Customer purchasing behavior may have substantial impact upon the outcome of a market. In particular, the forward-looking behavior of customers naturally interacts with the trade-in remanufacturing practice in which firms collect used products for remanufacturing by offering rebates that allow repeat customers to trade in used products for upgraded ones at a discount price. This paper studies how different intensities of strategic customer behavior impact the economic and environmental values of such trade-in remanufacturing practice. We demonstrate a new value of trade-in remanufacturing that it helps exploit the forward-looking behavior of strategic customers in the market, which is much more significant than the widely recognized revenue-generating and environmental benefits of remanufacturing. In particular, under trade-in remanufacturing, a firm may earn a higher profit when customers are more strategic. When customers are highly strategic, trade-in remanufacturing creates a tension between profitability and sustainability: On one hand, by exploiting the intensive forward-looking customer behavior, trade-in remanufacturing is quite valuable to the firm; on the other hand, with highly strategic customers, trade-in remanufacturing has a substantial negative impact on environment and social welfare, since it may give rise to a significantly higher production quantity without improving customer surplus. With nearly-myopic customers, however, trade-in remanufacturing benefits both the firm and the environment, since it motivates the firm to produce less in this case. Therefore, understanding the interactions between customer purchasing behavior and trade-in remanufacturing is important both to firms and policy makers. Finally, to resolve the above tension, we study how a social planner (e.g., the government) should design a public policy to maximize social welfare. The social optimum can be achieved by using a simple linear subsidy and tax scheme for all product versions. Such government policy counters strategic customer behavior and, thus, induces the social optimum independent of customer purchasing behavior.

Key words: customer behavior; trade-in rebates; remanufacturing; environment; government policy
1. Introduction

Remanufacturing is the rebuilding of a product to specifications of the original manufactured product using a combination of reused, repaired, and new parts (Johnson and McCarthy 2014). The initial purpose of remanufacturing was to recover the residual value of the components and materials from used products (see, e.g., Guide and Van Wassenhove 2009, Debo et al. 2005). More recently, with growing awareness of sustainability, the environmental advantages of remanufacturing have also been widely recognized. As a result, remanufacturing has been increasingly adopted in industry to enhance a firm’s competitive edge. According to the U.S. International Trade Commission (2012), an integral component of the closed-loop supply chain for remanufacturing is core collection, i.e., the process of obtaining used products from customers. A common practice for core collection is to provide trade-in rebates that encourage customers to return their used products. For example, Apple offers both in-store and online trade-in programs, which allow customers to exchange their used iPhones, iPads, and Macs for credits to purchase new ones (Apple Online Store 2015). Analogously, Amazon allows Kindle owners to trade in their old products for newer versions at a discount price (Copy 2011). More examples of using trade-in rebates for core collection have been reported in industries such as furniture, carpets, power tools, etc. (see Ray et al. 2005).

Remanufacturing through trade-in rebates has been lauded for various benefits. From an economic perspective, the value generated from remanufacturing and recycling could be quite significant. Xerox, which partly bases its remanufacturing on trade-in returns, has saved several hundred million dollars each year, which accounts for 40% to 65% of the company’s manufacturing costs (Savaskan et al. 2004). In 2015, Apple recovered more than a ton of gold from recycled iPhones (Goldman 2016). From the strategic perspective, trade-in rebates may improve firm profitability by elevating customer switching costs (Klemperer 1987), discouraging second-hand markets (Levinthal and Purohit 1989), increasing purchase frequency (Van Ackere and Reyniers 1995), and reducing inefficiencies arising from the lemon problem (Rao et al. 2009). From an environmental perspective, trade-in rebates encourage customers to return used products for remanufacturing and recycling, which helps reduce the consumption of natural resources and also alleviates the impact of waste on the environment. For example, Giutini and Gaudette (2003) document that annually remanufacturing saves 16 million barrels of crude oil and reduces 28 million tons of carbon dioxide emissions worldwide.

The trade-in program not only helps a firm collect used products from customers, but also grants price discounts to repeat customers who return their used products, thus enabling the firm to price discriminate the new and repeat customers (Van Ackere and Reyniers 1995). Clearly, the incentive brought by the trade-in option may affect customers’ purchasing strategy. For instance, a customer often needs to make the decision of whether to purchase immediately or to wait for
better opportunities (e.g., a price markdown or a new technology) in the future. A customer may be called strategic or forward-looking if she strategizes the purchasing decision to maximize her long-run utilities. In contrast, a myopic customer does not consider future opportunities and base their purchasing decision on the immediate utilities.

Consumer purchasing behavior can be quite complex in the real world. It has been empirically verified that customers exhibit a mixture of strategic and nonstrategic purchasing behaviors in various markets (see, e.g., Li et al. 2014, Hendel and Nevo 2013, Chevalier and Goolsbee 2009). So the actual customer purchasing behavior in a market should be somewhere between the two extremes of fully strategic and fully nonstrategic behaviors. Despite its complexity, understanding the customer purchasing behavior in a market is important, since whether customers are strategic or not has significant impact upon a firm’s operations strategy. For example, whereas responsive pricing could effectively exploit customer segmentation with myopic customers, it has potential adverse impact with strategic customers (Aviv and Pazgal 2008). As another example, in the durable goods and event ticketing markets, resale in the secondary market damages the profit when customers are myopic, but the profit will increase with the presence of the secondary/resale market if customers are forward-looking (see Chevalier and Goolsbee 2009, Su 2010, Cui et al. 2014).

Customer purchasing behavior naturally interacts with trade-in remanufacturing. More specifically, under trade-in remanufacturing, strategic customers will anticipate a possible future price discount in the form of a trade-in rebate, which is ignored by myopic customers. As a consequence, different customer purchasing behaviors may lead to drastically different market outcomes under trade-in remanufacturing. Although both strategic and nonstrategic customer behaviors have been widely acknowledged in the literature, it is not clear what role they will play under the trade-in remanufacturing setting.

The primary goal of this paper is to analyze the impact of customer purchasing behavior on the value of trade-in remanufacturing to different stakeholders. For this purpose, we develop a two-period model in which a profit-maximizing firm sells two generations of a product in an ex-ante uncertain market. We use the customer discount factor to model the intensity of their forward-looking behavior. If this discount factor is large, the intensity of strategic customer behavior is high, and the customers make their purchasing decisions with serious considerations of anticipated future utilities. Otherwise, the customer discount factor is low, so customers care little about future utilities and are myopic to a large extent. In the first period, the firm sells the first-generation product in the market. In the second period, the firm sells the second-generation product to new customers (who have not purchased in the first period); meanwhile, the firm offers trade-in rebates that allow repeat customers (who have purchased in the first period) to exchange used products
for new second-generation ones at a discount price. We explicitly model two benefits from remanufacturing and recycling used products: First, it generates economic value for the firm; second, it helps reduce the product’s negative impact on the environment.

A key message of our paper is that customer purchasing behavior has important implications for the value of trade-in remanufacturing. From the firm’s perspective, we find that the profit improvement from adopting trade-in remanufacturing is more significant under more strategic customers. In other words, the firm will have more incentives to use remanufacturing in a market with strategic rather than myopic customers. Furthermore, in contrast to the common belief that forward-looking behavior is detrimental to firm profit, we show the firm’s profit may increase with the intensity of strategic behavior under trade-in remanufacturing. This is because when the economic value of remanufacturing is strong enough, the trade-in rebate in the second period will be high enough to ensure a higher surplus for repeat customers than for new customers. As a result, customers are willing to pay a higher first-period price if they are more strategic, which could improve the total profit of the firm. These findings indicate that trade-in remanufacturing helps exploit strategic customer behavior, which is a new benefit that has not been explored before.

As for the environment, we find that the impact of trade-in remanufacturing also depends critically on customer behavior. With highly strategic customers, adopting trade-in remanufacturing may aggravate the negative impact on the environment. As discussed above, under intensive forward-looking customer behavior, the trade-in opportunity offers customers a strong incentive to purchase early, which prompts the firm to increase production quantities after adopting trade-in remanufacturing. The increased production quantities may outweigh the environmental advantage of remanufacturing under general circumstances. Hence, trade-in remanufacturing generally hurts the environment with highly strategic customers. In addition, it may decrease the total customer surplus as well as the social welfare. However, with a sufficiently low intensity of strategic customer behavior, trade-in remanufacturing can be beneficial to the environment. In this case, new customers would be more profitable than repeat customers in the second period; this drives the firm to decrease the first-period production quantity to serve more new customers in the second period. These results call for caution when adopting the trade-in remanufacturing strategy. In particular, understanding customer purchasing behavior in the market is essential in evaluating the strategy, both for the firm and for the environment.

From the above results we can see that with strategic customers, the adoption of trade-in remanufacturing may create a tension between profitability and sustainability. That is, trade-in remanufacturing can greatly improve firm profit but meanwhile worsen the environmental impact. This motivates us to study how government intervention can achieve the socially optimal outcome in a market with highly strategic customers. Recently, we have seen more government interventions
of markets based on environmental issues. For instance, in January 2015, the Chinese government released a policy to subsidize the use of remanufactured vehicle engines and transmissions (Chen 2015). Similarly, Scotland is calling for government subsidies to promote recycling and remanufacturing (The Recycler 2015).

We focus on the use of subsidy/tax policy by the government to regulate the practice of trade-in remanufacturing. The government is modeled as a central planner who aims to maximize the social welfare, i.e., the sum of firm profit and customer surplus less environmental impact. An intuitive policy observed in practice is to subsidize the firm/customers for selling/purchasing remanufactured products. However, we find that subsidizing remanufactured products alone actually hurts the environment, intensifies the tension between profitability and sustainability, and does not achieve the social optimum. This suggests that a haphazard policy design may lead to undesired outcomes. We thus consider a more comprehensive linear subsidy/tax scheme for the sales of both product generations and remanufacturing. We show that, for any forward-looking customer behavior intensity, such a linear subsidy/tax scheme, if designed properly, can alleviate the tension and induce the social optimum. The socially optimal government policy aims to provide incentives to counter strategic customer behavior and, thus, induces the social-welfare-maximizing equilibrium, which does not depend on how strategic customers are.

The rest of the paper is organized as follows. Section 2 reviews the related literature. The model and equilibrium analysis are presented in Section 3. In Sections 4 and 5, we analyze the value of trade-in remanufacturing for the firm and the environment, respectively. Section 6 characterizes the socially optimal government policy. This paper concludes with Section 7. All proofs are given in the Appendix.

2. Literature Review

The impact of customer purchasing behavior upon a firm’s operations decisions has received an increasing amount of attention in the operations management literature. Shen and Su (2007) provide a comprehensive review on customer behavior models in revenue management and auctions. Bensako and Winston (1990) compare the optimal pricing policy in the scenarios with fully strategic and fully myopic customers. They show that the presence of strategic customers drives a monopolist firm to charge a lower price at the initial stage of the sales season and to mark down less aggressively afterwards. Liu and Zhang (2013) extend this work to a competitive setting, and further demonstrate that, under horizontal competition, price skimming enhances the duopolists’ profits with myopic customers, but generally hurts their profits in the presence of strategic customers. In a revenue management framework, Aviv and Pazgal (2008) demonstrate that the responsive pricing strategy could effectively improve a monopolist’s revenue with myopic customers, but this strategy
could lead to significant revenue losses with strategic customers. In a follow-up work, Aviv et al. (2015) show that the benefits of responsive pricing and demand learning depend crucially on the nature of customer purchasing behavior: Their values tend to worsen when customers are strategic. Under a newsvendor framework, Cachon and Swinney (2009) shows that quick response could deliver a significantly higher value to a retailer in the presence of strategic customers than without them. Caldentey et al. (2016) consider a robust formulation of a monopolist’s pricing problem. The authors characterize the different pricing policies without knowing the customer valuation and arrival timing under different customer purchasing behaviors (i.e., fully strategic and fully myopic customers). When the market is segmented into customers with differentiated customer purchasing behaviors, Su (2007) shows that when high-value customers are less (more) strategic, the mark-down (mark-up) pricing strategy is optimal. The bottom line of this stream of research is that the effectiveness of an operations strategy is very sensitive to the customer purchasing behavior. In addition to the modeling works, there are several papers that empirically examine the customer purchasing behaviors in, for instance, the airline (Li et al. 2014), soft drink (Hendel and Nevo 2013), and textbook (Chevalier and Goolsbee 2009) industries. The empirical findings in this literature suggest that the market is likely to have a mixture of strategic and nonstrategic customers and the overall customer purchasing behavior is complex.

There is a rapidly growing stream of literature on remanufacturing and closed-loop supply chain management. A comprehensive review of this literature is given by Guide and Van Wassenhove (2009). Several papers study the optimal inventory policy with return flows of used products and exogenously given demand rate, price, and remanufacturability; see, e.g., Van der Laan et al. (1999), Toktay et al. (2000), and Gong and Chao (2013). More recently, researchers start to explicitly model some strategic issues related to remanufacturing, such as used product acquisition, demand segmentation, product cannibalization, and competition. Savaskan et al. (2004) study the optimal reverse channel structure for the collection of used products from customers. Ferguson and Toktay (2005) analyze the competition between new and remanufactured products (i.e., the cannibalization effect) and characterize the optimal recovery strategy. When remanufacturability is an endogenous decision, Debo et al. (2005) investigate a joint pricing and production technology selection problem of a manufacturer who sells a remanufacturable product to heterogeneous customers. Atasu et al. (2008) show that remanufacturing could serve as a marketing strategy to target customers in the green segment and, hence, enhance the profitability of the OEM. Galbreth et al. (2013) study how the rate of product innovation affects the firm’s reuse and remanufacturing decisions. Gu et al. (2015) investigate the quality design and environmental consequences of green consumerism with remanufacturing. The impact of trade-in rebates has also received some attention in the remanufacturing literature. For example, Ray et al. (2005) examine the value of price discrimination
for new and repeat customers with differentiated ages (and qualities) of the products returned through trade-ins for remanufacturing.

Government regulations on remanufacturing and other environmentally relevant operations issues have also been studied in the literature. For instance, Calcott and Walls (2000) show that, in a supply chain, a downstream disposal fee charged by the government may not ensure the social optimum, unless supplemented with upstream instruments. Ma et al. (2013) study the impact of a government consumption-subsidy program on a dual-channel closed-loop supply chain. Cohen et al. (2015) characterize the impact of demand uncertainty on government subsidies for green technology adoption.

There are a few papers that investigate trade-in rebates or multiple product introductions with the presence of forward-looking customers. Van Ackere and Reyniers (1995) show that, under strategic customer behavior, trade-ins can serve as a mechanism to achieve price commitment. Fudenberg and Tirole (1998) study the monopoly pricing of overlapping generations of a durable good with and without a second-hand market. In an infinite-horizon model setting, Rao et al. (2009) demonstrate that trade-in rebates can alleviate the inefficiencies arising from the lemon problem. Liang et al. (2014) prove that, in a setting with multiple production introductions, single rollover is more valuable when the new product’s innovation is low and the number of strategic customers is high. Lobel et al. (2015) study the new product launch strategy, and show that the technology release pre-commitment can lead to significant profit improvement under forward-looking customer behavior.

Our paper contributes to the aforementioned streams of research by studying the interactions between customer purchasing behavior and trade-in remanufacturing, and how such interactions affect the economic and environmental values of trade-in remanufacturing. We demonstrate that different intensities of forward-looking customer behavior have important implications on the value of trade-in remanufacturing both to the firm and to the environment. The government subsidy/tax scheme that induces the social optimum is also sensitive to customer purchasing behavior.

3. Model and Equilibrium Analysis

3.1. Model Setup

We consider a monopolist firm (he) in the market who sells a product to customers (she) in a two-period sales horizon. In the first period, the firm produces the first-generation product at a unit production cost $c_1$. The potential market size $X$, which is the total number of potential customers, is \textit{ex-ante} uncertain. Without loss of generality, $X$ is a continuous random variable with a distribution function $F(\cdot)$ and a density function $f(\cdot) = F'(\cdot)$. We assume that all customers arrive at the market at the beginning of period 1, but all our results and insights continue to hold
if there are new customers arriving in period 2. The customers are infinitesimal, each requesting at most one unit of the product in any period. Demand uncertainty is a common feature with new product introduction, but the firm can obtain more accurate demand information as the market matures. Hence, in period 2, the market uncertainty is resolved so the realized market size $X$ becomes known to the firm.

A customer’s valuation $V$ for the first-generation product is independently drawn from a continuous distribution with a distribution function $G(\cdot)$ supported on $[\underline{v}, \bar{v}]$ ($0 \leq \underline{v} < \bar{v}$). We call the customer with product valuation $V$ the type-$V$ customer. At the beginning of the sales horizon, each customer only knows the distribution of her own valuation $G(\cdot)$, but not the realization $V$. This assumption captures the customers’ uncertainties about product quality, and fits the situation where the product is brand new. In period 2, all customers observe their own type $V$. Customers who purchased the product in period 1 learn their type $V$ by consuming the product. Customers who did not purchase the product in period 1 learn its quality and fit (thus, their type $V$) through social learning platforms (e.g., Facebook and Amazon customer review systems). Hence, the customers are homogeneous ex ante (i.e., at the beginning of period 1), but heterogeneous ex post (i.e., at the beginning of period 2). This is a common setting in the models concerning customer purchasing behavior (see, e.g., Xie and Shugan 2001, Su 2009). We assume that the valuation distribution $G(\cdot)$ has an increasing failure rate, i.e., $g(v)/\bar{G}(v)$ is increasing in $v$, where $g(\cdot) = G'(\cdot)$ is the density function and $\bar{G}(\cdot) = 1 - G(\cdot)$. This is a mild assumption and can be satisfied by most commonly used distributions. Let $\mu := \mathbb{E}(V) > c_1$, i.e., in expectation, a customer’s valuation exceeds the production cost.

The firm offers an upgraded version of the product in period 2. This is a customary practice for product categories like consumer electronics, home appliances, and furniture. A type-$V$ customer has a valuation of $(1+\alpha)V$ for the upgraded second-generation product, where $\alpha \geq 0$ is exogenously given and captures the innovation level (e.g., the improved features) of the upgraded product. Accordingly, let the production cost of the second-generation product be $c_2$. To model product depreciation, we take the approach of Van Ackere and Reyniers (1995): If a type-$V$ customer has already bought the product in period 1, her valuation of continuing to consume the used product in period 2 is $(1-k)V$, where $k \in [0, 1)$ refers to the depreciation factor. Specifically, if $k = 0$, the product is completely durable; if $k$ approaches 1, the product is almost useless after the first period (either the product is worn out or the technology is obsolete). Therefore, the willingness-to-pay of a type-$V$ customer in period 2 is $(1+\alpha)V$ if she did not purchase the product in period 1 (i.e., a new customer), and is $(1+\alpha)V - (1-k)V = (k+\alpha)V$ if she purchased the product in period 1 (i.e., a repeat customer). We do not explicitly model the secondary market, but studying its impact would be an interesting direction for future research.
As widely recognized in the literature, the firm can generate revenue by extracting materials and components from used products (see, e.g., Savaskan et al. 2004, Ray et al. 2005). We now model the revenue-generating effect of remanufacturing. There are two types of remanufacturing in our model. First, the firm recycles the unsold first-generation products at the end of period 1. The recycled leftover inventory in the first period is remanufactured and can generate a net per-unit revenue $r_1$ ($r_1 < c_1$) for the firm. That is, no excess inventory is carried over to the second period. This assumption applies when the inventory holding cost is sufficiently high or the firm does not want to dilute the sales of the newer generation product, which is usually the case in the electronics market. Moreover, this assumption facilitates the technical tractability of our model, but would not affect the insights. In the Appendix, we present an extension where the firm may hold leftover inventory, upgrade some of it, and offer both product generations in the second period. The second type of remanufacturing is by using the returned products in period 2, i.e., customers who bought the product in period 1 can trade in the old product for a second-generation one at a discount price in period 2. The net revenue of remanufacturing from a used product in period 2 is $r_2$ ($r_2 < c_2$). Following Savaskan et al. (2004), we assume all remanufactured products are upgraded to the quality standards of new ones so customers cannot distinguish them from newly made products. Relaxing this assumption will not affect our qualitative insights.

The environmental impact of the product is the aggregate (negative) lifetime impact of the product on the environment. The total environmental impact is the production quantity of the product multiplied by the per-unit impact (see, e.g., Thomas 2011, Agrawal et al. 2012). Let $\kappa_1 > 0$ denote the unit environmental impact of the first-generation product. Analogously, we denote $\kappa_2 > 0$ as the unit environmental impact of the second-generation product. Such impact may refer to the use of natural resources, emission of harmful gases, and generation of solid wastes. Moreover, $\kappa_1$ and $\kappa_2$ can be estimated by the conventional life-cycle analysis (see, e.g., Agrawal et al. 2012). To model the environmental benefit of remanufacturing, let $\iota_1$ ($\iota_1 < \kappa_1$) be the unit environmental benefit of recycling the first-period leftover inventory, and $\iota_2$ ($\iota_2 < \kappa_2$) be that of recycling the used products through trade-in rebates. Here, $\iota_1$ and $\iota_2$ refer to the reductions in both the production environmental impact of the second-generation product and the end-of-use/end-of-life product disposals, by recycling and reusing the materials and components of first-generation products. The sequence of events unfolds as follows. At the beginning of period 1, the firm announces the price $p_1$ and decides the production quantity $Q_1$. Each customer observes $p_1$, but not $Q_1$, and decides whether to order the product or to wait until period 2. The first-period demand $X_1 \leq X$ is then realized, the firm collects his first-period revenue, and all customers stay in the market. Note that $X_1$ is determined by the collective effect of all customers’ purchasing decisions. If $X_1 \leq Q_1$, any customer who requests a product can get one in period 1. Otherwise, $X_1 > Q_1$, then the $Q_1$
products are randomly allocated to the demand, and \( X_1 - Q_1 \) customers have to wait due to the limited availability. At the end of period 1, the firm recycles and remanufactures the leftover inventory. At the beginning of period 2, the firm learns the realized total market size \( X \), and each individual customer learns her type \( V \). The firm then announces the price \( p_2^n \) for new customers as well as the trade-in price \( p_2^r \) (\( p_2^n - p_2^r \) is the trade-in rebate); all new customers decide whether to purchase the second-generation product, whereas all repeat customers decide whether to trade in their used products for new second-generation ones. Finally, the firm produces the second-generation products, recycles and remanufactures the used products from repeat customers, and collects the second-period revenue.

For notational convenience, we will use \( \mathbb{E} [\cdot] \) to denote the expectation operation, \( x \wedge y \) to denote the minimum of two real numbers \( x \) and \( y \), and \( \epsilon_1 \overset{d}{=} \epsilon_2 \) to denote that two random variables \( \epsilon_1 \) and \( \epsilon_2 \) follow the same distribution.

### 3.2. Customer Purchasing Behavior and Equilibrium Analysis

We use \( \delta \in (0, 1] \) to denote the risk-free discount factor of the market, which is also the discount factor of the firm. To study the impact of different customer purchasing behaviors, we denote \( \delta_c \) (\( \delta_c \in [0, \delta] \)) as the discount factor of customers, which measures the intensity of forward-looking behavior. The larger the \( \delta_c \), the more the customers care about future utilities, and the more strategic they are. Thus, for the rest of the paper, we call \( \delta_c \) the customer discount factor and the forward-looking behavior intensity interchangeably. If \( \delta_c = \delta \), customers are fully strategic and maximize their long-run utilities; if \( \delta_c = 0 \), customers are fully myopic and maximize their immediate utilities. Using customer discount factor to capture their forward-looking behavior intensity is a common approach in the literature (e.g., Levin et al. 2009, Chevalier and Goolsbee 2009). An alternative modeling approach is to assume there are two customer segments (strategic and myopic) in the market (e.g., Su 2007, Li et al. 2014). Under this approach, the intensity of forward-looking behavior can be captured by the proportion of strategic customers. Both approaches generate the same qualitative results and insights, so we will focus on the first approach for ease of exposition.

To characterize the market outcome, we adopt the rational expectation (RE) equilibrium framework. The RE equilibrium was proposed by Muth (1961) and has been widely used in the operations management literature (e.g., Su and Zhang 2008, 2009, Cachon and Swinney 2009, 2011). Using backward induction, we start with the subgame in period 2. There are \( X_2^n = X - (X_1 \wedge Q_1) \) new customers and \( X_2^r = X_1 \wedge Q_1 \) repeat customers in the market. Since period 2 is the final period in our model, customers with different intensities of forward-looking behavior adopt the same purchasing strategy therein. Hence, regardless of customer discount factor \( \delta_c \), the firm should adopt the same pricing strategy in period 2 as well. Given \((X_2^n, X_2^r)\), let \( p_2^n(X_2^n, X_2^r) \) and \( Q_2^n(X_2^n, X_2^r) \)
be the equilibrium price and production quantity for new customers in period 2. Analogously, we define \( p_2^*(X_1^n, X_2^n) \) and \( Q_2^*(X_1^n, X_2^n) \) as the equilibrium trade-in price and production quantity for repeat customers, and \( \pi_2(X_1^n, X_2^n) \) as the equilibrium second-period profit of the firm.

**Lemma 1.** (a) For any \((X_1^n, X_2^n)\), \( p_2^*(X_1^n, X_2^n) \equiv p_2^{n*} \) and \( p_2^*(X_1^n, X_2^n) \equiv p_2^* \), where \( p_2^* < p_2^{n*} \).  
(b) For any \((X_1^n, X_2^n)\), \( Q_2^*(X_1^n, X_2^n) = G \left( \frac{p_2^*}{1 + \alpha} \right) X_2^n \), and \( Q_2^*(X_1^n, X_2^n) = G \left( \frac{p_2^{n*}}{1 + \alpha} \right) X_2^n \).  
(c) For all \((X_1^n, X_2^n)\), \( \pi_2(X_1^n, X_2^n) = \beta_n^* X_2^n + \beta_r^* X_2^n \) for some positive constants \( \beta_n^* \) and \( \beta_r^* \).

Lemma 1 implies that both the equilibrium price for new customers and the equilibrium trade-in price are independent of the realized market size \((X_1^n, X_2^n)\). In particular, the firm offers positive trade-in rebates to repeat customers (i.e., \( p_2^* < p_2^{n*} \)). Moreover, the equilibrium profit of the firm in period 2, \( \pi_2(X_1^n, X_2^n) \), is linearly separable in \( X_2^n \) and \( X_2^n \), with the coefficients \( \beta_n^* \) and \( \beta_r^* \) capturing the expected unit profit from new and repeat customers, respectively.

We now analyze the equilibrium market outcome in period 1, starting with customers’ purchasing behavior. Each customer forms beliefs about the first-period product availability probability \( a \), the expected unit profit from new and repeat customers, respectively. The expected unit profit from trade-in rebates to repeat customers (i.e., \( a \), \( p_2^* \), and \( p_2^* \) are all nonnegative random variables. Based on the belief vector \((a_1, p_2^n, p_2^*) \) and the observed first-period price \( p_1 \), she computes the expected utility of making an immediate purchase, \( U_p := a_1 [E[V] + \delta_a E[(k + \alpha)V - p_2^n] + - p_1] + (1 - a_1) \delta_a E[(1 + \alpha)V - p_2^n] \), and the expected utility of waiting, \( U_w := \delta_a E[(1 + \alpha)V - p_2^n] \). Hence, the first-period reservation price of a customer with forward-looking behavior intensity \( \delta_a \) is given by \( \xi_a(\delta_a) := \max\{p_1 : U_p \geq U_w\} \), and she will make a purchase in period 1 if and only if \( p_1 \leq \xi_a(\delta_a) \). Following the standard approach in the literature (Xie and Shugan 2001, Su and Zhang 2008, Cachon and Swinney 2011), we assume that all customers will make a purchase in period 1 if \( p_1 \) equals their reservation price \( \xi_a(\delta_a) \). Thus, with customer discount factor \( \delta_a \), the first-period demand, \( X_1 \), is given by \( X_1 = X \cdot 1_{\{p_1 \leq \xi_a(\delta_a)\}} \).

Next, we consider the firm’s problem in period 1. For any customer discount factor \( \delta_a \), the firm does not know the customer reservation price \( \xi_a(\delta_a) \), but forms a belief \( r_1(\delta_a) \) about it. To maximize his expected profit, the firm sets the first-period price \( p_1(\delta_a) \) equal to the expected reservation price \( r_1(\delta_a) \), which is the highest price (the firm believes) customers are willing to pay in the first period. Thus, the firm believes that the first-period demand \( X_1 = X \). Thus, the second-period market size of new customers is \( X_1^n = (X - Q_1)^+ \), and that of repeat customers is \( X_2^n = X \wedge Q_1 \). Moreover, the firm sets the first-period production quantity \( Q_1 \) to maximize the total expected profit with customer discount factor \( \delta_a \), \( \Pi_f(Q_1|\delta_a) \), where \( \Pi_f(Q_1|\delta_a) = E[p_1(\delta_a)(X \wedge Q_1) - c_1 Q_1 + r_1(Q_1 - X)^+ + \delta \pi_2(X_1^n, X_2^n)] \), with \( p_1(\delta_a) = r_1(\delta_a) \), \( X_1^n = (X - Q_1)^+ \), and \( X_2^n = X \wedge Q_1 \).

Let \( \Re^* (\delta_a) := (p_1(\delta_a), Q_1(\delta_a), \xi_r(\delta_a), r_1(\delta_a), a_1(\delta_a), p_2^{n*}(\delta_a), p_2^*(\delta_a)) \) be an RE equilibrium given the customer forward-looking behavior intensity \( \delta_a \), which is formally defined in Definition 1 in the
Appendix. In a nutshell, \( \mathcal{R}_n^*(\delta_c) \) satisfies that all decisions are optimal given the beliefs, whereas all beliefs are consistent with the actual outcome. To characterize the RE equilibrium, we define an auxiliary variable \( m_1^*(\delta_c) := \mu + \delta(\beta^* - \beta_n^* + \delta_c(\sigma^*_r - \sigma_c^*)) \), where \( \sigma_r^* := \mathbb{E}((k + \alpha)V - p_r^*)^+ \) is the expected surplus of a repeat customer and \( \sigma_c^* := \mathbb{E}((1 + \alpha)V - p_c^*)^+ \) is the expected surplus of a new customer, respectively, in period 2. As will be clear in our subsequent analysis, \( m_1^*(\delta_c) \) is the first-period effective marginal revenue with customer discount factor \( \delta_c \). Based on Lemma 1, we can characterize the RE equilibrium market outcome in the following theorem.

**Theorem 1.** For any customer discount factor \( \delta_c \), there exists a unique RE equilibrium \( \mathcal{R}_n^*(\delta_c) \) exists with (a) \( p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_c^*) \); (b) \( Q_1^*(\delta_c) = F^{-1}(\frac{e_1(1 - \gamma)}{m_1^*(\delta_c) - r_1}) \); and (c) the expected total profit of the firm, \( \Pi_1^*(\delta_c) = (m_1^*(\delta_c) - r_1)\mathbb{E}(X \wedge Q_1^*(\delta_c)) - (c_1 - r_1)Q_1^*(\delta_c) + \delta \beta_c^*\mathbb{E}(X) \).

Theorem 1 shows that, for any customer discount factor \( \delta_c \), the equilibrium first-period price is the expected valuation of the first-generation product (i.e., \( \mu \)), plus the (discounted) expected surplus difference between a repeat customer and a new one in period 2 (i.e., \( \delta_c(\sigma_r^* - \sigma_c^*) \)). The equilibrium first-period production quantity, on the other hand, can be determined by the solution of a corresponding newsvendor problem.

### 3.3. Benchmark Model without Trade-in Remanufacturing

In the next two sections, we analyze the impact of customer purchasing behavior on the value of trade-in remanufacturing, both from the firm’s and from environmental perspectives. To facilitate our comparison, we introduce a benchmark model where the firm does not adopt trade-in remanufacturing. As a consequence, the firm charges the same price for all customers and recycles no used products for remanufacturing in period 2. We call this the No Trade-in Remanufacturing (NTR) model, which is denoted by the superscript “u” hereafter. We use \( p_2^u(X_2^u, X_2^r) \) to denote the equilibrium second-period pricing strategy of the firm in the NTR model, which does not depend on customer purchasing behavior. The characterization of \( p_2^u(\cdot, \cdot) \) is given in Lemma 3 in the Appendix. As in the base model, customers form beliefs about the product availability and the second-period price, and time their purchases. The firm, on the other hand, forms a belief about customers’ expected first-period willingness-to-pay, and bases his (first-period) price and production decisions on this belief. Given any customer discount factor \( \delta_c \), the RE equilibrium in the NTR model, \( \mathcal{R}_n^u(\delta_c) \), is formalized in Definition 2 in the Appendix. By the same argument in the proof of Theorem 1, we can show that a unique RE equilibrium exists with any customer forward-looking behavior intensity \( \delta_c \in [0, \delta] \) in the NTR model (Theorem 12 in the Appendix). Let \( (p_1^{*u}(\delta_c), Q_1^{*u}(\delta_c)) \) denote the equilibrium first-period price and production decisions of the firm with customer discount factor \( \delta_c \) in the NTR model (see Definition 2). Accordingly, the associated equilibrium total firm profit is denoted by \( \Pi_1^{*u}(\delta_c) \) with customer discount factor \( \delta_c \).
To conclude this section, we define a few notations that will prove useful throughout our analysis. Given the first-period production quantity $Q_1$, let $\sigma_n^u(Q_1) := E((k + \alpha)V - p_n^u(X_2^n, X_1^n)^+)$ and $\sigma_n^u(Q_1) := E((1 + \alpha)V - p_n^u(X_2^n, X_1^n)^+)$ denote the expected second-period surpluses for repeat and new customers in the NTR model. Clearly, $\sigma_n^u(\cdot)$ and $\sigma_n^* (\cdot)$ are the counterparts of $\sigma_n^*$ and $\sigma_n^*$ in the NTR model. If $Q_1^u(\delta_c) = 0$ or $Q_1^u(\delta_c) = 0$, the problem is reduced to an uninteresting one with no repeat customer on the market in period 2. In this case, neither customer purchasing behavior nor the adoption of trade-in remanufacturing matters. Thus, without loss of generality, we assume $Q_1^u(\cdot) > 0$ and $Q_1^u(\cdot) > 0$ for the rest of our paper.

4. Value of Trade-in Remanufacturing for the Firm

This section investigates the value of trade-in remanufacturing from the firm’s perspective. To begin with, we perform a sensitivity analysis with respect to the customer discount factor $\delta_c$ to unveil insights on the role of customer forward-looking behavior intensity.

**Theorem 2.** (a) Under trade-in remanufacturing, we have (i) $p_1^*(\delta_c)$ is strictly increasing (decreasing) in $\delta_c$ if $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$), (ii) $Q_1^u(\delta_c)$ is strictly increasing (decreasing) in $\delta_c$ if $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$), (iii) $\Pi_1^*(\delta_c)$ is strictly increasing (decreasing) in $\delta_c$ if $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$), and (iv) there exists a threshold $\bar{r} \geq \frac{1 - \epsilon}{1 + \alpha} c_2$, such that $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$) if and only if $r_2 > \bar{r}$ ($r_2 < \bar{r}$).

(b) Under no trade-in remanufacturing (i.e., the NTR model), we have (i) $p_1^{u*}(\delta_c)$ is strictly decreasing in $\delta_c$ on $[0, \delta_0]$ for some $\delta_0 > 0$, (ii) $Q_1^{u*}(\delta_c)$ is strictly decreasing in $\delta_c$, (iii) $\Pi_1^{u*}(\delta_c)$ is strictly decreasing in $\delta_c$, and (iv) $\sigma_1^{u*}(Q_1) < \sigma_n^{u*}(Q_1)$ for all $Q_1 \geq 0$.

Theorem 2 characterizes the condition under which the firm can earn a higher profit with more intensive forward-looking customer behavior: (a) Trade-in remanufacturing is adopted, and (b) Remanufacturing can deliver a sufficiently high economic value to the firm. The firm will be better off with more intensive strategic customer behavior if the expected second-period surplus of a repeat customer dominates that of a new customer. In this case, customers are willing to pay a higher first-period price if they are more strategic, resulting in a higher total profit for the firm. If remanufacturing can generate a revenue so high (i.e., $r_2 > \bar{r}$) that the trade-in rebate/price discount for repeat customers in period 2 ensures a higher expected surplus of repeat customers than that of new customers (i.e., $\sigma_1^* > \sigma_n^*$), then the firm will charge a higher price, produce more, and thus earn a higher profit with more strategic customers (i.e., with a higher $\delta_c$). We emphasize that both the trade-in option and the revenue-generating effect of remanufacturing are essential for the firm to benefit from more intensive strategic customer behavior: The former offers early purchase rewards to repeat customers, which can be well anticipated if customers are sufficiently strategic, whereas the latter brings in the additional benefit that guarantees a discount so deep that, in
expectation, a repeat customer enjoys a higher surplus than a new customer in period 2. If the firm adopts no trade-in remanufacturing or remanufacturing cannot deliver a high enough economic value, however, the expected repeat customer surplus will always be lower than the expected new customer surplus (i.e., \( \sigma^r_2 < \sigma^u_2 \) if \( r_2 < \bar{r} \); and \( \sigma^r(Q_1) < \sigma^u(Q_1) \) for all \( Q_1 \)). In these cases, the more intensive the strategic customer behavior, the more reluctant customers are to make an immediate purchase (in period 1), and the lower the production quantity, sales price, and firm profit will be. Therefore, our model delivers a new insight to the literature that trade-in remanufacturing can help exploit the strategic purchasing behavior of customers, so that the firm may enjoy a higher profit under more intensive forward-looking customer behavior. This also complements the findings in the existing literature that demonstrate strategic customer behavior may benefit the seller in some retail and airline settings (e.g., Su 2007, Li et al. 2014).

From Theorem 2, we can derive some actionable insights for practitioners. In a market with frequent new product introductions and intensive strategic customer behavior (e.g., the electronics market, see Song and Chintagunta 2003, Plambeck and Wang 2009), the firm should make the product more remanufacturable (i.e., increase \( r_2 \)) so that the intensive forward-looking customer behavior can be well leveraged by the trade-in remanufacturing program. On the other hand, if the firm does adopt trade-in remanufacturing and the product is sufficiently remanufacturable, it would be a good idea to induce more intensive strategic customer behavior by extensively advertising the trade-in opportunities in the market.

It can be shown that the equilibrium second-period price without trade-in remanufacturing, \( p^*_2(\cdot, \cdot) \), is bounded from below by the equilibrium second-period trade-in price \( p^*_2^\circ \), and from above by the equilibrium second-period price for new customers \( p^*_2^u \) (see Lemma 3 in the Appendix). Hence, trade-in remanufacturing improves the expected utility of strategic customers to make a purchase in the first period (i.e., \( \delta_c \sigma^*_r > \delta_c \sigma^u_r(Q_1) \) for all \( Q_1 \geq 0 \), and, meanwhile, decreases the benefit of waiting (i.e., \( \delta_c \sigma^*_n < \delta_c \sigma^u_n(Q_1) \) for all \( Q_1 \geq 0 \)). This implies that trade-in remanufacturing makes strategic customers more willing to purchase immediately than to wait until period 2. Our next result leverages this idea to characterize how strategic customer behavior intensity influences the value of trade-in remanufacturing to the firm. The following theorem compares the equilibrium prices and profits in the NTR model and those in the base model under different intensities of forward-looking purchasing behavior.

**Theorem 3.** (a) \( \sigma^*_r \geq \sigma^u_r(Q_1) \) and \( \sigma^n_2 < \sigma^n_2(Q_1) \) for all \( Q_1 > 0 \). (b) \( p^*_1(\delta_c) > p^{u_1}(\delta_c) \) for all \( \delta_c > 0 \). Moreover, \( p^*_1(\delta_c) - p^{u_1}(\delta_c) > \delta_c(\sigma^*_r - \mathbb{E}[(k + \alpha)V - p^*_2(\delta_c)]) \) for \( \delta_c > 0 \). (c) \( \Pi^*_1(\delta_c) > \Pi^{u_1}(\delta_c) \) for all \( \delta_c \in [0, \delta] \).
Theorem 3 shows that, for every customer discount factor \( \delta_c \), the firm charges a higher price and earns a higher profit under trade-in remanufacturing. We also establish a lower bound for the difference between the equilibrium first-period price in the base model and that in the NTR model, which is strictly increasing in the customer discount factor \( \delta_c \). In our extensive numerical experiments, the price difference \( p_1^*(\delta_c) - p_1^{**}(\delta_c) \) is always strictly increasing in the customer discount factor \( \delta_c \). As customers become more strategic, trade-in remanufacturing enables the firm to better exploit the more intensive forward-looking behavior.

We observe from Lemma 3 and Theorem 3 that there are three beneficial effects of trade-in remanufacturing that may improve firm profit: (a) the revenue-generating effect of remanufacturing, i.e., remanufacturing can recover the residual value of used products, (b) the price-discrimination effect of trade-in rebates, i.e., the differentiated prices for new and repeat customers help the firm exploit customer segmentation in period 2, and (c) the early-purchase inducing effect of trade-in rebates, i.e., the price discount for repeat customers attracts strategic customers to purchase in period 1 by offering them early-purchase rewards. The first two effects benefit the firm regardless of customer purchasing behavior, whereas the third effect improves its profit only if customers are strategic. In the following, we conduct extensive numerical experiments to quantify the third effect with different customer discount factor \( \delta_c \), and deliver insights on how the intensity of strategic customer behavior influences the value of trade-in remanufacturing to the firm.

The design of the numerical study is as follows. Let the customer valuation \( V \) follow a uniform distribution on \([0, 1]\) (\( \mu = \mathbb{E}(V) = 0.5 \)). The discount factor of the firm is \( \delta = 0.95 \), the unit environmental impact of the first-generation product is \( \kappa_1 = 1 \), and the unit environmental impact of the second-generation product is \( \kappa_2 = 0.75 \). To focus on the impact of customer behaviors, we set \( r_1 = r_2 = 0 \) (i.e., there is no revenue-generating effect associated with remanufacturing), and the unit environmental benefits of recycling/remanufacturing to be \( \iota_1 = 0 \) and \( \iota_2 = 0.3 \) (these two values will be useful when studying the environmental impact in Section 4.2). The unit production cost of the first-generation product is \( c_1 \in \{0.05, 0.1, 0.15, 0.2, 0.25\} \). The innovation level of the second-generation product is \( \alpha \in \{0, 0.05, 0.1, 0.15, 0.2\} \), and the unit production cost of the second-generation product is \( c_2 = 0.25(1 + \alpha) \in \{0.25, 0.2625, 0.275, 0.2875, 0.3\} \). We consider the depreciation factor \( k \in \{0.3, 0.4, 0.5, 0.6, 0.7\} \). The demand \( X \) follows a gamma distribution with mean 100 and coefficient of variation \( CV(X) \) taking values from the set \( \{0.5, 0.6, 0.7, 0.8, 0.9\} \). Thus, we have a total of 625 parameter combinations that cover a wide range of reasonable problem scenarios. The above problem scenarios form a subset of the extensive experiments we have conducted. Since the numerical findings are very robust, we will only present the numerical study described above for concision.
We examine three different customer discount factors corresponding to three different customer behaviors: (a) \( \delta_c = 0.95 \), which refers to the case with fully strategic customers; (b) \( \delta_c = 0.475 \), which refers to the case with partially strategic customers; and (c) \( \delta_c = 0 \), which refers to the case with fully myopic customers. We calculate the expected profits both in the base model (i.e., \( \Pi^*_1(\delta_c) \)) and in the NTR model (\( \Pi^*_2(\delta_c) \)) for each parameter combination. The metric of interest is \( \gamma(\delta_c) = (\Pi^*_1(\delta_c) - \Pi^*_2(\delta_c)) / \Pi^*_2(\delta_c) \times 100\% \), which measures the relative profit improvement of trade-in remanufacturing with customer discount factor \( \delta_c \). We evaluate \( \gamma(\delta_c) \) under the 625 parameter combinations with customer discount factors \( \delta_c \in \{0.95, 0.475, 0\} \). Under each problem instance, \( \gamma(0.95) \) and \( \gamma(0.475) \) are significantly higher than \( \gamma(0) \). Moreover, for most (more than 97%) of the problem instances, \( \gamma(0.95) \) is significantly higher than \( \gamma(0.475) \). We give the summary statistics of \( \gamma(\delta_c) \) in Table 1.

| \( \delta_c \) | Min 25th percentile Median 75th percentile Max Mean Standard deviation |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.95            | 5.8             | 19.9            | 28.3            | 40.1            | 61.6            | 30.2            | 13.1            |
| 0.475           | 6.6             | 11.4            | 16.6            | 22.5            | 30.5            | 17.1            | 6.1             |
| 0            | 0.008           | 1.0             | 2.5             | 4.6             | 11.7            | 3.1             | 2.5             |

Table 1  Summary Statistics of \( \gamma(\delta_c) \) (%)

Our numerical results deliver an important message on the value of trade-in remanufacturing to the firm: Trade-in remanufacturing has a much higher value to the firm with more strategic customers (for most of the problem instances, \( \gamma(\cdot) \) is significantly higher when \( \delta_c \) is larger). Recall that, with forward-looking customer behavior, trade-in remanufacturing has the additional benefit of inducing early purchases on top of the revenue-generating effect and the price-discrimination effect, both of which are independent of customer purchasing behavior. Therefore, our numerical results indicate that the early-purchase inducing effect of trade-in remanufacturing to exploit strategic customer behavior will be magnified if customers are more strategic. In particular, as long as customers are moderately strategic (i.e., \( \delta_c \) is moderately large), the value of trade-in remanufacturing to a firm mainly comes from its early-purchase inducing effect to exploit strategic customer behavior, rather than from the revenue-generating effect of remanufacturing or the price-discrimination effect of trade-in rebates that exploits customer segmentation.

5. Values of Trade-in Remanufacturing for the Environment

We proceed to examine the environmental value of trade-in remanufacturing under different customer purchasing behaviors. To begin with, we study how strategic customer behavior intensity impacts the environment under trade-in remanufacturing. In equilibrium, the total environmental impact should be the difference between the total environmental impact of production/disposal and the total environmental benefit of remanufacturing. Hence, the equilibrium environmental impact with trade-in remanufacturing is \( I^*_1(\delta_c) = E\{\kappa_1 Q^*_1(\delta_c) + \delta \kappa_2 (Q^*_2(X^*_2, X^*_2) + \)


Q_2^1(X_2^{2*}, X_2^{1*}) = \{Q_1^1(\delta_c) - X\}^+ - \delta_2 Q_2^1(X_2^{2*}, X_2^{1*})\}, \text{ where } X_2^{2*} = (X - Q_1^1(\delta_c))^+ \text{ and } X_2^{1*} = X \wedge Q_1^1(\delta_c).

**Theorem 4.** Suppose \(\kappa_1 \geq \delta \kappa_2 \bar{G}(\frac{p_2^*}{1 + \alpha})\). Then \(I^*_c(\delta_c)\) is strictly increasing (decreasing) in \(\delta_c\) if \(\sigma^*_c > \sigma^*_n (\sigma^*_c < \sigma^*_n)\).

Theorem 4 shows that, as long as the environmental impact of the first-generation product is not too low (i.e., \(\kappa_1 \geq \delta \kappa_2 \bar{G}(\frac{p_2^*}{1 + \alpha})\)), the negative environmental impact gets bigger (smaller) under more intensive strategic customer behavior if the expected second-period surplus of a repeat customer is higher (lower) than that of a new customer. Comparing Theorem 4 with Theorem 2(a) shows that the intensity of forward-looking customer behavior has exactly the opposite impact upon the environment to that on firm profit. This is because, when the firm earns a higher profit, the first-period production quantity also increases, thus leading to a worsened environment if the environmental impact of the first-generation product is reasonably large. The condition that \(\kappa_1 \geq \delta \kappa_2 \bar{G}(\frac{p_2^*}{1 + \alpha})\) is not restrictive in reality. In particular, it applies to the case where the newer generation product dominates the older generation in terms of environmental sustainability, i.e., \(\kappa_1 \geq \kappa_2 > \delta \kappa_2 \bar{G}(\frac{p_2^*}{1 + \alpha})\).

We now characterize how trade-in remanufacturing influences the first-period production quantity of the firm under different intensities of strategic customer behavior.

**Theorem 5.** (a) There exists a threshold \(\tilde{\delta}_q < \delta\), such that \(Q_1^1(\delta_c) > Q_1^{2*}(\delta_c)\) for all \(\delta_c \in (\tilde{\delta}_q, \delta]\). (b) Assume that \(c_1\) is sufficiently small (formally specified in the Appendix). There exists a threshold \(\tilde{\delta}_q (0 < \tilde{\delta}_q \leq \tilde{\delta}_q)\), such that \(Q_1^1(\delta_c) < Q_1^{2*}(\delta_c)\) for all \(\delta_c \in (0, \tilde{\delta}_q)\). (c) In particular, if \(\sigma^*_c > \sigma^*_n\), we have \(\tilde{\delta}_q = \tilde{\delta}_q\), i.e., \(Q_1^1(\delta_c) > Q_1^{2*}(\delta_c)\) for all \(\delta_c > \tilde{\delta}_q = \tilde{\delta}_q\) and \(Q_1^1(\delta_c) < Q_1^{2*}(\delta_c)\) for all \(\delta_c < \tilde{\delta}_q = \tilde{\delta}_q\).

Theorem 5 reveals contrasting effects of trade-in remanufacturing under different intensities of strategic customer behavior: If customers are highly strategic (i.e., \(\delta_c > \tilde{\delta}_q\)), trade-in remanufacturing increases the first-period production quantity of the firm \((Q_1^1(\delta_c) > Q_1^{2*}(\delta_c))\), whereas it may prompt the firm to produce less \((Q_1^1(\delta_c) < Q_1^{2*}(\delta_c))\) when the strategic customer behavior is not too intensive \((\delta_c < \tilde{\delta}_q)\). In particular, if the expected surplus of a repeat customer dominates that of a new customer in period 2 (i.e., \(\sigma^*_c > \sigma^*_n\)), we have a unique threshold such that trade-in remanufacturing increases (decreases) the equilibrium first-period production quantity whenever the customer discount factor \(\delta_c\) is above (below) this threshold. With highly strategic customers \((\delta_c > \tilde{\delta}_q)\), the early-purchase inducing effect of trade-in remanufacturing gives rise to a higher willingness-to-pay and, thus, induces a higher first-period production quantity. As a result, if customers are sufficiently forward-looking, the firm produces more in period 1 under trade-in remanufacturing.
customers are not very strategic (δc < δ̄), the early-purchase inducing effect is not significant, but
the price-discrimination effect of trade-in remanufacturing enables the firm to earn a higher aver-
age profit from new customers in period 2, which in turn drives the firm to lower the first-period
production quantity, thus enlarging the (potential) second-period market size of new customers.

By Theorem 5, trade-in remanufacturing impacts the equilibrium first-period production quan-
tity differently under different customer purchasing behaviors. How does customer purchasing
behavior affect the value of trade-in remanufacturing to the environment? The answer is given in
the next theorem, where we define \( I_{e}^{u}(δc) \) as the equilibrium total environmental impact in the
NTR model.

**Theorem 6.** Suppose \( κ_1 ≥ δ\kappa_2 \hat{G} \left( \frac{\nu^{*}}{1+α} \right) \).

(a) If \( δc > δ_q \), there exists a threshold \( \bar{t}_c > 0 \), such that \( I_{e}^{u}(δc) > I_{e}^{u*}(δc) \) if \( t_2 < \bar{t}_c \).

(b) If \( c_1 \) is sufficiently small and \( δc < δ_q \), there exists a threshold \( \bar{t}_c < κ_2 \), such that \( I_{e}^{u*}(δc) > I_{e}^{u}(δc) \) if \( t_2 > \bar{t}_c \).

When customers are highly strategic (\( δc > δ_q \)), trade-in remanufacturing encourages them to
recycle the used first-generation products more frequently, so they also purchase the product more
frequently. In this scenario, trade-in remanufacturing leads to a worsened outcome for the envi-
ronment if the unit environmental benefit of remanufacturing is not high enough to justify the
early-production inducing effect (i.e., \( t_2 < \bar{t}_c \)). When customers are not that forward-looking (\( δc < δ_q \)) and the unit production cost is sufficiently low, trade-in remanufacturing motivates the firm
to produce less in period 1 (see Theorem 5(b)). Hence, trade-in remanufacturing helps improve
the environment as long as the unit environmental benefit of remanufacturing is not too low (i.e.,
\( t_2 > \bar{t}_c \)). Theorem 6 reveals the significant impact of customer purchasing behavior on the environ-
mental value of trade-in remanufacturing. With highly strategic customers, the adoption of trade-in
remanufacturing is likely to be detrimental to the environment, whereas, with more myopic cus-
tomers, adopting trade-in remanufacturing may benefit both the firm and the environment. Some
papers in the literature (e.g., Debo et al. 2005, Galbreth et al. 2013, Gu et al. 2015) have also noted
that remanufacturing may increase the production quantity and thus the environmental impact.
Our analysis, however, demonstrates that the environmental value of trade-in remanufacturing
depends critically on the customer purchasing behavior.

We now numerically illustrate the environmental value of trade-in remanufacturing. We employ
the same numerical setup as in Section 4. Recall that \( I_{e}^{u}(δc) \) (\( I_{e}^{u*}(δc) \)) is the expected total envi-
ronmental impact for the scenario with customer discount factor \( δc \) in the base (NTR) model.
We are interested in the following metric: \( \eta(δc) := (I_{e}^{u}(δc) - I_{e}^{u*}(δc))/I_{e}^{u*}(δc) > 100\% \), referring to
the relative change in environmental impact after adopting trade-in remanufacturing. If \( \eta(δc) > 0 \),
trade-in remanufacturing increases the total negative impact and is, thus, detrimental to the environment. Otherwise, $\eta(\delta_c) \leq 0$, trade-in remanufacturing decreases the total environmental impact and improves the environment.

For $\delta_c \in \{0.95, 0.475, 0\}$, we evaluate $\eta(\delta_c)$ under the 625 parameter combinations detailed in Section 4 and obtain the following findings: (i) Under each parameter combination, $\eta(0.95)$ is significantly higher than $\eta(0.475)$, which is significantly higher than $\eta(0)$; and (ii) it exhibits a clear pattern that, as $\delta_c$ gets larger, $\eta(\delta_c)$ is more likely to become positive. Specifically, $\eta(0.95) < 0$ (i.e., trade-in remanufacturing benefits the environment with fully strategic customers) for 10 out of the 625 (i.e., 1.6%) problem instances, $\eta(0.475) < 0$ (i.e., trade-in remanufacturing benefits the environment with semi-strategic customers) for 194 out of the 625 (i.e., 31.0%) problem instances, and $\eta(0) < 0$ (i.e., trade-in remanufacturing benefits the environment with fully myopic customers) for 584 out of the 625 (i.e., 93.4%) problem instances. Table 2 summarizes the statistics of $\eta(\delta_c)$ for different values of $\delta_c$.

<table>
<thead>
<tr>
<th>$\delta_c$</th>
<th>Min</th>
<th>25th percentile</th>
<th>Median</th>
<th>75th percentile</th>
<th>Max</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95</td>
<td>-1.2</td>
<td>10.0</td>
<td>37.8</td>
<td>87.1</td>
<td>117.8</td>
<td>49.2</td>
<td>41.4</td>
</tr>
<tr>
<td>0.475</td>
<td>-3.9</td>
<td>-0.8</td>
<td>3.0</td>
<td>26.8</td>
<td>88.7</td>
<td>13.1</td>
<td>18.6</td>
</tr>
<tr>
<td>0</td>
<td>-10.2</td>
<td>-6.9</td>
<td>-5.5</td>
<td>-3.8</td>
<td>4.5</td>
<td>-5.0</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Table 2 Summary Statistics of $\eta(\delta_c)$ (%)

Table 2 confirms that the environmental value of trade-in remanufacturing is sensitive to customer purchasing behavior. It leads to much higher total environmental impact with more intensive strategic customer behavior ($\eta(0.95)$ is significantly higher than $\eta(0.475)$, which is significantly higher than $\eta(0)$). Though beneficial to the firm (see Table 1), the early-purchase inducing effect of trade-in remanufacturing gives rise to much higher production quantities under more intensive forward-looking customer behavior, thus dominating the recycling effect of remanufacturing and leading to a much worse outcome from an environmental perspective.

The above results suggest that customer purchasing behavior has opposing effects on the value of trade-in remanufacturing: More strategic customer behavior makes this strategy more attractive to the firm, but less desirable to the environment. In particular, trade-in remanufacturing may create a tension between firm profitability and environmental sustainability when customers are highly strategic, but benefits both the firm and the environment with myopic customers. Under high forward-looking customer behavior intensity ($\delta_c \approx \delta$), the early-purchase inducing effect dominates the environmental benefit of remanufacturing. In this case, the firm benefits from trade-in remanufacturing significantly, but the environment suffers from this strategy a lot. Thanks to the economic and environmental benefits of remanufacturing and the price-discrimination effect of
trade-in rebates, both the firm and the environment, however, would benefit from the adoption of trade-in remanufacturing when customers are nearly myopic ($\delta_c \approx 0$).

Although an increased production quantity means more pressure on the environment, it also increases the consumption level of the product. To conclude this section, we explore the impact of trade-in remanufacturing impacts on total customer surplus under different customer purchasing behaviors. For any customer discount factor $\delta_c$, we use $S^*_c(\delta_c)$ and $S^{u*}_c(\delta_c)$ to denote the equilibrium total customer surplus in the base and NTR models, respectively.

**Theorem 7.** (a) In the base model, we have $S^*_c(\delta_c) = \mathbb{E}[(a^*_n(\delta_c)(\delta - \delta_c)(\sigma^*_n - \sigma^*_{n*}) + \delta\sigma^*_{n*}(Q^*_{n*}(\delta_c)))X]$.

(b) In the NTR model, we have $S^{u*}_c(\delta_c) = \mathbb{E}[(a^*_n(\delta_c)(\delta - \delta_c)(\sigma^*_{u*}(Q^*_{u*}(\delta_c)) - \sigma^*_{n*}(Q^*_{n*}(\delta_c))) + \delta\sigma^*_{n*}(Q^*_{n*}(\delta_c)))X]$.

(c) There exists a threshold $\tilde{\delta}_c < \delta$, such that $S^*_c(\delta_c) < S^{u*}_c(\delta_c)$ for any $\delta_c \in (\tilde{\delta}_c, \delta]$.

Theorem 7 compares the total customer surpluses in the base model and the NTR model. In particular, Theorem 7(c) demonstrates that, when customers are sufficiently strategic (i.e., $\delta_c > \tilde{\delta}_c$), they are worse off with the adoption of trade-in remanufacturing ($S^*_c(\delta_c) < S^{u*}_c(\delta_c)$). Strategic customers can well perceive the potential price discounts offered by trade-in rebates, and adjust their first-period purchasing decisions accordingly. Therefore, if customers are sufficiently strategic, the firm can employ trade-in remanufacturing to exploit their forward-looking purchasing behavior, and extract their second-period surpluses when complemented with a wisely-designed first-period pricing and production strategy. Interestingly, Theorem 7(c) contrasts with our intuition that higher production quantities (see Theorem 5(a)) would lead to higher consumptions and thus a higher total customer surplus. Instead, our analysis shows that, under intensive strategic customer behavior, trade-in remanufacturing increases production quantities (and, thus, a worse environment) without improving customer surplus. Further, the earlier numerical study indicates that the social welfare (i.e., firm profit plus customer surplus less environmental impact) may decrease under trade-in remanufacturing as well.

To summarize, customer purchasing behavior plays an important role in the economic and environmental values of trade-in remanufacturing. In a market with not-so-strategic customers, trade-in remanufacturing benefits both the firm and the environment. With highly strategic customers, however, trade-in remanufacturing would be even more beneficial to the firm; meanwhile it seriously hurts the environment, decreases customer surplus, and possibly lowers social welfare. In this case, the value of trade-in remanufacturing is mainly about facilitating the firm to exploit the forward-looking behavior of strategic customers, which is much more significant than the widely recognized revenue-generating and environmental benefits of remanufacturing. Therefore, when making decisions related to trade-in remanufacturing, firms and policy-makers should keep in mind the customer purchasing behavior in the focal market.
6. Social Optimum and Government Intervention

With an increasing societal awareness of sustainability, how to regulate a market with environmental concerns has attracted more attention from government (e.g., Chen 2015, The Recycler 2015). As shown in Section 5, adopting trade-in remanufacturing may create a tension between firm profitability and environmental sustainability under intensive strategic customer behavior. In this section, we analyze how a policy-maker (e.g., the government) can design the public policy to resolve this tension and maximize the social welfare under different customer purchasing behaviors.

We first characterize the socially optimal market outcome by assuming that the government can set the prices and production levels in both periods, with an objective to maximize the social welfare. Let $W_s$ denote the social welfare, which is defined by the expected profit of the firm $\Pi_f$, plus the expected customer surplus $S_c$, net the expected environmental impact $I_e$, i.e.,

$$W_s = \Pi_f + S_c - I_e.$$

By backward induction, we start with the second-period pricing and production problem. As in the base model, customers’ second-period purchasing decisions are irrelevant to how strategic they are. For any given realized market size in period 2 ($X_2^n, X_2^r$), we use $(p_s^n(X_2^n, X_2^r), p_s^r(X_2^n, X_2^r))$ to denote the equilibrium second-period pricing strategy that maximizes the social welfare. Correspondingly, we denote $w_2(X_2^n, X_2^r)$ as the equilibrium second-period social welfare.

**Lemma 2.** (a) For any $(X_2^n, X_2^r)$, $p_s^n(X_2^n, X_2^r) \equiv p_s^{n*}$ and $p_s^r(X_2^n, X_2^r) \equiv p_s^{r*}$, where $p_s^{n*} = c_2 + \kappa_2$ and $p_s^{r*} = c_2 - r_2 + \kappa_2 - \iota_2$. Hence, $p_s^{n*} > p_s^{r*}$ if and only if $r_2 > 0$ or $\iota_2 > 0$.

(b) For $(X_2^n, X_2^r)$, $w_2(X_2^n, X_2^r) = \sigma_n^{**}X_2^n + \sigma_r^{**}X_2^r$, where $\sigma_n^{**} = E[(1 + \alpha)V - p_s^{n*}]^+$ and $\sigma_r^{**} = E[(k + \alpha)V - p_s^{r*}]^+$.

Lemma 2 implies that, regardless of customer purchasing behavior, the socially optimal second-period pricing strategy takes the form that the prices for new and repeat customers are equal to the respective net total unit production and environmental cost (i.e., $p_s^{n*} = c_2 + \kappa_2$ and $p_s^{r*} = c_2 - r_2 + \kappa_2 - \iota_2$). Moreover, the equilibrium social welfare is linear in the realized market size $(X_2^n, X_2^r)$. Here, the linear coefficient $\sigma_n^{**}$ ($\sigma_r^{**}$) is the equilibrium expected surplus of a new (repeat) customer, which is also the equilibrium unit social welfare of selling to new (repeat) customers in period 2.

In period 1, the government and customers base their decisions on rational beliefs. Let $(p_1^*(\delta_1), Q_1^*(\delta_1))$ denote the equilibrium first-period price and production quantity with customer discount factor $\delta_1$. As in our previous analysis, we introduce the first-period effective marginal welfare, $m_1^* := \mu + \delta(\sigma_r^{**} - \sigma_n^{*})$, which measures the marginal social welfare to produce in period 1. The following theorem characterizes the social welfare maximizing equilibrium outcome.
THEOREM 8. (a) With customer discount factor $\delta_c$, we have (i) $p_1^*(\delta_c) = \mu + \delta_c(\sigma^*_r - \sigma^*_n)$; (ii) $Q_1^*(\delta_c) = F^{-1}\left(\frac{c_1 + \kappa_1 r_1 - r_1 - u_1}{m_1 - r_1 - \delta_1}\right)$; and (iii) the equilibrium expected social welfare is $W_s^*(\delta_c) = (m_1^* - r_1 - \delta_1)E[X \wedge Q_1^*(\delta_c)] - (c_1 + \kappa_1 r_1 - r_1 - u_1)Q_1^*(\delta_c) + \delta \sigma^*_n E[X]$.

(b) We have (i) $p_1^*(\delta_c)$ is strictly increasing (decreasing) in $\delta_c$ if and only if $\sigma^*_r > \sigma^*_n$ ($\sigma^*_r < \sigma^*_n$). (ii) There exists a threshold $\bar{\delta}_2 > 0$, such that $\sigma^*_r > \sigma^*_n$ ($\sigma^*_r < \sigma^*_n$) if and only if $r_2 + \delta_2 > \bar{\delta}_2$ ($r_2 + \delta_2 < \bar{\delta}_2$). (iii) $Q_1^*(\delta_c)$ and $W_s^*(\delta_c)$ are independent of $\delta_c$.

Since the social planner needs to balance firm profit, customer surplus, and environmental impact, whereas the firm maximizes its own profit only, the social-welfare-maximizing equilibrium outcome may be quite different from the profit-maximizing one, as shown by comparing Theorem 8 with Theorem 1. In particular, if the unit environmental impacts, $\kappa_1$ and $\kappa_2$, are large, the social planner will set lower production quantities than the firm will do to limit the negative environmental impact. Theorem 8(b) characterizes how different intensities of strategic customer behavior influence the social-welfare-maximizing RE equilibrium outcome. Observe that the marginal welfare $m_1^*$ is independent of the customer discount factor $\delta_c$; so are the equilibrium social welfare $W_s^*(\delta_c)$ and the equilibrium first-period production quantity $Q_1^*(\delta_c)$. Thus, the social-welfare-maximizing equilibrium is essentially independent of the customer discount factor $\delta_c$. This is in sharp contrast to the equilibrium outcome in the base model, which depends critically on the customer purchasing behavior (Theorem 2). Similar to the profit-maximizing RE equilibrium, the equilibrium social-welfare-maximizing first-period price $p_1^*(\delta_c)$ depends on the forward-looking customer behavior intensity $\delta_c$.

We proceed to analyze how the government, whose objective is to maximize the expected social welfare $W_s$, could induce the firm, whose objective is to maximize his expected profit $\Pi_f$, to set the socially optimal prices and production quantities under different customer purchasing behaviors. A natural candidate policy is to subsidize the firm or customers for the remanufactured products (see, e.g., Mitra and Webster 2008, Chen 2015) to promote the environmentally friendly remanufacturing business practice. To model this subsidization policy, we assume that the government offers the firm a per-unit subsidy $s_r$ for remanufacturing leftover inventory and used products. The per-unit subsidy to the firm is without loss of generality, because all results and qualitative insights in this section continue to hold with the per-unit subsidy to customers, and the proportional subsidy 1 to either the firm or customers. For expositional ease, we take the approach of per-unit subsidy to the firm.

We first study the impact of such government subsidization policy for remanufactured products in the following theorem.

THEOREM 9. For any $(X_2^s, X_2^t)$ and any $\delta_c \in [0, \delta]$, we have (a) $p_2^*$ is decreasing in $s_r$, and $Q_2^*(X_2^s, X_2^t)$ is increasing in $s_r$; and (b) $p_1^*(\delta_c)$, $Q_1^*(\delta_c)$, $\Pi_f^*(\delta_c)$ and $I_s^*(\delta_c)$ are increasing in $s_r$. 

One of the main goals for the government to subsidize remanufacturing is to improve the environment (e.g., Chen 2015, The Recycler 2015). Theorem 9(b), however, suggests that if the government only subsidizes for remanufactured products, the environment actually suffers from this subsidization policy ($I^*_1(\delta_c)$ is increasing in $s_r$). Moreover, this subsidization scheme will further improve the firm profit ($\Pi^*_1(\delta_c)$ is increasing in $s_r$). Thus, subsidizing remanufactured products alone actually intensifies the tension between profitability and sustainability, which is opposite to the initial purpose of government intervention. This result follows from the rationale that subsidizing remanufactured products not only promotes the adoption of remanufacturing, but also increases the production levels of the first-generation product. The environment thus suffers from the increased production levels under the subsidy for remanufacturing alone. Therefore, the government should be careful about designing the subsidization policy, because haphazard subsidization for trade-in remanufacturing may result in an undesired outcome.

Motivated by the discrepancy between the intention and outcome of the intuitive government policy to subsidize remanufacturing, we consider an alternative policy that may either subsidize or tax different product versions. We assume that government subsidies (taxes) are provided (charged) for the sales of both generation products, as well as recycling/remanufacturing leftover inventory and used products. Specifically, let $s_g := (s_1, s_2, s_r)$ denote the subsidy/tax scheme. The government offers the firm a per-unit subsidy $s_1$ for the sales of the first-generation product, a per-unit subsidy $s_2$ for the sales of the second-generation product, and a per-unit subsidy $s_r$ for remanufacturing. If $s_i < 0$ ($i = 1, 2, r$), the government taxes the sales of respective product version or the remanufacturing of leftover inventory and used products.

The proposed government subsidization/taxation policy is quite general. For instance, the aforementioned most intuitive government subsidization policy for remanufacturing alone is a special case of this general subsidy/tax scheme with $s_1 = 0, s_2 = 0, s_r > 0$. Some special forms of the proposed policy have been discussed in the literature; see, e.g., Calcott and Walls (2000), Webster and Mitra (2007), Ma et al. (2013), and Wang et al. (2014). Starting from the year of 2011, the Chinese Ministry of Finance maintained a fund for the treatment of waste electrical and electronics equipment (WEEE), to which OEMs contribute in the form of taxes (see Xie and Bai 2010, Chinese Ministry of Finance 2012). This fund is used to subsidize the recycling and remanufacturing of used electrical and electronic products. This policy is also a special case of our general subsidy/tax scheme with $s_1, s_2 < 0$ and $s_r > 0$.

**Theorem 10.** For each customer discount factor $\delta_c \in [0, \delta]$, we can show:

(a) There exists a unique linear subsidy/tax scheme $s^*_g(\delta_c) = (s^*_1(\delta_c), s^*_2(\delta_c), s^*_r(\delta_c))$, under which the social-welfare maximizing RE equilibrium outcome is achieved.
(b) \( s_1^*(\delta) \) is decreasing in \( \kappa_1 \), \( s_1^*(\delta) \) is decreasing in \( \kappa_2 \), and \( s_2^*(\delta) \) is increasing in \( \nu_2 \).

(c) There exists a threshold vector \((\hat{\kappa}_1^*(\delta), \hat{\kappa}_2^*(\delta), \hat{\nu}_1^*(\delta), \hat{\nu}_2^*(\delta))\), such that (i) \( s_1^*(\delta) > 0 \) if \( s_1^*(\delta) < 0 \) if and only if \( \kappa_1 < \hat{\kappa}_1^*(\delta) \) (\( \kappa_1 > \hat{\kappa}_1^*(\delta) \)); (ii) \( s_2^*(\delta) > 0 \) if \( s_2^*(\delta) < 0 \) if and only if \( \kappa_2 < \hat{\kappa}_2^*(\delta) \) (\( \kappa_2 > \hat{\kappa}_2^*(\delta) \)); and (iii) \( s_2^*(\delta) > 0 \) if \( s_2^*(\delta) < 0 \) if and only if \( \nu_2 > \hat{\nu}_2^*(\delta) \). (\( \nu_2 < \hat{\nu}_2^*(\delta) \)).

Theorem 10 demonstrates that, for any intensity of forward-looking customer behavior \( \delta \), the government can use a simple linear subsidy/tax scheme, \( s_2^*(\delta) \), to induce the socially optimal outcome. The linear subsidy/tax policy \( s_2 \) has great flexibilities in controlling the margin of the firm and the willingness-to-pay of customers. Hence, the government can use this incentive scheme, if well designed, to regulate the market and ensure the firm sets the socially optimal prices and production quantities with any \( \delta \). More specifically, regardless of customer purchasing behavior, the government should provide a combined subsidy/tax scheme for the sales of both product generations and the recycle of leftover inventory and used products. Since some components in \( s_2^* \) may be negative, it is possible that the government will tax the firm on some product versions to discourage their sales (e.g., the Chinese government charges the OEM for the production of electrical and electronic products and subsidizes those who recycle and remanufacture e-wastes). This phenomenon results from the government’s goal of balancing the tradeoff between firm profit, customer surplus, and environmental impact. In particular, Theorem 10(c) shows that, with any forward-looking behavior intensity, the government should subsidize for (tax on) the sales of one product version if its unit environmental impact is below (above) a threshold. Analogously, subsidies (taxes) should be provided for (charged on) remanufacturing if its unit environmental benefit is above (below) a threshold.

Based on Theorem 10, we study the impact of customer purchasing behavior upon government policy to induce the social optimum by performing the sensitivity analysis of \( s_2^*(\delta) \) with respect to \( \delta \). We are also interested in how the total government cost to regulate the market gets influenced by customer purchasing behavior. For any customer discount factor \( \delta \), we denote \( C_g^*(\delta) \) as the equilibrium expected total government cost associated with the social-welfare-maximizing subsidy/tax scheme \( s_2^*(\delta) \).

**Theorem 11.** (a) The social-welfare-maximizing subsidy/tax scheme \( s_2^*(\delta) \) satisfies: (i) \( s_1^*(\delta) \) is strictly increasing (decreasing) in \( \delta \) if and only if \( \sigma_r^{**} < \sigma_n^{**} \) (\( \sigma_r^{**} > \sigma_n^{**} \)); and (ii) \( s_2^*(\delta) \) and \( s_2^*(\delta) \) are independent of \( \delta \).

(b) The threshold vector \((\hat{\kappa}_1^*(\delta), \hat{\kappa}_2^*(\delta), \hat{\nu}_1^*(\delta), \hat{\nu}_2^*(\delta))\) satisfies: (i) \( \hat{\kappa}_1^*(\delta) \) is strictly increasing (decreasing) in \( \delta \) if and only if \( \sigma_r^{**} < \sigma_n^{**} \) (\( \sigma_r^{**} > \sigma_n^{**} \)); and (ii) \( \hat{\kappa}_2^*(\delta) \) and \( \hat{\nu}_2^*(\delta) \) are independent of \( \delta \).

(c) \( C_g^*(\delta) \) is strictly increasing (decreasing) in \( \delta \) if and only if \( \sigma_r^{**} < \sigma_n^{**} \) (\( \sigma_r^{**} > \sigma_n^{**} \)), which is equivalent to \( r_2 + \nu_2 < \hat{\nu}_2 \) (\( r_2 + \nu_2 > \hat{\nu}_2 \)).
Theorem 11 sheds light on how strategic customer behavior intensity influences the optimal government subsidization policy. We find that the optimal subsidy/tax rates for the second-generation product and remanufacturing ($s_2^r(\delta_c)$ and $s_r^*(\delta_c)$) are independent of customer purchasing behavior. The optimal subsidy/tax rate for the first-generation product $s_1^*(\delta_c)$, however, is sensitive to customer behavior. The government should provide a higher subsidy/lower tax for the sales of the first-generation product with more strategic customers ($s_1^*(\delta_c)$ is increasing in $\delta_c$) if and only if the expected second-period surplus of a new customer dominates that of a repeat one (i.e., $\sigma_1^{**} < \sigma_n^{**}$). In this case, strategic customers are more reluctant to make an immediate purchase, and the government should provide more subsidies for the sales of the first-generation product to induce sufficient early purchases. As a result, the government is also more likely to subsidize the first-period production and the total cost to regulate the market increases as customers become more strategic. On the other hand, if $\sigma_1^{**} > \sigma_n^{**}$, a repeat customer enjoys a higher expected surplus in period 2, and thus strategic customers are more willing to purchase the product immediately in period 1. In this case, the government offers fewer subsidies for the sales of the first-generation product to discourage strategic customers from overconsumption in period 1. Equivalently, the government is also more likely to tax first-period production and the total cost to regulate the market decreases with more strategic customers. Theorem 11(c) further implies that more intensive strategic customer behavior has a negative (positive) impact upon the government if the total economic and environmental value of remanufacturing is low (high), i.e., the total government cost to regulate the market $C_g(\delta_c)$ is increasing (decreasing) in $\delta_c$ if $r_2 + \tau_2 < \bar{V}_2$ ($r_2 + \tau_2 > \bar{V}_2$). The above dichotomy implies that the socially optimal subsidy/tax scheme $s_r^*(\delta_c)$ serves as a mechanism to counter the effect of strategic customer behavior, in order to induce the social optimum, which is (essentially) independent of forward-looking customer behavior intensity $\delta_c$.

In summary, to alleviate the tension between profitability and sustainability and achieve the social optimum, it suffices for the government to use an easy-to-implement linear subsidy/tax scheme for the production of both product generations and remanufacturing. In particular, the socially optimal government policy aims to provide incentives to counter strategic customer behavior and induce the social-welfare-maximizing equilibrium independent of customer purchasing behavior.

7. Conclusion

In this paper, we develop an analytical model to study how different customer purchasing behaviors influence the economic and environmental values of trade-in remanufacturing. From the firm’s perspective, we identify a new benefit of trade-in remanufacturing, i.e., it helps exploit the forward-looking behavior of strategic customers. A trade-in rebate essentially offers an early purchase reward
and thus can deliver higher additional value when customers are highly strategic. In particular, with the adoption of trade-in remanufacturing and a sufficiently high economic benefit of remanufacturing, more intensive strategic customer behavior can increase the firm profit. In this case, the expected surplus of a repeat customer dominates that of a new customer, so more strategic customers care more about this benefit and have higher incentive to purchase early, thus improving the profit.

From an environmental perspective, the value of trade-in remanufacturing depends on the intensity of strategic customer behavior. With highly strategic customers, this business practice decreases the unit environmental impact, but increases the production quantities through the early-purchase inducing effect. Overall trade-in remanufacturing may have a significant negative impact on the environment. Moreover, under intensive strategic customer behavior, adopting trade-in remanufacturing decreases the customer surplus and social welfare. Hence, for a market with highly strategic customers, caution is needed when adopting trade-in remanufacturing, because it could be detrimental to the environment and the society. With not-so-strategic customers, however, trade-in remanufacturing leads to a lower first-period production quantity and, thus, generally improves the environment. Our results indicate that customer purchasing behavior plays an important role in the value of trade-in remanufacturing. Specifically, under intensive strategic customer behavior, trade-in remanufacturing creates a tension between firm profitability and environmental sustainability; however, under low strategic customer behavior intensity, it generally benefits both the firm and the environment.

To resolve the above tension caused by trade-in remanufacturing, we also study the government policy that balances firm profit, customer surplus, and environmental impact, and induces the social optimum. A natural candidate policy is to subsidize the remanufactured products. However, we find that despite its intention to protect the environment, such a policy fails to achieve the social optimum, harms the environment, and intensifies the tension. To achieve the socially optimal outcome, we show that it suffices for the government to employ a simple linear incentive scheme, which imposes either subsidies or taxes on the sales of both product generations as well as the remanufactured products. We characterize the impact of customer purchasing behavior upon the socially optimal government policy. Such policy counters strategic customer behavior and induces the social optimum, which does not depend on customer purchasing behavior. Our model also shows that it costs the government more (less) to regulate the market under more intensive strategic customer behavior if the total economic and environmental value of remanufacturing is low (high).

References
Apple Online Store. 2015. We will give credit for your device. Apple Website, https://www.apple.com/recycling/.


Appendix A: Equilibrium Definitions

In this section, we give the formal definitions of RE equilibria in both the base model and the NTR model for any customer discount factor $\delta_c$. Let $A(Q_1) := \mathbb{E}[X \land Q_1]/\mathbb{E}[X]$ ($Q_1 \geq 0$) be the availability function given the first-period production quantity $Q_1$ (see Su and Zhang 2009). Definitions 1 and 2 below formally define the RE equilibrium in the base model and the NTR model, respectively.

**Definition 1. (Base Model)** An RE equilibrium in the base model with customer discount factor $\delta_c$

$*$\footnote{The RE equilibrium in the base model with customer discount factor $\delta_c$ is characterized by $(p_1^*(\delta_c), Q_1^*(\delta_c), \xi^*(\delta_c), r_1^*(\delta_c), a_1^*(\delta_c), p_2^*(\delta_c), p_2^*(\delta_c))$ satisfies\footnote{a}:

(a) $p_1^*(\delta_c) = r_1^*(\delta_c) = \max_{Q_1 \geq 0} \Pi_f(Q_1|\delta_c)$ where $\Pi_f(\cdot|\cdot)$ is given in Lemma 4;

(b) $a_1^*(\delta_c)(V[\delta, \mathbb{E}[(V(k+\alpha)V - p_2^*(\delta_c))]+ - \xi^*(\delta_c)) + (1 - a_1^*(\delta_c))\delta_c\mathbb{E}[(1+\alpha)\delta - p_2^*(\delta_c)] + \delta_c\mathbb{E}[(1+\alpha)\delta - p_2^*(\delta_c)] + \delta_c\mathbb{E}[(1+\alpha)\delta - p_2^*(\delta_c)]$;

(c) $r_1^*(\delta_c) = \xi^*(\delta_c)$;

(d) $a_1^*(\delta_c) = A(Q_1^*(\delta_c))$; $p_2^*(\delta_c) \equiv (p_2^*, p_2^*)$, where $p_2^*$ and $p_2^*$ are given in Lemma 1.

**Definition 2. (NTR Model)** An RE equilibrium in the NTR model with customer discount factor $\delta_c$

$*$\footnote{The RE equilibrium in the base model with customer discount factor $\delta_c$ is characterized by $(p_1^*(\delta_c), Q_1^*(\delta_c), \xi^*(\delta_c), r_1^*(\delta_c), a_1^*(\delta_c), p_2^*(\delta_c))$ satisfies\footnote{c}:

(a) $p_1^*(\delta_c) = r_1^*(\delta_c) = \max_{Q_1 \geq 0} \Pi_f(Q_1|\delta_c)$, where $\Pi_f(\cdot|\cdot)$ is given in Lemma 4 and, in case of multiple optimizers, the smallest one is selected;

(b) $a_1^*(\delta_c)(V[\delta, \mathbb{E}[(V(k+\alpha)V - p_2^*(\delta_c))]+ - \xi^*(\delta_c)) + (1 - a_1^*(\delta_c))\delta_c\mathbb{E}[(1+\alpha)\delta - p_2^*(\delta_c)] + \delta_c\mathbb{E}[(1+\alpha)\delta - p_2^*(\delta_c)]$;

(c) $r_1^*(\delta_c) = \xi^*(\delta_c)$;

(d) $a_1^*(\delta_c) = A(Q_1^*(\delta_c))$; $p_2^*(\delta_c) \equiv (p_2^*, \ldots, p_2^*)$, where $p_2^*$ is characterized in Lemma 3.

In Definitions 1-2, conditions (a) and (b) follow from that the decisions are optimal given the beliefs. Conditions (c) and (d) follow from that the beliefs are rational and consistent with actual outcomes.

Appendix B: Equilibrium Characterization in the NTR Model

In this section, we give a brief characterization of the RE equilibrium in the NTR model $\mathcal{R}_n^*(\delta_c)$. To begin with, by backward induction, we present a lemma that characterizes the second-period equilibrium pricing and production strategy in the NTR model. In the NTR model, let $Q_1^*(X_2^n, X_2^*)$ and $Q_1^*(X_2^n, X_2^*)$ be the equilibrium production quantities for new and repeat customers, respectively. The following lemma is the counterpart of Lemma 1 in the NTR model.

**Lemma 3.** (a) For any $(X_2^n, X_2^*)$, $p_2^*(X_2^n, X_2^*) = \max_{p^2 \geq 0} \Pi_f(p_2^n|X_2^n, X_2^*)$, where

$$\Pi_f(p_2^n|X_2^n, X_2^*) := X_2^n(p_2^n - c_2)\hat{G}\left(\frac{p_2^n}{1+\alpha}\right) + X_2^*(p_2^n - c_2)\hat{G}\left(\frac{p_2^n}{k+\alpha}\right).$$

(b) For any $(X_2^n, X_2^*)$, $Q_1^*(X_2^n, X_2^*) = \hat{G}\left(\frac{p_2^n(X_2^n, X_2^*)}{1+\alpha}\right)X_2^n$, and $Q_1^*(X_2^n, X_2^*) = \hat{G}\left(\frac{p_2^n(X_2^n, X_2^*)}{k+\alpha}\right)X_2^*$.

(c) $p_2^*(X_2^n, X_2^*)$ is increasing in $X_2^n$ and decreasing in $X_2^*$. Moreover, for any $(X_2^n, X_2^*)$, $p_2^*(X_2^n, X_2^*) \leq p_2^*(X_2^*, X_2^*) < 0$. 


Let $\Pi_i^*(Q_1|\delta_\epsilon)$ be the expected profit (equilibrium first-period price) of the firm to produce $Q_1$ products in period 1 over that in period 2. We characterize $\Pi_i^*(\cdot|\cdot)$ in the following lemma.

**Lemma 4.** In the NTR model, we have $p_i^*(Q_1|\delta_\epsilon) = \mu + \delta_\epsilon(\sigma_n^*(Q_1) - \sigma_n^*(Q_2))$ and $\Pi_i^*(Q_1|\delta_\epsilon) = (m_i^*(Q_1|\delta_\epsilon) - r_1)E(X \land Q_1) - (c_1 - r_1)Q_1 + \delta R_2^*(Q_1)$, where $m_i^*(Q_1|\delta_\epsilon) = \mu + \delta_\epsilon(\beta_n^*(Q_1) - \beta_n^*(Q_2)) + \delta_\epsilon(\sigma_n^*(Q_1) - \sigma_n^*(Q_2))$, $\beta_n^*(Q_1) := E[h_i^*(p_i^*(X_2^*, X_2^*))|X_2^* = X \land Q_1]$, and $\beta_n^*(Q_2) := E[v_q^*(p_q^*(X_2^*, X_2^*))|X_2^* = X \land Q_1]$. Moreover, $\beta_n^*(\cdot)$ is increasing, whereas $\sigma_n^*(\cdot)$, $\beta_n^*(\cdot)$, and $m_i^*(\cdot)$ are decreasing in $Q_1$, respectively.

It is clear that, with first-period production quantity $Q_1$, $\beta_n^*(Q_1)$ and $\beta_n^*(Q_2)$ are the expected second-period unit profit from new and repeat customers in the NTR model, respectively. In addition, $m_i^*(Q_1|\delta_\epsilon)$ is the effective first-period marginal revenue, which measures the additional expected marginal revenue to produce the first-generation product in period 1 over that in period 2. $\beta_n^*(\cdot)$, $\beta_n^*(\cdot)$, and $m_i^*(\cdot)$ are the counterparts of $\beta_n^*(\cdot)$, $\beta_n^*(\cdot)$, and $m_i^*(\cdot)$ in the NTR model. As a corollary of Lemmas 3-4, the following theorem summarizes the equilibrium price and production quantity ($p_i^{*+}(\delta_\epsilon), Q_i^{*+}(\delta_\epsilon)$) in the NTR model.

**Theorem 12.** In the NTR model, for any customer discount factor $\delta_\epsilon$, a unique RE equilibrium $R_i^*(\delta_\epsilon)$ exists with (a) $Q_i^{*+}(\delta_\epsilon) = \arg\max_{Q_1 \geq 0} \Pi_i^*(Q_1|\delta_\epsilon)$; (b) $p_i^{*+}(\delta_\epsilon) = \mu + \delta_\epsilon(\sigma_n^*(Q_1^{*+}(\delta_\epsilon)) - \sigma_n^*(Q_1^{*+}(\delta_\epsilon)))$; and (c) the expected profit of the firm $\Pi_i^{*+}(\delta_\epsilon) = (m_i^*(Q_1^{*+}(\delta_\epsilon)|\delta_\epsilon) - r_1)E(X \land Q_1^{*+}(\delta_\epsilon)) - (c_1 - r_1)Q_1^{*+}(\delta_\epsilon) + \delta R_2^*(Q_1^{*+}(\delta_\epsilon))$.

**Appendix C: Proofs of Statements**

We use $h_i^*(\cdot)$ to denote the derivative operator of a single variable function $h_i(\cdot)$, $\partial_i h_2(\cdot)$ to denote the partial derivative operator of a multi-variable function, $h_2(\cdot)$, with respect to variable $x$, and $\partial_i x_1(x_1, x_2, \cdots, x_n)$ to denote the indicator function. For any multivariate continuously differentiable function $h_2(x_1, x_2, \cdots, x_n)$ and $x' := (x_1', x_2', \cdots, x_n')$ in $h_2(\cdot)$’s domain, $\forall i$, we use $\partial_i h_2(x_1, x_2, \cdots, x_n)$ to denote $\partial_i h_2(x_1, x_2, \cdots, x_n)|_{x=x'}$.

**Proof of Lemma 1: Part (a).** Given $(p_2, p_2')$ with $p_2 \leq p_2'$, a new customer will make a purchase if and only if $(1 + \alpha)\overline{V} \geq p_2'$, whereas a repeat customer will make a purchase if and only if $(k + \alpha)\overline{V} \geq p_2'$. Thus, the ex ante probability that a new customer will purchase the second-generation product is $\hat{G}(p_2'_{
frac{1}{1+\alpha}})$, whereas the probability that a repeat customer will join the trade-in program is $\hat{G}(p_2'_{
frac{1}{k+\alpha}})$. Therefore, conditioned on the realized market size $(X_2^*, X_2^*)$, the expected profit of the firm in period 2 is given by:

$$\Pi_2(p_2', p_2'|X_2^*, X_2^*) := X_2^*(p_2' - c_2)\hat{G}\left(\frac{p_2'}{1+\alpha}\right) + X_2^*(p_2' - c_2 + r_2)\hat{G}\left(\frac{p_2'}{k+\alpha}\right)
=X_2^*v_2^*(p_2') + X_2^*v_2^*(p_2'),$$

where $v_2^*(p_2') := (p_2' - c_2)\hat{G}(p_2'_{\nfrac{1}{1+\alpha}})$ and $v_2^*(p_2') := (p_2' - c_2 + r_2)\hat{G}(p_2'_{\nfrac{1}{k+\alpha}})$. We now show that $v_2^*(\cdot)$ is quasiconcave in $p_2'$, and $v_2^*(\cdot)$ is quasiconcave in $p_2'$. Note that

$$\partial_{p_2} v_2^*(p_2') = -\left(\frac{p_2' - c_2}{1+\alpha}\right)g\left(\frac{p_2'}{1+\alpha}\right) + \hat{G}\left(\frac{p_2'}{1+\alpha}\right)$$

and

$$\partial_{p_2} v_2^*(p_2') = -\left(\frac{p_2' - c_2 + r_2}{k+\alpha}\right)g\left(\frac{p_2'}{k+\alpha}\right) + \hat{G}\left(\frac{p_2'}{k+\alpha}\right).$$
Because \( g(v)/\hat{G}(v) \) is continuously increasing in \( v \), \( g\left(\frac{p^*_2}{k+\alpha}\right)/\hat{G}\left(\frac{p^*_2}{k+\alpha}\right) \) is continuously increasing in \( p^*_2 \) and \( g\left(\frac{p^*_2}{k+\alpha}\right)/\hat{G}\left(\frac{p^*_2}{k+\alpha}\right) \) is continuously increasing in \( p^*_2 \). Hence, \( \partial_{p^*_2} v^*_2(p^*_2) = 0 \) has a unique solution \( p^*_2 \) and \( \partial_{p^*_2} v^*_2(p^*_2) = 0 \) has a unique solution \( p^*_2 \). Clearly, \( v^*_2(\cdot) \) is strictly increasing on \([0,p^*_2)\) and strictly decreasing on \((p^*_2, +\infty)\) for \( i = n, r \). Therefore, for any realized \((X^*_2, X^*_2)\), \( X^*_2 v^*_2(\cdot) \) is quasiconcave in \( p^*_2 \), and \( X^*_2 v^*_2(\cdot) \) is quasiconcave in \( p^*_2 \). Thus, for any realized \((X^*_2, X^*_2)\), \((p^*_2(X^*_2, X^*_2), p^*_2(X^*_2, X^*_2)) = (p^*_2, p^*_2) \) maximizes \( \Pi_2(\cdot, X^*_2, X^*_2) \).

It remains to show that \( p^*_2 > p^*_2 \). Note that \( p^*_2 \) satisfies

\[
\left( \frac{p^*_2 - c_2}{1 + \alpha} \right) \frac{g\left(\frac{p^*_2}{p^*_2} \right)}{\hat{G}\left(\frac{p^*_2}{k+\alpha}\right)} = 1, \tag{2}
\]

and \( p^*_2 \) satisfies

\[
\left( \frac{p^*_2 - c_2 + r_2}{k + \alpha} \right) \frac{g\left(\frac{p^*_2}{p^*_2} \right)}{\hat{G}\left(\frac{p^*_2}{k+\alpha}\right)} = 1. \tag{3}
\]

Since \( k < 1, \frac{p^*_2 - c_2 + r_2}{k+\alpha} > \frac{p^*_2 - c_2}{1 + \alpha} \), and the increasing failure rate condition implies that \( g\left(\frac{p^*_2}{k+\alpha}\right)/\hat{G}\left(\frac{p^*_2}{k+\alpha}\right) \geq g\left(\frac{p^*_2}{1+\alpha}\right)/\hat{G}\left(\frac{p^*_2}{1+\alpha}\right) \). Thus,

\[
\left( \frac{p^*_2 - c_2 + r_2}{k + \alpha} \right) g\left(\frac{p^*_2}{p^*_2} \right) > \left( \frac{p^*_2 - c_2}{1 + \alpha} \right) g\left(\frac{p^*_2}{p^*_2} \right) = 1,
\]

and, hence, \( \partial_{p^*_2} v^*_2(p^*_2) < 0 \). Since \( v^*_2(\cdot) \) is quasiconcave, \( p^*_2 < p^*_2 \).

**Part (b).** Because all new customers with willingness-to-pay \((1 + \alpha)V \) greater than \( \bar{p}^*_2(X^*_2, X^*_2) \equiv p^*_2 \) would make a purchase. Hence,

\[
Q^*_2(X^*_2, X^*_2) = \mathbb{E}[X^*_2|1_{(1+\alpha)V \geq p^*_2}] = \hat{G}\left(\frac{p^*_2}{1+\alpha}\right)X^*_2.
\]

Analogously, all repeat customers with willingness-to-pay \((k + \alpha)V \) greater than \( \bar{p}^*_2(X^*_2, X^*_2) \equiv p^*_2 \) would make a purchase. Hence,

\[
Q^*_2(X^*_2, X^*_2) = \mathbb{E}[X^*_2|1_{(k+\alpha)V \geq p^*_2}] = \hat{G}\left(\frac{p^*_2}{k+\alpha}\right)X^*_2.
\]

**Part (c).** Since \( \pi_2(X^*_2, X^*_2) := \max\{\Pi_2(p^*_2, p^*_2, X^*_2, X^*_2) : 0 \leq p^*_2 \leq \bar{p}^*_2\} \), it follows immediately that

\[
\pi_2(X^*_2, X^*_2) = [\max v^*_2(p^*_2)]X^*_2 + [\max v^*_2(p^*_2)]X^*_2.
\]

To complete the proof, it remains to show that \( \beta^*_n = [\max v^*_2(p^*_2)] > 0 \) and \( \beta^*_r = [\max v^*_2(p^*_2)] > 0 \). By equations (2) and (3), we have \( p^*_2 > c_2 \geq 0 \), \( \hat{G}\left(\frac{p^*_2}{1+\alpha}\right) > 0 \), \( p^*_2 - c_2 + r_2 > 0 \), and \( \hat{G}\left(\frac{p^*_2}{k+\alpha}\right) > 0 \). Hence, \( \beta^*_n = (p^*_2 - c_2)\hat{G}\left(\frac{p^*_2}{1+\alpha}\right) > 0 \) and \( \beta^*_r = (p^*_2 - c_2 + r_2)\hat{G}\left(\frac{p^*_2}{k+\alpha}\right) > 0 \). Q.E.D.

**Proof of Theorem 1: Part (a).** Since \( \xi^*_n(\delta_c) \) satisfies that \( \mathcal{U}_n = \mathcal{U}_n \) (equivalently, Definition 1(b)), we have

\[
a^*_1(\delta_c)[\mathbb{E}[V] + \delta_c \mathbb{E}[(k+\alpha)V - p^*_2(\delta_c)]^+] + \xi^*_n(\delta_c) + (1 - a^*_1(\delta_c))\delta_c \mathbb{E}[(1+\alpha)V - p^*_2(\delta_c)]^+ = \delta_c \mathbb{E}[(1+\alpha)V - p^*_2(\delta_c)]^+.
\]
Direct algebraic manipulation yields that \( \xi^*_v(\delta_v) = \mu + \delta_v \mathbb{E}[(k + \alpha)V - p^*_v(\delta_v)]^+ - \delta_v \mathbb{E}[(1 + \alpha)V - p^*_v(\delta_v)]^+ \). Hence, by Definition 1 and Lemma 1(a),

\[
p^*_v(\delta_v) = r^*_v(\delta_v) = \frac{\mu + \delta_v \mathbb{E}[(k + \alpha)V - p^*_v(\delta_v)]^+ - \delta_v \mathbb{E}[(1 + \alpha)V - p^*_v(\delta_v)]^+}{\delta_v (\sigma^*_v - \sigma^*_v)}.
\]

**Part (b,c).** Plug \( p^*_v(\cdot) \) into \( \Pi_f(\cdot|\cdot) \) and we have:

\[
\Pi_f(Q_1|\delta_v) = p^*_v(\delta_v)\mathbb{E}(X \cap Q_1) - c_1 Q_1 + r_1 \mathbb{E}(Q_1 - X) + \delta \mathbb{E}\{\pi_2(X - (X \cap Q_1), X \cap Q_1)\} = (p^*_v(\delta_v) - r_1)\mathbb{E}(X \cap Q_1) - (c_1 - r_1) Q_1 + \delta \mathbb{E}(X) + \delta \mathbb{E}(X) + \delta \mathbb{E}(X) + \delta \mathbb{E}(X),
\]

where the second equality follows from \( (Q_1 - X)^+ = Q_1 - (X \cap Q_1) \), and the last from the identity \( m^*_v(\delta_v) = \mu + \delta \beta^*_v - \beta^*_v + \delta (\sigma^*_v - \sigma^*_v) \). Therefore, \( Q^*_v(\delta_v) \) is the solution to a newsvendor problem with marginal revenue \( m^*_v(\delta_v) - r_1 \), marginal cost \( c_1 - r_1 \), and demand distribution \( F(\cdot) \). Hence, \( Q^*_v(\delta_v) \) is quasiconcave and \( \Pi^*_v(\delta_v) = \Pi_f(Q^*_v(\delta_v)|\delta_v) = (m^*_v(\delta_v) - r_1)\mathbb{E}(X \cap Q^*_v(\delta_v)) - (c_1 - r_1) Q^*_v(\delta_v) + \delta \mathbb{E}(X) \). Q.E.D.

**Proof of Lemma 3:** Part (a). If the firm charges a single price \( p^*_2 \) in period 2, all new (repeat) customers with willingness-to-pay \( (1 + \alpha)V \) greater than \( p^*_2 \) will make a purchase (join the trade-in program). Hence, the second-period profit function of the firm \( \Pi_f^2(p^*_2|X^*_2, X^*_2) \) is given by

\[
\Pi_f^2(p^*_2|X^*_2, X^*_2) = X^*_2(p^*_2 - c_2)G\left(\frac{p^*_2}{1 + \alpha}\right) + X^*_2v^*_2(p^*_2 - c_2)G\left(\frac{p^*_2}{k + \alpha}\right)
\]

where \( v^*_2(p^*_2) := (p^*_2 - c_2)G\left(\frac{p^*_2}{k + \alpha}\right) \). Clearly, \( v^*_2(\cdot) \) has a unique maximizer \( \hat{p}^*_2 \), where \( \hat{p}^*_2 \geq p^*_2 \) with the inequality being strict if \( r_2 > 0 \). Moreover, \( \Pi_f^2(p^*_2|X^*_2, X^*_2) = \hat{\Pi}_2(p^*_2, p^*_2|X^*_2, X^*_2) \), where, by the proof of Lemma 1(a), \( \hat{\Pi}_2(p^*_2, p^*_2|X^*_2, X^*_2) := X^*_2v^*_2(p^*_2) + X^*_2v^*_2(p^*_2) \) is quasiconcave function of \( (p^*_2, p^*_2) \). Thus, the equilibrium second-period pricing strategy \( p^*_2(X^*_2, X^*_2) \) is the maximizer of the second-period profit function, i.e., \( p^*_2(X^*_2, X^*_2) = \arg\max_{p^*_2 \geq 0} \Pi_f^2(p^*_2|X^*_2, X^*_2) \). Note that since \( \hat{\Pi}_2(\cdot, \cdot|X^*_2, X^*_2) \) is quasiconcave in \( (p^*_2, p^*_2) \), \( \Pi_f^2(p^*_2|X^*_2, X^*_2) \) is also quasiconcave in \( p^*_2 \).

**Part (b).** Because all new customers with willingness-to-pay \( (1 + \alpha)V \) greater than \( p^*_2(X^*_2, X^*_2) \) would make a purchase. Hence,

\[
Q^*_u(X^*_2, X^*_2) = \mathbb{E}[X^*_21_{\{(1 + \alpha)V > p^*_2(X^*_2, X^*_2)\}}|X^*_2] = G\left(\frac{p^*_2(X^*_2, X^*_2)}{1 + \alpha}\right)X^*_2.
\]

Analogously, all repeat customers with willingness-to-pay \( (k + \alpha)V \) greater than \( p^*_2(X^*_2, X^*_2) \) would join the trade-in program. Hence,

\[
Q^*_u(X^*_2, X^*_2) = \mathbb{E}[X^*_21_{\{(k + \alpha)V > p^*_2(X^*_2, X^*_2)\}}|X^*_2] = G\left(\frac{p^*_2(X^*_2, X^*_2)}{k + \alpha}\right)X^*_2.
\]
Part (c). Observe that
\[\partial_{p_2^*} \Pi_2^*(p_2^*|X_2^*, X_2^*) = X_2^* \left[G \left(\frac{p_2^*}{1+\alpha}\right) - \left(\frac{p_2^* - c_2}{1+\alpha}\right) g \left(\frac{p_2^*}{1+\alpha}\right)\right] + X_2^* \left[G \left(\frac{p_2^*}{k+\alpha}\right) - \left(\frac{p_2^* - c_2}{k+\alpha}\right) g \left(\frac{p_2^*}{k+\alpha}\right)\right].\]
Since \(g(v)/G(v)\) is increasing in \(v\), \(\partial_{p_2^*} \Pi_2^*(p_2^*|X_2^*, X_2^*) < 0\) if \(p_2^* > p_2^{**}\), and \(\partial_{p_2^*} \Pi_2^*(p_2^*|X_2^*, X_2^*) > 0\) if \(p_2^* < p_2^{**}\). Thus,
\[p_2^*(X_2^*, X_2^*) \in [\hat{p}_2^*, p_2^{**}].\]
By the proof of Lemma 1(a), \(\hat{p}_2^* < p_2^{**}\). Since \(X_2^*, X_2^* > 0\), \(\partial_{p_2^*} \Pi_2^*(p_2^*|X_2^*, X_2^*) = X_2^* \left[G \left(\frac{p_2^*}{1+\alpha}\right) - \left(\frac{p_2^* - c_2}{1+\alpha}\right) g \left(\frac{p_2^*}{1+\alpha}\right)\right] > 0\) and \(\partial_{p_2^*} \Pi_2^*(p_2^*|X_2^*, X_2^*) = X_2^* \left[G \left(\frac{p_2^*}{k+\alpha}\right) - \left(\frac{p_2^* - c_2}{k+\alpha}\right) g \left(\frac{p_2^*}{k+\alpha}\right)\right] < 0\). Therefore, \(p_2^* \leq \hat{p}_2^* < p_2^*(X_2^*, X_2^*) < p_2^{**}\) for all \(X_2^*, X_2^* > 0\).

When \(p_2^* \in [\hat{p}_2^*, p_2^{**}]\), \(G \left(\frac{p_2^*}{1+\alpha}\right) - \left(\frac{p_2^* - c_2}{1+\alpha}\right) g \left(\frac{p_2^*}{1+\alpha}\right) \geq 0\) and \(G \left(\frac{p_2^*}{k+\alpha}\right) - \left(\frac{p_2^* - c_2}{k+\alpha}\right) g \left(\frac{p_2^*}{k+\alpha}\right) \leq 0\). Thus, \(\Pi_2^*(p_2^*|X_2^*, X_2^*)\) is increasing in \(X_2^*\) and decreasing in \(X_2^*\) if \(p_2^* \in [\hat{p}_2^*, p_2^{**}]\), i.e., \(\Pi_2^*(p_2^*|X_2^*, X_2^*)\) is supermodular in \((p_2^*, X_2^*)\) on the lattice \([\hat{p}_2^*, p_2^{**}] \times [0, +\infty)\), and submodular in \((p_2^*, X_2^*)\) on the lattice \([\hat{p}_2^*, p_2^{**}] \times [0, +\infty]\). Therefore, \(p_2^*(X_2^*, X_2^*)\) is continuously increasing in \(X_2^*\) and continuously decreasing in \(X_2^*\). Q.E.D.

**Proof of Lemma 4:** To compute \(\Pi_2^*(Q_1|\delta_1)\), we use the same argument as the proof of Theorem 1. Given the first-period production quantity \(Q_1\), the first-period equilibrium price \(p_1^*(Q_1|\delta_1)\) satisfies
\[a_1^*(\delta_1)(E[V]* + \delta_1 E[(k+\alpha)V - p_2^{**}(\delta_1)]^+ - p_1^*(Q_1|\delta_1)) + (1 - a_1^*(\delta_1))E[(1+\alpha)V - p_2^{**}(\delta_1)]^+ = \delta_1 E[(1+\alpha)V - p_2^{**}(\delta_1)]^+,\]
i.e.,
\[p_1^*(Q_1|\delta_1) = E[V] + \delta_1 E[(k+\alpha)V - p_2^{**}(\delta_1)]^+ - E[(1+\alpha)V - p_2^{**}(\delta_1)]^+ = \mu\] \[+ \delta_1 E[(k+\alpha)V - p_2^{**}(X_2^*, X_2^*)]^+ - E[(1+\alpha)V - p_2^{**}(X_2^*, X_2^*)]^+] = \mu + \delta_1 (\sigma_1^*(Q_1) - \sigma_1^*(Q_1)),\]
where \(X_2^* = (X - Q_1)^+\) and \(X_2^* = X \land Q_1\). Since \(\sigma_2^*(X_2^*, X_2^*) = \max_{p_2^* \geq 0} \Pi_2^*(p_2^*|X_2^*, X_2^*)\),
\[\sigma_2^*(X_2^*, X_2^*) = \max_{p_2^* \geq 0}[X_2^* (p_2^* - c_2) G \left(\frac{p_2^*}{1+\alpha}\right) + X_2^* (p_2^* - c_2) G \left(\frac{p_2^*}{k+\alpha}\right)\]
\[= X_2^* \epsilon_2^*(p_2^*|X_2^*, X_2^*) + X_2^* \epsilon_1^* G \left(\frac{p_2^*}{1+\alpha}\right),\]
where \(X_2^* = (X - Q_1)^+\), and \(X_2^* = X \land Q_1\). Since \(\beta_1^*(Q_1) = E[v_2^*(p_2^*|X_2^*, X_2^*)|X_2^* \land Q_1]\) and \(\beta_1^*(Q_1) = E[v_2^*(p_2^*|X_2^*, X_2^*)|X_2^* \land Q_1)]\), we have
\[\Pi_1^*(Q_1|\delta_1) = p_1^*(Q_1|\delta_1) E[X \land Q_1|c_1 Q_1 + r_1 E(X - Q_1)^+] + \delta_1 E[\sigma_2^*(X_2^*, X_2^*)]\]
\[= (p_1^*(Q_1|\delta_1) - r_1) E[X \land Q_1|c_1 - r_1 Q_1 + \delta_1 E[(X - Q_1)^+] + \epsilon_2^*(p_2^*|X_2^*, X_2^*)] + (E[X \land Q_1|c_1 - r_1 Q_1 + \delta_1 E[v_2^*(p_2^*|X_2^*, X_2^*)|X_2^* \land Q_1])]\]
\[= (m_1^*(Q_1|\delta_1) - r_1) E[X \land Q_1|c_1 - r_1 Q_1 + \delta_1 R_2^*(Q_1)],\]
where \(m_1^*(Q_1|\delta_1) = p_1^*(Q_1|\delta_1) + \delta_1 \beta_1^*(Q_1) - \beta_1^*(Q_1) = \mu + \delta_1 \beta_1^*(Q_1) - \beta_1^*(Q_1) + \delta_1 (\sigma_1^*(Q_1) - \sigma_1^*(Q_1)),\)
\[R_2^*(Q_1) = E[v_2^*(p_2^*(X_2^*, X_2^*))|X_2^* \land Q_1] = E[X - Q_1]^+\text{ and } X_2^* = X \land Q_1.\]

It remains to show that \(\beta_1^*(\cdot)\) is increasing, whereas \(\sigma_1^*(\cdot), \sigma_2^*(\cdot), \beta_1^*(\cdot)\) and \(R_2^*(\cdot)\) are decreasing in \(Q_1\), respectively. By Lemma 3, \(p_2^*(X_2^*, X_2^*) \geq \hat{p}_2^*\) is increasing in \(X_2^*\) and decreasing in \(X_2^*\). Hence, for any
realization of $X$, $p_2^γ((X - Q_1)^+, X \wedge Q_1)$ is decreasing in $Q_1$ and bounded from below by $\tilde{p}_2^γ$, which is the maximizer of $\hat{v}_2^γ(\cdot)$. Hence, for any realization of $X$, $\hat{v}_2^γ(p_2^γ((X - Q_1)^+, X \wedge Q_1))$ is increasing in $Q_1$. Thus, $β_2^γ(Q_1) := E[\hat{v}_2^γ(p_2^γ((X - Q_1)^+, X \wedge Q_1))]$ is increasing in $Q_1$. Analogously, for any realization of $X$, $p_2^γ((X - Q_1)^+, X \wedge Q_1)$ is decreasing in $Q_1$ and bounded from above by $p_2^γ$, which is the maximizer of $v_2^γ(\cdot)$. Hence, for any realization of $X$, $v_2^γ(p_2^γ((X - Q_1)^+, X \wedge Q_1))$ and $v_2^γ(p_2^γ((X - Q_1)^+, X \wedge Q_1))X$ are decreasing in $Q_1$. Thus, $β_2^δ(Q_1) := E[v_2^γ(p_2^γ((X - Q_1)^+, X \wedge Q_1))]$ and $E[v_2^γ(p_2^γ((X - Q_1)^+, X \wedge Q_1))X]$ are decreasing in $Q_1$. Since, for any realization of $X$, $p_2^γ((X - Q_1)^+, X \wedge Q_1)$ is decreasing in $Q_1$, $σ_2^γ(Q_1) = E[(k + α)V - p_2^γ((X - Q_1)^+, X \wedge Q_1)]$ and $σ_2^α(Q_1) = E[(1 + α)V - p_2^γ((X - Q_1)^+, X \wedge Q_1)]$ are decreasing in $Q_1$. Q.E.D.

**Proof of Theorem 12:** **Part (a)** follows directly from Definition 2. **Part (b, c)** follows from the proof of Lemma 4. Q.E.D.

**Proof of Theorem 2:** **Part (a).** It follows from Theorem 1(a) that $p_2^γ(\delta_γ) = μ + δ_γ(σ_γ^+ - σ_γ^-)$ and $n_2^γ(\delta_γ) = μ + (β_γ - β_γ^-) + δ_γ(σ_γ^+ - σ_γ^-)$ are strictly increasing (decreasing) in $δ_γ$ if $σ_γ^+ > σ_γ^- (σ_γ^+ < σ_γ^-)$. By Theorem 1(b), $Q_2^γ(\delta_γ) = F^{-1}(σ_2^γ(\delta_γ))$ is increasing (decreasing) in $δ_γ$ if and only if $σ_2^γ > σ_2^- (σ_2^γ < σ_2^-)$. Moreover, for any $Q_1$ and any $δ_γ > δ_γ$, $Γ(\frac{Q_1^+}{Q_1^-}) = (\delta_γ - δ_γ)(σ_γ^+ - σ_γ^-)E(X \wedge Q_1) > 0$ if and only if $σ_2^γ > σ_2^-$. Therefore, $Γ(\frac{Q_1^+}{Q_1^-}) = \max Γ(\frac{Q_1^+}{Q_1^-})$ if and only if $σ_2^γ > σ_2^-$. If, on the other hand, $σ_2^- < σ_2^γ$, it follows immediately from the same argument that $Γ(\frac{Q_1^+}{Q_1^-}) < 0$. Next, we show that $σ_2^γ > σ_2^- (σ_2^γ < σ_2^-)$ if and only if $r_2 > \bar{r} (r_2 < \bar{r})$. Observe that $v_2^γ(p_2^γ) = (p_2^γ - c_2 + r_2)G(\frac{p_2^γ}{1 + r_2})$ is submodular in $(p_2^γ, r_2)$, so $p_2^γ$ is decreasing in $r_2$. Moreover, $σ_2^γ = E[(k + α)V - p_2^γ]^+]$ is decreasing in $p_2^γ$. Hence, $σ_2^γ$ is increasing in $r_2$ and $σ_2^γ > σ_2^-$ if and only if $r_2 > \bar{r}$ for some $\bar{r}$. We now show that $\bar{r} \geq \frac{1 + \alpha}{\alpha}c_2$. It suffices to show that if $r_2 = \frac{1 + \alpha}{\alpha}c_2$, $σ_2^γ < σ_2^-$. If $r_2 = \frac{1 + \alpha}{\alpha}c_2$, $v_2^γ(p_2^γ) = (p_2^γ - \frac{1 + \alpha}{\alpha}c_2)G(\frac{p_2^γ}{1 + \alpha})$. It’s straightforward to check that $p_2^γ = \frac{k + α}{1 + α}p_2^γ$. Hence,

$$σ_2^γ - σ_2^-= E[(k + α)V - p_2^γ]^+ - E[(1 + α)V - p_2^γ]^+$$

$$= E[(k + α)V - \frac{k + α}{1 + α}p_2^γ]^+ - E[(1 + α)V - p_2^γ]^+$$

$$= - \frac{1 - k}{1 + α}E[(1 + α)V - p_2^γ]^+ < 0.$$  

**Part (b).** We first show (b-iv). By definition, $σ_2^α(Q_1) - σ_2^α(Q_1) = E[(k + α)V - p_2^γ(X_1^+, X_2^+)]^+ - E[(1 + α)V - p_2^γ(X_1^+, X_2^+)]^+$ since $k < 1$ and $p_2^γ(X_1^+, X_2^+) \in (p_2^γ, p_2^γ)$ (Lemma 3), $σ_2^α(Q_1) - σ_2^α(Q_1) < 0$ for all $Q_1 \geq 0$.

By Theorem 12, $p_2^γ(\delta_γ) = μ + δ_γ(α^γ(\delta_γ) - α^γ(\delta_γ^-))$ is continuously differentiable in $δ_γ$. Since the right derivative of $p_2^γ(\cdot)$ at $0$ is $\hat{p}_2^γ(\delta_γ) = σ_2^γ(\delta_γ)$, $σ_2^γ(\delta_γ) < 0$, there exists a positive threshold $δ_0 > 0$, such that $p_2^γ(\cdot)$ is strictly decreasing on $[0, δ_0]$.

To show that $Q_2^γ(\delta_γ)$ is decreasing in $δ_γ$, it suffices to show that $Q_2^γ(\delta_γ)$ is strictly submodular on a neighbourhood of $(Q_2^γ(\delta_γ), \delta_γ)$. Direct computation yields that $\partial_δ^1 Q_2^γ(\delta_γ) = (σ_2^γ(\delta_γ) - σ_2^γ(\delta_γ^-))E(X \wedge Q_1)$. Note that $σ_2^γ(\delta_γ) - σ_2^γ(\delta_γ^-) < 0$ and is decreasing in $Q_1$, whereas $E(X \wedge Q_1) > 0$ and is strictly increasing in $Q_1$ on a neighbourhood of $Q_2^γ(\delta_γ)$. It follows immediately that $\partial_δ^1 Q_2^γ(\delta_γ)$.
a neighbourhood of $Q_1^\ast(\delta)$. Therefore, $\Pi_y^\ast(Q_1^\ast(\delta))$ is strictly submodular on a neighbourhood of $(Q_1^\ast(\delta), \delta)$ and, thus, $Q_1^\ast(\delta)$ is strictly decreasing in $\delta$.

By the envelope theorem, $\delta_c(\Pi_y^\ast(\delta)) = (\sigma_1^\ast(Q_1^\ast(\delta)) - \sigma_n^\ast(Q_1^\ast(\delta)))E(X \wedge Q_1^\ast(\delta)) > 0$. Hence, $\Pi_y^\ast(\delta)$ is strictly increasing in $\delta$.

Q.E.D.

**Proof of Theorem 3: Part (a).** By Lemma 3, $p_2^\ast < p_2^y(X_2^*, X_2^*) < p_2^\ast$ with probability 1. Thus, if $Q_1 > 0$,

$$
\sigma_1^\ast = E[(k + \alpha)V - p_2^\ast] + E[(k + \alpha)V - p_2^y((X - Q_1)^+, X \wedge Q_1)] = \sigma_1^\ast(Q_1),
$$

and

$$
\sigma_n^\ast = E[(1 + \alpha)V - p_2^\ast] < E[(1 + \alpha)V - p_2^y((X - Q_1)^+, X \wedge Q_1)] = \sigma_n^\ast(Q_1).
$$

**Part (b).** By Theorem 1(b) Theorem 12(b), for all $\delta > 0$,

$$
p_1^\ast(\delta) - p_1^\ast(\delta) = \delta_1|\sigma_1^\ast - \sigma_n^\ast| - \delta_1|\sigma_1^\ast(Q_1^\ast(\delta)) - \sigma_n^\ast(Q_1^\ast(\delta))| = \delta_1|\sigma_1^\ast(Q_1^\ast(\delta)) - \sigma_n^\ast(Q_1^\ast(\delta))| - \delta_1|\sigma_1^\ast(Q_1^\ast(\delta)) - \sigma_n^\ast(Q_1^\ast(\delta))| > 0.
$$

(4)

Since $\delta_2^\ast E[(k + \alpha)V - p_2^\ast] > E[(k + \alpha)V - p_2^y] > 0$ and $p_2^\ast < p_2^y(X_2^*, X_2^*) < p_2^\ast$ with probability 1,

$$
\sigma_1^\ast(Q_1^\ast(\delta)) - \sigma_n^\ast(Q_1^\ast(\delta)) < E[(k + \alpha)V - p_2^\ast] + E[(1 + \alpha)V - p_2^\ast] = E[(k + \alpha)V - p_2^\ast] - \sigma_n^\ast(Q_1).
$$

Hence, by (4), $p_1^\ast(\delta) - p_1^\ast(\delta) > \delta_1|\sigma_1^\ast - \sigma_n^\ast| - \delta_1E[(k + \alpha)V - p_2^\ast] - \sigma_n^\ast$ for all $\delta > 0$.

**Part (c).** It is straightforward to compute that, for any $\delta \in [0, \delta]$ and $Q_1 > 0$,

$$
\Pi_y(Q_1|\delta) = \Pi_y(Q_1|\delta) = (p_1(\delta) - p_1(Q_1|\delta))E(X \wedge Q_1) + \delta E[\beta_n^\ast - \beta_n^\ast(Q_1)](X - Q_1)^+ + (\beta_n^\ast - \beta_n^\ast(Q_1))(X \wedge Q_1),
$$

$$
> \delta_1(\sigma_1^\ast - \sigma_n^\ast)E[(k + \alpha)V - p_2^\ast] + \delta E[\beta_n^\ast - \beta_n^\ast(Q_1)](X - Q_1)^+ + (\beta_n^\ast - \beta_n^\ast(Q_1))(X \wedge Q_1).
$$

Since $p_2^\ast < p_2^y$, $\beta_n^\ast \geq \beta_n^\ast(Q_1)$ and $\beta_n^\ast \geq \beta_n^\ast(Q_1)$ for any $Q_1 > 0$, $\sigma_1^\ast - E[(k + \alpha)V - p_2^\ast] > 0$ and, thus, $\Pi_y(Q_1|\delta) > \Pi_y(Q_1|\delta)$ for all $Q_1 > 0$ and $\delta \in [0, \delta]$. Therefore, $\Pi_y(\delta) = \max Q_1 \Pi_y(Q_1|\delta) > \max Q_1 \Pi_y(Q_1|\delta) = \Pi_y(\delta)$.

Q.E.D.

**Proof of Theorem 4:** Since $I^\ast(\delta) = E\{\kappa_1 Q_1^\ast(\delta) + \delta_2^\ast E(X - Q_1^\ast(\delta)) + \delta_2^\ast E(X \wedge Q_1^\ast(\delta))\} - \delta_1(\delta - X)^+$, where $X_2^* = (X - Q_1^\ast(\delta))$ and $X_2^* = X \wedge Q_1^\ast(\delta)$, we have

$$
I^\ast(\delta) = \kappa_1 Q_1^\ast(\delta) + \delta_2^\ast E(X - Q_1^\ast(\delta)) + \delta_2^\ast E(X \wedge Q_1^\ast(\delta)) - \delta_1(\delta - X)^+ + \delta_2^\ast E(X \wedge Q_1^\ast(\delta)) - \delta_1 E(Q_1^\ast(\delta) - X)^+
$$

$$
= I^\ast(\delta).
$$
where \( L(Q_1) := (\kappa_1 - \iota_1)Q_1 + \left[ \delta \kappa_2 G \left( \frac{p_{2}^*}{k + \alpha} \right) - \delta \kappa_2 G \left( \frac{p_{2}^*}{1 + \alpha} \right) + \iota_1 - \delta \iota_2 \right] \mathbb{E}(X \wedge Q_1) + \delta \kappa_2 \mathbb{E}(1 + \frac{p_{2}^*}{1 + \alpha}) \mathbb{E}[X]. \) If \( \kappa_1 \geq \delta \kappa_2 \left( \frac{p_{2}^*}{1 + \alpha} \right), \)

\[
I'_1(Q_1(\delta)) = (\kappa_1 - \iota_1) + \delta \kappa_2 \left( \frac{p_{2}^*}{k + \alpha} \right) - \delta \kappa_2 \left( \frac{p_{2}^*}{1 + \alpha} \right) + \iota_1 - \delta \iota_2 \mathbb{E}(X \geq Q_1(\delta)) > 0,
\]

where the first inequality follows from \( \kappa_1 > \iota_1 \) and \( \mathbb{P}(X \geq Q_1(\delta)) < 1 \), whereas the second inequality follows from the assumptions that \( \kappa_1 \geq \delta \kappa_2 \left( \frac{p_{2}^*}{1 + \alpha} \right) \) and \( \kappa_2 > \iota_2 \). Thus, by Theorem 2(a), if \( \sigma^*_n > \sigma^*_n \), \( Q_1^*(\delta) \) is strictly increasing in \( \delta \), so is \( I'_1(\delta) = L(Q_1^*(\delta)) \); if \( \sigma^*_n < \sigma^*_n \), \( Q_1^*(\delta) \) is strictly decreasing in \( \delta \), so is \( I'_1(\delta) = L(Q_1^*(\delta)). \) Q.E.D.

**Proof of Theorem 5: Part (a).** Since \( Q_1^*(\cdot) \) and \( Q_2^*(\cdot) \) are continuous in \( \delta \), it suffices to show that \( Q_1^*(\delta) > Q_2^*(\delta) \). We first show that \( m_1^*(Q_1|\delta) \) is decreasing in \( Q_1 \). Observe that \( m_1^*(Q_1|\delta) = \mu + \delta [U_r(1) - U_n(1)] \), where

\[
U_r(1) := \mathbb{E} \left[ (p_2(X^2_2, X^1_2) - c_2) G \left( \frac{p_2(X^2_2, X^1_2)}{k + \alpha} \right) + \mathbb{E}((k + \alpha) V - p_2(X^2_2, X^1_2))^+ \right],
\]

and

\[
U_n(1) := \mathbb{E} \left[ (p_2(X^2_2, X^1_2) - c_2) G \left( \frac{p_2(X^2_2, X^1_2)}{1 + \alpha} \right) + \mathbb{E}((1 + \alpha) V - p_2(X^2_2, X^1_2))^+ \right].
\]

Let \( u_\alpha(p) := (p - c_2) G(\frac{p}{1+\alpha}) + \mathbb{E}((k + \alpha) V - p)^+ = \mathbb{E}[(k + \alpha) V - c_2] 1_{1(1+\alpha) V \geq p} \) and \( u_n(\alpha) := (p - c_2) G(\frac{p}{1+\alpha}) + \mathbb{E}((1 + \alpha) V - p)^+ = \mathbb{E}[(1 + \alpha) V - c_2] 1_{1(1+\alpha) V \geq p} \). It’s clear that \( u_\alpha(\cdot) \) and \( u_n(\cdot) \) are continuously decreasing in \( p \). Moreover, \( U_r(1) = \mathbb{E}[u_\alpha(p_2(X^2_2, X^1_2))] \) and \( U_n(1) = \mathbb{E}[u_n(p_2(X^2_2, X^1_2))] \), where \( X^2_2 = (X - Q_1)^+ \) and \( X^1_2 = X \wedge Q_1 \). Since \( p_2(X^2_2, X^1_2) \) is increasing in \( X^2_2 \) and decreasing in \( X^1_2 \), it is stochastically decreasing in \( Q_1 \). Hence, it suffices to show that \( u_\alpha(p) - u_n(p) \) is increasing in \( p \). Observe that

\[
u_\alpha(p) - u_n(p) = -\int_0^{1/(k+\alpha)} ((1+\alpha) V - p) g(V) dV + \int_{1/(k+\alpha)}^\infty (1-k) V g(V) dV = -\int_0^{1/(1+\alpha)} ((1+\alpha) V - \max(p, (k+\alpha) V)) g(V) dV,
\]

which is continuously increasing in \( p \). Therefore,

\[
m_1^*(Q_1|\delta) = \mu + \delta (U_r(1) - U_n(1)) = \mu + \delta (\mathbb{E}[u_n(p_2(X^2_2, X^1_2))] - u_\alpha(p_2(X^2_2, X^1_2)))
\]

is continuously decreasing in \( Q_1 \).

We now show that \( m_1^*(Q_1|\delta) < m_1^*(\delta) \) for all \( Q_1 \). Observe that

\[
m_1^*(Q_1|\delta) - m_1^*(\delta) = \delta \mathbb{E}[u_\alpha(p_2(X^2_2, X^1_2)) - u_\alpha(p_2^*)] - \delta \mathbb{E}[u_n(p_2(X^2_2, X^1_2)) - u_n(p_2^*)].
\]

Because \( p_2^* \leq p_2^2(X^2_2, X^1_2) \leq p_2^* \) and \( u_\alpha(\cdot) \) and \( u_n(\cdot) \) are decreasing in \( p \), \( \delta \mathbb{E}[u_\alpha(p_2(X^2_2, X^1_2)) - u_\alpha(p_2^*)] \leq 0 \) and \( \delta \mathbb{E}[u_n(p_2(X^2_2, X^1_2)) - u_n(p_2^*)] \geq 0 \). Hence, \( m_1^*(Q_1|\delta) \leq m_1^*(\delta) \). Since \( k < 1 \), \( p_2^* < p_2^* \), one of the inequalities
$\mathbb{E}[u_n(p^2_n(X_2^s, X_2^b)) - u_n(p^2_n)] \leq 0$ and $\mathbb{E}[u_n(p^2_n(X_2^s, X_2^b)) - u_n(p^2_n)] \geq 0$ must be strict. Therefore, $m^*_n(Q_1) \delta < m^*_n(\delta)$ for all $Q_1 \geq 0$.

Next, we show that $Q^*_1(\delta) > Q^*_1(\delta)$. Observe that

$$\Pi^*_1(Q_1) - \Pi^*_1(Q_1) = (m^*_n(Q_1) - m^*_n(\delta))\mathbb{E}(X \cap Q_1) + \delta \mathbb{E}\left[\left(p^2_n(X_2^s, X_2^b) - c_2\right)G\left(\frac{p^2_n(X_2^s, X_2^b)}{1 + \alpha}\right) - \beta^*_n\right].$$

Let $\Pi(Q_1, 1) = \Pi^*_1(Q_1) - \Pi^*_1(Q, 1)$. Then,

$$\Pi(Q_1, 1) - \Pi(Q_1, 0) = (m^*_n(\delta) - m^*_n(Q_1))\mathbb{E}(X \cap Q_1) + \delta \mathbb{E}[\beta^*_n - (p^2_n(X_2^s, X_2^b) - c_2)G\left(\frac{p^2_n(X_2^s, X_2^b)}{1 + \alpha}\right)].$$

Since $m^*_n(\delta) \geq m^*_n(Q_1)$ and $m^*_n(Q_1)$ is decreasing in $Q_1$, $(m^*_n(\delta) - m^*_n(Q_1))\mathbb{E}(X \cap Q_1)$ is increasing in $Q_1$. Also note that for any realization of $X$, $p^2_n(X_2^s, X_2^b)$ and thus $(p^2_n(X_2^s, X_2^b) - c_2)G\left(\frac{p^2_n(X_2^s, X_2^b)}{1 + \alpha}\right)$ is decreasing in $Q_1$. Therefore, $\Pi(Q_1, 1) - \Pi(Q_1, 0)$ is increasing in $Q_1$. Hence, $\Pi(\cdot, \cdot)$ is supermodular on the lattice $[0, +\infty) \times \{0, 1\}$ and $Q^*_1(\delta) = \arg\max_{Q_1 \geq 0} \Pi^*_1(Q_1) \leq \arg\max_{Q_1 \geq 0} \Pi(Q_1) = Q^*_1(\delta)$. Since $m^*_n(\delta) > m^*_n(Q^*_1(\delta))$, $\partial_Q, \partial_{Q^*_1}(\delta) > \partial_Q, \partial_{Q^*_1}(\delta) = 0$. Since $\Pi(\cdot, \cdot)$ is concave in $Q_1$, $Q^*_1(\delta) > Q^*_1(\delta)$. Due to the continuity of $Q^*_1(\cdot)$ and $Q^*_1(\cdot)$ in $\delta$, there exists a threshold $\delta_q \leq \delta$, such that $Q^*_1(\delta_q) > Q^*_1(\delta_q)$ for all $\delta > \delta_q$.

**Part (b).** We first show that $m^*_n(Q_1) > m^*_n(Q_1)$ is increasing in $Q_1$. By Lemma 4, $\beta^*_n(\cdot)$ is increasing whereas $\beta^*_n(\cdot)$ is decreasing in $Q_1$. Therefore, $m^*_n(Q_1)$ is increasing in $Q_1$.

We now show that there exists a threshold $\bar{Q}_1$, such that $m^*_n(Q_1 | Q_1 > m^*_n(0) (m^*_n(Q_1) < m^*_n(0)) if Q_1 > \bar{Q}_1 (Q_1 < \bar{Q}_1)$. Let $\beta^*_n = \max_{p \geq 0} \hat{v}_n(p) = \lim_{Q_1 \rightarrow +\infty} \hat{v}_n(Q_1)$. Since $k < 1$, $\beta^*_n = \hat{v}_n(p_n) < \beta^*_n$. It’s clear that $\beta^*_n - \beta^*_n$ is increasing in $r_2$, with $\beta^*_n = \beta^*_n$ if $r_2 = 0$. Let $r_2 > 0$ be the threshold such that $\beta^*_n = \beta^*_n - \beta^*_n$. Hence, for all $r_2 < r_2$, $\beta^*_n - \beta^*_n < \beta^*_n - \beta^*_n$. Moreover, by the monotone convergence theorem,

$$\lim_{Q_1 \rightarrow +\infty} m^*_n(Q_1) | Q_1 > m^*_n(0) (m^*_n(Q_1) < m^*_n(0)) if Q_1 > \bar{Q}_1 (Q_1 < \bar{Q}_1).$$

Now we show there exists a $c_q > 0$ such that, if $c_1 < c_q$, $Q^*_1(0) < Q^*_1(0)$. It’s clear that $Q^*_1(0) \uparrow \bar{X}$ and $Q^*_1(0) \uparrow \bar{X}$ as $c_1 \downarrow 0$, where $X$ is the upper bound of the support of $X (X)$ may take the value of $+\infty$. Hence, there exists a $c_q > 0$ (dependent on $r_2$) such that $c_1 < c_q$, $Q^*_1(0) > \bar{Q}_1$ and $Q^*_1(0) > \bar{Q}_1$. Let $\tilde{\pi}_2(Q_1) = \mathbb{E}[\hat{v}_n(p_n(X_2^s, X_2^b))]$, where $X_2 = (X - Q_1)^+$ and $X_2 = X \cap Q_1$. It’s clear that $\tilde{\pi}_2(\cdot)$ is differentiable and, by the chain rule

$$\tilde{\pi}_2(Q_1) = \mathbb{E}[\hat{v}_n(p_n(X_2^s, X_2^b)) \hat{v}_n(X_2^s p_n(X_2^s, X_2^b) + \hat{v}_n(X_2^s p_n(X_2^s, X_2^b))].$$

As $Q_1 \rightarrow \bar{X}$, for any realization of $X \leq \bar{X}$, $\hat{v}_n(X_2^s, X_2^b)$ and $\hat{v}_n(X_2^s, X_2^b)$ converges to 0. Hence, by the dominated convergence theorem, there exits a threshold $\bar{Q}_1 \in [\bar{Q}_1, \bar{X}]$, such that $\tilde{\pi}_2(Q_1) \in [-\varepsilon P(X \geq Q_1), 0]$ for all $Q_1 \geq Q_1$, where $\varepsilon := (m^*_n(\bar{Q}_1) - m^*_n(\bar{Q}_1))/2 > 0$. Let $\tilde{c}_1(r_2) (0, \tilde{c}(r_2))$ be the threshold such that, if $c_1 < \tilde{c}_1(r_1)$, we have $Q^*_1, Q^*_1 > \bar{Q}_1$. Therefore,

$$\partial_Q, \Pi(Q_1) > Q^*_1(0) = (m^*_n(0) - r_1) \mathbb{P}(X \geq Q^*_1(0)) - (c_1 - r_1) < (m^*_n(Q^*_1(0)) - r_1) \mathbb{P}(X \geq Q^*_1(0)) - (c_1 - r_1) \leq (m^*_n(Q^*_1(0)) - r_1) \mathbb{P}(X \geq Q^*_1(0)) + \tilde{\pi}_2(Q^*_1(0)) - (c_1 - r_1) \leq \partial_Q, \Pi(Q^*_1(0)) = 0,$$
where the first inequality follows from \( m_1^\ast(Q_1^\ast(0)|0) - m_1^\ast(0) \geq (m_1^\ast(\hat{Q}|0) - m_1^\ast(0)) = 2\epsilon > \epsilon \), the second from \( \hat{\pi}_2^\ast(Q_1^\ast(0)) \in [-c_2^\ast P(X \geq Q_1^\ast(0)), 0] \), and the last from the monotonicity that \( m_1^\ast(\cdot|0) \) is increasing in \( Q_1 \). Because \( \Pi_1(\cdot|0) \) is concave in \( Q_1 \), \( Q_1^\ast(0) = \arg \max_{Q_1} \Pi_1(Q_1|0) < Q_2^\ast(0) \) follows immediately. Since \( Q_1^\ast(\delta_c) \) and \( Q_1^\ast(\delta_c) \) are continuous in \( \delta_c \), there exists a threshold \( \delta_{c_1} \) such that \( Q_1^\ast(\delta_c) < Q_1^\ast(\delta_{c_1}) \) for all \( \delta_c \in [0, \delta_{c_1}) \).

**Part (c).** By Theorem 2, \( Q_1^\ast(\delta_c) \) is strictly increasing in \( \delta_c \) if \( \sigma_c^* > \sigma_e^* \); whereas \( Q_1^\ast(\delta_c) \) is strictly decreasing in \( \delta_c \). Therefore, \( \delta_{c_1} = \delta_c \) if \( \sigma_c^* > \sigma_e^* \). Q.E.D.

**Proof of Theorem 6: Part (a).** Since \( \delta_c > \delta_e \), Theorem 5(a) implies that \( Q_1^\ast(\delta_c) > Q_1^\ast(\delta_e) \). Now we compute \( I^\ast(\delta_c) \). Given the market sizes \((X_2^*, X_3^*)\), the equilibrium total second-period production quantity, \( Q_2^*(X_2^*, X_3^*) \), is given by

\[
Q_2^*(X_2^*, X_3^*) = X_2^* G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) + X_3^* \tilde{G} \left( \frac{p_2^*(X_2^*, X_3^*)}{k + \alpha} \right).
\]

Therefore, following the same argument as in the proof of Theorem 4, we have

\[
I^\ast(\delta_c) = \mathbb{E} \{ \kappa_1 Q_1^\ast(\delta_c) - \ell_1(Q_1^\ast(\delta_c) - X) + \delta_2 Q_2^*(X_2^*, X_3^*) \}
\]

\[
= (\kappa_1 - \ell_1) Q_1^\ast(\delta_c) + \mathbb{E} \left[ (\delta_2 G \left( \frac{p_2^*(X_2^*, X_3^*)}{k + \alpha} \right) - \delta_2 \tilde{G} \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) + \ell_1) (X \wedge Q_1^\ast(\delta_c)) \right]
\]

\[
+ \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{k + \alpha} \right) X \right],
\]

where \( X_2^* = (X - Q_1^\ast(\delta_c))^+ \) and \( X_3^* = X \wedge Q_1^\ast(\delta_c) \). For any \( \delta_c, I^\ast(\delta_c) \) is strictly linearly decreasing in \( \ell_2 \). Thus, let \( \ell_c := \max \{ \ell_2 : I^\ast(\delta_c) > I^\ast(\delta_c) \} \). We have \( I^\ast(\delta_c) > I^\ast(\delta_c) \) if \( \ell_2 < \ell_c \). In particular, if \( \ell_2 = 0, Q_1^\ast(\delta_c) > Q_1^\ast(\delta_c), p_2^* < p_2^*(\cdot, \cdot) < p_2^*, \) and \( \kappa_1 \geq \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \) imply that

\[
(k_1 - \ell_1) Q_1^\ast(\delta_c) + \mathbb{E} \left[ (\ell_1 - \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \right)
\]

\[
> (k_1 - \ell_1) Q_1^\ast(\delta_c) + \mathbb{E} \left[ (\ell_1 - \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \right)
\]

\[
> (k_1 - \ell_1) Q_1^\ast(\delta_c) + \mathbb{E} \left[ (\ell_1 - \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \right)
\]

\[
+ \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{k + \alpha} \right) X \right] \}
\]

\[
\delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{k + \alpha} \right) X \right] \}
\]

Thus, for \( \ell_2 = 0 \), we follow the same argument as the proof of Theorem 4 to establish that

\[
I^\ast(\delta_c) = (k_1 - \ell_1) Q_1^\ast(\delta_c) + \mathbb{E} \left[ (\ell_1 - \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \right)
\]

\[
> (k_1 - \ell_1) Q_1^\ast(\delta_c) + \mathbb{E} \left[ (\ell_1 - \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \right)
\]

\[
+ \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \}
\]

\[
= I^\ast(\delta_c),
\]

i.e., \( I^\ast(\delta_c) > I^\ast(\delta_c) \) for \( \ell_2 = 0 \). Therefore, \( \ell_c > 0 \).

**Part (b).** Since \( \delta_c < \delta_{c_1} \), Theorem 5(b) implies that \( Q_1^\ast(\delta_c) < Q_1^\ast(\delta_e) \). Lemma 3 implies that \( p_2^* < p_2^*(\cdot, \cdot) < p_2^* \). Hence,

\[
(k_1 - \ell_1) Q_1^\ast(\delta_c) + \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \}
\]

\[
< (k_1 - \ell_1) Q_1^\ast(\delta_c) + \delta_2 \mathbb{E} \left[ G \left( \frac{p_2^*(X_2^*, X_3^*)}{1 + \alpha} \right) X \right] \}
\]

\[
= I^\ast(\delta_c),
\]
Let $\tilde{\iota}_c := (\tilde{G}(\frac{p_2^*}{k+\alpha}) - \tilde{G}(\frac{p_2^*}{k+\alpha}))/\tilde{\kappa}_2/\tilde{G}(\frac{p_2^*}{k+\alpha}) < \kappa_2$. If $\iota_2 > \tilde{\iota}_c$, since $Q_1^*(\delta_c) > Q_1^*(\delta_c)$ and $p_2^* < p_2^*(\cdot, \cdot) < p_2^*$, 

$$\mathbb{E}\left[\left(\iota_1 + \delta \kappa_2 \tilde{G}\left(\frac{p_2^*(X_2^*, X_2^*)}{k+\alpha}\right)\right) (Q_1^*(\delta_c) \wedge X)\right] > \left[\iota_1 + \delta(\kappa_2 - \iota_2) \tilde{G}\left(\frac{p_2^*}{k+\alpha}\right)\right] \mathbb{E}(Q_1^*(\delta_c) \wedge X) > \left[\iota_1 + \delta(\kappa_2 - \iota_2) \tilde{G}\left(\frac{p_2^*}{k+\alpha}\right)\right] \mathbb{E}(Q_1^*(\delta_c) \wedge X).$$

Therefore, if $\iota_2 > \tilde{\iota}_c$, 

$$I_1^*(\delta_c) = (\kappa_1 - \iota_1)Q_1^*(\delta_c) + \delta \kappa_2 \mathbb{E}\left[\tilde{G}\left(\frac{p_2^*(X_2^*, X_2^*)}{1+\alpha}\right) X - (X \wedge Q_1^*(\delta_c))\right] + \mathbb{E}\left[\iota_1 + \delta \kappa_2 \tilde{G}\left(\frac{p_2^*(X_2^*, X_2^*)}{1+\alpha}\right) (Q_1^*(\delta_c) \wedge X)\right] > (\kappa_1 - \iota_1)Q_1^*(\delta_c) + \delta \kappa_2 \tilde{G}\left(\frac{p_2^*}{1+\alpha}\right) \{\mathbb{E}[X] - \mathbb{E}(X - Q_1^*(\delta_c))^+\} + \mathbb{E}[\iota_1 + \delta(\kappa_2 - \iota_2) \tilde{G}\left(\frac{p_2^*}{k+\alpha}\right) (Q_1^*(\delta_c) \wedge X)] = I_1^*(\delta_c).$$

\textit{Q.E.D.}

\textbf{Proof of Theorem 7: Part (a).} To compute the equilibrium total customer surplus, $S_1^*(\delta_c)$, we observe that, a new customer’s (discounted) expected surplus in period 2 is $\delta \mathbb{E}((1+\alpha)V - p_2^*)^+ = \delta \sigma_1^*$, whereas that of a repeat customer is $\delta \mathbb{E}(k+\alpha)V - p_2^*)^+ = \delta \sigma_2^*$. Hence, the expected surplus of a customer with discount factor $\delta_c$ in the base model is given by:

$$a_1^*(\delta_c)(\mu - p_1^*(\delta_c) + \delta \sigma_1^*) + (1 - a_1^*(\delta_c))\delta \sigma_2^* = a_1^*(\delta_c)(\mu - \delta_c(\sigma_1^* - \sigma_2^*) + \delta \sigma_2^*) + (1 - a_1^*(\delta_c))\delta \sigma_2^* = a_1^*(\delta_c)(\delta - \delta_c)(\sigma_1^* - \sigma_2^*) + \delta \sigma_2^*,$$

where the first equality follows from $p_1^*(\delta_c) = \mu + \delta_c(\sigma_1^* - \sigma_2^*)$. Therefore, the equilibrium total customer surplus is given by $S_1^*(\delta_c) = \mathbb{E}\{a_1^*(\delta_c)(\delta - \delta_c)(\sigma_1^* - \sigma_2^*) + \delta \sigma_2^*)(X)\}.$

\textbf{Part (b).} To compute $S_1^*(\delta_c)$, we observe that the (discounted) expected surplus of a new customer in period 2 is $\delta \mathbb{E}((1+\alpha)V - p_2^*)^+ = \delta \sigma_1^*(Q_1^*(\delta_c))$, whereas that of a repeat customer is $\delta \mathbb{E}(k+\alpha)V - p_2^*)^+ = \delta \sigma_2^*(Q_1^*(\delta_c))$. Hence, the expected surplus of a customer with discount factor $\delta_c$ in the NTR model is given by:

$$a_1^*(\delta_c)(\mu - p_1^*(\delta_c) + \delta \sigma_1^*(Q_1^*(\delta_c))) + (1 - a_1^*(\delta_c))\delta \sigma_2^*(Q_1^*(\delta_c)) = a_1^*(\delta_c)(\mu - \delta_c(\sigma_1^*(Q_1^*(\delta_c)) - \sigma_2^*(Q_1^*(\delta_c))) + \delta \sigma_2^*(Q_1^*(\delta_c))) + (1 - a_1^*(\delta_c))\delta \sigma_2^*(Q_1^*(\delta_c)) = a_1^*(\delta_c)(\delta - \delta_c)(\sigma_1^*(Q_1^*(\delta_c)) - \sigma_2^*(Q_1^*(\delta_c))) + \delta \sigma_2^*(Q_1^*(\delta_c)),$$

where the first equality follows from $p_1^*(\delta_c) = \mu + \delta_c(\sigma_1^*(Q_1^*(\delta_c)) - \sigma_2^*(Q_1^*(\delta_c))).$ Therefore, the equilibrium total customer surplus is given by $S_1^*(\delta_c) = \mathbb{E}\{a_1^*(\delta_c)(\delta - \delta_c)(\sigma_1^*(Q_1^*(\delta_c)) - \sigma_2^*(Q_1^*(\delta_c))) + \delta \sigma_2^*(Q_1^*(\delta_c)))(X)\}.$

\textbf{Part (c).} Note that, when $\delta_c = \delta$, $S_1^*(\delta_c) = \mathbb{E}[\sigma_1^*(X)]$ and $S_2^*(\delta_c) = \mathbb{E}[\sigma_2^*(Q_1^*(\delta_c))X]$. As shown by Lemma 3(c), $\sigma_2^* < \sigma_2^*(Q_1^*(\delta_c))$. Hence, it follows immediately that $S_1^*(\delta_c) = \mathbb{E}[\sigma_1^*(Q_1^*(\delta_c))X] > \mathbb{E}[\sigma_2^*(X)] = S_2^*(\delta_c)$ for $\delta_c = \delta$. Since $S_1^*(\delta_c)$ and $S_2^*(\delta_c)$ are continuous in $\delta_c$, there exists a threshold $\hat{\delta}_c < \delta$ such that $S_1^*(\delta_c) > S_2^*(\delta_c)$ for $\delta_c \in (\hat{\delta}_c, \delta)$. \textit{Q.E.D.}
Proof of Lemma 2: Part (a). Let \( W_2(p_2^*, p_2^*)|X_2^+, X_2^-) \) be the expected social welfare in period 2 when the price for new customers is \( p_2^* \), and that for repeat customers is \( p_2^* \). Since all new (repeat) customers with valuation \( (1 + \alpha) V \geq p_2^* \) \( ((k + \alpha) V \geq p_2^* \) will make a purchase (trade the used products in), the firm profit equals \( (p_2^* - c_2) G(\frac{p_2^*}{1 + \alpha}) X_2^- + (p_2^* - c_2 + r_2) G(\frac{p_2^*}{k + \alpha}) X_2^+ \), the expected total customer surplus equals \( X_2^+ \mathbb{E}((1 + \alpha) V - p_2^*)^+ + X_2^- \mathbb{E}((k + \alpha) V - p_2^*)^+ \), and the total environmental impact equals \( \kappa_2 X_2^+ G(\frac{p_2^*}{1 + \alpha}) + (\kappa_2 - \mu_2) X_2^- G(\frac{p_2^*}{k + \alpha}) \). Therefore, \( W_2(p_2^*, p_2^*)|X_2^+, X_2^-) = X_2^+ w_n(p_2^*) + X_2^- w_r(p_2^*) \), where

\[
w_n(p_2^*) := (p_2^* - c_2 - \kappa_2) G(\frac{p_2^*}{1 + \alpha}) + \mathbb{E}((1 + \alpha) V - p_2^*)^+ = \mathbb{E}((1 + \alpha) V - c_2 - \kappa_2) 1_{(1 + \alpha) V \geq p_2^*},
\]

and

\[
w_r(p_2^*) := (p_2^* - c_2 + r_2 - \kappa_2 + \mu_2) G(\frac{p_2^*}{k + \alpha}) + \mathbb{E}((k + \alpha) V - p_2^*)^+ = \mathbb{E}((k + \alpha) V - c_2 - r_2 - \kappa_2 + \mu_2) 1_{(k + \alpha) V \geq p_2^*}.
\]

Thus, \( w_n(p_2^*) = \frac{p_2^* - c_2 - \kappa_2}{1 + \alpha} g(\frac{p_2^*}{1 + \alpha}) \) and \( w_r(p_2^*) = \frac{p_2^* - c_2 + r_2 - \kappa_2 + \mu_2}{k + \alpha} g(\frac{p_2^*}{k + \alpha}) \). It is straightforward to compute that \( w_n'(p_2^*) > 0 \) if \( p_2^* < c_2 + \kappa_2 \) and \( w_n'(p_2^*) < 0 \) if \( p_2^* > c_2 + \kappa_2 \). Analogously, \( w_r'(p_2^*) > 0 \) if \( p_2^* < c_2 + \kappa_2 - r_2 - \mu_2 \) and \( w_r'(p_2^*) < 0 \) if \( p_2^* > c_2 + \kappa_2 - r_2 - \mu_2 \). Hence, the unique maximizer of \( w_n(\cdot) \) is \( c_2 + \kappa_2 \), and the unique maximizer of \( w_r(\cdot) \) is \( c_2 + \kappa_2 - r_2 - \mu_2 \). Finally, it is straightforward to check that \( c_2 + \kappa_2 - r_2 + \mu_2 \leq c_2 + \kappa_2 \), with the inequality being strict if and only if \( r_2 > 0 \). Therefore, \( p_2^*(X_2^+, X_2^-) \equiv p_2^* = c_2 + \kappa_2 \) and \( p_2^*(X_2^+, X_2^-) \equiv p_2^* = c_2 + \kappa_2 - r_2 - \mu_2 \) for any realized \( X_2^+, X_2^- \), and \( p_2^* \geq p_2^* \) with inequality being straight if and only if \( r_2 > 0 \) or \( \mu_2 > 0 \).

Part (b). Plugging \( p_2^* \) and \( p_2^* \) into \( w_n(\cdot) \) and \( w_r(\cdot) \), respectively, we have \( w_2^*(p_2^*) = \mathbb{E}((1 + \alpha) V - p_2^*)^+ \) and \( w_2^*(p_2^*) = \mathbb{E}((k + \alpha) V - p_2^*)^+ \). Therefore, \( w_2(X_2^+, X_2^-) = X_2^+ \mathbb{E}((1 + \alpha) V - p_2^*)^+ + X_2^- \mathbb{E}((k + \alpha) V - p_2^*)^+ = \sigma_2^* X_2^+ + \sigma_2^* X_2^- \). \( Q.E.D. \)

It follows immediately from Lemma 2 that the second-period production quantity for new customers is

\[
Q_2^*(X_2^+, X_2^-) = \mathbb{E}[X_2^+ 1_{(1 + \alpha) V \geq p_2^*}]|X_2^-] = X_2^+ G(\frac{p_2^*}{1 + \alpha}),
\]

and that for repeat customers is

\[
Q_2^*(X_2^+, X_2^-) = \mathbb{E}[X_2^- 1_{(k + \alpha) V \geq p_2^*}]|X_2^-] = X_2^+ G(\frac{p_2^*}{k + \alpha}).
\]

Proof of Theorem 8: Part (a). It follows from the same argument as the proof of Theorem 1(a) that, we have \( p_1^*(\delta_1) = \mu + \delta_1 \mathbb{E}[(1 + \alpha) V - p_1^*]^+ - \mathbb{E}[(1 + \alpha) V - p_1^*]^+ = \mu + \delta_1 (\sigma_1^* - \sigma_1^*) \). Let \( W_s(Q_1|\delta_1) \) denote the expected total social welfare with first-period production quantity \( Q_1 \) and customer discount factor \( \delta_1 \). To compute \( W_s(Q_1|\delta_1) \), Since \( w_2(X_2^+, X_2^-) = \sigma_2^* X_2^+ + \sigma_2^* X_2^- \), we have

\[
W_s(Q_1|\delta_1) = p_1^*(\delta_1) \mathbb{E}(X \wedge Q_1) + (\mu - p_1^*(\delta_1)) \mathbb{E}(X \wedge Q_1) - (c_1 + \kappa_1) Q_1 + (r_1 + \mu_1) \mathbb{E}(Q_1 - X)^+
\]

\[
+ \delta \mathbb{E}(w_2(X - (X \wedge Q_1), X \wedge Q_1))
\]

\[
= (\mu - r_1 - \mu_1) \mathbb{E}(X \wedge Q_1) - (c_1 - r_1 + \kappa_1 - \mu_1) Q_1 + \delta \mathbb{E}(\sigma_2^*(X - (X \wedge Q_1)) + \sigma_2^*(X \wedge Q_1))
\]

\[
= (\mu_1 - r_1 - \mu_1) \mathbb{E}(X \wedge Q_1) - (c_1 - r_1 + \kappa_1 - \mu_1) Q_1 + \delta \mathbb{E}(\sigma_2^*) \mathbb{E}(X).
\]
Therefore, $Q_1^*(\delta_1)$ is the solution to a newsvendor problem with marginal revenue $m_1^* - r_1 - \iota_1$, marginal cost $c_1 + \kappa_1 - r_1 - \iota_1$, and demand distribution $F(\cdot)$. Hence, $Q_1^*(\delta_1) = F^{-1}(\frac{c_1 + \kappa_1 - r_1 - \iota_1}{m_1^* - r_1 - \iota_1})$, and the equilibrium social welfare is

$$W_1^*(\delta_1) = W_0(Q_1^*(\delta_1)|\delta_1) = (m_1^* - r_1 - \iota_1)E(X \cap Q_1^*(\delta_1)) - (c_1 - r_1 + \kappa_1 - \iota_1)Q_1^*(\delta_1) + \delta \sigma_n^* E(X).$$

**Part (b).** It follows immediately from part (a) that $p_1^*(\delta_1) = \mu + \delta (\sigma_1^* - \sigma_n^*)$ is strictly increasing (decreasing) in $\delta_1$ if and only if $\sigma_1^* > \sigma_n^* (\sigma_1^* < \sigma_n^*)$. Since $\sigma_1^* = E((k + \alpha)V - c_2 - \kappa_2 + r_2 + \iota_2)^+$ is strictly increasing in $r_2 + \iota_2$. Hence, let $\bar{V}_2 := \min\{r_2 + \iota_2 : \sigma_1^* \geq \sigma_n^*\}$. It follows immediately that $\sigma_1^* > \sigma_n^* (\sigma_1^* < \sigma_n^*)$ if and only if $r_2 + \iota_2 > \bar{V}_2 (r_2 + \iota_2 < \bar{V}_2)$. We also observe that $E((k + \alpha)V - c_2 - \kappa_2)^+ - E((1 + \alpha)V - c_2 - \kappa_2)^+ < 0$. Thus, $\bar{V}_2 > 0$. Finally, since $m_1^*$ is independent of $\delta_1$, $Q_1^*(\delta_1)$ and $W_1^*(Q_1^*(\delta_1))$ are also independent of the customer discount factor $\delta_1$.  

**Q.E.D.**

**Proof of Theorem 9:** **Part (a).** With the unit subsidy rate $s_r$ for remanufactured products, the expected per demand profit from repeat customers $v_2^r(p_2^r) = (p_2^r + s_r + s_2 - c_2 + r_2)G\left(\frac{p_2^r}{k + \alpha}\right)$. Since $\partial_{p_2^r} p_2^r = -\frac{1}{k + \alpha}g\left(\frac{p_2^r}{k + \alpha}\right) \leq 0$, $v_2^r(p_2^r)$ is submodular in $(p_2^r, s_r)$. Hence, $p_2^r = \arg\max_{p_2^r \geq 0} v_2^r(p_2^r)$ is continuously decreasing in $s_r$. Because $Q_2^r(X_2^r, X_2) = X_2^r G\left(\frac{p_2^r}{k + \alpha}\right)$ and $p_2^r$ is decreasing in $s_r$, $Q_2^r(X_2^r, X_2)$ is increasing in $s_r$.

**Part (b).** By definition, $\sigma_1^* = E((k + \alpha)V - p_1^*)^+$ is increasing in $p_1^*$ and, thus, increasing in $s_r$. Therefore, $p_1^*(\delta_1) = \mu + \delta (\sigma_1^* - \sigma_n^*)$ is increasing in $s_r$ for any $\delta_1$. Moreover, $\beta_1^* = \max\{(p_2^r - c_2 + r_2 + s_r)G\left(\frac{p_2^r}{k + \alpha}\right)\}$ is increasing in $s_r$. Thus, $m_1^*(\delta_1) = \mu + \delta (\beta_1^* - \beta_n^*) + \delta (\sigma_1^* - \sigma_n^*)$ is increasing in $s_r$. With the unit subsidy rate $s_r$ for remanufactured product,

$$\Pi_r(Q_1|\delta_1) = (m_1^*(\delta_1) - r_1 - s_r)E(X \cap Q_1) - (c_1 - r_1 - s_r)Q_1 + \delta \beta_n^* E(X),$$

Hence, $Q_1^*(\delta_1) = F^{-1}\left(\frac{c_1 - r_1 - s_r}{m_1^*(\delta_1) - r_1 - s_r}\right)$. The critical fractile $\frac{c_1 - r_1 - s_r}{m_1^*(\delta_1) - r_1 - s_r}$, is decreasing in $m_1^*(\delta_1)$ and $s_r$. Therefore, $Q_1^*(\delta_1)$ is increasing in $s_r$. For each $Q_1$ and $\delta_1$, $\Pi_r(Q_1|\delta_1)$ is increasing in $s_r$. Thus, $\Pi^*(\delta_1) = \max_{Q_1 \geq 0} \Pi_r(Q_1|\delta_1)$ is increasing in $s_r$. By the proof of Theorem 4, $I^*_r(\delta_1) = I_r(Q_1^*(\delta_1)|\delta_1)$. Since $L_r(Q_1|\delta_1)$ is increasing in $Q_1$ (see the proof of Theorem 4). Thus, $I^*_r(\delta_1)$ is increasing in $s_r$ as well.  

**Q.E.D.**

**Proof of Theorem 10:** **Part (a).** If $s_2^*(\delta)$ is the solution to $p_2^* = \arg\max_{p_2^r \geq 0} (p_2^r + s_2 - c_2)G\left(\frac{p_2^r}{k + \alpha}\right)$, it is clear that the subsidy/tax scheme with $s_2 = s_2^*(\delta)$ can induce the equilibrium price for new customers $p_2^*$. We now show that $s_2^*(\delta)$ exists. Since $v_2^r(p_2^r)$ is quasiconcave in $p_2^r$ for any $s_2$, the first-order condition $\partial_{p_2^r} v_2^r(p_2^r) = 0$ guarantees the optimal price for new customers. Moreover,

$$\partial_{p_2^r} v_2^r(p_2^*) = G\left(\frac{p_2^*}{1 + \alpha}\right) - \frac{p_2^* + s_2 - c_2}{1 + \alpha} g\left(\frac{p_2^*}{1 + \alpha}\right),$$

which is strictly decreasing in $s_2$. Hence, there exists a unique $s_2^*(\delta)$, such that $\partial_{p_2^r} v_2^r(p_2^*) = 0$, thus inducing the socially optimal equilibrium price for new customers $p_2^*$.

If $s_1^*(\delta)$ is the solution to $p_1^* = \arg\max_{p_1^r \geq 0} (p_1^r + s_1^* + s_1 - c_1 + r_1)G\left(\frac{p_1^r}{k + \alpha}\right)$, the subsidy/tax scheme with $s_r = s_1^*(\delta)$ can induce the equilibrium trade-in price for repeat customers $p_1^*$. We now show that $s_1^*(\delta)$
exists. Since \( v^*_1(p^*_2) \) is quasiconcave in \( p^*_2 \) for any \((s_2, s_e)\), the first-order condition \( \partial_{p^*_2} v^*_1(p^*_2) = 0 \) guarantees the optimal price for new customers. Moreover, if \( s_2 = s_2^*(\delta_e) \),

\[
\partial_{p^*_2} v^*_1(p^*_2) = G\left( \frac{p^*_2}{k + \alpha} \right) - \frac{p^*_2 + s_2^*(\delta_e) + s_e - c_2 + r_2}{k + \alpha} g\left( \frac{p^*_2}{k + \alpha} \right),
\]

which is strictly decreasing in \( s_e \). Hence, there exists a unique \( s^*_2(\delta_e) \), such that \( \partial_{p^*_2} v^*_1(p^*_2) = 0 \) if \( s_2 = s_2^*(\delta_e) \), thus inducing the socially optimal equilibrium trade-in price for repeat customers \( p^*_2 \).

Given the subsidy/tax scheme \((s_1, s_2^*(\delta_e), s^*_2(\delta_e))\), as shown above, the firm adopts the same second-period pricing strategy as the social welfare maximizing one: \((p^*_1, p^*_2)\). Hence, the first-period price should also be the same as the one which is socially optimal and characterized by Lemma 8(a): \( p^*_1(\delta_e) = \mu + \delta_e(\sigma^*_1 - \sigma^*_2) \).

Thus, the expected profit of the firm in period 1 is

\[
\Pi_1(Q_1) = (p^*_1(\delta_e) + s_1 - r_1)E(X \land Q_1) - (c_1 - r_1)Q_1 + \delta E[(X - X \land Q_1)(p^*_1 + s^*_2(\delta_e) - c_2)] G\left( \frac{p^*_2}{1 + \alpha} \right)
\]

\[
+ (X \land Q_1)(p^*_1 + s^*_2(\delta_e) - c_2 - r_2) G\left( \frac{p^*_2}{k + \alpha} \right)
\]

\[
= (m^*_1(s_1|\delta_e) - r_1)E(X \land Q_1) - (c_1 - r_1)Q_1 + \delta (p^*_1 + s^*_2(\delta_e) - c_2) G\left( \frac{p^*_2}{1 + \alpha} \right) E(X),
\]

where \( m^*_1(s_1|\delta_e) = s_1 + p^*_1(\delta_e) + \delta (\kappa_2 + s^*_2(\delta_e) - \kappa_2 s^*_2(\delta_e) - \kappa_2) G\left( \frac{p^*_2}{k + \alpha} \right) - (\kappa_2 + s^*_2(\delta_e)) G\left( \frac{p^*_2}{1 + \alpha} \right) \). Thus, \( \Pi_1(Q_1) \) has a unique optimizer \( F^{-1}\left( \frac{c_1 - r_1}{m^*_1(s_1|\delta_e) - r_1} \right) \). Moreover, as shown in Theorem 8, \( Q^*_1(\delta_e) \) is the unique solution to \( m^*_1(s_1|\delta_e) - r_1 = \frac{\kappa_1 s_2^*(\delta_e) - \kappa_2 s_2^*(\delta_e)}{m^*_1(s_1|\delta_e) - r_1} \), the optimal production quantity with the linear subsidy/tax scheme \( s_2^*(\delta_e) = (s_1^*(\delta_e), s_2^*(\delta_e), s^*_2(\delta_e)) \) is \( Q^*_1(\delta_e) \), which is the socially optimal first-period production quantity.

Part (b). We now show that \( s^*_2(\delta_e) \) is increasing in \( \kappa_2 \). As shown in part (a), \( s^*_2(\delta_e) \) satisfies \( G\left( \frac{p^*_2}{k + \alpha} \right) - \frac{p^*_2 + s^*_2(\delta_e) - c_2}{k + \alpha} g\left( \frac{p^*_2}{k + \alpha} \right) = 0 \), i.e.,

\[
s^*_2(\delta_e) = \frac{(k + \alpha)G\left( \frac{p^*_2}{k + \alpha} \right) - p^*_2 + c_2}{g\left( \frac{p^*_2}{k + \alpha} \right)} = \frac{(k + \alpha)G\left( \frac{c_2 + p^*_2}{k + \alpha} \right)}{g\left( \frac{c_2 + p^*_2}{k + \alpha} \right)} - \kappa_2.
\]

Because \( g(v)/G(v) \) is increasing in \( v \), \( s^*_2(\delta_e) \) is strictly decreasing in \( \kappa_2 \). Analogously, \( s^*_2(\delta_e) \) satisfies \( G\left( \frac{p^*_2}{k + \alpha} \right) - \frac{p^*_2 + s^*_2(\delta_e) + s_e - c_2 + r_2}{k + \alpha} g\left( \frac{p^*_2}{k + \alpha} \right) = 0 \), i.e.,

\[
s^*_2(\delta_e) = \frac{(k + \alpha)G\left( \frac{p^*_2}{k + \alpha} \right) - p^*_2 + s^*_2(\delta_e) + c_2 - r_2}{g\left( \frac{p^*_2}{k + \alpha} \right)} = \frac{(k + \alpha)G\left( \frac{c_2 + c_e + p^*_2 - r_2}{k + \alpha} \right)}{g\left( \frac{c_2 + c_e + p^*_2 - r_2}{k + \alpha} \right)} - s^*_2(\delta_e) - \kappa_2 + r_2.
\]

Because \( g(v)/G(v) \) is increasing in \( v \), \( s^*_2(\delta_e) \) is strictly increasing in \( \kappa_2 \).

Since \( s^*_1(\delta_e) \) satisfies \( m^*_1(s_1|\delta_e) - r_1 = \frac{\kappa_1 s_2^*(\delta_e) - \kappa_2 s_2^*(\delta_e)}{m^*_1(s_1|\delta_e) - r_1} \), the left-hand-side of which is strictly decreasing in \( s^*_1(\delta_e) \), whereas the right-hand-side of which is strictly increasing in \( \kappa_1 \). Therefore, \( s^*_1(\delta_e) \) is strictly decreasing in \( \kappa_1 \).

Part (c). Define \( \tilde{s}^*_2(\delta_e) \) as the solution to \( \frac{1 + \alpha)}{g\left( \frac{c_2 + p^*_2}{k + \alpha} \right)} = \kappa_2, \tilde{i}^*_e(\delta_e) \) as the solution to \( \frac{1 + \alpha)}{g\left( \frac{c_2 + c_e + p^*_2 - r_2}{k + \alpha} \right)} = \tilde{s}^*_2(\delta_e) - \kappa_2 + r_2 = 0 \), and \( \tilde{s}^*_2(\delta_e) \) as the solution to \( \frac{c_1 - r_1}{m^*_1(s_1|\delta_e) - r_1} = \frac{\kappa_1 s_2^*(\delta_e) - \kappa_2 s_2^*(\delta_e)}{m^*_1(s_1|\delta_e) - r_1} \). Since \( g(v)/G(v) \) is increasing in \( v \), \( \tilde{s}^*_2(\delta_e), \tilde{i}^*_e(\delta_e), \) and \( \tilde{s}^*_2(\delta_e) \) are well-defined and unique. Since \( s^*_2(\delta_e) \) is strictly decreasing in \( \kappa_2 \), \( s^*_2(\delta_e) \) is strictly increasing in \( \kappa_2 \), and \( s^*_2(\delta_e) \) is strictly decreasing in \( \kappa_1 \). Therefore, \( s^*_2(\delta_e) > 0 (s^*_1(\delta_e) < 0) \).
if and only if $\kappa_1 < \hat{\kappa}_1^*(\delta_e) \ (\kappa_1 > \hat{\kappa}_1^*(\delta_e))$, $s_2^*(\delta_e) > 0 \ (s_2^*(\delta_e) < 0)$ if and only if $\kappa_2 < \hat{\kappa}_2^*(\delta_e) \ (\kappa_2 > \hat{\kappa}_2^*(\delta_e))$, and $s_1^*(\delta_e) > 0 \ (s_1^*(\delta_e) < 0)$ if and only if $\nu_e > \hat{\nu}_2^*(\delta_e) \ (\nu_e < \hat{\nu}_2^*(\delta_e))$. \textit{Q.E.D.}

**Proof of Theorem 11: Part (a).** It follows immediately from the proof of Theorem 10(a) that $s_2^*(\delta_e)$ and $s_1^*(\delta_e)$ are independent of $\delta_e$. By its definition, $m_1^*(s_1^*(\delta_e))$, is strictly increasing in $s_1$, and increasing (decreasing) in $\delta_e$ if $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$). Therefore, $s_1^*(\delta_e)$ is strictly increasing (decreasing) in $\delta_e$ if $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$).

**Part (b)** follows directly from part (a) and the definition of $\tilde{\kappa}_1^*(\delta_e)$, $\tilde{\kappa}_2^*(\delta_e)$, and $\tilde{\nu}_2^*(\delta_e)$.

**Part (c).** We now compute $C_0^*(\delta_e)$:

\[
C_0^*(\delta_e) = \mathbb{E}\{s_2^*(\delta_e)(X \wedge Q_1^*(\delta_e)) + s_1^*(\delta_e)(Q_1^*(\delta_e) - X)^+ + \delta[s_2^*(\delta_e)Q_1^*(\delta_e) + (X - Q_1^*(\delta_e))^+, X \wedge Q_1^*(\delta_e)] + (s_1^*(\delta_e) + s_2^*(\delta_e))Q_1^*(\delta_e)(X - Q_1^*(\delta_e))^+, X \wedge Q_1^*(\delta_e))\n\]

\[
= (s_1^*(\delta_e) - s_2^*(\delta_e) - \delta s_1^*(\delta_e)G\left(\frac{P^*_1}{1 + \alpha}\right) + \delta s_2^*(\delta_e)G\left(\frac{P^*_2}{k + \alpha}\right)) + \delta s_1^*(\delta_e)G\left(\frac{P^*_1}{k + \alpha}\right)\mathbb{E}(X \wedge Q_1^*(\delta_e)) + s_2^*(\delta_e)Q_1^*(\delta_e) + \delta s_2^*(\delta_e)\mathbb{E}[X].
\]

Since $s_1^*(\delta_e)$, $s_2^*(\delta_e)$, and $Q_1^*(\delta_e)$ are independent of $\delta_e$, for any $\delta_e > \delta_e$, it follows immediately that

\[
C_0^*(\delta_e) - C_0^*(\delta_e) = (s_1^*(\delta_e) - s_1^*(\delta_e))\mathbb{E}(X \wedge Q_1^*(\delta_e)).
\]

By part (a), $s_1^*(\delta_e) > s_1^*(\delta_e) \ (s_2^*(\delta_e) < s_2^*(\delta_e))$ if and only if $\sigma_1^* > \sigma_n^*$ ($\sigma_1^* < \sigma_n^*$). Thus, $C_0^*(\delta_e) > C_0^*(\delta_e)$ ($C_0^*(\delta_e) < C_0^*(\delta_e)$) if and only if $s_1^*(\delta_e) > s_1^*(\delta_e) \ (s_2^*(\delta_e) < s_2^*(\delta_e))$. Therefore, by Theorem 8(b), $s_1^*(\delta_e) > s_1^*(\delta_e) \ (s_2^*(\delta_e) < s_2^*(\delta_e))$ if and only if $r_2 + \nu_e > \hat{V}_2 \ (r_2 + \nu_e < \hat{V}_2)$. \textit{Q.E.D.}

**Appendix D: Model with Inventory Carryover**

In our base model, we assume that the firm recycles and remanufactures all the excess inventory of the first-generation product at the end of the first period. In some scenarios, the firm carries the leftover first-generation products and, possibly, offer both generations of the product in the second period. Note that the firm may offer two product generations to the market only if the realized first-period demand $X_1$ is lower than the first-period production quantity $Q_1$. Otherwise, the firm only sells the second-generation product in period 2. For tractability, we assume that $V$ follows the uniform distribution on $[0, 1]$.

We begin our analysis with the pricing and production decisions in the second period. As in the base model, the optimal price and purchasing decisions are independent of customer purchasing behavior (equivalently, $\delta_e$). If $X_1 \geq Q_1$, there is no leftover first-generation products to sell in period 2. Hence, the equilibrium decisions are identical to those characterized in Lemma 1. On the other hand, if $X_1 < Q_1$, there are $X_2 = X_1$ repeat customers and $Y_2 = Q_1 - X_1$ leftover inventory in period 2. For a type-$V$ customer, her valuation for the second-generation product is $(k + \alpha)V$, and that for the first-generation product is $V - (1 - k)V = kV$. For any given $(X_2, Y_2)$, let $p_1^2(X_2, Y_2)$ and $p_2^2(X_2, Y_2)$ be the optimal prices for the first-generation and the second-generation products in period 2, respectively. Accordingly, the associated optimal sales quantities of the first- and second- generation product is $Q_1^2(X_2, Y_2)$ and $Q_2^2(X_2, Y_2)$, respectively. We allow repeat customers to trade in used first-generation products in exchange for new first-generation ones. The following lemma characterizes the second-period equilibrium decisions in the model with inventory carryover.
Lemma 5. Assume that $0 < X_1 < Q_1$ (i.e., $X_2^* > 0$ and $Y_2 > 0$).

(a) There exist two thresholds $\theta_1(X_2^*, Y_2)$ and $\theta_2(X_2^*, Y_2)$ ($0 \leq \theta_1(X_2^*, Y_2) \leq \theta_2(X_2^*, Y_2) \leq 1$), such that customers with valuations $V \in [\theta_1(X_2^*, Y_2), 1]$ will trade in their used first-generation products. Specifically, customers with valuations $V \in [\theta_1(X_2^*, Y_2), \theta_2(X_2^*, Y_2)]$ will purchase the first-generation product, and those with valuations $V \in [\theta_2(X_2^*, Y_2), 1]$ will purchase new second-generation ones. Hence, $Q_2^r(X_2^*, Y_2) = X_2^*(\theta_2(X_2^*, Y_2) - \theta_1(X_2^*, Y_2))$ and $Q_2^e(X_2^*, Y_2) = X_2^*(1 - \theta_2(X_2^*, Y_2))$. 

(b) $p_2^c(X_2^*, Y_2) = k\theta_1(X_2^*, Y_2)$ and $p_2^e(X_2^*, Y_2) = k\theta_2(X_2^*, Y_2) + \alpha \theta_2(X_2^*, Y_2)$.

(c) Let $\lambda := Y_2/X_2^*$. There exists a threshold $\bar{\lambda}$, such that if $\lambda > \bar{\lambda}$, $\theta_1(X_2^*, Y_2) = \theta_1^*$ and $\theta_2(X_2^*, Y_2) = \theta_2^*$ for some constants $\theta_1^*$ and $\theta_2^*$; and (ii) $Q_2^r(X_2^*, Y_2) = X_2^*(\theta_2^* - \theta_1^*) < Y_2$. Otherwise, $\lambda \leq \bar{\lambda}$, $Q_2^e(X_2^*, Y_2) = Y_2$.

Lemma 5 implies that, in the model with inventory carryover, the repeat customers with high valuations will purchase second-generation products (i.e., $V \geq \theta_2(X_2^*, Y_2)$), whereas those with moderate valuations (i.e., $V \in [\theta_1(X_2^*, Y_2), \theta_2(X_2^*, Y_2)]$) will purchase the first-generation ones. Moreover, if the leftover inventory level is high (i.e., $\lambda > \bar{\lambda}$), the customers purchase a fraction of the leftover first-generation products ($Q_2^r(X_2^*, Y_2) < Y_2$), and the purchasing thresholds $\theta_1(X_2^*, Y_2)$ and $\theta_2(X_2^*, Y_2)$ are independent of the $(X_2^*, Y_2)$. In this case, the firm will recycle and upgrade the rest of the leftover first-generation products. Otherwise, customers purchase all of the leftover first-generation products ($Q_2^e(X_2^*, Y_2) = Y_2$). For the case $Y_2 = 0$, we define $\theta_1(X_2^*, Y_2) = \theta_2(X_2^*, Y_2) = 1/2(1 + \alpha \rbrack_{X_2^*}X_2^*$), which is the minimal valuation of repeat customers who will join the trade-in program in the base model.

We now study the first-period RE equilibrium. For any customer discount factor $\delta_e$ and first-period production quantity $Q_1$, the first-period equilibrium price

$$p_1^c(Q_1|\delta_e) = \mu + \delta_1\delta_e[(k + \alpha)E(V - \theta_2(X \land Q_1,(Q_1 - X)\rbrack) + kE\{V - \theta_1(X \land Q_1,1)(Q_1 - X)\rbrack + \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack + \alpha \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack + \alpha \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack

Therefore, given the production quantity $Q_1$, the expected total profit $\Pi_1^e(Q_1|\delta_e)$ is given by

$$\Pi_1^e(Q_1|\delta_e) = p_1^e(Q_1|\delta_e)E(X \land Q_1) - c_1 Q_1 + r_1 E((Q_1 - X)\rbrack) - \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack + \theta_1(X \land Q_1,1)(Q_1 - X)\rbrack + \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack + \alpha \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack + \alpha \theta_2(X \land Q_1,1)(Q_1 - X)\rbrack

where we assume zero inventory holding cost for expositional ease. Hence, $Q_1^{i_c}(\delta_e) = \max_{Q_1 \geq 0} \Pi_1^e(Q_1|\delta_e)$ is the equilibrium production quantity, and $p_1^{i_c}(\delta_e) = p_1^e(Q_1^{i_c}(\delta_e)|\delta_e)$ is the equilibrium first-period price with customer discount factor $\delta_e$.

For the NTR model, we consider two cases: (a) $X \geq Q_1$ and (b) $X < Q_1$. In case (a) (i.e., $X \geq Q_1$), there’s no leftover inventory in period 1, the firm should adopt the same pricing strategy as $p_2^e(\cdot,\cdot)$. In case (b) (i.e., $X > Q_1$), there is no new customer in the market in period 2, so the pricing strategy is the same as the
one characterized in Lemma 5. Therefore, in the model with inventory carryover, the impact of customer purchasing behavior should be qualitatively the same as our base model where all excess inventory is recycled and remanufactured: More intensive strategic customer behavior makes trade-in remanufacturing a lot more attractive to the firm, but also a lot more detrimental to the environment. Because the second-period equilibrium pricing strategy is contingent on the realized market sizes \( X^n_x, X^n_2 \), and leftover inventory \( Y_2 \), the socially optimal (second-period) subsidy/tax scheme also depends on \( X^n_x, X^n_2, Y_2 \). Moreover, since the first-period production quantity optimization is no longer a newsvendor problem, the optimal subsidy/tax policy should be contingent on the production quantity \( Q \) and market size realization \( X \) as well. Hence, when inventory carryover is allowed, achieving social optimum is much more difficult, if not infeasible in practice.

**Proof of Lemma 5: Part (a).** For any prices \((p^2_1, p^2_2)\), the customers with valuation \( \{ V \in [0, 1] : (k + \alpha)V - p^2_1 \geq \max\{0, kV - p^2_1\}\} \) will purchase the second-generation product, whereas those with valuation \( \{ V \in [0, 1] : kV - p^2_1 \geq \max\{0, (k + \alpha)V - p^2_2\}\} = \{ V \in [0, 1] : V \in [p^2_1 / k, (p^2_2 - p^2_1)/\alpha]\} \) will purchase the first-generation product. Since \( V \) is uniformly distributed on \([0, 1]\), part (a) follows immediately.

**Part (b).** By the proof of part (a), \( \theta_1(X^n_x, Y_2) = p^2_2(X^n_x, Y_2)/k \) and \( \theta_2(X^n_x, Y_2) = (p^2_2(X^n_x, Y_2) - p^2_2(X^n_x, Y_2))/\alpha \). Direct algebraic manipulation yields that \( p^2_1(X^n_x, Y_2) = k\theta_1(X^n_x, Y_2) \) and \( p^2_2(X^n_x, Y_2) = k\theta_1(X^n_x, Y_2) + \alpha\theta_2(X^n_x, Y_2) \).

**Part (c).** The optimal thresholds \( (\theta_1(X^n_x, Y_2), \theta_2(X^n_x, Y_2)) \) is the solution to the following convex program:

\[
(\theta_1(X^n_x, Y_2), \theta_2(X^n_x, Y_2)) = \arg \max_{0 \leq \theta_1 \leq \theta_2 \leq 1} [(k\theta_1 + r_2)(\theta_2 - \theta_1) + (k\theta_2 + \alpha\theta_2 - c_2 + r_2)(1 - \theta_2)]X^n_x
\]

s.t. \((\theta_2 - \theta_1)X^n_x \leq Y_2\).

It is straightforward to show, by concavity, that \( (\theta_1(X^n_x, Y_2), \theta_2(X^n_x, Y_2)) \) uniquely exists, and is determined by the ratio \( \lambda = Y_2/X^n_x \). In particular, let \((\theta_1^*, \theta_2^*)\) be the solution to (5) without the constraint \((\theta_2 - \theta_1) \leq \lambda \). Let \( \bar{\lambda} := \theta_2^* - \theta_1^* \geq 0 \). If \( Y_2/X^n_x > \bar{\lambda} \), the constraint \((\theta_2 - \theta_1) \leq \lambda \) is non-binding and thus \((\theta_1(X^n_x, Y_2), \theta_2(X^n_x, Y_2)) = (\theta_1^*, \theta_2^*) \). If \( Y_2/X^n_x \leq \bar{\lambda} \), the constraint \((\theta_2 - \theta_1) \leq \lambda \) is binding and thus \( Q^n_2(X^n_x, Y_2) = (\theta_2(X^n_x, Y_2) - \theta_1(X^n_x, Y_2))X^n_x = Y_2\). \( Q.E.D. \)