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Portfolio optimization under asset pricing anomalies

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Abstract

Fama and French (1993) find that the SMB and the HML factors explain much of the cross-section stock returns that are unexplained by the CAPM, whereas Daniel and Titman (1997) show that it is the characteristics of the stocks that are responsible rather than the factors. But both arguments are largely based only on expected return comparisons, and little is known about how important each of the two explanations matters to an investor's investment decisions in general and portfolio optimization in particular. In this paper, we show that a mean-variance maximizing investor who exploits the asset pricing anomaly of the CAPM can achieve substantial economic gain than simply holding the market index. Indeed, using monthly Japanese data on the first 50 largest stocks over the period 1980–1997, we find the optimized portfolio constructed from characteristics-based model is the best performing one and has monthly returns more than 0.81 percent (10.16 percent annualized) over the Nikkei 225 index with no greater risk.

JEL classification: C13; C53; G14

Keywords: Factors; Characteristics; Portfolio selection; Momentum

1. Introduction

There is a growing literature on the major asset pricing anomaly that the realized cross-section stock returns are not consistent with the predictions of the basic capital asset pricing

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model (CAPM) of Sharpe (1964) and Lintner (1965). Among the competing explanations, Fama and French (1993) find that the anomaly is largely driven by the SMB and the HML portfolios (two zero net investments in which the first is long in small firms and short large ones and the second is long in high book-to-market firms and short low book-to-market ones), whereas Daniel and Titman (1997) show that it is the characteristics of the stocks that are responsible. But both arguments are largely based on expected return comparisons, and little is known about how important each of the two explanations matters to an investor's investment decisions in general and portfolio optimization in particular. Intuitively, if an investor can do much better in terms of profit making by incorporating the true nature of the anomaly into his investment decision, an economic value for the anomaly will be apparent. In other words, if the SMB and the HML portfolios were the true driving sources for the anomaly, it is likely that the investor can do better in his investment by using the SMB and the HML portfolios than by using the characteristics of the stocks. On the other hand, if all the claimed features of an anomaly matter little in both the investor's portfolio decision and the associated results, one may take a somewhat extreme view of Black (1993) that "most of the so-called anomalies that have plagued the literature on investments seems likely to be the result of data-mining." Hence, a study on how an investor may utilize an anomaly not only provides insights on the economic significance of the anomaly, but also help to identify competing explanations for the anomaly.

In this paper, we show how a mean-variance maximizing investor can exploit the asset pricing anomaly of the CAPM to achieve substantial economic gain than simply holding the market index. While many well-known asset pricing anomalies[1] may be analyzed in an analogous utility maximizing framework, we choose to study the CAPM anomaly due to its close relation with all the basic asset pricing models and its wide applications in both investments and corporate capital budgeting decisions. The assumption of a mean-variance investor is to simplify the analysis, and the case of a more general utility function may be solved numerically from the first order conditions as reviewed by Duffie (1988). In assessing the economic importance of the anomaly, we analyze two scenarios. The first is where the investor makes his investment decision based on the CAPM. Following Markowitz (1952), Sharpe (1964) and Lintner (1965), it is well-known that the investor should hold a portfolio of a riskless investment and the market portfolio. In the second scenario, the investor exploits the anomaly by making a dynamic portfolio choice decision based on the time-varying investment opportunity set. More specifically, each of the earlier explanations for the anomaly provides a unique way for the investor at each time t to forecast the means and variances of the security returns, and the forecasts are then used to form his optimal portfolio at time t. All mean-variance maximizing investors who exploit the asset pricing anomaly will hold only portfolios of a riskless investment, and the weight on the optimized portfolio depends on their degrees of risk aversion. Hence, it will suffice to show that the optimized portfolio outperforms the market index substantially in order to prove that the anomaly makes a significant economic difference in investors' investments.

[1] Such as the size anomaly of Banz (1981) where small firms earn abnormal returns and the momentum anomaly of Jegadeesh and Titman (1993) where individual security returns are found to have significant momentum.

Our approach for portfolio optimization and assessing asset pricing anomaly is a special case of the general utility maximizing approach for evaluating the economic importance of a hypothesis which has gained popularity recently. For example, Kandel and Stambaugh (1996) find that a statistically insignificant stock return predictability can actually make an economically significant contribution into an investor's asset allocation decision. But their studies are different from ours as they examine the predictability of the market index and its relation to asset allocation decision. In contrast, we analyze the cross-section of stock return anomaly and its implications for an investor's mean-variance portfolio choice. Another related study is Brennan and Xia (2001) who show how the SMB and the HML factors translate into forecasts for the drift of a diffusion with known variances and how a CRRA investor can use this to improve his asset allocation.[2] In contrast, our analysis focuses on portfolio choices with both unknown means and variances, but restricts the investor's utility be of the quadratic type. In terms of variance forecasting methodology, our procedures reply on similar intuitive approaches of Chan et al. (1999). While they are solely interested in the second moments, we are also interested in forecasting the first moments as well in forming optimal portfolios.

The rest of the paper is organized as follows. Section 2 describes the methodology and the models, and explains how various factors and portfolios are constructed. Section 3 describes the data and reports the main results. Section 4 examines further on the empirical results. The last section concludes the paper.

2. The portfolio choice

In this section, we outline first the well-known Markowitz's (1952) framework for a mean-variance maximizing investor to choose his optimal portfolio given the means and variances. Then we show how the CAPM anomaly and the competing explanations help provide forecasts of the means and variances.

2.1. The investor's problem

Given the means and variances of stock returns, it is well-known that the two fund separation theorem applies for mean-variance maximizing investors. Assume there is a riskless asset, then they hold only a portfolio of the riskless asset and the tangency portfolio on the mean-variance frontier. The allocation to the optimized portfolio depends on the risk-aversion of an individual investor. Hence, for the purpose of the optimal portfolio choice, we need only to be concerned about how the tangency portfolio is constructed given N stocks under consideration.

The mean-variance maximizing investor is interested in finding the best combination of the N stocks that has the smallest return variance given any level of required expected return. Let $R_t = (r_{1t}, \ldots, r_{Nt})'$ denote the column vector of returns on the N stocks in excess

[2] Also related is Pástor and Stambaugh (2000) who focus on how Bayesian prior beliefs on the Fama and French (1993) and the characteristics models affect an investor's portfolio choice.

of the riskfree rate at time t, and denote the mean and covariance matrix of the excess returns by μ and Σ, respectively. If $R_{pt}=W'R_t$ is the portfolio to be chosen, where $W=(w_1,\ldots,w_N)'$ denotes the weights assigned to the stocks. Then the optimal investment decision is to find a set of optimal weights, W^*, that minimizes the variance of the portfolio $\mathrm{Var}(R_{pt})=W'\Sigma W$, given a level of expected return $E(R_{pt})=\mu'W=\mu_0$. Formally, the investor's portfolio choice requires solving the following quadratic optimization problem:

$$\mathrm{Min}\frac{1}{2}W'\Sigma W, \quad \text{s.t.} \quad \mu'W=\mu_0 \quad W\geq 0. \tag{1}$$

Notice that the usual restriction that the weights sum to one is not imposed because we have represented the problem in terms of excess expected returns. In addition, we assume non-negative weights to disallow short-selling. There are two reasons for this. First, no-short-selling is a more realistic situation for most investors in practice. Indeed, as investors have to pay 102 percent (the short-sell proceeds plus 2 percent cash) of the capital for short-selling in the real world, it can be prohibitively costly to investors (Reed (2001)). Second, from a theoretical point of view, a rational investor with margin requirements (regulation T) will not short-sell a stock unless its price is far away from its theoretical value (Cuoco and Liu (2000)).

The solution to the above problem gives the highest Sharpe ratio among all possible combinations of stocks. However, implementing the Markowitz's mean-variance optimization procedure requires the inputs of expected returns and covariance matrix of the stocks in practice. In reality, these true parameters are not known and have to be estimated by using the data. This will depend on the models the investor bears in mind. If an investor is a true believer of the CAPM, he will hold a portfolio of the market index and the riskless asset. If he believes the presence of the asset pricing anomaly of the CAPM, with the assumption of a mean-variance maximizing agent, he will forecast the means and variances accordingly and incorporate such forecasts into his determination of the optimal portfolio. Of course, the quality of the forecasts will depend on the models he is using. In what follows, we consider mainly three models: a naive one, the Fama and French model and the Daniel and Titman (1997) characteristics model. Notice that our optimization experiments are predictive in nature because the portfolio weights are obtained based on past information alone. The subsections below describe how we estimate the expected returns and the covariance matrix for each of the three models and their simple variations.

2.2. *A naive model*

For a non-believer of any asset pricing models, such as the CAPM, the simplest approach seems to take the means and variances as time-varying parameters and estimate them from past returns over a given time window. Following Chan et al. (1999) and others in this area of research, we take the window (estimation period) as the past 60 months. However, during each time window, the returns are treated as if they were independent and identically distributed so that the means and variances can be estimated with standard procedures.

Specifically, the means or expected returns are the average of the realized returns:

$$\bar{R} = \frac{1}{60}\sum_{s=1}^{60} R_{t-s}, \tag{2}$$

This is our naive model for estimating the means at time $t + 1$ to be used for constructing the optimal portfolio at time t.

Similarly, the variances and covariances can be replaced by their sample counterpart in the optimization procedure, i.e.,

$$\hat{\text{cov}}(r_i, r_j) = \frac{1}{59}\sum_{k=1}^{60}(r_{i,t-k} - \bar{r}_i)(r_{j,t-k} - \bar{r}_j), \tag{3}$$

where the summation is taken over the past 60 months (the estimation period). However, the estimated covariance matrix will have the singularity problem when the number of assets N exceeds 60. Despite this problem, the optimization (1) usually has a solution in practice. Notice that a solution without the non-negative weight constraints is impossible as the well-known analytical solution depends on the inverse of the covariance matrix explicitly. Nevertheless, the naive covariance matrix estimation can be refined in two simply ways, as shown later in Section 2.4, to yield nonsingular estimators.

2.3. Factor models

As pointed out earlier, it is simply infeasible to estimate an unconstrained $N \times N$ positive definite matrix Σ in the case where N is greater than the number of estimation periods. A formal statistical procedure is to impose a factor structure. A k-factor model for the asset returns can be represented in matrix form as follows:

$$R_t = BF_t + E_t, \tag{4}$$

where B is an $N \times K$ matrix of factor loadings on the stocks, and F_t is a $K \times 1$ vector of factors, and E_t is the N-vector of error terms or residuals. Under the factor model, the expected returns are straightforwardly estimated:

$$\bar{R} = \hat{B}\bar{F}, \tag{5}$$

where $\hat{B}$ is the estimates of the factor loadings, and $\bar{F}$ is the sample average of the factor realizations, $\bar{F} = 1/60\sum_{k=1}^{60} F_{t-k}$. The estimate of Σ is also easily obtained by replacing the population parameters below with their sample estimates

$$\Sigma = B\Omega B' + D \tag{6}$$

where $\Omega = \text{cov}(F_t)$ is the covariance matrix of the factors and D is the covariance matrix of the residuals which is often assumed to be diagonal.

Connor and Korajczyk's (1988) asymptotic principal components method or the standard factor analysis approach may be used to both extract the factors and estimate the parameters in (4). However, a factor analysis is not our focus here and we consider only some economically based factors in the factor model. The first one is the standard market model,

$$r_{\mathrm{it}} = \beta_{i1} R_{\mathrm{mt}} + e_{\mathrm{it}}, \tag{7}$$

where R_{mt} is the returns on the market. This is a one-factor model with the return on the market as the sole factor, a simple version of the CAPM that serves as a useful benchmark for other factor models. The second model we analyze is the well-known Fama and French (1993) three-factor model,

$$r_{\mathrm{it}} = \beta_{i1} R_{\mathrm{mt}} + \beta_{i2} \mathrm{SMB}_t + \beta_{i3} \mathrm{HML}_t + e_{\mathrm{it}}, \tag{8}$$

where, in addition to the market factor R_{mt}, there are two additional factors, the SMB and the HML portfolios which are two zero net investments in which the first is long in small firms and short large ones and the second is long in high book-to-market firms and short low book-to-market ones (their detailed construction in terms of the Japanese data is given in the next section). The Fama and French model is of our primary interest as one of the objectives of the paper is to assess its implications to investor's investment decision, and, in particular, to compare it with the characteristics-based model of Daniel and Titman (1997) to see which exerts more economic significance in the investor's bottom line. For interest of comparison, we consider yet another factor model,

$$r_{\mathrm{it}} = \beta_{i1} R_{\mathrm{mt}} + \beta_{i2} \mathrm{SMB}_t + \beta_{i3} \mathrm{HML}_t + \beta_{i4} \mathrm{MOM}_t + e_{\mathrm{it}}. \tag{9}$$

This is a four-factor model by adding one momentum factor, MOM_t, into the Fama and French three-factor model. The momentum factor is a mimicking portfolio constructed following Chan et al. (1999).

2.4. Characteristics model

Daniel and Titman (1997) and Daniel et al. (2001) document that the characteristics model better explains the cross-sectional variations in expected stock returns in both the US and the Japanese markets than the Fama and French model. We follow Daniel et al. (1997) by constructing 125 three-way sorting portfolios on size, book-to-market ratio (BM), and past returns performance. Basically, in each year all stocks in the market are grouped into five size portfolios, and within each size portfolio the stocks are further sorted to five portfolios according to their book-to-market ratios. Finally, within each size-BM portfolio the stocks are further divided into five portfolios based on their past 12 month's returns. Once the groupings are complete, value-weighted returns for the subsequent 12 months for each of 125 the portfolios are calculated. The procedure is repeated every year. After the characteristic portfolios are formed, the expected return of a stock in a certain year can be calculated as the average of the past 60-month's returns of the "characteristic" portfolio to which it belongs.

While the characteristics model provides estimates for the expected returns, it is unclear how it can estimate the second moments. CKL (1999) adopt an ad hoc regression approach to calculate the covariance matrix. Following their intuitive practice, we employ a formula similar to their sample covariance estimate, except that the "sample average" is replaced with the "characteristic counterpart." Specifically, the covariance between stocks i and j is given as:

$$\hat{\text{cov}}(r_i, r_j) = \frac{1}{59}\sum_{k=1}^{60}(r_{i,t-k} - \hat{r}_i)(r_{j,t-k} - \hat{r}_j), \tag{10}$$

where $\hat{r}_i$ is the estimate of the expected return for stock i based on the characteristic model. Since estimating the covariances this way is somehow ad hoc, we will also combine the estimates of expected return from the characteristic model with the estimates of covariances from other models to see how differences in the covariances estimates affect the optimization performance. In addition to the above methods, we also propose the use of two naive procedures, which we refer to as "industry diagonal" and "block diagonal." The industry diagonal procedure estimates the covariance matrix as we outlined in the naive model, but ignores the covariances between stocks of different industries. Since the number of stocks we consider will be $N = 50$, 100 and 200, ignoring some covariances may effectively reduce the number of parameters estimated if the covariances among stocks of different industries are indeed ignorable.

The "block diagonal" procedure divides the sample stocks into a few blocks based on sorting by correlations, and ignores the correlations between different blocks. More specifically, when $N > 50$, we can divide the stocks into $N/50$ blocks. For example, if $N = 100$, we will have two blocks. We can then estimate Σ by assuming it is block-diagonal and each block is estimated by using (3). The question now becomes how we divide the stocks into blocks. A simple way is to start randomly with any given stock and find the next 49 stocks that have the greater absolute correlation values with it. Then we get the first block and have $(N - 50)$ stocks remaining. Continuing the same process, we obtain the next block up to the final one. Notice that given the length of the estimation window and the number of assets, it is impossible to estimate an unconstrained $N \times N$ positive definite matrix Σ. The simplifying assumptions impose restrictions on the free parameters of Σ and make the estimation doable. Both of these two naive procedures give rise to different estimates of Σ and are both non-singular. For easy reference, we will call the resulted optimized portfolios are from the characteristic-block-diagonal and characteristic-industry-diagonal models.

3. The data and main results

The portfolio optimization procedures are applied to monthly returns including dividends on common stocks listed on both sections of the Tokyo Stock Exchange (TSE) from January 1975 to December 1997. As there are no risk-free rates in Japan that are comparable to the US Treasury bill rates, following Chan et al. (1991), we use a combined series of the call money rate (from January 1975 to November 1977) and the 30-day Gensaki (repo) rate (from December 1977 to December 1997) as the risk-free interest rate. The dividends data and the market capitalization, and the series used for constructing the risk-free rates are all available from the database compiled by the PACAP Research Center at the University of Rhode Island.

3.1. Factors, characteristics and benchmarks

To construct the Fama–French SMB and the HML factors, following Fama and French (1993), we form the factor portfolios based on sorting the stocks on market size (SZ) and book-to-market ratio (BM). A detailed description on the construction of Fama–French three factors in the Japanese market can be found in Daniel et al. (2001).[3] As our analysis is predictive in nature, we need to ensure that the accounting data that we use in forming the optimized portfolios are publicly available at the time of portfolio formation, and so we form portfolios on the first trading day of October, and hold them for exactly 1 year. The reason for this is because most firms listed on the TSE have March as the end of their fiscal year and the accounting information becomes publicly available before September. For portfolios formed in October of year t, we use the book equity (BE) of a firm at the fiscal year-end that falls between April of year $t-1$ and March of year t. BM is set to equal the ratio of BE to the market equity at the end of March of year t, and SZ is set to equal the market equity at the end of September of year t. The portfolios are rebalanced every year as in Fama and French (1993).

The characteristics-based portfolios are constructed in the way as described in Section 2.4. Now, the market portfolio is needed to assess whether or not the CAPM anomaly matters in a mean-variance maximizing investor's investment decision, and to compare which of the explanations, the SMB and the HML factors or the characteristics-based portfolios, are of more economic importance in the portfolio optimization. We use the value-weighted index of all stocks listed on the TSE, compiled also by the PACAP Research Center, as the market portfolio. As the Nikkei 225 index is the most widely quoted Japanese market index, we also provide it for comparison. In addition, we create a value-weighted index of N stocks to see the robustness of the potential gains from portfolio optimization. There is an issue of deciding the value of N and which N stocks. In what follows, we examine three cases: $N = 50$, 100, and 200. A number of N not too large reflects the practice of many mutual funds that usually hold only about 100 or so stocks to track the market index. This may have many institutional reasons such as liquidity and resources available to study the chosen stocks. Theoretically, the more stocks, the better the mean-variance frontier, and hence the higher the expected return of the tracking portfolio. However, this is true only in sample and with known parameters. As it is the out-of-sample performance that matters, and the parameters are never known in the real world, a too large N can only lead to less accurate estimate for the covariance matrix and hence reduce the out-of-sample performance.

Now, given N, there is still a problem of how to select N stocks from the entire stock market. One strategy is to select randomly with equal probability. For example, Chan et al. (1999) use such an approach to draw 250 stocks randomly in their evaluation of various covariance forecasting procedures which are applied to reduce portfolio tracking errors. A potential problem with this strategy is that there is a substantial probability to get quite a few small firms in the portfolio. Problems with small firms, such as illiquidity and limited holding may bias the analysis in certain ways. However, a simple solution to the stock selection problem does not seem to exist. Realizing this, we select the top N largest stocks

[3] We are grateful to John Wei for providing us the Japanese data on the Fama–French factors.

as our sample, where N is set to be 50, 100, and 200. From a practical point of view, this is a plausible strategy, though there is no strong reason not to select the next largest as it may have higher diversification value. If it is truly the case, then the optimized portfolio is likely to under-state the impact of the importance of the pricing anomaly. Hence, the selection may not be a problem as we do find the optimized portfolio performs substantially better. An under-statement will not affect the qualitative nature of the conclusion. Furthermore, if the problem of under-statement does exist, it should affect both the SMB and the HML factors or the characteristics-based portfolios, and hence the effect on the comparison between them is minimal.

3.2. *Performance of the optimized portfolios*

Recall that we have three major models: the naive model, the factor-based models, and the characteristics model. In implementing the portfolio optimization process, we select in September of each year from 1980 to 1996 the largest 50, 100 and 200 largest stocks listed on the TSE. Each of the models provides a forecast of next year's expected returns and covariance matrix for the selected stocks. The estimates are then used as inputs to solve the quadratic programming problem (1) to obtain the non-negative optimized portfolio weights on all stocks. Given the optimized weights, the buy-and-hold returns on the portfolio for the next 12 months are then calculated. After the calculation, the whole procedures are repeated for the next year, and year after year for the whole sample. As the Japanese market has become bearish after 1990, we also perform subperiod analysis to examine how various models perform under different market conditions. The performance of the optimization results using each model is reported in Table 1, for the full sample period, and for the two sub-periods as well. As a comparison, the result is compared with five major benchmarks: the value-weighted market portfolio, the Nikkei 225 index, and three weighted portfolios of N stocks (equal-weighted, price-weighted and value-weighted).[4] Table 1 provides the results of the means, standard deviations and the Sharpe ratios for the benchmark indices and the optimized portfolios over the entire test period and the two sub-periods.

Panel A of Table 1 presents the results on the full sample period. Of the five benchmarks, the equal-weighted portfolio of 100 stocks has the highest Sharpe ratio of 0.0848, and a highest average monthly return of 0.49 percent, whereas the Nikkei 225 index has the lowest Sharpe ratio of 0.0429 and a lowest monthly return of 0.25 percent over the full sample period. Here, the Sharpe ratio is calculated based on realized monthly portfolio returns over the sample period. Fig. 1 presents the time-series plots of the cumulative returns of the major benchmarks. The figure shows that all benchmarks have close time-series behavior, with the Nikkei 225 index being dominated by other indices over time.[5] The market portfolio, which is the valued-weighted TSE index, has an average monthly

[4] The price-weighted portfolio is compiled so that the result can be compared with the Nikkei 225 index, which is also a price-weighted index. As a major consideration in selecting its constituents in Nikkei 225 is liquidity, which is different from the well-documented anomalies such as size and BM, our choices of stocks do not fully conform to those of the Nikkei 225 constituents. For the case of $N = 200$, about 50 percent of our stocks are Nikkei 225 components during the sample period.

[5] The better performance of our price-weighted indexes over the Nikkei 225 implies that large stocks have better subsequent performance over the "liquid" Nikkei constituents.

Table 1
Performance of optimal portfolios based on forecasting models

Model	N = 50				N = 100				N = 200			
	Mean (percent)	H_u	S.D. (percent)	Sharpe ratio	Mean (percent)	H_u	S.D. (percent)	Sharpe ratio	Mean (percent)	H_u	S.D. (percent)	Sharpe ratio
Panel A: full sample: 198010–199709												
Market	0.35	–	5.54	0.0635	0.35	–	5.54	0.0635	0.35	–	5.54	0.0635
Nikkei 225	0.25	–	5.84	0.0429	0.25	–	5.84	0.0429	0.25	–	5.84	0.0429
Value-weighted	0.48	0.81	6.52	0.0737	0.47	1.02	6.14	0.0770	0.45	1.14	5.88	0.0760
Equally-weighted	0.49	1.07	6.30	0.0774	0.49	1.69	5.81	0.0848	0.45	1.63	5.49	0.0824
Price-weighted	0.49	0.47	7.25	0.0675	0.49	0.52	6.75	0.0721	0.46	0.49	6.28	0.0739
Naïve	0.12	−0.76	5.83	0.0199	0.17	−0.56	5.95	0.0293	−0.13	−1.66	5.69	−0.0220
1-factor	0.46	0.40	6.12	0.0759	0.47	0.47	5.49	0.0853	0.39	0.18	5.60	0.0705
3-factor	0.62	0.84	6.83	0.0908	0.81	1.50	6.37	0.1270	0.73	1.37	6.45	0.1128
4-factor	0.61	0.80	6.66	0.0910	0.71	1.17	6.55	0.1089	0.70	1.23	6.42	0.1082
Characteristic	1.06	2.47*	5.94	0.1779	0.91	2.17*	5.10	0.1777	0.83	2.03*	5.30	0.1563
Panel B: 198010–198909												
Market	1.26	–	4.30	0.2931	1.26	–	4.30	0.2931	1.26	–	4.30	0.2931
Nikkei 225	1.11	–	3.98	0.2795	1.11	–	3.98	0.2795	1.11	–	3.98	0.2795
Value-weighted	1.37	0.43	6.17	0.2222	1.38	0.66	5.62	0.2466	1.36	0.82	5.16	0.2639
Equally-weighted	1.39	0.64	5.82	0.2392	1.45	1.50	5.02	0.2893	1.42	1.67	4.33	0.3279
Price-weighted	1.07	−0.75	5.71	0.1879	1.04	−1.13	4.95	0.2109	1.07	−1.05	4.40	0.2426
Naïve	1.06	−0.65	5.20	0.2039	1.13	−0.44	4.85	0.2329	1.00	−0.93	4.58	0.2182
1-factor	1.62	1.16	6.70	0.2424	1.61	1.74	5.68	0.2834	1.58	2.63*	4.98	0.3172
3-factor	1.86	1.38	6.94	0.2676	1.84	1.58	5.66	0.3245	1.59	0.99	5.25	0.3024
4-factor	1.88	1.42	7.03	0.2680	1.79	1.45	5.79	0.3087	1.53	0.84	5.13	0.2984
Characteristic	1.92	1.77	5.44	0.3531	1.76	1.45	4.60	0.3826	1.33	0.21	4.31	0.3086
Panel C: 198910–199709												
Market	−0.67	–	6.54	−0.1024	−0.67	–	6.54	−0.1024	−0.67	–	6.54	−0.1024
Nikkei 225	−0.72	–	7.30	−0.0985	−0.72	–	7.30	−0.0985	−0.72	–	7.30	−0.0985
Value-weighted	−0.52	0.77	6.78	−0.0769	−0.55	0.79	6.57	−0.0842	−0.58	0.74	6.47	−0.0902
Equally-weighted	−0.53	0.91	6.68	−0.0794	−0.59	0.72	6.44	−0.0911	−0.63	0.43	6.41	−0.0990
Price-weighted	−0.17	0.95	8.65	−0.0192	−0.14	1.09	8.32	−0.0167	−0.21	1.08	7.84	−0.0274

Naïve	−0.95	−0.49	6.33	−0.1495	−0.90	−0.41	6.86	−0.1314	−1.39	−1.39	6.52	−0.2133
1-factor	−0.84	−0.34	5.13	−0.1634	−0.82	−0.30	4.99	−0.1635	−0.94	−0.56	5.97	−0.1570
3-factor	−0.77	−0.22	6.47	−0.1191	−0.35	0.64	6.93	−0.0502	−0.24	0.92	7.49	−0.0319
4-factor	−0.83	−0.37	5.93	−0.1403	−0.50	0.33	7.16	−0.0693	−0.25	0.87	7.54	−0.0326
Characteristic	0.09	1.70	6.35	0.0134	−0.05	1.60	5.48	−0.0097	0.26	3.12*	6.20	0.0422

At the end of September of each year from 1980 to 1996 three samples of stocks (biggest 50, biggest 100 and biggest 200) are selected from eligible common stock issues on the PACAP database for Japan. Forecasts of means and covariances of monthly excess returns are generated from different models, using the prior 60 months of data for each stock. Based on each model's forecasts of means and covariances, a quadratic programming procedure is used to find the optimal portfolio. These weights are then applied to form buy-and-hold portfolio returns until the next September, when the forecasting and optimization steps are repeated and the portfolio is reformed. For each procedure summary statistics are presented in the Exhibit for the unannualized mean excess return and unannualized standard deviation for the returns realized on the portfolio, and the unannualized Sharpe ratio. The hypothesis is that the expected rate or return on underlying portfolio is equal to that on market portfolio. An asterisk denotes statistical significance at the 5% level.

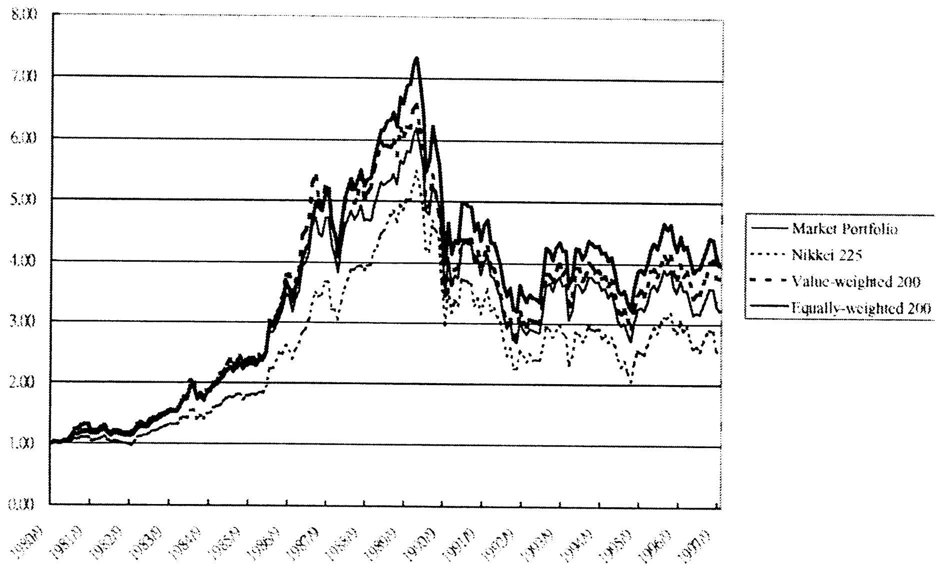

Fig. 1. Time-series plots of the cumulative returns of the benchmarks.

return of 0.35 percent (the first row) and a monthly standard deviation of 5.54 percent. This gives rise to a Sharpe ratio of only 0.0635. In contrast, with as few as 50 stocks, the optimized portfolio based on the characteristics model has a much higher mean of 1.06 percent (the last row), which is more than three times over the market return. Moreover, this optimized portfolio has a standard deviation of 5.94 which is not too different from that of the market, 5.54. As a result, the characteristics optimized portfolio has a Sharpe ratio of 0.1779, which is almost three times as large as that of the market. To confirm this fact statistically, the column under H_μ reports the t-statistic for testing whether or not the expected return of an optimized portfolio is equal to that of the market portfolio. The t-test is based on Newey and West's (1987) robust estimate of the standard error which allows for serial correlation and conditional heteroscedasticity. As the t-statistic has a value of 2.47, it shows that the optimized portfolio based on the characteristics model significantly outperforms the benchmark in terms of the returns.

The optimization based on the naive model has the worst performance among all models. Indeed, for $N = 50$, the naive model obtains an average monthly return of only 0.12 percent versus 0.35 percent on the market, while the risk (standard deviation) is 5.83 percent versus 5.54 percent. As to the Fama–French three-factor model, the return of 0.62 percent is about twice as much as the market. With the inclusion of an additional momentum factor, the four-factor model is doing even slightly worse with a return of 0.61 percent. This suggests that inclusion of momentum factor does not yield significant gain in investors' portfolio optimization decision, although it is well documented that the Japanese market persistently displays contrarian profitability. The failure of the momentum factor to contribute to portfolio optimization may be explained by the fact that the optimization decision is subject to short-sale restriction, while implementation of momentum or contrarian strategies requires simultaneously long and short of different stocks. But both are better than the 0.46 percent of the single-factor CAPM. Nevertheless, all of the three-factor models have higher risks than the market, but they only slightly outperform the market in terms of the Sharpe performance measure.

When we allow the number of assets N, to increase from 50 to 100 and 200, the results are somewhat different. Except for the characteristics model, both the naive and the factor-based models appear to perform better when $N = 100$ than when $N = 200$. For the naive model, the resulting optimized portfolio with $N = 200$ even yields a negative average return of -0.13 percent. For the characteristics model, Panel A of Table 1 indicates that the optimized portfolio with $N = 50$ is the best among the three sets of stocks, while the portfolio with $N = 200$ has the worst performance. The results seem to be confusing because one would have expected a better performance with a larger basket of stocks. But this is true only when the out-of-sample parameters on the means and covariances are estimated with high precisions. With a larger N, a much larger number of parameters have to be estimated, thereby causing greater sampling variances on the parameter estimates. Hence, it is clear that although increasing the number of stocks may expand the potential performance of the optimized portfolio, there is also a danger of losing too much efficiency in estimation.[6]

In our case here, the results suggest that the marginal cost of "losing degrees of freedom" exceeds the marginal benefit of including more assets when the number of assets goes beyond 100. Overall, although most models, except the naive model, outperform the markets in terms of the realized returns and Sharpe ratios, only the characteristics model significantly outperforms the market over the full sample. An explanation for better performance of the characteristics model with $N = 50$ is that the parameters of the largest 50 stocks are better predicted by the characteristics model, hence assuring a better out-of-sample performance. We will explore this issue further in the next section.

To further assess the performance, Fig. 2A through 2C plot the cumulative returns on the market portfolio and on the optimized portfolios for the full sample period. It is seen that the naive model is the worst one as its cumulative returns fall below those of the market for most of the time. As expected, the characteristics optimized portfolio has the best end-of-the-period return, which is 1189.23 percent, earning an annualized return of 15.68 percent over the entire 18-year sample period! In contrast, the second ranked model, which is the three-factor model, only yields a cumulative return of 745.30 percent, equivalent to an annualized return of 12.54 percent.

It is well-known that the Japanese stock market has experienced a severe bear market since 1990. This is also readily seen from Fig. 1. In practice, many investors believe that the behavior of the bull market is very much different from that of the bear market. Academically, we have the well-known asymmetric effect that stocks tend to be more correlated in a down market than an up one. Hence, it is of interest to examine how the optimized portfolios perform in both bull and bear markets.

[6] Since the number of parameters in the covariance matrix is $N(N + 1)/2$, where N is the number of stocks, the number of parameters to be estimated in the covariance matrix for the 100-stock case is 5050, but increases dramatically to 20,100 for the 200-stock case.

Panel B of Table 1 presents the results over the first sub-period 1980–1989, during which Japan was experiencing a bull market. With $N = 50$, the results show that the characteristics model and all of the factor models yielded higher average returns than all benchmarks, including the market portfolio, but only characteristics model has a higher

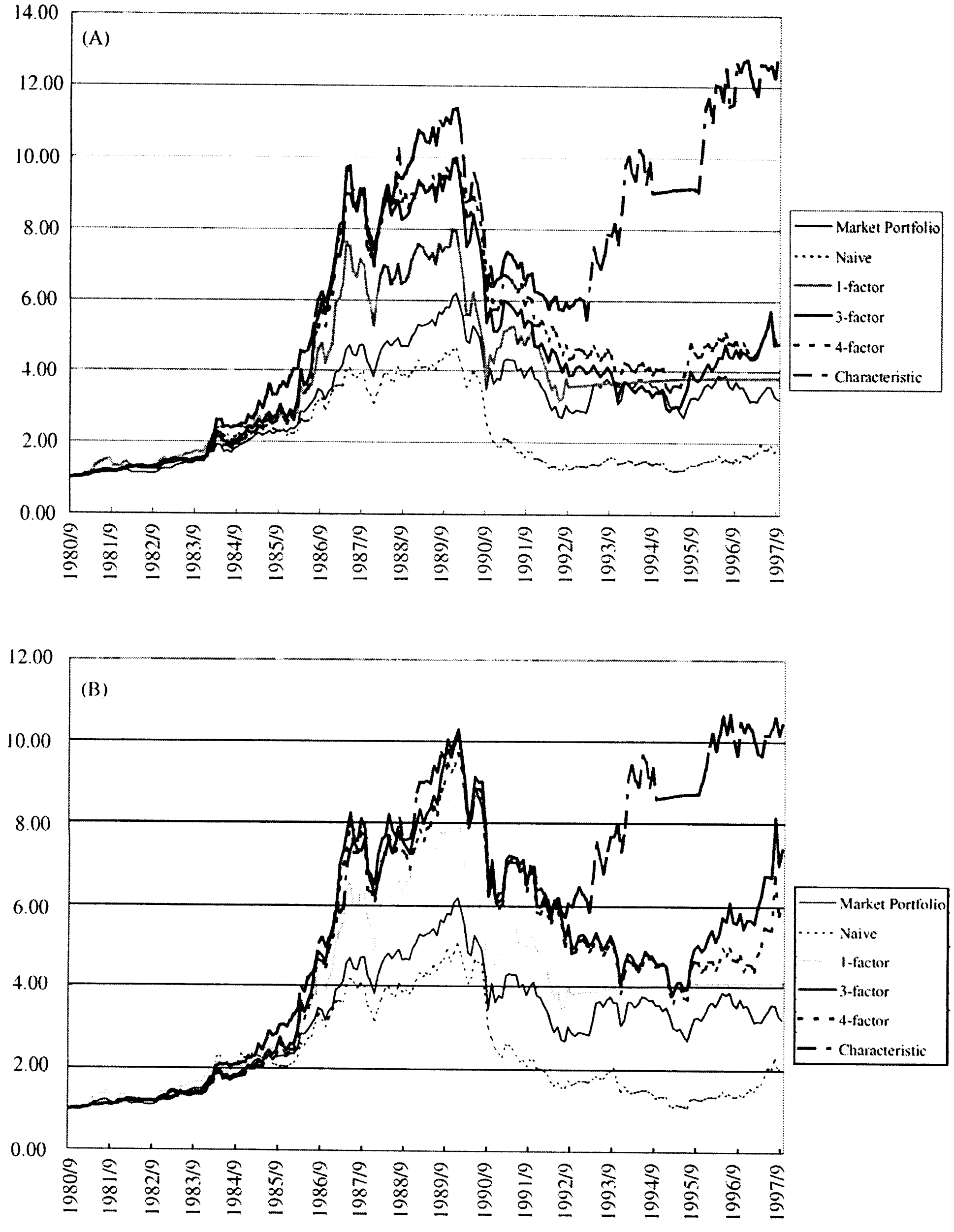

Fig. 2. Time-series plots of the cumulative returns of the market portfolio and optimized portfolio on various models.

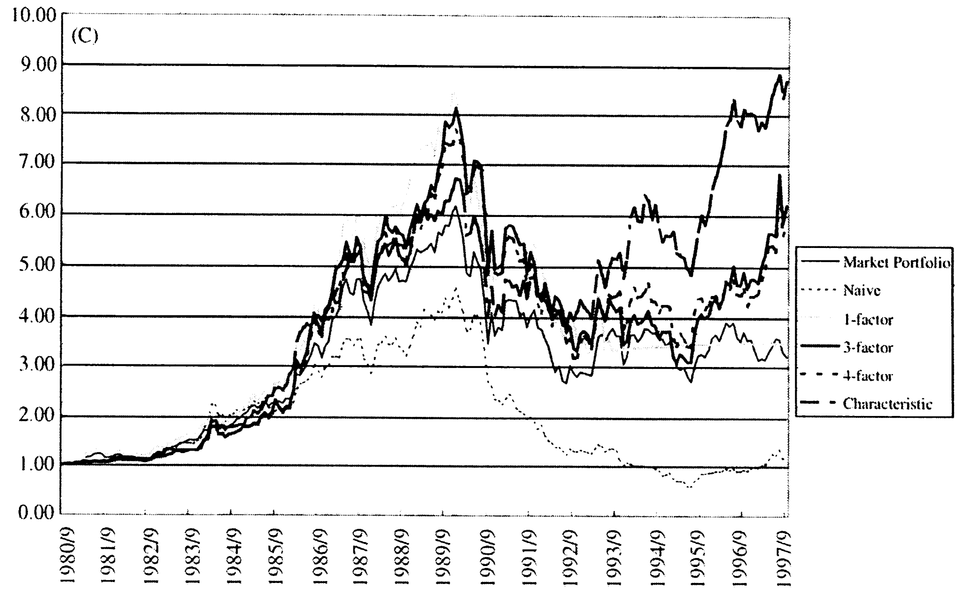

Fig. 2. (*Continued*).

Sharpe ratio. The optimized portfolio based on the naive model is still the worst among all portfolios. With $N = 100$, the results are about the same. But with $N = 200$, the factor models perform better than the characteristics model. Overall, however, the difference in performance, measured in terms of either returns or Sharpe ratios does not appear to be very significant during this bull market.

Panel C of Table 1 presents the results over the second sub-period 1989–1997. It is surprising that during this bear market, almost all portfolios yield negative returns. The single-factor CAPM performs worse than the market regardless of the number of assets in the optimized portfolios. The three- and four-factor models perform better when N increases from 50 to 100 and 200. The optimized portfolios based on the characteristics model are the only portfolios yielding positive returns. Although not reported here, the cumulative returns of the optimized portfolio based on the naive model persistently fall below those of other portfolios, whereas those of the characteristics model persistently stay above others. The results appear to suggest that the characteristics model may have provided better prediction of the parameters than the factor models. In particular, the characteristics models outperform the Fama–French three-factor model in terms of portfolio optimization, which seems to echo the findings of Daniel and Titman (1998) and Daniel et al. (2001). But notice that the performance of a model in the context of mean-variance optimization relies on the predictive precision on the expected returns and covariances. Hence, it is not clear yet whether the better performance of the characteristics model is due to its better prediction in the first moments or in the second moments or in both. We will explore this question in detail in the next section.

4. What makes the optimized portfolio do better?

The empirical results of the previous section show that the optimized portfolio based on the characteristics model performs the best over the market. The question one will ask is then: what are the fundamental sources that derive the results? In our simplified mean-variance framework, only the forecasted means and variances enter into the portfolio optimization problem. Hence, it is of interest to examine how the forecasted means and variances from each of the five models (including naive, characteristics, and three-factor-based models) perform over time as this may shed light on why the optimized portfolio based on the characteristics model performs so well.

At the end of September in each year from 1980 to 1996, the forecast of expected stock return for each of the N assets from each of the five models is compared to the sample average return realized over a subsequent 12-month period. Table 2 reports the summary statistics of the average absolute forecast error for estimates of expected returns from each of the five models. The absolute forecasting error for a stock is calculated as the absolute difference between realized and forecasted values of pairwise expected returns. To measure the precision of the forecasts on expected returns, we also calculate the correlation between the in-sample forecasts and the out-of sample averages of all stocks. A slope is also

Table 2
Performance of mean forecasting models

N	Model	Absolute forecast error					Correlation	Slope
		Mean	S.D.	5-th	95-th	Maximum		
Full sample: 198010–199709								
50	Naive	0.0269	0.0159	0.0054	0.0533	0.0735	−0.1669 (0.1435)	−0.2656 (0.1435)
	1-factor	0.0254	0.0151	0.0057	0.0519	0.0661	0.0516 (1.0000)	2.2475 (1.0000)
	3-factor	0.0253	0.0157	0.0047	0.0537	0.0676	−0.0216 (0.6291)	0.0226 (0.6291)
	4-factor	0.0253	0.0155	0.0051	0.0534	0.0672	−0.0253 (0.6291)	−0.0475 (0.6291)
	Characteristic	0.0248	0.0147	0.0056	0.0507	0.0653	0.1026 (0.1435)	0.5744 (0.1435)
100	Naive	0.0253	0.0160	0.0049	0.0535	0.0807	−0.1430 (0.3323)	−0.2146 (0.3323)
	1-factor	0.0237	0.0151	0.0045	0.0518	0.0710	0.0266 (1.0000)	1.4943 (1.0000)
	3-factor	0.0240	0.0154	0.0045	0.0535	0.0727	−0.0033 (1.0000)	−0.0859 (1.0000)
	4-factor	0.0241	0.0154	0.0043	0.0534	0.0727	−0.0155 (1.0000)	−0.2015 (1.0000)
	Characteristic	0.0235	0.0147	0.0046	0.0504	0.0734	0.1038 (0.0490)	0.4962 (0.0490)
200	Naive	0.0252	0.0170	0.0045	0.0558	0.0977	−0.1450 (0.0127)	−0.2258 (0.0127)
	1-factor	0.0232	0.0159	0.0033	0.0522	0.0903	0.0172 (0.6291)	1.2806 (0.6291)
	3-factor	0.0239	0.0163	0.0035	0.0544	0.0887	0.0007 (1.0000)	−0.0628 (1.0000)
	4-factor	0.0240	0.0162	0.0037	0.0544	0.0887	−0.0050 (0.6291)	−0.1535 (0.6291)
	Characteristic	0.0233	0.0155	0.0042	0.0517	0.0895	0.0917 (0.0127)	0.3829 (0.0127)

At the end of September of each year from 1980 to 1996 three samples of stocks (biggest 50, biggest 100 and biggest 200) are selected from eligible common stock issues on the PACAP database for Japan. Forecasts of expected returns are generated from five models, based on the prior 60 months of data for each stock. Forecasts are then compared against the realized sample means estimated over the subsequent 12 months. The last estimation period ends in 1997. Summary statistics are provided for the distribution of the absolute difference between realized and forecasted value of means. Also reported are the Pearson correlation between forecasts and realization, and the slope coefficient in the regression of realization on forecasts. The numbers in parentheses are the t-statistics for tests of the hypothesis that the parameter of interest is equal to zero.

calculated by running simple linear regression of out-of-sample average returns on in-sample forecasts.

Table 2 shows that for $N = 50$, the estimates of expected returns based on the characteristics model have the smallest absolute forecast error of 0.0248 among all five models. Measured in terms of the correlation and the slope, the characteristic model also has the best performance among all models. The correlation and slope from the naive, three- and four-factor models are even negative. Statistically, however, none of the five models show significant predictive power in expected returns.

For $N = 100$ and 200, the results show that characteristic model has the smallest forecast error, and is the only model for which the correlation and the slope are significantly different from zero statistically. For the naive model, the correlation and the slope are always negative, and for $N = 200$, they are even significantly different from zero with a P-value of 0.0127. This result provides an indirect evidence for the observation that Japanese stock returns are negatively autocorrelated. Hence, it is not surprising that the naive model performs so poorly. Measured in terms of the forecast error, the one-factor CAPM has the second best performance, the Fama–French three factor the third, while the naive model has the worst performance. Although not reported here, the subperiod results are similar except that the significance of the statistics is lowered because of smaller sample sizes.

Table 3 presents the summary statistics on the forecasted covariances from each model. The numbers are close to the results reported in Table 3 of CKL (1999). For both sets of stocks, the naive model and the characteristics model display the largest standard deviations of all models, a result also similar to CKL (1999). The one-factor CAPM model has the smallest standard deviation of 0.19, 0.17 and 0.16 percent among all models for

Table 3
Properties of forecasted covariances

N	Model	Mean	S.D.	Minimum	5-th	95-th	Maximum
Full sample: 198010–199709							
50	Naïve	0.0034	0.0028	−0.0039	−0.0002	0.0088	0.0158
	1-factor	0.0032	0.0019	−0.0002	0.0007	0.0068	0.0105
	3-factor	0.0034	0.0022	−0.0009	0.0006	0.0074	0.0114
	4-factor	0.0034	0.0022	−0.0012	0.0004	0.0074	0.0115
	Characteristic	0.0035	0.0029	−0.0037	−0.0002	0.0090	0.0162
100	Naive	0.0030	0.0024	−0.0049	−0.0002	0.0076	0.0164
	1-factor	0.0029	0.0017	−0.0005	0.0008	0.0061	0.0111
	3-factor	0.0030	0.0019	−0.0013	0.0006	0.0065	0.0118
	4-factor	0.0030	0.0019	−0.0017	0.0005	0.0065	0.0118
	Characteristic	0.0030	0.0025	−0.0047	−0.0002	0.0077	0.0170
200	Naive	0.0027	0.0023	−0.0059	−0.0004	0.0069	0.0183
	1-factor	0.0026	0.0016	−0.0010	0.0007	0.0056	0.0116
	3-factor	0.0027	0.0017	−0.0019	0.0005	0.0058	0.0122
	4-factor	0.0027	0.0017	−0.0023	0.0004	0.0059	0.0123
	Characteristic	0.0027	0.0023	−0.0056	−0.0004	0.0070	0.0190

At the end of September of each year from 1980 to 1996 three samples of stocks (biggest 50, biggest 100 and biggest 200) are selected from eligible common stock issues on the PACAP database for Japan. Forecasts of monthly return covariances are generated from five models, based on the prior 60 months of data for each stock. The number in the Exhibit are the time-series average of the annual statistics.

Table 4
Performance of covariance forecasting models

N	Model	Absolute forecast error					Correlation	Slope
		Mean	S.D.	5-th	95-th	Maximum		
Full sample: 198010–199709								
50	Naive	0.0033	0.0029	0.0002	0.0090	0.0192	0.4492 (0.0003)	0.6886 (0.0003)
	1-factor	0.0032	0.0029	0.0002	0.0090	0.0196	0.3614 (0.0003)	0.8084 (0.0003)
	3-factor	0.0032	0.0029	0.0002	0.0089	0.0194	0.3825 (0.0003)	0.7462 (0.0003)
	4-factor	0.0032	0.0029	0.0002	0.0089	0.0194	0.3875 (0.0003)	0.7491 (0.0003)
	Characteristic	0.0033	0.0030	0.0002	0.0091	0.0193	0.4491 (0.0003)	0.6733 (0.0003)
100	Naive	0.0030	0.0027	0.0002	0.0082	0.0230	0.3991 (0.0001)	0.6544 (0.0001)
	1-factor	0.0030	0.0027	0.0002	0.0081	0.0231	0.3145 (0.0003)	0.7040 (0.0003)
	3-factor	0.0030	0.0027	0.0002	0.0080	0.0230	0.3435 (0.0003)	0.7162 (0.0003)
	4-factor	0.0030	0.0027	0.0002	0.0081	0.0230	0.3486 (0.0003)	0.7185 (0.0003)
	Characteristic	0.0031	0.0027	0.0002	0.0082	0.0231	0.3991 (0.0001)	0.6412 (0.0001)
200	Naive	0.0031	0.0028	0.0002	0.0084	0.0313	0.3088 (0.0001)	0.5328 (0.0001)
	1-factor	0.0031	0.0028	0.0002	0.0083	0.0314	0.2306 (0.0003)	0.5103 (0.0003)
	3-factor	0.0030	0.0028	0.0002	0.0082	0.0315	0.2627 (0.0001)	0.5980 (0.0001)
	4-factor	0.0031	0.0028	0.0002	0.0082	0.0315	0.2675 (0.0001)	0.6009 (0.0001)
	Characteristic	0.0031	0.0028	0.0002	0.0084	0.0314	0.3080 (0.0001)	0.5216 (0.0001)

At the end of September of each year from 1980 to 1996 three samples of stocks (biggest 50, biggest 100 and biggest 200) are selected from eligible common stock issues on the PACAP database for Japan. Forecasts of monthly return covariances are generated from five models, based on the prior 60 months of data for each stock. Forecasts are then compared against the realized sample covariances estimated over the subsequent 12 months. The last estimation period ends in 1997. Summary statistics are provided for the distribution of the absolute difference between realized and forecasted value of pairwise covariances. Also reported in the Pearson correlation between forecasts and realization, and the slope coefficient in the regression of realization on forecasts. The numbers in parentheses are the P-value for tests of the hypothesis that the parameter of interest is equal to zero.

$N = 50$, 100 and 200, respectively. The results for the subperiods, not reported here, are about the same. However, the average covariances are much larger (two to three times larger in general) during the 1990's than during the 1980's.

Table 4 compares each model's forecasts of the covariances with realized sample covariances estimated over the subsequent 12 months. The reported average absolute forecast errors of the covariances are calculated similarly to those in Table 2. The results show that the absolute forecast errors are about the same for all five models. The correlation and the slope are also highly significant for all models. The results for the subperiods are also similar. The results suggest that the covariance estimates are not sensitive to the choices of models. Nevertheless, of all the five models, the covariance estimates from the naive model and from the characteristics model have the highest correlation with their out-of-sample realizations.

Overall, as the differences in covariance estimates appear to be minor for different models, it appears that the optimization performance would have been mostly affected by the precision of the estimates on the expected returns. Also, the empirical results show that optimization based on the characteristics model outperform other models. A question of interest, then, is to ask whether its better performance is due to its precise prediction in expected returns or in covariances. To check this, we replace the covariance estimates from

Table 5
Performance of optimal portfolios based on characteristic-based models

Model	N = 50				N = 100				N = 200			
	Mean (percent)	H_u	S.D. (percent)	Sharpe ratio	Mean (percent)	H_u	S.D. (percent)	Sharpe ratio	Mean (percent)	H_u	S.D. (percent)	Sharpe ratio
Market	0.35	–	5.54	0.0635	0.35	–	5.54	0.0635	0.35	–	5.54	0.0635
Nikkei 225	0.25	–	5.84	0.0429	0.25	–	5.84	0.0429	0.25	–	5.84	0.0429
Value-weighted	0.48	0.81	6.52	0.0737	0.47	1.02	6.14	0.0770	0.45	1.14	5.88	0.0760
Equally-weighted	0.49	1.07	6.30	0.0774	0.49	1.69	5.81	0.0848	0.45	1.63	5.49	0.0824
Price-weighted	0.49	0.47	7.25	0.0675	0.49	0.52	6.75	0.0721	0.46	0.49	6.28	0.0739
Characteristic	1.06	2.47*	5.94	0.1779	0.91	2.17*	5.10	0.1777	0.83	2.03*	5.30	0.1563
Characteristic – block diagonal	–	–	–	–	0.92	2.30*	5.07	0.1825	0.87	2.52*	5.34	0.1632
Characteristic – industry diagonal	0.91	2.29*	6.17	0.1470	0.76	1.91	5.57	0.1364	0.74	2.40*	5.42	0.1362
Characteristic + Naïve	1.02	2.33*	5.95	0.1716	0.90	2.06*	5.07	0.1776	0.86	2.00*	5.33	0.1604
Characteristic + 1-factor	1.11	2.02*	6.83	0.1621	0.88	1.68	5.60	0.1565	1.02	2.23*	5.90	0.1724
Characteristic + 3-factor	1.13	2.14*	6.66	0.1694	0.91	1.88	5.26	0.1731	0.97	2.10*	5.54	0.1749
Characteristic + 4-factor	1.10	2.10*	6.62	0.1660	0.89	1.81	5.33	0.1663	0.97	2.16*	5.53	0.1751

At the end of September of each year from 1980 to 1996 three samples of stocks (biggest 50, biggest 100 and biggest 200) are selected from eligible common stock issues on the PACAP database for Japan. Forecasts of means and covariances of monthly excess returns are generated from different models, using the prior 60 months of data for each stock. Based on each model's forecasts of means and covariances, a quadratic programming procedure is used to find the optimal portfolio. These weights are then applied to form buy-and-hold portfolio returns until the next September, when the forecasting and optimization steps are repeated and the portfolio is reformed. For each procedure summary statistics are presented in the Exhibit for the unannualized mean excess return and unannualized standard deviation for the returns realized on the portfolio, and the unannualized Sharpe ratio. The hypothesis is that the expected rate or return on underlying portfolio is equal to that on market portfolio. An asterisk denotes statistical significance at the 5% level.

the characteristic model with various alternative estimates outlined earlier in Section 1, and estimates from other models to see if the combination can further improve upon the performance of the characteristic model. The results are provided in Table 5. Table 5 shows that for the full sample period and with $N = 50$, all optimized portfolios significantly outperform the market portfolio, regardless of the use of various covariance estimates. The use of covariance estimates from factor models does yield higher average returns, but the resulting Sharpe ratios are never greater than that of the "plain-version" characteristics model. For example, although the combination of return estimates from the characteristics model and covariance estimates from the 3-factor model yields a higher monthly return of 1.13 percent (which is higher than 1.06 percent, the return from the plain-version model) with the resulting Sharpe ratio of 0.1694 is slightly smaller than that of the plain version, which is 0.1779. For subperiods, the results are about the same. Use of covariance estimates based on the "industry-diagonal" model only yields a monthly return of 0.91 and a Sharpe ratio of 0.1470 for the full-sample period.

The results with N equal to 100 and 200 are about the same as those with $N = 50$. That is, although use of alternative covariance estimates does improve upon the performance of the characteristics model in terms of out-of-sample realized returns or Sharpe ratios, the difference does not appear to be substantial. This confirms our conjecture that the optimization is probably more sensitive to the choices of expected return estimates, and less sensitive to the differences in covariance estimates. Yet there is another problem: why is it that the returns with $N = 50$ are higher than those with $N = 100$ or 200? Since the optimization is not that sensitive to the covariance estimates, the better performance of the characteristic model with a fewer number of assets are likely due to its better prediction of the expected returns on the top largest stocks as confirmed by Table 5.

Overall, our results show that portfolio optimization based on estimates from the characteristics model outperform other models, including the naive model, the one-factor CAPM, the Fama–French three-factor model, and a four-factor which includes an additional momentum factor. The results are robust in both bull and bear markets. Further investigation reveals that the performance of the characteristics model is attributed to its better prediction in expected returns, rather than in the estimates on covariances.

5. Conclusion

This paper takes a new approach to examine the economic importance of asset-pricing models from an investor's portfolio optimization perspective. In contrast to the usual cross-sectional analysis of stock returns which focuses on the comparison of expected returns, our approach examines also the variances and covariances as well as their joint impact on portfolio performance. This approach can be regarded as a hybrid of the realistic practical optimization procedure of Chan et al. (1999) (which focuses on forecasting covariances only) with the emerging literature, such as Kandel and Stambaugh (1996), Brennan and Xia (2001), and Pástor and Stambaugh (2000), on analyzing investment choices from the perspective of a non-representative agent.

Our empirical results shed some light on whether the factor or characteristics based models are better at explaining the cross-section of stock returns. Using Japanese data over the period 1980–1997, we find that an investor who believes Fama and French (1993) factor model would have obtained a much higher return than holding the market index, but also with a much higher risk. In contrast, if the investor optimizes his portfolio according to the characteristics-based model of Daniel and Titman (1997) and Daniel et al. (2001), and if the first 50 largest stocks were used, he would have had a monthly return of more than 0.81 percent (10.16 percent annualized) over the Nikkei 225 index with no greater risk. Moreover, this overwhelmingly better performance is also robust to various formations of the portfolio and various estimators of the covariances. In short, our findings seem to support the results of Daniel et al. (2001) in a different direction that, like the US evidence, the Japanese market is better described by using the characteristic-based models than by using the factor models.

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