Economics Letters 108 (2010) 184–186

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

How much stock return predictability can we expect from an asset pricing model?

Guofu Zhou *,[1]

Washington University in St. Louis, United States

ARTICLE INFO

Article history:
Received 28 September 2009
Received in revised form 27 April 2010
Accepted 5 May 2010
Available online 12 May 2010

Keywords:
Predictive regression
R-squared
Forecasting stock return

JEL classification:
C22
G11
G12

ABSTRACT

We provide a new upper bound on the *R*-squared of a predictive regression of stock returns on predictable variables, tightening substantially Ross's (2005) bound. An empirical application illustrates that while Ross's bound is not binding, our bound does.

1. An improved bound on predictability

Forecasting stock returns is of great interest to both academics and practitioners in finance. Consider a general predictive regression,

$$R_{t+1} = \mu(I_t) + \epsilon_{t+1}, \tag{1}$$

where R_{t+1} is the return on an asset or on a portfolio of N risky assets in excess of the riskfree rate, I_t is the information available at time t, $\mu(I_t) = E(R_{t+1}|I_t)$ is the predictive component of the future excess return, and ϵ_{t+1} is the residual. A special case of Eq. (1) is the popular predictive regression of the market,

$$R_{t+1} = \alpha + \beta z_t + \epsilon_{t+1}, \tag{2}$$

where R_{t+1} is the market excess return or equity risk premium, and z_t is the predictable variable, such as the dividend yield, used to predict the equity risk premium. There is a huge literature on predictability of the equity risk premium. For example, Fama and Schwert (1977) and Campbell (1987) are early studies that use various economic variables to forecast the market. Subsequently, Ferson and Harvey (1991), and Ang and Bekaert (2007), among others, find varying degrees of predictability with various predictable variables. Recently, Campbell and Thompson (2008), Cochrane (2008), and Rapach et al. (2010) provide further evidence even on out-of-sample predictability.

How much predictability should we expect given an asset pricing model? While studies such as Kirby (1998), Ferson and Harvey (1999), Stambaugh (1999), Avramov (2004), and Pástor and Stambaugh (2009) analyze the implications of rational asset pricing on return predictability, Ross (2005, p. 56) provides a simple and elegant bound on the R^2 of the predictive regression.

Lemma. (Ross, 2005, p. 56) Assume that the riskfree rate is constant, then

$$R^2 \le (1 + R_f)^2 \mathrm{Var}(m), \tag{3}$$

where R_f is the riskfree rate and m is any stochastic discount factor that prices the assets.

Since R^2 is a common measure of predictability, the above Ross's Lemma provides a simple bound in terms of the stochastic discount factor. The constant interest rate assumption is not a problem since $1 + R_f$ does not vary much relative to 1, and since one obtains similar results with no substantial differences if a higher rate is used on the right hand side. Under an additional auxiliary assumption that the Var(m) is no greater than 5 times the observed market risk aversion on asset returns, Ross (2005, p. 56) further shows, based on typical sample moments of the market, that the R^2 of the market excess return regression cannot be greater than 0.25%, 1.9%, and 7.9% for daily, weekly and monthly returns, respectively. However, for commonly used monthly regressions, a bound of 7.9% is almost never close to being reached even with some of the best

* Corresponding author. Olin Business School, Washington University, 1 Brookings Dr., St. Louis, MO 63130, United States. Tel.: +1 314 935 6384.
E-mail address: zhou@wustl.edu.

[1] I am grateful to Dashan Huang, Raymond Kan, Steve Ross, and an anonymous referee for valuable comments.

0165-1765/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.econlet.2010.05.008

predictors, as demonstrated in our application below. Hence, it is of interest to tighten the bound.

Before providing our improved bound, we discuss first the stochastic discount factor (SDF), which is not only useful for understanding Ross's Lemma, but also necessary for the proof below. The discount factor or the state-price density or the pricing kernel, is a random variable $m = m_{t+1}$ that satisfies

$$E[r_{j,t+1} m_{t+1} | I_t] = 1, \quad j = 1, \ldots, N, \tag{4}$$

where $r_{j,t+1}$ is the gross returns on the j-th asset at time $t+1$. Under very general conditions, any asset pricing model is a specification of m_{t+1} (see, e.g., Cochrane, 2001). The unconditional version of Eq. (4), in a vector form, is

$$E[rm] = 1_N, \tag{5}$$

where r is an N-vector of the gross returns, and 1_N is an N-vector of ones.

While Eq. (5) is the pricing restriction that the SDF implied by a given asset pricing model must satisfy, there can be other SDFs that make the restriction hold. In particular, the specific function of the asset returns below also serves as a SDF,

$$m_0 = \mu_m + (1_N - \mu_m \mu)' \Sigma^{-1} (r - \mu), \tag{6}$$

where $\mu_m = E[m]$ is the mean of m, and μ and Σ are the mean and covariance matrix of the N risky asset returns. The m_0 is called a default SDF since it prices the assets by construction without requiring the validity of any asset pricing model. We assume as usual that μ is not proportional to 1_N in order to avoid the trivial case. In addition, the N risky assets are assumed to be nonredundant so that Σ is nonsingular.

Let $x = (x_1, \ldots, x_K)'$ be a vector of K state variables. Consider the linear regression of m_0 on x,

$$m_0 = \alpha + \beta' x + \epsilon_0. \tag{7}$$

By construction, we have $E[\epsilon_0] = 0$ and $\mathrm{Cov}[\epsilon_0, x] = 0$. However, we make a slightly stronger assumption that

$$E[\epsilon_0 | x] = 0. \tag{8}$$

A sufficient condition for this to hold is when the returns and the state variables are jointly elliptically distributed (see, e.g., Muirhead, 1982, p. 36). Tu and Zhou (2004) demonstrate that the t-distribution, a special case of the elliptical, fits the return data well, so Assumption (8) does not seem too restrictive. Under this condition, Kan and Zhou (2007) provide a bound that improves the well-known Hansen–Jagannathan bound. Here we provide our improved bound on predictability under the same assumption.

With above preparations and with results from Kan and Zhou (2007), we can now improve Ross's bound by a factor, that is, we prove that

Proposition. Under Assumption (8) and the conditions of Ross (2005),

$$R^2 \le \rho^2_{x,m_0} (1 + R_f)^2 \mathrm{Var}[m(x)], \tag{9}$$

where R_f is the riskfree rate, ρ_{x,m_0} is the multiple correlation between the state variable x and the default SDF m_0, and $m(x)$ is the SDF or pricing kernel of an given asset pricing model.

Since the correlation ρ_{x,m_0} is always no greater than 1, our bound is in general an improvement of Ross's. In fact, because ρ_{x,m_0} is usually much less than one, and is of order from 0.10 to 0.15 in our later application, our bound can then be much tighter than Ross's, and is potentially binding in many applications.

Proof. Based on Kan and Zhou (2007), we have

$$\mathrm{Var}[m(x)] \ge \frac{1}{\rho^2_{x,m_0}} \mathrm{Var}[m_0] \tag{10}$$

or

$$\mathrm{Var}[m_0] \le \rho^2_{x,m_0} \mathrm{Var}[m(x)]. \tag{11}$$

Since m_0 is a SDF, the original Ross bound holds for m_0. One can also directly verify this by following Ross's (2005) derivations. Applying Eq. (11) to the Ross bound with m_0, we obtain the desired result. Q.E.D.

It should be pointed out that our new bound is obtained at a cost. Ross's bound is applicable to *all* SDFs, and ours is applicable only to those that are a function (with an unknown parametric form) of the given state variables. However, in the same spirit of Kan and Zhou (2007), the latter set of SDFs is of practical interest. For an asset model in practice, one has to specify what the state variables are and how they drive the dynamics of asset returns. For example, most asset pricing models are consumption-based, that is, $m(x)$ is a function of the aggregate consumption, and there is a huge literature on it (Cochrane, 2001). Our bound will be useful to answer the question whether or not the consumption-based asset pricing models can explain a given amount of predictability observed in the data.

2. An empirical application

Consider now an application of the bound in the popular predict regression,

$$R_{t+1} = \alpha + \beta z_t + \epsilon_t,$$

where R_{t+1} is the return on the S&P500 index in excess of the riskfree rate (which is approximated by the Treasury-bill rate, and z_t is one of the 10 predictors: dividend-price ratio, earnings-price ratio, book-to-market, T-bill rate, default yield spread, term spread, net equity issuance, inflation, long-term return, or stock variance. Welch and Goyal (2008) provide a detailed description of the data. The 10 predictors are those commonly used to forecast stock returns, and are consistently available from December 1926 to December 2008 at Goyal's website.

Table 1 provides the results. The R^2's are ranging from 0.0121% to 0.7059%. Stock returns are notoriously difficult to predict, and it is typical that the R^2's are very small in the predictive regressions. Ross (2005) first provides an upper bound on the variance of all SDFs, and then his earlier bound can be applied to bind the R^2. However, his bound on R^2 for the monthly regression, as is computed in his book, is 7.9%, far away from binding any of the realized R^2's based on real data.

On the other hand, ρ_{x,m_0} is typically low, and hence it helps to improve the bound sharply. Following Kan and Zhou (2007), we consider a standard choice of x as the consumption growth rates, and two sets of pricing assets. The first set is a single value-weighted market index of the NYSE, and the second set is the Fama and French (1993) 25 size and book-to-market ranked portfolios. This gives rise to two SDFs, m_0^a and m_0^b, that correspond to two asset pricing models, respectively. Based on Table 1 of Kan and Zhou (2007), we have $|\rho_{x,m_0^a}| \le 0.15$ and $|\rho_{x,m_0^b}| \le 0.10$. Hence, we can compute the numerical values of the new bounds, reported in the seventh and last columns of Table 1. Except three cases, the bounds fail to bind the R^2's of the predictive regressions. Since any one violation is a rejection of the theory, the low predictability found in the data is still too high to be consistent with the two specified asset pricing models.

There are at least three reasons for the violation of the bounds. First, the underlying asset pricing models, the standard ones, use a single state variable. More state variables may be added that can potentially increase ρ_{x,m_0}, and hence make the new bound less binding. But research is clearly

Table 1
Predictive regressions and bounds on R^2. The table reports OLS estimation results and bounds on the R^2 for the predictive regression model, $R_{t+1}=\alpha+\beta x_t+\epsilon_t$, where x_t is the predictor given in the first column. The next three columns are standard regression results. The fifth column is the Ross (2005) bound for the R^2. The sixth column is the correlation between the predictor and the default pricing kernel, followed by the new bound in the last column. The values for R^2 and its bounds are all in percentage points.

Predictor	α (t-stats)	β (t-stats)	R^2	Ross bound	ρ_{x,m_0^A}	New bound A	ρ_{x,m_0^B}	New bound B
Dividend-price ratio	0.0040 (1.68)	−0.0002 (−0.87)	0.0766	7.9000	0.1500	0.1777	0.1000	0.0790
Earnings-price ratio	0.0032 (1.43)	−0.0000 (−0.42)	0.0178	7.9000	0.1500	0.1777	0.1000	0.0790
Book-to-market	−0.0079 (−1.81)	0.0177 (2.64)	0.7059	7.9000	0.1500	0.1777	0.1000	0.0790
T-Bill rate	0.0058 (2.03)	−0.0828 (−1.42)	0.2047	7.9000	0.1500	0.1777	0.1000	0.0790
Default yield spread	−0.0015 (−0.45)	0.3680 (1.46)	0.2170	7.9000	0.1500	0.1777	0.1000	0.0790
Term spread	−0.0001 (−0.02)	0.1704 (1.22)	0.1516	7.9000	0.1500	0.1777	0.1000	0.0790
Net equity issuance	0.0054 (2.35)	−0.1406 (−1.91)	0.3720	7.9000	0.1500	0.1777	0.1000	0.0790
Inflation	0.0040 (2.01)	−0.5306 (−1.59)	0.2560	7.9000	0.1500	0.1777	0.1000	0.0790
Long-term return	0.0021 (1.16)	0.1078 (1.40)	0.1997	7.9000	0.1500	0.1777	0.1000	0.0790
Stock variance	0.0029 (1.48)	−0.1055 (−0.34)	0.0121	7.9000	0.1500	0.1777	0.1000	0.0790

required to analyze how ρ_{x,m_0} can be improved and how better asset models can be developed. Second, there may be structural breaks in the specified models over the long term under our study. For example, Welch and Goyal (2008), and Rapach et al. (2010) find strong evidence of fairly frequent breaks in the predictive regression. Third, there may be small sample problems in measuring the exact distribution of the R^2's. While studies on any of the three causes are of interest, we leave them elsewhere due to the substantial amount of research required.

3. Conclusion

The degrees of stock return predictability that an asset pricing model allows for is an important and interesting question in economics and investment practice. Ross (2005) provides an elegant bound on the R^2 of predictive regressions. However, his bound is too loose to be binding in applications. In this paper, we provide a simple way to tighten the bound by a scaler that measures the correlation of the state variables of an asset pricing model with the default pricing kernel. Since the correlations are low with many commonly used state variables, our bound can tighten Ross's bound substantially. In an application with the use of two popular consumption-based asset pricing models, we find that our new bound binds most of the R^2s in the predictive regressions when the predictor is one of the 10 popular ones: dividend-price ratio, earnings-price ratio, book-to-market, T-bill rate, default yield spread, term spread, net equity issuance, inflation, long-term return, or stock variance. In other words, the predictability found in the data cannot be explained, based on our new bound, by the asset pricing models. We leave explorations for the causes as future research.

References

Ang, A., Bekaert, G., 2007. Return predictability: is it there? Review of Financial Studies 20, 651–707.
Avramov, D., 2004. Stock return predictability and asset pricing models. Review of Financial Studies 17, 699–738.
Campbell, J.Y., 1987. Stock returns and the term structure. Journal of Financial Economics 18, 373–399.
Campbell, J.Y., Thompson, S.B., 2008. Predicting the equity premium out of sample: can anything beat the historical average? Review of Financial Studies 21, 1509–1531.
Cochrane, J.H., 2001. Asset Pricing. Princeton University Press.
Cochrane, J.H., 2008. The dog that did not bark: a defense of return predictability. Review of Financial Studies 21, 1533–1575.
Fama, E.F., Schwert, G.W., 1977. Asset returns and inflation. Journal of Financial Economics 5, 115–146.
Fama, E.F., French, K.R., 1993. Common risk factors in the returns on stocks and bonds. Journal of Financial Economics 33, 3–56.
Ferson, W.E., Harvey, C.R., 1991. The variation in economic risk premiums. Journal of Political Economy 99, 385–415.
Ferson, W.E., Harvey, C.R., 1999. Conditioning variables and the cross section of stock returns. Journal of Finance 54, 1325–1360.
Kan, R., Zhou, G., 2007. A new variance bound on the stochastic discount factor. Journal of Business 79, 941–961.
Kirby, C., 1998. The restrictions on predictability implied by rational asset pricing models. Review of Financial Studies 11, 343–382.
Muirhead, R.J., 1982. Aspects of Multivariate Statistical Theory. Wiley, New York.
Pástor, L., Stambaugh, R.F., 2009. Predictive Systems: Living with Imperfect Predictors. Journal of Finance 64, 1583–C1628 2000.
Rapach, D.E., Strauss, J.K., Zhou, G., 2010. Out-of-sample equity premium prediction: combination forecasts and links to the real economy. Review of Financial Studies 23, 821–C862.
Ross, S.A., 2005. Neoclassical Finance. Princeton University Press.
Stambaugh, R.F., 1999. Predictive regressions. Journal of Financial Economics 54, 375–421.
Tu, J., Zhou, G., 2004. Data-generating process uncertainty: what difference does it make in portfolio decisions? Journal of Financial Economics 72, 385–421.
Welch, I., Goyal, A., 2008. A comprehensive look at the empirical performance of equity premium prediction. Review of Financial Studies 21, 1455–1508.