

# Online Appendix

## Unspanned Global Macro Risks in Bond Returns

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In this Internet Appendix, we include auxiliary material about the following:

Out-of-sample excess bond return predictions for the LMF (Section 1)

Forecasting GDP growth with the GMF (Section 2)

MTSMs with the unspanned GMF (Section 3)

Forward term premia (Section 3.1)

Economic value of the GMF predictability (Section 3.2)

Estimates of model parameters (Section 3.3)

# 1 Out-of-sample predictability of the LMF

This section examines the out-of-sample predictability of excess bond returns in four industrialized countries—the United States, the United Kingdom, Japan, and Germany. To avoid look-ahead bias, we generate out-of-sample forecasts of excess bond returns using a recursive expanding estimation method. In the out-of-sample exercise, the GMF and LMF are recursively constructed, using only information upon time  $t$ . All predictive regression parameters are estimated just using information available up to the quarter of forecast formation. More specifically, we estimate and forecast recursively, using data from the first observation to the time that the forecast is made, beginning in 1995:Q1 and extending through 2018:Q4.

We investigate whether the LMF, extracted from a panel of local real-time macroeconomic variables, is able to predict international bond risk premia. The following table reports the forecasting performance of the LMF. We find that the LMF consistently delivers negative  $R_{OS}^2$ s in all markets under consideration, though only two of these  $R_{OS}^2$ s are significant at the conventional level when assessed using the Diebold-Mariano test. The HLN and MSPE-adjusted statistics suggest similar level of significance. Overall, our findings suggest that the LMF are not able to predict international bond returns.

Recent empirical literature frequently documents a significant predictive power for bond risk premia of macro factor, but they rarely account for data revisions and publication lags in macroeconomic data. As demonstrated recently by Ghysels, Horan, and Moench (2017), such features may create a wedge between bond returns implied by vintage data and revised macro factors, implying that a large fraction of the predictability documented by Ludvigson and Ng (2009) may be driven by revision and publication lag components that are unavailable to an investor in real-time. The

Table 1: Out-of-Sample Bond Return Predictions: LMF

	US	UK	Japan	Germany
	$R_{OS}^2(\%)$	$R_{OS}^2(\%)$	$R_{OS}^2(\%)$	$R_{OS}^2(\%)$
$rx_t^{(2)}$	-24.7 (0.14) [0.17]	-14.3 (0.21) [0.19]	-7.58* (0.11) [0.09]	-2.52 (0.45) [0.38]
$rx_t^{(3)}$	-15.3 (0.18) [0.20]	-13.5 (0.18) [0.17]	-8.37 (0.18) [0.24]	-1.40 (0.41) [0.39]
$rx_t^{(4)}$	-12.4 (0.24) [0.27]	-16.9* (0.14) [0.13]	-17.4 (0.20) [0.24]	-0.95 (0.39) [0.45]
$rx_t^{(5)}$	-7.53 (0.32) [0.29]	-17.3** (0.09) [0.08]	-19.6 (0.15) [0.16]	-1.69 (0.30) [0.28]

The table reports the out-of-sample  $R^2$  statistics for log excess bond returns on the  $n$ -year long-term Treasury bond over the 1995:Q1-2018:Q4 period. Statistical significance for the out-of-sample  $R^2$  is based on the Diebold-Mariano statistic. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively. The numbers in "()" and "[ ]" indicate  $p$ -value evaluated based on the Harvey, Leybourne, and Newbold (1997) small-sample bias corrected test and the MSPE-adjusted test.

LMF is constructed using real-time macro factors, this may account for why our findings differ from the empirical studies suggesting a predictive power of macro factors.

## 2 Macro forecasts with the GMF

Seeking to understand the economic mechanism behind the link between the GMF and international bond returns, we examine the predictive ability of the GMF for real economic activity, above and beyond the information contained in yield-curve factors. Rational paradigms almost always imply that asset returns should be driven by economic fundamentals. Along this line of reasoning, the essence of the spanning puzzle is whether the yield curve contains all information for economic fundamentals related to bond pricing.

We measure economic activity by real year-over-year GDP growth rates. Data on real GDP is seasonally adjusted from the FRED database. The regression is as follows:

$$GDP_{i,t} = a_i + b_i GMF_t + \mathbf{b}' PC_{i,t} + w_t, \quad (1)$$

where  $PC_t$  are the first three principal components of bond yields. The parameter of interest is  $b_i$ . If it is significant, it implies that the GMF has additional predictive power for economic fundamentals, thus providing insights on the source of the GMF's predictive about for bond returns. Empirically, we find that  $b_i$  is respectively 0.65, 0.32, 0.37, and 0.24 in the US, Japanese, UK, and Germany bond markets, which are consistently significant at the 5 percent significance level.

### 3 MTSMs with the unspanned GMF

#### 3.1 Forward term premia

We illustrate the differences between the forward term premiums implied by our MTSM with and without the unspanned GMF in the following figure. Specifically, we plot the fitted forward term premium from Model 2  $FTP^{4,1}$ , the difference between the one-year forward rate starting in four years and the expected one-year spot rate four years from today. The results are similar to those of  $FTP^{2,1}$ .

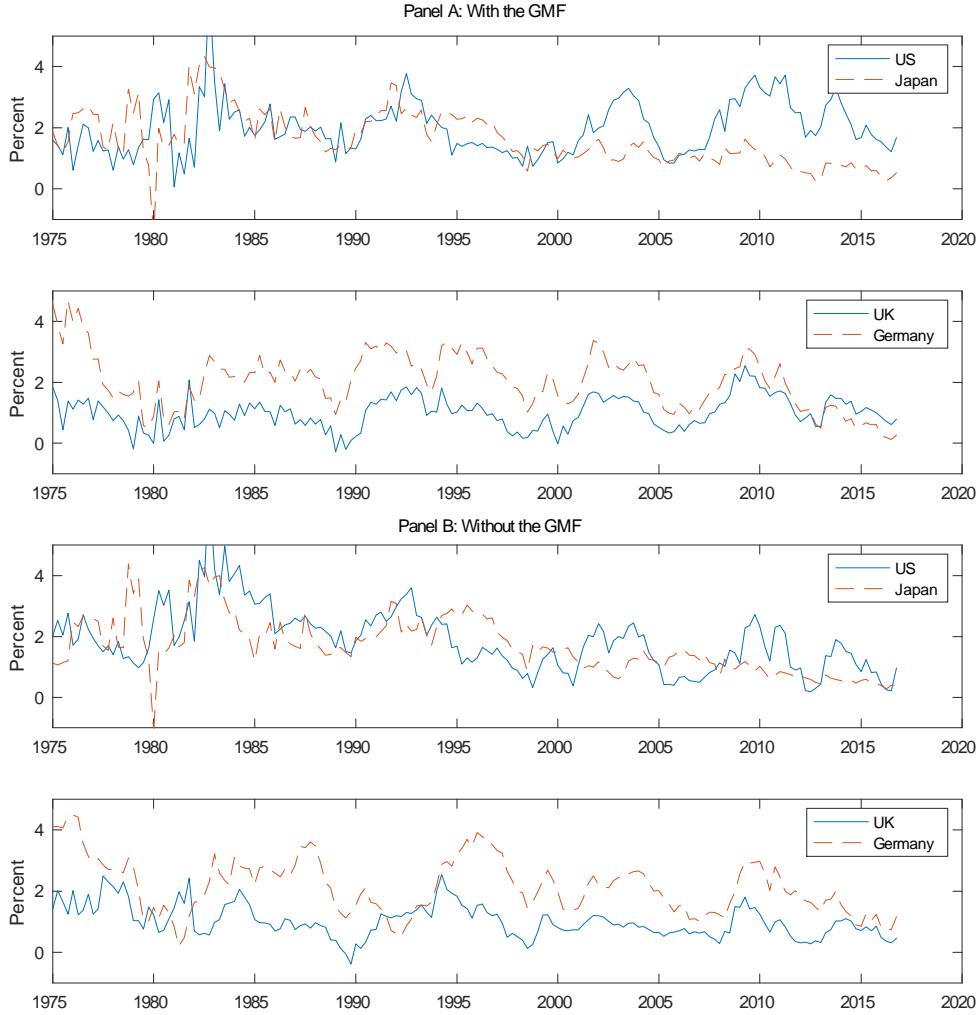


Figure 1: **Forward term premiums.** This figure depicts the forward term premiums,  $FTP_t^{4,1}$ , defined as the difference between the forward rate from four years to five years from the present and the expectation for the one-year spot rate four years from the present. We use  $FTP_t^{4,1}$  implied by our model with the unspanned GMF in Panel A and without the unspanned GMF in Panel B. We compute  $FTP_t^{4,1}$  from Model 2 for the UK, Germany and Japan.

### 3.2 Economic value

We assess the economic value of the GMF predictability. Following Campbell and Thompson (2008) and Welch and Goyal (2008), we assume that a mean-variance investor with relative risk aversion parameter  $\gamma$  allocates her portfolio quarterly between a long-term bond and a risk-free short-term bond using forecasts of excess bond returns  $\widehat{rx}_{t+1}^{(n)}$  and their variances  $\widehat{\sigma}_{t+1}^2{}^{(n)}$ . The investor will choose a long-term bond share of

$$\widehat{w}_t = \frac{1}{\gamma} \frac{\widehat{rx}_{t+1}^{(n)}}{\widehat{\sigma}_{t+1}^2{}^{(n)}}.$$

The following table reports the average annualized returns for the investor's portfolios where the forecasts of excess returns and variances are based on the MSTMs with and without the unspanned GMF. We set  $\gamma = .3$  and  $w_t$  is restricted to be between -50% and 150% to prevent extreme investments.

Table 2: Mean-variance portfolio returns

	Long-term bond			
	2-Year	3-Year	4-Year	5-Year
US	5.29%	5.76%	6.06%	5.94%
	5.33%	5.59%	5.69%	5.51%
UK	6.57%	7.25%	7.66%	7.92%
	6.46%	6.98%	7.20%	7.32%
Japan	2.54%	3.00%	3.21%	3.47%
	2.60%	2.86%	3.00%	3.29%
Germany	4.75%	5.50%	6.07%	6.45%
	4.82%	5.51%	5.89%	6.04%

The table reports the average annualized returns from a portfolio of a long-term bond and a risk-free short-term bond by a mean-variance investor with a relative risk aversion parameter of three. The numbers in the top (bottom) cells are based on our preferred MTSM with (without) the unspanned GMF. The sample period is 1995:Q1-2018:Q4.

### 3.3 Model estimates

This section presents maximum likelihood estimates of persistence parameters, and intercept and feedback parameters. When we estimate two model specifications for the UK, Japan and Germany, we find that the persistence parameters  $(r_{\infty}^Q, \lambda^Q)$  are nearly identical across models. As a result, we find the model implied bond yields are also (essentially) indistinguishable across the two models examined.



Table 3: Persistence Parameters: US

	Estimates	Std. Err
$r_\infty^Q$	0.1643	0.0177
$\lambda_1^Q$	0.9758	0.0002
$\lambda_2^Q$	0.9762	0.0021
$\lambda_3^Q$	0.8368	0.0019

The table reports maximum likelihood estimates of persistence parameters under risk-neutral measure  $Q$ : the long-run mean of the short rate  $r_\infty^Q$ ; the eigenvalues of the feedback matrix under  $Q$ ,  $\lambda^Q$ , which control the  $Q$ -rates of the factors' mean reversion. Asymptotic standard errors are provided.

Table 4: Intercept and Feedback Parameters: US

$Z$	$\mu$	$\phi$				
		$PC1$	$PC2$	$PC3$	$LMF$	$GMF$
$PC1$	-0.0011 [0.0578]	0.9896 [0.0196]	0.0346 [0.0625]	-0.0953 [0.2787]	-0.0066 [0.0155]	0.0253 [0.0183]
$PC2$	0.0015 [0.0399]	-0.0076 [0.0135]	0.8775 [0.0432]	0.5339 [0.1926]	-0.0190 [0.0107]	-0.0075 [0.0127]
$PC3$	0.0038 [0.0123]	0.0004 [0.0042]	0.0087 [0.0133]	0.9217 [0.0593]		
$LMF$	0.5745 [0.2873]	-0.2208 [0.0974]	0.3809 [0.3108]	-3.4064 [1.3857]	-0.3298 [0.0772]	0.3289 [0.0911]
$GMF$	-0.2146 [0.2533]	0.1846 [0.0859]	-0.1191 [0.2740]	-0.0463 [1.2218]	0.0409 [0.0681]	0.7105 [0.0803]

The table reports maximum likelihood estimates of intercept  $\mu$  and feedback parameters  $\phi$  for the state variables  $Z$  under physical measure  $P$ , i.e.  $E_t^P[Z_{t+1}] = \mu + \phi Z_t$ . State variables  $Z$  include principal components of local yields, the GMF and LMF in Model 1, and include principal components of local and US yields, and the GMF in Model 2. Zeros correspond to the restrictions on model parameters based on our model selection criterion. Asymptotic standard errors are provided.

Table 5: Persistence Parameters: UK

	Model 1		Model 2	
	Estimates	Std. Err.	Estimates	Std. Err.
$r_\infty^Q$	0.0004	0.0050	0.0027	0.0052
$\lambda_1^Q$	0.9872	0.0011	0.9837	0.0011
$\lambda_2^Q$	0.9716	0.0018	0.9736	0.0018
$\lambda_3^Q$	0.7553	0.0015	0.7521	0.0015

The table reports maximum likelihood estimates of persistence parameters under risk-neutral measure  $Q$ : the long-run mean of the short rate  $r_\infty^Q$ ; the eigenvalues of the feedback matrix under  $Q$ ,  $\lambda^Q$ , which control the  $Q$ -rates of the factors' mean reversion. Asymptotic standard errors are provided.

Table 6: Intercept and Feedback Parameters: UK

Panel A: Model 1									
$Z$	$\mu$	$\phi$							
		$PC1$	$PC2$	$PC3$	$LMF$	$GMF$			
$PC1$	0.0019 [0.0380]	0.9892 [0.0164]	0.0122 [0.0468]	-0.1550 [0.1769]	-0.0147 [0.0144]	0.0388 [0.0161]			
$PC2$	0.0074 [0.0244]	0.0007 [0.0105]	0.9155 [0.0301]	0.3241 [0.1137]	0.0071 [0.0092]	-0.0196 [0.0103]			
$PC3$	-0.0026 [0.0114]	0.0006 [0.0049]	0.0220 [0.0141]	0.8523 [0.0531]					
$LMF$	0.1463 [0.2049]	-0.0062 [0.0886]	-0.3800 [0.2526]	-0.2052 [0.9548]	-0.0234 [0.0775]	0.0000 [0.0867]			
$GMF$	-0.6594 [0.1656]	0.2823 [0.0716]	0.3825 [0.2041]	0.7623 [0.7715]	0.0447 [0.0627]	0.6875 [0.0700]			
Panel B: Model 2									
$Z$	$\mu$	$\phi$							
		$PC1$	$PC2$	$PC3$	$PC1^{us}$	$PC2^{us}$	$PC3^{us}$	$LMF$	$GMF$
$PC1$	0.0019 [0.0744]	0.9882 [0.0397]	0.0363 [0.0512]	-0.1552 [0.1814]	-0.0003 [0.0379]	0.0157 [0.0170]			
$PC2$	0.0078 [0.0481]	0.0008 [0.0257]	0.8784 [0.0331]	0.3583 [0.1172]			-0.0107 [0.0096]	-0.0177 [0.0112]	
$PC3$	-0.0028 [0.0224]	0.0005 [0.0120]	0.0230 [0.0154]	0.8524 [0.0546]					
$PC1^{us}$	-0.1783 [0.0934]	0.0882 [0.0499]	0.0707 [0.0642]	-0.2358 [0.2276]	0.8869 [0.0476]	0.0074 [0.0213]	-0.0078 [0.0186]	0.0051 [0.0178]	0.0254 [0.0218]
$PC2^{us}$	-0.1139 [0.2383]	0.0905 [0.1273]	-0.0626 [0.1639]	-0.4461 [0.5810]	-0.1894 [0.1215]	0.7657 [0.0544]	0.1005 [0.0474]	-0.0169 [0.0454]	0.0930 [0.0556]
$PC3^{us}$	0.1832 [0.3285]	-0.0884 [0.1755]	0.0865 [0.2259]	-1.1694 [0.8008]	0.0688 [0.1675]	0.0671 [0.0750]	0.5533 [0.0653]	-0.0213 [0.0626]	0.0318 [0.0767]
$LMF$	0.0771 [0.4035]	-0.0056 [0.2156]	-0.1612 [0.2775]	-0.2996 [0.9837]	0.0588 [0.2058]	0.1670 [0.0921]	-0.0111 [0.0802]	-0.3337 [0.0769]	-0.0712 [0.0942]
$GMF$	-1.5092 [0.3193]	0.7370 [0.1705]	0.5207 [0.2196]	0.6543 [0.7783]	-0.4421 [0.1628]	0.1324 [0.0729]	0.0898 [0.0634]	0.0438 [0.0608]	0.6895 [0.0745]

The table reports maximum likelihood estimates of intercept  $\mu$  and feedback parameters  $\phi$  for the state variables  $Z$  under physical measure  $P$ , i.e.  $E_t^P[Z_{t+1}] = \mu + \phi Z_t$ . State variables  $Z$  include principal components of local yields, the GMF and LMF in Model 1, and include principal components of local and US yields, and the GMF in Model 2. Zeros correspond to the restrictions on model parameters based on our model selection criterion. Asymptotic standard errors are provided.

Table 7: Persistence Parameters: Japan

	Model 1		Model 2	
	Estimates	Std. Err.	Estimates	Std. Err.
$r_\infty^Q$	0.0830	0.0572	0.0777	0.0494
$\lambda_1^Q$	0.9987	0.0155	0.9873	0.0133
$\lambda_2^Q$	0.8922	0.0000	0.8944	0.2203
$\lambda_3^Q$	0.8912	0.0153	0.8812	0.2221

The table reports maximum likelihood estimates of persistence parameters under risk-neutral measure  $Q$ : the long-run mean of the short rate  $r_\infty^Q$ ; the eigenvalues of the feedback matrix under  $Q$ ,  $\lambda^Q$ , which control the  $Q$ -rates of the factors' mean reversion. Asymptotic standard errors are provided.

Table 8: Intercept and Feedback Parameters: Japan

Panel A: Model 1									
$Z$	$\mu$	$\phi$							
		$PC1$	$PC2$	$PC3$	$LMF$	$GMF$			
$PC1$	0.0023	0.9719	-0.1502	0.3108	0.0093	0.0141			
	[0.0186]	[0.0133]	[0.0838]	[0.4263]	[0.0096]	[0.0107]			
$PC2$	0.0006	0.0089	0.9093	0.5187	0.0030	-0.0113			
	[0.0099]	[0.0071]	[0.0445]	[0.2263]	[0.0051]	[0.0057]			
$PC3$	-0.0021	-0.0030	-0.0136	0.7787					
	[0.0028]	[0.0020]	[0.0128]	[0.0650]					
$LMF$	0.2518	-0.1823	1.4025	-6.5775	-0.0750	0.0651			
	[0.1495]	[0.1070]	[0.6748]	[3.4337]	[0.0772]	[0.0859]			
$GMF$	-0.3284	0.3356	-0.6734	1.9271	-0.1157	0.3838			
	[0.1225]	[0.0877]	[0.5531]	[2.8143]	[0.0632]	[0.0704]			

Panel B: Model 2									
$Z$	$\mu$	$\phi$							
		$PC1$	$PC2$	$PC3$	$PC1^{us}$	$PC2^{us}$	$PC3^{us}$	$LMF$	$GMF$
$PC1$	0.0023	0.9753	-0.0078	-0.3210	-0.0095	-0.0151			
	[0.0339]	[0.0330]	[0.1073]	[0.4813]	[0.0242]	[0.0119]			
$PC2$	0.0006	0.0087	1.0234	-0.2374			-0.0060		
	[0.0176]	[0.0171]	[0.0557]	[0.2498]			[0.0058]		
$PC3$	0.0020	0.0032	0.0193	0.7553					
	[0.0049]	[0.0048]	[0.0156]	[0.0702]					
$PC1^{us}$	0.1417	-0.1076	0.3629	-1.1045	0.8836	0.0093	0.0024	-0.0121	-0.0344
	[0.0621]	[0.0605]	[0.1966]	[0.8825]	[0.0443]	[0.0219]	[0.0203]	[0.0184]	[0.0217]
$PC2^{us}$	0.1489	-0.0826	0.9727	2.7151	-0.1995	0.7473	0.1194	-0.0723	-0.1193
	[0.1560]	[0.1521]	[0.4943]	[2.2183]	[0.1115]	[0.0550]	[0.0511]	[0.0463]	[0.0546]
$PC3^{us}$	-0.2058	0.4089	0.7608	-0.4254	0.2358	0.0510	0.5003	-0.0829	-0.0275
	[0.2191]	[0.2136]	[0.6940]	[3.1143]	[0.1565]	[0.0772]	[0.0717]	[0.0651]	[0.0767]
$LMF$	0.6341	-0.6613	1.8784	9.4434	-0.5735	-0.1665	0.2314	-0.2864	0.0057
	[0.2559]	[0.2495]	[0.8107]	[3.6384]	[0.1828]	[0.0902]	[0.0838]	[0.0760]	[0.0896]
$GMF$	-0.6806	0.6090	-1.8545	-1.3943	0.2539	-0.1093	-0.0421	-0.0425	0.6557
	[0.2177]	[0.2123]	[0.6896]	[3.0946]	[0.1555]	[0.0768]	[0.0712]	[0.0646]	[0.0762]

The table reports maximum likelihood estimates of intercept  $\mu$  and feedback parameters  $\phi$  for the state variables  $Z$  under physical measure  $P$ , i.e.  $E_t^P[Z_{t+1}] = \mu + \phi Z_t$ . State variables  $Z$  include principal components of local yields, the GMF and LMF in Model 1, and include principal components of local and US yields, and the GMF in Model 2. Zeros correspond to the restrictions on model parameters based on our model selection criterion. Asymptotic standard errors are provided.

Table 9: Persistence Parameters: Germany

	Model 1		Model 2	
	Estimates	Std. Err.	Estimates	Std. Err.
$r_{\infty}^Q$	0.1272	0.0007	0.1296	0.0007
$\lambda_1^Q$	0.9845	0.0003	0.9841	0.0003
$\lambda_2^Q$	0.8480	0.0208	0.8490	0.0000
$\lambda_3^Q$	0.8478	0.0209	0.8480	0.0003

The table reports maximum likelihood estimates of persistence parameters under risk-neutral measure  $Q$ : the long-run mean of the short rate  $r_{\infty}^Q$ ; the eigenvalues of the feedback matrix under  $Q$ ,  $\lambda^Q$ , which control the  $Q$ -rates of the factors' mean reversion. Asymptotic standard errors are provided.

Table 10: Intercept and Feedback Parameters: Germany

Panel A: Model 1										
$Z$	$\mu$	$\phi$								
		$PC1$	$PC2$	$PC3$	$LMF$	$GMF$				
$PC1$	-0.0550 [0.0322]	0.9986 [0.0143]	0.0063 [0.0347]	0.2920 [0.1733]	-0.0044 [0.0093]	0.0345 [0.0097]				
$PC2$	0.0702 [0.0261]	-0.0163 [0.0116]	0.9035 [0.0282]	-0.0614 [0.1408]	0.0022 [0.0076]	-0.0133 [0.0079]				
$PC3$	0.0132 [0.0097]	0.0018 [0.0043]	0.0017 [0.0104]	0.8515 [0.0520]						
$LMF$	0.2300 [0.2674]	-0.2159 [0.1189]	-0.4403 [0.2886]	2.0098 [1.4394]	-0.0850 [0.0772]	-0.0148 [0.0806]				
$GMF$	-0.5424 [0.2252]	0.1937 [0.1002]	0.6037 [0.2431]	0.3313 [1.2123]	-0.0686 [0.0651]	0.6852 [0.0679]				
Panel B: Model 2										
$Z$	$\mu$	$\phi$								
		$PC1$	$PC2$	$PC3$	$PC1^{us}$	$PC2^{us}$	$PC3^{us}$	$LMF$	$GMF$	
$PC1$	0.0119 [0.0476]	1.0014 [0.0308]	-0.0034 [0.0369]	-0.1894 [0.1732]	-0.0097 [0.0214]	0.0177 [0.0106]			0.0322 [0.0109]	
$PC2$	-0.0216 [0.0382]	-0.0160 [0.0247]	0.9070 [0.0296]	0.5875 [0.1390]			0.0176 [0.0075]		-0.0120 [0.0088]	
$PC3$	0.0238 [0.0142]	0.0016 [0.0092]	-0.0067 [0.0110]	0.7779 [0.0517]						
$PC1^{us}$	-0.0769 [0.0928]	-0.0167 [0.0601]	-0.0242 [0.0719]	0.8611 [0.3378]	0.9677 [0.0418]	-0.0130 [0.0206]	-0.0023 [0.0182]	-0.0242 [0.0180]	0.0404 [0.0213]	
$PC2^{us}$	0.2783 [0.2365]	-0.2292 [0.1533]	-0.0614 [0.1834]	0.3652 [0.8614]	0.0263 [0.1065]	0.7409 [0.0526]	0.1031 [0.0465]	-0.0600 [0.0460]	0.0854 [0.0543]	
$PC3^{us}$	-0.3314 [0.3267]	-0.0258 [0.2117]	0.1830 [0.2533]	2.4276 [1.1898]	0.0170 [0.1471]	0.0712 [0.0726]	0.5869 [0.0642]	-0.0092 [0.0635]	0.0156 [0.0750]	
$LMF$	0.0238 [0.3963]	-0.1043 [0.2569]	-0.3465 [0.3074]	2.2226 [1.4436]	-0.0491 [0.1784]	0.1090 [0.0881]	0.0788 [0.0779]	-0.3768 [0.0770]	-0.0529 [0.0910]	
$GMF$	-0.3875 [0.3341]	0.0293 [0.2166]	0.6825 [0.2591]	0.4733 [1.2170]	0.1605 [0.1504]	0.0686 [0.0742]	0.0085 [0.0657]	-0.0726 [0.0649]	0.6752 [0.0767]	

The table reports maximum likelihood estimates of intercept  $\mu$  and feedback parameters  $\phi$  for the state variables  $Z$  under physical measure  $P$ , i.e.  $E_t^P[Z_{t+1}] = \mu + \phi Z_t$ . State variables  $Z$  include principal components of local yields, the GMF and LMF in Model 1, and include principal components of local and US yields, and the GMF in Model 2. Zeros correspond to the restrictions on model parameters based on our model selection criterion. Asymptotic standard errors are provided.