Abstract
Do large shareholders monitor firms on behalf of minority shareholders, or share control with other insiders to maximize the gains of the controlling group? We show how firm characteristics and governance laws determine the role of large shareholders in corporate governance. If the value of investment opportunities are hard for insiders to evaluate, letting a large shareholder monitor a single controlling shareholder is efficient because shared control creates disagreement costs that are more likely to destroy profitable opportunities than preventing bad investments. In contrast, sharing control is efficient if investment opportunities are hard for outsiders to evaluate, in countries that poorly protect minority shareholders, and when external financing requirements are large.
I Introduction

Since Berle and Means (1932), an extensive literature has investigated the consequences to firm value of a separation between ownership and control. In most of this literature, ownership structure matters because part of the firm’s value consists of benefits of control that are not enjoyed by outside investors: the insiders’ ability to appropriate corporate assets for personal use, empire building motives in the selection of projects, etc. To preserve private benefits, insiders may lead the firm to, for instance, forego profitable projects (e.g., down sizing) at a cost to minority shareholders.

Accompanying academics and practitioners have searched for mechanisms to protect minority shareholders. In particular, Shleifer and Vishny (1986) argue that the monitoring of investment decisions by large outside shareholders is an important mechanism to protect the value of minority shares. Thanks to the size of their equity stakes, these large shareholders have incentives to go to court to overturn business decisions that are against the interests of outside investors, or to mount a proxy fight to replace an unfriendly management team. The presence of a large outside investor, therefore, may prevent insiders from diluting the rights of minority shareholders.

Yet, monitoring by outside investors also entails efficiency costs. As Pagano and Roell (1998), Bolton and Von Thadden (1998), and Burkart, Gromb, and Panunzi (1997) argue, outside investors have incentives to block business decisions that reduce verifiable cash flows even if this loss is more than offset by an increase of private benefits of control. This excessive monitoring elicits incentives for alternative mechanisms to protect the rights of minority shareholders.

In this paper, we argue that entrepreneurs may find it optimal to share control with other large shareholders, rather than letting them destroy valuable private benefits of control through excessive monitoring. Under shared control, a governance structure arises in which multiple controlling shareholders enjoy private benefits and minority shareholders can no longer count on the presence of a peer that monitors corporate decisions on their behalf. Still, sharing control may increase firm value, for two reasons. First, it increases the equity stake of the decision makers, making them internalize firm value to a greater extent. This equity effect reduces incentives for business decisions that increase private benefits at a high efficiency cost. Second,
ex-post bargaining problems among controlling shareholders may prevent business decisions that are in the collective interest of the controlling group but harm minority shareholders. In an extreme case, a corporate governance structure can arise in which the controlling group owns all shares, and thus internalize all the expropriation costs.

Sharing control, however, is not always efficient. Bargaining problems may result in corporate paralysis, reducing the firm’s overall efficiency and possibly hurting the minority shareholders as well. A trade-off thus exists between the excessive monitoring of outside investors and the net bargaining costs associated with a governance structure with multiple controlling shareholders.

By solving this trade-off, we show that sharing control dominates monitoring in firms with investment opportunities that are hard for outsiders to evaluate. In these firms, controlling shareholders should have an informational advantage in evaluating projects, while monitoring is more likely to harm firm value than overturning inefficient business decisions. In contrast, monitoring by outside investors is efficient if the investment opportunities yield private benefits that are hard for insiders to evaluate. Here, outside monitors ignore noisy signals of the value of the private benefits that could make firms under shared control lose profitable investment opportunities. As we shall argue later, these results imply that shared control should be pervasive in family firms. Moreover, conditioned on a governance structure with shared control, the model predicts that the controlling shareholders should have similar business backgrounds.

Firm characteristics do not fully explain the role of large shareholders in corporate governance, though. Governance law is also an important determinant of the costs and benefits of monitoring and sharing control. In particular, our model implies that shared control is more common in countries with legal systems that offer weak protection to minority shareholders. The intuition for this implication is as follows. An improvement of governance law can be thought as a shift in the investment opportunities that reduces the importance of the private benefits vis-à-vis the public cash flows. At first glance, this shift reduces the controlling shareholders’ incentives for projects that increase private benefits but inefficiently harm minority shareholders. This alignment of incentives is vacuous, nonetheless, because controlling groups can replicate it by an increase of the controlling stake that makes them internalize the verifiable cash flow more fully. An improvement of governance law, therefore, does not increase the
value of a firm under shared control. In contrast, laws that protect minority shareholders make it easier for monitors to block corporate decisions that harm minority shareholders, increasing firm value accordingly. Hence, while governance laws enhance the value of monitoring, they are much less important for firms under shared control.

But is there evidence of ownership structures with shared control? Laeven and Levine (2004) show that about one-third of 865 firms in 13 Western Europe countries—and over forty percent of firms with one large shareholder—have two or more owners holding more than 10 percent of the voting rights each. Moreover, firm value (proxied by Tobin’s Q) increases with the equity stake of a second large shareholder only if the gap in voting rights between the first and the second largest shareholder is small, as one would expect to occur in governance structures under shared control. These findings confirm in a broader sample previous research by Volpin (2002) that shows that, from 1987 to 1996, 15 percent of the firms listed in the Milan Stock Exchange were controlled by large shareholders that entered in explicit agreements to vote as a block (voting syndicates). Clearly, these agreements are meaningful only in firms under shared control. More importantly, Volpin shows that managerial turnover is more sensitive to performance in firms with voting syndicates, consistent with our argument that shared control plays an important role in corporate governance.

The papers closest to ours are Bennedsen and Wolfenzon (2000) and Aghion and Bolton (1992). Bennedsen and Wolfenzon show that the presence of a large outside shareholder forces the controlling group to amass a greater equity stake or else control may be lost. The larger control stake increases efficiency because it makes the controlling group internalize more of the firm’s value. Bennedsen and Wolfenzon ignore bargaining problems within the controlling group and focus on the coalition games that determine the size of the controlling stake. In contrast, we ignore the coalition games and focus on the bargaining problems among controlling shareholders.

1In the US, Black and Gilson (1997) report that shareholders’ agreements similar to voting syndicates are often present in the venture capital industry. Yet, these shareholders’ agreements are not limited to partnerships and start-up companies; they also seem to be pervasive in public firms that experienced privately negotiated block transactions. The Securities and Exchange Commission (SEC) requires that firms with publicly traded securities disclose the presence of contractual agreements such as shareholders’ agreements in form 13-D. We searched for the presence of shareholders’ agreements among a random sample of 115 public firms announcing privately negotiated block transactions of 5% or more from 1996 to 2000. These privately negotiated transactions triggered board rights agreements, veto power provisions, or voting agreements in 53 of the 115 firms (46 percent).
As in our paper, Aghion and Bolton (1992) look for the optimal ownership structure of a firm whose initial shareholder is credit constrained. In their paper, a single shareholder – the initial entrepreneur – enjoys private benefits of control, which lead to conflicting objectives between the entrepreneur and the remaining shareholders. Our paper departs from Aghion and Bolton by letting multiple controlling shareholders enjoy private benefits of control. In this setting, we obtain new trade-offs that link firm characteristics and governance laws to the choice of the ownership structure, and we show that, in contrast to Aghion and Bolton, shared control may be an efficient mechanism to protect minority shareholders.

The remainder of the paper is organized as follows. Section II describes the model. Section III characterizes firm value under shared control and unilateral control with monitoring. Section IV shows how firm characteristics and governance laws determine the optimal role of large shareholders in corporate governance. Section V discusses the stability of ownership structures with shared control and the robustness of our results. And section VI concludes the paper. Proofs of the propositions that are not present in the text can be found in the appendix.

II Framework

Our starting point is a firm whose single shareholder seeks outside investors to finance new projects. For simplicity, all agents are risk-neutral and the risk-free interest rate is zero.

A Timing

Figure 1: Timing of events

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Choice of ownership structure</td>
</tr>
<tr>
<td>1</td>
<td>Information is released</td>
</tr>
<tr>
<td>2</td>
<td>Investment decision</td>
</tr>
<tr>
<td>3</td>
<td>Cash flows realize</td>
</tr>
</tbody>
</table>
Figure 1 summarizes the timing and the main events of the model. At time $t = 0$, an investment opportunity becomes available and the initial shareholder (entrepreneur) looks for outside investors to finance the cost $I$ of the project. To create a link between the investment opportunity and the firm’s ownership structure, we assume that both the entrepreneur and the firm have exhausted their debt capacities. Financing the project thus requires attracting new shareholders through an equity issue which, for simplicity, makes the existing debt safe. (Assuming that the firm’s debt is safe after the equity issue let us ignore the existing debt in the analysis.)

Additional information about the project’s payoff is released at time $t = 1$, and the final decision on the investment is made at time $t = 2$. Cash flows realize at $t = 3$, when the firm is liquidated.

**B Ownership structure**

The initial shareholder’s problem is to choose an ownership structure that maximizes the firm’s value conditioned on raising the investment requirement $I$. To finance the project, the initial shareholder has three alternatives. He may keep full control over business decisions by selling equity to minority investors who will act passively, with no say on business decisions. Alternatively, the initial entrepreneur may attract a large investor who will finance all of the investment requirement. Or he may sell equity to minority investors and the large investor as well.\(^2\) We rule out, however, investors who can afford the whole firm. Otherwise, the first best would be trivially obtained by selling the whole firm to a single investor, who would internalize all of the costs and benefits of business decisions.

One of the main goals of this paper is to explore the relation between firm value and the role that large shareholders play in the governance structure. As such, we consider two types of large shareholders: a large shareholder who does not directly participates on the management of the firm but may monitor business decisions on behalf of the minority shareholders, and a large shareholder who shares control with the entrepreneur.

\(^2\)Although all the arguments of this paper apply to ownership structures with more than two large shareholders, the analysis is simpler if we restrict attention to two large shareholders. Zwiebel (1995) and Faccio and Lang (2002) report that, in the US and West Europe, ownership structures with more than two large shareholders are rare.
Our modelling of the large shareholder who acts as a monitor is standard. At a private cost, the monitor can assure a probability \( m \in [0, 1] \) that the court will overturn business decisions that are against the interests of the minority shareholders. We parameterize the cost of monitoring by the function \( c(m) = \rho m^2 \), where \( \rho \) is a positive parameter. The cost of monitoring increases with the probability \( m \) that the court will be convinced to act against the control group. Moreover, the marginal cost depends on a parameter \( \rho \) that is related to the nature of the firm. For instance, evaluating the appropriateness of undertaking a project with a high volatility of verifiable returns (e.g., an R&D project) is likely to be more difficult than a typical project. Courts should then be more reluctant to overturn managements’ decisions related to an R&D project; a reluctance that we model as a larger \( \rho \).

The way we model shared control is standard as well. As in Aghion and Bolton (1992), we model control by the authority to veto projects.\(^3\) In case of shared control, therefore, a project will be undertaken if and only if both controlling shareholders agree with the investment. Volpin (1992) shows that governance structures with veto power do in fact exist. In particular, 15 percent of the public firms in the Milan Stock Exchange are controlled by large shareholders that form a voting syndicate whose members agree to vote together. Although voting syndicates can decide on their actions either by unanimously or by majority rule, Volpin (1992, page 65)) reports that only the former is legally binding. Voting syndicates, therefore, give their members veto power, as we assume in our model.

C  Cash flows

Ownership structure is relevant to the firm’s value only if shareholders may have different incentives to undertake the project. As in the modern literature on the theory of firm (e.g., Grossman and Hart (1986), and Hart and Moore (1990)), we obtain conflicting incentives on the investment decision by introducing nonverifiable cash flows – the private benefits of control – which are fully captured by the shareholders who run the firm, that is, the controlling shareholders.

Accordingly, the project’s cash flows consist of two parts: the verifiable cash flow, \( I + y \),

\(^3\)In Aghion and Bolton (1992), future actions must be chosen by unanimous consent when more than one investor is in control.
which is the sum of the investment requirement $I$ and its return $y$, and the private benefit component, $b$. Nonetheless, the project does not necessarily increase either the verifiable cash flows or the private benefits of control. For instance, downsizing the firm may increase profits and yet reduce the utility of a controlling shareholder who is an empire builder, in which case $b < 0$. Conversely, expanding the firm may increase the private benefits of an empire builder at a loss of profits, $y < 0$.

Although there are reasons to believe that the number of controlling shareholders may have an impact on the distribution of the project’s payoffs, the direction of this effect is uncertain.\textsuperscript{4} Hence, we assume that the private benefits do not depend on the number of controlling shareholders. Likewise, the private benefits of the project do not depend on the control stake. Intuitively, the distribution of the project’s payoffs, $(b, y)$, reflects the investment opportunities, which do not depend on the ownership structure. These two assumptions imply that $\sum_{i=1}^{2} b_i = b$, where $b$ is the private benefit of the project in an ownership structure with a single controlling shareholder, and $b_i$ is the private benefit of controlling shareholder $i \in \{1, 2\}$ under shared control. It then follows that the ownership structure has an impact on firm value only by influencing the control group’s incentives to take advantage of the investment opportunities.

Finally, to rule out uninteresting solutions to the conflicts between the controlling group and the other shareholders, we do not allow for the controlling group to commit to never undertake the project or to always undertake it.\textsuperscript{5}

\section*{D Information structure}

When the initial shareholder chooses the ownership structure (time $t = 0$), the verifiable cash flow $y$ is known by the initial entrepreneur and the investors (but not by the court), while the total private benefits of the project $b$ is a random variable with a publicly known distribution. If the initial entrepreneur remains as the single controlling shareholder after financing the

\textsuperscript{4}A large number of controlling shareholders may increase the efforts of unlocking private benefits. If so, the private benefits should stochastically increase with the number of controlling shareholders. But, a large number of controlling shareholders may also lead to a destructive fight for private benefits. Hence, private benefits may stochastically decrease with the number of controlling shareholders.

\textsuperscript{5}This assumption can be justified on the basis that we analyze one of several investment opportunities (the one with conflicts of interests), which make the never-invest and always-invest strategies too costly for shareholders.
project, he learns the private benefits of control of the project before making the investment decision (time $t = 1$). Otherwise, the controlling shareholders learn their own private benefits from the project, $b_i$ (possibly negative) at $t = 1$. By their very nature, however, the benefits of control of each controlling shareholder are likely to be privately known. Therefore, we assume that controlling shareholder $i \in \{1, 2\}$ observes only a noisy signal $s_j$ of the private benefit of the controlling shareholder $j \neq i$. The noisy signals $s_1$ and $s_2$ are observed by both controlling shareholders (but not by outside investors), and they satisfy $b_j = s_j + \epsilon_j$ with $\epsilon_j \in [-\frac{\epsilon}{2}, \frac{\epsilon}{2}]$ and $\epsilon > 0$ for $j \in \{1, 2\}$.

Conditioned on $s_j$, the true private benefit $b_j$ is uniformly distributed in the interval $[s_j - \frac{\epsilon}{2}, s_j + \frac{\epsilon}{2}]$. Hence, the realization of $s_j$ implies that controlling shareholder $i$’s posterior about $b_j$ is independent of $b_i$. Note also that we do not impose restrictions on the distribution of the signals $(s_1, s_2)$. As such, our results do not rely on how the private benefits of control are shared. In particular, the model is consistent with a sharing rule that gives the smallest fraction of the total private benefits (possibly zero) to the controlling shareholder with the lowest equity stake.

### III Firm Value under Monitoring and Shared Control

Left unchecked, the entrepreneur has incentives to overinvest if private benefits are high enough to offset his share of the negative verifiable return of an inefficient project. In turn, incentives for underinvestment arise if the project is efficient, but the entrepreneur’s share of the verifiable payoffs does not offset the loss of private benefits. Anticipating an inefficient investment policy, investors will buy equity only at reduced prices, implying that the entrepreneur pays the cost of his sub-optimal incentives. It is in the best interest of the entrepreneur, therefore, to look for mechanisms that constrain his investment decisions. This section analyzes two such mechanisms: monitoring by an outside investor and shared control. We shall characterize firm value under these two mechanisms, showing that shared control yields a compromise between monitoring and a governance structure that gives full discretion to the entrepreneur.
A  Firm value with monitoring

Suppose that the initial entrepreneur wants to stay as the single controlling shareholder but, recognizing the efficiency costs of being left unchecked, finances the investment requirement by selling a fraction $\beta$ of the shares to a large investor who, at a personal cost $c(m) = \frac{\rho m^2}{2}$, can overturn business decisions that harm minority shareholders with probability $m \in [0, 1]$. As we shall now show, for each equity stake $\beta$, there is a monitoring level $m(\beta)$ that maximizes the monitor’s expected utility. Anticipating the monitor’s best reaction, the entrepreneur can finance the investment requirement by selling to the monitor an equity stake $\beta(m^*)$ that induces a value-maximizing monitoring level $m^*$. To characterize this value-maximizing level of monitoring, we first solve the monitor’s problem for any given equity stake $\beta$:

$$\max_{\{m \in [0, 1]\}} \beta m |y| - \frac{\rho m^2}{2}. \quad (1)$$

To understand problem (1), recall that, in our model, the entrepreneur may act against the interests of the monitor in two ways. He may invest in a project with negative verifiable returns (e.g., a value-decreasing acquisition driven by empire building concerns), imposing on the monitor a loss of $-\beta y$. And he may pass up a project with a positive verifiable return, at a cost of $\beta y$ to the monitor. We can thus write the monitor’s cost of an investment decision that is against their interests as $\beta |y|$, where $|y|$ is the absolute value of the verifiable return. Taking into account the monitor’s cost, a level of monitoring that blocks the investment decisions with probability $m$ thus increases his expected utility by $\beta m |y| - \frac{\rho m^2}{2}$.

The first order condition of problem (1), which is also sufficient, gives us the monitor’s optimal level of monitoring: $m(y, \beta) = \min\{\frac{\beta}{\rho} |y|, 1\}$. As expected, the incentives to monitor increase with the equity stake, $\beta$, and decrease with the parameter cost of monitoring, parameterized by $\rho$. Hence, given $\rho$, there is a one-to-one mapping between the level of monitoring that maximizes firm value and the monitor’s equity stake. We can thus obtain the firm’s value under optimal monitoring by solving for the monitor’s equity stake that maximizes firm value.

---

6There are ways for an initial entrepreneur to convince investors to become large shareholders in the firm. The entrepreneur may, for example, sell shares privately at lower prices, or disclose information that would normally remain privy. Likewise, an entrepreneur may make it difficult for outside investors to buy a too large equity stake by issuing dual class shares or by adopting antitakeover defenses that are triggered by an undesirable build up of shares by an outsider. Accordingly, we assume that the initial entrepreneur can choose the monitor’s equity stake $\beta$. 
To do so, consider first that the project’s verifiable cash flow is negative, i.e., $y < 0$.

The monitor always tries to block a project that reduces verifiable cash flows, being successful with probability $m(y, \beta)$. Of course, the monitor’s strategy is vacuous if the entrepreneur has no incentive to undertake the project either. Hence, monitoring makes sense only if, despite the negative verifiable cash flow, private benefits make it worthwhile for an entrepreneur to undertake the project. This will be the case if $b + \alpha^1 y > 0$, where $\alpha^1$ is the equity stake of the entrepreneur. If this condition is satisfied, the firm value with monitoring is $(y + b) (1 - m(y, \beta)) - \frac{\rho m(y, \beta)^2}{2}$. And so the monitor’s stake that maximizes firm value solves:

$$
\max_{\beta \in [0, 1-\alpha^1]} E \left[ (y + b) (1 - m(y, \beta)) - \frac{\rho m(y, \beta)^2}{2} | b > -\alpha^1 y \right] P(b > -\alpha^1 y),
$$

(2)

where $m(y, \beta)$ is the probability that the monitor will block the project, which, from program (1), is $m(y, \beta) = \min\left\{ \frac{\alpha^1}{\rho} | y \right\}$.

The program that solves for the optimal monitor’s stake with projects that increase verifiable cash flows ($y > 0$) is analogous. If, despite the positive verifiable cash flow, the undertaking of the project leads to a sufficiently large loss of private benefits ($b + \alpha^1 y < 0$), then the entrepreneur may pass up the project. In this case, the monitor will try to force the undertaking of the project, succeeding it with the probability $m(y, \beta)$. It then follows that we can write the entrepreneur’s problem as

$$
\max_{\beta \in [0, 1-\alpha^1]} E \left[ (y + b) | b > -\alpha^1 y \right] P(b > -\alpha^1 y) + E \left[ (y + b)m(y, \beta) - \frac{\rho m(y, \beta)^2}{2} | b < -\alpha^1 y \right] P(b < -\alpha^1 y),
$$

(3)

where the first term in the objective function is firm value conditioned on being in the entrepreneur’s interest to undertake the project while the second term is firm value conditioned on the monitor’s forcing the firm to undertake the project despite its being against the entrepreneur’s interest.

Proposition 1 below characterizes the solutions of programs (2) and (3).

**Proposition 1** Monitoring is valuable if there are overinvestment problems, that is, $b + y < 0$
and \( y < 0 \), where \( b = E \left[ b \mid (b + \alpha^1 y) > 0 \right] \), or underinvestment problems, \( \overline{b} + y > 0 \) and \( y > 0 \), where \( \overline{b} = E \left[ b \mid b < -\alpha^1 y \right] \). For a firm with overinvestment problems, the optimal equity stake of the monitor and the firm’s value are, respectively,

\[
\beta^* = \begin{cases} 
\frac{1}{y}(b + y) & \text{if } b + y < 0, \\
0 & \text{if } (b + y) > 0
\end{cases}
\quad \text{and } V_m = \begin{cases} 
V_{uc} + \frac{1}{2\rho} (b + y)^2 P(b > -\alpha^1 y) & \text{if } (b + y) < 0 \\
V_{uc} = (b + y) P(b > -\alpha^1 y) & \text{if } (b + y) > 0
\end{cases}
\]

For a firm with underinvestment problems, the optimal equity stake of the monitor and the firm’s value are, respectively,

\[
\beta^* = \begin{cases} 
\frac{1}{y} (\overline{b} + y) & \text{if } (\overline{b} + y) > 0, \\
0 & \text{if } (\overline{b} + y) < 0
\end{cases}
\quad \text{and } V_m = \begin{cases} 
V_{uc} + \frac{1}{2\rho} (\overline{b} + y)^2 P(b < -\alpha^1 y) & \text{if } (\overline{b} + y) > 0 \\
V_{uc} = (\overline{b} + y) P(b > -\alpha^1 y) & \text{if } (\overline{b} + y) < 0
\end{cases}
\]

The intuition for Proposition 1 is quite simple. For projects that destroy verifiable cash flows, overinvestment is the problem that the monitor will try to avoid. In this case, monitoring is efficient if, conditioned on being optimal for the entrepreneur to undertake the project, the expect payoff of the project is negative, that is, \( b + y < 0 \). Conversely, \( b + y > 0 \) implies that, in expectation, monitoring is not efficient because it is more likely that the project is valuable when the entrepreneur wants to undertake it. Now, for projects that increase verifiable cash flows, underinvestment is the problem that the monitor will try to avoid. Here, monitoring is valuable if it is more likely that the project is efficient when the entrepreneur does not want to undertake it, that is, \( \overline{b} + y < 0 \).

Our next task is to analyze the entrepreneur’s alternative for monitoring: sharing control.

**B Firm value with shared control**

A vast literature on corporate law (e.g., O’Neal and Thompson (1992)) discusses conflicts of interest within a controlling group that may lead to deadlock problems. For instance, the Wall Street Journal of May 13, 1998 (page B10) reports that Ted Turner, then Vice Chairman and the largest individual shareholder (11 percent) of Time Warner, had for a second time vetoed the sale of the group’s legal channel, Court TV, to Discovery Communications Inc. Allegedly, Ted Turner was concerned with a new owner transforming the legal channel into a competitor to CNN, the flagship of Turner Broadcasting’s own cable channel and also a member of the
Time-Warner group. According to the Wall Street Journal, Mr. Turner prevailed over Gerald Levin, Time Warner’s Chairman, who did not internalize the consequences to CNN of the sale of Court TV to Discovery as much as Mr. Turner. Sharing control, therefore, opens the door for bargaining problems.

In our paper, conflicts of interest between the controlling shareholders arise if \( b_i + \alpha_i y > 0 \) and \( b_j + \alpha_j y < 0 \) for \( i \neq j \), where \( b_i + \alpha_i y \) is controlling shareholder \( i \)'s valuation for the project. In this case, the firm undertakes the project only if controlling shareholder \( i \) convinces the opposing shareholder \( j \) not to use his veto power. The investment decision amounts to a bargaining game.

By their very nature, the benefits of control of each controlling shareholder are likely to be privately known. We thus model the investment decision under shared control as a bargaining game under asymmetric information. For ease of exposition, we use a simple and well known mechanism – first analyzed by Chatterjee and Samuelson (1983) – to solve the controlling shareholders’ bargaining game. All of the qualitative results of our paper, however, hold if we use the direct mechanism approach of Myerson and Satterthwaite (1983) to solve the controlling shareholders’ bargaining problem.\(^7\)

In the mechanism of Chatterjee and Samuelson, which is a natural generalization of the Nash bargaining solution to a setting with imperfect information, the controlling shareholders simultaneously announce their valuations of the project – call them \( V_i \) for \( i \in \{1, 2\} \). The project is undertaken if and only if \( V_1 + V_2 \geq 0 \), in which case the two controlling shareholders split their announced benefits. The split of the announced benefits is implemented by a transfer, \( t \), from the first controlling shareholder (the initial one) to the second one. If the project is not undertaken, no side payment is required. The transfer thus solves \( V_1 - t = V_2 + t \) or \( t = \frac{V_1 - V_2}{2} \). As Lemma 1 shows, transfers of shares can support any incentive-compatible side-payment \( t \) if the equity value that arises from the existing assets is sufficiently large.

**Lemma 1** Assume that the equity value from the assets in place, \( V_0 + I \), satisfies

\[
V_0 + I \geq -y + \max_{i \in \{1, 2\}} \frac{\bar{b}_i + \alpha_i \bar{y}}{\alpha_i - \bar{\alpha}},
\]

\(^7\)In an earlier version of our paper – Rodney White Center of Financial Research Working Paper #05-1999 – we use the direct mechanism approach to solve the controlling shareholders’ bargaining problem.
where \( y \) is the minimum verifiable return, \( \bar{y} \) is the maximum verifiable return, \( \bar{b}_i \) is the maximum private benefit of controlling shareholder \( i \), \( \alpha_i \) is the equity stake of controlling shareholder \( i \), and \( \bar{\alpha} \) is the minimum equity stake that allows a shareholder to capture private benefits of control. Then transfers of shares can implement any incentive-compatible payment of the bargaining game.

A quick inspection of condition (4) shows that, for \( \bar{\alpha} \) (the minimum equity stake that gives control) sufficiently small, there is a large enough value of the existing shares that allows for transfer of shares to implement payments of the bargaining game. As such, from now on we shall assume that condition (4) is satisfied, in which case we can proceed as if the controlling shareholders could afford the transfer payments. \(^8\)

Take then \( \alpha_1 \) and \( \alpha_2 \) as given and assume, from Lemma 1, that the controlling shareholders can use their shares to make transfer payments in the bargaining game. The transfer payment \( t = V_1 - V_2 \) implies that, conditioned on the investment being made, the two controlling shareholders gain by shading their valuations of the project. Of course, reducing the announced valuation will also increase the chances that the project will not be undertaken (remember that the investment happens if and only if \( V_1 + V_2 \geq 0 \)). When shading their valuations, each controlling shareholder will weigh a higher gain in the event that the project is undertaken against a higher probability that a valuable project is foregone.

To solve this trade-off, we look for a Bayesian equilibrium in which the announcements of the controlling shareholders depend on their own valuations for the project, \( b_i + \alpha_i y \), and their guesses of the announcement of the other controlling shareholder. The Bayesian equilibrium is described by a pair of functions, \((V_1(b_1 + \alpha_1 y, s_1, s_2), V_2(b_2 + \alpha_2 y, s_1, s_2))\), such that the announcement of the first controlling shareholder, \( V_1(b_1 + \alpha_1 y, s_1, s_2) \), solves

\[
\max \int_{V_2^{-1}(-V_1(s_1, s_2) - \alpha_2 y)}^{s_2 + \frac{V_1 - V_2}{2}} (b_1 + \alpha_1 y - V_1 - \frac{V_1 - V_2(b_2 + \alpha_2 y, s_1, s_2)}{2}) f_2(b_2 | s_2) db_2. \tag{5}
\]

The objective function in program (5) is the expected payoff of the initial shareholder given his announcement of \( V_1 \); his true valuation of the project, \( b_1 + \alpha_1 y \); and the signals \( s_1 \) and \( s_2 \). This payoff is uncertain, for two reasons. First, the transfer payment \( t = \frac{V_1 - V_2}{2} \) depends

\(^8\)Section V shows that the main result of the paper – the efficiency of shared control – holds if condition (4) is not satisfied.
on the second controlling shareholder’s announcement, \(V_2\), which is a function of his unknown valuation for the project. Second, announcing \(V_1\) will block the project if \(V_1 + V_2 < 0\), or equivalently, \(V_2 < -V_1\). Hence, the lower the announced \(V_1\), the higher the chances that the project will not be undertaken. In fact, given \(V_1\), the lowest \(V_2\) that leads to the acceptance of the project solves \(V_1 + V_2(b_2 + \alpha_2y, s_1, s_2) = 0\), which implies that \(V_1 + V_2(b_2 + \alpha_2y, s_1, s_2) > 0\) if and only if \(b_2 + \alpha_2y > V_2^{-1}(-V_1, s_1, s_2)\), where \(V_2^{-1}(\cdot)\) is the inverse function of \(V_2(b_2 + \alpha_2y, \cdot)\).\(^9\) Therefore, the expectation of the initial shareholder’s payoff is taken with respect to \(b_2\) (using the density that the signal \(s_2\) induces, \(f_2(b_2|s_2)\)) for values higher than the cut-off \(V_2^{-1}(-V_1, s_1, s_2) - \alpha_2y\).

Analogous to program (5), the optimal announcement of the second controlling shareholder solves

\[
\max_{V_2} \int_{V_1^{-1}(-V_2,s_1,s_2) - \alpha_1y}^{s_1 + \frac{\alpha_1y}{2}} (b_2 + \alpha_2y + \frac{V_1(b_1 + \alpha_1y, s_1, s_2) - V_2}{2}) f_1(b_1|s_1) db_1. \tag{6}
\]

Proposition 2 characterizes the solution of the bargaining game for best responses \(V_1(.)\) and \(V_2(.)\) that are linear functions of the controlling shareholders’ own valuations.

**Proposition 2** Suppose that the project’s verifiable return is \(y \in [y, \bar{y}]\); the private benefit of controlling shareholder \(i \in \{1, 2\}\) is \(b_i \in [b_i, \bar{b}_i]\); the signals of the private benefits are \(s_i = b_i + \epsilon_i\); \(\epsilon_i \in [-\frac{\epsilon}{2}, \frac{\epsilon}{2}]\); and the equity value from the existing assets, \(V_0 + I\), satisfies condition (4). Then, there is a Bayesian Equilibrium in which the investment is undertaken if and only if

\[
b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{4}(s_1 + s_2 + (\alpha_1 + \alpha_2)y + \epsilon). \tag{7}
\]

The left-hand side of the investment rule (equation (7)) is the combined valuation of the controlling shareholders. In the absence of bargaining problems, a project will be undertaken if and only if the combined valuation is positive, that is, \(b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq 0\). Under imperfect information, however, the project will be accepted only if the combined valuation exceeds \(\frac{1}{4}(s_1 + s_2 + (\alpha_1 + \alpha_2)y + \epsilon)\). Using \(s_i = b_i + \epsilon_i\), this decision rule can be re-written as

\[
b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2). \tag{8}
\]

\(^9\)It can be shown that, in any Bayesian Equilibrium, the functions \(V_i(.)\) and \(V_2(.)\) increase with the valuation of the project. If these functions are not strictly increasing, \(V_i^{-1}(x)\) should be understood as the minimum valuation of the project that makes the controlling shareholder \(i\) announce \(x\).
Since $\epsilon_i$ is uniformly distributed in the interval $[-\frac{\epsilon}{2}, \frac{\epsilon}{2}]$, $\epsilon - \epsilon_1 - \epsilon_2$ is strictly positive with probability 1. As a result, the controlling shareholders will pass up projects that are in their collective interest if the payoffs satisfy $0 < b_1 + b_2 + (\alpha_1 + \alpha_2)y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$. We have thus shown that

**Proposition 3** With shared control, the controlling shareholders will not undertake projects that are against their collective interest. However, they may pass up projects that would have increased the sum of their expected payoffs.

The intuition for Proposition 3 is straightforward. Both controlling shareholders have incentives to shade their valuations for the project. After all, the transfer paid by the initial shareholder increases with his announcement of the project’s value, $V_1$, and decreases with the other controlling shareholder’s announcement, $V_2$. Shading the valuations, though, increases the chances that the project is not undertaken. Bargaining under asymmetric information, therefore, biases the investment decision against the undertaking of the project.

Rejecting projects that are in the collective interest of the controlling shareholders may well be efficient, though. Disagreements among the controlling shareholders may prevent them from undertaking a project that, although in their collective interest, inefficiently harm minority shareholders. In other words, bargaining problems associated with shared control mitigate overinvestment problems.

Unfortunately, ex-post bargaining problems may also exacerbate underinvestment problems. For instance, an efficient project $(b + y > 0)$ may fail to pass the hurdle for investment under shared control (i.e., $b_1 + b_2 + (\alpha_1 + \alpha_2)y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$). Clearly, whether the benefits of shared control overcome its costs or not depends on the control structure $(\alpha_1, \alpha_2)$. Our next task, therefore, is to characterize the control structure that maximizes firm value under shared control.

The investment policy is $b + \alpha^2y = b_1 + b_2 + (\alpha_1 + \alpha_2)y < \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$, where the control stake $\alpha^2 = \alpha_1 + \alpha_2$ and $b = b_1 + b_2$. Since we assume that $\epsilon_1$ and $\epsilon_2$ are independently and uniformly distributed in the interval $[-\epsilon/2, \epsilon/2]$, straightforward calculations yield that, given $b$ and $y$, the probability that the two controlling shareholders agree to undertake the project is

$$\frac{1}{\epsilon^2} \int_{-\epsilon/2}^{\epsilon/2} \int_{-\epsilon/2}^{\epsilon/2} I(b + \alpha^2y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)) \, de_1 \, de_2,$$

where $I(b + \alpha^2y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2))$ is an indicator function that takes value one if the hurdle for undertaking the project is satisfied. It
then follows that firm value under a control stake $\alpha^2$ is

$$V_{sc}(\alpha^2) = \int (b + y) \left[ \frac{1}{\epsilon} \int_{-\epsilon/2}^{\epsilon/2} \int_{-\epsilon/2}^{\epsilon/2} I(b + \alpha^2 y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2))d\epsilon_1d\epsilon_2 \right] f(b)db,$$  \hspace{1cm} (9)$$

where $f(b)$ is the density function of the sum of the private benefits of control. The probability that the controlling shareholders undertake the project is an increasing function of their collective payoffs $(b + \alpha^2 y)$. Lemma 2 obtains a closed form solution for the probability that the project is undertaken.

**Lemma 2** The value under shared control is $V_{sc}(\alpha^2) = \int (b + y) \Gamma\left(\frac{b + \alpha^2 y}{\epsilon}\right) f(b)db$, where $\alpha^2$ is the controlling stake, and

$$\Gamma(x) = \begin{cases} 
1 & \text{if } x \geq \frac{2}{3} \\
1 - \frac{1}{2}(3x - 2)^2 & \text{if } x \in \left[\frac{1}{3}, \frac{2}{3}\right] \\
\frac{1}{2}(3x)^2 & \text{if } x \in \left[0, \frac{1}{3}\right] \\
0 & \text{if } x \leq 0 
\end{cases}.$$

From Lemma 2, it is rather simple to obtain the control stake that maximizes firm value under shared control. Proposition 4 below characterizes the optimal control stake.

**Proposition 4** Shared control with no minority shareholders achieves the first best if $P(b+y \in (0, \frac{2\epsilon}{3})) = 0$. Otherwise, shared control may distort the investment policy and the optimal controlling stake, $\alpha^*$, satisfies:

(i) $1 - \frac{2e}{3|y|} < \alpha^* < 1$ if $y < 0$ (i.e., undertaking the project yields private benefits and decrease in verifiable cash flows). In this case, the presence of minority shareholders increases firm value under shared control.

(ii) $\alpha^* = 1$ if $y > 0$ (i.e., undertaking the project yields loss of private benefits and increases in verifiable cash flows). In this case, it is not optimal to sell equity to minority shareholders.

As we have already argued, shared control implies ex-post bargaining costs that make it more difficult for the firm to undertake the project. As a result, underinvestment is the only
inefficiency that may arise. And it will arise if the project is valuable, that is \( b + y > 0 \), but the hurdle for undertaking the project is not met: \( b + \alpha^2 y < \frac{1}{\delta}(\epsilon_1 - \epsilon_2) \). In the absence of minority shareholders, \( \alpha^2 = 1 \), these two conditions are equivalent to \( b + y \in \left( 0, \frac{1}{\delta}(\epsilon_1 - \epsilon_2) \right) \).

Hence, a sufficient condition for shared control to be first best – and for \( \alpha^2 = 1 \) to be optimal – is that, conditioned on being profitable, projects are likely to be highly profitable. Formally, this happens if the event \( b + y \in \left( 0, \frac{2}{\delta} \right) \) has zero probability.

Assume now that \( P(b + y \in (0, \frac{2}{\delta})) > 0 \). Then shared control leads to underinvestment. More interestingly, Proposition (4) shows that, in the case where undertaking projects decrease verifiable cash flow, allowing for minority shareholders mitigates the underinvestment problem. The intuition is straightforward. Bargaining problems within the controlling group is the reason for underinvestment under shared control. Clearly, these bargaining problems are less severe if the controlling group can shift some of the costs of undertaking the project to minority shareholders. But this is what happens if the verifiable project cash flow is negative; the controlling group captures all of the benefits of control while the minority shareholders bear the cost \(- (1 - \alpha^2)y\). It is thus not surprising that the optimal ownership structure with sharing control implies the presence of minority shareholders. In contrast, the controlling group cannot shift to minority shareholders part of the project cost if verifiable project cash flows increase. The optimal control structure then gives full ownership to the controlling shareholders so that they internalize as much as possible the cost of passing up profitable projects.

We are now ready to compare unilateral and shared control.

### IV Comparing Monitoring and Shared Control

What is the role of large shareholders in corporate governance? Do they monitor the management on behalf of minority shareholders and other outside investors, or simply capture benefits that are privy to those that control the firm? This section addresses these questions by comparing the values of two types of firm: a firm with a single controlling shareholder overseen by the classical monitor (unilateral control) and a firm with two large shareholders who share control over the investment decisions. We show that the relative efficiency of unilateral and shared control depends not only on firm characteristics but also on the governance law.
A Firm characteristics

Whether it is optimal for large shareholders to monitor the firm (unilateral control) or join the controlling group (shared control) it depends on firm characteristics. To see why, we must assess the costs and benefits of these two governance structures. Consider first a firm under shared control. From Proposition 3, controlling shareholders will not voluntarily undertake projects that are against their collective interest. Yet, they may pass up efficient projects because of an uneven distribution of private benefits between the controlling shareholders. Underinvestment, therefore, is the main source of inefficiency in a governance structure with shared control. An uneven distribution of private benefits does not suffice for underinvestment to obtain, though. Knowing that the private benefits are unevenly distributed, the most favored controlling shareholder knows that some compensation is warranted or else the other controlling shareholder will veto the project. Underinvestment problems require, therefore, frictions on the controlling shareholders’ ability to square off their differences.

One such a friction is asymmetry of information on the controlling shareholders’ private benefits from the projects. As is well known, information asymmetry may lead to inefficient bargaining, allowing for underinvestment problems to obtain in equilibrium. In our model, asymmetric information between the controlling shareholders increases with the bound $\epsilon$ of the support of the distribution of the private benefits. One would expect, therefore, that underinvestment problems increase with $\epsilon$. Accordingly, Proposition 5 shows that, under shared control, firm value is non-increasing on this measure of asymmetry of information.

**Proposition 5** Firm value with shared control is non-increasing in the degree of information asymmetry $\epsilon$ between the controlling shareholders.

Propositions 1 and 5 summarize the determinants of firm value with monitoring and shared control, respectively. From Proposition 1, firm value with monitoring decreases with costs that make it more difficult for outsiders to evaluate the value added of the project to the minority shareholders. These costs, which are summarized in our model by the parameter $\rho$, do not play a role under shared control. In turn, Proposition 5 implies that, unlike governance structures centered around outside monitors, shared control is less efficient in firms with investment opportunities that are more difficult for insiders to evaluate. We have thus established:
Proposition 6 Large shareholders are more likely to monitor the firm on behalf of outside investors in firms with investment opportunities in projects that are harder for insiders to evaluate (large $\epsilon$). In turn, large shareholders are more likely to join the control group in firms with investment opportunities in projects that are hard for outsiders to evaluate (large $\rho$).

From Proposition 6, an entrepreneur should not simply look for a deep-pocket investor to share control with. It is in the entrepreneur’s interest to find an investor with similar background. Conceivably, a similar background should decrease as much as possible the level of asymmetry of information (small $\epsilon$), increasing the value of sharing the control. Assuming that family members are likely to have common backgrounds, our model thus predicts that shared control should be pervasive in family businesses. In contrast, finding a deep-pocket investor with similar background should be difficult in firms that are built around a high-tech entrepreneur. In these firms, Proposition 6 predicts that a large shareholder should play the role of the classical outside monitor.

Finally, Proposition 7 below tells us that shared control is more likely to be efficient in firms with large financing requirements. The intuition is quite simple. When the investment requirement increases, a single controlling shareholder must sell a larger amount of minority shares to finance the project. The lower control stake makes the initial shareholder internalize a smaller fraction of the firm’s value, increasing his incentives to distort the investment policy and making it harder for a large shareholder to monitor the entrepreneur. Firm value with monitoring should thus decrease with the investment requirement. In contrast, a larger financing requirement does not necessarily imply a reduction of the control stake under shared control. The initial shareholder can raise extra funds by selling more of his control shares to the other controlling shareholder, while keeping the control stake constant. Large financing requirements, therefore, do not reduce the value of firms under shared control.

Proposition 7 Firm value under shared control does not depend on the financing requirement size ($I$), while firm value with monitoring decreases with the financing requirement size ($I$). Therefore, the likelihood of shared control versus monitoring increases with the investment size.

While this section compares the relative efficiency of shared control and monitoring control across firms, our model also yields implications on the relative efficiency of shared control in
countries with different judicial systems. As we show next, shared control is more likely to be efficient in countries that offer weak protection to minority shareholders.

B Governance law

Is there a relation between governance laws and the role that large shareholders play in corporate governance? To answer this question, we consider a change in governance law that makes it harder for controlling shareholders to dilute the value of minority shareholders, and we study the impact of such change in the relative efficiency of shared and unilateral control.

Consider first the situation in which undertaking the project increases private benefits relative to the status quo \( (b > 0) \). In this case, a natural way of modeling an improvement of governance law is by assuming that the change reduces the value of the private benefits to \( b - g \), while increasing the verifiable cash flows to \( y + g \), where \( g \geq 0 \). In other words, the improvement of governance law constrains the firm’s ability to harm minority shareholders.

It is not surprising that such an improvement of governance law increases firm value under the classical monitor. For one, the improvement increases the value of minority shares for any level of monitoring, letting the entrepreneur raise the financing requirement with fewer shares; the entrepreneur’s equity stake thus rises, making him internalize firm value to a greater extent and moving the investment decision closer to the first best. Moreover, an improvement in governance law increases the relative importance of verifiable cash flows relative to private benefits which better aligns the interest of the entrepreneur with minority shareholders.

More surprisingly, the improvement in governance laws to not alter the value of a firm under shared control. To see why, suppose that undertaking the project yields private benefits \( b > 0 \) but harms minority shareholders, that is, \( y < 0 \) (if \( y > 0 \) there would be no conflict of interest). Lemma 2 shows that, without the improvement in governance law, firm value under shared control is \( V_{sc}(0) = \int (b + y) \Gamma \left( \frac{b + \alpha^2 y}{\epsilon} \right) f(b) db \), where \( \alpha^2 \) is the optimal controlling stake. By transferring value from the private benefits to the verifiable cash flows, firm value under shared control changes to \( V_{sc}(g) = \max_{\alpha \in [\alpha_0, 1]} \int (b + y) \Gamma \left( \frac{(b - g) + \alpha(y + g)}{\epsilon} \right) f(b) db \). The controlling stake that solves this problem is \( \alpha^2(g) = \alpha^2 - \frac{(1 - \alpha^2) y}{|y + g|} < \alpha^2 \), that is, the improvement of the governance law reduces the controlling stake; a reduction that weakens the value-enhancing
effect of the improvement of the law because it makes the controlling group internalize less of the firm value. Indeed, one can easily check that plugging $\alpha^2(g)$ into $V_{sc}(g)$ yields $V_{sc}(g) = V_{sc}(0)$.\(^{10}\)

Hence, an improvement in the governance law leads to larger minority stakes but there is no overall change in firm value. Intuitively, the improvement of the law is vacuous because controlling shareholders can more efficiently change their preferences vis-à-vis public and private cash flows by properly choosing the size of the minority stake. An improvement of governance law, therefore, increases the value of a firm under a classical monitor but has no impact on the value of a firm under shared control.

Similar results hold for the situation in which undertaking the project decreases private benefits relative to the status quo ($b < 0$). However, in this case the natural way of modeling an improvement of governance laws is by assuming that it reduces the value of the private benefits by a smaller amount, in line with the idea that there are less private benefits to be destroyed. Accordingly, upon the improvement of the law, we assume that the project destroys private benefits by $b + g \leq b$ while it increases verifiable cash flows by $y - g \geq y$, that is, that $g \leq 0$. We show in the proposition below the following result.

**Proposition 8** Firm value under shared control does not depend on the quality of governance law and the optimal controlling stake is non-increasing in the quality of governance laws. In contrast, an improvement of governance law increases firm value with monitoring. Hence, shared control is more likely to prevail in countries with legal systems that offer weak protection to minority shareholders.

Proposition 8 is consistent with recent research that associates underdeveloped capital markets to judicial systems that offer weak protection to minority shareholders. Weak governance laws increases the relative efficiency of sharing control as opposed to unilateral control to finance investment requirements. Moreover, from Proposition 4, selling minority shares

\(^{10}\text{Plugging the optimal controlling stake into the expression for firm value above yields}\)

$$V_{sc}(g) = \int (b + y) \Gamma \left( \frac{(b - g) + \alpha^2(g)(y + g)}{\epsilon} \right) f(b) db = \int (b + y) \Gamma \left( \frac{b + \alpha^2 y}{\epsilon} \right) f(b) db = V_{sc}(0).$$
to finance investment requirements is suboptimal if the project increases verifiable cash flows that cannot be diverted from minority shareholders. As such, our model predicts that, in countries that offer weak protection to minority shareholders, firms that go public are likely to have investment opportunities that facilitate the diversion of value from minority shareholders. The low capitalization of stocks in emerging markets, therefore, arise in our model as a self-selection phenomenon.

V Discussion

A The stability of shared control: Ex post changes in the control stake

A lower equity stake makes controlling shareholders internalize more of the private benefits of control and less of the firm’s verifiable cash flows. Thus, the stock price prices should fall when a controlling shareholder tries to sell some of his or her shares. This stock price reaction curbs some of the controlling shareholders’ desire to reduce their equity stake. Yet, while a drop in the equity value is shared with the minority shareholders, the controlling shareholders capture all of the increase in private benefits that ultimately explains the lower equity value. Depending on the joint distribution of the private benefits and the verifiable cash flows, the stock price reaction may not fully curb ex post incentives for lowering the control stake. As such, one may wonder wether shared control is a stable governance structure. As we argue below, supermajority rules can prevent ex post incentives to reduce the control stake from breaking down an ex-ante optimal ownership structure based on shared control.

Suppose that an ex ante optimal ownership structure requires that a fraction $\alpha^*$ of the shares be held by the controlling group. For a given voting structure, the stake $\alpha^*$ is associated with a number of votes, say $v$. The initial shareholder can avoid ex-post incentives to reduce the controlling stake below $\alpha^*$ by giving control to a group of investors who holds a fraction $v$ of the votes. With this mechanism, which can be interpreted as a supermajority rule, the controlling shareholders cannot divest below $v$ without bearing the risk of losing control.
B Dissolving the controlling group

In the absence of credit constraints, a buyout is a natural mechanism to eliminate bargaining problems that, although ex-ante optimal, are ex-post inefficient from the perspective of the controlling shareholders. In our model, the second controlling shareholder may have some debt capacity left after his purchasing of the control stake. If so, he may try to acquire full control by making an offer for the remaining control shares.

Consider then that a controlling shareholder can make a buyout offer between the time that he learns his valuation of the project and the time that the investment decision has to be made. We ask whether there is an incentive compatible direct mechanism that lets the controlling shareholders dissolve their partnership with probability 1. If so, ex-post bargaining problems can be solved by a buyout, and the temporary presence of multiple controlling shareholders would not increase value.

Proposition 9 shows that the same asymmetry of information that prevents the controlling shareholders from agreeing with the investment decision may prevent them from dissolving the control group.

Proposition 9 (Dissolving the controlling group) Assume that controlling shareholder \( j \) receives a signal \( s_i \) of the private benefits of controlling shareholder \( i \neq j \), with \( b_i = s_i - \frac{\epsilon}{2} \), \( \bar{b}_i = s_i + \frac{\epsilon}{2} \), and \( \sum_{i=1}^{2}(b_i + \alpha_i y) < 0 < \sum_{i=1}^{2}(\bar{b}_i + \alpha_i y) \). Then there is no ex-post efficient mechanism that dissolves the controlling group after the controlling shareholders have privately learned their valuations.

Proposition 9 departs from Cramton, Gibbons, and Klemperer (1987), who argue that a partnership can always be efficiently dissolved if the equity holdings are evenly spread across several partners. (Proposition 9 can be generalized to more than two controlling shareholders.) The way we model the private benefits of control is the key to explaining the difference in the results. In Cramton, Gibbons, and Klemperer, the value of the firm to each controlling shareholder is proportional to the fraction of shares that they own. As a result, in an evenly distributed ownership structure, the cost of extracting a truthful announcement of the firm’s value decreases with the number of controlling shareholders. In our model, a controlling shareholder may have large private benefits of control in spite of an evenly distributed ownership structure.
A buyout is just one example of a coalition of shareholders that can overturn an existing control structure. In principle, any member of a controlling group may be co-opted to participate in a new coalition that aims to defeat the incumbent controlling group. Bennedsen and Wolfenzon (2000) argue that the size of the controlling stake is determined by these coalition games, which our model ignores. As it turns out, shareholders’ agreements can prevent exclusions and defections that unravel the controlling group. For instance, voting agreements in Italy assure that each member of the agreement – the voting syndicate – will vote together. Voting syndicates, therefore, can prevent coalition games from unravelling governance structures based on shared control.

C Incentive contracts

So far, we have ignored incentive schemes that align the interests of minority and controlling shareholders. One could argue, however, that incentive contracts are a cheaper and more efficient mechanism to protect minority shareholders than shared control. Incentive contracts – the argument goes – prevent an inefficient dilution of the rights of minority shareholders, without imposing the bargaining costs associated with shared control.

Suppose then that the initial entrepreneur decides to remain as the single controlling shareholder, committing to an incentive scheme before selling minority shares to finance the project. In our framework, an efficient incentive scheme pays the entrepreneur an amount \( F(y) \), which is a function of the verifiable cash flows. (Because private benefits are not verifiable by a court, the compensation scheme cannot be contingent on \( b \).) Under this compensation scheme, the entrepreneur will invest if and only if his total payoff from the project, which now includes the compensation \( F(y) \), is bigger than or equal to zero, i.e., \( \alpha_1 y + b + F(y) \geq 0 \).

Of course, the first best investment rule is still \( y + b \geq 0 \); invest if and only if the project increases firm value. Thus, the incentive contract implements the first best if and only if \( \alpha_1 y + F(y) + b = y + b \), which implies that \( F(y) = (1 - \alpha_1)y \). But then the controlling shareholder captures all of the project’s cash flows, leaving no incentives for outside investors to pay \( I \) for minority shares.\(^{11}\) With unilateral control, therefore, incentive contracts cannot

\(^{11}\)Given our assumption that the firm is credit constrained (i.e., \( V_0 < I \)), investors cannot count on the assets in place to recover all of the investment \( I \). Assuring the market return for the minority shares thus requires that outside investors capture some of the project’s verifiable returns.
obtain the first best while letting the entrepreneur finance the whole project with minority shares. In other words, unilateral control implies an inefficient investment policy despite optimal contracting. And we conclude that allowing for incentive contracts does not change the main result of this paper, namely, sharing control may dominate unilateral control by protecting minority shareholders while preserving valuable private benefits of control.

D Bargaining under credit constraint

In the bargaining game under shared control, we have assumed that the controlling shareholders can use shares as side payments. Yet, controlling shareholders may be unwilling to dispose of their shares if, for example, lowering the equity stake makes a controlling shareholder vulnerable to a control fight.\textsuperscript{12} In this case, the bargaining problems are magnified, but, as we show next, the main result of the paper holds: Shared control may dominate ownership structures with a single controlling shareholder.

Let us then assume that controlling shareholders cannot use shares as side payments in the bargaining over the investment decision. As in Aghion and Bolton (1992), the firm will undertake the project only if there is unanimity in the control group. Under such investment rule, the controlling shareholders do not gain anything by falsifying their private benefits from the project. Without loss of generality, we can therefore assume that the controlling shareholders bargain over the investment decision with complete information on the private benefits of control.

It then follows that ruling out transfers changes the bargaining over the investment decision in two ways: First, it eliminates the controlling shareholders’ incentives to hide their private benefits. In the Chatterjee-Samuelson mechanism, those incentives increase the hurdle for accepting the project. Second, the lack of the transfer payments makes it essential that the project increases the expected payoff of both controlling shareholders. As such, projects that are highly profitable for one (and only one) controlling shareholder will not be implemented because we no longer allow for a side-payment that convinces the opposing controlling shareholder to accept the project.

\textsuperscript{12}More formally, transfers of shares will not support the transfer payments that are asked for in the Chatterjee-Samuelson mechanism if, contrary to Lemma 1, the minimum number of shares to retain control is large while the value of the project, $I$, is low.
While the enhanced focus on the controlling shareholders’ individual well being clearly reduces the overinvestment problem (at the possible cost of an increase in the underinvestment problem), eliminating the controlling shareholders’ incentives to hide their private benefits reduces the hurdle for the acceptance of the project, magnifying the overinvestment problem. We thus have that ruling out transfers implies conflicting effects on the firm’s investment decisions.

Still, shared control is likely to dominate unilateral control in firms in which overinvestment is the main source of agency costs. The reason is as follows. Regardless of the side payments, the bargaining over the investment decision is likely to induce ex post inefficiencies. When side payments are possible, the inefficiency in our model stems from the controlling shareholders’ incentives to hide their private benefits. When side payments are ruled out, a controlling shareholder cannot offer part of his private benefits to a controlling shareholder who opposes the project. In any of these two cases, sharing control decreases the likelihood of overinvestment problems and increases the likelihood of underinvestment problems. Accordingly, shared control is efficient when the costs of increasing underinvestment problems are more than offset by the benefits of reducing overinvestment problems; a condition that is satisfied when the probability that the project is inefficient is sufficiently large.

Nonetheless, allowing for transfers is not a harmless assumption. To see why, suppose that the total private benefits from the project and the verifiable cash-flows can take only two points, $b \in \{-\hat{b}, \hat{b}\}$ and $y \in \{-\hat{y}, \hat{y}\}$, and the project is efficient if and only if $b > 0$. In this extreme case, unilateral control is efficient regardless of the level $I$ of financing requirements. But, in the absence of transfers, shared control will be inefficient (regardless of $I$) if only one of the controlling shareholders will capture the private benefits. In this case, the controlling shareholder with private benefits cannot prevent the other controlling shareholder from inefficiently vetoing the project when $b = \hat{b}$ and the verifiable cash-flow is negative. In the absence of transfers, therefore, shared control may not be more likely to be efficient in firms with large financing requirements, as predicted by Proposition 7.
VI Conclusion

In the corporate control literature, large shareholders are usually assumed to monitor managers on behalf of all shareholders. As La Porta, Lopez-de-Silanes, and Shleifer (1999) document, however, large shareholders often participate in the management. Accordingly, Pagano and Roell (1998) suggest that an optimal ownership structure may require multiple large shareholders: It takes a large shareholder to monitor a large shareholder in control. Yet, a vast literature on corporate law does not view large shareholders as monitoring each other on behalf of minority shareholders. Instead, large shareholders are perceived as decision makers who seek to influence corporate decisions in a way that favors their personal agendas.

This paper argues that firm characteristics and governance laws determine the role that large shareholders play in corporate governance. In firms whose investment opportunities that are hard for insiders to evaluate, sharing control creates bargaining problems that exacerbate the risk of corporate paralysis. Control should not be divided, and, as in the corporate control literature, monitoring by a large outside investor arises as the most efficient way to protect minority shareholders. In contrast, sharing control increases efficiency in countries with legal systems that offer weak protection to minority shareholders, in firms with projects that are hard for outsiders to evaluate, and when external financing requirements are large. In these cases, multiple large shareholders should participate in the firm’s management, as assumed in the corporate law literature.
References


Appendix

Proof of Proposition 1:
i) case $y < 0$. The value of the firm gross of monitoring costs is

$$V_m(\beta) = E \left[ (y + b) (1 - m(y, \beta)) - \frac{\rho m(y, \beta)^2}{2} | b > -\alpha^1 y \right] P(b > -\alpha^1 y) =$$

$$= \left[ (y + E [b | (b + \alpha^1 y) > 0]) (1 - m(y, \beta)) - \frac{\rho m(y, \beta)^2}{2} \right] P(b > -\alpha^1 y)$$

where $m(y, \beta) = \min\{\frac{\beta}{\rho} |y|, 1\}$ (note that if $b + \alpha^1 y < 0$, the manager does not undertake the project and there is no monitoring).

Define $\underline{b} = E \left[ b | (b + \alpha^1 y) > 0 \right]$. The problem with monitoring is $\max_{\beta \in [0, 1 - \alpha^1]} V_m(\beta)$, and the maximum is $V_m$, the firm value under monitoring. Note that

$$\frac{d}{d\beta} \left( (y + \underline{b}) \left( 1 - \frac{\beta}{\rho} |y| \right) - \rho \left( \frac{\beta}{\rho} |y| \right)^2 \right) = -\frac{1}{\rho} |y| (\underline{b} + y) - \frac{1}{\rho^2} \beta |y|^2$$

(and the second derivative is negative). Therefore monitoring is valuable, i.e. $V_m > V_m(0) = V_{uc}$, whenever $(\underline{b} + y) < 0$.

The precise value for the optimal stake is, if $\frac{1}{y} (\underline{b} + y) < 1 - \alpha^1$ and $\frac{1}{y} (\underline{b} + y) \leq \frac{\rho}{|y|}$ (interior condition constraints), equal to

$$\beta^* = \begin{cases} \frac{1}{y} (\underline{b} + y) & \text{if } (\underline{b} + y) < 0 \\ 0 & \text{if } (\underline{b} + y) > 0 \end{cases}$$

(note that if $(\underline{b} + y) < 0$ then $\beta^* < 1$), and the optimal value is, when both constraints are satisfied,

$$V_m = \begin{cases} \frac{1}{2\rho} (\underline{b} + y)^2 P(b > -\alpha^1 y) + V_{uc}^* & \text{if } (\underline{b} + y) < 0 \\ (\underline{b} + y) P(b > -\alpha^1 y) = V_{uc} & \text{if } (\underline{b} + y) > 0. \end{cases}$$

ii) Case $y > 0$. Let $\bar{b} = E [b | b < -\alpha^1 y]$.

$$V_m(\beta) = E \left[ (y + b) (1 - m(y, \beta)) - \frac{\rho m(y, \beta)^2}{2} | b > -\alpha^1 y \right] P(b > -\alpha^1 y) =$$

30
\[
\begin{align*}
= \left[ (y + E[b] (b + \alpha^1 y) > 0) \right] (1 - m(y, \beta) - \frac{\rho m(y, \beta)^2}{2}) P(b > -\alpha^1 y),
\end{align*}
\]

where \( m(y, \beta) = \min \{ \frac{\beta}{\rho} y, 1 \} \). A similar argument as above yields that the optimal stake is

\[
\beta^* = \begin{cases} 
\frac{1}{\beta} (\bar{b} + y) & \text{if } (\bar{b} + y) > 0 \\
0 & \text{if } (\bar{b} + y) < 0,
\end{cases}
\]

where \( \bar{b} = E[b|b < -\alpha^1 y] \) and the optimal value is

\[
V_m = \begin{cases} 
V_{uc} + \frac{1}{2\rho} (\bar{b} + y)^2 P(b < -\alpha^1 y) & \text{if } (\bar{b} + y) > 0 \\
(\bar{b} + y) P(b > -\alpha^1 y) = V_{uc} & \text{if } (\bar{b} + y) < 0.
\end{cases}
\]

\[\Box\]

**Proof of Lemma 1:** Let \( V_0 + I \) be the value of equity that arises from the assets in place (i.e., excluding the project). In case the project is undertaken, the verifiable cash flow increases by \( y \). Since the verifiable cash flow \( y \) is known by both controlling shareholders, they agree that controlling shareholder \( j \)'s wealth increases by \( \alpha(y + V_0 + I) \geq 0 \) if he receives a fraction \( \alpha \) of the equity stake of controlling shareholder \( i \). Given that \( \alpha \) suffices for any controlling shareholder to retain control, controlling shareholder \( i \) can offer up to \( (\alpha - \alpha)(y + V_0 + I) \) as a side payment in any bargaining game. If \( y \) is the minimum verifiable cash flow, then \( (\alpha - \alpha)(y + V_0 + I) \) is a transfer that controlling shareholder \( i \) can afford with probability 1.

Now, let \( \bar{V}_i \equiv \bar{b}_i + \alpha_i \bar{y} \) be the controlling shareholder \( i \)'s maximum valuation for the project, with \( \bar{b}_i \) an upper bound on the private benefits of controlling shareholder \( i \) and \( \bar{y} \) an upper bound on the project’s verifiable cash flow. Clearly, controlling shareholder \( i \) will not announce a valuation that implies a transfer larger than \( \bar{V}_i \). In equilibrium, therefore, the transfer payment is bounded by \( \bar{b}_i + \alpha_i \bar{y} \). Hence, a sufficient condition for transfers of shares to implement any incentive-compatible bargaining mechanism is that, with probability 1, any controlling shareholder \( i \in \{1, 2\} \) can afford the upper bound on the transfer payment by transferring an amount of shares that does not lead to a loss of control. Formally,

\[
(\alpha_i - \alpha)(y + V_0 + I) \geq \bar{b}_i + \alpha_i \bar{y}, \text{ for } i \in \{1, 2\},
\]

31
which implies condition (4) when \( \alpha_i > \alpha_i^* \) for \( i \in \{1, 2\} \).

\[\Box\]

**Proof of Proposition 2**: From Lemma 2, condition 4 implies that the controlling shareholders can honor any incentive compatible payment that may arise from the Chatterjee-Samuelson mechanism by transferring their shares. Thus, let \( V_i(x_i, s_1, s_2) \) be the best announcement of controlling shareholder \( i \in \{1, 2\} \), where \( x_i = b_i + \alpha_i y \) is controlling shareholder \( i \)'s valuation of the project. To simplify the notation, we will henceforth omit the arguments \( s_1 \) and \( s_2 \) in \( V_i(x_i, s_1, s_2) \). In addition, let \( u_i \equiv s_i + \frac{x_i}{2} + \alpha_i y \) and \( l_i \equiv s_i - \frac{x_i}{2} + \alpha_i y \) be, respectively, the upper and lower bounds of controlling shareholder \( i \)'s valuation of the project given the signal \( s_i \) and the verifiable return \( y \).

Standard arguments in the mechanism design literature show that, in any Bayesian equilibrium, \( V_i(x_i) \) increases with the valuation of the project \( x_i \). Moreover, Lemma 3, below, shows that the equilibrium announcements must satisfy a system of differential equations.

**Lemma 3** In any Bayesian equilibrium in which the best policies \( V_1(x_1) \) and \( V_2(x_2) \) are differentiable, the following linked differential equations hold:

\[
V_1^{-1}(-V_2(x_2)) + V_2(x_2) = \frac{1}{2} \frac{(1 - F_2(x_2))}{f_2(x_2)} V'_2(x_2) \tag{10}
\]

\[
V_2^{-1}(-V_1(x_1)) + V_1(x_1) = \frac{1}{2} \frac{(1 - F_1(x_1))}{f_1(x_1)} V'_1(x_1), \tag{11}
\]

where \( F_i(x_i) \) and \( f_i(x_i) \) are, respectively, the distribution and the density of \( x_i = b_i + \alpha_i y \) conditioned on the signals \( s_1 \) and \( s_2 \).

**Proof.** Given \( V_1 \), the minimal announcement \( V_2^* \) that implies the undertaking of the project must satisfy \( V_1 + V_2^* = 0 \). Since \( V_2(x_2) \) increases with \( x_2 \), \( V_2^* \) induces a cutoff for the valuation \( x_2 \) of the second controlling shareholder: \( V_1 + V_2(x_2^*) = 0 \Rightarrow x_2^* = V_2^{-1}(-V_1) \). The expected payoff of the initial shareholder given an announcement \( V_1 \) and a valuation \( x_1 \) is then equal to

\[
\Pi_1(V_1, x_1) = \int_{V_2^{-1}(-V_1)}^{u_2} [x_1 - \frac{1}{2}(V_1 - V_2(x_2))] f_2(x_2) dx_2.
\]

32
Assume first that any small perturbation from \( V_1(x_1) \) affects the probability that the project will be undertaken. Then \( V_1 \) maximizes the expected payoff off the initial shareholder if and only if

\[
\frac{\partial \Pi_1(V_1, x_1)}{\partial V_1} = -(x_1 - V_1) f_2(V_2^{-1}(-V_1)) \frac{dV_2^{-1}(-V_1)}{dV_1} - \frac{1}{2} \int_{V_2^{-1}(-V_1)}^{u_2} f_2(x_2) dx_2 = 0.
\]

If \( V_1 \) is an optimal response, \( V_1 + V_2(x_2^*) = 0 \) implies that the initial shareholder’s valuation, \( x_1^* \), that led to \( V_1 \) solves \( V_1(x_1^*) + V_2(x_2^*) \Rightarrow x_1^* = V_1^{-1}(-V_2(x_2^*)) \). Plugging \( x_1^* \) into equation (12) yields equation (10):

\[
V_1^{-1}(-V_2(x_2^*)) + V_2(x_2^*) - \frac{1}{2} \frac{(1 - F_2(x_2^*))}{f_2(x_2^*)} V_2'(x_2^*) = 0.
\]

The proof that equation (11) holds when any small perturbation of \( V_2 \) affects the chances that the project will be undertaken is analogous.

Suppose now that a perturbation of \( V_1(x_1) \) does not change the probability that the project will be undertaken. In particular, \( x_1 \) may be so large that, given \( V_1(.) \) and \( V_2(.) \), the project will be undertaken regardless of the announcement of the second controlling shareholder. In this case, there is a \( x_2^* \in [l_2, u_2] \) such that \( V_1(l_1) + V_2(x_2^*) = 0 \). Still, the differential equation associated with the initial shareholder’s announcement remains unchanged, as we show below.

\[
\Pi_1^*(V_1, x_1) = \int_{V_2^{-1}(-V_1)}^{x_2^*} [x_1 - \frac{1}{2}(V_1 - V_2(x_2))] f_2(x_2) dx_2 + \int_{x_2^*}^{u_2} [x_1 - \frac{1}{2}(V_1 - V_2(x_2))] f_2(x_2) dx_2.
\]

\[
\frac{\partial \Pi_1^*(V_1, x_1)}{\partial V_1} = -(x_1 - V_1) f_2(V_2^{-1}(-V_1)) \frac{dV_2^{-1}(-V_1)}{dV_1} - \frac{1}{2} \int_{V_2^{-1}(-V_1)}^{x_2^*} f_2(x_2) dx_2 - \frac{1}{2} \int_{x_2^*}^{u_2} f_2(x_2) dx_2.
\]
which is the first order condition that yields equation (10).

A second boundary case happens when the valuation of a controlling shareholder is so low that it blocks the project regardless of the announcement of the other controlling shareholder. To characterize this situation, let $x_1^{**}$ be the minimum valuation of the initial shareholder when the second controlling shareholder’s announcement is as large as possible, that is, $V_2(u_2)$. Then, $V_1(x_1^{**}) + V_2(u_2) = 0$, and the project will not be undertaken for any $x_1 < x_1^{**}$. Since the project will not be undertaken, the announcement of the initial shareholder is irrelevant. It is then optimal to set $V_1(x_1)$ satisfying equation (10) for $x_1 < x_1^{**}$ with the understanding that the project will not be undertaken. Similarly, $V_1(u_1) + V_2(x_2^{**}) = 0$ implies that the project will not be undertaken for $x_2 < x_2^{**}$, and we can assign $V_2(x_2)$ satisfying equation (11).

The proof of the Proposition follows from equations (10) and (11). Conditioned on $s_i$, $b_i$ is uniformly distributed in the interval $[l_i - \alpha_i y, u_i - \alpha_i y]$. Standard computations then show that, conditioned on $s_i$, the hazard rate of the random variable $x_i \equiv b_i + \alpha_i y$ is \( \frac{(1-F_i(x_i))}{f_i(x_i)} = u_i - x_i \).

Plugging this hazard ratio into equations (10) and (11) yields

\[
V_1^{-1}(-V_2(x_2)) = \frac{1}{2} (u_2 - x_2) V_2'(x_2) - V_2(x_2)
\]

\[
V_2^{-1}(-V_1(x_1)) = \frac{1}{2} (u_1 - x_1) V_1'(x_1) - V_1(x_1).
\]

Assume now that there is a solution for the above system of differential equations that is linear in the valuation $x_i$, that is, $V_1(x_1) = Ax_1 + B$ and $V_2(x_2) = Cx_2 + D$. Thus

\[
V_1^{-1}(-(Cx_2 + D)) = \frac{1}{2} (u_2 - x_2) C - (Cx_2 + D)
\]

\[
V_2^{-1}(-(Ax_1 + B)) = \frac{1}{2} (u_1 - x_1) A - (Ax_1 + B).
\]

Plugging $V_1^{-1}(-(Cx_2 + D)) = \frac{-(Cx_2 + D) - B}{A}$, $V_2^{-1}(-(Ax_1 + B)) = \frac{-(Ax_1 + B) - D}{C}$, and collecting terms gives us

\[
\left( \frac{-C}{A} + \frac{3}{2}C \right) x_2 - \frac{B + D}{A} = \frac{1}{2} Cu_2 - D
\]

(13)
\[
\left( \frac{-A}{C} + \frac{3}{2} A \right) x_1 - \frac{B + D}{C} = \frac{1}{2} A u_1 - B.
\]

This system of equations must hold for all values of \( x_1 \) and \( x_2 \), which requires that \(- \frac{A}{C} + \frac{3}{2} A = 0 \Rightarrow A = \frac{2}{3} \) and \(- \frac{A}{C} + \frac{3}{2} A = 0 \Rightarrow C = \frac{2}{3} \). Plugging \( A = \frac{2}{3} \) and \( C = \frac{2}{3} \) into the system of equations (13) obtains

\[
-(B + D) = \frac{2}{9} u_2 - \frac{2}{3} D
\]
\[
-(B + D) = \frac{2}{9} u_1 - \frac{2}{3} B
\]

Solving this system of equation gives us \( D = -\frac{1}{3} u_1 + \frac{1}{12} u_2, \ B = \frac{1}{12} u_1 - \frac{1}{4} u_2 \). The optimal announcements of the controlling shareholders as a function of their valuations are then

\[
V_1 (x_1) = \frac{2}{3} x_1 + \frac{1}{12} u_1 - \frac{1}{4} u_2,
\]
\[
V_2 (x_2) = \frac{2}{3} x_2 + \frac{1}{12} u_2 - \frac{1}{4} u_1.
\]

From above, \( V_1 (x_1) + V_2 (x_2) \geq 0 \) is equivalent to \( \frac{2}{3} x_1 + \frac{1}{12} u_1 - \frac{1}{4} u_2 + \frac{2}{3} x_2 + \frac{1}{12} u_2 - \frac{1}{4} u_1 \geq 0 \), which implies \( x_1 + x_2 \geq \frac{1}{6} (u_1 + u_2) \). Plugging \( x_i = b_i + \alpha_i y \) and \( u_i = s_i + \frac{\alpha}{2} + \alpha_3 y \) into this last inequality yields \( b_1 + b_2 + (\alpha_1 + \alpha_2) y \geq \frac{1}{4} \{ s_1 + s_2 + (\alpha_1 + \alpha_2) y + \epsilon \} \).

We now show that the investment rule holds in the boundaries as well. If, for instance, \( x_2 \) is large enough to imply the investment regardless of the announcement of the initial shareholder, Lemma 3 shows that \( V_2 = V_2 (x_2^*) \) for \( x_2 \geq x_2^* \), where \( x_2^* \) solves \( V_1 (l_1) + V_2 (x_2^*) = 0 \). Thus, \( V_1 (x_1) + V_2 (x_2) \geq V_1 (x_1) + V (x_2^*) \geq 0 \Rightarrow V_1 (x_1) + V_2 (x_2) \geq 0 \Rightarrow b_1 + b_2 + (\alpha_1 + \alpha_2) y \geq \frac{1}{4} \{ s_1 + s_2 + (\alpha_1 + \alpha_2) y + \epsilon \} \). Conversely, \( V_1 (x_1) + V_2 (x_2) < 0 \Rightarrow V_1 (x_1) + V (x_2^*) < 0 \), which is not consistent with the assumption that the investment will happen for \( x_2 \geq x_2^* \) with probability 1. Therefore, investing if and only if \( b_1 + b_2 + (\alpha_1 + \alpha_2) y \geq \frac{1}{4} \{ s_1 + s_2 + (\alpha_1 + \alpha_2) y + \epsilon \} \) is optimal when some realization of \( x_2 \) implies the undertaking of the project regardless of the realization of \( x_1 \). The same argument can be used to show that \( b_1 + b_2 + (\alpha_1 + \alpha_2) y \geq \frac{1}{2} \{ s_1 + s_2 + (\alpha_1 + \alpha_2) y + \epsilon \} \) characterizes the optimal investment rule when a large \( x_1 \) implies the undertaking of the project regardless of \( x_2 \). Finally, Lemma 3 shows that \( V_1 (x_1) \) and \( V_2 (x_2) \) are optimal announcements when \( x_1 \) and \( x_2 \) are such that the probability that the project will be undertaken is zero. Moreover, \( V_1 (x_1) + V_2 (x_2) < 0 \) in these cases. Hence, the investment
rule $b_1 + b_2 + (\alpha_1 + \alpha_2)y \geq \frac{1}{3}(s_1 + s_2 + \alpha y + \epsilon)$ still applies.

\[ \square \]

**Proof of Lemma 2:** Let $K_\epsilon(x)$ be the following integral:

$$K_\epsilon(x) = \frac{1}{\epsilon^2} \int_{-\epsilon/2}^{\epsilon/2} \int_{-\epsilon/2}^{\epsilon/2} I(\epsilon_1 + \epsilon_2 > x)de_1de_2.$$ 

Clearly, for all $x \geq \epsilon$, $K_\epsilon(x) = 0$, and for all $x \leq -\epsilon$, $K_\epsilon(x) = 1$. In the interval $[0, \epsilon]$, $K_\epsilon(x) = \frac{1}{\epsilon^2} \int_{x-\epsilon/2}^{\epsilon/2} (\epsilon - (x - \epsilon_2))de_2 = \frac{(\epsilon-x)^2}{2\epsilon^2}$, and in the interval $[-\epsilon, 0]$, $K_\epsilon(x) = \frac{1}{\epsilon^2} \int_{x+\epsilon/2}^{\epsilon/2} \epsilon de_2 + \frac{1}{\epsilon^2} \int_{-\epsilon/2}^{x+\epsilon/2} (\epsilon - (x - \epsilon_2))de_2 = 1 - \frac{(\epsilon+x)^2}{2\epsilon^2}$.

Note that $\Gamma(\frac{\epsilon}{\epsilon}) = K_\epsilon(\epsilon - 3x)$, where $\Gamma(x)$ is the function defined in the statement of the Lemma. Therefore, since $b + \alpha^2 y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2)$ is equivalent to $\epsilon_1 + \epsilon_2 > \epsilon - 3(b + \alpha^2 y) = x$, we have that $\Gamma \left( \frac{b+\alpha^2 y}{\epsilon} \right) = \frac{1}{\epsilon^2} \int_{-\epsilon/2}^{\epsilon/2} \int_{-\epsilon/2}^{\epsilon/2} I(b + \alpha^2 y > \frac{1}{3}(\epsilon - \epsilon_1 - \epsilon_2))de_1de_2$, which concludes the proof.

\[ \square \]

**Proof of Proposition 4:** The paragraph following the statement of the proposition demonstrates that if $P(b + y \in (0, \frac{2}{3}x)) = 0$, then share control with $\alpha^2 = 1$ achieves the first-best. Assume now that $P(b + y \in (0, \frac{2}{3}x)) > 0$. Since $V_{sc}(\alpha^2) = \int (b + y) \Gamma(\frac{b+\alpha^2 y}{\epsilon})f(b)db$ and $\Gamma(x)$ is a continuous and differentiable function, so is $V_{sc}(\alpha^2)$. An interior optimum for the control stake must thus satisfy $\frac{dV_{sc}}{d\alpha^2} = \frac{d}{d\alpha^2} \int (b + y) \Gamma(\frac{b+\alpha^2 y}{\epsilon})f(b)db = 0$ with $\frac{d\Gamma}{d\alpha^2} \geq 0$ for a corner solution at $\alpha^2 = 1$. To evaluate $\frac{dV_{sc}}{d\alpha^2}$, we consider two cases ($y < 0$ and $y > 0$) after noting that $\Gamma'(x)$ is a continuous and positive triangular function that is equal to

$$\Gamma'(x) = \begin{cases} 
0 & \text{if } x \geq \frac{2}{3} \\
6 - 9x & \text{if } x \in \left[ \frac{1}{3}, \frac{2}{3} \right] \\
9x & \text{if } x \in \left[ 0, \frac{1}{3} \right] \\
0 & \text{if } x \leq 0.
\end{cases}$$

i) Case $y < 0$. 

36
(i.a) At \( \alpha^2 = 1 \), \( \frac{dV_{sc}}{d\alpha^2} < 0 \): \( \frac{dV_{sc}}{d\alpha^2} \big|_{\alpha^2=1} = \frac{y}{2} \int (b+y) \Gamma'(\frac{b+y}{y}) f(b) db < 0 \), because \( P(b+y \in (0, \frac{\alpha}{3}) > 0 \) and \( \Gamma'(\frac{b+y}{y}) > 0 \) in the region \( b+y \in (0, \frac{\alpha}{3}) \) and is zero elsewhere.

(i.b) At any \( \alpha^2 \leq 1 - \frac{2\alpha}{3|y|} \), \( \frac{dV_{sc}}{d\alpha^2} \geq 0 \): for all \( b+\alpha^{2}y \in (0, \frac{\alpha}{3}) \) then \( b+y \in ((1-\alpha^2) y, \frac{2\alpha}{3} + (1-\alpha^2) y) \), where \( \frac{2\alpha}{3} + (1-\alpha^2) y \leq 0 \). Therefore, \( \Gamma'(\frac{b+\alpha^{2}y}{y}) \geq 0 \), and if \( \Gamma'(\frac{b+\alpha^{2}y}{y}) > 0 \) then \( b+y \leq 0 \).

Items (i.a), (i.b), and the continuity of \( V_{sc} (\alpha^2) \) imply that the optimum \( \alpha^2 \) is in the range \( (1 - \frac{2\alpha}{3|y|}, 1) \).

ii) Case \( y > 0 \).

We show that \( V_{sc} (\alpha^2) = \int (b + y) \Gamma(\frac{b+\alpha^{2}y}{y}) f(b) db \) is non-decreasing in \( \alpha^2 \). Because \( \Gamma(x) \) is a non-decreasing function in \( x \), \( \Gamma(\frac{b+\alpha^{2}y}{y}) \) is non-decreasing in \( \alpha^2 \) \( (y > 0) \). Since shared control cannot imply overinvestment, \( b+y = 0 \) implies \( \Gamma(\frac{b+\alpha^{2}y}{y}) = 0 \). Hence, without loss of generality we can assume \( (b+y) f(b) \geq 0 \), implying that \( (b+y) \Gamma(\frac{b+\alpha^{2}y}{y}) f(b) \) is non-decreasing in \( \alpha^2 \).

\( \square \)

**Proof of Proposition 5:** Let the optimal value under shared control be \( V_{sc}(\epsilon) = V_{sc}(\epsilon, \alpha^*(\epsilon)) \), where \( \alpha^*(\epsilon) \) is the optimal stake with \( \epsilon \). We must analyze two cases.

i) Case \( y > 0 \). We have shown in the text that \( \alpha^*(\epsilon) = 1 \) so \( V_{sc}(\epsilon) = \int (b + y) \Gamma(\frac{b+\alpha^{2}y}{y}) f(b) db \).

But note that \( \Gamma(\frac{\epsilon}{y}) \) is non-increasing function of \( \epsilon \) for all \( b+y \geq 0 \), and \( \Gamma(\frac{b+\alpha^{2}y}{y}) = 0 \) for all \( b+y < 0 \). Therefore, \( V_{sc}(\epsilon) \) is non-increasing in the disagreement measure \( \epsilon \).

ii) Case \( y < 0 \). Our goal is to show that at any point \( \epsilon = \epsilon_0 \) the derivative \( \frac{dV_{sc}(\epsilon)}{d\epsilon} \big|_{\epsilon=\epsilon_0} \leq 0 \). Let \( \alpha^*(\epsilon) \) be the optimal solution, and let the function \( \alpha(\epsilon) \) be implicitly defined by \( \Gamma(\frac{b+\alpha(\epsilon)y}{y}) = \Gamma(\frac{b+\alpha^*(\epsilon)y}{y}) \). Note that \( \alpha(\epsilon_0) = \alpha^*(\epsilon_0) \), and \( \alpha(\epsilon) \) exists and is uniquely determined by the monotonicity of \( \Gamma(\cdot) \). By the the envelope theorem, we have that \( \frac{dV_{sc}(\epsilon, \alpha(\epsilon))}{d\epsilon} \big|_{\epsilon=\epsilon_0} = \frac{dV_{sc}(\epsilon, \alpha^*(\epsilon_0))}{d\epsilon} = \frac{dV_{sc}(\epsilon)}{d\epsilon} \big|_{\epsilon=\epsilon_0} \).

Since at the optimum, which is an interior solution by proposition 4, \( \frac{dV_{sc}(\epsilon, \alpha^*(\epsilon))}{d\alpha} = 0 \).

Define \( g_{\epsilon}(b) = \Gamma(\frac{b+\alpha^*(\epsilon)y}{y}) \). The derivative is \( \frac{dV_{sc}(\epsilon, \alpha(\epsilon))}{d\epsilon} \big|_{\epsilon=\epsilon_0} = \int (b+y) \frac{d\Gamma}{d\alpha}(\frac{b+\alpha(\epsilon)y}{y}) f(b) db \). We now show that \( \frac{d\Gamma}{d\alpha}(\frac{b+\alpha(\epsilon)y}{y}) \big|_{\epsilon=\epsilon_0} \leq 0 \) for all \( b \geq -y \), and \( \frac{d\Gamma}{d\alpha}(\frac{b+\alpha(\epsilon)y}{y}) \big|_{\epsilon=\epsilon_0} \geq 0 \) for \( b \leq -y \) (note that this
proves that \( \frac{dV_{sc}(\epsilon, \alpha(\epsilon))}{d\epsilon} \bigg|_{\epsilon=\epsilon_0} \leq 0 \). The derivative \( \frac{dg_{e}(b)}{d\epsilon} \bigg|_{\epsilon=\epsilon_0} \) is equal to

\[
\frac{dg_{e}(b)}{d\epsilon} \bigg|_{\epsilon=\epsilon_0} = \frac{\partial \Gamma(\frac{b+\alpha(\epsilon_0)\epsilon}{\epsilon_0})}{\partial x} \left( \alpha'(\epsilon_0) \epsilon_0 y - (b + \alpha(\epsilon_0) y) \right).
\]

where \( \frac{\partial \Gamma(\frac{b+\alpha(\epsilon_0)\epsilon}{\epsilon_0})}{\partial x} \geq 0 \), which implies that \( \frac{dg_{e}(b)}{d\epsilon} \bigg|_{\epsilon=\epsilon_0} \) is decreasing in \( b \). But note that at \( b = -y \),

\[
\frac{dg_{e}(-y)}{d\epsilon} \bigg|_{\epsilon=\epsilon_0} = 0,
\]

because by the definition of \( \alpha(\epsilon) \), \( g_\epsilon(-y) = g_{\epsilon_0}(-y) \) for all \( \epsilon \). Finally this yields that \( \frac{dV_{sc}(\epsilon)}{d\epsilon} \bigg|_{\epsilon=\epsilon_0} \leq 0 \), which concludes the proof.

**Proof of Proposition 7:** With a single controlling shareholder, a larger investment requirement implies a larger equity sale to minority shareholders. The larger minority stake increases the incentives to distort the investment policy, leading to larger efficiency costs and a lower value. Therefore, a large financing requirement reduces firm value if control is not shared.

In contrast, firm value does not depend on the investment requirement \( I \) under shared control. To see this, let \( \alpha_1 + \alpha_2 \) be an optimal controlling stake given \( I \) and consider an increase of the investment requirement to \( I' > I \). The initial shareholder can finance \( I' \) without changing the controlling stake: simply sell more of his own shares to the second controlling shareholder. Inspection of the investment rule under shared control (equation (7)) reveals that the decision of undertaking the project depends only on the aggregate control stake, \( \alpha_1 + \alpha_2 \). Hence, if a controlling stake \( \alpha_1 + \alpha_2 \) is optimal for investment requirement \( I \), then it must remain optimal for an investment requirement \( I' > I \).

It then follows that, contrary to firms with a single controlling shareholder, the financing requirement does not affect the value of firms with shared control. As such, the relative efficiency of shared control increases with the financing requirement, making it easier for sharing control to dominate unilateral control.

**Proof of Proposition 8:** We first consider unilateral control and then address the situation with shared control.
i) **Unilateral control:** Consider first that the project harms minority shareholders, that is, \( y < 0 \). If, in addition, the project destroys private benefits, then unilateral control achieves the first best (the entrepreneur rejects the project with probability one) and changes in governance laws are irrelevant. We can thus restrict our attention to projects that produce private benefits, \( b > 0 \), at the cost of verifiable cash flows \( (y < 0) \). In this case, an improvement of governance laws amounts to a decrease in the private benefits to \( b - g \) with an increase in the verifiable cash flow to \( y + g \), with \( g \geq 0 \).

In this setting, monitoring cannot be valuable upon the improvement in the governance law if it wasn’t prior to it. To see why, recall from Proposition 1 that a necessary and sufficient condition for monitoring to be valuable in this case is that the expected value of the project conditioned on the undertaking of the project be negative, that is, \( y + E[b|b + \alpha^I y > 0] < 0 \), where \( \alpha^I \) is the equity stake of the single controlling shareholder. Intuitively, a project that harms minority shareholders induces a monitor to block it. Hence, monitoring is efficient if, conditioned on the entrepreneur’s choice of undertaking the project \( (b + \alpha^I y > 0) \), it enhances firm value. Suppose then that monitoring is not efficient prior to the improvement, \( y + E[b|b + \alpha^I y > 0] \geq 0 \), and that an improvement in governance law increases the verifiable cash flow to \( y + g \) while reducing private benefits to \( b - g \).

The improvement in the governance law may change the optimal ownership structure under unilateral control in two ways. First, it increases the value of minority shares for any level of monitoring, letting the entrepreneur raise the financing requirement with fewer shares. The entrepreneur’s equity stake thus rises, making him internalize firm value to a greater extent and moving the investment decision closer to the first best. Clearly, this equity effect increases firm value conditioned on the undertaking of the project, and thus \( y + E[b|b + \alpha^I y > 0] \geq 0 \) implies \( y + E[b|b + \alpha^I(g)y > 0] \geq 0 \), where \( \alpha^I(g) \) is the entrepreneur’s stake conditioned on the improvement in governance law. The equity effect, therefore, makes it easier for the entrepreneur to rule out monitoring.

In addition to the equity effect, an improvement in governance law increases the relative importance of private benefits and verifiable cash flows to the entrepreneur. To see this, fix the equity stake of the entrepreneur at \( \alpha^I (\text{its level without the improvement}) \) and define the expected private benefit conditioned on the entrepreneur’s decision to undertake the project as
\[ b(g) = E[b - g|b > -\alpha f y + g(1 - \alpha_1)] = -g + E[b|b > -\alpha f y + g(1 - \alpha_1)]. \] This latter expectation increases with \( g \) because a positive value for \( g(1 - \alpha f) \) truncates the support of the private benefits at a higher cut-off. Hence, it follows that \( E[b|b > -\alpha f y + g(1 - \alpha_1)] \) is larger than \( E[b|b > -\alpha f y] \) and \( b(g) + (y + g) = E[b|b > -\alpha f y + g(1 - \alpha f)] + y \geq E[b|b > -\alpha f y] - g. \) And we conclude that if monitoring is not beneficial to start with \( (E[b|b > -\alpha f y] - g \geq 0) \), then it cannot be beneficial either when governance improves.

A similar line of reasoning shows that an improvement in governance law also decreases the incentives for monitoring when the project does not harm minority shareholders, \( y > 0 \). Since unilateral control achieves the first best if the project also creates private benefits \( b > 0 \), governance law can only change firm value if \( y > 0 \) and \( b < 0 \). In this setting, an improvement of governance laws amounts to a reduction of the verifiable cash flow by \( |g| \) and an increase in the private benefits by \( |g| \), or equivalently, the verifiable cash flow becomes \( y + g \) and the private benefits \( b - g \), with \( g \leq 0 \).

We now show that the improvement in the governance law enhances firm value with unilateral control. To do so, we consider only projects that increases private benefits \( b > 0 \) but harm minority shareholders \( y < 0 \), and evaluate the derivative of the firm value at \( g = 0 \). (The proof for \( b < 0 \) and \( y > 0 \) is analogous.)

As we have already argued, the improvement of the governance law increases the value of the minority shareholders allowing the entrepreneur to increase the control stake; an increase that is value enhancing. As such, the proof is complete if we show that the change in the relative importance of public and private benefits does not undo the increase in value that stems from the equity effect. In the analysis that follows, therefore, we can keep the controlling stake fixed at \( \alpha f \). In this setting, the investment decision condition changes to \( b - y \geq -\alpha_1(y + g) \), or equivalently \( b \geq -\alpha_1 y + g(1 - \alpha_1) \). The value with governance \( g \) and no monitoring is

\[
V_{uc}(g) = \int_{-\alpha_1 y + g(1 - \alpha_1)}^{b+y} f(b)db.
\]

The derivative is

\[
\frac{dV_{uc}(g)}{dg} \bigg|_{g=0} = -(1 - \alpha_1)(b+y)f(b) \geq 0,
\]

where \( b = -\alpha_1 y \).

In turn, firm value under unilateral control and an optimal level of positive monitoring (given by the monitor’s stake \( \beta^*(g) \)) is

\[
V_m(g) = \int_{-\alpha_1 y + g(1 - \alpha_1)}^{b+y} \left[(y + b)(1 - m(y + g, \beta^*(g))) - \frac{\rho m(y + g, \beta^*(g))^2}{2}\right] f(b)db.
\]
We now show that if \((\bar{b} + y) < 0\), then \(0 \leq \frac{dV_m(g)}{dg} \bigg|_{g=0}\), using throughout that \(F(g) = \int_{L(g)}^{H(g)} f(b, g) db\) implies that

\[
\frac{dF(g)}{dg} = \int_{L(g)}^{H(g)} \frac{\partial f(b, g)}{\partial g} db + H'(g) f(H(g), g) - L'(g) f(L(g), g).
\]

Define \(F(y, b, \beta, g) = \left[(y + b) (1 - m(y + g, \beta)) - \frac{\mu (y + g, \beta)^2}{2}\right] = (y + b) \left(1 - \frac{\beta}{\rho} |y + g|\right) - \rho \left(\frac{\beta |y + g|}{2}\right)^2\).

The derivative is \(\frac{dV_m(g)}{dg} \bigg|_{g=0} = \int_{-\alpha_1 y} - \alpha_1 y) \beta^* (0) = \int_{-\alpha_1 y} - \alpha_1 y) f(b) db = 0\) the derivative \(\frac{dF(y, b, \beta, g)}{dg} \bigg|_{g=0} = \frac{\beta}{\rho} ((b + y) - y\beta)\) and

\[
\int_{-\alpha_1 y} - \alpha_1 y) \frac{\partial F(y, b, \beta^*, 0)}{\partial g} db = 0,
\]

because \(\beta^* = \frac{1}{y} (\bar{b} + y)\), \(\bar{b} = E[b | b > -\alpha_1 y] = \int_{-\alpha_1 y} - \alpha_1 y) f(b) db / P(b > -\alpha_1 y)\).

Now we show that \(0 \geq F(y, b, \beta^*, 0) \geq (\bar{b} + y)\): First \(F(y, b, \beta^*, 0) - (y + b) = -(y + b) \frac{\beta^*}{\rho} |y| - \frac{\rho (\frac{\beta^*}{\rho} |y|)^2}{2} = (y + b) \frac{\beta^*}{\rho} y - \rho \frac{(\frac{\beta^*}{\rho} y)^2}{2} = \left(-\frac{\beta^* y}{2\rho}\right) \left(y \beta^* - 2y - 2b\right) = \left(-\frac{\beta^* y}{2\rho}\right) \left(\bar{b} + y - 2y - 2b\right) \geq 0\), because \(\bar{b} \geq b\); Second, note that \(F(y, b, \beta^*, 0) = (y + b) \left(\alpha^2 - \frac{\beta^*}{\rho} \alpha^2\right) - \frac{\rho (\frac{\beta^*}{\rho} |y|)^2}{2} \leq 0\). This completes the proof that \(0 \leq \frac{dV_m(g)}{dg} \bigg|_{g=0} \leq \frac{dV_m(g)}{dg} \bigg|_{g=0}\).

ii) **Shared control:** Note that the controlling stake is a choice variable (the financing can be accomplished by selling shares to another controlling shareholder; keeping the controlling stake fixed).

ii.a) Case \(y > 0\): The optimum controlling stake is \(\alpha^2 = 1\). Therefore, the value under shared control is: \(V_{sc} = \int (b + y) \Gamma \left(\frac{b + y}{\epsilon}\right) db = V_{sc} (g) = \int (b + y) \Gamma \left(\frac{(b - y) + (y + g)}{\epsilon}\right) f(b) db\).

ii.b) Case \(y < 0\): The value under shared control is

\[
V_{sc} (g) = \max_{\alpha \in [\alpha_1, 1]} \int (b + y) \Gamma \left(\frac{(b - g) + \alpha (y + g)}{\epsilon}\right) f(b) db = \max_{\alpha \in [\alpha_1, 1]} \int (b + y) \Gamma \left(\frac{b + \alpha y + g (1 - \alpha)}{\epsilon}\right) f(b) db.
\]

The optimum controlling stake changes to \(\alpha^2 (g) = \alpha^2 - \frac{(1 - \alpha^2) g}{|y + g|}\), and \(V_{sc} (g) = V_{sc} (0)\).
Proof of Proposition 9: Assume by absurd that, for any \((b, y)\), there is a controlling shareholder \(i(b, y)\) who can successfully acquire full control. From the Revelation Principle, we can ignore the signals of the private benefits, restricting attention to direct mechanisms, in which the controlling shareholder \(i(b, y)\) pays \(t_j(b, y)\) to the controlling shareholder \(j \neq i(b, y)\).

The new single controlling shareholder internalizes all of the private benefits, investing if and only if \(b + \alpha y > 0\), where \(\alpha \equiv \sum_{i=1}^{2} \alpha_i\). Conditioned on the existence of the buy out mechanism, this investment rule can be replicated in the ownership structure with multiple controlling shareholders. If \(x\) is the probability that the controlling group undertakes the project, set \(x(b, y) = 1\) if and only if \(b + \alpha y > 0\), with transfers \(t_j(b, y)\) for \(j \neq i(b, y)\), and \(t_i(b, y) = -t_j(b, y)\). Thus, we have obtained a direct mechanism for the investment decision that, under shared control, is ex-post efficient.

Standard arguments in the mechanism design literature show that an ex-post efficient mechanism \((x(.), t(.))\) must satisfy the following inequality

\[
I = \int \int \sum_{i=1}^{2} \left( b_i + \alpha_i y - \frac{1 - F_i(b_i)}{f_i(b_i)} \right) \prod_{k=1}^{2} f(b_k) db_k \geq 0. \tag{14}
\]

Consider the following change of variables: \(x_1 = -(b_1 + \alpha_1 y), x_2 = b_2 + \alpha_2 y\). Let the density and cumulative distribution of \(x_i\) be, respectively, \(f_i\) and \(F_i\) (by an abuse of notation) with support in the interval \([x_i, \overline{x}_i]\) where \(\underline{x}_1 = -(\overline{b}_1 + \alpha_1 y), \overline{x}_1 = -(\overline{b}_1 + \alpha_1 y), \underline{x}_2 = b_2 + \alpha_2 y,\) and \(\overline{x}_2 = \overline{b}_2 + \alpha_2 y\). One can easily check that the assumption of the proposition implies \(\underline{x}_2 < \overline{x}_1\) and \(\overline{x}_2 < \underline{x}_1\). Using the formula for the integral with a transformation of variables we have that,

\[
I = \int_{\underline{x}_2}^{\overline{x}_2} \int_{\underline{x}_1}^{\min\{\overline{x}_2, \overline{x}_1\}} \left( [x_2 - \frac{1 - F_2(x_2)}{f_2(x_2)}] - [x_1 + \frac{F_1(x_1)}{f_1(x_1)}] \right) f_1(x_1) f_2(x_2) dx_1 dx_2 \tag{15}
\]

Myerson and Satterthwaite (1983) show that the above integral is negative under the assumptions of the Proposition. It then follows that the ex-post efficient mechanism to dissolve the partnership is not feasible.

\[\square\]