market).

From this discussion, it is obvious that the number of potential bad firms and the distribution of their costs of seeking funds would be key factors. So long as the supply of firms (and especially bad firms) is somewhat inelastic, however, our main results would be unaffected: very optimistic or pessimistic beliefs might not be sustainable in equilibrium, but there would still be a range of equilibrium priors supporting the different regimes we have analyzed.

### 6.2 Explicit Detection of Fraud

We have assumed that fraud has a fixed cost \( f \). This is consistent with a model in which fraud has some fixed effort cost \( \varepsilon \), after which it may be detected by the authorities with fixed probability \( \alpha \) and fixed punishment (if caught) \( P \), such that \( \varepsilon + \alpha P = f \). In practice, however, this formulation is overly simplistic. If a firm is actually funded, transaction data is generated and future performance may be scrutinized and compared to earlier reports; thus, the authorities may find it easier to catch fraud committed by firms that are actually funded. Similarly, investors who monitor should have a better chance of detecting possible fraud than do investors who rely completely on the free signal.

Accordingly, suppose that the probability of being caught after committing fraud, \( \alpha \), varies directly with the probability that the (bad) firm is funded and with the probability that the firm is monitored: with probability \( \omega > 0 \), the regulatory authorities (such as the SEC) catch fraudulent firms that are funded without being monitored, and with probability one, investors catch fraudulent firms that they monitor. (Implicitly, we are assuming that the authorities cannot investigate all funded transactions.) Then the probability of being caught is

\[
\alpha = (\beta + \delta)(\kappa_h \omega + \lambda_h) + (1 - \beta - \delta)(\kappa_l \omega + \lambda_l),
\]

where once more \( \kappa_s \) is the probability of getting...
unmonitored funding when the free signal is $s$ and $\lambda_s$ is the probability of being monitored when the free signal is $s$. Given an effort cost $\varepsilon$ for committing fraud, it follows that the total cost of committing fraud is still $f = \varepsilon + \alpha P$, but now $\alpha$ depends on the probabilities with which firms are monitored and with which they are given unmonitored funding.

This does not affect the basic outline of our results. To see why, note that the firm’s decision to commit fraud is based on the gain from committing fraud versus the cost. In our simple model, the gain is the expected increase in the chance of getting unmonitored funding times the control benefit, or $\delta (\kappa_h - \kappa_\ell) C$; the cost is $f = \varepsilon + \alpha P$. It is easy to show that if $\delta C < \varepsilon + (\beta + \delta) \omega P$, the benefit $\delta (\kappa_h - \kappa_\ell) C$ is always less than the cost $\varepsilon + \alpha P$, so fraud is never attractive. Consistent with our emphasis before, we will assume that $\delta C$ exceeds $\varepsilon + (\beta + \delta) \omega P$ so that fraud is in fact possible.

First, consider the case where monitoring costs are so high that investors never monitor ($m > \overline{m}$). It is easy to show that, in this region, the boundaries of the five regimes are precisely as before. For example, in the No-Trust regime, $\kappa_h = \kappa_\ell = 0$, so there is no incentive to commit fraud. The boundary between this regime and the Skeptical regime is the point at which unmonitored funding for high-signal firms is just attractive, assuming the probability of fraud is zero; this occurs when the posterior $\hat{\mu}_h(0)$ satisfies $\hat{\mu}_h(0) = \mu_1(m)$. This condition does not depend on the cost of fraud, and so the boundary of the No-Trust regime is unaffected by the form of the cost of fraud.

Similarly, in the Fund-Everything regime, $\kappa_h = \kappa_\ell = 1$, so again there is no incentive to commit fraud. The boundary between this regime and the Optimistic regime is determined by the condition $\hat{\mu}_\ell(0) = \mu_3(m)$. Again, this condition does not depend on the cost of fraud, and so this boundary is unaffected by the form of the cost of fraud. Furthermore, the condition $\delta C > \varepsilon + (\beta + \delta) \omega P$ guarantees that it is feasible to have a Trust-Signals regime, and then arguments along the lines just given prove that the boundaries of the Skeptical, Trust-Signals, and Optimistic regimes will be as in Proposition 3. Nevertheless, in the Skeptical and Optimistic regimes the probabilities with which investors provide unmonitored finance will be affected by the form of the cost of fraud, since the probability $\alpha$ of detecting fraud depends on these probabilities.\textsuperscript{14}

\textsuperscript{14} In particular, in the Skeptical regime, the probability $\kappa_h$ with which high-signal firms are given unmonitored finance will be higher than it would be if the cost of fraud $f$ was equal to $\varepsilon$ alone. In the Optimistic regime, the probability $\kappa_\ell$ with which high-signal firms are given unmonitored finance will be lower than it would be if the
If monitoring costs are low enough to permit monitoring \((m < \bar{m})\), matters are slightly more complex. Now, the fact that investors who monitor always catch fraud may shift the lower boundary of the region where fraud occurs with some probability. To see why, note that if investors always monitored all firms in the No-Trust regime, then a manager who committed fraud would be caught for certain and thus face cost \(\varepsilon + P\). This may exceed the maximum benefit of fraud, which is \(\delta C\). In this case, investors might actually be able to scale back monitoring, providing high-signal firms with unmonitored finance some of the time, without provoking fraud. Eventually, if the probability of unmonitored finance is high enough (and so the probability of monitored finance is low enough), some fraud will be attractive. The upshot is that part of the Skeptical regime may now be free of fraud. Indeed, if \(P\) is high enough, even the Trust-Signals regime may be partially free of fraud, the reason being that monitoring of low-signal firms may be enough to deter fraud.

From this discussion, it follows that the main qualitative effect of having the cost of fraud reflect the probability of being caught (and, in particular, the probability of being monitored) is that when monitoring costs are sufficiently low, the region where fraud is possible may shrink further, with less fraud in regions with lower priors. This would reinforce the link between fraud and “good times.”

### 6.3 Good Firms and Fraud

We have assumed that only managers of bad firms commit fraud. We now discuss how our results would be affected if managers of good firms could commit fraud. In a nutshell, there would be little change in our results in four of the five regimes — Skeptical, Trust-Signals, Optimistic, and Fund-Everything — but behavior in the No-Trust regime might be affected.

To see this, suppose that a good firm can commit fraud at cost \(f'\), in which case its chance of producing a high signal goes from \(\gamma\) to \(\gamma + \delta'\), where \(f' \geq f\) and \(\delta' < \delta\). We assume that the cost of fraud is higher for good firms because, in a less stylized model, managers of good firms should have more to lose from being caught than managers of bad firms. For example, in a multiperiod setting, managers of good firms might find that being caught committing fraud cost of fraud \(f\) was equal to \(\varepsilon\) alone. Essentially, the possibility of catching fraud raises the cost of fraud; this means that the benefit \(\delta(\kappa_h - \kappa_l)C\) and thus the difference \(\kappa_h - \kappa_l\) must be larger in order to get the manager to be indifferent between committing fraud and not committing fraud.
ruins their chances of getting funding in the future—e.g., from SEC penalties. In that case, a good manager may be better off taking a higher chance of sending low signals now and not getting funding for current expansion, since she can return to the market the following period and try again. Similarly, we assume that fraud is more effective for bad firms because fraud should have a higher expected impact on bad firms’ results than on good firms’ results.

Now consider when a good firm would commit fraud. As for bad firms, the good firm’s goal of fraud is to reduce the chance of being denied funding. If there is no investor monitoring \( m > \bar{m} \), committing fraud increases a good firm’s chance of getting funding by \( \delta'(\kappa_h - \kappa_\ell) \), as compared with \( \delta(\kappa_h - \kappa_\ell) \) for bad firms. It follows that good firms have weaker incentives to commit fraud than do bad firms, and so the probability with which they commit fraud will be weakly lower than that with which bad firms commit fraud. All else equal, if good firms do commit fraud, the free signal actually becomes more informative, since a high signal is now more likely to come from a good firm. Since high signals are more attractive, this actually increases the incentives for bad firms to commit fraud. Nevertheless, the main thrust of our results would not be affected.

Suppose instead that investor monitoring is feasible \( m < \bar{m} \). In this case, the incentives for fraud differ qualitatively between good firms and bad firms. Bad firms wish to avoid being monitored, since this reveals them as bad; good firms do not mind being monitored, since this reveals them as good. It follows that in any regime where low-signal firms are monitored more frequently than high-signal firms, bad firms will have strictly more incentive to commit fraud than do good firms.

By contrast, if low-signal firms are monitored less frequently than high-signal firms, and firms are denied funding if they are not monitored, incentives reverse. (This corresponds to the sub-region marked “Monitor High Signals” in Figure 3, and its extension into the regions where fraud is possible.) Now, bad firms are not interested in committing fraud, because even a high signal cannot get them unmonitored funding, but good firms wish to be monitored so that they can prove their type. It follows that in this region, good firms may commit fraud with higher probability than bad firms.

Because this tends to occur for lower priors on the probability that firms are good, this runs counter to our main result that fraud is more prevalent for better priors. Nevertheless, this result must be taken with a grain of salt, since it requires that good firms who commit fraud
are monitored and then not penalized by investors or the authorities for committing fraud. In practice, this seems unlikely. The act of committing fraud is not only a signal of incentives but a signal of a manager’s ethics. In a less stylized model, finding out that a manager was willing to commit fraud in order to alter investor incentives is likely to be a bad signal for the future – after all, what will this manager be willing to do when the firm is truly in bad shape? If honesty is to be preferred in general, investors as well as the authorities may wish to replace a fraudulent manager now even if the underlying firm is good. In this case, good firms’ incentives to commit fraud so as to be monitored disappear, and we are back to the situation analyzed in the base model.

To summarize this discussion, allowing good managers to commit fraud only has a major impact on our results when both monitoring costs and priors are relatively low, so that good firms may wish to commit fraud in order to boost their chances of being monitored. Nevertheless, this relies on the simplicity of our single-period model. In a model that incorporates multiple periods, investors and regulatory authorities are likely to wish to penalize fraudulent managers even if their firms prove to have good prospects. If this is the case, managers at good firms will have lower incentives to commit fraud even when priors are low, and the qualitative results of our base model continue to apply.

7 Conclusion

We have presented a simple model of incentives for firms to commit fraud in order to get funds from investors. Despite its simplicity, the model can motivate several patterns of behavior, such as changes in the prevalence of fraud over the business cycle and across different sectors and counterintuitive effects of changes in monitoring costs and investor priors. It also has some implications for policy on disclosure standards.
8 References


Appendix: Proofs

A.1 Proof of Proposition 1

Investing without monitoring dominates not investing iff \( V_U > V_N \iff \hat{\mu} N_g - (1 - \hat{\mu}) N_b > 0 \iff \hat{\mu} > \frac{N_b}{N_b + N_g} \). Monitoring and investing in the good firm dominates not investing iff \( V_M > V_N \iff \hat{\mu} N_g - m > 0 \iff \hat{\mu} > \frac{m}{N_g} \). Investing without monitoring dominates monitoring and investing in the good firm iff \( V_U > V_M \iff \hat{\mu} N_g - (1 - \hat{\mu}) N_b > \hat{\mu} N_g - m \iff \hat{\mu} > 1 - \frac{m}{N_b} \). Threshold for \( m \): monitoring is dominated if \( \hat{\mu} \leq \frac{m}{N_g} \) and \( \hat{\mu} \geq 1 - \frac{m}{N_b} \); combine \( \hat{\mu} = \frac{m}{N_g} \) and \( \hat{\mu} = 1 - \frac{m}{N_b} \), which yields \( 1 - \frac{m}{N_b} = \frac{m}{N_g} \), and the definition of \( m \).

A.2 Proof of Proposition 2

The cut-offs for the five regimes can equivalently be defined using cut-offs for the posterior beliefs. Recall from (2) that

\[
\hat{\mu}_\ell (0) < \hat{\mu}_\ell (1) < \hat{\mu}_h (1) < \hat{\mu}_h (0).
\]

These four cut-offs in the interval \([0, 1]\) define the five regimes, depending on the location of \( \mu_3 (m) \) in relation to the four cut-offs (for example, the Fund-Everything regime has \( \hat{\mu}_\ell (0) > \mu_3 (m) \)).

- The proofs for the Fund-Everything and No-Trust regimes are straightforward.

- The Optimistic regime: \( \phi \in (0, 1] \) such that \( \mu_3 (m) < \hat{\mu}_\ell (\phi) \) cannot be an equilibrium. If it was, \( \ell \) signals would not be monitored, so there would be no benefit from committing fraud, i.e. \( \phi = 0 \). Similarly, \( \phi \in [0, 1) \) such that \( \mu_3 (m) > \hat{\mu}_\ell (\phi) \) cannot be an equilibrium. If it was, \( \ell \) signals would be either monitored or rejected, while \( h \) signals receive unmonitored financing; so there would be an incentive to increase \( \phi \). So in equilibrium, the bad firm chooses \( \phi \in (0, 1) \) such that with a signal \( \ell \),

\[
V_U = V_M \iff \hat{\mu}_\ell (\phi) = \mu_3 (m) \iff \phi = \frac{1}{\delta} \left( 1 - \beta - (1 - \gamma) \frac{\mu}{1 - \mu N_b - m} \right). \quad (A1)
\]

Next, \( \kappa_h < 1 \) cannot be an equilibrium, since \( \mu_3 (m) < \hat{\mu}_h (\phi) \forall \phi \). Therefore, \( \kappa_h = 1 \) and \( \lambda_h = 0 \).
\(\kappa_\ell = 1\) cannot be an equilibrium. If it was, there would be no incentive for bad firms to commit fraud, and therefore firms with a signal \(\ell\) should not receive unmonitored financing. Similarly, \(\kappa_\ell + \lambda_\ell < 1\) cannot be an equilibrium. If it was, \(\ell\) signals would be rejected with positive probability. But that is not optimal for the investor since \(\hat{\mu}_\ell (\phi) = \mu_3 > \mu_1\), i.e. she strictly prefers monitoring an \(\ell\) signal to rejecting it. Next, \(\lambda_\ell = 1, \kappa_\ell = 0\) cannot be an equilibrium. If it was, bad firms would commit fraud with certainty. So in equilibrium, the investor chooses \(\lambda_\ell\) and \(\kappa_\ell\) such that

\[
(\beta + \delta) C + (1 - \beta - \delta) \kappa_\ell C - f = \beta C + (1 - \beta) \kappa_\ell C \iff \kappa_\ell = 1 - \frac{f}{\delta C}.
\]

- The Trust-Signals regime: \(\hat{\mu}_\ell (0) < \hat{\mu}_\ell (1) < \mu_3 (m) < \hat{\mu}_h (1) < \hat{\mu}_h (0)\), so \(\ell\) signals are rejected or monitored while \(h\) signals are financed without monitoring. By assumption, \(\delta C > f\), so it pays for a bad firm to increase \(\phi\) up to one. Signals \(\ell\) are monitored iff

\[
\hat{\mu}_\ell (1) \geq \mu_1 (m) \iff \frac{\mu}{\mu + (1 - \mu) \frac{1 - \beta - \delta}{1 - \gamma}} \geq m \frac{N_g}{N_g} \iff \mu \geq \frac{m \frac{1 - \beta - \delta}{1 - \gamma}}{1 + \frac{\gamma - \beta - \delta}{1 - \gamma} m \frac{N_g}{N_g}}.
\]

- The Skeptical regime: \(\phi \in (0, 1]\) such that \(\mu_3 (m) > \hat{\mu}_h (\phi)\) cannot be an equilibrium. If it was, all firms would be either monitored or rejected, and there would be no benefit from committing fraud. Similarly, \(\phi \in [0, 1)\) such that \(\mu_3 (m) < \hat{\mu}_h (\phi)\) cannot be an equilibrium. If it was, \(h\) signals would receive unmonitored financing, while \(\ell\) signals would be either monitored or rejected, giving bad firms an incentive to increase \(\phi\). So in equilibrium, the bad firm chooses \(\phi\) such that with a signal \(h\),

\[
V_U = V_M \iff \hat{\mu}_h (\phi) = \mu_3 (m) \iff \phi = \frac{1}{\delta} \left( \frac{\mu}{1 - \mu} - \frac{m}{N_h - m \gamma} \right).
\]  

If \(\phi\) is such that \(\hat{\mu}_h (\phi) = \mu_3 (m)\), the investor is indifferent between monitored and unmonitored finance for \(h\) signals, and she prefers either option to rejecting an \(h\) signal; therefore \(\lambda_h + \kappa_h = 1\). The investor mixes between monitored and unmonitored finance for \(h\) signals, such that a bad firm is indifferent between committing fraud and not:

\[
(\beta + \delta) \kappa_h C - f = \beta C \kappa_h \iff \kappa_h = \frac{f}{\delta C}.
\]
So $\lambda_h = 1 - \kappa_h = 1 - \frac{f}{\delta C}$. Finally, $\kappa_{\ell} > 0$ cannot be an equilibrium. If it was, then $\hat{\mu}_{\ell}(\phi) \geq \mu_3(m) = \hat{\mu}_h(\phi)$, contradiction. So bad firms with an $\ell$ signal cannot expect to get financing at all. In equilibrium, $\ell$ signals are monitored iff

$$\hat{\mu}_{\ell}(\phi) \geq \mu_1(m) \iff \mu \geq \frac{m}{N_0 \left(1 - \frac{N_h}{N_0} m/(N_0 - m)\right)}.$$

(and rejected otherwise).

\section*{A.3 Proof of Proposition 3}

- The proofs for the Fund-Everything, Trust-Signals and No-Trust regimes are straightforward.

- The Optimistic regime: $\kappa_h = 1$ since $\mu_2 < \hat{\mu}_h(1) < \hat{\mu}_h(0)$. Next, $\phi = 0$ cannot be an equilibrium. The investor would not finance with a signal $\ell$, since $\hat{\mu}_\ell(0) < \mu_2$. But then a bad would firm prefer to increase $\phi_2$ above zero, since $\delta C > f$. Similarly, $\phi = 1$ cannot be an equilibrium. The investor would finance with any signal, so there would be no need to invest $f$. Next, $\kappa_{\ell} = 0$ cannot be an equilibrium. All bad firms would commit fraud with certainty, and the investor should then provide unmonitored finance for either signal, since $\mu_2 < \hat{\mu}_\ell(1)$. Finally, $\kappa_{\ell} = 1$ cannot be an equilibrium. Bad firms would not commit fraud, and the investor should then reject $\ell$ signals, since $\hat{\mu}_\ell(0) < \mu_2$. So the equilibrium must be in mixed strategies for both players. The bad firm chooses $\phi$ such that with a signal $\ell$,

$$V_U = V_N \iff \hat{\mu}_\ell(\phi) N_g - (1 - \hat{\mu}_\ell(\phi)) N_b = 0 \iff \phi = \frac{1}{\delta} \left(1 - \beta - (1 - \gamma) \frac{\mu N_g}{1 - \mu N_b}\right).$$

The investor chooses $\kappa_{\ell}$ such that

$$(\beta + \delta) C + (1 - \beta - \delta) \kappa_{\ell} C - f = \beta C + (1 - \beta) \kappa_{\ell} C \iff \kappa_{\ell} = 1 - \frac{f}{\delta C}.$$

- The Skeptical regime: $\phi = 0$ cannot be an equilibrium. The investor would not finance with a signal $\ell$, since $\hat{\mu}_\ell(0) < \mu_2$. But then a bad would firm prefer to increase $\phi_2$ above zero, since $\delta C > f$. Similarly, $\phi = 1$ cannot be an equilibrium. If it was, the investor would not finance any firm, so there would be no need to commit fraud. Next, $\kappa_h = 0$
cannot be an equilibrium. No firm would be financed, and therefore bad firms would not commit fraud; but then the investor should finance all $h$ signals, since $\mu_2 < \hat{\mu}_h(0)$. Finally, $\kappa_h = 1$ cannot be an equilibrium. Bad firms would have an incentive to commit fraud with probability 1; but then the investor should reject all signals, since $\hat{\mu}_h(1) < \mu_2$. So the equilibrium must be in mixed strategies for both players. The bad firm chooses $\phi$ such that with a signal $h$,

$$V_U = V_N \iff \hat{\mu}_h(\phi) N_g - (1 - \hat{\mu}_h(\phi)) N_b = 0 \iff \phi = \frac{1}{\delta} \left( \frac{\mu \gamma N_g}{1 - \mu N_b} - \beta \right).$$

The investor chooses $\kappa_\ell$ such that

$$(\beta + \delta) \kappa_h C - f = \beta \kappa_h C \iff \kappa_h = \frac{f}{\delta C}.$$  

\textbf{A.4 Proof of Proposition 4}

The conditional probabilities are derived in Proposition 2. The ex-ante probability of fraud is calculated as $(1 - \mu) \phi$ in each regime.

\textbf{A.5 Proof of Proposition 5}

Follows immediately from (A2).

\textbf{A.6 Proof of Proposition 6}

Follows immediately from (A1).

\textbf{A.7 Proof of Proposition 7}

Part (i) follows immediately from (A1) and (A2). Part (ii) follows from an inspection of the regime boundaries as defined in Section 4.2 (immediately before Proposition 2).