Responsive Pricing of Fashion Products: The Effects of Demand Learning and Strategic Consumer Behavior

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Abstract

This paper studies the potential benefits of responsive pricing and demand learning to sellers of seasonal fashion goods. As typical in such markets, demand uncertainty is high at the beginning of a season, but there is a potential opportunity to learn about demand via early sales observations. Additionally, while the consumers have general preference for purchasing a fashion product earlier rather than later in the season, they may exhibit strategic behavior – contemplating the benefits of postponing their purchase in anticipation of end-of-season discounts. Our results demonstrate that the benefits of responsive pricing, in comparison to a benchmark case of a fixed-price policy, depend sharply on the nature of the consumers’ behavior. Interestingly, in stark contrast to markets of myopic consumers, when the consumers are all strategic the benefits of responsive pricing tend to worsen when there is a higher potential for learning. We explain this counter-intuitive outcome by pointing to two phenomena: the spread effect and information shaping. For example, sellers of fashion products that consider upgrading their pricing systems to incorporate "accurate response" strategies (i.e., integrating learning and responsive pricing) should be aware of the possibility that such action might lead them to a new and potentially worse equilibrium, particularly when there is a higher opportunity to learn. Despite the fact that price commitment completely eliminates the seller’s ability to learn, it appears to increasingly dominate responsive pricing as the portion of strategic consumers in the market increases. But while performing better than responsive pricing, a price commitment policy is typically limited in performing effective discrimination. Finally, we studied the potential benefits of quick response strategies – ones that embed both dynamic pricing and quick inventory replenishment during the sales season – and found that they are particularly significant under strategic consumer behavior. We explain this result by arguing that quick response provides the seller with a real option that serves as an effective implicit threat to the consumers: encouraging them to buy earlier at premium prices rather than wait for discounts at the end of the season.

Key words: demand learning; dynamic pricing; strategic consumer behavior; revenue management; game theory
1 Introduction

Dynamic pricing of fashion products is a complex, but a potentially rewarding revenue management activity (see, e.g., Elmaghraby and Keskinocak (2003), van Ryzin and Talluri (2005), and Aviv and Vulcano (2012) for comprehensive reviews). The complexity arises due to several factors, such as the need to take capacity levels into account, the relatively-high degree of market uncertainty, and the nature of the consumers' purchasing behavior. Research in this area has studied two important facets of this topic: learning, and strategic consumer behavior.

Learning in dynamic pricing systems is of particular relevance to fashion products (see, e.g., Heun (2001)). For example, Fisher (2009) argues that in retail settings of innovative products, a seller can learn about demand from early sales. Based on data gathered from a skiwear supply chain, he observed that the demand forecast error could be reduced from 55% to merely 8% after utilizing just two weeks of sales information. The implication of this observation on the potential value of dynamic pricing is significant. When selling a fashion product, a retailer can benefit from the ability to change the price later in the sales season in response to sales observations - in other words, the ability to act upon information; see research papers such as Balvers and Cosimano (1990), Aviv and Pazgal (2005), Besbes and Zeevi (2009), Araman and Caldentey (2009), Farias and Van Roy (2010), as well as the case study Ghemawat and Nueno (2003).

Unfortunately, in many retail environments, consumers are cognizant of the retailers’ pricing strategies, or at least they are aware of the possibility that price discounts may be offered later in the season. Consequently, the consumers may exhibit “strategic behavior” – i.e., making a choice between purchasing the product at a premium price early in the season, or postponing their purchasing decisions to a later time in the season when a price discount may be offered. Research on this topic in the economics literature goes back to the seminal paper of Coase (1972), and other key papers such as Stokey (1979) and Landsberger and Meilijson (1985). Over the past decade, the study of the effects of strategic consumer behavior on dynamic pricing of finite capacities has experienced significant growth; see, e.g., Su (2007), Elmaghraby et al. (2008), Aviv and Pazgal (2008), and Su and Zhang (2008, 2009). One of the key messages raised by the latter literature is that a retailer’s ability to vary prices during a sales season can work to its detriment when the market consists of strategic consumers. Specifically, knowing that the retailer has the flexibility to change prices, consumers expect that the retailer will offer markdowns later in the season, which in turn encourages them to avoid
paying the premium prices charged at the beginning of the sales season. In many cases, this phenomenon can turn a dynamic pricing policy to sub-perform even a fixed-price policy. For brevity of exposition, we refer the interested reader to Aviv and Vulcano (2012) for a detailed survey of the existing literature on strategic consumer behavior, and to Netessine and Tang (2009) which contains several review chapters dedicated to this particular subject.

The above contradicting messages, one in favor of responsive pricing (due to learning), and one against it (due to strategic consumer behavior), leave a serious question opened – are responsive pricing strategies effective or counter-productive when selling fashion-like products? We use the title responsive pricing to emphasize that the seller can adjust prices in reaction to early sales information. Similarly, we use the title fashion-like products to emphasize our focus on markets characterized by high degree of uncertainty, but with an opportunity to (partially) resolve it via early sales observations. To address the above question, we propose an integrative theoretical model that reflects both learning and strategic consumer behavior. Our model is stylized in nature, and considers a monopolist retailer that sells a finite quantity of a given fashion product over a two-period horizon. The seller selects a price for the first period, based on limited demand information. Then, the seller adjusts the price in the second period, based on the sales made in the first period. We utilize a Bayesian learning mechanism to model the demand learning process, through which information about the average market size for the product evolves. As we shall discuss later, this learning process brings up an important connection between pricing decisions and the value of information extracted from sales data. Finally, our model considers a heterogeneous pool of strategic consumers whose valuations are drawn from a uniform distribution.

Our model enables us to examine four interesting subjects pertaining to the use of responsive pricing strategies. First, we analyze the influence of the degree of resolvable market uncertainty on the effectiveness of responsive pricing. Without strategic consumers in the market, the more one can learn about the market in the first period, the more one is interested in being able to act upon such information. However, this logic breaks down in face of strategic consumer behavior. Interestingly, we find that strategic consumer behavior becomes increasingly detrimental as the market provides a higher potential for learning. We explain this counter-intuitive outcome by pointing to two phenomena: the spread effect and the active learning effect. Furthermore, we argue that the latter effect is driven by information shaping – the conscious collective attempt of the consumers to influence the seller’s interpretation of the market size. Second, we constructed
a hypothetical model in which we eliminated the seller’s ability to learn from sales, and surprisingly, we observe that the seller’s expected revenue performance actually improves. This outcome is of potential interest to sellers of fashion products who consider upgrading their pricing systems to incorporate “accurate response” strategies (i.e., integrating learning and responsive pricing). Third, we explore the effectiveness of price commitment strategies. Our results suggest that despite the fact that price commitment completely eliminates the seller’s ability to learn, it appears to increasingly dominate responsive pricing as more strategic consumers are present in the market. Moreover, the most significant benefits of commitment were observed for the cases in which the degree of market uncertainty was high. However, while performing better than responsive pricing, when the market size is highly uncertain and when the seller’s inventory is sufficiently large, price commitment is very limited in its ability to perform effective discrimination. Finally, we studied the potential benefits of quick response strategies; ones that embed both dynamic pricing and quick inventory replenishment during the sales season. Our results suggest that the option to replenish inventory close to the end of the season serves as an effective implicit threat by the seller to the consumers in a way that discourages strategic waiting. Consistent with the earlier paper of Cachon and Swinney (2009) we also argue that the benefits of quick response strategies might be significantly underestimated in the presence of markets with strategic consumers.

The rest of the paper is organized as follows. In §2 we introduce and analyze the main model. Section 3 presents our numerical study and our key discussions and insights. In §4 we discuss markets in which only a portion of the consumers behave strategically. Section 5 explores the effectiveness of price commitment strategies, and §6 studies the potential benefits of quick response strategies. The paper’s conclusions are presented in §7. All proofs are given in the Appendix.

2 Main Model

We develop a game theoretic model to study the research questions listed in the introduction. Consider a seller who sells $Q$ units of a product to consumers in a relatively short season, divided into two periods: a regular selling period (we shall refer to it as the “first period”), and a final sales period (the “second period”). At the beginning of the first period, the seller sets a price, $p_1$. Then, at the beginning of the second period, the seller can change the price to $p_2$, after observing the initial sales.

To reflect the demand uncertainty as well as the seller’s ability to learn from sales, we adopt a special type
of statistical framework known as the \textit{Gamma-Poisson model}. Specifically, we assume that the number of consumers in the market is drawn from a Poisson distribution with a mean value of $\Lambda$, that is unknown. The value of $\Lambda$ – the average market size – has a prior Gamma distribution with a mean $E[\Lambda]$ and a coefficient of variation denoted by $CV \doteq \sqrt{\text{Var}(\Lambda)}/E[\Lambda]$. Clearly, because sales realizations depend on the market size, the seller is able to resolve part of the uncertainty about $\Lambda$ after observing the number of units sold in the first period. Note that the apriori (i.e., before sales observation) mean and standard deviation of the demand are equal to $E[\Lambda]$ and $\sqrt{E[\Lambda] + E^2[\Lambda] \cdot CV^2}$, respectively.

Among the possible alternatives for statistical models of demand (such as the Normal-Normal or the Beta-Binomial models; see e.g., Raiffa and Schlaifer 2000) the Gamma-Poisson model is appropriate, as it allows to specify large demand uncertainty scenarios that are often observed in fashion product sales. For example, the well-known article of Fisher (1997) reports that the demand forecasts for new products offered by a fashion skiwear designer (Sport Obermeyer) often err by as much as 200%. For the sake of illustration, suppose that $E[\Lambda] = 10$, and that the apriori standard deviation of the demand is equal to 100%-200% of that mean demand. This would translate to $CV$ levels in between 1 and 2.

The consumers in the market are heterogeneous in their valuations, which are independently drawn from a uniform distribution on $[0,1]$. If a consumer chooses to wait for the sales period, then his valuation for the product will be discounted by a factor $\delta \in (0,1]$. Such a decline in valuation reflects a situation where a product consumed later in the selling season provides a lower utility (e.g., purchasing a bathing suit at the end of a summer), or a situation where consumers are “style-conscious”. Unlike perishable products for which one would expect low $\delta$ values, or durable goods for which one would expect relatively high $\delta$ values, we believe that fashion-like products are generally associated with mid-levels of $\delta$. In our main model, the consumers are \textit{strategic} in the sense that they select the timing of their purchases by comparing the expected surpluses associated with “buy now” vs. “wait” (to the second period) decisions. When appropriate, we also discuss and present results for the case in which the consumers in the market are all \textit{myopic}; namely, they do not consider future pricing in their purchasing decisions.

Our analyses are based on the study of the following sequential game. In the first stage, the seller posts the price $p_1$. Thereafter, the consumers and the seller become engaged in a competitive situation (the \textit{subgame}) in which the consumers decide on whether to buy in the first period or wait for a possible discount, and the seller determines its second-period price as a function of the quantity sold in the first period. We
analyze the subgame by solving for a Nash equilibrium between the seller and the consumers. All problem parameters are common knowledge in this game. In addition, we assume that the seller and the consumers are risk neutral and aim at maximizing their expected payoff (revenue and surplus, respectively). Since the seller can set the second-period price in response to the initial sales and leftover inventory information, we refer to the dynamic pricing strategy as “responsive pricing.”

In the next two subsections, we first conduct the equilibrium analysis of the subgame (Section 2.1); then we study the seller’s optimal pricing decision in the first stage (Section 2.2).

2.1 Analysis of the Subgame

The analysis of the subgame is challenging due to two complicating factors. First, the consumers’ purchasing decisions must be based on the seller’s second-period pricing policy, which in turn depends on the consumers’ purchasing strategy. Second, as in most models that study strategic consumer behavior, the consumers also compete against each other because their decisions affect the product availability in both periods. Any equilibrium in the subgame requires that each player adopts the optimal (payoff-maximizing) strategy given the rest of the players’ strategies. The equilibrium analysis proceeds in two steps as follows. First, we analyze the consumers’ and the seller’s problems in the subgame separately; then we characterize the equilibrium by tying the two parties’ problems together. Since the initial level of inventory $Q$ and the first-period price $p_1$ are common knowledge at this stage, we generally avoid referring to them explicitly in the analysis of this section.

2.1.1 The Consumers’ Purchasing Problem

Let $\{p_2(q) : q = 0, \ldots, Q\}$ be any arbitrary second-period pricing strategy adopted by the seller, where $q$ denotes the remaining inventory immediately after the first period sales. The consumers need to decide whether to purchase in the first period. Specifically, a consumer with valuation $v$ compares between the expected surplus gained from an immediate purchase and the expected surplus associated with waiting for the second period. Both assessments of the surpluses are complicated by the fact that the consumers must weigh in the likelihood that a unit will be available to them during these periods. Theorem 1 below establishes that for any arbitrary pricing strategy of the seller and any arbitrary purchasing behavior of the other consumers in the market, it is optimal for an individual consumer to adopt a so-called threshold policy.
Theorem 1 Consider a consumer with a given valuation \( v \). For any initial inventory level \( Q \), pricing scheme \( \{p_1, p_2(\cdot)\} \), and any set of purchasing rules adopted by all other consumers in the market, it is optimal for that consumer to follow a threshold policy \( \theta \). Namely, the consumer will attempt to purchase a unit at the price \( p_1 \) if \( v > \theta \); otherwise, the consumer will wait for the second period. Furthermore, the optimal threshold value is unique.

[All proofs appear in the Appendix.] Note that given the specification of the information structure in our model, the probabilistic assessment of the purchasing rules adopted by all consumers in the market must be the same from each consumer’s perspective. Therefore, Theorem 1 implies that, in equilibrium, all consumers must adopt the same unique threshold value. Thus, mixed-strategy equilibria are not predicted in this game.

Consider a setting in which all consumers adopt a threshold purchasing policy \( \theta \), under a given pricing strategy \( \{p_1, p_2(\cdot)\} \). Let \( X(\theta) \) be the number of immediate purchases in the first period, which follows a Poisson distribution with a mean of \( \Lambda \cdot (1 - \theta) \). Below, we consider a particular consumer with valuation \( v \), and evaluate the expected surplus that is gained by each of the two possible actions. There are two common approaches for modeling the expected surplus values: the conditional\(^1\) and the unconditional probability calculations. Under the unconditional approach (see, e.g., Yin et al. 2009), which we adopt in this paper, a consumer evaluates the current surplus by multiplying the potential surplus \( (v - p_1) \) by the probability that a unit would be available in the current period if he demands it. This is naturally the case if a consumer buys the product online and there is no indication of product availability, or if the consumer considers travelling to the store to purchase the product on the advertised price.

To calculate the probability of a unit being available to an individual consumer during a given period, we consider the following model setup. Suppose that all consumers interested in purchasing the product arrive during a very small interval of time during at the very beginning of the first period, according to a Poisson process with a rate \( a = \Lambda \cdot (1 - \theta) \). Then, from any individual consumer’s perspective, the size of the rest of the market (say, \( Y \)) remains Poisson with the same parameter. Furthermore, suppose that if demand cannot be fully satisfied during a given period, all available units are randomly allocated among the consumers (who

\(^1\) Under the conditional approach, we assume that the consumer can verify the availability of a unit when making the purchasing decision. If a unit is available, then the immediate surplus would be \( (v - p_1) \). But if the customer decides to wait, the expected surplus needs to be calculated based on the information that a unit is currently available. In other words, the expression in (2) must be replaced by conditional expectation. We refer the reader to Osadchiy and Vulcano (2010) where such approach is adopted.
attempt to buy) with equal probabilities. Consequently, the likelihood that a unit is allocated to a given consumer can be written using the expression

\[ H(Q,a) = \mathbb{E} \left[ \frac{Q}{\max \{Q,Y+1\}} \right] = \sum_{y=0}^{Q-1} a^y e^{-a/y} + \sum_{y=Q}^{\infty} \left( \frac{Q}{y+1} a^y e^{-a/y} \right) \]

Hence, if the consumer makes a buy-now decision, then he expects to gain a surplus of

\[ s_1(v|\theta) = (v - p_1) \cdot \mathbb{E}_{\Lambda} [H(Q,\Lambda (1-\theta))] , \tag{1} \]

If the consumer decides to wait for the second period, his expected surplus would be:

\[ s_2(v|\theta) = \mathbb{E}_{\Lambda} \left[ \mathbb{E}_{X|\theta} \left[ (\delta v - p_2 (Q - X(\theta)))^+ \cdot H \left( Q - X(\theta), \Lambda (\theta - p_2 (Q - X(\theta)) / \delta)^+ \right) \right] \right] . \tag{2} \]

(Note that both functions depend on \( \{Q,p_1,p_2(\cdot)\} \), but we omit those parameters for brevity of exposition.) The expression within the inner expectation is once again a product of the surplus that could be gained from a purchase in the second period and the likelihood that a unit will be available in that period. Note that all (and only) consumers in the segment \( (p_2/\delta, \theta) \) will be interested in buying the product in the second period, if a price \( p_2 \) is set at that time.

As shown in the proof of Theorem 1, the consumer’s best action is given by a unique threshold value, say \( \theta^*(\theta) \), for which \( s_1(v|\theta) > s_2(v|\theta) \) for all \( v > \theta^*(\theta) \), and \( s_1(v|\theta) \leq s_2(v|\theta) \) for all \( v \leq \theta^*(\theta) \). The following proposition states that an equilibrium in the consumers’ game always exists.

**Proposition 1** For any given \( Q \) and pricing scheme \( \{p_1,p_2(\cdot)\} \), there exists a Nash equilibrium \( \theta^* \in [p_1,1] \) in the consumers’ game, where all consumers adopt the same \( \theta^* \) threshold purchasing policy.

We emphasize that the equilibrium threshold \( \theta^* \) may not be unique in the consumers’ game for some arbitrary pricing strategies. In fact, we have found numerical examples in which the game possesses up to three equilibria. This issue need not be of a concern. First, the technical analysis of the subgame between the seller and the consumers remains the same, regardless of the predicted equilibria in the consumers’ game. Second, as we demonstrate later, no multiple equilibria appeared in our analysis of the game where optimal prices (rather than arbitrary) are used.

### 2.1.2 The Seller’s Second-Period Pricing Problem

We now turn to the analysis of the seller’s best pricing policy \( p_2(\cdot) \) in the second period, in response to any given consumer threshold policy \( \theta \), and in the presence of demand learning. In the second period, prices are
determined on the basis of the first-period sales and the anticipated consumer purchasing behavior. Clearly, the first-period sales not only affect the remaining quantity, but also influence the seller’s perception of the market size. Suppose that the seller updates its belief about the expected market size, \( \Lambda \), using a Bayesian updating scheme, and recall that \( \Lambda \) has a Gamma prior distribution. In particular, the characteristics of the posterior distribution of \( \Lambda \) can be fully determined using two variables: the first-period sales (say, \( x \)), and the consumers’ policy parameter (say, \( \theta \)). Note that although the first period price (\( p_1 \)) and the initial inventory (\( Q \)) influence learning, we do not need to include them explicitly in the updating mechanism, since the knowledge of \( x \) and \( \theta \) is sufficient. By construction, the updated distribution of the market size is still Poisson, and we refer to that posterior random variable as \( \hat{\Lambda} (x, \theta) \).

Let \( q \) be the remaining quantity at the beginning of the second period. Then, the seller’s pricing problem at that time can be written in the following form:

\[
\pi_2^* (q, x, \theta) = \max_{p_2 \in [0, \delta]} \left\{ p_2 \cdot E_{\hat{\Lambda}(x, \theta)} \left[ N \left( q, \hat{\Lambda} (x, \theta) \cdot (\theta - p_2/\delta) \right) \right] \right\},
\]

where \( N (q, a) \) is defined as the expected number of units sold, when there are \( q \) units available and the number of consumers interested in purchasing is Poisson-distributed with a mean of \( a \). Theorem 2 below establishes that the function within the maximization problem (3) is strictly concave and possesses a unique and interior solution in \([0, \delta] \).

**Theorem 2** The objective function in (3) is strictly concave in \( p_2 \). The optimal solution to (3), \( p_2^* (q, x, \theta) \), lies in the interior of the range \([0, \delta] \), and satisfies the first-order condition

\[
E_{\hat{\Lambda}(x, \theta)} \left[ N \left( q, \hat{\Lambda} (x, \theta) \cdot (\theta - p_2/\delta) \right) - p_2 \cdot \frac{\hat{\Lambda} (x, \theta) \cdot (\theta - p_2/\delta)}{\delta} \sum_{y=0}^{q-1} P_y \left( \hat{\Lambda} (x, \theta) \cdot (\theta - p_2/\delta) \right) \right] = 0,
\]

where \( P_y (\lambda) \) is the probability mass function at \( y \) of Poisson distribution with mean \( \lambda \). Furthermore, the optimal solution \( p_2^* (q, x, \theta) \) is increasing in \( \theta \) and \( x \), and decreasing in \( q \).

Theorem 2 demonstrates that the second-period optimal prices are monotonically increasing in the consumer threshold policy \( \theta \). This result can be viewed as a combination of two effects, which we qualitatively explain below. The first effect is direct – a larger \( \theta \), for given values of \( q \) and \( x \), means that the seller is selling the same quantity but to a larger segment of customers (with equal or higher valuations); hence, the optimal second-period price is expected to increase. The second effect is indirect, driven by the learning mechanism. For any given \( x \), a larger \( \theta \) leads the seller to statistically estimate the value of \( \Lambda \) to be higher. Consequently,
the seller is expecting a larger number of customers to be interested in purchasing the product in the second period; in turn, this leads to a similar effect on the optimal second-period price. The increase in \( x \) can be explained using precisely the latter argument. The monotonicity of the optimal price in \( q \) – higher inventory leads to lower second-period prices for given \( x \) and \( \theta \) – is also intuitive, given that the learning effect is kept constant.

By combining the above analyses of the consumers’ and the seller’s problems, we are ready to prove the existence of subgame equilibrium for any arbitrary first period price \( p_1 \). Again, in any equilibrium \( \{\theta^*, p_2^*(\cdot)\} \), each player must adopt its optimal strategy in response to all other players’ strategies.

**Proposition 2** For any arbitrary first period pricing \( p_1 \), there exists a Nash equilibrium in the subgame between the consumers and the seller.

Unfortunately, due to the complexity of the players’ payoff functions, we were unable to analytically prove that the subgame possesses a unique equilibrium. However, we will fully address this standing concern shortly, in Proposition 3 below.

### 2.2 Optimal Pricing in the First Stage

The seller’s first-period pricing decision is an optimization problem that involves a search for the best price \( p_1^* \), given the initial level of inventory \( Q \). This problem is quite challenging even without the significant complexity introduced by the presence of strategic consumer behavior. Note that the seller must optimally settle the following trade-off. On one hand, the seller may wish to charge a relatively high initial price so as to collect high revenues in the first period; on the other hand, a high price limits the volume of early sales, which constrains the seller’s learning ability and hence imposes a negative effect on future revenues. This dilemma is referred to as the *exploitation-versus-exploration* trade-off in the literature; see, e.g., Araman and Caldentey (2009). In our model, when setting the initial price \( p_1 \), the seller must take into account the subgame discussed in the previous section. In particular, for any given choice of \( p_1 \) and any predicted consumer behavior \( \theta \), the seller’s revenue performance can be calculated using the conditional expectation form

\[
\pi^R(Q, p_1, \theta^*) = p_1 \cdot E_{\Lambda} [N (Q, \Lambda \cdot (1 - \theta^*))] + E_{\Lambda} \left[ \sum_{x=0}^{Q-1} \pi_2^* (Q - x, x, \theta^*) \cdot P_x (\Lambda \cdot (1 - \theta^*)) \right].
\]
(We use the superscript $R$ to denote responsive pricing.) Thus, the seller’s first-price decision problem can be stated as follows:

$$\pi^R(Q) = \max_{p_1 \in [0,1]} \{ \pi^R(Q, p_1, \theta^*(p_1)) \}$$

(5)

where $\theta^*(p_1)$ is the predicted consumer behavior in the subgame (again, the parameter $Q$ is omitted for convenience). To solve this problem, we could have treated $\pi^R(Q)$ as a constrained non-linear optimization on $p_1$. However, we took an alternative approach in which we present the seller’s pricing problem as a segmentation problem. That is, instead of choosing $p_1$, the seller chooses the $\theta$ value to separate the consumers who will buy or wait in the first period. This way, the seller’s decision problem is presented as an optimization over the consumers’ threshold value $\theta$. The significant advantage of this approach is that for any given $\theta$, the determination of the second-period price becomes independent of $p_1$, and is done via an optimization procedure using (4). Once the second-period prices are identified, we simply use the indifference equation

$$s_1(\theta|\theta) = s_2(\theta|\theta),$$

(6)

to set $p_1$, and evaluate the expected revenue using the equation

$$\pi^R(Q|\theta) = p_1 \cdot E_\Lambda [N(Q, \Lambda \cdot (1 - \theta))]$$

$$+ E_\Lambda \left[ \sum_{x=0}^{Q-1} p_2(Q - x, x, \theta) \cdot N(Q - x, \Lambda \cdot \left( \theta - \frac{p_2(Q - x, x, \theta)}{\delta} \right)) \cdot P_x(\Lambda \cdot (1 - \theta)) \right].$$

(7)

we then sequentially calculated the expected revenue $\pi^R(Q|\theta)$ over the range $\theta \in [0,1]$ to identify the best segmentation (say $\theta^*$) that leads to the optimal expected revenue for the seller; i.e., $\theta^* = \arg\max_\theta \pi^R(Q|\theta)$. Interestingly, we have noticed throughout our numerical studies (see §3), without exception, that the first-period price that makes the marginal consumer with valuation $\theta$ indifferent is strictly increasing in $\theta$. The implication of this numerical observation is significant, in view of the next proposition.

**Proposition 3** If $p_1(\theta)$, the implicit function defined by (6), is strictly increasing in $\theta$, then there is a unique Nash equilibrium in the subgame analyzed in Section 2.1.

We can now conclude that for each and every case studied in our extensive numerical study, the subgame between the seller and consumers (see §2.1), possesses a unique equilibrium.
3 The Benefits of Responsive Pricing: A Numerical Study

To measure the potential benefits of responsive pricing, we follow a standard approach. As a baseline, we consider a setting in which the seller employs a single price across the season. Clearly, in such situations, strategic consumer behavior is irrelevant. Moreover, the pricing problem practically reduces to a single-period price optimization as follows:

$$
\pi^F (Q) = \max_{p \in [0,1]} \{\mathbb{E}_\Lambda [p \cdot N (Q, \Lambda \cdot (1 - p))]\}.
$$

Consequently, one can gauge the percentage benefit of responsive pricing by the metric

$$
\frac{\pi^R (Q)}{\pi^F (Q)} - 1.
$$

Due to the complexity of our model setting, closed-form expressions for the seller’s expected revenue functions are practically impossible to obtain. Therefore, we have conducted a numerical study that spans across 3,000 instances, covering all possible parameter combinations of: (i) Fifteen levels of the degree of uncertainty about the average market size, measured by the coefficient of variation $CV = \sqrt{\text{Var} (\Lambda) / \mathbb{E}[\Lambda]} \in \{0.2, 0.4, 0.6, \ldots, 3\}$; (ii) ten levels of the consumer discount factor $\delta \in \{0.1, 0.2, ..., 1\}$; and (iii) twenty levels of the initial inventory $Q \in \{1, 2, 3, \ldots, 20\}$. For all instances, and without any significant loss of scope, we held the average value of the market size fixed at $\mathbb{E}[\Lambda] = 10$.

3.1 Preliminary Results

Imagine for a moment a market consisting of myopic consumers only. In this case, responsive pricing provides the seller with a valuable option. Not only that sales realized in the first period result in up-to-date inventory information (inventory status update), but the seller is also able to resolve part of the uncertainty regarding the market size (demand learning) prior to setting the second-period price. Therefore, it is obvious that responsive pricing dominates a fixed-price policy. For example, across all 3,000 scenarios in our study, the benefit of responsive pricing in the case of myopic consumers fell in the range (+0.8%, +33.3%), with an average value of +13.2%. In contrast, the corresponding values for the case of strategic consumers were (−13.0%, +9.6%), and −5.6%, respectively. In 86% of the instances (2,584 out of 3,000) the benefits of responsive pricing were negative.

Consistent with the extant literature (e.g., Aviv and Pazgal 2008 and Lai et al. 2010), we find that the benefits of responsive pricing are highly dependent on the consumers’ discount rate $\delta$; see Figure 1. The

\[\text{We have also tested a set of additional 12,000 instances with } \mathbb{E}[\Lambda] = 50. \text{ For this set, we extended the values of } Q \text{ ranging from 1 to 80. The results based on this augmented study were fully aligned with our reported observations, and hence we omit them.}\]
pattern observed in the figure for the myopic consumers case is intuitive. As $\delta$ increases, the valuations of the consumers in the second period become proportionally higher, and hence the seller can extract higher revenues under responsive pricing. As expected, the benefits of responsive pricing are significantly affected by strategic consumer behavior; see the right-hand-side of Figure 1. But here, the value of $\delta$ has two opposing effects. The first, positive effect, is similar to that discussed in the case of myopic consumers. The second, negative effect, stems from the fact that a higher $\delta$ value corresponds to a higher willingness to wait among strategic consumers (i.e., more patient consumers). Together, these effects lead to a non-monotonic pattern of the benefits with respect to $\delta$, with the worst impact predicted for mid values of $\delta$ (say $0.5 - 0.8$). As explained in the introduction, mid values of $\delta$ are reasonably expected in markets of fashion goods.

3.2 The Adverse Impact of Responsive Pricing in Fashion Markets Consisting of Strategic Consumers

Confining our attention to mid-values of $\delta$ and our core model of strategic consumers, we have taken a closer look at the benefits of responsive pricing as a function of the $CV$ value and the level of inventory. Hereafter, we shall generally refer to fashion markets as ones characterized by mid $\delta$ values, low-to-medium levels of inventory (see further discussion in §3.8), and relatively large $CV$ values. For example, Figure 2 below shows the benefits of responsive pricing for the case of $\delta = 0.7$. Interestingly, we found that responsive pricing tends to become increasingly counter-productive as the value of $CV$ increases. In other words, it is in those settings in which the seller has a larger opportunity to learn from early sales realizations, that responsive pricing might
Figure 2: The benefits of responsive pricing under strategic consumer behavior \((\pi^R(Q)/\pi^E(Q) - 1)\), for the case \(\delta = 0.7\).

significantly hurt revenue performance rather than contribute. This observation has important ramifications for sellers of fashion goods that consider adopting dynamic pricing decision technologies. When the market consists of myopic consumers only, the benefits (excluding the system deployment costs) are expected to be high, but on the other extreme – when the consumers are all strategic – the impact is expected to be substantially negative. This obviously emphasizes the critical importance of understanding the extent to which strategic consumer behavior persists in the specific market; see, e.g., Li et al. (2012) as well as §4 in this paper.

To better understand the drivers behind the negative benefits observed in our study, we propose the following methodology. Note that the value of responsive pricing is driven by two key factors: (i) the mere ability to offer two prices, to which we shall refer as the base value of segmentation; and (ii) the ability to observe information, to which we shall refer as the value of information. To measure the base value of segmentation, we consider a hypothetical setting in which the seller adopts an open-loop pricing decision rule. As commonly interpreted in the literature (e.g., Puterman 2005), an open-loop policy does not require the observation of the system’s state (sales data in our paper) except for the very beginning state (i.e., initial inventory position). In this case, the seller utilizes a planning process that specifies the prices for both periods upfront, but without a commitment power. To avoid a potential confusion, we emphasize that the second-period price is still determined by identifying a Nash Equilibrium in the subgame. In particular, the
seller’s best response to any given consumers’ purchasing strategy \( \theta \), can be calculated as follows:

\[
\max_{p_2 \in [0, \theta]} \left\{ p_2 \cdot E_{\Lambda} \left[ N(Q, \Lambda \cdot (1 - p_2/\delta)) - N(Q, \Lambda \cdot (1 - \theta)) \right] \right\}.
\]

The complete equilibrium analysis of this setting is identical to that of the main model stated in §2, and we refer to this model as the case of dynamic pricing with no sales information, whose performance is denoted by \( \pi^{NI}(Q) \). By comparing the latter value to the revenue performance of a fixed-price strategy, we obtain the base value of segmentation: \( \pi^{NI}(Q)/\pi^F(Q) - 1 \); in other words, it is the benefit the seller can gain by merely utilizing two prices, but without using sales information. Similarly, we define the value \( (\pi^R(Q) - \pi^{NI}(Q))/\pi^F(Q) \) as the value of information, where the division by \( \pi^F(Q) \) is used in order to obtain a standardized reference basis:

\[
\text{Benefits of Responsive Pricing} = \left( \frac{\pi^R(Q) - \pi^F(Q)}{\pi^F(Q)} \right) = \left( \frac{\pi^{NI}(Q) - \pi^F(Q)}{\pi^F(Q)} \right) + \left( \frac{\pi^R(Q) - \pi^{NI}(Q)}{\pi^F(Q)} \right)
\]

\[
= \text{Base Value of Segmentation} + \text{Value of Information}
\]

3.3 The Base Value of Segmentation and the Spread Effect

When the consumers are all \textit{myopic}, the base value of segmentation is always positive. Across all 3,000 instances in our numerical study, this value ranged between 0.9% and 33.3%, with an average of 12.8%. In stark contrast but as expected, strategic consumer behavior can dramatically affect the base value of segmentation. Under such market setting, this value was mostly negative (about 86% of the instances), and ranged between \(-11.1\%\) and \(9.6\%\), with an average of \(-4.8\%\).

In a market consisting of myopic consumers, the base value of segmentation is driven by the fact that a \textit{non-contingent (on sales)} but \textit{rationally-expected} two-price scheme allows the seller to target two types of consumers: charge a relatively high price in the first period in order to exploit scenarios in which high-valuation consumers arrive to the store. Then charge a reduced price in the second period to gain revenues from lower-valuation consumers. Let us look at Figure 3 below as a representative illustration of our results. Restricting our attention to the case of myopic consumers, presented on the left-hand-side of the figure, we can see that the base value of segmentation increases as a function of \( Q \) in a concave fashion. However, the \( CV \) value has two opposing effects on the base value of segmentation. When \( Q \) is small (say \( Q = 1 \)), and \( CV \) gets larger, the seller increasingly benefits from an aggressive price \textit{betting strategy}, under which a high first-period price is posted and followed by a significant markdown in the second period. This is merely due
to the fact that high market size scenarios are more likely to happen. As $Q$ increases, the betting strategy becomes less appropriate. Here, a larger $CV$ may negatively affect the base value of segmentation by the virtue of uncertainty in the decision-making process. In other words, with increased uncertainty regarding the market size, the seller’s ability to accurately segment the market becomes impaired.

Let us turn our attention to markets consisting of strategic consumers, presented on the right-hand-side of Figure 3. First, consider the smallest value of $Q$ (i.e., $Q = 1$). When the spread in the possible market size scenarios increases (i.e., higher $CV$ values), the “extreme” scenarios of very small market size and very large market size realizations are more likely to happen. This has an interesting effect on the expected surplus values associated with “buy now” and “wait” decisions. If the market size is very large, both surplus values become negligible, due to the low likelihood of obtaining the unit. But if the market size is small, a consumer is likely to find a unit available in the second period at the reduced price. In fact, with a larger $CV$-value and lack of a sale in the first period, the seller is rationally expected to offer a drastic discount. In sum, a higher market size uncertainty makes waiting preferable. As we increase the value of $Q$, the consumers face a lower shortage risk in the second period, and they are more inclined to wait (see, e.g., Liu and van Ryzin 2008). In fact, the base value of segmentation becomes independent of $CV$, as the shortage risk declines to zero (see, e.g., the cases of $Q = 15$ and $Q = 20$ in the figure).

Let us define the degree of strategic behavior in equilibrium by the ratio $(\theta - p_1) / (1 - p_1)$; namely, the percentage of consumers that attempt to wait for sales, among those who can afford the price $p_1$. Figure 4 below shows the value of this measure as a function of $CV$ for different levels of inventory. As can be seen, the degree of strategic behavior is generally increasing in $CV$ value, and the difference is particularly large.
under the smallest value of $Q$, as we argued above. (Note that the graphs are non-monotone for larger $Q$ values. As we explained before, this is merely due to the fact that the influence of $Q$ and $CV$ on the shortage risk is intertwined.) We consequently define the following phenomenon.

**Definition (The Spread Effect):** *In markets consisting of strategic consumers, the degree of strategic behavior in equilibrium tends to increase as a function of the market-size uncertainty ($CV$). The increase is particularly significant under small levels of inventory ($Q$).*

### 3.4 The Value of Information

As expected, we found that when all consumers are *myopic*, the value of information was always non-negative. However, the values observed were quite small, ranging between 0.0% and +2.1%, with an average of +0.5%, across all 3,000 instances. Obviously, for $Q = 1$, information has no value since the only situation that leads to pricing in the second period is if there were no sales in the first period. Similarly, when $Q$ approaches infinity, the value of information declines to zero, as the sales in the first period have no practical effect on the second-period price. The value of information under the myopic consumers setting is relatively small in comparison to the base value of segmentation. This means that the benefits of dynamic pricing are driven almost solely by the ability to charge two prices, with the “responsiveness” feature typically yielding no further significant advantage. For example, Gallego and van Ryzin (1994) – in a model with myopic consumers and no learning element – report that a fixed-price policy performs almost the same as
an optimal responsive pricing policy based on real-time sales information; see §3.3 there. To reconcile our
results with the research papers on dynamic pricing with learning (see references in the introduction), we
note two important characteristics of our setting: first, we study a two-period setting, having an opportunity
to incorporate learning only at the very end of the sales season. Second, pricing in the second period is
focused on the segment of consumers with valuations below $p_1$, further limiting the seller’s ability to take
advantage of learning. We shall come back to the latter point shortly, for the case of strategic consumers.

When all consumers are strategic, the value of information was mainly negative (82% of the 3,000 in-
stances), ranging between $-3.7\%$ and $+3.0\%$, with an average of $-0.8\%$. Among all 3,000 instances, the
value of information exceeded 1% only under the special case in which the consumer’s $\delta$ was at the highest
level ($\delta = 1$), the $CV$ value was large ($CV > 1$), and the inventory level was in a medium value range (relative
to the spread of the demand distribution). To explain this outcome, note that under the aforementioned
conditions, the consumers are highly patient and are expected to be strongly strategic (i.e., follow a high
$\theta$ threshold value). The seller hence reaches the second period with a market of remaining with residual
valuations in the range $[0, \delta \theta]$. Thus, market size information can be very useful for setting the second-period
price optimally. Interestingly, because the residual market valuations in the second period is lower in the
myopic case than in the strategic case (i.e., $[0, \delta p_1]$ vs. $[0, \delta \theta]$), the value of information was higher in the
all-strategic consumers case than in the all-myopic consumers case.

As we shift our attention back to our focal range of medium $\delta$ levels, we note that the value of information
under strategic consumer behavior is negative for all $Q > 1$ and sufficiently large degrees of market uncer-
tainty (e.g., $CV > 0.5$); see Figure 5 below as a representative illustration of our results. In other words, the

![Figure 5: The value of information $((\pi^R(Q) - \pi^N(Q))/\pi^F(Q))$ under myopic (left chart) and strategic (right chart)
consumer behavior, for selected values of $Q$ and $CV$, and $\delta = 0.7$.](image-url)
ability to act upon information might have a negative influence on the seller’s expected revenue performance, when faced with strategic consumers. Generally, this is not necessarily a surprising result. Existing research indeed demonstrates that responsive pricing policies that utilize inventory status information can underperform fixed-price policies in face of strategic consumers. However, in contrast the extant research (e.g., Aviv and Pazgal 2008, Cachon and Swinney 2009) our modeling framework allows us to clearly separate the negative effect of strategic consumer behavior into two parts: the base value of segmentation, and the value of information. Interestingly, when the market size uncertainty is higher, which brings an attractive opportunity to learn via sales observations, the value of information might become counter-productive. This outcome stands in contrast to the situation in the case of all-myopic consumers, where the value of information tends to increase in $CV$; see the left- and right-hand-side of Figure 5. This leads us to the next part of our study.

3.5 Can Learning be Bad in the Sales of Fashion Goods?

To better understand why the value of information can be negative in the sales of fashion goods (again, restricting our attention to the regime of medium levels of $\delta$, and high $CV$ values), we have conducted an additional study. Note that the value of information is driven by two inter-dependent types of information: the up-to-date inventory status at the beginning of the second period (value of inventory information), and the revised forecast of the average market size at that same time (value of demand learning). To achieve some separation between these two inter-related information pieces, and in particular to isolate the impact of demand learning, we constructed an auxiliary model which hypothetically assumes that the seller is committed to not (or unable to) update demand information using the first-period sales information. Instead, the seller will continue to rely on the prior distribution of $\Lambda$, rather than the updated distribution $\hat{\Lambda}(x, \theta)$, when setting the second-period prices, according to (4). The consumers are fully aware of this particular limitation. We refer to this auxiliary model by the title “no-learning” and denote its expected revenue performance by $\pi^{NL}(Q)$.

Figure 6 below shows the values of $(\pi^{R}(Q) - \pi^{NL}(Q)) / \pi^{F}(Q)$ for the same subset of scenarios considered in the figures presented in the previous sections. As expected, the value of demand learning is non-negative when the consumers are myopic. However, for the set of scenarios exhibited in the table, all values are negative for all-strategic consumers, meaning that the seller’s performance would be better if we
forced it to use the original demand forecasting scenarios (and likelihoods), without updating them on the basis of early sales readings. As can be seen in the table, for most scenarios, and particularly those with high degree of resolvable market uncertainty, the seller might be able to perform better if it could credibly commit not to update its demand forecast. Looking at this from a slightly different angle, a seller that develops a sales-based forecasting process to support responsive pricing strategies may find itself settling in an equilibrium that yields a lower revenue performance.

3.6 The Adverse Impact of Active Demand Learning: Exploration (Seller) and Information Shaping (Consumers)

To appreciate the combined impact of demand learning and strategic consumer behavior on the value of demand learning, recall the two contradicting effects mentioned in the introduction. On one hand, learning enables the seller to react to market information. On the other hand, strategic consumers may respond to that capability in a way that negatively affects the seller’s performance.

Let us first consider the seller’s perspective. It is easy to see that the seller can impact its demand learning process via a single lever only – the first-period price. In our model, the seller optimally takes this consideration into account. More specifically, in its attempt to generate maximal revenue performance, the seller considers the subtle way in which the first-period price affects the consumers’ purchasing behavior, which in turn affects its learning process and effectiveness in charging the right price in the second period. The trade-off associated with the ways in which the ability to collect immediate revenue gains versus the ability to learn, affect pricing decisions, is named *exploitation vs. exploration* in the economic and revenue
management literature.

Turning to the consumers, note that in determining their purchasing strategy, they also must consider the effect of their decisions on the seller’s ability to learn about the market size. For example, a large $\theta$ implies that a (statistically) larger number of consumers will deprive the seller from sales information and consequently limit the ability of the seller to resolve market uncertainty. Obviously, this affects the seller’s pricing in the second period, and for the consumers it impacts the expected surplus associated with a “wait” decision. Before we proceed, let us define the following phenomenon.

**Definition (Information Shaping):** In markets consisting of strategic consumers, information shaping refers to the conscious collective attempt of the consumers to influence the seller’s interpretation of the market size.

In using the term “collective attempt” we obviously refer to the consumers’ selection of the value of $\theta$ in equilibrium. Here, it is also useful to recall our observation in §2.2 that the seller’s first-period pricing problem can be translated into a segmentation problem; namely, an optimal selection of $\theta$. Thus, in effect, learning is solely controlled via the model parameter $\theta$, but the ways in which the seller and the consumers affect learning are non-trivially intertwined. In this section, we study the way in which the conscious considerations of how $\theta$ affects learning – exploration (by seller) and information shaping (by consumers) – plays a role in the overall value of demand learning. Below, we propose an auxiliary model that neutralizes the conscious consideration of the impact that $\theta$ has on learning. Specifically, we shall look at a model in which learning takes place and affects the seller’s and consumers’ decisions in equilibrium. However, in this model, neither the seller nor the consumers believe that they can affect the seller’s demand learning process.

Consider a hypothetical case in which the seller cannot use sales information in updating its demand forecast. Instead, the seller is guaranteed to observe the (aggregate) number of consumers in the market with valuations in the range $[\bar{\theta}, 1]$, but just prior to the end of the season (i.e., prior to setting $p_2$). The seller can use that information to update the demand forecast. For instance, if $\bar{\theta} = 0$, the model reflects a situation in which the seller perfectly observes the total number of consumers (but not their individual valuations) in the market prior to setting $p_2$. In contrast, when $\bar{\theta} = 1$, the model becomes equivalent to the no-learning model presented in the previous section.

The auxiliary model is considerably more complex than our original model, and for the sake of clarity of exposition, we refer the interested reader to the end of the Appendix for a brief overview of the tech-
nical details of our analysis. Most importantly, for any exogenously given value $\tilde{\theta}$, we predict the market equilibrium given by the pair $\{p^*_1(\tilde{\theta}), \theta^*(\tilde{\theta})\}$ (and the resulting $p_2$ menu, which is less crucial for our discussion at the moment). We then apply a search procedure over $\tilde{\theta} \in [0,1]$ to identify fixed points of the type $\theta^*(\tilde{\theta}) = \tilde{\theta}$. Assuming that a fixed point indeed exists (which happened at a unique value of $\tilde{\theta}$ for each one of the instances we studied), the model would predict a rational expectation equilibrium in which the consumers behave according to a policy $\theta$ and the seller would observe the number of consumers in the range $[\tilde{\theta}, 1]$, which is in fact equal to the sales quantity. Note that in the equilibrium predicted for the hypothetical model, we have neutralized the seller’s and consumers’ ability to actively influence learning. In other words, learning takes place, but in a passive way only. Let us use the notation $\pi^{PL}(Q)$ as the predicted expected revenue in equilibrium for this auxiliary model ($PL$ stands for “Passive Learning”).

Table 1 below shows the values of active learning (exploration and information shaping), defined by the measure $(\pi^R(Q) - \pi^{PL}(Q)) / \pi^F(Q)$, for the subset of scenarios considered in the previous subsections. As can be seen, the results do not exhibit any particularly strong pattern with respect to the model parameter, but they suggest that the active learning interplay between the seller and the consumers leads to a loss of 1% to 2% in expected revenue performance. Since exploration is controlled by the seller, this leads us to the conjecture that information shaping – the conscious attempt of the consumers to influence demand learning via their purchasing decisions – brings to a loss that is at least as strong as that just mentioned.

3.7 The Expected Market Size

Recall that our numerical study has focused on a unique value of the expected market size $E[\Lambda] = 10$. In many fashion goods settings, this is a reasonably realistic order of magnitude for the market size; in other situations, the expected market size could vary. By fixing the expected market size at a given level, we were able to explore the impact of the inventory-to-mean-market-size ratio, $Q/E[\Lambda]$, but primarily gain speed...
in our analyses. For example, a typical evaluation of market equilibrium takes 6,000 times longer if we increase E[\lambda] by a factor of 100. As the market grows in order of magnitude, one may wonder if the same phenomena discussed earlier would continue to exist. For example, could it be that the negative effect of demand learning takes place only in markets characterized by a small number of consumers, as only then each consumer’s decision is sizeable enough to affect the seller’s perception of the market? To address this issue, we conducted a numerical study including two different values of the mean market size E[\lambda] ∈ {10, 1000}, covering all possible combinations of ten inventory-to-mean-market-size ratios Q/E[\lambda] ∈ {0.1, 0.2, ..., 1}, two levels of degree of uncertainty CV ∈ {1, 2}, and four levels of consumers’ δ ∈ {0.5, 0.6, 0.7, 0.8}. For example, Table 2 shows the influence of the market size on three key metrics: the benefits of responsive pricing, the base value of segmentation, and the value of demand learning, for the case of δ = 0.7, CV = 2, and five values Q/E[\lambda]. Overall, our study indicates that changing the average market size does not qualitatively influence our managerial results and insights. Moreover, it seems that the increase in the average market size could even amplify the negative effects of demand learning; for example, the negative impact of demand learning worsens from a loss of 4.48% when E[\lambda] = 10, to a loss of 7.13% when E[\lambda] = 1000, for Q/E[\lambda] = 0.2. To summarize, the presence of a large number of consumers in the market (large E[\lambda]) does not mitigate the adverse consequences of the demand learning effect, due to the fact that the expected market equilibrium is an outcome of the collective action of all consumers in the market.

<table>
<thead>
<tr>
<th>Q/E[\lambda]</th>
<th>Average Market Size = 10</th>
<th>Average Market Size = 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benefits of Responsive Pricing</td>
<td>Base Value of Segmentation</td>
</tr>
<tr>
<td></td>
<td>(\pi^\text{\text{r}} - \pi^\text{\text{f}})/\pi^\text{\text{f}}</td>
<td>(\pi^\text{\text{N}} - \pi^\text{\text{f}})/\pi^\text{\text{f}}</td>
</tr>
<tr>
<td>0.2</td>
<td>-6.48%</td>
<td>-5.57%</td>
</tr>
<tr>
<td>0.4</td>
<td>-8.71%</td>
<td>-6.67%</td>
</tr>
<tr>
<td>0.6</td>
<td>-9.98%</td>
<td>-7.39%</td>
</tr>
<tr>
<td>0.8</td>
<td>-10.72%</td>
<td>-7.93%</td>
</tr>
<tr>
<td>1</td>
<td>-11.14%</td>
<td>-8.36%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q/E[\lambda]</th>
<th>Average Market Size = 10</th>
<th>Average Market Size = 1000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benefits of Responsive Pricing</td>
<td>Base Value of Segmentation</td>
</tr>
<tr>
<td></td>
<td>(\pi^\text{\text{r}} - \pi^\text{\text{f}})/\pi^\text{\text{f}}</td>
<td>(\pi^\text{\text{N}} - \pi^\text{\text{f}})/\pi^\text{\text{f}}</td>
</tr>
<tr>
<td>0.2</td>
<td>-9.03%</td>
<td>-5.81%</td>
</tr>
<tr>
<td>0.4</td>
<td>-9.64%</td>
<td>-6.86%</td>
</tr>
<tr>
<td>0.6</td>
<td>-10.05%</td>
<td>-7.56%</td>
</tr>
<tr>
<td>0.8</td>
<td>-10.34%</td>
<td>-8.08%</td>
</tr>
<tr>
<td>1</td>
<td>-10.56%</td>
<td>-8.48%</td>
</tr>
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</table>

Table 2: The influence of mean market size on the benefits of responsive pricing, the base value of segmentation, and the value of demand learning, for the case δ = 0.7 and CV = 2. For brevity, the argument Q is omitted from the \pi-functions in the titles.
3.8 Optimal Capacity Choice – Illustrations and Discussion

As the initial inventory plays a significant role in explaining the benefits of responsive pricing, it may be of interest to examine the case where the seller selects not only the optimal prices, but also the best level of inventory \( Q \). For brevity, the nature of this section is primarily illustrative, and our approach is standard. We consider a given per-unit cost, \( c \), and identify the inventory level and best pricing strategy that optimize the seller’s expected profit. For example, in our main model, we have searched for \( \Pi^R = \max_Q \{\pi^R (Q) - cQ\} \).

Similarly, we calculated the optimal expected profit for the fixed-price case (denoted by \( \Pi^F \)), and for a benchmark case in which the seller utilizes responsive pricing in a market consisting of myopic consumers (denoted by \( \Pi^M \)). For example, we have considered 4 scenarios, spanned by the combinations of \( \delta = 0.7 \), two cost parameter values: \( c = 0.15 \) and 0.25, and two \( CV \) values: 1 and 2. Our results are summarized in Table 3 below. First, this table demonstrates that the influence of strategic consumer behavior on optimal profits can be quite severe. Moreover, as the value of \( CV \) increases, the difference between the benefits in the myopic-consumers and strategic-consumers settings tends to increase. In particular, under relatively high per-unit costs, higher \( CV \)-values result in small-to-medium optimal inventory levels. This is exactly where the effect of strategic consumer behavior tends to be the most detrimental.

<table>
<thead>
<tr>
<th>CV</th>
<th>( \varepsilon = 0.15 )</th>
<th>( \varepsilon = 0.25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( Q^*, p_1, \text{Markup, Profit [II]} )</td>
<td>( Q^*, p_1, \text{Markup, Profit [II]} )</td>
</tr>
<tr>
<td>Fixed</td>
<td>5</td>
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</tr>
<tr>
<td>Myopic</td>
<td>7</td>
<td>0.703</td>
</tr>
<tr>
<td>Strategic</td>
<td>5</td>
<td>0.595</td>
</tr>
<tr>
<td>2</td>
<td>( Q^*, p_1, \text{Markup, Profit [II]} )</td>
<td>( Q^*, p_1, \text{Markup, Profit [II]} )</td>
</tr>
<tr>
<td>Fixed</td>
<td>4</td>
<td>0.706</td>
</tr>
<tr>
<td>Myopic</td>
<td>5</td>
<td>0.777</td>
</tr>
<tr>
<td>Strategic</td>
<td>3</td>
<td>0.665</td>
</tr>
</tbody>
</table>

Table 3: The benefits of responsive pricing under myopic and strategic consumer behavior, when inventory levels are chosen optimally for the case \( \delta = 0.7 \).

4 The Degree of Strategic Consumer Behavior in the Market

As shown above (see Figure 1), the benefits of responsive pricing depend heavily on the extent to which strategic consumer behavior is prevalent in the market. Indeed, our primary purpose in this paper is to juxtapose the two extreme cases – of all myopic and all strategic consumers – to derive interesting insights about the interplay between demand learning and strategic consumer behavior. Nonetheless, the consider-
ation of markets characterized by a mixture of strategically and myopically behaving consumers has taken place in several key papers in the Management Science literature; see, e.g., Su (2007), Cachon and Swinney (2009), Levin et al. (2009), Lai et al. (2010), and Mersereau and Zhang (2012).

To study markets in which not all consumers are strategic, we propose a variant of our model where a portion (say, $\alpha$) of the consumers are strategic and the rest are myopic. In particular, consider a case where each individual consumer arriving to the store behaves strategically with probability of $\alpha$, and suppose that the consumers are statistically independent in this regard. Such setting lends itself to a relatively easy modification of our analysis, technically stemming from the fact that the market size can be separated into two parts that are Poisson-distributed and statistically independent: one with a mean of $\alpha \lambda$ (strategic consumers), and one with a mean of $(1 - \alpha) \lambda$ (myopic consumers). For the details of the analysis of this model, we refer the reader to the Appendix.

Using this modified model, we conducted a numerical study that spans across 1760 scenarios, covering all possible combinations of twenty levels of the initial inventory $Q \in \{1, 2, \ldots, 20\}$, two levels of degree of uncertainty $CV \in \{1, 2\}$, four levels of the consumer discount factor $\delta \in \{0.5, 0.6, 0.7, 0.8\}$, and eleven values of the expected portion of strategic consumers in the market $\alpha \in \{0, 0.1, \ldots, 1\}$. To be consistent with our main model, the average value of the market size for all instances was fixed at $E[\Lambda] = 10$. The benefits of responsive pricing over the fixed pricing (i.e., $\pi^R/\pi^F - 1$) ranged between $-11.28\%$ and $22.15\%$, with an average value of $1.26\%$. For example, Table 4 shows a subset of the results, for the case where $\delta = 0.7$ and $CV = 2$. As discussed before, the benefits of responsive pricing decrease as consumers are more likely to behave strategically (i.e. $\alpha$ increases). Most impressive, in our opinion, is the magnitude of dependence of

<table>
<thead>
<tr>
<th>Initial Inventory</th>
<th>Benefits of Responsive Pricing ($\pi^R/\pi^F$)</th>
<th>Base Value of Segmentation ($\pi^N/\pi^F$)</th>
<th>Value of Demand Learning ($\pi^R/\pi^N$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha=0.2$</td>
<td>$\alpha=0.5$</td>
<td>$\alpha=0.8$</td>
</tr>
<tr>
<td>1</td>
<td>7.99%</td>
<td>0.58%</td>
<td>-2.96%</td>
</tr>
<tr>
<td>5</td>
<td>8.99%</td>
<td>-1.56%</td>
<td>-7.21%</td>
</tr>
<tr>
<td>10</td>
<td>9.25%</td>
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</tr>
<tr>
<td>15</td>
<td>9.26%</td>
<td>-3.82%</td>
<td>-9.23%</td>
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<tr>
<td>20</td>
<td>9.20%</td>
<td>-4.30%</td>
<td>-9.40%</td>
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</tbody>
</table>

Table 4: The benefits of responsive pricing as a function of the expected portion of strategic consumers in the market ($\alpha$) and inventory ($Q$), presented for the case of $CV = 2$, $\delta = 0.7$, and $E[\Lambda]= 10$. 
the benefits of responsive pricing with respect to $\alpha$ – across all the range $[0, 1]$ – emphasizing the necessity of understanding the consumers mixture, and developing empirical methodologies for measuring the extent to which strategic behavior is likely to occur in markets of fashion goods. The rest of our results exhibit a similar pattern with respect to all other $\delta$ and $CV$ value combinations, and are therefore excluded from the table.

Consistent with our discussion in the previous section, we also observed that both the spread effect and the demand learning effect increase with the expected portion of strategic consumers ($\alpha$). Intuitively, it is expected that the information shaping effect would diminish as more myopic consumers are present in the market, since these consumers already convey market size information to the seller; thus, strategic waiting becomes less effective in influencing the seller’s perception of the market size.

5 The Effectiveness of Price Commitment

In this section, we explore whether price commitment can be used as an effective tool for counteracting strategic consumer behavior. The promise behind price commitment is that it ties the hands of the seller in the second period; consequently, consumers realize that their purchasing decisions will not affect the second-period price, and therefore, their incentive to wait can be controlled and reduced. But commitment in settings where market size uncertainties can be resolved via sales observations might be radically counter-productive.

Consider a setting in which the first-period and second-period prices, $p_1$ and $p_2$, can be credibly announced in advance. It is easy to verify that it is always optimal for the seller to set $p_2 < \delta p_1$; otherwise, the seller effectively utilizes a fixed-price policy. In fact, since fixed-price policies can be viewed as a special case of price commitment, this means that the optimal price commitment policy always dominates the optimal fixed-price policy. In light of our earlier results, this observation immediately establishes that price commitment strategies are generally of potential value.

Clearly, for any given pair of announced prices, it follows directly from Theorem 1 that only threshold policies can hold in equilibrium. The following theorem characterizes the consumers’ purchasing behavior under a price-commitment policy. Let $\theta^*$ be the unique solution to

$$p_1 = \theta + \delta (1 - \theta) - (\delta - p_2) \frac{E_{\Lambda} [H (Q, \Lambda (1 - p_2/\delta))] \quad (9)}{E_{\Lambda} [H (Q, \Lambda (1 - \theta))]}.\]

**Theorem 3** For any pair of credibly announced prices $(p_1, p_2)$ such that $p_1 > p_2/\delta$, there exists a unique
threshold policy, $\theta \geq p_1$, in the consumer game. Furthermore, $\theta$ is given by

$$
\theta = \begin{cases} 
\theta^* \text{ given by (9)} & \text{if } p_1 \leq 1 - \frac{(\theta - p_2) E_\Lambda[H(Q, \Lambda | 1 - p_2 / \delta)]}{E_\Lambda[H(Q, \theta)]} \\
1 & \text{otherwise}
\end{cases}.
$$

(10)

The significance of the condition in the first line of (10) is that it allows us to establish a bound on the first-period price. If $p_1$ is too high, all consumers wait, and the seller is not expected to generate any revenues in the first period. As a result, the seller can drop the price exactly to that bound, without loss of revenues. This argument allows us to focus on Equation (9) for calculating $p_1$ in our optimization procedure, as stated below. For any value of $\theta \in [0, 1]$, we conduct a search for a value $p_2 \in (0, \theta \delta)$ such that $p_1$ is set according to the equation above, and the following expected revenue is maximized:

$$
\pi^C (Q) \doteq \max_{\theta \in [0, 1]} \left\{ p_2 \cdot E_\Lambda [N (Q, \Lambda \cdot (1 - p_2 / \delta))] + (p_1 - p_2) \cdot E_\Lambda [N (Q, \Lambda \cdot (1 - \theta))] \right\}
$$

(We use the superscript ‘C’ for commitment.) To understand the objective function stated above, note that we separate the revenue into two parts: $p_2$ collected from the total sales made in both periods, and a premium of $(p_1 - p_2)$ collected from the sales made in the first period only.

As discussed above, price commitment enables the seller to regain part of the loss of market power caused by the dynamic competitive environment of the responsive pricing model, but it cannot eliminate strategic consumer behavior when the seller employs price discrimination (i.e., when $p_2 < \delta p_1$). In other words, the seller’s revenue performance could still be dramatically lower than that which can be achieved in a market composed of myopic consumers. Among all 3,000 instances, the percentage benefit measure $(\pi^C (Q) / \pi^R (Q) - 1)$ ranges between $-2.3\%$ and $+15.0\%$, with an average of $+6.6\%$. As discussed in §3.4, the extreme results (both negative and positive) were observed for the high $CV$ values, with the nature of the impact highly sensitive to $\delta$. For $\delta = 1$, the impact of price commitment was negative, whereas for mid values of $\delta \in [0.5, 0.8]$ the impact of price commitment was highly positive. In summary, price commitment is most helpful in the cases where strategic consumer behavior has its most adverse impact on revenue performance. But while beneficial, it does not fully eliminate the negative impact of such consumer behavior.

5.1 Commitment vs. Contingency - Further Discussion and Analysis

The price commitment strategy discussed above maintains two important characteristics: commitment, and lack of contingency option. As we observed, the commitment provides a seller with a possible advantage.
However, the lack of contingency is potentially counter-productive to the seller. To understand the degree to which the two opposing forces affect the benefits of price commitment strategies, we studied an auxiliary model in which a seller deploys a sales-dependent price commitment strategy. In this model, the seller commits in advance to a price function $p_2(Q - x)$, rather than to a single price $p_2$, where $x$ is the number of units sold in the first period. Note that unlike the simple price-commitment model, this hypothetical model is responsive to early sales. Let us denote its optimal expected revenue performance by $\pi^{RC}(Q)$ (with the superscript standing for both ‘responsive’ and ‘commitment’). With a commitment to the second-period price function, the subgame in §2.1 maintains a Stackelberg game form, where the seller enjoys the benefits of being the leader.

We have developed an optimization procedure to identify the best responsive pricing for our auxiliary model. Technically, some pricing schemes $\{p_1, p_2(\cdot)\}$ could induce multiple equilibria ($\theta$-values) in the consumers’ game; see also the discussion in §2.1.1. Thus, when searching for the optimal announced sales-dependent pricing policy, we evaluated the seller’s expected revenue for all possible equilibria points ($\theta$-values). Nevertheless, we found that for all the numerical instances, the seller’s optimal announced sales-dependent prices $\{p_1^*, p_2^*(\cdot)\}$ always induced a unique equilibrium in the consumers’ game.

As expected, a sales-dependent price commitment strategy dominates both the simple price-commitment strategy, and the responsive price strategy (i.e., our main model, where no form of commitment exists). Among all 3,000 instances, the percentage benefit measure $(\pi^{RC}(Q) / \pi^R(Q) - 1)$ ranges between 0.0% and +25.6%, with an average of +9.9%. The marginal benefits of the contingency feature, i.e. $(\pi^{RC}(Q) - \pi^C(Q)) / \pi^F(Q)$ – taken again in reference to the baseline value $\pi^F$ ranges between 0.0% and +15.6%, with an average of +3.0%. A deeper examination of our results suggests that in order to increase revenue performance via discrimination, some sort of contingency capability – with commitment – is needed. Specifically, when the initial inventory is relatively small, and the level of $CV$ is at a medium level, the simple-price commitment $\{p_1, p_2\}$ performs similarly to the hypothetical sales-dependent price-commitment policy. But as the levels of inventory and $CV$ increase, discrimination via a larger set of possible second-period prices becomes valuable, particularly for large $\delta$ values.

28
5.2 Optimal Choice of Inventory – Revisited

Similarly to §3.8, we have also considered settings in which the seller makes an optimal selection of the initial inventory level $Q$. Table 5 below displays the results pertaining to the same illustrative scenarios that we examined earlier (see Table 3). This numerical study echoes the results we have found earlier:

Table 5: The value of price commitment and the value of contingency under commitment, under strategic consumer behavior, when inventory levels are chosen optimally. All 4 scenarios share the same value of $\delta = 0.7$.

<table>
<thead>
<tr>
<th>CV</th>
<th>C</th>
<th>Responsive Pricing</th>
<th>Price Commitment with Single Price</th>
<th>Price Commitment with Sales-dependent Prices</th>
<th>Value of Commitment ($\Pi^C/\Pi^F-1$)</th>
<th>Marginal Value of Contingency ($\Pi^C/\Pi^F-1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>5</td>
<td>0.792</td>
<td>5</td>
<td>0.894</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>3</td>
<td>0.434</td>
<td>3</td>
<td>0.485</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
<td>3</td>
<td>0.266</td>
<td>4</td>
<td>0.342</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>1</td>
<td>0.073</td>
<td>1</td>
<td>0.091</td>
<td>2</td>
</tr>
</tbody>
</table>

price commitment can be substantially beneficial for modest-to-high $Q$ and $CV$ values, and practical price commitment policies (i.e., those with no sales-dependent price menu) can be valuable to a seller. Nevertheless, when $CV$ is high – i.e., when the seller can resolve a high degree of market size uncertainty by reading early sales – price commitment considerably suffers from its lack of ability to respond to sales.

6 Quick Response: The Effect of an Inventory Replenishment Option

Our analysis thus far has focused on fashion retail settings in which the seller can utilize responsive pricing for a fixed inventory of a product, brought to the store prior to the beginning of the season. Nonetheless, some fashion retail supply chains may possess the operational capacity to procure additional inventory during the season, in response to the sales and demand signals for the product. For a rich background and a review of the relevant research, we refer the reader to Cachon and Swinney (2009). In that paper, the authors present a model of a market consisting of a mixture of myopic and strategic consumers, and study the benefits of Quick Response (QR) strategies in which the seller can obtain perfect demand information prior to the beginning of the season, and procure additional inventory (instantaneously) on the basis of such information but at a higher cost per unit. One of their main findings is that QR may become significantly more valuable to the seller as the proportion of strategic consumers in the market increases. They explain this by the fact that the seller’s ability to observe demand and respond to it – something valuable already in the all-myopic
consumers setting – becomes even more valuable in the presence of strategic consumers, due to a rationing effect (e.g., if the seller learns that the demand is low, then he would source less units, thus mitigating the adverse consequence of strategic consumer behavior).

In this section, we report on results stemming from an alternative model variant of QR in which the inventory replenishment takes place after the interaction between the seller and the consumers has begun, and the consumers endogenously affect the seller’s perception of the market. Specifically, we constructed a model in which the seller has an option to replenish its inventory – in a quick fashion – at the end of the first period. After observing the first-period demand, and prior to setting the price at the beginning of the second period, the seller can order additional units of the product. Such units would be expeditiously procured and delivered at a higher per-unit cost rate than that available prior to the beginning of the season. For the details of the analysis of this model, we refer the reader to the Appendix.

Obviously, the seller is able to utilize the demand information gathered during the first period to refine its understanding of the market potential, and accordingly determine its replenishment quantity and second-period price. When the consumers are strategic, we find that the loss in expected profit (due to strategic behavior) is mitigated under QR, in comparison to the original (no-QR) setting (see the results in the appendix in and around Table 6). We argue that QR provides a strategic option to the seller, in the following sense. In the no-QR case, the seller must make an upfront commitment to inventory; thereafter, it becomes engaged with the consumers in the pricing-and-purchasing game. The negative outcome of such interaction is known by now. With QR, the seller does not make a full commitment to inventory. This gives rise to a market equilibrium that is settled in the presence of an implicit threat by the seller to the consumers. When less consumers purchase the product in the first period, and particularly if they are successful in their information shaping attempt – to create a perception of a poor market – the seller would respond by not ordering additional units. Of course, the consumers who are now faced with increased shortage risk and no hope for significant price markdowns at the end of the season would be leaning more towards immediate purchasing at premium prices. In summary, we argue that as more strategic consumers are present in the market, responsive inventory becomes more effective than responsive pricing.

We also found that the benefits of QR are higher in the case of all-strategic consumers than in the case of all-myopic consumers, across all scenarios (see again, some representative numerical results in the appendix). This result is consistent with the finding of Cachon and Swinney (2009), who argue that the benefits of QR
might be significantly under-estimated in the presence of markets with strategic consumers, if such behavior is not taken into account. Furthermore, we conjecture that by using a QR strategy, the seller becomes less susceptible to an error regarding the estimation of the portion of strategic consumers in the market. The seller can order first a small level of inventory, observe the market, and then react by ordering a proper amount of additional inventory. If the market seems to be poor, responding by ordering a small number of units (or none) would be proper regardless of the nature of the consumers.

7 Conclusion and Discussion

Over the past two decades, the subject of responsive pricing has received an increasing level of attention from both practitioners and academics. Two major factors that affect the value of price segmentation via responsive pricing are demand learning and strategic consumer behavior. Existing research on strategic consumer behavior demonstrates that the ability to dynamically change prices over a course of a season can be counter-productive to a seller’s revenue performance. In contrast, research on demand learning emphasizes the critical (positive) role of responsive pricing in increasing revenue performance via actionable information; namely, recording early sales, updating the forecast of the market condition, and appropriately changing the price in reaction to the newly available information. Although the effects of the above two factors have been studied extensively, there has been little research on optimal pricing under the presence of both. This paper aims to fill this void in the literature.

We study the benefits of responsive pricing, with a primary focus on fashion-like products, characterized by relatively large demand uncertainty, an opportunity to learn about the demand from sales observations, medium levels of the consumers’ discount factor, and modest levels of inventory deployment. Our results overwhelmingly demonstrate that the benefits of responsive pricing, in comparison to a benchmark case of a fixed-price policy, depend sharply on the nature of the consumers’ behavior. In stark contrast to markets of myopic consumers, when the consumers are all strategic the benefits tend to worsen (in fact, they become mostly negative) when there is a higher potential for learning. We explain this counter-intuitive outcome by pointing to two phenomena: the spread effect and the active learning effect. The spread effect is the phenomenon in which a higher degree of market size uncertainty makes the expected surplus associated with a wait-for-markdown decision increasingly more attractive than that associated with a purchase-now decision, due to the higher spread in market size scenarios. The active learning effect refers to the way in which
market equilibrium is influenced by the seller’s demand exploration (via pricing) and by the consumers’ act of information shaping. We define information shaping as the conscious collective attempt of the consumers to influence the seller’s interpretation of the market size via their purchasing decisions. For example, by waiting for markdowns, the consumers withhold sales information from the seller, possibly leading the seller to interpret the market as weak and hence reduce prices later.

The information shaping effect has the following ramification. Sellers of fashion products that consider upgrading their pricing systems to incorporate “accurate response” strategies (i.e., integrating learning and responsive pricing), should be aware of the possibility that this might lead them to a new and potentially worse equilibrium, particularly when there is a higher opportunity to learn. We demonstrated this insight using a hypothetical model in which we eliminated the seller’s ability to learn from sales (§3.5). However, we do not wish to make a case against learning. In contrast, we argue that sellers in fashion markets should extensively seek market signals other than mere sales figures, to base their forecasting process on (Fisher and Rajaram (2000)). For example, our model presented in §3.6 is readily available to study the performance of responsive pricing in a market where the number of consumers is fully visible to the seller (use $\tilde{\theta} = 0$, ibid). The reliance on indirect demand data may considerably mitigate the degree of the information shaping effect. For example, Zara, a well-known “fast-fashion” Spanish retailer relies on store managers and supporting information technology to gather and analyze customer preference data. In particular, store managers provide Zara’s design teams with information that is not captured in the data gathered by their automated point-of-sales system (Ghemawat and Nueno (2003)).

Despite the fact that price commitment completely eliminates the seller’s ability to learn, it appears to increasingly dominate responsive pricing as more strategic consumers are present in the market. Moreover, the most significant benefits of commitment were observed for the cases in which the degree of market uncertainty is high. However, it is noteworthy that in cases where the market size is highly uncertain, and when the seller’s inventory is large, price commitment may be highly limited in its ability to perform effective discrimination due to its lack of ability to prescribe sales-dependent price changes.

Cachon and Swinney (2009, 2011) study the value of quick response in the presence of strategic consumers. For example, in their earlier paper they find that the ability to quickly replenish inventory at the very beginning of the selling period is dramatically more beneficial in a market of strategic consumers than in a market of myopic consumers. A key driver behind this result is that by better matching supply with
demand, the intensity of strategic consumer behavior can be considerably mitigated. Building upon this interesting insight, we studied the potential benefits of quick response strategies – ones that embed both dynamic pricing and quick inventory replenishment during the sales season. We also find that the benefits of quick response strategies might be significantly underestimated in the presence of markets with strategic consumers. We explain that the option to replenish inventory close to the end of the season serves as an effective implicit threat by the seller to the consumers in a way that discourages strategic waiting. If less consumers purchase the product at the premium price, and particularly if they are successful in their information shaping attempt, the seller would likely respond by not ordering additional units. But the consumers, now faced with increased shortage risk and no hope for significant price markdowns at the end of the season, would be leaning more towards immediate purchasing at premium prices. Similarly to Cachon and Swinney (2009), we conjecture that a quick-response seller may be less susceptible to an error regarding the estimation of the portion of strategic consumers in the market. The seller can order first a small level of inventory, observe the market, and then react by ordering a proper amount of additional inventory. If the market seems to be poor, responding by ordering a small number of units (or none) would be proper regardless of the nature of the consumers.

The results reported above highlight opportunities for further research. Our model considers a case where a seller can resolve uncertainty regarding the mean market size. While technically challenging, this framework may be expanded to consider settings where uncertainty about price elasticity could also be learned via early sales; see, e.g., Aviv and Pazgal (2005) for such treatment in the case of myopic consumers. We speculate that when strategic consumers are present in the market, the above phenomena – the spread effect and information shaping – would still play a major role, and perhaps even intensify. On one hand, sellers can derive important information from sales data to effectively price in the second period. However, in equilibrium, the sellers’ ability to learn and deploy responsive pricing may put them at significant disadvantage due to the aforementioned effects. Other obvious directions of research include empirical studies of strategic consumer behavior in fashion markets where demand learning opportunities exist. For example, one type of study may involve an estimation of the extent to which strategic behavior is prevalent (again, see Li et al. 2012). Or one can also propose and validate novel models for consumer behavior. Recall that in our model consumers are surplus maximizers, are fully rational, and privy to all model information. Another theme of research is the study of competitive settings involving more than a single seller (see, e.g., Levin et al. (2009))
and Liu and van Ryzin (2008)). It is not upfront obvious if the spread and information shaping effects would be diminished or intensified when demand is split among several retailers.

References


Appendix: Proofs

Proof of Theorem 1. Let us use the expression “focal consumer” to describe the particular consumer considered in the theorem. Recall that the surplus gained from an immediate purchase is \( v - p_1 \), but the expected gain from a buy-now decision must take into account the likelihood of obtaining this product, given by \( P[A_1] \), where \( A_1 \) is the event under which the focal consumer will get a unit in the first period. Let us denote the expected surplus from a buy-now decision by \( S_1(v) = (v - p_1) \cdot 1 \{ A_1 \} \). On the other hand, if the focal consumer decides to wait, his expected surplus would be \( S_2(v) = (v - p_2(Q - X))^+ \cdot 1 \{ A_2 \} \), where \( X \) is the number of all other consumers that purchase a unit during the first period, and \( A_2 \) is the event under which the focal consumer will get a unit in the second period if he demands one. Note that \( X, A_1, \) and \( A_2 \) are generally correlated with each other; however, they are both independent of the focal consumer’s valuation \( v \). Next, observe that both \( E_{A_1} [S_1(v)] \) and \( E_{X,A_2} [S_2(v)] \) are continuous functions of \( v \), with \( E_{X,A_2} [S_2(0)] \geq 0 > E_{A_1} [S_1(0)] = -p_1 \cdot P[A_1] \), where \( P[A_1] \) is strictly positive for any positive initial inventory level. There are two possible cases: (i) if the two functions never cross in \([0,1]\), then it is
obviously optimal for the focal consumer to wait. This is equivalent to adopting a threshold policy with \( \theta = 1 \); (ii) if the two functions meet at a given point \( \theta \) (i.e., \( E_{A_1} [S_1 (\theta)] = E_{X,A_2} [S_2 (\theta)] \)), we show below this crossing point is unique. To see this, note that

\[
0 \leq \frac{\partial}{\partial v} E_{X,A_2} [S_2 (v)] \leq \frac{\partial}{\partial v} E_{X,A_1} [(\delta v - p_2 (Q - X)) \cdot 1 \{A_2\}] = \delta \cdot P [A_2] < P [A_1] = \frac{\partial}{\partial v} E_{A_1} [S_1 (v)]
\]  

The strict inequality straightforwardly follows from \( P [A_2] < P [A_1] \), which holds for any meaningful pricing policy, and establishes the uniqueness of \( \theta \). Consequently, it is optimal for the focal consumer to purchase a unit during the first period if \( v > \theta \), and wait if \( v \leq \theta \).

The Allocation Probability Function \( H \). To simplify our analysis, we consider the following model setup. Suppose that all consumers in the market arrive during a very small interval of time during at the very beginning of the first period, according to a Poisson process with a rate \( a \). Then, from any individual consumer’s perspective, the size of the rest of the market remains Poisson with the same parameter \( a \). Furthermore, suppose that if demand cannot be fully satisfied during a given period, all available units are randomly allocated among the consumers (who attempt to buy) with equal probabilities. Consequently, 

\[
H (q, a) = \sum_{x=0}^{q-1} P_x (a) + \sum_{x=q}^{\infty} \left( \frac{a}{x+1} P_x (a) \right)
\]

is the likelihood that a unit is allocated to a particular individual among \( X + 1 \) consumers, where \( X \) has a Poisson distribution with mean \( a \) \( (P_x (a) = a^x e^{-a} / x!) \) and \( q \) is the units available.

**Proof of Proposition 1.** The proposition follows directly from the continuity of \( \theta^* (\theta) \) (utilizing the Implicit Function Theorem), and Brouwer’s fixed-point theorem.

**Proof of Theorem 2.** Note that the first-order and second-order derivatives of the objective function in (3) are given by:

\[
E_{A_1} \left[ N \left( q, \frac{\hat{\lambda} (x, \theta) \cdot (\theta - p_2 / \delta)}{\delta} \right) - \frac{\hat{\lambda} (x, \theta)}{\delta} \sum_{y=0}^{q-1} P_y \left( \frac{\hat{\lambda} (x, \theta) \cdot (\theta - p_2 / \delta)}{\delta} \right) \right]
\]

and

\[
-\frac{\partial}{\partial (x, \theta)} \left[ 2 \cdot \frac{\hat{\lambda} (x, \theta)}{\delta} \left( \sum_{y=0}^{q-1} P_y \left( \frac{\hat{\lambda} (x, \theta) \cdot (\theta - p_2 / \delta)}{\delta} \right) \right) \right] < 0,
\]

respectively. Strict concavity follows immediately from the latter equation. But given that the objective function is equal to zero under both \( p_2 = 0 \) and \( p_2 = \delta \theta \), the value of \( p_2 \) that maximizes (3) lies strictly within that range, with the first-order condition (4) holding.

To prove the monotonicity of \( p_2^* \) with respect to \( \theta \) (and \( x \)), we utilize a monotone comparative statics result by Athey (2002)\(^3\). Let us present the objective function in (3) in the form 

\[
\int_0^\infty p_2 N \left( q, \lambda (\theta - \frac{p_2}{\delta}) \right) f (\lambda | \theta, x) d\lambda,
\]

where \( \tilde{f} \) is the distribution function of \( \tilde{A} \), conditioned on \( x \) and \( \theta \) (a Gamma distribution with \( \tilde{m} = m + x \) and \( \tilde{\beta} = \beta + 1 - \theta \)). We further transform the latter expression into \( g(w, \theta) \geq \int_{0}^{\infty} w \delta N(q, \gamma(1 - w)) \tilde{f}(\gamma \theta, x) \, d\gamma \) by letting \( \gamma = \lambda \theta \) and \( w = \frac{P}{\delta} \), and demonstrate that \( w^*(\theta) = \arg \max_w \{ g(w, \theta) \} \) increases in \( \theta \). Using Theorem 2 of Athey (2002), it is sufficient to prove that \( w \delta N(q, \gamma(1 - w)) \) satisfies the single-crossing property in \((w, \gamma)\) and that \( \tilde{f}(\gamma \theta, x) \) is a log-supermodular function with respect to \( \theta \) (and \( x \)). The log-supermodularity property follows from the fact that \( \tilde{f}(\gamma \theta, x) \) maintains a monotone likelihood ratio, which follows from

\[
\frac{\tilde{f}(\gamma \theta, x)}{\tilde{f}(\gamma \theta, x)} = \left( \frac{\gamma}{\gamma_1} \right)^{m+x-1} e^{-(\gamma_2 - \gamma)(\beta + 1 - 1)}
\]

being an increasing function of \( \theta \) (and \( x \)). To prove the single crossing property, we show that \( \Delta(\gamma) \geq w_2 N(q, \gamma(1 - w_2)) - w_1 N(q, \gamma(1 - w_1)) \) crosses zero at most once in the range \( \gamma > 0 \). In particular, we obtain a stronger result: if \( \Delta'(\gamma') = 0 \) for some \( \gamma' \), then \( \Delta(\gamma) \) is strictly increasing in the range \( \gamma > \gamma' \). To this end, note that

\[
\Delta'(\gamma) = w_2 (1 - w_2) \sum_{k=0}^{q-1} q_k (\gamma(1 - w_2)) - w_1 (1 - w_1) \sum_{k=0}^{q-1} q_k (\gamma(1 - w_1))
\]

where \( \lim_{\gamma \to \infty} \Delta'(\gamma) = 0 \). Furthermore, consider the second- and third-order derivatives of \( \Delta(\gamma) \):

\[
\Delta''(\gamma) = w_1 (1 - w_1)^2 q_k (\gamma(1 - w_1)) - w_2 (1 - w_2)^2 q_k (\gamma(1 - w_2))
\]

\[
\Delta'''(\gamma) = w_1 (1 - w_1)^2 \left[ \frac{q - 1}{\gamma} - (1 - w_1) \right] q_k (\gamma(1 - w_1))
\]

\[
-w_2 (1 - w_2)^2 \left[ \frac{q - 1}{\gamma} - (1 - w_2) \right] q_k (\gamma(1 - w_2))
\]

Now, suppose that \( \Delta''(\tilde{\gamma}) = 0 \) for some \( \tilde{\gamma} > \gamma' \). Then, the third-order derivative at that point becomes

\[
\Delta'''(\tilde{\gamma}) = \left( \frac{q - 1}{\tilde{\gamma}} - (1 - w_1) \right) \Delta''(\tilde{\gamma})
\]

\[
- (w_2 - w_1) w_2 (1 - w_2)^2 q_k (\gamma(1 - w_2))
\]

\[
< 0
\]

This establishes that \( \Delta'(\gamma) \) is a strongly quasi-concave function according to Mas-Colell et al. (1995). Moreover, given that \( \Delta'(\gamma') = 0 \) and \( \lim_{\gamma \to \infty} \Delta'(\gamma) = 0 \), we conclude that \( \Delta'(\gamma) > 0 \) for all \( \gamma > \gamma' \). As explained above, this establishes that \( w^*(\theta) \) increases in \( \theta \), from which the last part of the theorem trivially follows, in view of \( p_2^2(\theta) = \delta \theta w^*(\theta) \). To prove the last property, we rewrite the objective function of (3) in the form \( h(r, q) = \int_{0}^{\infty} \delta (\frac{r}{\theta} - 1) \cdot N(q, z) \tilde{f}(\frac{z}{\theta} \theta, x) \, dz \), where \( r = \theta - p_2/\delta \) and \( z = \lambda (\theta - p_2/\delta) \). To prove that \( p_2^2 \) is monotonically decreasing with respect to \( q \), it is equivalent to demonstrate that \( r^*(q) \)

---

4 Excluding \( \gamma = 0 \) will not affect our result, as the probability on \( \gamma = 0 \) has a zero mass.

Then, let 

From which it is easy to see that 

Proof of Proposition 2. The consumers’ best response function may not even be upper semi-continuous, so we cannot apply the standard approaches (e.g., fixed point theorems) for proving existence. Instead, we prove the existence of equilibrium by construction. Recall that for any value of \( \theta \in [0,1] \), there exists a unique second-period price response \( \{p^*_2(Q-x,x,\theta)\} \). Moreover, once these values are calculated, one can determine the unique value: 

\[
p_1(\theta) = \theta - s_2(\theta|\theta)/E_\lambda \left[H(Q,\Lambda(1-\theta))\right] \leq \theta.
\]

Hence, by construction, the subgame has an equilibrium for any initial price \( p_1 \) in the set \( A = \{p = p_1(\theta) : \theta \in [0,1]\} \). Now, suppose that there is a value \( p_1 \in [0,1] \) which is not in the set \( A \). This means that for such \( p_1 \) value, \( s_1(\theta|\theta) < s_2(\theta|\theta) \) for all \( \theta \in [0,1] \). In particular, this inequality holds for \( \theta = 1 \), which means that, together with \( \{p^*_2(Q-x,x,1)\} \) it forms an equilibrium.  

\[
= \arg\max_q \{h(r,q)\} \text{ is increasing in } q. \text{ We establish the latter by verifying that } N(q,z) \text{ maintains a monotone likelihood ratio for } z_2 > z_1, \text{ and that } (\frac{q}{z_1} - 1) \hat{f}(\frac{z}{\theta}|\theta,x) \text{ satisfies the single-crossing property in } (r,z). \text{ To see that the first property holds, note that}

\[
\frac{N(q+1,z_2)}{N(q,1,z_1)} N(q,z) = \frac{1 + \left(\sum_{x=q+1}^{\infty} P_x(z_2)\right)}{1 + \left(\sum_{x=q+1}^{\infty} P_x(z_1)\right)} N(q,z)
\]

Then, let \( A_q(z) = e^z \sum_{x=0}^q P_x(z) \), \( A_q(z) = e^z - A_{q-1}(z) \), and observe that

\[
\frac{\sum_{x=q+1}^{\infty} P_x(z)}{N(q,z)} = \frac{A_{q-1}(z)}{A_q(z)} \left[ zA_{q-1}(z) + qA_{q+1}(z) \right]^{-1}.
\]

Thus, if \( \phi(z) = zA_{q-1}(z)/A_{q+1}(z) \) is decreasing in \( z \), it will imply that the expression in (12) is larger or equal to 1; hence, the monotone likelihood property of \( N(q,z) \) will follow. To this end, observe that

\[
\phi'(z) \cdot A_{q+1}(z)^2 = \left[A_{q-1}(z) + zA'_{q-1}(z)\right] A_{q+1}(z) - zA_{q+1}(z) A_{q-1}(z)
\]

\[
= A_{q-1}(z) A_{q+1}(z) + zA_{q-2}(z) A_{q+1}(z) - zA_q(z) A_{q-1}(z)
\]

\[
= A_{q+1}(z) \cdot \left[A_{q-1}(z) - z e^z P_{q-1}(z) - z e^z P_q(z) A_{q-1}(z)\right]
\]

\[
= \sum_{y=q+1}^{2q} \left(\sum_{s=0}^{y-q-1} \frac{1}{s!(y-s)!} - \frac{1}{q!(y-q-1)!}\right) z^y
\]

\[
+ \sum_{y=2q+1}^{\infty} \left(\sum_{s=0}^{y-q-1} \frac{1}{s!(y-s)!} - \frac{1}{(q-1)!(y-q)!}\right) z^y
\]

and since all polynomial coefficients in the expressions above are negative, we have \( \phi'(z) < 0 \). The single crossing property of \( \left(\frac{\theta}{r_2} - 1\right) \hat{f}(\frac{z}{r_2}|\theta,x) \) requires that for any \( r_2 > r_1 \), the difference function

\[
\Delta(z) = \left(\frac{\theta}{r_2} - 1\right) \hat{f}(\frac{z}{r_2}|\theta,x) - \left(\frac{\theta}{r_1} - 1\right) \hat{f}(\frac{z}{r_1}|\theta,x)
\]

crosses zero at most once from below the zero line. To see this, note that

\[
1 - \left(\frac{\theta}{r_1} - 1\right) \hat{f}(\frac{z}{r_1}|\theta,x) / \left[\left(\frac{\theta}{r_2} - 1\right) \hat{f}(\frac{z}{r_2}|\theta,x)\right] = 1 - \left(\frac{\theta - r_1}{\theta - r_2}\right) \left(\frac{r_2}{r_1}\right)^{m+x} \cdot e^{-(\beta+1-\theta)(\frac{r_1}{r_2} - 1)} z
\]

From which it is easy to see that \( \Delta(0) < 0 \), and \( \Delta'(z) > 0 \).
Proof of Proposition 3. Let \( \{\theta_1, \{p^*_1(\cdot, \theta_1)\}\} \) and \( \{\theta_2, \{p^*_2(\cdot, \theta_2)\}\} \) are subgame equilibriums for the first period prices \( p^1_1 \) and \( p^1_2 \) respectively, and \( \theta_1 < \theta_2 \). As the implicit function \( p_1(\theta) \), solved by (6), increases in \( \theta \), we must have \( p^1_1 < p^1_2 \), which implies that for an arbitrary first period price the subgame between consumers and the seller possesses an unique equilibrium according to the construction of Proposition 2. ■

Proof of Theorem 3. Let \( u_1(\theta) \doteq s_1(\theta|\theta) \) and \( u_2(\theta) \doteq s_2(\theta|\theta) \), and observe that

\[
 u_2(\theta) = (\delta - p_2) \cdot E_A \left[ \sum_{x=0}^{Q-1} \frac{H(Q-x, \Lambda(\theta - p_2/\delta)) \cdot P_x(\Lambda(1-\theta))}{\Lambda} \right] 
\]

\[
 = E_A \left[ \frac{\delta}{\Lambda} \sum_{x=0}^{Q-1} N(Q-x, \Lambda(\theta - p_2/\delta)) \cdot P_x(\Lambda(1-\theta)) \right] 
\]

\[
 = E_A \left[ \frac{\delta}{\Lambda} [N(Q, \Lambda(1-p_2/\delta)) - N(Q, \Lambda(1-\theta))] \right] 
\]

\[
 = (\delta - p_2) E_A [H(Q, \Lambda(1-p_2/\delta))] - \delta(1-\theta) \cdot E_A [H(Q, \Lambda(1-\theta))] 
\]

From the last equation, it follows that (9) is equivalent to \( u_1(\theta) = u_2(\theta) \). Next, note that

\[
u_1(p_1) = 0 < (\delta p_1 - p_2) \cdot E_A \left[ \sum_{x=0}^{Q-1} \frac{H(Q-x, \Lambda(p_1 - p_2/\delta)) \cdot P_x(\Lambda)}{\Lambda} \right] = u_2(p_2)
\]

in view of \( p_1 > p_2/\delta \). Furthermore,

\[
u'_1(\theta) = E_A [H(Q, \Lambda(1-\theta))] + (\theta - p_1) \cdot \frac{\partial}{\partial \theta} E_A [H(Q, \Lambda(1-\theta))] > E_A [H(Q, \Lambda(1-\theta))]
\]

(this inequality trivially follows from the fact that the first-period allocation probability is increasing in \( \theta \)), and

\[
u'_2(\theta) = -\frac{\partial}{\partial \theta} \left( \frac{\delta E_A \left[ \frac{1}{\Lambda} N(Q, \Lambda(1-\theta)) \right]}{E_A [H(Q, \Lambda(1-\theta))]^2} \right) = \delta \cdot E_A [E_{X(\theta)} [\Pr \{X(\theta) < Q\}] | \Lambda] < E_A [H(Q, \Lambda(1-\theta))].
\]

Therefore, \( u'_1(\theta) > u'_2(\theta) \), and hence the lines cross exactly once in view of \( u_1(1) > u_2(1) \), the equivalent of the second inequality in (10). ■

Lemma A. For any given value \( \theta \in (0, 1) \), and any \( p_2 \in (0, \theta \delta) \), the value of \( p_1 \) calculated by (9) satisfies condition (10). ■

Proof of Lemma A. First, note that

\[
(\delta - p_2) \frac{E_A [H(Q, \Lambda(1-p_2/\delta))]}{E_A [H(Q, \Lambda(1-\theta))]} = \delta(1-\theta) \frac{E_A \left[ \frac{1}{\Lambda} N(Q, \Lambda(1-p_2/\delta)) \right]}{E_A \left[ \frac{1}{\Lambda} N(Q, \Lambda(1-\theta)) \right]} > \delta(1-\theta)
\]

for any \( p_2 < \delta \theta \). Hence,

\[
p_1 = \theta + \delta (1-\theta) \left[ 1 - \frac{E_A \left[ \frac{1}{\Lambda} N(Q, \Lambda(1-p_2/\delta)) \right]}{E_A \left[ \frac{1}{\Lambda} N(Q, \Lambda(1-\theta)) \right]} \right] < \theta
\]

(13)
Additionally, from (9) it is easy to see that \( p_1 \) is increasing in \( \theta \), since \( E_{\Lambda} [H(Q, \Lambda(1 - \theta))] \) is strictly increasing in \( \theta \). Similarly, from (13) it is easy to see that \( p_1 \) is also strictly increasing in \( p_2 \). Consequently, for a given value of \( p_2 \), \( p_1 \) obtains its minimal level if \( \theta = p_2 / \delta \); plugging this value in (13) yields \( p_1 > p_2 / \delta \).

In the same spirit, plugging in \( \theta = 1 \) in (9), establishes the upper bound inequality in condition (10).

**A brief overview of the analysis of the auxiliary model presented in §3.6**

The analysis of the auxiliary model is highly complex and tedious. Thus, we provide only a brief overview of our numerical procedure below. Let \( z \) be the number of consumers in the market with valuations equal or larger than \( \tilde{\theta} \). Using this information, the seller needs to infer the statistical distribution of \( D(p_2 / \delta, \theta|z, \tilde{\theta}) \), defined as the number of customers in the segment \([p_2 / \delta, \theta)\), for any arbitrary values of \( p_2 / \delta \) and \( \theta \) (with \( p_2 / \delta < \theta \)). Three cases arise:

\[
D\left( \frac{p_2}{\delta}, \theta|z, \tilde{\theta} \right) \sim \begin{cases} 
\text{Bin} \left( z, \frac{\theta - p_2 / \delta}{1 - \theta} \right) + \text{Poisson} \left( \Lambda \left( z, \tilde{\theta} \right) \cdot \left( \theta - p_2 / \delta \right) \right) & \tilde{\theta} \leq \frac{p_2}{\delta} \\
\text{Poisson} \left( \Lambda \left( z, \tilde{\theta} \right) \cdot \left( \theta - p_2 / \delta \right) \right) & \tilde{\theta} > \frac{p_2}{\delta}
\end{cases}
\]

and the second-period pricing problem can be expressed as follows:

\[
\bar{\pi}_2 (q, z, \theta|\tilde{\theta}) = \min_{p_2 < \delta \theta} \left\{ p_2 \cdot \mathbb{E} \left[ \min \left\{ q, D \left( \frac{p_2}{\delta}, \theta|z, \tilde{\theta} \right) \right\} \right] \right\}
\]

Continuing in a way similar to our analysis of the main model, we next characterized the expected surplus functions, with \( s_1 (v|\theta) \) being identical to that of the original model. In contrast, the expression for the expected surplus associated with a wait decision \( s_2 (v|\theta, \tilde{\theta}) \) becoming significantly more complex than before. For example, the function \( s_2 (\theta|\theta, \tilde{\theta}) \), which has a pivotal role in identifying the consumers’ threshold policy \( \theta^* (\tilde{\theta}) \) is computed according to three cases:

(i) \( p_2 / \delta < \theta \leq \tilde{\theta} \). Here, let’s set \( Y = N[\theta, \tilde{\theta}], X = N[\theta, 1], Z = N[\tilde{\theta}, 1] \).

\[
s_2 (\theta|\theta, \tilde{\theta}) = \int_{\Lambda} \sum_{Z \sim \text{Poisson}(\Lambda(1 - \theta))} \sum_{Y \sim \text{Poisson}(\Lambda(\theta - \theta))} \left( \delta \theta - p_2 \left( Q - Y - Z, \theta|Z, \tilde{\theta} \right) \right)^+ \cdot \hat{H} \left( Q - Y - Z, \text{Poisson} \left( \Lambda \left( \theta - p_2 \left( Q - Y - Z, \theta|Z, \tilde{\theta} \right) \right) / \delta \right)^+ \right) \mathbb{P} (Z|\Lambda) \mathbb{P} (Y|\Lambda) f (\Lambda) d \Lambda
\]

where \( N[a, b] \) is defined as the number of arriving consumers with valuations in the range \([a, b]\), and

\[
\hat{H} \left( q, \text{P} \mathbb{P} P \right) = \sum_{i=0}^{\infty} \frac{q}{\max(1 + \frac{1}{q}, P_i)}
\]

(ii) \( \tilde{\theta} \leq p_2 / \delta < \theta \). Here, we set \( Y = N[\theta, \tilde{\theta}], X = N[\theta, 1], Z = N[\tilde{\theta}, 1] \).

\[
s_2 (\theta|\theta, \tilde{\theta}) = \int_{\Lambda} \sum_{X \sim \text{Poisson}(\Lambda(1 - \theta))} \sum_{Y \sim \text{Poisson}(\Lambda(\theta - \theta))} \left( \delta \theta - p_2 \left( Q - X, \theta|X + Y + 1, \tilde{\theta} \right) \right)^+ \cdot \hat{H} \left( Q - X, \text{Bin} \left( \frac{\theta - p_2 \left( Q - X, \theta|X + Y + 1, \tilde{\theta} \right) / \delta}{\theta - \theta} \right) \right) \mathbb{P} (X|\Lambda) \mathbb{P} (Y|\Lambda) f (\Lambda) d \Lambda
\]
(iii) $p_2/\delta < \bar{\theta} < \theta$. Here, we set $Y = N[\bar{\theta}, \theta,$ $X = N[\theta, 1], Z = N[\bar{\theta}, 1]$. 

$$s_2 (\theta|\tilde{\theta}, \tilde{\theta}) = \int_{\bar{\Lambda}} \sum_{X \sim \text{Poisson}(\Lambda(1-\theta))} \sum_{Y \sim \text{Poisson}(\Lambda(\theta-\bar{\theta}))} (\delta \theta - p_2 (Q - X, \theta|X = Y + 1, \bar{\theta}))^+ \cdot \tilde{H} \left( Q - X, Y + \text{Poisson} \left( \Lambda (\theta - p_2 (Q - X, \theta|X = Y + 1, \bar{\theta}) / \delta)^+ \right) \right) \Pr (X|\Lambda) \Pr (Y|\Lambda) f(\Lambda) d\Lambda$$

In deviation from the segmentation procedure used in our main model, here we utilize a search over the information threshold $\bar{\theta}$. Specifically, we systematically vary $\bar{\theta}$ over the range $[0, 1]$ using steps of 0.01 (we found that using steps of 0.001 dramatically increase the run time by a factor of more than one thousand, without any noticeable increase in accuracy). Then, for each value of $\bar{\theta}$ we identify (if at all) a pair of values $(p_1, \theta = \bar{\theta})$ that forms an equilibrium in the model. Obviously, as the seller can select $p_1$ freely, we pick the equilibrium point that would maximize the seller’s expected revenue performance.

**Analysis of the model described in §4 (a market consisting of both strategic and myopic consumers)**

Similarly to our analysis in the main model, we can demonstrate that, in equilibrium, the strategic consumers follow a purchasing policy with a unique threshold $\theta$. Additionally, for any given consumers’ threshold $\theta$ and premium price $p_1$, the seller’s second-period profit function (of the leftover inventory $q$, and the first-period sales $x$) is given by

$$\pi_2 (\theta, p_1, q, x) = \max_{0 \leq p_2 \leq \delta \theta} \{\pi_2 (p_2|\theta, p_1, q, x)\},$$

where

$$\pi_2 (p_2|\theta, p_1, q, x) = \left\{ \begin{array} {c} E_{\tilde{\Lambda}} \left[ p_2 \cdot N \left( q, \alpha \tilde{\Lambda} \cdot (\theta - p_2 / \delta) \right) \right] ; \text{ if } \delta p_1 < p_2 \leq \delta \theta \\
E_{\tilde{\Lambda}} \left[ p_2 \cdot N \left( q, \tilde{\Lambda} \cdot (\alpha (\theta - p_2 / \delta) + (1 - \alpha) (p_1 - p_2 / \delta)) \right) \right] ; \text{ if } 0 \leq p_2 \leq \delta p_1 \end{array} \right.$$ 

where $\tilde{\Lambda}$ is the updated mean market size and follows a Gamma distribution with the pair of parameters $(m + x, \beta + \alpha (1 - \theta) + (1 - \alpha) (1 - p_1))$. The reason we distinguish between the two cases of the second-period price $p_2$ is that when $\delta p_1 < p_2 \leq \delta \theta$, only strategic consumers will purchase in the second period, as myopic consumers whose valuations are higher than $p_2$ have already purchased in the first period. In contrast, when $p_2 \leq \delta p_1$, both strategic and myopic consumers will attempt to purchase in the second period.

Continuing one step further towards the identification of a market equilibrium, we restructure the expected surplus functions for a strategic consumer with valuation $v$. To this end, consider a market where all strategic consumers follow a purchasing policy $\theta$, and let $p_2^* (x)$ be the solution to the above second-period seller’s optimization problem (for brevity, we omit $p_1$ as it is known prior to the consumers’ choices, and $q$ as it can be uniquely determined by $Q$ and $x$). Thus, the expected surplus for an immediate purchase is given
by
\[ S_1 (v|\theta) = (v - p_1) E_\Lambda [H (Q, \Lambda (\alpha (1 - \theta) + (1 - \alpha) (1 - p_1)))] , \]
and that for a wait decision is given by
\[ S_2 (v|\theta) = E_\Lambda [E_{X(\theta,p_1)} [(\delta v - p_2^* (X (\theta,p_1))) \cdot H (Q - X (\theta), \Lambda \cdot g (\theta, p_2^* (X (\theta,p_1)))) |\Lambda]] , \]
where
\[ g (\theta, p_2) = \begin{cases} \alpha (\theta - p_2/\delta) & ; \delta p_1 < p_2 \leq \delta \theta \\ \alpha (\theta - p_2/\delta) + (1 - \alpha) (p_1 - p_2/\delta) & ; 0 \leq p_2 \leq \delta p_1 \\ 0 & ; \text{o.w.} \end{cases} \]
Finally, the seller’s total revenue function for any given purchasing policy \( \theta \), can be expressed by
\[ \pi (Q) = \max_{0 \leq p_1 \leq 1} \left\{ E_\Lambda [p_1 \cdot N (Q, \Lambda (\alpha (1 - \theta) + (1 - \alpha) (1 - p_1)))] + E_\Lambda [E_{X(\theta,p_1)} [\pi_2 (\theta, p_1, Q - X (\theta,p_1), X (\theta,p_1))]] \right\} \]
Unfortunately, unlike our main model, the latter problem does not possess the same separation property that can transform the seller’s decision problem into an easily-tractable segmentation problem (note that the second-period profit depends on both \( p_1 \) and \( \theta \)). Consequently, we employ a tedious price optimization process, in which for any given \( p_1 \), we identify the Nash equilibrium between consumers and the seller iteratively: we search for the equilibrium \( \theta \) from \( p_1 \) to 1. For any given \( \theta \), we identify the seller’s optimal second-period price by maximizing \( \pi_2 (p_2|\theta, p_1, q, x) \). Then, using \( \theta \) and the resulting optimal value of \( p_2 \), we calculate the unique supporting \( \tilde{p}_1 \) by setting \( S_1 (\theta|\theta) = S_2 (\theta|\theta) \). If \( p_1 = \tilde{p}_1 \), then \( \theta \) and \( p_2 \) are the equilibrium for the fixed \( p_1 \). Otherwise, we increase \( \theta \) and try the above processes again. If the above procedure fails to identify a subgame equilibrium for \( \theta \in [p_1, 1] \), then it must be the case that all strategic consumers wait in the first period. In this case, we set \( \theta = 1 \) and evaluate the seller’s second-period profit accordingly. Finally, we use a line-search method to determine the optimal first-period price that maximizes the seller’s total expected revenue.


We propose a model of Quick Response (QR), as discussed in §6. In this model, the seller has an option to replenish its inventory – in a quick fashion – at the end of the first period. After observing the first-period demand, and prior to setting the price at the beginning of the second period, the seller can obtain additional units of the product, expeditiously delivered, at a per-unit cost \( c' \) that is normally higher than that charged for units purchased prior to the start of the season (denoted by \( c \)). We assume that the seller observes the demanded number of units during the first period, which includes units not sold, in case of a shortage. Consumers that demand the product during the first period, but do not get it, remain in the market. They can demand the product at the end of the season if the inventory is replenished, and if they find the end-of-season price acceptable.
To study the quick response (QR) setting, we searched for a rational expectation equilibrium in which all consumers follow a threshold policy. Let \( \pi^{QR}(Q) \) denote the expected revenue performance under QR, if the seller’s initial inventory is \( Q \), and let

\[
\Pi^{QR} = \max_Q \{ \pi^{QR}(Q) - cQ \}
\]

be the optimal profit function under the QR setting (see similar definition for our main model in §3.8). As done before, we begin by analyzing the seller’s second-period profit for a given threshold \( \theta \). However, this time, we look at the demanded quantity \( x \), which can possibly be larger than the initial inventory level \( Q \). Similarly, we note that the leftover inventory \( q = Q - x \) can be a negative number. The seller’s objective is to identify the best order quantity \( q_2^*(\theta, q, x) \) and price \( p_2^*(\theta, q, x) \). The decision problem can be written as follows.

\[
\pi^{QR}_2(\theta, q, x) = \begin{cases} 
\max_{c' \leq p_2 \leq \delta \theta} \left\{ p_2 \cdot E_{\hat{\Lambda}(x, \theta)} \left[ N \left( q_2 + q, \hat{\Lambda}(x, \theta) \cdot (\theta - \frac{p_2}{\delta}) \right) \right] - c' \cdot q_2 \right\} & q \geq 0 \\
\max_{c' \leq p_2 \leq \delta \theta} \left\{ -q \left( p_2 - c' \right) + E_{\hat{\Lambda}(x, \theta)} \left[ p_2 \cdot N \left( q_2, \hat{\Lambda}(x, \theta) \cdot (\theta - \frac{p_2}{\delta}) \right) \right] - c' \cdot q_2 \right\} & q < 0
\end{cases}
\]

Note that when \( q < 0 \), two possibilities arise: First, as reflected by the first maximization expression in the equation above, is the case where the seller sets the second-period price in the range \([c', \delta \theta]\) and targets all of the consumers that attempted to purchase in the first period \((-q)\) in addition to the consumers with (second-period) valuations in the range \([p_2, \delta \theta]\). The second option the seller has is to exclusively target the high-valuation consumers, by selecting a second-period price above \( \delta \theta \). In such case,

\[
\pi^{exc.}_2(\theta, q, x) = \max_{p_2 \in [\delta \theta, \delta \theta]} \left\{ p_2 \cdot E \left[ \min \left\{ q_2, \text{Binomial} \left( -q, \frac{1 - \frac{p_2}{\delta}}{1 - \theta} \right) \right\} \right] - c' q_2 \right\}
\]

Under the above optimal inventory and pricing decisions in the second-period, the consumers may be exposed to rationing (allocation to consumers is settled on a completely random basis). The exact expressions for the rationing probabilities are straightforward but tedious, and we therefore use the condensed formulation \( r_2(\theta, q, x) \) to denote the probability of assigning a unit to a randomly selected consumer that attempts to purchase in the second period. We are now ready to present the expected surplus functions for a focal consumer who makes a purchase decision in the first period. Note that the value of \( s_1(v, \theta) \) is different than before, as it now includes the possibility of a shortage and inventory replenishment. Specifically,

\[
s_1(v|\theta) = E_{\Lambda} \left[ E_{X(\theta)} \left[ (v - p_1) \frac{Q}{\max \{1 + X(\theta), Q\}} + \left( 1 - \frac{Q}{\max \{1 + X(\theta), Q\}} \right) \cdot \omega(v, \theta, X(\theta) + 1) \right] \right]
\]
where $\omega(v, \theta, x) \equiv (\delta v - p^*_2(\theta, Q - x, x))^+ \cdot r_2(\theta, Q - x, x)$. Similarly, the expected surplus associated with a wait decision is given by

$$s_2(v|\theta) = E \left[ E_{X(\theta)} \left[ \omega(v, \theta, X(\theta) + 1) \right] \right]$$

When a solution to the equation $s_1(\theta|\theta) = s_2(\theta|\theta)$ exists, the value of $p_1$ as a function of $\theta$ can be determined immediately and uniquely. As in the original model, when no solution exists, the unique equilibrium in the model is one in which all consumers wait (i.e., $\theta = 1$). Additionally, once an equilibrium is identified for a given value of $p_1$, we can calculate the seller’s optimal expected revenue as follows:

$$\pi^{QR}(Q) \equiv \max_{0 \leq p_1 \leq 1} \left\{ E_A \left[ p_1 \cdot N(Q, A \cdot (1 - \theta)) \right] + E_A \left[ E_{X(\theta)} \left[ \pi^{QR}_2(\theta, Q - X(\theta), X(\theta)) \right] \right] \right\}$$

Finally, since we are interested in identifying the optimal profit function under the QR setting, we search for the optimal initial inventory level to obtain $\Pi^{QR}$. The optimal profit under QR and myopic consumers is also of interest to us. The evaluation of this setting is similar in spirit to the analysis above, yet significantly simpler; thus, the details are omitted.

We used our QR model in a numerical study that spans across 128 scenarios, covering all possible combinations of two levels of degree of uncertainty $CV \in \{1, 2\}$, four levels of consumer’s $\delta \in \{0.5, 0.6, 0.7, 0.8\}$, two replenishment cost per-unit $c \in \{0.15, 0.25\}$, four levels of $c' - c \in \{0.00, 0.05, 0.10, 0.15\}$ (this is the cost premium the seller pays for a quick replenishment), and both cases of all-myopic and all-strategic consumers. We again held the average value of the market size fixed at $E[A] = 10$. Table 6 shows the percentage benefits of responsive pricing (over fixed pricing). The numbers in the shaded cells on the table provide the case of no-QR (our original model) as a reference. For the QR setting, we show the value $\Pi^{QR}/\Pi_{F/QR} - 1$, where $\Pi_{F/QR}$ is the optimal seller’s expected profit under QR and fixed-pricing. For the no-QR case, we show the value $\Pi^{R}/\Pi_{F} - 1$, as before. Let us first consider the case of all-myopic consumers, presented on the top part of the table. Our results suggest that when $\delta$ is large, it is sufficiently optimal for the seller to use the inventory lever (via QR) only, without the pricing lever. For instance, consider the scenario ($\delta = 0.8, CV = 2, c = 0.15, c' = c$); here, responsive pricing adds a value of 1.15% compared to 53.08% if QR was not available. The seller can simply utilize the demand information gathered during the first period to establish its understanding of the market potential, and then control its profit via the inventory decision lever. When $\delta$ gets smaller, the seller can utilize the inventory lever less effectively, as pushing sales to the end of the season decreases the revenue potential due to the mere decline in consumers’ valuations. When the consumers are strategic, the situation reverses, and the loss in expected profit (due to strategic behavior) appears to be mitigated under QR, in comparison to the original (no-QR) setting.

We have also looked at the benefits of QR, as given by the value $\Pi^{QR}/\Pi^{R} - 1$ for the same range of
Table 6: The benefits of responsive pricing over fixed pricing in the presence of Quick Response.

<table>
<thead>
<tr>
<th>CV \ δ</th>
<th>QR premium cost/ unit (c^c)</th>
<th>0.15</th>
<th>0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>+0.00</td>
<td>+0.05</td>
<td>+0.10</td>
</tr>
<tr>
<td>0.5</td>
<td>12.54%</td>
<td>15.14%</td>
<td>17.79%</td>
</tr>
<tr>
<td>0.6   !</td>
<td>12.12%</td>
<td>14.90%</td>
<td>18.28%</td>
</tr>
<tr>
<td>0.7</td>
<td>10.16%</td>
<td>14.15%</td>
<td>17.36%</td>
</tr>
<tr>
<td>0.8</td>
<td>7.53%</td>
<td>11.57%</td>
<td>15.92%</td>
</tr>
<tr>
<td>0.5</td>
<td>5.74%</td>
<td>9.01%</td>
<td>14.00%</td>
</tr>
<tr>
<td>0.6</td>
<td>3.48%</td>
<td>6.20%</td>
<td>10.30%</td>
</tr>
<tr>
<td>0.7</td>
<td>1.29%</td>
<td>3.77%</td>
<td>6.59%</td>
</tr>
<tr>
<td>0.8</td>
<td>1.15%</td>
<td>1.55%</td>
<td>3.72%</td>
</tr>
</tbody>
</table>

scenarios presented above. The results are very consistent with the picture arising from Table 6. As expected, QR is always valuable in the case of myopic consumers, ranging from 2.41% under \( \{ c = 0.15, c' - c = 0.15, \delta = 0.5, CV = 1 \} \), and increasing in the parameters \( (\delta, CV) \) while decreasing in \( c' - c \), up to 155.12% under \( \{ c = 0.15, c' - c = 0.00, \delta = 0.8, CV = 2 \} \). When \( c = 0.25 \), the numbers range in the same pattern from 0.00% to 420.62%. When consumers are all strategic, we found that the benefits of QR are even better than those of the myopic-consumers case, across all scenarios. In fact, the results follow the same pattern with respect to \( \delta, CV, \) and \( c' - c \). The benefits range from 4.81% to 348.85% when \( c = 0.15 \), and from 0.00% to 964.22% when \( c = 0.25 \).