

# Manufacturing \& Service Operations Management 

Publication details, including instructions for authors and subscription information: http:// pubsonline.informs.org

# Inventory Commitment and Monetary Compensation Under Competition 

Junfei Lei, Fuqiang Zhang, Renyu Zhang, Yugang Yu

## To cite this article:

J unfei Lei, Fuqiang Zhang, Renyu Zhang, Yugang Yu (2023) Inventory Commitment and Monetary Compensation Under Competition. Manufacturing \& Service Operations Management

Published online in Articles in Advance 03 Aug 2023
. https:// doi. org/ 10.1287/ msom. 2021.0411
Full terms and conditions of use: https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-andConditions

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2023, INFORMS
Please scroll down for article-it is on subsequent pages

## informs.

With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.
For more information on INFORMS, its publications, membership, or meetings visit http:// www. informs.org

# Inventory Commitment and Monetary Compensation Under Competition 

Junfei Lei, ${ }^{\text {a }}$ Fuqiang Zhang, ${ }^{\text {b }}$ Renyu Zhang, ${ }^{\mathrm{c}, *}$ Yugang Yu ${ }^{\text {d }}$<br>${ }^{\text {a }}$ INSEAD, 77300 Fontainebleau, France; ${ }^{\text {b }}$ Olin Business School, Washington University in St. Louis, St. Louis, Missouri 63130; ${ }^{\text {c }}$ CUHK Business School, The Chinese University of Hong Kong, Hong Kong; ${ }^{\text {d School of Management, University of Science and Technology of China, }}$ Hefei 230026, China<br>*Corresponding author<br>Contact: junfei.lei@insead.edu, (DD https:// orcid.org /0000-0002-5875-0221 (JL); fzhang22@wustl.edu, © https:// orcid.org/0000-0003-2918-2613 (FZ); philipzhang@cuhk.edu.hk, © https:// orcid.org/0000-0003-0284-164X (RZ); ygyu@ustc.edu.cn, (D) https:// orcid.org /0000-0003-2882-5584 (YY)

Received: August 30, 2021
Revised: April 19, 2022; April 23, 2023; July 6, 2023
Accepted: July 9, 2023
Published Online in Articles in Advance: August 3, 2023
https://doi.org/10.1287/msom.2021.0411
Copyright: © 2023 INFORMS


#### Abstract

Problem definition: Inventory commitment and monetary compensation are widely recognized as effective strategies in monopoly settings when customers are concerned about stockouts. To attract more customer traffic, a firm reveals its inventory availability information to customers before the sales season or offers monetary compensation to placate customers if the product is out of stock. This paper investigates these two strategies when retailers compete on both price and inventory availability. Methodology/results: We develop a game-theoretic framework to analyze the strategic interactions among the retailers and customers and draw the following insights. First, both inventory commitment and monetary compensation may lead to a prisoner's dilemma. Although these strategies are preferred regardless of the competitor's price and inventory decisions, the equilibrium profit of each retailer could be lower in the presence of inventory commitment or monetary compensation because they intensify the competition between the retailers. Second, we find that market competition may hurt social welfare compared with a centralized setting by reducing the product availability in equilibrium. The inventory commitment and monetary compensation strategies further intensify the competition between the retailers, therefore causing an even lower social welfare. Managerial implications: Our study shows that, although inventory commitment and monetary compensation improve retailers' profit and social welfare under monopoly, these strategies should be used with caution under competition.


Funding: F. Zhang is grateful for the financial support from the National Natural Science Foundation of China [Grants 71929201, 72131004]. R. Zhang is grateful for the financial support from the Hong Kong Research Grants Council General Research Fund [Grant 14502722] and the National Natural Science Foundation of China [Grant 72293560/72293565]. Y. Yu is grateful for the financial support from the National Natural Science Foundation of China [Grant 71921001].
Supplemental Material: The online appendix is available at https://doi.org/10.1287/msom.2021.0411.

Keywords: inventory availability • retail competition • inventory commitment • monetary compensation

## 1. Introduction

Inventory commitment and monetary compensation are widely adopted marketing strategies for firms when consumers are worried about potential stockouts. For example, a retailer may choose to reveal its inventory stocking quantity to customers (see Su and Zhang 2009). Many e-commerce firms, including BestBuy.com and Neweggs.com, offer real-time availability information in the store and online. Target and Walmart also allow consumers to check the inventory availability of a particular product at local stores using the zip code and depart-ment-class-item number (Chelsey 2022). Moreover, consumers have access to technologies and applications that help them track product availability information. For instance, TrackAToy offers availability information for
many retailers if consumers enter the product's name on the web page (see http://www.trackatoy.com/). An alternative strategy to placate consumers is to offer monetary compensation if the product is out of stock when they visit the store. Sloot et al. (2005) find that monetary compensation, such as discount coupons, rain checks, and additional services are effective in placating consumers experiencing stockouts. Bhargava et al. (2006) report that MAP LINK (the United States's largest map distributor), VERGE (a U.S. media network publisher), and IntelliHome (a U.S. smart home technology company) offer discounts of $2 \%, 5 \%$, and $10 \%$, respectively, for all backlogged items. Many car dealers provide price reductions if the automobile consumers choose is out of stock, whereas restaurants offer free dishes if consumers'
original choices are sold out. Online retailers usually waive delivery fees if items are backlogged.

The strategies of inventory commitment and monetary compensation improve firms' profits in a monopoly market environment (see, e.g., Su and Zhang 2009). In the first place, the two strategies attract demand as they signal an assurance of high inventory quantity in stock. Consumers would not accept necessary up-front costs (i.e., time) to patronize a firm if the product they are looking for is out of stock. Moreover, the monetary compensation reimburses consumers when the product is out of stock, which encourages them to visit the firms even if there is a certain probability that the product is unavailable. These two strategies become even more important in a competitive marketplace as product availability is a key leverage to capture market demand, especially in the era of e-commerce and online shopping. For example, in December 2011, BestBuy. com canceled some online orders because of the overwhelming demand for hot product offerings. Soon after the cancellation, many customers moved to Amazon. com with a click of button as reported by TradeGecko (Tao 2014).

Being aware of the importance of inventory availability, firms may compete aggressively to attract market demand by committing to high inventory quantity in stock and/or offering high compensation upon stockouts. Although the strategies of inventory commitment and monetary compensation are acknowledged to benefit firms in monopoly settings, there is little research studying their effectiveness in a competitive marketplace. On one hand, these strategies provide incentives for customers to visit the retailers and enhance their competitive edge. On the other hand, it is also possible that firms will battle to overcommit inventory quantities and/or provide higher compensation in order to win a larger market share under competition. This phenomenon is analogous to the "price war" in many industries that has economically devastated many small businesses. For example, as major airlines went toe-to-toe in matching and exceeding one another's reduced fares, the whole industry recorded a higher volume of air travel as well as an alarming record of profit losses (see https:// hbr.org/2000/03/how-to-fight-a-price-war). Similarly, the two marketing strategies, if adopted by competing firms, may result in an escalated competition on product availability and eventually lead to excess inventory in stock throughout the whole industry.

In view of the potential alarming impact of overcompetition on product availability, this paper examines the inventory commitment and monetary compensation strategies under market competition. We model two competing retailers as newsvendors located at the endpoints of a Hotelling line market. Customers are uniformly distributed on the Hotelling line. As in the standard Hotelling model, customers incur a search cost
to patronize a retailer. The closer a customer is located to a retailer, the less search cost the customer incurs. Before demand is realized, each retailer sets its price and inventory order quantity to maximize the expected profit. The prices are observable to customers and the other retailer, whereas the inventory order quantity is each retailer's private information. Individual customers choose which retailer to patronize based on product price, search cost, and belief about inventory availability. Under the inventory commitment strategy, a retailer credibly reveals its inventory order information to the public, whereas under the monetary compensation strategy, a retailer compensates the customers who cannot get the product because of stockouts.

We adopt the rational expectation equilibrium (REE) framework to study the strategic interactions between retailers and customers under competition. A fraction of customers may switch to the other retailer once the focal one runs out of inventory. The retailers are competing on both price and inventory availability. In particular, the retailers' trade-off is between decreasing price (which implies low inventory availability) and increasing inventory availability (which requires a high price). We characterize the market equilibrium and deliver the following insights.

First, inventory commitment and monetary compensation may decrease retailers' profit under market competition. In the monopoly setting, it is shown that the inventory commitment and monetary compensation strategies benefit the retailer in the presence of strategic customers because they help mitigate the stockout risk for customers (Su and Zhang 2009). However, under market competition, if the inventory commitment option is available to retailers, a prisoner's dilemma may arise. Specifically, although both retailers have incentives to commit to an inventory order quantity, the equilibrium profits of both retailers may decrease if inventory commitment is adopted. Revealing inventory information to customers intensifies market competition and results in inventory overcommitment (overstocking), reducing profits for both retailers. Likewise, monetary compensation may prompt the retailer to overcompensate customers so as to signal high product availability, thus backfiring on the retailers and hurting their profits.

Second, we find that market competition may hurt social welfare in this problem setting. It is welldocumented in the economics literature that competition can increase social welfare (see, e.g., Stiglitz 1981). In our problem setting, one may intuit that competition enhances social welfare as well because competition lowers equilibrium prices. Our results, however, indicate that market competition leads to lower product availability and, therefore, reduces the social welfare. Moreover, inventory commitment and monetary compensation strategies, whereas improving social welfare
in a monopoly market (e.g., Su and Zhang 2009), induce more aggressive market competition between retailers, which further decreases social welfare.

The remainder of this paper is organized as follows. Section 2 positions this paper in the relevant literature. The model is introduced in Section 3. Sections 4 and 5, respectively, study the value of the inventory commitment and monetary compensation strategies under inventory availability competition. The analysis of customer surplus and social welfare is given in Section 6. Finally, Section 7 concludes this paper. All the proofs are presented in the online appendix.

## 2. Literature Review

The impact of inventory availability is extensively studied in the operations management literature. Dana and Petruzzi (2001) consider a newsvendor model in which consumers are concerned about inventory availability and can choose whether to visit the firm. Su and Zhang (2008) introduce the strategic waiting behavior of customers into the newsvendor setting and investigate the impact of such behavior on a firm's pricing and stocking decisions. Liu and Van Ryzin (2008) demonstrate that one can mitigate the strategic waiting behavior by limiting inventory availability over repeated selling horizons. Since then, there have been a growing number of operations studies that involve strategic customer behavior and availability considerations in various settings. For example, Su and Zhang (2009) and Cachon and Feldman (2015) further include a search cost for the stockout-conscious customers. Cachon and Swinney $(2009,2011)$ focus on the value of quick response under strategic customer behavior. Prasad et al. (2014), Li and Zhang (2013), and Wei and Zhang (2018a) investigate the advance selling strategy in which product availability may affect customers' optimal timing of purchase. Allon and Bassamboo (2011) use a cheap talk framework to quantify the value of providing inventory availability information to customers. Liang et al. (2014) examine a firm's product rollover strategies under consumers' forward-looking behavior. With variable assortment depth, Bernstein and Martínez-de Albéniz (2016) study the optimal dynamic product rotation strategy in the presence of strategic customers. Tereyagoglu and Veeraraghavan (2012) study a retailer's problem when selling to conspicuous consumers whose consumption utility depends on the availability of the product. Finally, Gao and Su (2016) study the role of inventory availability in the context of omni-channel retailing. Wei and Zhang (2018b) provide a recent review of this line of research. Despite the fast growth of this topic, the majority of studies in the literature focus on single-firm settings; our paper, instead, contributes to the literature by studying the impact of product availability in a competitive setting.

In a competitive marketplace, if a stockout occurs at one firm, unsatisfied demand may switch to the other firms. Such stockout-based substitution has also received significant attention in the operations management literature. Lippman and McCardle (1997) propose several ways to model demand allocation between competing newsvendors and show that competition leads to overstocking relative to the centralized solution. Netessine and Rudi (2003) develop a tractable model to compare inventory management under centralized versus decentralized control. Several studies extend the static substitution model to dynamic ones; see, for example, Bassok et al. (1999), Shumsky and Zhang (2009), and Yu et al. (2015). This line of research does not explicitly model individual customer behavior, which is a key focus of our work. Therefore, our paper differs from this research in terms of both the model setting and insights.

Another stream of papers studies the competition in product availability in the economics literature. Carlton (1978) is among the first to formally consider the issue of product availability in a competitive market and argues that only an equilibrium outcome with zero firm profit will arise. As a follow-up to Carlton's (1978) work, Deneckere and Peck (1995) consider a game in which firms can decide on both price and capacity and demonstrate that a pure-strategy equilibrium exists if and only if the number of firms is sufficiently large. Lei (2015) studies a similar integrated newsvendor and Hotelling model but with asymmetric unit costs. He finds that firms with the lowest unit cost may survive in the long run. Along this line of research, Daughety and Reinganum (1991) and Dana (2001) are the closest to our paper. Specifically, Daughety and Reinganum (1991) consider a setting in which consumers have imperfect information on both price and stocking levels at firms. An important finding is that, in equilibrium, the duopoly price is lower than the monopoly price if consumers' search cost is low, whereas the duopoly price is the same as the monopoly price if consumers' search cost is high. In contrast, we find that retailers may charge a strictly higher price to signal high product availability and, thus, attract more demand in the presence of market competition. Dana (2001) adopts a newsvendor setup to model retailers competing on product availability. It is shown that the retailers can enjoy a positive profit (i.e., they can charge a price higher than marginal cost) even though the products are perfectly substitutable because the retailers can signal a high probability of product availability using a high price. Our paper also uses a similar newsvendor paradigm but with several important differences. First, we use the Hotelling setup to incorporate heterogeneous travel costs of customers, which leads to different insights. Second, we examine the impact of availability competition on customer surplus, whereas Dana (2001) focuses on the equilibrium outcome from the firms'
perspectives. Finally, we also study the effectiveness of operational strategies such as stockout compensations and inventory commitment, which are absent from the preceding economics literature.

The economics literature also studies the impact of competition on customer surplus. For example, Brynjolfsson et al. (2003) summarize two channels of how the market competition of product variety improves consumer surplus. In their study of the online bookstores market, the increased product variety competition induces around $\$ 3$ million more consumer welfare in 2000. In the food industry, Hausman and Leibtag (2007) empirically verify that the entry of new business and the expansion of existing business improve average consumer surplus by approximately $25 \%$. Goolsbee and Petrin (2004) show that the competition between direct broadcast satellites and cable leads to a consumer welfare gain of $\$ 2.5$ billion for satellite buyers and $\$ 3$ billion for cable subscribers. Our contribution to this literature is that we demonstrate the adverse effect of market competition on social welfare if customers are concerned about inventory availability.

## 3. Model

To study the inventory commitment and monetary compensation strategies under inventory availability competition, we build our model upon the classic newsvendor and Hotelling frameworks. The newsvendor setup captures the key features of demand uncertainty and perishable inventory, which are common for a retail setting in which the inventory availability concern is most relevant. The Hotelling model highlights the competition between the retailers under heterogeneous tastes/preferences of the customers. These salient features are often ignored in the literature studying inventory availability. Moreover, because of demand uncertainty, customers may patronize the other retailer upon the stockout of the first retailer the customer visits, which we refer to as the customer switching behavior. We first study a base model without customer switching behavior. Then, we extend the base model by considering the customer switching behavior, which we refer to as the focal model.

### 3.1. Base Model Without Customer Switching

We model the market as a Hotelling line with a unit length, denoted by $\mathcal{M}=[0,1]$. Two retailers, $R_{i}(i=1,2)$, are located at the two endpoints of the Hotelling market $\mathcal{M}$. Without loss of generality, we assume $R_{i}$ is located at $i-1(i=1,2)$. Each retailer sells a substitutable product that has the same procurement cost $c$. Retailer $R_{i}$ chooses a stock quantity $q_{i}$ and charges a price $p_{i}$ to maximize its expected profit.

In the market $\mathcal{M}$, customers are uniformly distributed over the interval [0,1]. Each customer has an infinitesimal
mass and purchases at most one unit of the product. The valuation of the product to all customers is homogeneous and denoted $v$. The aggregate market demand $D$ (i.e., the total mass of the Hotelling line) is uncertain and follows a known distribution $F(\cdot)$. We assume that the demand distribution has an increasing failure rate, which can be satisfied by most commonly used distributions. For conciseness, we define $\mathbb{E}[\cdot]$ as the expectation operation and $x \wedge y:=\min (x, y)$ as the minimum operation. To visit a retailer, each customer incurs a search cost that increases linearly with distance. More specifically, the search cost of a customer located at $x \in$ $\mathcal{M}$ to visit $R_{1}$ (respectively, $R_{2}$ ) is $s x(s(1-x))$, where $s$ is the unit distance search cost. The search cost can also be interpreted as a disutility for a customer traveling to a retailer to obtain the product: the longer the distance between the customer and the focal retailer, the larger the disutility the customer incurs to purchase this product. Furthermore, to highlight the competition between the two retailers, we assume that the unit distance search cost $s$ is not too high such that all customers consider visiting the focal retailer as well as switching to the alternative retailer upon stockout. ${ }^{1}$ Finally, each customer aims to maximize the customer's expected payoff by choosing a retailer to visit.

The sequence of events unfolds as follows. At the beginning of the sales season, each retailer $R_{i}$ simultaneously decides its stocking quantity $q_{i}$ and announces the retail price $p_{i}$. Both the inventory level and the price cannot be adjusted throughout the sales horizon. Customers observe the prices $\left(p_{1}, p_{2}\right)$ but not the inventory levels ( $q_{1}, q_{2}$ ), and decide which retailer to visit (or not to visit any of them). The demand $D_{i}$ for retailer $R_{i}$ is realized as a result of customers' cumulative purchasing decisions. If $D_{i} \leq q_{i}$, all customers requesting the product can get one. Otherwise, $D_{i}>q_{i}$, stockout occurs, and customers not receiving the product leave the market. Finally, the transactions occur, and the retailers collect the revenues.

### 3.2. Base Model Equilibrium

Next, we analyze the equilibrium of the base model without customer switching. To this end, we adopt the REE concept, which is commonly used in the gametheoretic analytical models in the operations literature (see, e.g., Cachon and Swinney 2009, Li and Jain 2016, Anand and Goyal 2019, Aviv et al. 2019). Under the REE, customers, upon observing the prices $\left(p_{1}, p_{2}\right)$, form beliefs about inventory availability and make purchasing decisions to maximize their own expected utilities, whereas retailers (at the beginning of the sales horizon) base their pricing and inventory decisions on the anticipation of customers' cumulative purchasing behaviors to maximize profits. Furthermore, under equilibrium, both the customers' beliefs about inventory availability and the retailers' anticipations should be consistent with
the actual outcomes. The formal definition of REE is specified subsequently.
3.2.1. Customers' Problem. We first analyze the customers' problem. Consider a customer located at $x \in \mathcal{M}$. The customer's surplus to visit $R_{1}$ is $v-p_{1}-s x(-s x)$ if the product is in stock (out of stock). A similar analysis can be applied if the customer visits $R_{2}$. The customer gains zero surplus if the customer does not visit any retailer. Because customers cannot observe retailers' inventory status, they form a belief about it (see Dana 2001). To facilitate the analysis, we assume customers form beliefs about the (unobservable) inventory availability probability instead of order quantity because the influence of inventory stocking quantity on the expected utility (and, thus, the purchasing behavior) of a customer boils down to the availability probability it induces. Specifically, let $\theta_{i}\left(p_{1}, p_{2}\right) \in[0,1]$ be the in-stock probability of $R_{i}$ given the price, where $i=1,2$. Thus, the expected utility of a customer located at $x$ to visit $R_{1}\left(R_{2}\right)$ is $\mathcal{U}_{1}(x):=\left(v-p_{1}\right) \theta_{1}\left(p_{1}, p_{2}\right)-s x\left(\mathcal{U}_{2}(x):=\left(v-p_{2}\right)\right.$ $\left.\theta_{2}\left(p_{1}, p_{2}\right)-s(1-x)\right)$.

Customers base their purchasing decisions on the beliefs of product availability. More specifically, a customer chooses to visit the retailer from which the customer can earn a higher nonnegative expected payoff. Because a customer is infinitesimal, without loss of generality, a customer located at $x$ patronizes $R_{i}$ if $\mathcal{U}_{i}(x) \geq$ $\mathcal{U}_{-i}(x)(i=1,2)$. Therefore, there exists a threshold $x\left(p_{1}, p_{2}\right)$ such that a customer located at $x$ patronizes $R_{1}$ if $x \leq x\left(p_{1}, p_{2}\right)$ and patronizes $R_{2}$ if $x \geq x\left(p_{1}, p_{2}\right)$. Simple algebraic manipulation yields

$$
\begin{aligned}
& x\left(p_{1}, p_{2}\right) \\
& :=\mathcal{P}_{[0,1]}\left(\frac{1}{2}-\frac{\left(v-p_{2}\right) \theta_{2}\left(p_{1}, p_{2}\right)-\left(v-p_{1}\right) \theta_{1}\left(p_{1}, p_{2}\right)}{2 s}\right) \\
& \quad \in[0,1],
\end{aligned}
$$

where $\mathcal{P}_{[0,1]}(x)=\max \{0, \min \{x, 1\}\}$ is the projection onto interval $[0,1]$. Note that we focus on the more interesting case with market share competition in this paper by assuming that the search cost $s$ is not too high so that the market is fully covered in equilibrium.
3.2.2. Retailer's Problem. Next, we analyze the retailer's pricing and inventory problem. Each retailer strategizes its price and inventory decisions in anticipation of customers' purchasing behaviors (thus, its market share). Specifically, the demand for $R_{1}\left(R_{2}\right)$ is $x\left(p_{1}, p_{2}\right) D$ $\left(\left(1-x\left(p_{1}, p_{2}\right)\right) D\right)$. Given the competitor's price, the retailer $R_{i}$ 's profit maximization problem is

$$
\max _{\left(p_{i}, q_{i}\right)}\left\{p_{i} \mathbb{E}\left(\alpha_{i}\left(p_{1}, p_{2}\right) D \wedge q_{i}\right)-c q_{i}\right\}
$$

where $\alpha_{1}\left(p_{1}, p_{2}\right)=x\left(p_{1}, p_{2}\right)$ and $\alpha_{2}\left(p_{1}, p_{2}\right)=1-x\left(p_{1}, p_{2}\right)$ represent the respective market shares. Therefore, given
price $p_{i}$, retailer $R_{i}$ 's optimal inventory order strategy is the newsvendor solution: $q_{i}=\alpha_{i}\left(p_{1}, p_{2}\right) F^{-1}\left(\frac{p_{i}-c}{p_{i}}\right)$.
To characterize the REE, we need to specify the offequilibrium customer belief on inventory availability (see, e.g., Dana 2001). Moreover, we refine the offequilibrium belief to rule out implausible equilibria. Consistent with the equilibrium refinement strategy of Dana (2001), customers rationally believe that the retailers are stocking the optimal amount of inventory given any observed price. Specifically, given the price ( $p_{1}, p_{2}$ ), the customers believe that the inventory order quantity of retailer $R_{i}$ is $q_{i}=\alpha_{i}\left(p_{1}, p_{2}\right) F^{-1}\left(\frac{p_{i}-c}{p_{i}}\right)$. Conditioned on the existence of a customer, belief about the total demand for the retailer $R_{i}$ is a random variable with probability density function $g_{i}\left(y \mid p_{1}, p_{2}\right):=\frac{y}{\alpha_{i}\left(p_{1}, p_{2}\right) \mu}$ $f\left(\frac{y}{\alpha_{i}\left(p_{1}, p_{2}\right)}\right)$, where $\mu:=\mathbb{E}[D]$ (see, e.g., Dana 2001, Su and Zhang 2009). Because the customers simultaneously decide whether and which retailer to patronize, each customer holds an identical belief about the inventory availability for $R_{i}$. Therefore, the belief of the customers about $R_{i}$ 's inventory availability probability is $\theta_{i}\left(p_{1}, p_{2}\right)=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_{i}-c}{p_{i}}\right)\right) \mathrm{d} F(y)$. This belief is also supported by the uniform rationing rule; that is, upon stockout, the retailer's inventory is randomly allocated to each customer who visits it. Note that the product availability belief only depends on the price of the focal retailer. We remark that this is driven by our equilibrium refinement rule that customers believe retailers stock the optimal newsvendor inventory, which induces a service level that depends on the price of the focal retailer only. For the subsequent analysis, we use $\theta^{*}\left(p_{i}\right)$ $:=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_{i}-c}{p_{i}}\right)\right) \mathrm{d} F(y)=\theta_{i}\left(p_{1}, p_{2}\right)$ to denote customers' belief about $R_{i}$ 's inventory availability.
3.2.3. Equilibrium. We are now ready to characterize the equilibrium price and inventory decisions of the retailers. Under the REE and given the competing retailer's price $p^{\prime}$, the focal retailer's best price response, $p^{*}\left(p^{\prime}\right)=\arg \max _{0 \leq p \leq v} \Pi_{i}\left(p, p^{\prime}\right)$, can be solved by the following:

$$
\left.\begin{array}{rl}
\max _{0 \leq p \leq v} & \Pi\left(p, p^{\prime}\right)
\end{array}=p_{i} \mathbb{E}\left[\alpha\left(p, p^{\prime}\right) D \wedge q\left(p, p^{\prime}\right)\right]-c q\left(p, p^{\prime}\right)\right] \text { s.t. } \quad q\left(p, p^{\prime}\right)=\alpha\left(p, p^{\prime}\right) F^{-1}\left(\frac{p-c}{p}\right), ~ 子 r\left(p, p^{\prime}\right)=\frac{1}{2}+\frac{(v-p) \theta^{*}(p)-\left(v-p^{\prime}\right) \theta^{*}\left(p^{\prime}\right)}{2 s} .
$$

In particular, if the equilibrium outcome is symmetric, the two retailers charge the same equilibrium price, and thus, we have $p^{*}=p^{*}\left(p^{*}\right)$. The next proposition
characterizes the existence and uniqueness of the REE in the base model.

Proposition 1. There exists a unique REE in the base model. The equilibrium is symmetric and denoted ( $p^{*}, q^{*}$, $\left.\theta^{*}(\cdot)\right)$. Moreover, we have $p^{*}=\arg \max _{0 \leq p_{i} \leq v} \Pi_{i}\left(p_{i}, p^{*}\right), q^{*}$ $=\frac{1}{2} F^{-1}\left(\frac{p^{*}-c}{p^{*}}\right)$, and $\theta^{*}\left(p^{*}\right)=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p^{*}-c}{p^{*}}\right)\right) \mathrm{d} F(y)$. Each retailer covers half of the entire market, that is, $\alpha\left(p^{*}, p^{*}\right)=\frac{1}{2}$.

Proposition 1 demonstrates that a unique REE exists. Furthermore, the REE is symmetric. In particular, the two retailers cover the entire market competitively, each with a $50 \%$ market share. Note that this symmetric equilibrium outcome shares a similar structure as in the standard Hotelling model without demand uncertainty (see Lemma 3 in Online Appendix B). We may compare the equilibrium price of our model to that of the standard Hotelling model.
Proposition 2. The equilibrium price in our base model is higher than that in the Hotelling model, that is, $p^{*} \geq p_{d}^{*}$, where $p_{d}^{*}=s+c$ is the equilibrium price of the Hotelling model (see Lemma 3 in Online Appendix B).

Compared with the standard Hotelling model with deterministic demand, the retailers offer a higher equilibrium price when customers are concerned about product availability. The intuition of this result can be explained as follows. In the Hotelling model, the two retailers compete on offering a low price to attract customers. However, in our base model, the retailers compete on both price and product availability simultaneously. Because a high price signals high product availability, prompting the retailers to raise the price, the competition on price is alleviated. Therefore, the equilibrium price in our model is higher than that of the Hotelling model. This result also implies that inventory availability can serve as an operational lever to gain a competitive edge in the market, which softens the price competition between the retailers and, thus, leads to higher prices under equilibrium.

### 3.3. Focal Model with Customer Switching

In this section, we present our focal model with the customer switching behavior upon stockout. In this model, there are two customer segments in the market: nonswitching and switching customers. We define these two customer segments as follows:

- The nonswitching customers share identical behaviors as those characterized in the base model. If the product is out of stock at their focal retailer, the nonswitching customers leave the market directly.
- The switching customers consider visiting the competing retailer $R_{3-i}$ for substitutes upon the stockout of the focal retailer $R_{i}(i=1,2)$.

The probability for a customer to be of the switching (nonswitching) type is denoted by $\gamma \in(0,1)(1-\gamma)$, which is irrespective of the customer's location $x$. Furthermore, to highlight the impact of customer switching, we assume in the focal model that the unit search cost $s$ is sufficiently small such that any switching customer visits the competing retailer for substitutes upon the stockout of the focal retailer.

The switching behavior makes our problem more challenging. It is documented in the literature that general dynamic demand substitution problems can be intractable, and thus, approximation approaches are needed (see, e.g., Mahajan and Van Ryzin 2001, Karaesmen and Van Ryzin 2004). One approach to analyzing our problem is to approximate it with a two-stage model accounting for the customer switching behavior upon stockout. Specifically, in the first stage, all customers visit their focal retailers, which satisfy the demand using the initial in-stock inventory. In the second stage, the nonswitching customers leave the market. In contrast, the switching customers visit the competing retailer for substitutes if the focal retailer is out of stock. Finally, the retailers satisfy the demand from the customers who switch from their competitors using the remaining inventory left from the first stage. As can be seen, customers may hold two beliefs on the inventory availability probability for each retailer in the two-stage model. One is the ex ante belief on retailer inventory availability in the first stage when demand is satisfied by the retailer's initial inventory in stock. The other is the ex post belief on the retailer's inventory availability in the second stage with the knowledge that the demand is satisfied by using the remaining inventory after satisfying the demand in the first stage. We provide a detailed analysis of the ex ante and ex post beliefs on the retailer's inventory availability in Online Appendix C. However, the market equilibrium analysis based on the two-stage model is technically intractable. To restore tractability, we make the following assumption for the focal model with customer switching.
Assumption 1. (a) The travel time along the Hotelling line is negligible; that is, a switching customer can immediately visit the other retailer upon the stockout of the focal retailer. (b) The retailers apply the same uniform rationing rule to all customers because they cannot distinguish the switching customers from the original customers.

The key implication of Assumption 1 is that the switching and original customers arrive and make purchase decisions simultaneously, so they share the same probability of getting the product from a retailer. Therefore, all the customers hold the same belief on a retailer's product availability regardless of their location and switching status. As a consequence, given the open price information ( $p_{1}, p_{2}$ ), all customers hold the same inventory availability belief $\theta_{i}\left(p_{1}, p_{2}\right)$ for retailer
$R_{i}, i=1,2$. Intuitively, Assumption 1 does not change the nature of the switching behavior, but makes customers more likely to switch. Therefore, we conjecture that relaxing Assumption 1 leads to weaker but similar qualitative insights.

We now examine switching customers' decision problems given their availability beliefs under Assumption 1. Similar to the base model, let $\theta_{i}\left(p_{1}, p_{2}\right)$ represent the in-stock probability of $R_{i}$ given prices, where $i=1,2$. Consider a representative switching customer at location $x$, facing stockout at $R_{1}\left(R_{2}\right)$. The customer then switches to $R_{2}\left(R_{1}\right)$ for a substitute and earns a payoff $\left(v-p_{2}\right) \theta_{2}\left(p_{1}, p_{2}\right)-s(1-x)\left(\left(v-p_{1}\right) \theta_{1}\left(p_{1}, p_{2}\right)-s x\right)$. Therefore, the expected net surplus of the customer from switching to $R_{2}\left(R_{1}\right)$ upon the stockout at $R_{1}\left(R_{2}\right)$ is $\mathcal{U}_{12}(x)=\left(v-p_{2}\right) \quad \theta_{2}\left(p_{1}, p_{2}\right)-s(1-x)\left(\mathcal{U}_{21}(x)=\left(v-p_{1}\right)\right.$ $\left.\theta_{1}\left(p_{1}, p_{2}\right)-s x\right)$. Next, we examine the switching customer's choice of visiting the focal retailers by evaluating the customer's expected ex ante utility. For a switching customer at location $x$, the expected utility to visit $R_{1}$ $\left(R_{2}\right)$ with the product being available is $v-p_{1}-s x$ $\left(v-p_{2}-s(1-x)\right)$. Instead, if the product is out of stock, the customer may switch to $R_{2}\left(R_{1}\right)$ with an expected surplus $-s x+\mathcal{U}_{12}(x)\left(-s(1-x)+\mathcal{U}_{21}(x)\right) .{ }^{2}$ Hence, the expected total utility of a switching customer located at $x$ to visit $R_{1}\left(R_{2}\right)$ first is $\mathcal{U}_{1}(x)=\left(v-p_{1}\right) \theta_{1}\left(p_{1}, p_{2}\right)-s x+$ $\left(1-\theta_{1}\left(p_{1}, p_{2}\right)\right) \mathcal{U}_{12}(x)\left(\mathcal{U}_{2}(x)=\left(v-p_{2}\right) \theta_{2}\left(p_{1}, p_{2}\right)-s(1-\right.$ $\left.x)+\left(1-\theta_{2}\left(p_{1}, p_{2}\right)\right) \mathcal{U}_{21}(x)\right)$. Because a customer opts to first visit the focal retailer from which the customer can earn a higher expected (total) utility and switches to the competing retailer upon stockout, the customer patronizes $R_{i}$ first if $\mathcal{U}_{i}(x) \geq \mathcal{U}_{3-i}(x)$. Based on these arguments, there exists a threshold

$$
\begin{aligned}
& x_{s}\left(p_{1}, p_{2}\right) \\
& :=\mathcal{P}_{[0,1]}\left(\frac{\theta_{1}\left(p_{1}, p_{2}\right)}{\theta_{1}\left(p_{1}, p_{2}\right)+\theta_{2}\left(p_{1}, p_{2}\right)} .\right. \\
& \left.\quad\left(1+\frac{\left(p_{2}-p_{1}\right) \theta_{2}\left(p_{1}, p_{2}\right)}{s}\right)\right) \in[0,1]
\end{aligned}
$$

such that a switching customer located at $x$ first patronizes $R_{1}\left(R_{2}\right)$ and then switches to $R_{2}\left(R_{1}\right)$ upon stockout if $x \leq$ $x_{s}\left(p_{1}, p_{2}\right)\left(x>x_{s}\left(p_{1}, p_{2}\right)\right)$, where $\mathcal{P}_{[0,1]}(x)=\max \{0, \min \{x$, $1\}\}$ is the projection on to interval $[0,1]$.

Finally, following the same paradigm of rational expectation equilibrium, we derive that a customer's belief on retailer $R_{i}$ 's inventory availability probability is given by $\theta_{i}\left(p_{1}, p_{2}\right)=\theta^{*}\left(p_{i}\right)(i=1,2)$. For tractability, we focus on the symmetric REE in the presence of customer switching. Specifically, we consider the case in which, under equilibrium, both retailers charge the same price $p_{s}^{*}$ and capture the same market size $\alpha_{s}^{*}$ and the customers hold the same beliefs about product availability $\theta^{*}\left(p_{s}^{*}\right)$. Therefore, we have $\alpha_{s}^{*}=\alpha_{s}\left(p_{s}^{*}\right)=\gamma$ $\left(1-\frac{1}{2} \theta^{*}\left(p_{s}^{*}\right)\right)+\frac{1-\gamma}{2}$.

We are now ready to characterize the symmetric equilibrium price and inventory decisions of the retailers for the focal model with customer switching. Under the REE, the equilibrium price $p_{s}^{*}=\arg \max _{0<p<v} \Pi_{s}\left(p, p_{s}^{*}\right)$ is solved by the following:

$$
\begin{align*}
\max _{0 \leq p \leq v} \quad \Pi_{s}\left(p, p_{s}^{*}\right) & =p \mathbb{E}\left[\alpha_{s}\left(p, p_{s}^{*}\right) D \wedge q\left(p, p_{s}^{*}\right)\right]-c q\left(p, p_{s}^{*}\right) \\
\text { s.t. } \quad q\left(p, p_{s}^{*}\right)= & \alpha_{s}\left(p, p_{s}^{*}\right) F^{-1}\left(\frac{p-c}{p}\right), \\
\alpha_{s}\left(p, p_{s}^{*}\right) & =\gamma \alpha_{1}\left(p, p_{s}^{*}\right)+(1-\gamma) \alpha_{2}\left(p, p_{s}^{*}\right), \\
\alpha_{1}\left(p, p_{s}^{*}\right)= & \frac{\theta^{*}(p) \theta^{*}\left(p_{s}^{*}\right)}{\theta^{*}(p)+\theta^{*}\left(p_{s}^{*}\right)}\left(1+\frac{\left(p_{s}^{*}-p\right) \theta^{*}\left(p_{s}\right)}{s}\right) \\
& +\left(1-\theta^{*}\left(p_{s}^{*}\right)\right), \\
\alpha_{2}\left(p, p_{s}^{*}\right) & =\frac{1}{2}+\frac{(v-p) \theta^{*}(p)-\left(v-p_{s}^{*}\right) \theta^{*}\left(p_{s}^{*}\right)}{2 s} . \tag{2}
\end{align*}
$$

Note that $\alpha_{1}\left(p, p_{s}^{*}\right)$ and $\alpha_{2}\left(p, p_{s}^{*}\right)$ represent the market sizes from switching and nonswitching customers, respectively. In particular, $\alpha_{2}\left(p, p_{s}^{*}\right)$ is exactly the same as the market share function, $\alpha\left(p, p^{*}\right)$, in (1). Compared with the problem formulation (1) without customer switching, $\alpha_{s}\left(p, p^{*}\right)$ in Problem (2) represents the total market size from both the switching and nonswitching customers. Therefore, the retailer's total demand includes nonswitching customers, switching customers who first visit the retailer, and switching customers who switch to the retailer for substitutes. The next proposition characterizes the REE in the focal model.

## Proposition 3.

a. There exists a unique symmetric $R E E\left(p_{s}^{*}, q_{s}^{*}, \theta^{*}(\cdot)\right)$ in the focal model.
b. Under equilibrium, we have $p_{s}^{*}=\arg \max _{0 \leq p \leq v} \Pi(p$, $\left.p_{s}^{*}\right)\left(\operatorname{see}(2)\right.$ when $\left.\frac{\left(v-p_{s}^{*}\right) \theta^{*}\left(p_{s}^{*}\right)}{s} \geq 1\right), q_{s}^{*}=\alpha_{s}^{*} F^{-1}\left(\frac{p_{s}^{*}-c}{p_{s}^{*}}\right), \alpha_{s}^{*}=$ $\gamma\left(1-\frac{\theta^{*}\left(p_{s}^{*}\right)}{2}\right)+\frac{1-\gamma}{2}$, and $\theta^{*}\left(p_{s}^{*}\right)=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_{s}^{*}-c}{p_{s}^{*}}\right)\right)$ $f(y) \mathrm{d} y$.

If $\gamma=0$ (i.e., all customers belong to the nonswitching type), the symmetric REE in Proposition 3 reduces to the one characterized by Proposition 1 in the base model without customer switching. Note that Proposition 3 focuses on the symmetric REE. The focal model with substitution-driven customer switching may have asymmetric equilibria. Consider the case in which $R_{1}$ charges a high price and $R_{2}$ charges a low price. As a consequence, $R_{1}\left(R_{2}\right)$ induces a small (large) market size and attracts a fraction (all) of the customers who face stockout at the other retailer (i.e., when $\frac{\left(v-p_{s}^{*}\right) \theta^{*}\left(p_{s}^{*}\right)}{s}$ $<1$ ). An asymmetric equilibrium may be sustained in
this setting with certain model primitives. For example, the two retailers charge two different prices $p_{1}^{*}>p_{2}^{*}$ in an asymmetric equilibrium with $\alpha_{1}^{*}<\alpha_{2}^{*}$. Moreover, it is possible that the switching customers search for substitutes from one retailer only, and thus, the profits of the two retailers may not necessarily be equal under the asymmetric REE. For the rest of this paper, we focus our analysis on the inventory commitment and monetary compensation strategies under the symmetric REE in the model with customer switching. This enables us to capture the essential implications of customer switching without getting trapped by technical intractability. To conclude this section, we remark that Proposition 2 can be readily extended to the focal model with the same intuition, that is, $p_{s}^{*} \geq p_{d}^{*}$. In the focal model with stockout-driven substitution, the inventory availability remains an operational lever that softens the price competition and, hence, increases the equilibrium prices.

## 4. Inventory Commitment

Inventory commitment is a commonly used ex ante strategy (i.e., it is used before demand realization) in the presence of availability-concerned customers (see, e.g., Su and Zhang 2009). Under this strategy, the retailer should credibly announce its order quantity to the public. For example, Amazon.com recently provided a "lightning deal" platform to allow retailers to promote their products. A salient feature of lightning deals is that sellers have to announce the amount of inventory to customers. In particular, a customer can see a real-time status bar on the web page of the seller indicating the current price, inventory, and percentage of units that are already claimed by other customers. In some other circumstances, a retailer has to publicize its inventory information to customers even if it is not willing to do so by itself. For instance, the affiliated stores of Great Clips (a hair salon franchise in the United States and Canada) post real-time information of available slots online. Customers can check the anticipated waiting times of all stores in their area and add their names to the waitlist before actually visiting the salon. In this case, the competing franchised stores are forced to reveal their available inventory information.

It is shown in the literature that the inventory commitment strategy benefits the monopoly retailer (e.g., Su and Zhang 2009). In this section, we strive to analyze this strategy in a competitive market. Our results imply that the inventory commitment strategy may lead to an undesirable prisoner's dilemma: although both retailers voluntarily reveal their inventory information under equilibrium, the equilibrium profit of each retailer is lower than in the focal model in which the retailers
cannot credibly announce the order quantity information. Therefore, the inventory commitment strategy may not serve as an effective tool for retailers in a competitive market.

We now formally model the inventory commitment strategy in our duopoly market. We use subscript $v$ to represent the model with inventory commitment. At the beginning of the sales horizon, the competing retailers first decide whether to reveal the inventory information to the public (i.e., whether to adopt the inventory commitment strategy). Then, the retailers announce prices and order inventory accordingly. If a retailer commits to publicizing its inventory information, it truthfully announces its order quantity to the whole market. Finally, the customers observe the prices of the retailers and the amount of inventory ordered by the retailer that adopts the inventory commitment strategy and decides which retailer to visit. As in the focal model, we adopt the REE framework to analyze the equilibrium market outcome. There are three cases to consider: (i) both retailers do not reveal the inventory order quantities, which is essentially the focal model; (ii) both retailers adopt the inventory commitment strategy; (iii) one retailer adopts the inventory commitment strategy, whereas the other one does not reveal its inventory. Section 3 presents a detailed analysis for case (i). Here, we analyze cases (ii) and (iii).

### 4.1. Both Retailers Adopt the Inventory Commitment Strategy

Under inventory commitment, individual customers do not need to form beliefs about inventory availability, but directly optimize their purchasing decisions after observing both prices and inventory stocking quantities. Specifically, after observing retailer $R_{i}$ 's price $p_{i}$ and stocking quantity $q_{i}$, where $i \in\{1,2\}$, customers estimate the in-stock probability of each retailer conditional on the customer's existence. Similar to the focal model, there exists a threshold for nonswitching customers, $x\left(p_{1}, q_{1}, p_{2}, q_{2}\right)$, such that the nonswitching customers with $x \leq x\left(p_{1}, q_{1}, p_{2}, q_{2}\right) \quad\left(x>x\left(p_{1}, q_{1}, p_{2}, q_{2}\right)\right)$ visit retailer $R_{1}$ only (retailer $R_{2}$ only). For switching customers, there exists another threshold, $x_{s}\left(p_{1}, q_{1}, p_{2}, q_{2}\right)$, such that the switching customers with $x \leq x_{s}\left(p_{1}, q_{1}, p_{2}\right.$, $\left.q_{2}\right)\left(x>x\left(p_{1}, q_{1}, p_{2}, q_{2}\right)\right)$ visit retailer $R_{1}\left(R_{2}\right)$ first and then switch to retailer $R_{2}\left(R_{1}\right)$ upon stockout. Here, we focus on the case in which the search cost $s$ is sufficiently low to induce full market coverage with competition and customer switching. As in the focal model, a retailer's total market size includes nonswitching customers who visit the retailer directly, switching customers who first visit the retailer, and switching customers who switch from the competing retailer because of stockout. Specifically, the market size of $R_{1}\left(R_{2}\right)$ is $\alpha_{1}=\gamma\left[x_{s}\left(p_{1}, q_{1}, p_{2}, q_{2}\right)+\left(1-x_{s}\right.\right.$ $\left.\left.\left(p_{1}, q_{1}, p_{2}, q_{2}\right)\right)\left(1-\theta_{2}\right)\right]+(1-\gamma) x\left(p_{1}, q_{1}, p_{2}, q_{2}\right)\left(\alpha_{2}=\gamma[1\right.$
$\left.-x_{s}\left(p_{1}, q_{1}, p_{2}, q_{2}\right)+x_{s}\left(p_{1}, q_{1}, p_{2}, q_{2}\right)\left(1-\theta_{1}\right)\right]+(1-\gamma)[1-$ $\left.\left.x\left(p_{1}, q_{1}, p_{2}, q_{2}\right)\right]\right)$. Algebraic manipulation yields

$$
\left\{\begin{align*}
\alpha_{1, v}= & \gamma\left\{\frac{\theta_{1} \theta_{2}}{\theta_{1}+\theta_{2}}\left(1+\frac{\theta_{2}}{s}\left(p_{2}-p_{1}\right)\right)+\left(1-\theta_{2}\right)\right\}  \tag{3}\\
& +(1-\gamma)\left\{\frac{1}{2}+\frac{\left(v-p_{1}\right) \theta_{1}-\left(v-p_{2}\right) \theta_{2}}{2 s}\right\} \\
\alpha_{2, v} & =\gamma\left\{1-\frac{\theta_{1}^{2}}{\theta_{1}+\theta_{2}}\left(1+\frac{\theta_{2}}{s}\left(p_{2}-p_{1}\right)\right)\right\} \\
& +(1-\gamma)\left\{\frac{1}{2}+\frac{\left(v-p_{2}\right) \theta_{2}-\left(v-p_{1}\right) \theta_{1}}{2 s}\right\},
\end{align*}\right.
$$

where the subscript $v$ denotes the model under the inventory commitment strategy and $\gamma$ represents the portion of switching customers in the market. Similar to the focal model, it suffices to characterize the perceived inventory availability probabilities at the purchasing thresholds. Define $\theta_{1, v}\left(\theta_{2, v}\right)$ as the perceived inventory availability probability for a customer located at the threshold to visit $R_{1}\left(R_{2}\right)$. Then, we have $\theta_{i, v}=\frac{1}{\mu} \int_{y=0}^{\infty}$ $\left(y \wedge\left(\frac{q_{i}}{\alpha_{i, v}}\right)\right) f(y) \mathrm{d} y$, where $\alpha_{i, v}$ is the market size of retailer $R_{i}$ defined by (3). Denote retailer $R_{i}$ 's profit as $\Pi_{v}\left(p_{i}, q_{i}\right)=p_{i} \mathbb{E}\left[\alpha_{i, v}\left(p_{i}, q_{i}\right) D \wedge q_{i}\right]-c q_{i}$. As discussed, we focus on the symmetric $\operatorname{REE}\left(p_{v}^{*}, q_{v}^{*}, \theta_{v}^{*}(\cdot)\right)$, where $p_{v}^{*}$ is the equilibrium price, $q_{v}^{*}$ is the equilibrium order quantity, and $\theta^{*}(\cdot)$ is the equilibrium belief of product availability.

We focus on the case when the search cost $s$ is sufficiently small, so the market is fully covered by the two retailers with competition and all switching customers switch to the competing retailer upon stockout. The two retailers compete on price and order quantity to win the market. The following proposition characterizes the equilibrium outcome if both retailers commit to revealing their inventory information under market competition.

Proposition 4. If both retailers adopt the inventory commitment strategy, the following statements hold:
a. There exists a unique symmetric $\operatorname{REE}\left(p_{v}^{*}, q_{v}^{*}, \theta_{v}^{*}(\cdot)\right)$.
b. Under equilibrium, we have $\left(p_{v}^{*}, q_{v}^{*}\right)=\arg \max _{0 \leq p \leq v, q \geq 0}$ $\Pi_{v}(p, q)$ subject to the constraints $\left(p_{v}^{*}-p\right) \theta_{v}=s x_{s}\left(1+\theta_{v} / \theta\right)$ $-s / \theta$ and $(v-p) \theta-s x=\left(v-p_{v}^{*}\right) \theta_{v}-s(1-x)$, where $\theta=$ $\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge\left(\frac{q}{\alpha}\right)\right) f(y) \mathrm{d} y, \theta_{v}=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge\left(\frac{q_{v}^{*}}{\alpha^{\prime}}\right)\right) f(y) \mathrm{d} y, \alpha=\gamma$ $\left(x_{s}+\left(1-x_{s}\right)\left(1-\theta_{v}\right)\right)+(1-\gamma) x$, and $\alpha^{\prime}=\gamma\left(1-x_{s}+x_{s}\right.$ $(1-\theta))+(1-\gamma)(1-x)$. Moreover, each retailer's market size is $\alpha_{v}^{*}=\gamma\left(1-\frac{1}{2} \theta_{v}^{*}\right)+\frac{1-\gamma}{2}$, where $\theta_{v}^{*}=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge\left(\frac{q_{v}^{*}}{\alpha_{v}^{*}}\right)\right)$ $f(y) \mathrm{d} y$.

Proposition 4 implies that the equilibrium outcome of the scenario in which both retailers adopt the inventory commitment strategy shares the same structure as
that of the model with customer switching formulated by Equation (2).

### 4.2. Incentive for Inventory Commitment

Next, we show that inventory commitment is a dominating strategy for each retailer. As a consequence, the equilibrium outcome is that both retailers voluntarily publicize their inventory, charge the price $p_{v}^{*}$, and order $q_{v}^{*}$ units of inventory as prescribed by Proposition 4.

With retailer $R_{1}$ as the focal retailer, we consider both the case in which retailer $R_{2}$ credibly announces $q_{2}$ and the case in which retailer $R_{2}$ does not reveal its order quantity. Given retailer $R_{2}$ 's price and inventory decision $\left(p_{2}, q_{2}\right)$, we use $\Pi_{i, j}(i, j \in\{d, v\})$ to denote the maximum profit of retailer $R_{1}$ if it adopts strategy $i$ and retailer $R_{2}$ adopts strategy $j$, where subscript $d$ refers to no inventory commitment and subscript $v$ refers to inventory commitment. For example, $\Pi_{d, v}$ refers to the maximum profit of retailer $R_{1}$ if it does not adopt the inventory commitment strategy and retailer $R_{2}$ adopts this strategy. The derivations of $\Pi_{i, j}(i, j \in\{v, d\})$ are given in the proof of Lemma 1 in Online Appendix D.

Lemma 1. For any $\left(p_{2}, q_{2}\right)$ set by retailer $R_{2}$, we have $\Pi_{v, d}>\Pi_{d, d}$ and $\Pi_{v, v}>\Pi_{d, v}$.

Lemma 1 suggests that, if the competing retailers can credibly reveal their inventory information to the market, adopting the inventory commitment strategy would be a dominating strategy for each of the retailers regardless of the price and inventory decisions of the competitor. Therefore, the equilibrium outcome of the market under the inventory commitment option is that both retailers voluntarily reveal their inventory order quantity. Lemma 1 also reveals an important actionable insight for firms in a competitive market in which customers are concerned about inventory availability: credibly communicating the inventory stocking information to customers helps gain an edge for such firms. Our next result examines the profit implication of the inventory commitment strategy under market competition. We use $\Pi_{v}^{*}\left(\Pi^{*}\right)$ to denote the equilibrium profit of a retailer with (without) the inventory commitment strategy.

Proposition 5. If the retailers have the option to credibly announce their inventory information, the following statements hold:
a. Under equilibrium, both retailers $R_{1}$ and $R_{2}$ adopt the inventory commitment strategy.
b. There exist a threshold $\bar{s}_{v}$ for the search cost and a threshold $\bar{c}_{v}$ for the unit procurement cost such that, if $s<\bar{s}_{v}$ and $c<\bar{c}_{v}$, then we have $\Pi_{v}^{*}<\Pi^{*}$.

As shown in Proposition 5, if both the search cost $s$ and the procurement cost $c$ are low (i.e., $s<\bar{s}_{v}$ and $c<\bar{c}_{v}$ ) and if the inventory commitment strategy is adopted, the inventory stocking quantity can directly
influence the purchasing behaviors of the customers. Therefore, the competition between retailers may be intensified by this strategy. The retailers may overcommit to inventory in a competitive market, thus reducing the profit of each retailer. Recall that, in our focal model, the stocking quantity is not observable to customers but can be signaled by price, so the only competitive leverage of a retailer is the prevailing price it charges. However, if the retailers can commit to their preannounced inventory order quantities, they have more flexibility to influence demand. Furthermore, the signaling power of price is diluted if the inventory information is directly available to customers. In particular, when the unit cost $c$ is high, the inventory commitment strategy helps the retailers increase the willingness to pay of the customers, thus attracting higher demand. On the other hand, if the unit cost is low, this strategy may backfire by triggering an overcommitment of stocking quantity. If, in addition, the market competition is intense (i.e., $s<\bar{s}_{v}$ ), each retailer aggressively orders a large amount of inventory to attract customers, which, in turn, exacerbates market competition and decreases the profits of both retailers $\left(\Pi_{v}^{*}<\Pi^{*}\right.$ when $c<\bar{c}_{v}$ and $\left.s<\bar{s}_{v}\right)$. Therefore, when the procurement $\operatorname{cost} c$ and the search cost $s$ are both low, the retailers are actually worse off in the presence of the inventory commitment option because of the induced inventory overcommitment and intensified market competition. Lemma 1 and Proposition 5 together deliver a new and interesting insight that the inventory commitment strategy may give rise to a prisoner's dilemma under market competition. Although this strategy is preferred by either retailer regardless of the competitor's inventory and price decisions, the retailers would be worse off if both adopt the inventory commitment strategy.

Our analysis demonstrates that the inventory commitment strategy does not always benefit the retailers under competition, which is in sharp contrast to the monopoly setting. There is a large body of research focusing on the inventory commitment strategy. A central message in the literature is that the inventory commitment strategy is beneficial for retailers. For example, Cachon and Swinney (2011) and Liu and Van Ryzin (2008) propose two-stage models to explore how to use availability information to manipulate customers' expectations and, thus, induce them to buy early. In a competitive market setting, revealing inventory information to customers may lead to a higher equilibrium price and, as a result, improves the firms' profits (see Carlton 1978, Dana 2001, Dana and Petruzzi 2001). In a supply chain setting, Su and Zhang (2008) demonstrate that the firm's profit can be improved by promising either that the available inventory will be limited (quantity commitment) or that the price will be kept high (price commitment). In a monopoly setting, Su and Zhang (2009) further show that the inventory
commitment strategy offers customers information to more accurately assess their chances of securing the product. Thus, the inventory commitment strategy increases customers' willingness to pay and improves the profit of a monopoly firm. In a Hotelling competition setting, however, our results demonstrate that the inventory commitment strategy may give rise to a prisoner's dilemma and hurt the retailers.

Our results also deliver actionable insights for e-tailers. In today's digitalized business environment, customers have easy access to extensive product information with almost zero search cost (i.e., a very small s in our model). For example, customers can easily search online for product alternatives as well as price and inventory availability information. Our analysis shows that, although such information transparency attracts customers to visit retailers more frequently, the retailers may hurt themselves by revealing too much inventory availability information as a consequence of intensified market competition. This is because the inventory availability information has to be disclosed to the entire market instead of being limited to the intended customers of the retailers (i.e., the customers whose location is closest to the retailers). Granados and Gupta (2013) summarize two practical approaches to present inventory information: (i) a retailer may only disclose whether a product is in stock or not, or (ii) it may choose to publicize its inventory stocking level only when it is low. Both approaches reveal the retailer's inventory availability information in an imperfect way to prevent its competitors from using such information to improve their margins (i.e., stocking more inventory to attract demand; see Dewan et al. 2007). We indeed strengthen this insight by demonstrating that retailers should be cautious about triggering the war of implementing the fully transparent inventory strategy (i.e., the inventory commitment strategy) because the other competitors will copy the strategy, and eventually, it will backfire for all retailers when the customer search cost is low.

We complement our theoretical analysis with numerical experiments to further illustrate the impact of the inventory commitment strategy. We compare the equilibrium profits and stocking quantities in models with and without inventory commitment. In our numerical experiments, we set $\gamma=s=0.1, v=10$, and the market demand $D$ follows a Gamma distribution with mean 90 and standard deviation 30. Figures 1 and 2 plot the equilibrium profits and order quantities, respectively, for the focal model and the model with inventory commitment. Figure 1 shows that the equilibrium profit of a retailer is lower in the presence of inventory commitment whenever the ordering cost $c$ is low. Figure 2 further demonstrates that, with inventory commitment, the retailers order much more than they would have without revealing the inventory availability information to the market.

Figure 1. (Color online) Retailer Profits in Equilibrium


To conclude this section, we remark that implementing the inventory commitment strategy relies heavily on the retailers' credibility in the market. That is, the retailers should be able to credibly reveal their order quantity information to their competitors and their customers in the market. Otherwise, if the retailers fail to credibly convince the market, the effect of inventory commitment is diluted. In the next section, we analyze an ex post monetary compensation strategy that is applicable even without such commitment power.

## 5. Monetary Compensation

In this section, we proceed to analyze the widely used monetary compensation strategy, which is an ex post strategy. After customers visit a retailer and find that the product is out of stock, the retailer compensates

Figure 2. (Color online) Retailer Order Quantities in Equilibrium

them for such inconvenience. This strategy could reassure the customers in the presence of potential stockouts, thus motivating customers to visit the retailer. In practice, the compensation is offered in the form of coupons, gift cards, price discounts for future orders, and free shipping opportunities. For example, FoodLand offers consumers a rain check for out-of-stock items. ${ }^{3}$ The simplest and most direct compensation strategy is to placate customers for stockouts with cash, which we refer to as the monetary compensation strategy. In this section, we focus on studying the effect of monetary compensation under competition and substitution-based customer switching.

The monetary compensation strategy has proven beneficial to a monopoly retailer (see Su and Zhang 2009). In a competitive market, however, the story is different. Our analysis shows that, when monetary compensation is an option, competing retailers (voluntarily) overcompensate customers to attract higher demand, which, in turn, decreases their profits compared with the baseline setting in which monetary compensation is not allowed.

To model the monetary compensation strategy, we assume that each retailer offers a compensation $m_{i} \geq 0$ $(i \in\{1,2\})$ to customers who face stockouts. The special case in which $m_{i}=0$ refers to that $R_{i}$ does not offer monetary compensation. So both retailers have the flexibility to decide whether to offer monetary compensation upon stockouts and the amount of compensation. As in the focal model, customers observe the retailers' prices and monetary compensation terms, but not their stocking quantities. The retailers set the price and stocking quantity to maximize their profits, whereas customers choose to purchase the product to maximize their expected surplus. In particular, each nonswitching customer decides to visit a focal retailer only, and each switching customer chooses a focal retailer to make a purchase first and then switches to the other retailer for substitutes upon stockout. Following the same equilibrium analysis paradigm as in the focal model and the model with inventory commitment, we consider the symmetric REE in the model with monetary compensation. We use the subscript $c$ to denote the model with monetary compensation.

We first reexamine the purchase decisions of the nonswitching customers. For a nonswitching customer located at $x$, the customer visits the retailer that yields a higher nonnegative expected payoff and receives monetary compensation upon stockout. Hence, the customer's expected payoff is $\left(v-p_{1}\right) \theta_{1}+m_{1}\left(1-\theta_{1}\right)-s x((v$ $\left.\left.-p_{2}\right) \theta_{2}+m_{2}\left(1-\theta_{2}\right)-s(1-x)\right)$, where $\theta_{1}\left(\theta_{2}\right)$ represents the customer's belief about $R_{1}{ }^{\prime} \mathrm{s}\left(R_{2}\right.$ 's) inventory availability probability. Indeed, a rigorous definition of the inventory availability probability is $\theta_{i}\left(p_{1}, p_{2}, m_{1}, m_{2}\right)$, $i \in\{1,2\}$, which is a function of prices and monetary compensations. For conciseness, we drop the argument
and use $\theta_{i}$ to represent retailer $R_{i}^{\prime}$ 's inventory availability probability $(i \in\{1,2\})$ in the analysis.

Next, we examine the purchase decisions of the switching customers. If the product is out of stock at the focal retailer $R_{1}\left(R_{2}\right)$, a switching customer (located at $x$ ) switches to retailer $R_{2}$ for a substitute with an expected surplus $\mathcal{U}_{1,2}=\left(v-p_{2}\right) \theta_{2}+m_{2}\left(1-\theta_{2}\right)-s(1-x)\left(\mathcal{U}_{2,1}=\right.$ $\left.\left(v-p_{1}\right) \theta_{1}+m_{1}\left(1-\theta_{1}\right)-s x\right)$. Note that we assume a sufficiently small search cost $s$ to ensure that all switching customers switch for substitutes upon stockout. Similarly, each switching customer chooses to first visit a focal retailer that yields a higher nonnegative expected payoff and receive monetary compensation upon stockout. Hence, the customer's total expected payoff is $\mathcal{U}_{1}=$ $\left(v-p_{1}\right) \theta_{1}+m_{1}\left(1-\theta_{1}\right)-s x+\mathcal{U}_{1,2}\left(1-\theta_{1}\right)\left(\mathcal{U}_{2}=\left(v-p_{2}\right)\right.$ $\left.\theta_{2}+m_{2}\left(1-\theta_{2}\right)-s(1-x)+\mathcal{U}_{2,1}\left(1-\theta_{2}\right)\right)$ if the customer visits $R_{1}\left(R_{2}\right)$ first.

Now, we are ready to formulate retailer $R_{i}$ 's decision problem, where $i=1,2$ :

$$
\begin{aligned}
& \max _{\left(p_{i}, m_{i}, q_{i}\right)} \Pi_{i, c}\left(p_{i}, m_{i}, q_{i}\right) \\
& =p_{i} \mathbb{E}\left(\alpha_{i, c} D \wedge q_{i}\right)-m_{i} \mathbb{E}\left(\alpha_{i, c} D-q_{i}\right)^{+}-c q_{i}
\end{aligned}
$$

where market size $\alpha_{1, c}$ and $\alpha_{2, c}$ are the following:

$$
\left.\left\{\begin{align*}
& \alpha_{1, c}= \gamma\left\{\frac{\theta_{2} \theta_{1}\left(\frac{\left(p_{2}+m_{2}\right)-\left(p_{1}+m_{1}\right)}{\theta_{1}+\theta_{2}} \theta_{2}+1\right)}{s}\right)  \tag{4}\\
&\left.+\frac{\theta_{2}\left(m_{1} \theta_{2}-m_{2} \theta_{1}\right)}{s\left(\theta_{1}+\theta_{2}\right)}-\left(1-\theta_{2}\right)\right\} \\
&+(1-\gamma)\left\{\frac{1}{2}+\frac{\left[v-\left(p_{1}+m_{1}\right)\right] \theta_{1}-\left[v-\left(p_{2}+m_{2}\right)\right]}{\theta_{2}+\left(m_{1}-m_{2}\right)}\right. \\
& 2 s
\end{align*}\right\}, ~ \begin{array}{rl}
\alpha_{2, c}= & \gamma\left\{\frac{\theta_{2} \theta_{1}}{\theta_{1}+\theta_{2}}\left(\frac{\left(p_{1}+m_{1}\right)-\left(p_{2}+m_{2}\right)}{s} \theta_{1}+1\right)\right. \\
& \left.+\frac{\theta_{1}\left(m_{2} \theta_{1}-m_{1} \theta_{2}\right)}{s\left(\theta_{1}+\theta_{2}\right)}-\left(1-\theta_{1}\right)\right\}
\end{array}\right\} .
$$

Thus, retailer $R_{i}$ orders the newsvendor quantity $q_{i, c}$ $=\alpha_{i}^{c} F^{-1}\left(\frac{p_{i}+m_{i}-c}{p_{i}+m_{i}}\right)$, where $i=1$, 2 . Recall that the retailers adopt uniform rationing, so the in-stock probability for a customer is $\theta_{i, c}=\theta_{c}^{*}\left(p_{i}+m_{i}\right)=\frac{1}{\mu} \int_{y=0}^{\infty} \min \left(y \wedge F^{-1}\right.$ $\left.\left(\frac{p_{i}+m_{i}-c}{p_{i}+m_{i}}\right)\right) \mathrm{d} F(y)$. Here, the customer's belief in retailer
$R_{i}$ 's inventory availability depends on ( $p_{i}, m_{i}$ ) through the effective margin $p_{i}+m_{i}$.

We denote the symmetric REE as $\left(p_{c}^{*}, m_{c}^{*}, q_{c}^{*}, \theta_{c}^{*}(\cdot)\right)$, where $p_{c}^{*}$ is the equilibrium price, $m_{c}^{*}$ is the equilibrium compensation, $q_{c}^{*}$ is the equilibrium order quantity, and $\theta_{c}^{*}(\cdot)$ is the equilibrium product availability. Moreover, we focus on the case in which the search cost $s$ is sufficiently small. The following proposition characterizes the REE in the presence of monetary compensation.

Proposition 6. For the model with monetary compensation, the following statements hold:
a. There exists a unique symmetric REE, which we denote as $\left(p_{c}^{*}, m_{c}^{*}, q_{c}^{*}, \theta_{c}^{*}(\cdot)\right)$.
b. We have $\left(p_{c}^{*}, m_{c}^{*}\right)=\arg \max _{0 \leq p \leq v, m \geq 0} \Pi_{c}(p, m, q)$ subject to $q=\alpha(p, m) F^{-1}\left(\frac{p+m-c}{p+m}\right)$ and $\alpha(p, m)$ is given by Equation (4). In equilibrium, we have $q_{c}^{*}=\alpha_{c}^{*} F^{-1}\left(\frac{p_{c}^{*}+m_{c}^{*}-c}{p_{c}^{*}+m_{c}^{*}}\right)$, $\theta_{c}^{*}=\frac{1}{\mu} \int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_{c}^{*}+m_{c}^{*}-c}{p_{c}^{*}+m_{c}^{*}}\right)\right) \mathrm{d} F(y)$, and each retailer has market size $\alpha_{c}^{*}=\gamma\left(1-\frac{1}{2} \theta_{c}^{*}\right)+\frac{1-\gamma}{2}$.

To examine the impact of the monetary compensation on the retailers' profit, we denote the equilibrium profit of a retailer in the model with monetary compensation as $\Pi_{c}^{*}$ (the equilibrium profit of the retailers in the focal model is $\Pi^{*}$ ). The following proposition shows that monetary compensation may hurt the retailers if the market competition is intense.

Proposition 7. For the model with monetary compensation, there exists a critical threshold $\bar{s}_{c}$ such that, if $s<\bar{s}_{c}$, we have $\Pi_{c}^{*}<\Pi^{*}$.

Proposition 7 delivers an interesting message that, if the retailers have the option to offer monetary compensations upon stockout, they will earn a lower profit as long as the competition is sufficiently intense ( $s \leq \bar{s}_{c}$ ). This is in contrast with the effect of monetary compensation in the monopoly setting, which always benefits the retailer (see Su and Zhang 2009). By offering monetary compensation, the retailer, on one hand, is equipped with another lever in the competitive landscape; however, on the other hand, it competes more aggressively through direct subsidies to customers upon stockout. If the search cost, $s$, is large, the former effect dominates, which results in a higher profit in the presence of monetary compensation. If the search cost, $s$, is small, however, the latter effect dominates, and monetary compensation leads to severe competition, which, in turn, diminishes the profit of each retailer. As a consequence, if the market competition is already fierce (i.e., $s$ is small), the monetary compensation option further intensifies the competition and hurts the retailers. In a similar spirit to the classical Hotelling model (see Lemma 3 in Online Appendix D), the intensified competition induced by stockout compensations drags the

Figure 3. (Color online) Retailer Profits in Equilibrium

equilibrium effective margin $p_{c}^{*}+m_{c}^{*}$ down to the marginal cost $c$ as $s$ approaches zero. Hence, if the unit travel cost $s$ is sufficiently small (i.e., the model proposed by Dana 2001), both retailers may earn zero profit in the presence of the monetary compensation option.

We next complement the finding in Proposition 7 with numerical results to further illustrate the impact of the monetary compensation strategy. In Figure 3, we compare the two retailers' equilibrium profits in models with and without monetary compensation. In Figure 4, we plot the retailers' monetary compensation in the equilibrium with monetary compensation. In our numerical studies, we set $\gamma=0.1, c=5$, and $v=10$, and the market demand $D$ follows a Gamma distribution with mean 90 and standard deviation 30. Figure 3 shows that the equilibrium profit is lower in the presence of monetary

Figure 4. (Color online) Equilibrium Monetary Compensation

compensation when the unit search cost $s$ is low. Figure 4 further shows that the retailers compensate customers with a considerable amount of monetary compensation when the unit search cost $s$ is low, which indicates that the monetary compensation leads to overcompensation when the market is highly competitive.

In the existing literature, many studies demonstrate that retailers can extract more profit by offering monetary compensation in a monopoly market. To convince customers of inventory availability, retailers adopt monetary compensation as a self-punishment mechanism upon stockouts. With such a mechanism, customers anticipate a high service level and increase their willingness to pay, which, in turn, boosts the firm's profit. For example, Su and Zhang (2009) show that monetary compensation can increase the retailer's product availability in a monopoly model. For a competitive market environment, Kim et al. (2004) demonstrate that a capacity reward program benefits the firms when market demands are nonstationary across periods. By offering this program, firms can effectively reduce excess capacities when market demand is low and, thus, avoid intense price competition. Besides such a short-run effect, it is widely believed that monetary compensation also has a long-run effect to expand a firm's market share. Compensating customers upon stockouts has a positive effect on customers' shopping experience and, thus, cultivates customer loyalty. In other words, by purposefully providing compensations for stockouts, retailers have the potential to increase their demand in the long run (see, e.g., Bhargava et al. 2006). Kim et al. (2001) further validate this viewpoint by showing that the firms should apply the most inefficient rewards (i.e., monetary compensation) if the market consists of a small portion of pricesensitive customers. Albeit the monetary compensation strategy has all these benefits, our results (i.e., Proposition 7), nevertheless, deliver a new insight that this strategy may backfire and lead to profit losses for the retailers. Similar results are also shown by Kopalle and Neslin (2001) when firms compete in a market with relatively fixed sizes.

We also remark that offering monetary compensation may cause a free-rider issue. Specifically, customers who are not interested in purchasing the product may still visit the retailer with the hope of being compensated as long as the travel cost is not too high. These customers are referred to as free riders. The free-riding behavior creates a moral hazard issue so that retailers can hardly recognize their true customers. Fortunately, many marketing strategies and new technology tools can be used to alleviate or even eliminate the freeriding issue. For example, retailers may ask customers to claim their desired product in order to be eligible for compensation upon stockout. If the claimed product is out of stock and no substitute can match the
customer's need, then a monetary compensation is offered. Otherwise, the customers cannot receive the monetary compensation. Another mechanism the retailers can use is to solicit more information from customers through cheap talk. Once the retailer verifies a customer's true motivation for purchasing the product, a monetary compensation can be awarded. Therefore, throughout our analysis, we assume that the free-riding behavior is negligible. This is consistent with the business practice in various industries in which retailers effectively compensate customers' stockouts to induce a higher demand (see, e.g., Bhargava et al. 2006, Su and Zhang 2009).

Finally, we note that both inventory commitment and monetary compensation can be viewed as offering options that appeal to customers. Other business strategies offered by competing firms to attract customers and induce higher demand are also studied in the literature. For example, Chen et al. (2001) show that individual marketing by two competing firms can lead to a win-win competition even if the firms behave noncooperatively and the market does not expand. Shin and Sudhir (2010) examine whether a firm should use behavior-based pricing (BBP) to discriminate between its own and competitors' customers in a competitive market. The paper finds that it is optimal to reward one's own customers under symmetric competition, and BBP can increase profits with fully strategic and forward-looking consumers. Kim et al. (2001) show that reward (promotion) programs weaken price competition because firms gain less from undercutting their prices, so the equilibrium prices go up in this case. In sum, whereas the strategies may benefit the competing firms for various reasons, we show that inventory commitment and monetary compensation intensify competition and may lead to a prisoner's dilemma and a lose-lose outcome.

## 6. Social Welfare Implications

In this section, we study two important questions regarding the social welfare of a competitive market. First, how does market competition impact social welfare? Second, what are the social welfare implications of inventory commitment and monetary compensation under competition?

We begin our analysis by quantifying the average customer surplus and social welfare in different models, starting with the focal model. Note that we focus on the setting with full market competition and customer switching. Now, we introduce the average customer surplus for switching customers and nonswitching customers, respectively. The switching customers first visit their focal retailers and then switch to the competing retailer for substitutes. Under equilibrium, the expected surplus for a switching customer at $x$ is $\mathcal{U}_{s}(x)=\left(v-p_{s}^{*}\right)$ $\theta\left(p_{s}^{*}\right)-s x+\left[\left(v-p_{s}^{*}\right) \theta\left(p_{s}^{*}\right)-s(1-x)\right]\left(1-\theta\left(p_{s}^{*}\right)\right)$. Because
the two retailers are symmetric and the customers are uniformly distributed along the Hotelling line, the average surplus for switching customers is $2 \int_{0}^{1 / 2} \mathcal{U}_{s}(x)$ $d x=\left(v-p_{s}^{*}\right) \theta\left(p_{s}^{*}\right)\left(2-\theta\left(p_{s}^{*}\right)\right)-s\left(1-\frac{3}{4} \theta\left(p_{s}^{*}\right)\right)$. In contrast, the nonswitching customers visit their focal retailers only and leave the market upon stockout. Therefore, the expected surplus for a nonswitching customer at $x$ is $\mathcal{U}(x)=\left(v-p_{s}^{*}\right) \theta\left(p_{s}^{*}\right)-s x$, which provides the nonswitching customers' average surplus $2 \int_{0}^{1 / 2} \mathcal{U}(x) d x=\left(v-p_{s}^{*}\right) \theta$ $\left(p_{s}^{*}\right)-\frac{s}{4}$. Recall that the market consists of $\gamma$ portion of switching customers and $1-\gamma$ portion of nonswitching customers; the average surplus for all customers is $C S^{*}$ $=2 \gamma \int_{0}^{1 / 2} \mathcal{U}_{s}(x) d x+2(1-\gamma) \int_{0}^{1 / 2} \mathcal{U}(x) d x=\left(v-p_{s}^{*}\right) \theta\left(p_{s}^{*}\right)-\frac{s}{4}$ $+\gamma\left(1-\theta\left(p_{s}^{*}\right)\right)\left[\left(v-p_{s}^{*}\right) \theta\left(p_{s}^{*}\right)-\frac{3 s}{4}\right]$, where $p_{s}^{*}$ is the equilibrium price characterized by Proposition 3. The social welfare is the summation of the two retailers' profits and total customers' surplus; therefore, we have social welfare $S W^{*}=2 \Pi\left(p_{s}^{*}\right)+\mu \times C S^{*}=v \mu \theta\left(p_{s}^{*}\right)-c F^{-1}\left(\frac{p_{s}^{*}-c}{p_{s}^{*}}\right)$ $-\frac{\mu s}{4}+\gamma\left(1-\theta\left(p_{s}^{*}\right)\right)\left[v \mu \theta\left(p_{s}^{*}\right)-c F^{-1}\left(\frac{p_{s}^{*}-c}{p_{s}^{*}}\right)-\frac{3 \mu s}{4}\right]$. Note that the equilibrium price $p_{s}^{*}$ plays a key role in determining the average customer surplus and social welfare as it explicitly influences the order quantity and inventory availability.

To explore the impact of inventory availability competition, we introduce a benchmark model in which retailers at the two endpoints of the Hotelling line belong to a single firm and are managed in a centralized fashion. The firm optimizes price and inventory decisions of the two retailers to maximize their total profits. To ensure fair comparison, again, the firms engage in the full market coverage, and customers switch upon stockout because of the small unit search cost $s$. In the subsequent analysis, we use subscript $b$ to denote the benchmark model. Analogous to the analysis of the focal model, the average customer surplus in the benchmark model is $C S_{b}^{*}=\left(v-p_{b}^{*}\right) \theta\left(p_{b}^{*}\right)-\frac{s}{4}+\gamma\left(1-\theta\left(p_{b}^{*}\right)\right)\left[\left(v-p_{b}^{*}\right) \theta\left(p_{b}^{*}\right)-\frac{3 s}{4}\right]$, and the social welfare is $S W_{b}^{*}=v \mu \theta\left(p_{b}^{*}\right)-c F^{-1}\left(\frac{p_{b}^{*}-c}{p_{b}^{*}}\right)-$ $\frac{\mu s}{4}+\gamma\left(1-\theta\left(p^{*}\right)\right)\left[v \mu \theta\left(p_{b}^{*}\right)-c F^{-1}\left(\frac{p_{b}^{*}-c}{p_{b}^{*}}\right)-\frac{3 \mu s}{4}\right]$, where $p_{b}^{*}$ $=v-\frac{s}{\theta\left(p_{p}^{*}\right)}$. It is worth noting that the customer surplus and social welfare share the same structure for the cases with and without competition but with different equilibrium prices. Therefore, the key to understanding the impact of competition boils down to analyzing how it affects the equilibrium prices. The following lemma characterizes the impact of equilibrium price on customer surplus and social welfare.

## Lemma 2. The following statements hold:

a. The average customer surplus functions, $\operatorname{CS}^{*}(p)$ and $\operatorname{CS}_{b}^{*}(p)$, are concave in price $p$. In particular, the equilibrium price in the model with competition and customer switching,
$p_{s}^{*}$, satisfies the condition $p_{s}^{*} \in[\hat{p}, v)$, where $\hat{p}=\arg \max _{p}$ $C S^{*}(p)$.
b. The social welfare functions, $S W^{*}(p)$ and $S W_{b}^{*}(p)$, are concave in price $p$.

As shown by Lemma 2(a), the expected customer surplus functions in both models are concave in price. Moreover, the equilibrium price in the focal model is lower bounded by $\hat{p}$, which is the price that maximizes the average expected customer surplus. As a result, the customer's expected surplus is concavely decreasing in price under equilibrium. Lemma 2(b) shows that the social welfare functions are concave in price. Hence, as price increases, it first improves social welfare as the high price signals a high product availability; later, social welfare declines as the retailers may overstock the product.

The impact of competition on customer surplus and social welfare is a well-studied topic in the economics literature. A general insight from this literature is that competition improves customer surplus. For example, Brynjolfsson et al. (2003) summarize two mechanisms that drive market competition on product variety to improve consumer surplus. Increased market competition lowers market prices and expands product lines, both of which lead to increased customer surplus. However, the economics literature does not have a conclusive answer on how competition affects social welfare. Although many researchers show that competition may potentially improve social welfare, how market competition between firms could influence social welfare is still an open question because the benefits from customer surplus may not dominate the losses from firm profits (e.g., Stiglitz 1981). Our model incorporates the competition on both price and inventory availability. Recall that a high price can signal high product availability under equilibrium. Therefore, it is unclear a priori whether competition drives retailers to lower prices to directly attract customers or to increase prices to indirectly signal high product availability. The following proposition addresses this question and characterizes the conditions under which either effect dominates.

Proposition 8. Given full market coverage with competition and customer switching, we have (a) $C S^{*} \geq C S_{b}^{*}$ and (b) $S W^{*} \leq S W_{b}^{*}$.

Proposition 8 shows that market competition benefits customers but hurts social welfare. This differs from the insight in some economics literature that competition increases social welfare (see Stiglitz 1981). To understand the rationale of Proposition 8, we identify two opposing effects. The first effect is referred to as the pricing effect, under which competition drives retailers to charge lower prices as a promotion to attract customers. The second effect is called the product availability effect, under which competition drives
retailers to signal high inventory availability by increasing the prices. Specifically, as shown in Proposition 8(a), the retailers compete on capturing more market share by offering higher customer surplus, and thus, the market competition is beneficial to the customers. However, because the average customer surplus is decreasing in equilibrium price (see Lemma 2(a)), the retailers compete to offer lower prices in the market competition (the pricing effect dominates). In contrast, the social welfare might be increasing in equilibrium price (see Lemma 2(b)) as a high equilibrium price signals a high equilibrium product availability. Consequently, when retailers are competing on offering a lower price to attract more market share, the product availability decreases and, thus, social welfare decreases. In other words, although market competition improves the average customer surplus, the loss from retailers dominates the benefit from customers, so social welfare declines.

Another question we wish to address in this paper is how inventory commitment and monetary compensation strategies impact social welfare under competition. We now explore whether these two strategies can improve the average consumer surplus and social welfare under market competition. The equilibrium average consumer surplus and social welfare functions under the inventory commitment strategy are given by $C S_{v}^{*}=\left(v-p_{v}^{*}\right) \theta\left(p_{v}^{*}\right)-$ $\frac{s}{4}+\gamma\left(1-\theta\left(p_{v}^{*}\right)\right)\left[\left(v-p_{v}^{*}\right) \theta\left(p_{v}^{*}\right)-\frac{3 s}{4}\right]$ and $S W_{v}^{*}=v \mu \theta\left(p_{v}^{*}\right)$ $-c F^{-1}\left(\frac{p_{v}^{*}-c}{p_{v}^{*}}\right)-\frac{\mu s}{4}+\gamma\left(1-\theta\left(p_{v}^{*}\right)\right)\left[v \mu \theta\left(p_{v}^{*}\right)-c F^{-1}\left(\frac{p_{v}^{*}-c}{p_{v}^{*}}\right)-\right.$ $\left.\frac{3 \mu s}{4}\right]$, respectively, where $v$ represents the case of inventory commitment strategy. Similarly, the equilibrium average consumer surplus and social welfare functions under the monetary compensation strategy are given by $C S_{c}^{*}=\left(v-p_{c}^{*}\right) \theta\left(p_{c}^{*}\right)-\frac{s}{4}+\gamma\left(1-\theta\left(p_{c}^{*}\right)\right)\left[\left(v-p_{c}^{*}\right) \theta\left(p_{c}^{*}\right)-\frac{3 s}{4}\right]$ and $S W_{c}^{*}=v \mu \theta\left(p_{c}^{*}\right)-c F^{-1}\left(\frac{p_{c}^{*}-c}{p_{c}^{*}}\right)-\frac{\mu s}{4}+\gamma\left(1-\theta\left(p_{c}^{*}\right)\right)[v \mu$ $\left.\theta\left(p_{c}^{*}\right)-c F^{-1}\left(\frac{p_{c}^{*}-c}{p_{c}^{*}}\right)-\frac{3 \mu s}{4}\right]$, respectively, where $c$ represents the case of monetary compensation strategy. Note that the compensation term $m_{c}^{*}$ will not directly affect the social welfare as it is a cash transfer between the retailers and customers. However, the compensation $m_{c}^{*}$ does impact the equilibrium average consumer surplus because customers who face stockout are compensated.

Proposition 9. Under the inventory commitment or monetary compensation strategies, we have (a) $C S_{v}^{*} \geq C S^{*}$ and (b) $C S_{c}^{*} \geq C S^{*}$.

Proposition 9 shows that, although inventory commitment and monetary compensation do not necessarily benefit retailers under competition, these strategies are always beneficial to customers. Both strategies provide incentives to attract customers to patronize the retailers and, as a consequence, benefit the customers
once adopted by the retailers. It is also shown in the operations literature that inventory commitment and monetary compensation strategies improve social welfare in a monopoly market (e.g., Su and Zhang 2009). However, we demonstrate in the following proposition that these strategies may induce the retailers to compete more aggressively on inventory availability, which turns out to further decrease social welfare under market competition.
Proposition 10. The following statements hold:
a. Under the inventory commitment strategy, there exists a threshold $s_{v w}$ such that $S W_{v}^{*}<S W^{*}$ for $s<s_{v w}$.
b. Under the monetary compensation strategy, there exists a threshold $s_{c w}$ such that $S W_{c}^{*}<S W^{*}$ for $s<s_{c w}$.

Different from Proposition 9, Proposition 10 shows that inventory commitment and monetary compensation strategies may hurt social welfare under intense competition. Recall from Propositions 5 and 7 that, under intense competition, both strategies backfire and decrease the profit and inventory availability probability of the retailers. A similar rationale applies to Proposition 10 as well. Because the inventory commitment and monetary compensation strategies provide an alternative channel in which the retailers could compete for market share, the equilibrium price and product availability may decline when market competition is intense. As a result, social welfare decreases as well. Combining Propositions $5,7,9$, and 10 , we find that inventory commitment and monetary compensation strategies always make customers better off but retailers worse off under intense market competition with the former dominating the latter, so social welfare decreases under these strategies in this case.

These findings provide some practical insights for the central planner (e.g., industry regulator or the government). According to Lemma 2(b), social welfare is concave in price, so the central planner could set a price floor to restore the maximum social welfare (i.e., the maximum price in the market is set at the social welfare-maximizing one). ${ }^{4}$ Indeed, a properly set price floor may increase retailer profit by mitigating price competition, which also induces higher equilibrium product availability and eventually improves social welfare. It is worth noting that the price floor also benefits the customers in the long run. Because the market competition lowers the retailer profit under equilibrium, the retailers may tacitly coordinate to avoid marketing competition and, thus, charge a high price in the repeated game (e.g., the benchmark equilibrium price without demand uncertainty, $p_{b}^{*}$ ), which eventually hurts the customer's surplus. A carefully chosen price floor ensures retailer profit under competition and, consequently, increases the cost of deviating to tacit coordination (see Dufwenberg et al. 2007).

## 7. Conclusion

Inventory commitment and monetary compensation are widely observed in practice. The literature shows that these strategies could mitigate strategic customer behavior and enhance firm profit in a monopoly setting. This paper examines these strategies in a competitive setting when retailers compete on both price and inventory availability. Customers concerned about inventory availability may choose which retailer to patronize. Combining the newsvendor and Hotelling frameworks, we investigate the strategic interactions among retailers and customers. We derive market equilibrium price and inventory availability and quantify the impact of these strategies on firms' profitability, average consumer surplus, and social welfare. There are two main results from this research.

First, we find that both strategies may lead to a prisoner's dilemma: although a retailer would benefit from either strategy regardless of the competitor's price and inventory decisions, both inventory commitment and monetary compensation intensify market competition and hurt the retailers in a competitive market. This is in contrast to the common wisdom that these strategies improve retailer profit under monopoly. Specifically, the inventory commitment strategy may dilute the signaling power of price, thus leading to overstock of inventory for the competing retailers, whereas the monetary compensation strategy tends to overcompensate customers. That is, both strategies intensify market competition and, thus, reduce the profit of both retailers.

Second, our results indicate that, with customers' product availability concerns, competition may decrease equilibrium retail prices compared with a centralized setting, which decreases product availability and social welfare. This contrasts the insight in the literature that competition generally improves social welfare. Furthermore, inventory commitment and monetary compensation may further intensify competition between the retailers and, as a consequence, decrease product availability and hurt social welfare. Therefore, although inventory commitment and monetary compensation are beneficial in monopoly settings, both retail firms and social planners should exert caution when applying these strategies in competitive market environments.

## Acknowledgments

The authors thank the department editor, Mahesh Nagarajan; the anonymous associate editor; and two referees for their constructive comments that helped significantly improve the paper.

## Endnotes

${ }^{1}$ If the unit search cost $s$ is too high, then either the model reduces to two monopoly markets without competition or the customers will not consider switching upon stockout. The former situation is uninteresting, whereas the latter is essentially the base model. If $s$ is
moderate, then only some of the customers switch. The analysis and insights in this setting are similar to those in our base model, so we omit the details for brevity.
${ }^{2}$ We detail the derivation of the customer search cost as follows. Assume a customer is at location $x$ and the customer's focal retailer is $R_{1}$. (1) If the product is in stock, the total travel distance from the $x$ to the focal retailer and then back to $x$ is $2 x$. (2) If the product is out of stock, the total travel distance should be (i) the distance from the origin point $x$ to the first destination $R_{1}$ plus (ii) the distance from $R_{1}$ to the competing retailer $R_{2}$ plus (iii) the distance from $R_{2}$ to the origin point $x$. Therefore, the total travel distance is $x+1+(1-x)=2$. Therefore, if the product is out of stock in the focal retailer, the extra travel distance for a customer located at $x$ is $2-2 x=2(1-x)$. Without loss of generality, by redefining the unit search cost as $s / 2$ per unit distance, the customer's extra search cost is $s(1-x)$ upon the stockout at the focal retailer.
${ }^{3}$ See https://www.foodland.com/if-i-have-coupon-product-out-stock-may-i-receive-rain-check-product for more details.
${ }^{4}$ Let $p^{*}$ be the equilibrium price in the base model (the model without customer switching) and let $p_{b}^{*}$ be the price that achieves the maximum social welfare. According to the proof of Proposition 8, we have $p^{*}<p_{b}^{*}$. Because the social welfare function is concave in price (see Online Lemma 3), to restore the maximum social welfare, the market planner sets a price floor that equals $p_{b}^{*}$. As a result, the retailers stop competing on offering lower prices to attract more market share at price $p=p_{b}^{*}$.

## References

Allon G, Bassamboo A (2011) Buying from the babbling retailer? The impact of availability information on customer behavior. Management Sci. 57(4):713-726.
Anand KS, Goyal M (2019) Ethics, bounded rationality, and IP sharing in IT outsourcing. Management Sci. 65(11):5252-5267.
Aviv Y, Wei MM, Zhang F (2019) Responsive pricing of fashion products: The effects of demand learning and strategic consumer behavior. Management Sci. 65(7):2982-3000.
Bassok Y, Anupindi R, Akella R (1999) Single-period multiproduct inventory models with substitution. Oper. Res. 47(4): 632-642.
Bernstein F, Martínez-de Albéniz V (2016) Dynamic product rotation in the presence of strategic customers. Management Sci. 63(7):2092-2107.
Bhargava HK, Sun D, Xu SH (2006) Stockout compensation: Joint inventory and price optimization in electronic retailing. INFORMS J. Comput. 18(2):255-266.
Brynjolfsson E, Hu Y, Smith MD (2003) Consumer surplus in the digital economy: Estimating the value of increased product variety at online booksellers. Management Sci. 49(11):1580-1596.
Cachon GP, Feldman P (2015) Price commitments with strategic consumers: Why it can be optimal to discount more frequently...than optimal. Manufacturing Service Oper. Management 17(3):399-410.
Cachon GP, Swinney R (2009) Purchasing, pricing, and quick response in the presence of strategic consumers. Management Sci. 55(3):497-511.
Cachon GP, Swinney R (2011) The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior. Management Sci. 57(4):778-795.
Carlton DW (1978) Market behavior with demand uncertainty and price inflexibility. Amer. Econom. Rev. 68(4):571-587.
Chelsey (2022) How to use brickseek to check prices \& inventory at Walmart, target, and other retailers. Hip2Save Online (September 21), https://hip2save.com/tips/brickseek-walmart-target/.

Chen Y, Narasimhan C, Zhang ZJ (2001) Individual marketing with imperfect targetability. Marketing Sci. 20(1):23-41.
Dana JD (2001) Competition in price and availability when availability is unobservable. RAND J. Econom. 32(3):497-513.
Dana JD, Petruzzi NC (2001) Note: The newsvendor model with endogenous demand. Management Sci. 47(11):1488-1497.
Daughety AF, Reinganum JF (1991) Endogenous availability in search equilibrium. RAND J. Econom. 22(2):287-306.
Deneckere R, Peck J (1995) Competition over price and service rate when demand is stochastic: A strategic analysis. RAND J. Econom. 26(1):148-162.
Dewan RM, Freimer ML, Jiang Y (2007) A temporary monopolist: Taking advantage of information transparency on the web. J. Management Inform. Systems 24(2):167-194.
Dufwenberg M, Gneezy U, Goeree JK, Nagel R (2007) Price floors and competition. Econom. Theory 33(1):211-224.
Gao F, Su X (2016) Omnichannel retail operations with buy-online-and-pick-up-in-store. Management Sci. 63(8):2478-2492.
Goolsbee A, Petrin A (2004) The consumer gains from direct broadcast satellites and the competition with cable tv. Econometrica 72(2):351-381.
Granados N, Gupta A (2013) Transparency strategy: Competing with information in a digital world. Management Inform. Systems Quart. 37(2):637-641.
Hausman J, Leibtag E (2007) Consumer benefits from increased competition in shopping outlets: Measuring the effect of WalMart. J. Appl. Econometrics 22(7):1157-1177.
Karaesmen I, Van Ryzin G (2004) Overbooking with substitutable inventory classes. Oper. Res. 52(1):83-104.
Kim BD, Shi M, Srinivasan K (2001) Reward programs and tacit collusion. Marketing Sci. 20(2):99-120.
Kim BD, Shi M, Srinivasan K (2004) Managing capacity through reward programs. Management Sci. 50(4):503-520.
Kopalle PK, Neslin S (2001) The economic viability of frequency reward programs in a strategic competitive environment. Preprint, submitted April 11, https://dx.doi.org/10.2139/ssrn. 265431.

Lei J (2015) Market equilibrium and social welfare when considering different costs. Internat. J. Econom. Res. 6(2):60-77.
Li C, Zhang F (2013) Advance demand information, price discrimination, and preorder strategies. Manufacturing Service Oper. Management 15(1):57-71.
Li KJ, Jain S (2016) Behavior-based pricing: An analysis of the impact of peer-induced fairness. Management Sci. 62(9):2705-2721.
Liang C, Cakanyildirim M, Sethi SP (2014) Analysis of product rollover strategies in the presence of strategic customers. Management Sci. 60(4):1022-1056.
Lippman SA, McCardle KF (1997) The competitive newsboy. Oper. Res. 45(1):54-65.
Liu Q, Van Ryzin GJ (2008) Strategic capacity rationing to induce early purchases. Management Sci. 54(6):1115-1131.
Mahajan S, Van Ryzin G (2001) Stocking retail assortments under dynamic consumer substitution. Oper. Res. 49(3):334-351.
Netessine S, Rudi N (2003) Centralized and competitive inventory models with demand substitution. Oper. Res. 51(2):329-335.
Prasad A, Stecke K, Zhao X (2014) Advance selling by a newsvendor retailer. Production Oper. Management 20(1):129-142.
Shin J, Sudhir K (2010) A customer management dilemma: When is it profitable to reward one's own customers? Marketing Sci. 29(4):671-689.
Shumsky R, Zhang F (2009) Dynamic capacity management with substitution. Oper. Res. 57(3):671-684.
Sloot LM, Verhoef PC, Franses PH (2005) The impact of brand equity and the hedonic level of products on consumer stockout reactions. J. Retailing 81(1):15-34.

Stiglitz JE (1981) Potential competition may reduce welfare. Amer. Econom. Rev. 71(2):184-189.
Su X, Zhang F (2008) Strategic customer behavior, commitment, and supply chain performance. Management Sci. 54(10):1759-1773.
Su X, Zhang F (2009) On the value of commitment and availability guarantees when selling to strategic consumers. Management Sci. 55(5):713-726.
Tao R (2014) Out of stock problems? Walmart, Nike, and Best Buy had them too, but here's how you can do better. Accessed August 1, 2021, https://www.tradegecko.com/blog/out-of-stock-problems-and-solutions-walmart-nike-bestbuy-case-studies.

Tereyagoglu N, Veeraraghavan S (2012) Selling to conspicuous consumers: Pricing, production, and sourcing decisions. Management Sci. 58(12):2168-2189.
Wei MM, Zhang F (2018a) Advance selling to strategic consumers: Preorder contingent production strategy with advance selling target. Production Oper. Management 27(7):1221-1235.
Wei MM, Zhang F (2018b) Recent research developments of strategic consumer behavior in operations management. Comput. Oper. Res. 93:166-176.
Yu Y, Chen X, Zhang F (2015) Dynamic capacity management with general upgrading. Oper. Res. 63(6):1372-1389.

