

A Trend Factor: Any Economic Gains from Using Information over Investment Horizons?

In this online appendix, we provide a detailed proof of Theorem 1 of the paper. We consider first the informed investor's optimization problem.

To solve the HJB equation, it will be useful to write out explicitly the investment opportunity defined by the excess return of one share of the stock:

$$dQ = (D - rP)dt + dP. \quad (1)$$

The information set and the investment opportunity for the informed investor are given in the following lemma:

Lemma 01: For the informed investor i , the information set is given by the state variables $\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T$, which satisfies the following stochastic differential equation (SDE):

$$d\Psi^i = e_{\Psi}^i \Psi^i dt + \sigma_{\Psi}^i dB_t^i, \quad (2)$$

where B_t^i is a 5-dimensional Brownian Motion, and $e_{\Psi}^i, \sigma_{\Psi}^i \in R^{5 \times 5}$ constant matrices. Further, the investment opportunity defined in Equation (1) satisfies the SDE:

$$dQ^i = (D - rP)dt + dP = e_Q^i \Psi^i dt + \sigma_Q^i dB_t^i, \quad (3)$$

with $e_Q^i \in R^{5 \times 1}$ and $\sigma_Q^i \in R^{5 \times 1}$.

Proof of Lemma 01:

The state variables of the economy are given by

$$\begin{aligned} dD_t &= (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \\ d\pi_t &= \alpha_{\pi}(\bar{\pi} - \pi_t)dt + \sigma_{\pi} dB_{2t}, \\ d\theta_t &= -\alpha_{\theta} \theta_t dt + \sigma_{\theta} dB_{3t}. \end{aligned} \quad (4)$$

Thus the set that determines the informed investor's opportunity set is

$$\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T, \quad (5)$$

which satisfies the following vector SDE,

$$d\Psi^i = e_{\Psi}^i \Psi^i dt + \sigma_{\Psi}^i dB_t^i, \quad (6)$$

where B_t^i is a 5-dimensional Brownian Motion, e_{Ψ}^i and $\sigma_{\Psi}^i \in R^{5 \times 5}$ are constant matrices,

$$e_{\Psi}^i = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha_D & 1 & 0 & 0 \\ \alpha_{\pi} \bar{\pi} & 0 & -\alpha_{\pi} & 0 & 0 \\ 0 & 0 & 0 & -\alpha_{\theta} & 0 \\ p_0 & p_1 & p_2 & p_3 & p_4 - \alpha \end{pmatrix}, \quad (7)$$

and

$$\sigma_{\Psi}^i = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_D & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\pi} & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\theta} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (8)$$

The investment opportunity is then

$$dQ^i = (D_t - rP_t)dt + dP_t \equiv e_Q^i \Psi^i dt + \sigma_Q^i dB_t^i, \quad (9)$$

where e_Q^i and σ_Q^i are

$$e_Q^i = (p_0(p_4 - r) + p_2\alpha_{\pi}\bar{\pi}, 1 + p_1(p_4 - r - \alpha_D), p_1 + p_2(p_4 - r - \alpha_{\pi}), p_3(p_4 - r - \alpha_{\theta}), p_4(p_4 - r - \alpha)), \quad (10)$$

and

$$\sigma_Q^i = (0 \quad p_1\sigma_D \quad p_2\sigma_{\pi} \quad p_3\sigma_{\theta} \quad 0). \quad (11)$$

The result implies Lemma 01. QED.

With the aid of Lemma 01, we are now ready to prove a more complete form of Lemma 1.

Proposition 1: The HJB equation has a solution of the form:

$$J^i(W^i, D_t, \pi_t, \theta_t, A_t; t) = -e^{-\rho t - rW^i - \frac{1}{2}\Psi^{iT}V^i\Psi^i}, \quad (12)$$

with $\Psi^i = (1, D_t, \pi_t, \theta_t, A_t)^T$, and $V^i \in R^{5 \times 5}$ a positive definite symmetric matrix. The optimal demand of the informed investor for stock is given by

$$\eta^i = f^i \Psi^i = f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t, \quad (13)$$

where $f_0^i, f_1^i, f_2^i, f_3^i$ and f_4^i are constants.

Proof of Proposition 1

To prove Proposition 1, we conjecture a solution for the portfolio demand of the informed investor as a linear function of state variables as in Equation (13), and conjecture accordingly the value function be of the following form,

$$J^i(W^i, D_t, \pi_t, \theta_t, A_t; t) = -e^{-\rho t - rW^i - \frac{1}{2}\Psi^{iT}V^i\Psi^i}. \quad (14)$$

Substituting this into the HJB equation, we obtain

$$\eta = f^i \Psi^i, \quad (15)$$

where

$$f^i = \frac{1}{r}(\sigma_Q^i \sigma_Q^{iT})^{-1}(e_Q^i - \sigma_Q^i \sigma_{\Psi}^{iT} V^i) \quad (16)$$

with V^i a symmetric positive satisfying

$$V^i \sigma_{\Psi}^i \sigma_{\Psi}^{iT} V^{iT} - (\sigma_Q^i \sigma_Q^{iT})^{-1} (e_Q^i - \sigma_Q^i \sigma_{\Psi}^{iT} V^i)^T (e_Q^i - \sigma_Q^i \sigma_{\Psi}^{iT} V^i) + r V^i - (e_{\Psi}^{iT} V^i + V^i e_{\Psi}^i) + 2k \delta_{11}^{(5)} = 0, \quad (17)$$

where $k \equiv [(r - \rho) - r \ln r] - \frac{1}{2} T r (\sigma_{\Psi}^{iT} \sigma_{\Psi}^i V^i)$ and

$$[\delta_{(11)}^{(5)}]_{ij} = \begin{cases} 1, & i = j = 1 \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

This yields Proposition 1. QED.

Proposition 1 says that, given the model assumptions, the informed investor's demand for stock is a linear function of the fundamental variables, D_t, π_t, θ_t , as well as the technical variable A_t . The result is very similar to the one in Wang (1993) except that the estimation error therein is now replaced by A_t , the trading signal of the technical trader.

Now we turn our attention to the case of technical traders who are in a different situation from the informed investors in that they face a different information set and a different perceived investment opportunity. In particular, they do not observe state variable π_t as informed investors do. However, they do know the dynamics of the processes. In our setting, they use MA signal of past stock prices together with the other observables (D_t, P_t) to infer π_t , that is, they learn about π_t from its projection onto their information set $\Psi^u = (1, D_t, P_t, A_t)$. Specifically, they use the following linear regression to infer π_t , denoted as π_t^u ,

$$\pi_t^u = \beta_0 + \beta_1 D_t + \beta_2 P_t + \beta_3 A_t + \sigma_u u_t. \quad (19)$$

This is the optimal estimation for the unobservable π_t given the information set Ψ^u . With the insights from Wang (1993), we assume that technical traders, who do not observe π_t , know the process that drives its dynamics, and hence they can infer the unconditional linear regression coefficients in Equation (19). The details for computing the parameters $\beta_0, \beta_1, \beta_2, \beta_3$ and σ_u are as follows:

Based on (19), the regression slope, $\beta \equiv (\beta_1, \beta_2, \beta_3)$, is given by

$$\beta = \text{Var}^{-1} \text{Cov}, \quad (20)$$

where $\text{Var} \in R^{3 \times 3}$ and $\mu \in R^{3 \times 1}$ be the variance and mean of vector $Y_t = (D_t, P_t, A_t)$, and $\text{Cov} \in R^{1 \times 3}$ the covariance between π_t and (D_t, P_t, A_t) . Consider first how to compute $\text{Var}(Y_t)$ and $\text{Cov}(\pi_t, Y_t)$. Given the price relation, we have $Y = FX$ with

$$F \equiv \begin{pmatrix} 0 & 0 & 1 & 0 \\ p_2 & p_3 & p_1 & p_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (21)$$

Then it follows that

$$\text{Var} = FCF^T, \quad \mu = Fm^T, \quad \text{Cov} = e_1 CF^T, \quad (22)$$

where m and C are the mean and covariance matrix of X , respectively, and $e_1 \equiv (1, 0, 0, 0)$.

To compute the mean and covariance matrix of $X_t = (\pi_t, \theta_t, D_t, A_t)$, we, based on (4) and the pricing relation, have the following dynamics,

$$\begin{aligned} d\pi_t &= \alpha_\pi(\bar{\pi} - \pi_t)dt + \sigma_\pi dB_{2t}, \\ d\theta_t &= -\alpha_\theta\theta_t dt + \sigma_\theta dB_{3t}, \\ dD_t &= (\pi_t - \alpha_D D_t)dt + \sigma_D dB_{1t}, \\ dA_t &= [p_0 + p_1 D_t + p_2 \pi_t + p_3 \theta_t + (p_4 - \alpha)A_t]dt, \end{aligned}$$

which is an affine system. Let $\alpha_1 \equiv \alpha - p_4$. We now consider the following transform of X_t ,

$$\Phi(u, x, t, T) = E_t[e^{u \cdot X_T}] = e^{A(t) + B(t) \cdot X_t}, \quad (23)$$

where $A(t)$ and $B(t)$ satisfy the ODE system below,

$$\frac{dB(t)}{dt} = -K_1^T B(t), \quad B(T) = u, \quad (24)$$

$$\frac{dA(t)}{dt} = -K_0 \cdot B(t) - \frac{1}{2}B(t)^T H_0 B(t), \quad A(T) = 0, \quad (25)$$

with

$$K_0 = \begin{pmatrix} \alpha_\pi \bar{\pi} \\ 0 \\ 0 \\ p_0 \end{pmatrix}, \quad K_1 = \begin{pmatrix} -\alpha_\pi & 0 & 0 & 0 \\ 0 & -\alpha_\theta & 0 & 0 \\ 1 & 0 & -\alpha_D & 0 \\ p_2 & p_3 & p_1 & -\alpha_1 \end{pmatrix}, \quad H_0 = \begin{pmatrix} \sigma_\pi^2 & 0 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 & 0 \\ 0 & 0 & \sigma_D^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (26)$$

Note that the elements of the covariance matrix of X_t are the quadratic terms of X_t in Taylor expansion of Equation (23).

To solve Equations (24) and (25) analytically, we denote by U and Λ the eigenvectors and eigenvalues of K_1^T , ie,

$$UK_1^T = \Lambda U, \quad U^{-1}U = UU^{-1} = I, \quad \Lambda = \text{diag}(\lambda_1, \dots, \lambda_4). \quad (27)$$

Then, due to the special form of K_1^T , the eigenvalues are all negative,

$$\lambda_1 = -\alpha_\pi, \quad \lambda_2 = -\alpha_\theta, \quad \lambda_3 = -\alpha_D, \quad \lambda_4 = -\alpha_1.$$

Therefore, we have

$$B_t = U^{-1}e^{\Lambda(T-t)}Uu, \quad (28)$$

and

$$\begin{aligned} A_t &= \int_0^t K_0^T B_s ds + \frac{1}{2} \int_0^t B_s^T H_0 B_s ds \\ &= K_0^T U^{-1} \left[\int_0^t e^{\Lambda s} ds \right] Uu + \frac{1}{2} (Uu)^T \left[\int_0^t e^{\Lambda s} (U^{-1})^T H_0 U^{-1} e^{\Lambda s} ds \right] Uu, \end{aligned} \quad (29)$$

where we have used Equation (28).

Since we are only interested in the limit case when $T \rightarrow \infty$, and the elements of Λ are negative, the only term with non-zero limit in the exponent of Equation (23) is the second term of A_t in Equation (29). To compute the second term of A_t , we define

$$H \equiv \int_0^t e^{\Lambda s} (U^{-1})^T H_0 U^{-1} e^{\Lambda s} ds.$$

It is easy to show that the elements of H , denoted as H_{ij} for $i, j = 1, 2, 3, 4$, can be computed as

$$H_{ij} = -\frac{1}{\lambda_i + \lambda_j} [(U^{-1})^T H_0 U^{-1}]_{ij}. \quad (30)$$

Then the covariance matrix C of X_t and mean of X_t can be written as

$$C = U^T H U, \quad m = -K_0^T U^{-1} \Lambda^{-1} U.$$

In addition, the mean and covariance of Y_t can be computed as in Equation (22). Then the regression coefficients in (20) can be readily computed. Moreover,

$$\beta_0 = m e_1^T - \beta^T \mu, \quad \sigma_u^2 = e_1 C e_1^T - \beta^T (\text{Var}) \beta. \quad (31)$$

Once the coefficients β_i 's are determined, by matching the coefficients of both sides of

$$p_2 \hat{\pi}_t + p_3 \hat{\theta}_t = p_2 \pi_t + p_3 \theta_t = P_t - p_0 - p_1 D_t - p_4 A_t, \quad (32)$$

we obtain

$$\begin{aligned} p_2 \beta_0 + p_3 \gamma_0 &= p_0, \\ p_2 \beta_1 + p_3 \gamma_1 &= -p_1, \\ p_2 \beta_2 + p_3 \gamma_2 &= 1, \\ p_2 \beta_3 + p_3 \gamma_3 &= -p_4. \end{aligned} \quad (33)$$

Hence, we find

$$\gamma_0 = \frac{-p_0 - p_2 \beta_0}{p_3}, \quad \gamma_1 = \frac{-p_1 - p_2 \beta_1}{p_3}, \quad \gamma_2 = \frac{1 - p_2 \beta_2}{p_3}, \quad \gamma_3 = \frac{-p_4 - p_2 \beta_3}{p_3}. \quad (34)$$

This accomplishes the task.

Then, given the pricing equation, technical traders infer their estimation of the state variable θ_t, θ_t^u , from the price via

$$\begin{aligned} \theta_t^u &= \frac{1}{p_3} [P_t - (p_0 + p_1 D_t + p_2 \pi_t^u + p_4 A_t)] \\ &= \gamma_0 + \gamma_1 D_t + \gamma_2 P_t + \gamma_3 A_t - \frac{p_2}{p_3} \sigma_u u_t, \end{aligned} \quad (35)$$

where the parameters $\gamma_0, \gamma_1, \gamma_2$ and γ_3 are given above.

Define

$$\hat{\pi}_t = \beta_0 + \beta_1 D_t + \beta_2 P_t + \beta_3 A_t, \quad (36)$$

$$\hat{\theta}_t = \gamma_0 + \gamma_1 D_t + \gamma_2 P_t + \gamma_3 A_t. \quad (37)$$

The dynamics of D_t for technical traders is then

$$\begin{aligned} dD_t &= (\hat{\pi}_t + \sigma_u u_t - \alpha_D D_t) dt + \sigma_D dB_{1t} \\ &= (\hat{\pi}_t - \alpha_D D_t) dt + \sigma_D dB_{1t} + \sigma_u dZ_t \\ &= (\hat{\pi}_t - \alpha_D D_t) dt + \hat{\sigma}_D dB_{1t}^u, \end{aligned} \quad (38)$$

where Z_t is defined as $Z_t = \int_0^t u_s ds$, which is another independent Brownian motion with u_t the white noise in regression equation (19). The third equality in Equation (38) has used

$$\hat{\sigma}_D dB_{1t}^u = \sigma_D dB_{1t} + \sigma_u dZ_t, \quad (39)$$

with

$$\hat{\sigma}_D^2 = \sigma_D^2 + \sigma_u^2. \quad (40)$$

We define another state variable

$$\Lambda_t = p_2 \pi_t + p_3 \theta_t, \quad (41)$$

which is observable by technical traders through observing the equilibrium price and dividend. We have

$$\begin{aligned} d\Lambda_t &= (p_2 \alpha_\pi (\bar{\pi} - \hat{\pi}_t) - p_3 \alpha_\theta \hat{\theta}_t) dt \\ &\quad + p_2 (\sigma_\pi dB_{2t} - \alpha_\pi \sigma_u dZ_t) + p_3 (\sigma_\theta dB_{3t} + \alpha_\theta \frac{p_2}{p_3} \sigma_u dZ_t) \\ &= (p_2 \alpha_\pi (\bar{\pi} - \hat{\pi}_t) - p_3 \alpha_\theta \hat{\theta}_t) dt + \hat{\sigma}_\Lambda dB_{2t}^u, \end{aligned} \quad (42)$$

with

$$\begin{aligned} \hat{\sigma}_\Lambda dB_{2t}^u &= p_2 (\sigma_\pi dB_{2t} - \alpha_\pi \sigma_u dZ_t) + p_3 (\sigma_\theta dB_{3t} + \alpha_\theta \frac{p_2}{p_3} \sigma_u dZ_t) \\ &= p_2 \sigma_\pi dB_{2t} + p_3 \sigma_\theta dB_{3t} + (\alpha_\theta - \alpha_\pi) p_2 \sigma_u dZ_t \end{aligned} \quad (43)$$

and

$$\hat{\sigma}_\Lambda^2 = (p_2 \sigma_\pi)^2 + (p_3 \sigma_\theta)^2 + (\alpha_\theta - \alpha_\pi)^2 p_2^2 \sigma_u^2. \quad (44)$$

Based on Equations (39) and (43), the correlation between dB_{1t}^u and dB_{2t}^u , defined as

$$\text{Var}(dB_{1t}^u, dB_{2t}^u) \equiv \varrho dt,$$

can be written as

$$\varrho = \frac{p_2 \sigma_u^2 (\alpha_\theta - \alpha_\pi)}{\hat{\sigma}_D \hat{\sigma}_\Lambda}. \quad (45)$$

With the above discussions, we can summarize the investment opportunity faced by technical traders as in the following lemma:

Lemma 02: The state variable set observed by the technical trader, $\Psi^u = (1, D_t, P_t, A_t)^T$, follows a stochastic differential equation

$$d\Psi^u = e_\Psi^u \Psi^u dt + \sigma_\Psi^u dB_t^u, \quad (46)$$

where $B_t^u = (0, B_{1t}^u, B_{2t}^u, 0)$, with B_{1t}^u and B_{2t}^u defined in Equations (39) and (43), and e_Ψ^u and σ_Ψ^u given by

$$e_\Psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \beta_0 & \beta_1 - \alpha_D & \beta_2 & \beta_3 \\ q_0 & q_1 & q_2 & q_3 \\ 0 & 0 & 1 & -\alpha \end{pmatrix}, \quad \sigma_\Psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_D & 0 & 0 \\ 0 & p_1 \hat{\sigma}_D + \varrho \hat{\sigma}_\Lambda & \sqrt{1 - \varrho^2} \hat{\sigma}_\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (47)$$

The investment opportunity is

$$\begin{aligned} dQ^u &= (D - rP)dt + dP \\ &= e_Q^u \Psi^u dt + \sigma_Q^u dB_t^u, \end{aligned}$$

with e_Q^u and σ_Q^u defined as

$$e_Q^u = (q_0 \quad 1 + q_1 \quad q_2 - r \quad q_3), \quad \sigma_Q^u = (0 \quad p_1 \hat{\sigma}_D + \varrho \hat{\sigma}_\Lambda \quad \sqrt{1 - \varrho^2} \hat{\sigma}_\Lambda \quad 0). \quad (48)$$

Proof of Lemma 02

Given $\hat{\pi}_t$ and $\hat{\theta}_t$ in Equations (36) and (37), and the dynamics of Λ_t in Equation (42), we obtain

$$\begin{aligned} dP_t &= p_1 dD_t + d\Lambda_t + p_4 dA_t \\ &= p_1 (\hat{\pi}_t - \alpha_D D_t) dt + p_1 \hat{\sigma}_D dB_{1t}^u + d\Lambda_t + p_4 dA_t \\ &= [q_0 + q_1 D_t + q_2 P_t + q_3 A_t] dt + p_1 \hat{\sigma}_D dB_{1t}^u + \hat{\sigma}_\Lambda dB_{2t}^u, \end{aligned} \quad (49)$$

where

$$\begin{aligned} q_0 &= p_1 \beta_0 + p_2 \alpha_\pi (\bar{\pi} - \beta_0) - \alpha_\theta p_3 \gamma_0, \\ q_1 &= p_1 (\beta_1 - \alpha_D) - p_2 \alpha_\pi \beta_1 - p_3 \alpha_\theta \gamma_1, \\ q_2 &= p_1 \beta_2 - p_2 \alpha_\pi \beta_2 - p_3 \alpha_\theta \gamma_2 + p_4, \\ q_3 &= p_1 \beta_3 - p_2 \alpha_\pi \beta_3 - p_3 \alpha_\theta \gamma_3 - p_4 \alpha. \end{aligned}$$

Applying further Equations (34), we obtain

$$\begin{aligned} q_0 &= p_1 \beta_0 + p_2 \alpha_\pi \bar{\pi} + p_0 (\alpha_\pi + \alpha_\theta), \\ q_1 &= p_1 (\beta_1 - \alpha_D + \alpha_\pi + \alpha_\theta), \\ q_2 &= p_1 \beta_2 + (p_4 - \alpha_\pi - \alpha_\theta), \\ q_3 &= p_1 \beta_3 + p_4 (\alpha_\pi + \alpha_\theta - \alpha). \end{aligned} \quad (50)$$

The parameters $\hat{\sigma}_D^2$, $\hat{\sigma}_\Lambda^2$ and ϱ are defined in Equations (40), (44) and (45), and σ_u^2 is defined in Equation (31).

Combined with Equation (38) and

$$dA_t = (P_t - \alpha A_t)dt, \quad (51)$$

we obtain the dynamics for $\Psi^u = (1, D_t, P_t, A_t)^T$, which follows the following SDE,

$$d\Psi^u = e_\psi^u \Psi^u dt + \sigma_\Psi^u dB_t^u, \quad (52)$$

where

$$e_\psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \beta_0 & \beta_1 - \alpha_D & \beta_2 & \beta_3 \\ q_0 & q_1 & q_2 & q_3 \\ 0 & 0 & 1 & -\alpha \end{pmatrix}, \quad (53)$$

and

$$\sigma_\psi^u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \hat{\sigma}_D & 0 & 0 \\ 0 & p_1 \hat{\sigma}_D + \varrho \hat{\sigma}_\Lambda & \sqrt{1 - \varrho^2} \hat{\sigma}_\Lambda & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (54)$$

The investment opportunity is

$$dQ^u = (D_t - rP_t)dt + dP_t \equiv e_Q^u \Psi dt + \sigma_Q^u dB_t^u, \quad (55)$$

obtained via Equation (49), where e_Q^u and σ_Q^u are given by

$$e_Q^u = (q_0 \quad 1 + q_1 \quad q_2 - r \quad q_3), \quad (56)$$

and

$$\sigma_Q^u = (0 \quad p_1 \hat{\sigma}_D + \varrho \hat{\sigma}_\Lambda \quad \sqrt{1 - \varrho^2} \hat{\sigma}_\Lambda \quad 0). \quad (57)$$

Then we have the lemma. QED.

The intuition behind the portfolio solution method is that, in order to derive a linear equilibrium price in closed form, the demand function, and hence the investment opportunity set, should be in the state variable set. By adding a state variable A_t to the functional form of the price, the investment opportunity set will be in the state variable set. To solve the technical trader u 's optimization problem, let W^u be the wealth of the technical trader, η^u be the holding of stock and c^u be the consumption. Then the optimization problem is

$$\max_{\eta^u, c^u} E \left[- \int_t^\infty e^{-\rho s - c(s)} ds \middle| \mathcal{F}_t^u \right] \text{ s.t. } dW^u = (rW^u - c^u)dt + \eta^u dQ^u. \quad (58)$$

Let $J^u(W^u, \Psi^u; t)$ be the value function, then it solves the following HJB equation,

$$0 = \max_{c^u, \eta^u} \left[-e^{-\rho t - c^u} + J_W^u(rW^u - c^u + \eta^u e_Q^u \Psi^u) + \frac{1}{2} \sigma_Q^u \sigma_Q^{uT} \eta^{u2} J_{WW} + \eta^u \sigma_Q^u \sigma_\Psi^{uT} J_{W\Psi}^u - \rho J^u + (e_\Psi^u \Psi^u)^T J_\Psi^u + \frac{1}{2} \sigma_\Psi^u J_{\Psi\Psi}^u \sigma_\Psi^{uT} \right]. \quad (59)$$

The solution is provided by the following:

Proposition 2: Equation (59) has a solution of the following form,

$$J^u(W^u, D_t, P_t, A_t; t) = -e^{-\rho t - rW^u - \frac{1}{2} \Psi^{uT} V^u \Psi^u}, \quad (60)$$

with

$$\Psi^u = (1, D_t, P_t, A_t)^T, \quad (61)$$

and $V^u \in R^{4 \times 4}$ a positive definite symmetric matrix. The technical trader u 's optimal demand for stock is given by

$$\eta^u = f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t, \quad (62)$$

where f_0^u, f_1^u, f_2^u and f_3^u are constants.

Proof of Proposition 2

To prove Proposition 2, we conjecture a solution for the portfolio demand by the informed investors as linear function of state variable set Ψ as in Equation (62), and conjecture accordingly the value function

$$J^u(W^u, D_t, P_t, A_t; t) = -e^{-\rho t - rW^u - \frac{1}{2} \Psi^{uT} V^u \Psi^u}. \quad (63)$$

Substituting this into the HJB equation, we obtain

$$\eta = f^u \Psi^u, \quad (64)$$

where

$$f^u = \frac{1}{r} (\sigma_Q^u \sigma_Q^{uT})^{-1} (e_Q^u - \sigma_Q^u \sigma_\Psi^{uT} V^u) \quad (65)$$

with V^u a symmetric positive definite matrix satisfying

$$V^u \sigma_\Psi^u \sigma_\Psi^{uT} V^{uT} - (\sigma_Q^u \sigma_Q^{uT})^{-1} (e_Q^u - \sigma_Q^u \sigma_\Psi^{uT} V^u)^T (e_Q^u - \sigma_Q^u \sigma_\Psi^{uT} V^u) + rV^u - (e_\Psi^{uT} V^u + V^u e_\Psi^u) + 2k \delta_{11}^{(4)} = 0, \quad (66)$$

$k \equiv [(r - \rho) - r \ln r] - \frac{1}{2} Tr(\sigma_\Psi^{uT} \sigma_\Psi^u V^u)$ and

$$[\delta_{(11)}^{(4)}]_{ij} = \begin{cases} 1, & i = j = 1 \\ 0, & \text{otherwise.} \end{cases} \quad (67)$$

This implies Proposition 2. QED.

Proposition 2, which contains Lemma 2 as a special case, says that the technical trader's demand for stock is a linear function of state variables, D_t, P_t , observable to them, and A_t , the technical indicator they use. Note that, in contrast, the price P_t is not in the demand function of the informed investor because θ_t and P_t are equivalent in terms of information content. On the other hand, the technical trader observes neither π_t nor θ_t , and hence she can only pin down her demand through the price function P_t . Indeed, to the technical trader, P_t provides new information.

Finally, we can prove Theorem 1 in detail.

Proof of Theorem 1

Given Equations (13) and (62), the demands of stock by the informed and technical investors, the market clearing condition requires

$$\eta^i + \eta^u = 1 + \theta_t,$$

or equivalently,

$$(1 - w)[f_0^i + f_1^i D_t + f_2^i \pi_t + f_3^i \theta_t + f_4^i A_t] + w[f_0^u + f_1^u D_t + f_2^u P_t + f_3^u A_t] = 1 + \theta_t. \quad (68)$$

Substitute the price functional P_t into above, and by matching coefficients of state variables, we obtain

$$\begin{aligned} (1 - w)f_0^i + w(f_0^u + f_2^u p_0) &= 1, \\ (1 - w)f_1^i + w(f_1^u + f_2^u p_1) &= 0, \\ (1 - w)f_2^i + w(f_2^u p_2) &= 0, \\ (1 - w)f_3^i + w(f_2^u p_3) &= 1, \\ (1 - w)f_4^i + w(f_2^u p_4) &= 0. \end{aligned} \quad (69)$$

The solution to Equation (69) determines the coefficients p_0, p_1, p_2, p_3 and p_4 for the price functional. It is easy to show that a unique solution exists under general conditions. This implies that Theorem 1 holds. Q.E.D.