Brand Spillover as a Marketing Strategy

When a weak-brand firm and a strong-brand firm source from a common contract manufacturer, the weak-brand firm may advertise this relationship to promote its own product. This paper investigates whether the weak-brand firm should use such brand spillover as a marketing strategy and how this decision depends on the firms' characteristics and market conditions. We develop a game theoretic model consisting of one contract manufacturer and two firms with asymmetric brand power. The contract manufacturer determines the wholesale prices for the two firms and then each firm decides whether to source from the contract manufacturer. If both firms outsource to the contract manufacturer, then the weak-brand firm may choose whether to promote its product through brand spillover. Although brand spillover improves the attractiveness of the weak-brand firm's product at no cost, we find that the weak-brand firm should not use brand spillover if (1) its original brand power is sufficiently low or (2) the contract manufacturer does not have a significant cost advantage. Interestingly, the adoption of brand spillover by the weak-brand firm can benefit all three parties under certain circumstances. Nevertheless, when the contract manufacturer has a significant cost advantage, in equilibrium the strong-brand firm will be hurt by brand spillover and hence should take actions to prevent it.

Key words: brand spillover, marketing strategy, brand attractiveness, sourcing strategy

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1. Introduction

"*If you are*
a contract manufacturer for well-known brands,
with ISO quality management systems, CE or other certifications,
who can provide superior and professional OEM or ODM services,
please contact us!"[1]

This is a quote from the website of Netease Yanxuan, an online retailer that sells packaged mass consumption goods such as bedclothes, kitchenware, and personal care products.[2] The parent company, Netease, is a top five Chinese internet company with a market value of more than $30 billion. Yanxuan means "strictly selected" in Chinese. The company claims that its mission is to select superior products for Chinese consumers with a rigorous standard.

In China, as people's income and purchasing power rise, the upper-middle class population has been expanding rapidly over the past decade. Currently, this segment of the population not only

[1] http://you.163.com/help#business
[2] http://english.cntv.cn/2016/07/23/VIDEUeIvaNUK8ozJoZoqs8yU160723.shtml

buys more but also demands increasingly high-quality products. At the same time, thousands of contract manufacturers are producing high-quality products for famous brands, and many of these products are sold to Chinese consumers with a high profit margin because of the brand premium. Yanxuan's quote above shows that the company is eager to capture this burgeoning market in China by working with the contract manufacturers of leading international brands. Some of Yanxuan's products are to some extent similar to the leading brands' products in appearance and design. Interestingly, Yanxuan highlights the international brands, such as Adidas, Coach, and MUJI, when advertising its own products in certain categories (see its homepage http://you.163.com/). Yanxuan claims that it sells the same quality products as international brand companies by using "the same materials, the same factories, and the same workers". Seeing such advertisements, consumers who believe that the products of famous brands are of high quality are likely to trust that the products of a nascent brand such as Yanxuan are of high quality as well. In other words, brand reputation from a strong-brand firm spills over to a weak-brand rival due to the disclosure that their products originate from a common contract manufacturer; we refer to this phenomenon as brand spillover.

Despite the increasing presence of online retailing in China, MINISO, a fast fashion chain store established in 2013 that specializes in household and consumer goods (including cosmetics, stationery, toys, and kitchenware), is gaining tremendous popularity.[3] It has enjoyed explosive growth across Asia and garnered revenues of $1.5 billion in 2016. The secret to the company's great success, revealed by co-founder Guofu Ye, is its cooperation with suppliers of leading brands and provision of high-quality goods at affordable prices. In many media articles, the company advertises that its perfume product line is created by a perfumer from Givaudan, a leader in the fragrance industry that serves as the long-term supplier of some well-known brands such as Chanel, Dior, and Gucci. Similarly, the company highlights its collaboration with the world's top tableware supplier, Jiacheng Groups, which is also the supplier of the famous brand Zwilling.

Yanxuan and MINISO are two typical examples in which companies adopt brand spillover as a marketing strategy. The brand spillover phenomenon has been observed not only in the retail industry but also in other industries, including automobiles and consumer electronics. Chery (or Qirui), a Chinese automobile manufacturer, once promoted that its FlagCloud (or Qiyun) model had used a BMW engine.[4] In fact, this engine comes from Tritec, even though it is also used in the BMW Mini series.[5] When launching a new generation of its smartphone, Smartisan, a Chinese high-tech start-up, highlighted that it adopted high-quality components from the suppliers of Samsung

[3] https://en.wikipedia.org/wiki/Miniso

[4] http://news.163.com/special/reviews/autoindustry.html

[5] https://en.wikipedia.org/wiki/Tritec_engine

(Liang 2014).

Although Yanxuan and MINISO are not the first to leverage competitors' brand power, they are unique in that they developed a marketing strategy that focuses on brand spillover. That is, they systematically seek cooperation with the contract manufacturers of leading international brands to foster brand spillover.

While Yanxuan and MINISO adopt brand spillover as their marketing strategy, some firms choose not to do so. JD.com, one of China's biggest e-commerce and logistics companies, now sells its own branded goods under a new line called Jingzao, which by pronunciation means "finely made". The new brand focuses on what is termed the "normcore" segment (normal-looking clothes) and soft furnishings with natural hues.[6] Like Yanxuan and MINISO, Jingzao aims to provide high-quality products at fair prices; however, it does not reveal any information on product webpages about whether its contract manufacturers are also producing for leading brands. In fact, our investigation shows that many of Jingzao's products are sourced from contract manufacturers that also produce for leading brands.[7]

The above observations suggest that different approaches have been used by firms to market their high-quality products. Some firms try to take advantage of the brand spillover strategy, i.e., they publicize the connection to leading brands through the use of the same materials, the same factory, and the same workers. However, other firms choose not to reveal the information about their contract manufacturers, which eliminates the possibility of brand spillover. Why do firms choose different approaches to manage brand spillover, and when should a firm use brand spillover as a marketing strategy? This is the first research question we address.

When approached by new brands such as Yanxuan and MINISO, contract manufacturers need to decide whether to produce for firms that may want to take advantage of the brand spillover effect. This decision is not a straightforward one. Intuitively, a leading brand may not be willing to share its most valuable intangible asset (i.e., the brand name) with a potential competitor and thus may impose pressure on the contract manufacturer. In addition, the leading brand can also prevent brand spillover via insourcing. How should the contract manufacturer deal with such a multilateral relationship, and what sourcing structure should it induce? This is the second research question we explore.

The expanding middle-class consumers in emerging markets can to some extent afford leading brands, but they are still price sensitive. By using the brand spillover strategy, entrants such

[6] https://www.weekinchina.com/2018/01/finely-made/

[7] https://baijiahao.baidu.com/s?id=1655623283163894053&wfr=spider&for=pc

as Yanxuan and MINISO may encroach on the market of those leading brands, creating serious competition between them. Will the leading brands become worse off and/or is it necessary for them to fight against brand spillover? The answers to these questions are not clear because the contract manufacturer may use different pricing strategies depending on the presence of brand spillover. If the contract manufacturer wants to maintain its relationship with the strong-brand firm, and simultaneously supplies to the relatively weak-brand firm, it must provide favorable terms to the strong-brand firm. That is, although strong-brand firms face tougher competition, they might also enjoy more attractive prices from contract manufacturers. Therefore, it is not immediately clear how brand spillover influences the profits of leading brands.

In addition, some local firms in emerging economies have been increasingly investing in R&D to improve the performance of their products (Casey 2014), which narrows the gap between those traditionally regarded as low-end firms and leading international brand firms. Will the rise of weak brands hurt high-end competitors? The impacts of brand spillover and the weak-brand's attractiveness on the leading brand's profitability constitute the third set of questions we investigate.

Through the above questions, we study the implications of brand spillover from the weak-brand firm's, the strong-brand firm's, and the contract manufacturer's perspectives. To obtain a better understanding of the brand spillover phenomenon, we develop a model with one contract manufacturer (CM) and two downstream competing firms, Firm S and Firm W. Firm S is a strong-brand firm, for whose products consumers have high willingness-to-pay because of the brand premium. Compared to Firm S, Firm W is relatively weaker in brand power, and hence consumers have lower willingness-to-pay for its products—similar to those offered by Yanxuan and MINISO. The CM needs to decide the wholesale prices to charge each firm, and each firm decides whether to source from the CM or to insource at a fixed marginal cost. If both Firm S and Firm W source from the same CM, Firm W needs to decide whether to feature brand spillover to promote the perceived attractiveness of its product to consumers. If Firm S and Firm W do not both source from the CM, then brand spillover does not exist.

We first derive the equilibrium sourcing structure that the CM induces and the firms' profits when Firm W plans to use brand spillover whenever possible. The results when Firm W does not use brand spillover can be derived as a special case. By comparing Firm W's profit with brand spillover to that without brand spillover, we derive Firm W's optimal brand spillover strategy.

The results can be summarized as follows. When Firm W's brand power is too low, it should not use brand spillover; otherwise, the CM is not willing to produce for Firm W. If the CM does not have a significant cost advantage, Firm W should not use brand spillover either; otherwise, to keep Firm S as a customer, the CM has to charge an overly high wholesale price to Firm W but an overly low wholesale price to Firm S. When the magnitude of brand spillover plays an active role, Firm W should adopt the brand spillover strategy for intermediate levels of spillover. When Firm W adopts the brand spillover strategy, all firms, including Firm S, can be better off compared to the case without brand spillover. Furthermore, as Firm W climbs up the value chain and its brand power improves, Firm S can benefit once Firm W switches its strategy from not using brand spillover to using brand spillover.

The remainder of this paper is organized as follows. Section 2 reviews the related literature, and Section 3 sets up the model. We derive the equilibrium outcome in Section 4 and analyze the implications of brand spillover in Section 5. Finally, we conclude in Section 6. All proofs are presented in the appendices.

2. Literature Review

The marketing literature on brands and branding has empirically documented the effect of brand spillover within a single firm, e.g., by leveraging the equity in established brands, a firm can proliferate brand extensions relatively easily (Balachander and Ghose 2003, Swaminathan et al. 2012, Thorbjørnsen et al. 2016). Brand spillover between two firms has been investigated when they are presented as a brand alliance to consumers, e.g., an airline company and a bank jointly branding a credit card (Simonin and Ruth 1998, Yang et al. 2009, Cobbs et al. 2015). Brand spillover between competing firms has also been verified; for example, a brand scandal, a product recall, or a bankruptcy filing will spill over and negatively affect competing brands (Roehm and Tybout 2006, Borah and Tellis 2016, Ozturk et al. 2019). However, there are only a few papers studying the implications of these spillover effects. Fazli and Shulman (2018) find that when selling in a product market results in a spillover effect on another product market, competing brands can benefit from a negative market spillover and be hurt by a positive spillover. In this paper, motivated by the phenomenon that some firms leverage brand spillover via a common contract manufacturer as a marketing strategy, we study a positive spillover from the strong brand to the weak brand, and investigate the impacts of brand spillover on the contract manufacturer and the two competing brands.

Theoretical studies on brand management consider mainly competitive strategies between brands, especially on pricing and positioning (e.g., Carpenter 1989, Villas-Boas 2004, and Caldieraro 2016). A stream of literature in this field focuses on how to compete with counterfeiters from the perspective of authentic brands. For example, Sun et al. (2010) propose the optimal component-based technology transfer strategy in the presence of potential imitators. Qian and Xie (2014) develop an automated nonparametric data fusion approach to study counterfeit purchase behaviors and suggest ways to counter counterfeits. Cho et al. (2015) examine the effectiveness of competing strategies when the counterfeiter is either deceptive or nondeceptive. Qian et al. (2015) identify the conditions under which branded incumbents should focus on improving the searchable quality of their products in response to entries by counterfeiters. Gao et al. (2017) investigate the impact of counterfeits on luxury brands when the product attributes include physical resemblance and product quality and consumers have both consumption and status utility. Pun and DeYong (2017) use a two-period model to study the competition with copycats when customers are strategic. In contrast, our research investigates whether or not to leverage brand spillover from a weak brand's perspective. Since the existence of brand spillover in this paper depends on the premise that both firms outsource to the same CM, the CM plays a critical role in profit allocation among the firms. Consequently, brand spillover can benefit all parties, i.e., both firms and the CM. In other words, the strong brands may wish to embrace rather than boycott brand spillover under certain conditions.

This research is also related to firms' sourcing strategies under competition. In our paper, the weak-brand firm's brand spillover strategy depends critically on the sourcing structure. Normally, firms prefer outsourcing out of cost efficiency consideration. However, in a competitive environment, firms might outsource to a CM with no cost advantage compared to insourcing because of strategic considerations (Cachon and Harker 2002, Arya et al. 2008, and Liu and Tyagi 2011). Using a bargaining framework, Feng and Lu (2012) demonstrate that competing firms that outsource to a lower-cost supplier in equilibrium actually become worse off than they would be if both of them insource. Our work differs significantly from the existing literature by incorporating the possibility of brand spillover. We find that the weak brand should leverage brand spillover only if the cost disadvantage of insourcing is large enough.

Most of the sourcing literature examines sourcing decisions from the downstream firm's perspective. However, several studies, such as Venkatesh et al. (2006) and Xu et al. (2010), take the upstream player's perspective on the strategic design of supply chain structure. Similarly, in our paper, the upstream CM strategically sets wholesale prices to induce a certain sourcing equilibrium. Differently, this paper considers potential brand spillover in the CM's sourcing structure decision, and focuses on the weak brand's brand spillover strategy instead of the equilibrium sourcing structure.

Another related stream of literature investigates the impact of technology spillover, which is usually modeled as the effect of demand enhancement or cost reduction due to a competitor's investment. Sourcing strategy in light of technology spillover remains understudied. Van Long (2005) and Chen and Chen (2014) examine a downstream firm's outsourcing decision with potential technology spillover to a CM when they compete in the final market. Our work differs from these two papers in that we consider the interaction among one CM and two competing firms instead of between one CM and one firm. Also, the potential one-way brand spillover is from the strong brand to the weak brand only if these brands outsource to the same CM. Wang et al. (2014) and Agrawal et al. (2016) investigate the impact of technology spillover when competing firms invest in a shared CM. In these two papers, the supply chain structure is exogenously given, i.e., the firms always outsource and technology spillover can be two-way from either firm to the other. However, in our paper, the supply chain structure is endogenous, i.e., each firm can choose between outsourcing and insourcing, and there is only one-way brand spillover.

Furthermore, technology spillover in these papers is assumed to exist regardless of the firms' actions. In contrast, the weak-brand firm in this paper can choose whether or not to leverage brand spillover as a marketing strategy. The primary objective of this work is to address the issues of when brand spillover is a sensible marketing strategy and how it affects different firms' profitability.

3. Model

Consider two firms (indexed by $i \in \{S, W\}$) competing in the same market. The firms differ in the strength of their brand names. Specifically, consumers have different willingness to pay for the two firms' products. Suppose that consumers' perceived brand power of Firm i is θ_i, $\theta_S > \theta_W$. That is, Firm S has a strong brand, whereas Firm W has a weak brand. We follow the literature to model the competition between the two firms. Specifically, we adopt the following variation of the Cournot competition model:

$$p_i = \theta_i - q_S - q_W,$$

where q_i and p_i, $i \in \{\text{S, W}\}$, are the selling quantity and selling price of Firm i, respectively. This inverse demand function can be derived based on utility functions that are quadratic in product quantities (Dixit 1979) and has been widely used in the literature (see, e.g., Levinthal and Purohit 1989, Purohit 1994, Bhaskaran and Ramachandran 2011, Grahovac et al. 2015, and Arya et al. 2021). In Subsection 6.1, we develop a model of vertical differentiation, and demonstrate that all qualitative results are preserved.

Both firms aim to offer high-quality products, though they differ in the strength of their brand names. We assume that they have the same cost efficiency for in-house production. Let c denote the unit production cost for both firms (i.e., the insourcing cost). We have also studied the case of heterogenous production costs and found that all the results are qualitatively preserved. There is a CM from whom the firms can source. The CM is more efficient in production than the firms.[8] For analytical transparency, the unit production cost of the CM is normalized to 0. Note that even though the CM has a cost advantage and may offer a wholesale price lower than the firms' own production cost, whether a firm should insource or outsource production is not a trivial question due to strategic considerations. Let (O, O), (O, I), (I, O), and (I, I) denote the four possible sourcing structures chosen by the two firms, where the first (second) letter denotes Firm S's (Firm W's) sourcing decision and O (I) stands for the outsourcing (insourcing) decision. We use superscripts oo, oi, io, and ii to refer to these sourcing structures, respectively.

When at least one firm insources, brand spillover cannot happen. When both firms outsource to the CM (i.e., under the structure (O, O)), there is a one-way brand spillover from Firm S to Firm W if Firm W decides to use the brand spillover strategy.[9] However, if Firm W does not use the brand spillover strategy (i.e., the firm does not advertise the outsourcing structure), the spillover effect will be negligible because consumers are generally not sophisticated enough to infer the firms' sourcing strategies. If brand spillover occurs, it can improve the consumers' willingness to pay for Firm W's product.

Without loss of generality, we assume the consumers' perceived brand power for Firm S is $\theta_S = 1$. For Firm W, the original brand attractiveness is $\theta_W = \theta < 1$; however, when both firms outsource to the CM and Firm W adopts the brand spillover strategy, θ_W increases to $\theta_W = \theta + \alpha(1 - \theta)$, where $\alpha \in [0, 1]$ represents the level of brand spillover, with $\alpha = 0$ indicating no brand spillover and $\alpha = 1$ indicating the highest level of brand spillover. In the Yanxuan example, if consumers believe that the CM who supplies to both Yanxuan and the leading brand will use the same materials and follow similar production processes, the brand spillover effect will generally be strong (i.e., α is large). The magnitude of α also depends on the categories or attributes of the product. For example, production of bedding sets is more standard and involves less technology, and hence consumers tend to believe that those produced by a leading brand's CM are similarly attractive as the leading brand's products. In this case, brand spillover is typically strong. However, although

[8] An alternative interpretation is that the firms do not have in-house production capacity and have to rely on outsourcing, and among all potential suppliers, the CM is the most efficient in delivering the target quality.

[9] There might be a two-way brand spillover effect. That is, the use of brand spillover might also influence consumers' perceived attractiveness of Firm S's product. However, compared with the impact on Firm W's product, the impact on Firm S's product is much less significant. We conjecture that as long as the total impact of brand spillover on the supply chain is positive, that is, either the impacts on Firm W's and Firm S's products are both positive (the impact on Firm S's product might be positive due to increased brand awareness out of Firm W's advertisement (Qian 2014)), or the positive impact on Firm W's product outweighs the negative impact on Firm S's product, the findings of this study can be qualitatively preserved.

production of high-fashion clothing is also standard, brand spillover in this category is weak as consumers care about the brand names more than the function of the products. For mother and baby products, consumers are more risk averse, and brand spillover should be weak as well. The level of brand spillover depends largely on the products' characteristics, and hence α is treated as an exogenous parameter in this paper.

All parties engage in a three-stage game. First, Firm W chooses its marketing strategy, i.e., whether to adopt the brand spillover strategy. Second, the CM announces wholesale prices w_S to Firm S and w_W to Firm W. Third, the firms make their sourcing decisions simultaneously, either outsourcing (i.e., accepting the CM's wholesale price) or insourcing (i.e., producing in-house at the unit cost of c). The firms' decisions will give rise to the corresponding sourcing structure. Finally, the firms simultaneously decide the quantities supplied to the market and sell their products at market-clearing prices. All firms are risk-neutral and try to maximize their own profits.

Note that Firm W's decision regarding brand spillover occurs before the CM's wholesale price decisions for two reasons: First, the CM has to take into account Firm W's brand spillover strategy in its wholesale price decisions. In other words, one may assume that the CM sets wholesale prices contingent on Firm W's brand spillover strategy. Second, the CM's wholesale prices are short-term decisions and can be modified over time; however, Firm W's decision regarding whether to use brand spillover is often a long-term, strategic decision.

To be more specific, in the first stage, Firm W essentially decides whether to make the commitment of not using brand spillover. If it does not make such a commitment (or is unable to make a credible commitment), then after the wholesale prices are offered by the CM, under the structure (O, O), it is always optimal for Firm W to use brand spillover ex post to improve its brand attractiveness. Therefore, the case of Firm W not making any commitment is equivalent to "adopting the brand spillover strategy" in the first stage, and brand spillover will occur under the structure (O, O). If Firm W commits to not using brand spillover, then there will not be brand spillover under the structure (O, O). To make the commitment of not using brand spillover credible to the CM, Firm W can sign a contract with the CM specifying that the wholesale prices are contingent on Firm W's brand spillover strategy. In that case, if Firm W claims not to use brand spillover, but later on uses brand spillover under (O, O), e.g., advertising who the CM is after production is completed, the CM can charge a sufficiently higher price to Firm W based on the signed contract. Such a contracting device allows Firm W to make its commitment credible at the outset.

To derive Firm W's optimal marketing strategy in the first stage, we need to analyze two subgames: one subgame where Firm W does not make any commitment (or is unable to make a credible commitment), and one subgame where Firm W commits to not using brand spillover.

To differentiate the cases with and without brand spillover under (O, O), we denote the structure (O, O) with brand spillover as (O, Ob), where “b” denotes “brand spillover” and the structure (O, O) without brand spillover as (O, On), where “n” denotes “no brand spillover”.

The CM can influence the firms’ sourcing strategies by charging different wholesale prices. Wholesale price contracts have been widely used in contract manufacturing for simplicity and are also commonly adopted in the research literature (see, e.g., Arya et al. 2008, 2015, Liu and Tyagi 2011, and Wu and Zhang 2014). The CM is allowed to differentiate the wholesale prices for the two firms because the products with different brand names are viewed as different products, although they might be similar in design and material.

Note that if Firm W’s original brand power, θ, is sufficiently low, Firm W will be driven out of the market, and then neither competition nor brand spillover will occur. To exclude this trivial situation, we assume in our model that θ is high enough, i.e., $\theta \geq \underline{\theta} = \max\left\{\frac{2+7c}{7}, \frac{1+c}{2}\right\}$, such that each firm’s optimal quantity is always positive. The derivation of $\underline{\theta}$ is available in Appendix B.

Based on the above model setup, the CM’s and the firms’ profit functions are given by

$$\Pi_{CM} = \begin{cases} 0, & \text{if both firms insource,} \\ w_i q_i, & \text{if only Firm } i \text{ outsources,} \\ w_S q_S + w_W q_W, & \text{if both firms outsource,} \end{cases}$$

$$\Pi_i = \begin{cases} (\theta_i - q_S - q_W - c)\, q_i, & \text{if Firm } i \text{ insources,} \\ (\theta_i - q_S - q_W - w_i)\, q_i, & \text{if Firm } i \text{ outsources,} \end{cases} \quad i = S, W.$$

4. Equilibrium Analysis

In this section, we first consider the scenario with brand spillover (i.e., when Firm W decides to adopt the brand spillover strategy in the first stage) and obtain the optimal decisions and profits for the firms. Note that the scenario without brand spillover (i.e., Firm W commits to not using brand spillover) is equivalent to the scenario with brand spillover at $\alpha = 0$. By comparing Firm W’s profits in the two scenarios, we identify the condition under which Firm W should adopt the brand spillover strategy.

We begin with the optimal sourcing structure that the CM wants to induce. Under a given sourcing structure, we derive the CM’s wholesale price decisions, the firms’ quantity decisions, and the resulting CM’s profit. Then, by comparing the CM’s profits under different sourcing structures, we identify the CM’s preferred sourcing structure. The following lemma excludes two sourcing structures that cannot be optimal for the CM.

LEMMA 1. *The CM never prefers the sourcing structure (I, I) or (I, O).*

It is clear that the CM will never induce both firms to insource since the CM obtains zero profit under the sourcing structure (I, I). In addition, between the sourcing structures (O, I) and (I, O) where only one firm outsources, the CM will always induce Firm S rather than Firm W to outsource; that is, (O, I) should be preferred to (I, O) because working with the firm with a more attractive brand allows the CM to charge a higher wholesale price and produce a larger quantity. Therefore, the CM's problem boils down to charging the optimal wholesale prices to induce either the structure (O, I) or (O, O).

4.1. Results Under (O, I)

Under the structure (O, I), Firm S outsources while Firm W insources. In this case, the firms' profits are

$$\Pi_S^{oi}(w_S^{oi}) = \left(1 - q_S^{oi} - q_W^{oi} - w_S^{oi}\right) q_S^{oi},$$
$$\Pi_W^{oi} = \left(\theta - q_S^{oi} - q_W^{oi} - c\right) q_W^{oi}.$$

At the production stage, for a given wholesale price w_S^{oi}, it can be shown that each firm's profit is concave in its production quantity. From the first-order conditions, we derive the quantity decisions as follows:

$$q_S^{oi} = \frac{1}{3}\left(2 - \theta + c - 2w_S^{oi}\right),$$
$$q_W^{oi} = \frac{1}{3}\left(2\theta - 1 - 2c + w_S^{oi}\right).$$

Anticipating the above quantity responses, the CM's optimization problem over w_S^{oi} is

$$\max_{w_S^{oi}} \Pi_{CM}^{oi} = w_S^{oi} q_S^{oi},$$
$$s.t.\ \Pi_S^{oi}(w_S^{oi}) = \left(p_S^{oi} - w_S^{oi}\right) q_S^{oi} \geq \Pi_S^{ii} = \left(p_S^{ii} - c\right) q_S^{ii},$$

where the constraint guarantees Firm S to choose outsourcing instead of insourcing. Solving this optimization problem leads to the CM's optimal pricing policy in the following lemma.

LEMMA 2. *Under the sourcing structure (O, I), the CM's optimal wholesale price is* $w_S^{oi} = \min\left\{c, \frac{1}{4}(2 - \theta + c)\right\}$. *That is, there exists a threshold* $c^{oi} = \frac{1}{3}(2 - \theta)$ *such that* $w_S^{oi} = c$ *if* $c \leq c^{oi}$, *and* $w_S^{oi} = \frac{1}{4}(2 - \theta + c)$ *otherwise.*

In line with our intuition, a higher insourcing cost allows the CM to charge a higher wholesale price to Firm S (i.e., w_S^{oi} is increasing in c). However, the wholesale price cannot be higher than Firm S's insourcing cost, since otherwise Firm S prefers insourcing. Therefore, if c is small enough, the wholesale price is bounded by Firm S's participation constraint, i.e., $w_S^{oi} = c$.

With the CM's optimal pricing policy, we can derive the firms' profits and conduct a comparative statics analysis. Under the structure (O, I), there is no brand spillover and the impact of θ (Firm W's brand attractiveness) on the firms' profits is straightforward. As θ increases, Firm W is better off but the competing Firm S is worse off. Since the CM's profit under (O, I) is solely from producing for Firm S, the CM is also worse off as θ increases.

4.2. Results Under (O, Ob)

Now we derive the firms' optimal decisions and profits provided that Firm W adopts the brand spillover strategy and the CM induces the structure (O, Ob). Under (O, Ob),

$$\begin{aligned}\Pi_S^{oo}(w_S^{oo}, w_W^{oo}) &= (1 - q_S^{oo} - q_W^{oo} - w_S^{oo})\, q_S^{oo},\\ \Pi_W^{oo}(w_S^{oo}, w_W^{oo}) &= (\theta + \alpha\,(1-\theta) - q_S^{oo} - q_W^{oo} - w_W^{oo})\, q_W^{oo}.\end{aligned}$$

At the production stage, given w_S^{oo} and w_W^{oo}, the optimal quantity responses out of the first-order conditions are

$$\begin{aligned}q_S^{oo} &= \frac{1}{3}\,(2 - \theta - \alpha + \alpha\theta - 2w_S^{oo} + w_W^{oo})\,,\\ q_W^{oo} &= \frac{1}{3}\,(2\theta + 2\alpha - 2\alpha\theta - 1 + w_S^{oo} - 2w_W^{oo})\,.\end{aligned}$$

The CM's optimization problem over the wholesale prices is

$$\begin{aligned}\max_{w_S^{oo}, w_W^{oo}}\ & \Pi_{CM}^{oo} = w_S^{oo} q_S^{oo} + w_W^{oo} q_W^{oo},\\ s.t.\ & \Pi_S^{oi}(w_S^{oo}) \geq \Pi_S^{ii},\\ & \Pi_S^{oo}(w_S^{oo}, w_W^{oo}) \geq \Pi_S^{io}(w_W^{oo}),\\ & \Pi_W^{oo}(w_S^{oo}, w_W^{oo}) \geq \Pi_W^{oi}(w_S^{oo}).\end{aligned}$$

The constraints ensure that both firms will choose outsourcing. The first two constraints guarantee that Firm S is better off by choosing outsourcing regardless of Firm W's choice. The third constraint guarantees that Firm W is better off by choosing outsourcing given that Firm S chooses outsourcing. Note that the first constraint is needed to exclude (I, I) as an equilibrium sourcing structure. By solving this optimization problem, we obtain the following lemma.

Lemma 3. *Under the sourcing structure (O, Ob):*

(a) It is optimal for the CM to charge Firm S

$$w_S^{oo} = \min\{c - \frac{1}{2}\alpha\,(1-\theta)\,, \frac{1}{2}\}. \tag{1}$$

That is, there exists a threshold $c_2^{oo} = \frac{1}{2}\,(1 + \alpha\,(1-\theta))$ *such that* $w_S^{oo} = c - \frac{1}{2}\alpha\,(1-\theta)$ *if* $c \leq c_2^{oo}$*, and* $w_S^{oo} = \frac{1}{2}$ *otherwise.*

(b) If $c \leq c_2^{oo}$, it is optimal for the CM to charge Firm W

$$w_W^{oo} = \min\{c + \alpha(1-\theta), \frac{1}{4}(2c + 2\theta - 1 + \alpha(1-\theta))\}. \tag{2}$$

That is, there exists a threshold $c_1^{oo} = \frac{1}{2}(2\theta - 1 - 3\alpha(1-\theta))$ such that $w_W^{oo} = c + \alpha(1-\theta)$ if $c \leq c_1^{oo}$, and $w_W^{oo} = \frac{1}{4}(2c + 2\theta - 1 + \alpha(1-\theta))$ if $c_1^{oo} < c \leq c_2^{oo}$.

If $c > c_2^{oo}$, it is optimal for the CM to charge Firm W

$$w_W^{oo} = \frac{1}{2}(\theta + \alpha(1-\theta)). \tag{3}$$

The intuition behind Lemma 3 is similar to that behind Lemma 2. If c is small enough, the CM's optimal wholesale price charged to each firm is bounded by the firm's participation constraint. In the absence of brand spillover, the upper bounds of the wholesale prices to both firms are equal to their insourcing costs. Under the structure (O, Ob), the direct effect of brand spillover is an improvement in Firm W's brand attractiveness, because of which Firm W is more likely to outsource, whereas the bar for Firm S to outsource rises. As a result, the CM has to offer a more attractive wholesale price to Firm S, which makes the upper bound of the wholesale price to Firm S lower than the insourcing cost, i.e., $w_S^{oo} = c - \frac{1}{2}\alpha(1-\theta) < c$. On the other hand, the upper bound of the wholesale price to Firm W is higher than the insourcing cost, i.e., $w_W^{oo} = c + \alpha(1-\theta) > c$. As the insourcing cost increases, Firm W's participation constraint will first be relaxed. Therefore, for a moderate insourcing cost, only the wholesale price to Firm S is bounded.

It is worth noting that the threshold c_1^{oo} is always lower than $\frac{1}{2}$, and the threshold c_2^{oo} is always higher than $\frac{1}{2}$. This implies that if $c < \frac{1}{2}$, the wholesale price to Firm S is always bounded; otherwise, if $c \geq \frac{1}{2}$, the wholesale price to Firm W is always not bounded. Moreover, Firm W's original brand power θ and the level of brand spillover α have opposing effects on these two thresholds. That is, c_1^{oo} increases in θ and decreases in α, whereas c_2^{oo} decreases in θ and increases in α. The intuition is explained as follows. As θ decreases or α increases, brand spillover leads to a more significant improvement in Firm W's brand attractiveness, which increases the upper bound of the wholesale price to Firm W, and hence the wholesale price to Firm W is less likely to be bounded. The same logic applies to the wholesale price to Firm S but in the opposite way.

With the optimal wholesale prices, we can derive the firms' profits under the structure (O, Ob) and conduct a comparative statics analysis. We first study the effect of Firm W's original brand power θ on the firms' profits.

It is intuitive that given a fixed sourcing structure, as θ increases, the improved brand attractiveness makes Firm W better off and Firm S worse off. However, the effect on the CM's profit is less clear. As the CM sells to both downstream firms, will it always benefit from improved brand attractiveness by one of the downstream firms? The following proposition shows that this conjecture is not necessarily true in the presence of brand spillover.

PROPOSITION 1. *Under the sourcing structure (O, Ob), the CM's profit is decreasing in* θ *if and only if* $\theta \geq \max\left\{\frac{1+2c+3\alpha}{2+3\alpha}, \frac{9\alpha+4\alpha^2+2c}{2\alpha(5+2\alpha)}\right\}$.

By Lemma 3, it is clear that the wholesale price charged to Firm S is not decreasing in θ (see Equation 1), and that charged to Firm W decreases in θ only when the wholesale price is bounded (see Equations 2 and 3). We also know from the intuition behind Lemma 3 that if w_W^{oo} is bounded, i.e., $c \leq c_1^{oo}$, then w_S^{oo} is bounded as well. The condition $c \leq c_1^{oo}$ can be rewritten as $\theta \geq \frac{1+2c+3\alpha}{2+3\alpha}$.

The bounded wholesale price to Firm W is $w_W^{oo} = c + \alpha(1-\theta)$ and that to Firm S is $w_S^{oo} = c - \frac{1}{2}\alpha(1-\theta)$. Note that w_W^{oo} decreases in θ twice as fast as w_S^{oo} increases in θ. This is because brand spillover has a direct effect in improving Firm W's brand attractiveness, based on which the CM extracts all the benefit from Firm W via adjusting w_W^{oo}; from Firm S's perspective, the downside of brand spillover is partially dampened by the increasing wholesale price charged to Firm W, and hence the CM does not need to adjust w_S^{oo} as significantly as w_W^{oo} when θ changes. Isolating the impacts of θ on the bounded wholesale prices, we can see a possibility that the CM's profit decreases in θ.

In addition to wholesale prices, the relative market size of the two firms also plays a role. When θ is sufficiently large, i.e., $\theta \geq \frac{9\alpha+4\alpha^2+2c}{2\alpha(5+2\alpha)}$, Firm W's market share is not small, and hence as θ increases, the CM's loss of profit by charging lower wholesale prices to Firm W outweighs the gain of charging higher wholesale prices to Firm S. Therefore, if and only if $\theta \geq \max\left\{\frac{1+2c+3\alpha}{2+3\alpha}, \frac{9\alpha+4\alpha^2+2c}{2\alpha(5+2\alpha)}\right\}$ (which gurantees (i) the wholesale prices to both firms are bounded, and (ii) Firm W's market size is sufficiently large), the CM's profit decreases in θ.

Next, we study the effect of brand spillover level α on the firms' profits. With brand spillover, increasing α leads Firm W's brand to be more attractive. Does this always benefit Firm W and hurt Firm S? How is the CM's profit affected?

PROPOSITION 2. *Under the sourcing structure (O, Ob): (a) Firm S's profit is increasing in* α *if and only if* $\alpha > \frac{2c-1}{1-\theta}$*; (b) Firm W's profit is decreasing in* α *if and only if* $\alpha < \frac{2\theta-2c-1}{3(1-\theta)}$*; (c) the CM's profit is increasing in* α *if* $c > \min\{\frac{1}{2}, \frac{8-7\theta}{8}\}$*; otherwise, the CM's profit is increasing in* α *if and only if* $\alpha < \frac{5\theta-4}{4(1-\theta)}$ *or* $\alpha > \frac{5-6c-4\theta}{1-\theta}$.

Proposition 2 (a) and (b) jointly reveal a counterintuitive finding about the effect of α. That is, brand spillover improves Firm W's brand attractiveness, but a higher level of brand spillover could hurt Firm W and benefit Firm S. Note that, as α increases, by Equation (1) the wholesale price to Firm S changes from $\frac{1}{2}$ to $c - \frac{1}{2}\alpha(1-\theta)$ if $\alpha > \frac{2c-1}{1-\theta}$ (which can be rewritten as $c < c_2^{oo}$), and by Equation (2) the wholesale price to Firm W changes from $\frac{1}{4}(2c + 2\theta - 1 + \alpha(1-\theta))$ to $c + \alpha(1-\theta)$ if $\alpha < \frac{2\theta - 2c - 1}{3(1-\theta)}$ (which can be rewritten as $c < c_1^{oo}$). In other words, the counterintuitive finding about the effect of α on one firm holds true if and only if the wholesale price to the firm is bounded. Here, with a bounded wholesale price, the firm can only obtain a profit that is equal to its outside option, i.e., the profit of insourcing at the unit cost of c.

We first explain the impact of α on Firm S's outside option. By Lemma 3, when the CM induces the structure (O, Ob), the wholesale price to Firm W is always increasing in α. For the bounded wholesale price to Firm S, Firm S's profit under (O, Ob) is the same as that under (I, O), given the same wholesale price charged to Firm W. If Firm S chooses insourcing (which is its outside option), its profit increases in α because the rival's unit cost (i.e., the CM's wholesale price to Firm W) increases in α. Thus, the CM who wants to induce Firm S to outsource takes the value of Firm S's outside option into account by compensating Firm S with a lower wholesale price. When brand spillover is already strong (corresponding to the bounded wholesale price case), as α increases, the compensation is more significant and Firm S actually benefits from it.

Following similar reasoning, under the structure (O, Ob), Firm W's outside option is decreasing in α because by Lemma 3, the wholesale price to Firm S decreases in α when the wholesale price to Firm W is bounded. Consequently, in this case (i.e., $\alpha < \frac{2\theta - 2c - 1}{3(1-\theta)}$), the CM optimizes the wholesale price $w_W^{oo} = c + \alpha(1-\theta)$ so that the benefit of improved brand attractiveness $\alpha(1-\theta)$ is fully acquired by the CM. As α increases, the decrease of the wholesale price to Firm S leads to a competitive disadvantage for Firm W. This is the reason why Firm W does not necessarily benefit from a higher level of brand spillover.

Proposition 2(c) states that stronger brand spillover benefits the CM when $c > \min\{\frac{1}{2}, \frac{8-5\theta}{8}\}$. This is because a high insourcing cost implies that both firms' participation constraints can be easily satisfied, and the CM can extract the most benefit from the improvement in Firm W's brand attractiveness.

However, when the insourcing cost is not sufficiently high, stronger brand spillover imposes two opposing effects on the CM's profit: On one hand, it allows the CM to set a higher wholesale price for Firm W, which is the positive effect; on the other hand, it forces the CM to set a lower wholesale price for Firm S, which is the negative effect. As a result, the CM's profit could be either increasing or decreasing in α, contingent on the two firms' optimal quantity responses. When Firm

W's optimal quantity is larger, it is more likely that the positive effect dominates and the CM's profit increases in α.

When $\alpha < \frac{2\theta-2c-1}{3(1-\theta)}$ (equivalently, $c < c_1^{oo}$), the wholesale price to Firm W is bounded and increasing rapidly in α (see Equation 2), and then Firm W's optimal quantity decreases in α. Thus, for sufficiently small α values (i.e., $\alpha < \frac{5\theta-4}{4(1-\theta)}$), Firm W's optimal quantity is relatively large, and then the positive effect of charging a higher wholesale price to Firm W is dominant. Consequently, the CM's profit is increasing in α.

When $\alpha \geq \frac{2\theta-2c-1}{3(1-\theta)}$ (equivalently, $c \geq c_1^{oo}$), the wholesale price to Firm W is not bounded and increases relatively slowly in α (see Equations 2 and 3); hence, Firm W's optimal quantity, in contrast to the case of $\alpha < \frac{2\theta-2c-1}{3(1-\theta)}$, is increasing in α because of the improved brand attractiveness. For sufficiently large α values (i.e., $\alpha > \frac{5-6c-4\theta}{1-\theta}$), Firm W's optimal quantity is large enough so that the positive effect of charging a higher wholesale price to Firm W dominates, and then the CM's profit is increasing in α.

Overall, interestingly, when there is weak brand spillover, both Firm S and Firm W are worse off if the level of brand spillover increases, but the CM benefits from it. When the strength of brand spillover is above a certain level, all three parties benefit from a higher level of brand spillover, which implies that Firm S does not always have to fight against the increasing brand spillover to Firm W.

4.3. Results Under (O, On)

The last sourcing structure we analyze is (O, On), where both firms outsource and Firm W commits to not using brand spillover. From the mathematical perspective, the scenario without brand spillover is equivalent to the scenario with brand spillover at $\alpha = 0$. Thus, substituting $\alpha = 0$ into the results under (O, Ob), we obtain the results under (O, On). Since there is no brand spillover under the sourcing structure (O, On), the upper bound of the wholesale price to each firm is equal to the insourcing cost c. The effect of Firm W's brand power θ on the CM's profit is different from the case with brand spillover. Recall that under (O, Ob), the bounded wholesale price to Firm W decreases in θ; as a result, the CM's profit might decrease in θ. However, in the absence of brand spillover, under (O, On) the optimal wholesale prices are always weakly increasing in θ, and thus the CM's profit is always higher with a larger θ.

4.4. Equilibrium Sourcing Structure

In this subsection, we derive the equilibrium sourcing structure by comparing the CM's profits under (O, I) and (O, O). We consider the scenario with brand spillover versus that without brand spillover.

Define $T_1 = \theta - \frac{1}{2} - \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2}$, $T_2 = \theta - \frac{1}{2} + \frac{3}{2}\alpha - \frac{3}{2}\alpha\theta - \frac{1}{2}\sqrt{10\alpha^2(1-\theta)^2 + 2 - 8\theta(1-\theta) - 16\alpha + 36\alpha\theta - 20\alpha\theta^2}$, and $T_3 = \theta - \frac{1}{2} + \frac{3}{2}\alpha - \frac{3}{2}\alpha\theta + \frac{1}{2}\sqrt{10\alpha^2(1-\theta)^2 + 2 - 8\theta(1-\theta) - 16\alpha + 36\alpha\theta - 20\alpha\theta^2}$.

PROPOSITION 3. *It is optimal for the CM to induce the sourcing structure (O, O) if (i)* $T_1 < c < \min\{c^{oi}, c_1^{oo}\}$*, (ii)* $\max\{c_1^{oo}, T_2\} < c < \min\{c^{oi}, T_3\}$*, or (iii)* $c \geq c^{oi}$*. Otherwise, it is optimal for the CM to induce the sourcing structure (O, I).*

Proposition 3 presents the equilibrium sourcing structure for general $\alpha \geq 0$. More specifically, the equilibrium outcome when Firm W does not make any commitment is given by Proposition 3 with $\alpha > 0$, and the equilibrium outcome when Firm W commits to not using brand spillover is given by the proposition with $\alpha = 0$.

We next investigate how the presence of brand spillover influences the equilibrium sourcing structure. When Firm W commits to not using brand spillover, i.e., $\alpha = 0$, the conditions for the CM to induce both firms to outsource in Proposition 3 can then be rewritten as (i) $c < \min\{c^{oi}, c_1^{oo}\}$, (ii) $c_1^{oo} < c < c^{oi}$, or (iii) $c \geq c^{oi}$, which always hold. That is, it is always optimal for the CM to induce the sourcing structure (O, On) if Firm W commits to not using brand spillover. This result is due to the CM's high production efficiency. The CM's cost advantage allows it to induce both firms to outsource by charging wholesale prices that are not higher than the firms' production cost, c.

However, when Firm W does not make any commitment, i.e., there will be brand spillover under the structure (O, O) ($\alpha > 0$), Proposition 3 demonstrates that (O, I) could be the induced sourcing structure under certain parameter settings. This implies that brand spillover can fundamentally change the sourcing structure. Figure 1 illustrates the equilibrium sourcing structure in (θ, c) space for three representative α values. Each dashed curve separating the regions (O, Ob) and (O, I) corresponds to a specific α value. As mentioned in the model section, when θ is too small, Firm W will be driven out of the market. The region in the upper left of the solid lines in Figure 1 represents the trivial case in which neither competition nor brand spillover will occur.

In the case of $c \geq c^{oi}$ (Proposition 3(iii)), the wholesale prices to both firms are not bounded. Then, the CM does not need to lower the wholesale price to induce Firm S to outsource, and hence, the CM is always better off under (O, Ob).

In the case of $c < c^{oi}$, the wholesale prices are bounded under (O, Ob) due to brand spillover and the firms' participation constraints. In order to induce Firm S to outsource, the CM has to set a wholesale price lower than c. Since Firm W accepts a wholesale price higher than c, the overall

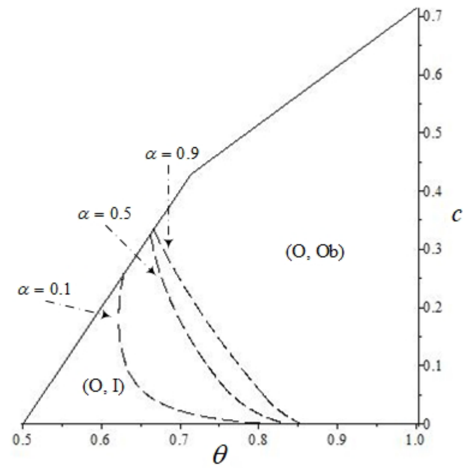

Figure 1 **Equilibrium sourcing structure: The dashed curves separate region (O, I) and region (O, Ob).**

effect of inducing structure (O, Ob) rather than (O, I) depends on the difference between Firm S's and Firm W's market potential. When θ is not sufficiently high, the equilibrium quantity of Firm S is significantly higher than that of Firm W, and then the loss due to the low wholesale price to Firm S outweighs the gain due to the high wholesale price to Firm W. Consequently, the CM can be worse off under (O, Ob) and thus should induce the sourcing structure (O, I).

In Figure 1, we find that for relatively low c values, as θ exceeds a certain value, the equilibrium sourcing structure switches from (O, I) to (O, Ob). Furthermore, region (O, Ob) is smaller for larger α. That is, it is more likely for the CM to collaborate with both firms when Firm W's original brand power is higher or when brand spillover is weaker. Under (O, Ob), the wholesale price to Firm S is decreasing in α and increasing in θ (see Lemma 3). Thus, for stronger brand spillover or lower original brand power of Firm W, it becomes more difficult or costly for the CM to satisfy Firm S's participation constraint under (O, Ob); therefore, the CM is more likely to induce the sourcing structure (O, I).

5. Implications of Brand Spillover

With the equilibrium analysis in the previous section, we are now ready to study the implications of the brand spillover effect. This section consists of three parts: Section 5.1 characterizes Firm W's optimal brand spillover strategy (i.e., when to adopt the brand spillover strategy). Section 5.2 sheds light on how Firm W's brand spillover strategy affects different parties' profits. Finally, Section 5.3 examines the impact of Firm W's increasing brand power.

5.1. Firm W's Optimal Brand Spillover Strategy

Section 4 shows that when Firm W commits to not using brand spillover ($\alpha = 0$), it is always optimal for the CM to induce the sourcing structure (O, On), whereas when Firm W adopts the

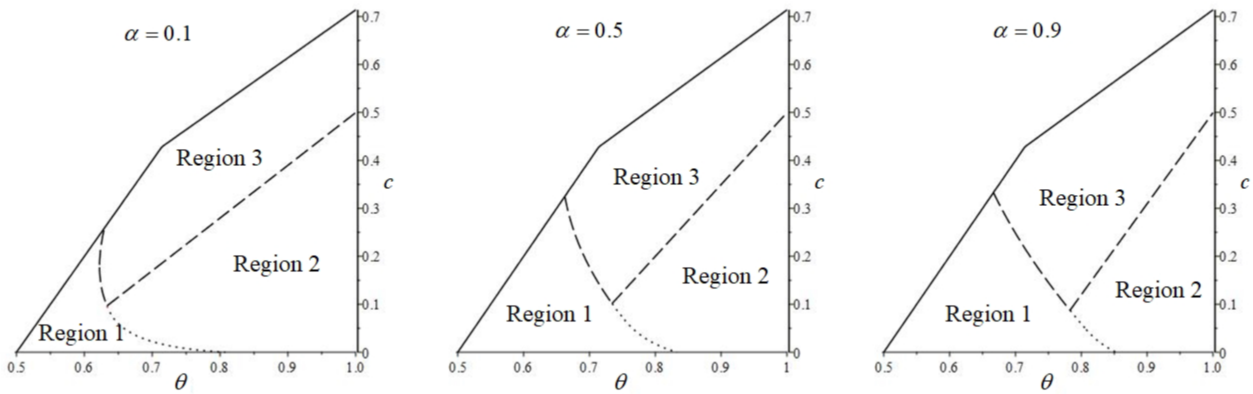

Figure 2 **Firm W adopts the brand spillover strategy only in Region 3.**

brand spillover strategy with an exogenous level α, the CM should induce either (O, Ob) or (O, I), as illustrated in Figure 1. Proposition 4 examines whether Firm W should adopt the brand spillover strategy in the first stage.

PROPOSITION 4. *Firm W should commit to not using brand spillover as a marketing strategy if and only if (i) the equilibrium sourcing structure without commitment is (O, I), or (ii)* $c < \theta - 1/2 - \alpha + \alpha\theta$ *(equivalently,* $\alpha < \frac{2\theta-1-2c}{2(1-\theta)}$*).*

Proposition 4 is illustrated in Figure 2. As characterized by Proposition 3 and shown in Figure 1, when both the insourcing cost c and Firm W's original brand attractiveness θ are low (i.e., Region 1 in Figure 2, which corresponds to the case of Proposition 4(i)), the induced sourcing structure by the CM when Firm W does not make any commitment (equivalent to adopting the brand spillover strategy whenever possible) is (O, I). However, if Firm W commits to not using brand spillover, the CM will induce the structure (O, On), where the wholesale price offered to Firm W is lower than the insourcing cost c. As Firm W incurs a higher unit cost under (O, I) than under (O, On), it should commit to not using brand spillover to induce (O, On) in this case. Region 1 will expand as α increases. That is, for situations where the effect of brand spillover is quite strong (i.e., large α or small θ) or it is difficult for the CM to induce both firms to source from it (i.e., small c), it is better for Firm W to commit to not using brand spillover. This finding implies that for nascent brands without any established reputation, the intent to use brand spillover will backfire because the suppliers of leading brands will not collaborate with them due to the high cost of pleasing leading brands while supplying to both firms.

In Regions 2 and 3 of Figure 2, depending on Firm W's brand spillover strategy in the first stage, the equilibrium sourcing structure will be either (O, Ob) or (O, On). However, brand spillover in such regions is not always beneficial to Firm W. On one hand, brand spillover has a direct positive

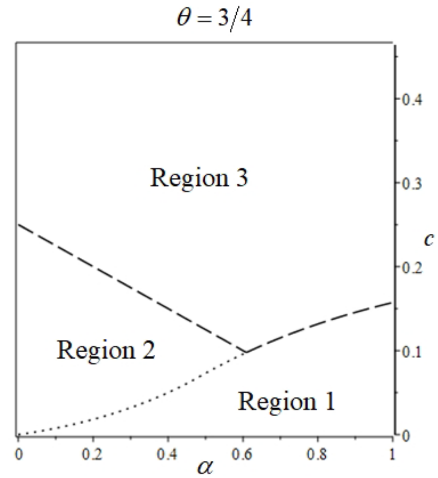

Figure 3 **Effect of α on Firm W's brand spillover strategy (Firm W adopts the brand spillover strategy in Region 3.)**

effect on Firm W's profit because of improved brand attractiveness. On the other hand, the CM's wholesale price to Firm W always increases and that to Firm S decreases in the presence of brand spillover, which has a negative effect on Firm W's profit. The wholesale prices are influenced by brand spillover to a greater extent for a lower insourcing cost since in this case, the participation constraint for Firm S is more difficult to satisfy and the CM would charge an even lower wholesale price to Firm S but an even higher wholesale price to Firm W. As a consequence, the negative effect of wholesale prices is dominant if the insourcing cost is low enough (i.e., $c < \theta - 1/2 - \alpha + \alpha\theta$, which corresponds to Region 2); in this case, Firm W should commit to not using brand spillover. In Region 3 with $c > \theta - 1/2 - \alpha + \alpha\theta$, the positive effect of improved brand attractiveness for Firm W dominates, and hence, Firm W should adopt the brand spillover strategy in the first stage.

Note that the condition for Region 3, $c > \theta - 1/2 - \alpha + \alpha\theta$, can be rewritten as $\alpha > \frac{2\theta-1-2c}{2(1-\theta)}$, which means that given the CM is willing to supply to Firm W who uses brand spillover, Firm W can benefit from the brand spillover strategy only when brand spillover is sufficiently strong. However, in order for the CM to be willing to collaborate with Firm W, the brand spillover level cannot be too high. That is, the significance of brand spillover presents a two-sided effect: It cannot be too high in order for the collaboration with CM and brand spillover to happen, but meanwhile, it cannot be too low in order for brand spillover to be beneficial to Firm W. Figure 3 provides a visual illustration of the impact of stronger brand spillover and shows that its impact on Firm W's brand spillover strategy presents multiple patterns depending on the other parameters. For large c values, it is always optimal to adopt the brand spillover strategy, regardless of the brand spillover level. For small c values, not using brand spillover is the optimal strategy. For intermediate

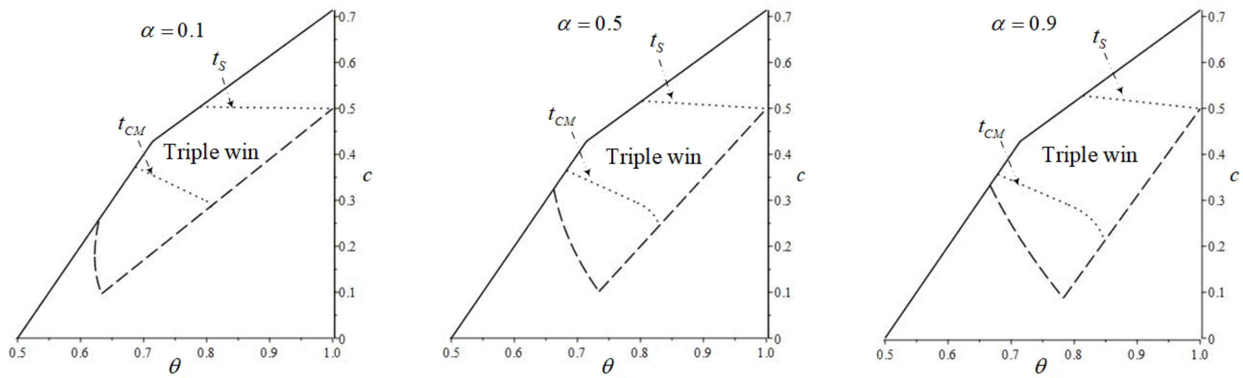

Figure 4 The impact of brand spillover on firms' profits

values of c (e.g., for c between 0.1 and 0.15 in the figure), Firm W's brand spillover strategy is not monotonic in α: Firm W commits to not using brand spillover for small and large α values, but for intermediate α values, it adopts the brand spillover strategy.

5.2. Impact of Adopting the Brand Spillover Strategy

Firm W is better off using brand spillover in Region 3. How does Firm W's adoption of the brand spillover strategy affect Firm S's and the CM's profits? Define

$$t_S = (3 + \alpha - \alpha\theta)/6,$$
$$t_{CM} = \begin{cases} \theta - 1/2 - 3\alpha(1-\theta)/2 + \sqrt{\alpha(4 - 9\theta + 5\theta^2 + 2\alpha(1-\theta)^2)}, & \text{if } c < c_1^{oo}, \\ (10 - \alpha - 8\theta + \alpha\theta)/12, & \text{otherwise.} \end{cases}$$

We can show that $t_S > t_{CM}$.

Proposition 5. *In Region 3 of Figure 2, where Firm W adopts the brand spillover strategy, brand spillover benefits Firm S if and only if $c < t_S$, and benefits the CM if and only if $c > t_{CM}$.*

Proposition 5 implies that Firm W's adoption of the brand spillover strategy can benefit all the firms, which occurs when $t_{CM} < c < t_S$, as illustrated in Figure 4. The triple-win area will expand as α increases. Under the structure (O, Ob), all production is carried out by the cost-efficient CM. Brand spillover does not incur additional costs to the supply chain but increases Firm W's brand attractiveness and hence the total profit of the supply chain. The CM plays a critical role in reallocating the total profit. It can charge Firm W a higher wholesale price to share the benefit of brand spillover and simultaneously charge Firm S a lower wholesale price to satisfy Firm S's participation constraint. Consequently, under certain conditions (i.e., c is moderate), Firm W's adoption of the brand spillover strategy can benefit all the firms.

In Region 3, if c is small ($c < t_{CM}$), brand spillover would hurt the CM because in this case, the wholesale price to Firm W is not bounded (i.e., Firm W's willingness to pay is greater than the ideal wholesale price charged by the CM), but the wholesale price to Firm S is bounded. Revisiting Lemma 3, we find that with brand spillover, the optimal wholesale price to Firm S $w_S^{oo} = c - \alpha(1-\theta)/2$ is decreased by $\alpha(1-\theta)/2$, whereas the optimal wholesale price to Firm W $w_W^{oo} = (2c + 2\theta - 1 + \alpha(1-\theta))/4$ is increased by $\alpha(1-\theta)/4$ compared to the no brand spillover case (i.e., $\alpha = 0$). Therefore, in the case with brand spillover, the wholesale price to Firm S is bounded but to Firm W is not, so the CM is worse off overall because the loss at Firm S outweighs the benefit at Firm W.

In Region 3, if c is large ($c > t_S$), brand spillover is detrimental to Firm S. In this case, due to Firm S's significant cost disadvantage, the wholesale price to Firm S is not bounded by Lemma 3. That is, Firm S cannot share the benefit of brand spillover through a lower wholesale price from the CM. Consequently, brand spillover hurts Firm S because it improves the attractiveness of the competitor's product. This implies that Firm S should take actions to prevent brand spillover, such as requiring the competitor to remove its brand name in advertisements.

5.3. Impact of Firm W's Rising Brand Power

Rising local brands have attracted more attention in recent years in emerging markets. This means that the perceived difference between those local brands and leading international brands narrows over time, which can be captured by increasing the value of θ. Recall that under a given sourcing structure, with or without brand spillover, Firm W always benefits from its rising brand attractiveness, whereas the opposite is true for Firm S. However, once the increase in θ leads to a change in Firm W's brand spillover strategy, the effect on Firm S's profit can be positive.

PROPOSITION 6. *As θ increases, if Firm W's optimal strategy switches from not using brand spillover to using brand spillover, then Firm S's profit can increase as a result of the switch.*

Proposition 6 reveals an interesting impact of the brand spillover strategy change induced by the increase in θ. As the above analysis shows, Firm W prefers not to use brand spillover in Region 1 of Figure 2, and the resulting sourcing structure is (O, On). In Region 3, Firm W prefers to adopt the brand spillover strategy and the resulting sourcing structure induced by the CM is (O, Ob). Note that as θ increases, the parameter setting switches from Region 1 to Region 3 or Region 2. In both Regions 1 and 2, Firm W prefers not to use brand spillover, and the resulting structures are both (O, On). Then, each firm's strategy and profit are continuous across the two regions, and increasing θ hurts Firm S. However, if the parameter setting switches from Region 1 to Region 3, Firm W's strategy changes from not using brand spillover to using brand spillover. Once Firm W uses brand spillover, the CM has to charge Firm S a lower wholesale price if Firm S's participation constraint is binding, which is the case when $c < t_S$ in Proposition 5. Therefore, Firm S can benefit from the switch in Firm W's brand spillover strategy caused by increasing θ. That is, as weak-brand firms improve their brand power, the incumbent strong-brand firms may actually be better off.

6. Extensions

6.1. Vertical Differentiation Model

In this extension, we investigate a vertical differentiation model to check the robustness of the results. A consumer's utility from purchasing Firm i's product with price p_i is $U_i = v\theta_i - p_i$, $i \in \{S, W\}$, where v denotes the consumer's willingness to pay for the brand. We model consumer heterogeneity by assuming that v is uniformly distributed over $[0,1]$ with unit density. Each consumer purchases at most one unit of the product that offers a higher, non-negative utility. In this vertical differentiation model, we can derive the following linear inverse demand functions for the two firms using the above consumer utility function. Let the superscript V denote the results in this extension.

$$p_S^V = 1 - q_S - \theta_W q_W,$$
$$p_W^V = \theta_W \left(1 - q_S - q_W\right).$$

The rest of the model remains the same as in the main model. Next, we characterize the equilibrium results for the vertical differentiation model. The expressions of the thresholds involved in the following propositions can be found in the appendix.

Proposition 7. *Consider the vertical differentiation model.*

(a) The equilibrium sourcing structure is (O, O) if (i) $T_1^V < c < \min\{c^{oiV}, c_1^{ooV}\}$, *(ii)* $\max\{c_1^{ooV}, T_2^V\} < c < \max\{c^{oiV}, T_3^V\}$, *or (iii)* $c \geq c^{oiV}$*; otherwise, it is (O, I).*

(b) Firm W should commit to not using brand spillover if and only if (i) the equilibirum sourcing structure without commitment is (O, I), or (ii) $c < t_W^V$*;*

(c) The use of brand spillover benefits Firm S if and only if $c < t_S^V$*, and benefits the CM if and only if* $c > t_{CM}^V$*.*

Proposition 7 is illustrated in Figure 5. We find that all the qualitative results in the main model carry over to the vertical differentiation model. Specifically, the equilibrium sourcing structure is (O, I) in Region V1, and (O, O) in other regions. Firm W should commit to not using brand spillover in Regions V1 and V2. Provided that Firm W should use brand spillover in equilibrium, there is a "triple-win" area in which all firms benefit from the brand spillover. Moreover, when θ increases, if Firm W's optimal strategy switches from not using brand spillover to using brand spillover, then Firm S's profit can increase as a result of the switch.

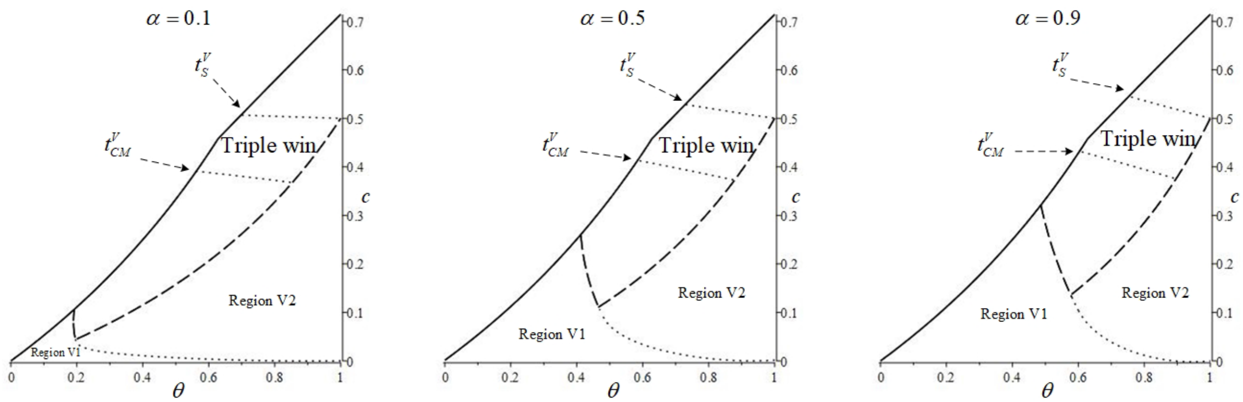

Figure 5 The equilibrium results in the model of vertical differentiation

The only notable change of result lies in the feasible range of the parameter θ, Firm W's original brand power. Recall that, to ensure that Firm W is not driven out of the market, our study requires θ to be high enough, i.e., $\theta > \max\left\{\frac{2+7c}{7}, \frac{1+c}{2}\right\}$ in the main model, which defines a feasible range of $\theta \in \left[\frac{1}{2}, 1\right]$. However, in the vertical differentiation model, the requirement turns out to be $\theta > \max\left\{\frac{2c}{1+c}, 3+\frac{c}{2}-\frac{1}{2}\sqrt{36-20c+c^2}\right\}$, which defines a new feasible range of $\theta \in [0,1]$. The difference in the feasible range is attributed to the different competition intensities in the two models. The competition intensity in the vertical differentiation model is measured by the attractiveness ratio of the two brands, $\theta_W/\theta_S = \theta_W \leq 1$. The variation of the Cournot competition model has a competition intensity of 1, which corresponds to the most intense competition. As a result, Firm W is more likely to be driven out of the market in the variation of Cournot competition model, and hence a higher θ is required.

6.2. Firm S as the Stackelberg Leader

In this extension, we consider the setting where Firm S has greater power in wholesale pricing. That is, Firm S first announces the wholesale price w_S. Then, the CM makes a take-it-or-leave-it decision. If the CM accepts the wholesale price, Firm S outsources to the CM; otherwise, Firm S insources. Next, the CM announces the wholesale price w_W and then Firm W decides whether to source from the CM or insource. Finally, Firm S decides its quantity q_S and Firm W decides q_W simultaneously.[10] The following proposition characterizes the equilibrium sourcing structure, Firm W's optimal strategy, and the impact of brand spillover. Let the superscript P denote the results in this extension.

[10] We have also studied the case where Firm S is the Stackelberg leader in both wholesale price and quantity decisions. The qualitative results are similar to the case where Firm S is the Stackelberg leader only in the wholesale price decision. In particular, the more powerful Firm S is, the more likely it suffers from brand spillover.

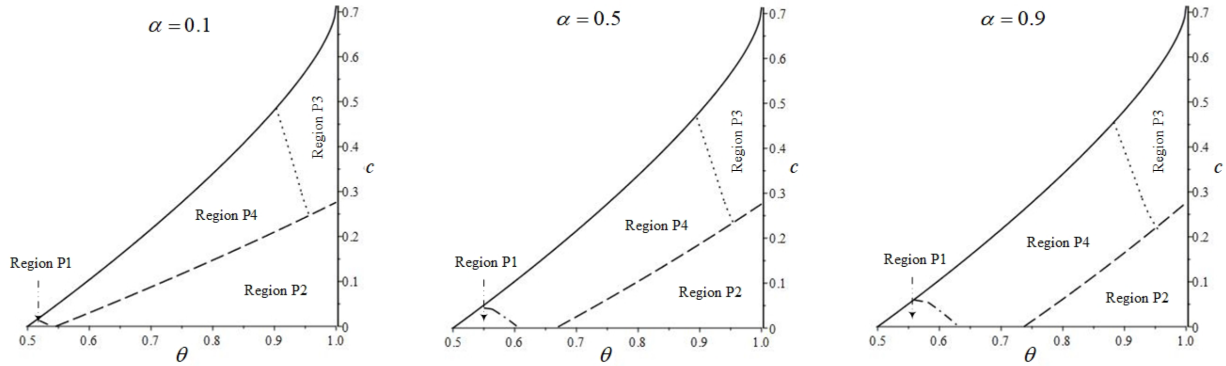

Figure 6 The equilibrium results when Firm S is the Stackelberg leader

Proposition 8. *Consider the scenario where Firm S acts as the Stackelberg leader.*

(a) The equilibrium sourcing structure is (O, O) if (i) $T_1^P \le c < c^{ioP}$*, or (ii)* $c \ge \max\{c^{ioP}, T_2^P\}$*; otherwise, it is (I, O).*

(b) Firm W should commit to not using brand spillover if and only if $c < t_W^P$*.*

(c) The use of brand spillover benefits Firm S if and only if $c > t_S^P$*.*

Proposition 8 is illustrated in Figure 6. Firm S should choose to insource if the insourcing cost c and Firm W's original brand attractiveness θ are both low (i.e., Region P1 in Figure 6). Note that Region P1 disappears in the absence of brand spillover (i.e., $\alpha = 0$).

Proposition 3 and Proposition 8(a) together show that regardless of Firm S's power in contracting, the equilibrium sourcing structure is (O, O) if c is sufficiently high. However, if c is sufficiently low, the sourcing structure in equilibrium is (I, O) instead of (O, I) in this extension. This is because the CM, as a follower, always induces Firm W to outsource, and hence Firm S, as a leader, has to insource to avoid brand spillover. Recall that in the main model, the powerful CM will induce Firm W instead of Firm S to insource to eliminate brand spillover. This implies that Firm S is more likely to suffer from brand spillover when it is in a more powerful position. In this extension, Firm S faces a tradeoff between a higher cost and tougher competition. Intuitively, the negative impact of a higher insourcing cost is insignificant when c is low, and the negative impact of brand spillover due to outsourcing is more significant when θ is low. Consequently, Firm S chooses to insource if both c and θ are low.

Proposition 8(b) reveals that Firm W will commit to not using brand spillover as a marketing strategy when c is low and θ is high (i.e., Region P2 in Figure 6). In this region, brand spillover hurts Firm W. As discussed in the main model, brand spillover may have a negative impact on Firm W when the wholesale price to Firm S is decreased and that to Firm W is increased in the presence of brand spillover. In Region P2 with low c and high θ, the wholesale price to Firm W is bounded (i.e., $w_W^{ooP} = c + \alpha(1-\theta)$), in which case the benefit of brand spillover is all appropriated by the CM, and then brand spillover is detrimental to Firm W. Therefore, Firm W should commit to not using brand spillover in this region. In Region P1 of Figure 6, the sourcing structure is (I, O), and hence Firm W does not need to commit (which is equivalent to adopting the brand spillover strategy whenever possible).

Proposition 8(c) identifies the conditions when brand spillover benefits both firms (i.e., Region P3 in Figure 6). Note that, the wholesale price to Firm S, which is decided by Firm S, is always bounded in order to satisfy the CM's participation constraint. That is, the CM obtains the same profit under (O, O) and (I, O), which is not affected by brand spillover. The benefit of brand spillover will be shared between Firm S and Firm W. Firm S benefits from brand spillover if c is sufficiently high, because as c increases, the wholesale price to Firm W increases.

In the main model, we find that Firm S can benefit from Firm W's rising brand power if and only if the increase in θ induces a change of Firm W's brand spillover strategy. Similarly, in this extension, taking Firm W's brand spillover strategy as given, Firm S's profit is always decreasing in θ. However, if an increase in θ leads Firm W to switch from using brand spillover (i.e., Region P4 in Figure 6) to not using brand spillover (i.e., Region P2 in Figure 6), Firm S's profit can increase. This is because in Region P4, Firm W uses brand spillover, which hurts Firm S, but in Region P2, Firm W does not use brand spillover; that is, Firm S's profit jumps across the boundary from Region P4 to Region P2.

7. Conclusion

Motivated by the increasing use of brand spillover in practice, this paper develops a game theoretic model with one contract manufacturer (CM) and two sourcing firms (Firm S with a strong brand and Firm W with a weak brand) to investigate whether the weak-brand firm should adopt the brand spillover strategy and how the decision depends on the firm's original brand power, the level of brand spillover, and the CM's cost advantage.

We find that Firm W should use brand spillover when its original brand power is not too low and the CM has a sufficient cost advantage over the downstream firms. When Firm W's brand is too weak, the CM, in order to induce the participation from Firm S, will not be willing to produce for Firm W if Firm W uses brand spillover. If the CM does not have a sufficient cost advantage, it would be difficult for the CM to induce Firm S to source from it. In this case, once Firm W uses brand spillover, the CM has to charge Firm S (Firm W) an overly low (high) wholesale price, which actually hurts Firm W.

Note that, in situations where it is not optimal to use brand spillover as a marketing strategy, Firm W should make a commitment of not using brand spillover upfront; otherwise Firm W will always adopt the brand spillover strategy whenever possible. To make the commitment credible, Firm W can sign a contract with the CM specifying that the wholesale prices are contingent on Firm W's brand spillover strategy.

The impact of the brand spillover level on Firm W's strategy choice presents different patterns, depending on Firm W's brand power and the CM's cost advantage. When the brand spillover level plays an active role, it cannot be too small or too large in order for Firm W to adopt the brand spillover strategy. Strong brand spillover may discourage the CM from collaborating with Firm W because otherwise, it is very costly for the CM to please Firm S. With weak brand spillover, even though the CM is willing to work with Firm W, the benefit from improved brand attractiveness can be offset by the increased wholesale price to Firm W and the decreased wholesale price to Firm S; as a result, Firm W will be better off by committing to not using brand spillover.

Firm W's use of brand spillover can lead to a triple win for all the firms under certain conditions. This may happen because the benefit of the improved brand power for Firm W due to brand spillover can be shared among the three firms through appropriate wholesale pricing by the CM.

It is generally believed that the rising brand power of new entrants in emerging economies imposes competitive pressure on leading international brands. This paper confirms the conventional wisdom by showing that Firm S's profit usually decreases in Firm W's original brand power. However, if rising brand power leads Firm W to switch from not using brand spillover to using brand spillover, Firm S can benefit from such improved brand power of a competitor. These findings imply that under certain conditions, contract manufacturers and strong-brand firms should embrace rather than boycott brand spillover.

Despite the potential benefits, from Firm S's perspective, the downside of brand spillover should never be ignored. We find that when the CM has a sufficient cost advantage, Firm W's adoption of brand spillover hurts Firm S. In the extension where Firm S acts as the Stackelberg leader, Firm S is more likely to be hurt by brand spillover, although it has the first-mover advantage. In such cases, Firm S should take actions to prevent brand spillover, such as requiring the competitor not to use its brand name in advertisements.

This study can be extended in several directions. For example, the current paper focuses on quantity competition between firms. Whether the results will hold under price competition remains to be studied. This paper analytically shows the conditions for the weak-brand firm to adopt the brand spillover strategy. In practice, the weak-brand firm's brand power varies and different products differ in terms of contract manufacturers' cost advantage and brand spillover level. It would be interesting to empirically test our theoretical predictions and investigate how brand power and product nature drive firms' brand spillover strategies.

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Appendices to "Brand Spillover as a Marketing Strategy"

Appendix A: Proofs

Proof of Lemma 1

Under sourcing structure (I, I) (i.e., both firms insource), the game becomes a straightforward Cournot duopoly game. The two firms' optimal quantities are derived as follows: $q_S^{ii} = \frac{2-\theta-c}{3}$ and $q_W^{ii} = \frac{2\theta-1-c}{3}$. The three firms' profits are

$$\begin{aligned}\Pi_{CM}^{ii} &= 0,\\ \Pi_S^{ii} &= \left(\frac{2-\theta-c}{3}\right)^2,\\ \Pi_W^{ii} &= \left(\frac{2\theta-1-c}{3}\right)^2.\end{aligned}$$

Under (I, I), the CM obtains zero profit. Under other structures, the CM obtains a non-negative profit. Thus, the sourcing structure (I, I) is never preferred by the CM.

Under the structure (I, O) (i.e., the CM induces only Firm W to outsource while Firm S insources), for a given w_W^{io}, the firms' optimal quantities are $q_S^{io}\left(w_W^{io}\right) = \frac{2-\theta-2c+w_W^{io}}{3}$, and $q_W^{io}\left(w_W^{io}\right) = \frac{2\theta-1+c-2w_W^{io}}{3}$.

Then, the firms' profits are

$$\begin{aligned}\Pi_S^{io}\left(w_W^{io}\right) &= \left(\frac{2-\theta-2c+w_W^{io}}{3}\right)^2,\\ \Pi_W^{io}\left(w_W^{io}\right) &= \left(\frac{2\theta-1+c-2w_W^{io}}{3}\right)^2.\end{aligned}$$

Given the firms' quantity responses for a given w_W^{io}, the CM's optimization problem is

$$\begin{aligned}&\max_{w_W^{io}} \Pi_{CM}^{io} = w_W^{io} q_W^{io},\\ &\text{s.t. } \Pi_W^{io}\left(w_W^{io}\right) \geq \Pi_W^{ii}.\end{aligned}$$

The constraint guarantees that Firm W will accept w_W^{io} and outsource, which is equivalent to $w_W^{io} \leq c$. Thus, the CM's optimal wholesale price to Firm W is $w_W^{io} = \min\left\{c, \frac{2\theta-1+c}{4}\right\}$. That is, there exists a threshold $c^{io} = \frac{2\theta-1}{3}$ such that for $c < c^{io}$, $w_W^{io} = c$ and $\Pi_{CM}^{io} = \frac{c(2\theta-1-c)}{3}$, and for $c \geq c^{io}$, $w_W^{io} = \frac{2\theta-1+c}{4}$ and $\Pi_{CM}^{io} = \frac{(2\theta-1+c)^2}{24}$.

Similarly, under (O, I), there exists a threshold $c^{oi} = \frac{1}{3}(2-\theta)$ such that for $c < c^{oi}$, $\Pi_{CM}^{oi} = \frac{c(2-\theta-c)}{3}$, and for $c \geq c^{oi}$, $\Pi_{CM}^{oi} = \frac{(2-\theta+c)^2}{24}$.

Now we compare the CM's profits under (I, O) and (O, I) to determine its preference.

If $c < c^{io}$, then $\Pi_{CM}^{io} = \frac{c(2\theta-1-c)}{3}$ and $\Pi_{CM}^{oi} = \frac{c(2-\theta-c)}{3}$. Thus, $\Pi_{CM}^{io} - \Pi_{CM}^{oi} = -\frac{c(1-\theta)}{3} < 0$.

If $c^{io} \leq c < c^{oi}$, then $\Pi_{CM}^{io} = \frac{(2\theta-1+c)^2}{24}$ and $\Pi_{CM}^{oi} = \frac{c(2-\theta-c)}{3}$. Let $\Delta_1 = \Pi_{CM}^{io} - \Pi_{CM}^{oi}$. We have $\frac{\partial^2\Delta_1}{\partial c^2} = \frac{3}{4} > 0$. Thus, Δ_1 is convex in c. In addition, when $c = c^{io}$, $\Delta_1 = -\frac{(1-\theta)(2\theta-1)}{3} < 0$, and when $c = c^{oi}$, $\Delta_1 = -\frac{(1-\theta)(7+\theta)}{24} < 0$. Therefore, $\Delta_1 < 0$ in this case.

If $c \geq c^{oi}$, then $\Pi_{CM}^{io} = \frac{(2\theta-1+c)^2}{24}$ and $\Pi_{CM}^{oi} = \frac{(2-\theta+c)^2}{24}$. Thus, $\Pi_{CM}^{io} - \Pi_{CM}^{oi} = -\frac{(1-\theta)(1+\theta+2c)}{24} < 0$.

In conclusion, the CM prefers structure (O, I) to (I, O). That is, given one firm outsources, the CM is better off inducing Firm S rather than Firm W to outsource. ■

Proof of Lemma 2

Under (O, I), the CM's profit is $\Pi_{CM}^{oi} = w_S^{oi} q_S^{oi} = w_S^{oi}\left(\frac{1}{3}\left(2-\theta+c-2w_S^{oi}\right)\right)$. Clearly, Π_{CM}^{oi} is concave in w_S^{oi}. The first-order condition leads to $w_S^{oi} = \frac{1}{4}(2-\theta+c)$. In order for (O, I) to be the equilibrium structure, the optimal wholesale price $w_S^{oi} \le c$ must hold so that Firm S chooses outsourcing. Therefore, the optimal price $w_S^{oi} = min\{c, \frac{1}{4}(2-\theta+c)\}$. Solving $c = \frac{1}{4}(2-\theta+c)$, we have $c = \frac{1}{3}(2-\theta)$. That is, we have the threshold $c^{oi} = \frac{1}{3}(2-\theta)$ in the lemma. ■

Proof of Lemma 3

Under (O, Ob), the CM's profit is $\Pi_{CM}^{oo} = w_S^{oo} q_S^{oo} + w_W^{oo} q_W^{oo} =$
$w_S^{oo}\left(\frac{1}{3}\left(2-\theta-\alpha+\alpha\theta-2w_S^{oo}+w_W^{oo}\right)\right) + w_W^{oo}\left(\frac{1}{3}\left(2\theta+2\alpha-2\alpha\theta-1+w_S^{oo}-2w_W^{oo}\right)\right)$. Clearly, Π_{CM}^{oo} is jointly concave in w_S^{oo} and w_W^{oo}. The first-order conditions lead to $w_S^{oo} = \frac{1}{2}$ and $w_W^{oo} = \frac{1}{2}(\theta+\alpha-\alpha\theta)$.

However, the optimal wholesale prices must satisfy the firms' participation constraint under (O, O), i.e., they choose outsourcing to the CM.

Given w_S, w_W, and that Firm W chooses outsourcing, Firm S's profit by choosing outsourcing is $\frac{1}{9}(2-\theta-\alpha+\alpha\theta-2w_S+w_W)^2$, and its profit by choosing insourcing is $\frac{1}{9}(2-\theta-2c+w_W)^2$. Therefore, solving $\frac{1}{9}(2-\theta-\alpha+\alpha\theta-2w_S+w_W)^2 \ge \frac{1}{9}(2-\theta-2c+w_W)^2$, we derive that Firm S will choose outsourcing if and only if $w_S \le \frac{1}{2}(2c-\alpha+\alpha\theta)$.

Given w_S, w_W, and that Firm W chooses insourcing, Firm S will choose outsourcing if and only if $w_S \le c$.

As a result, regardless of Firm W's sourcing strategy, Firm S will choose outsourcing if $w_S \le \frac{1}{2}(2c-\alpha+\alpha\theta)$. Solving $\frac{1}{2} = \frac{1}{2}(2c-\alpha+\alpha\theta)$, we have $c = \frac{1}{2}(1+\alpha-\alpha\theta)$. That is, we have the threshold $c_2^{oo} = \frac{1}{2}(1+\alpha-\alpha\theta)$ in the lemma.

Similarly, given w_S and w_W, if Firm S chooses outsourcing, Firm W will choose outsourcing if and only if $w_W \le c+\alpha-\alpha\theta$.

If $c \le c_2^{oo}$, the optimal w_S is bounded. Substituting $w_S = \frac{1}{2}(2c-\alpha+\alpha\theta)$ into the CM's profit function and solving the first-order condition lead to the optimal $w_W = \frac{1}{4}(2c+2\theta-1+\alpha-\alpha\theta)$. Solving $\frac{1}{4}(2c+2\theta-1+\alpha-\alpha\theta) = c+\alpha-\alpha\theta$, we have $c = \frac{1}{2}(2\theta-3\alpha+3\alpha\theta-1)$. That is, we have the threshold $c_1^{oo} = \frac{1}{2}(2\theta-3\alpha+3\alpha\theta-1)$ in the lemma.

If $c > c_2^{oo}$, the optimal w_S is not bounded, and the solution $\frac{1}{2}(\theta+\alpha-\alpha\theta)$ is always smaller than the upper bound of the wholesale price to Firm W, $c+\alpha-\alpha\theta$. Thus, we have the optimal $w_W^{oo} = \frac{1}{2}(\theta+\alpha-\alpha\theta)$ in this case.

Combining these results gives Lemma 3. ■

Proof of Proposition 1

Under (O, Ob), if $c \le c_1^{oo}$, which can be rewritten as $\theta \ge \frac{1+2c+3\alpha}{2+3\alpha}$, then $\frac{\partial^2 \Pi_{CM}^{oo}}{\partial \theta^2} = -\frac{1}{3}(5+2\alpha)\alpha < 0$. Setting $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta} = 0$ leads to $\theta = \frac{9\alpha+4\alpha^2+2c}{2\alpha(5+2\alpha)}$. That is, in this case $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta} \ge 0$ if and only if $\theta \le \frac{9\alpha+4\alpha^2+2c}{2\alpha(5+2\alpha)}$.

If $c_1^{oo} < c \le c_2^{oo}$, then $\frac{\partial^2 \Pi_{CM}^{oo}}{\partial \theta \partial c} = -\frac{1}{2}c < 0$, so $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta}$ decreases in c. Setting $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta} = 0$ leads to $c = \frac{\alpha\theta^2-8\alpha\theta+4\theta+9\alpha-\alpha^2}{6\alpha}$, which is greater than c_2^{oo}. That is, in this case $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta} > 0$ always holds.

If $c > c_2^{oo}$, then $\frac{\partial^2 \Pi_{CM}^{oo}}{\partial \theta^2} = \frac{1}{3}(1-\alpha)^2 > 0$. Setting $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta} = 0$ leads to $\theta = \frac{1-2\alpha}{2(1-\alpha)}$, which is less than $\underline{\theta}$. That is, in this case $\frac{\partial \Pi_{CM}^{oo}}{\partial \theta} > 0$ always holds.

To summarize, if $\theta \le \frac{1+2c+3\alpha}{2+3\alpha}$, i.e., $c \ge c_1^{oo}$, Π_{CM}^{oo} always increases in θ; if $\theta > \frac{1+2c+3\alpha}{2+3\alpha}$, Π_{CM}^{oo} increases in θ if and only if $\theta \le \frac{9\alpha+4\alpha^2+2c}{2\alpha(5+2\alpha)}$. Combining these two results leads to the proposition. ■

Proof of Proposition 2

Under (O, Ob), $c \le c_1^{oo}$ is equivalent to $\alpha \le \frac{2\theta-2c-1}{3(1-\theta)}$, $c_1^{oo} < c \le c_2^{oo}$ is equivalent to $\alpha \ge \max\left\{\frac{2\theta-2c-1}{3(1-\theta)}, \frac{2c-1}{1-\theta}\right\}$, and $c > c_2^{oo}$ is equivalent to $\alpha < \frac{2c-1}{1-\theta}$.

(a) If $c \le c_1^{oo}$, $\frac{\partial \Pi_S^{oo}}{\partial \alpha} = \frac{2}{9}(1-\theta)(2-\theta-c+\alpha-\alpha\theta) > 0$. If $c_1^{oo} < c \le c_2^{oo}$, $\frac{\partial \Pi_S^{oo}}{\partial \alpha} = \frac{1}{72}(1-\theta)(7-6c-2\theta+\alpha-\alpha\theta) > 0$. If $c > c_2^{oo}$, $\frac{\partial \Pi_S^{oo}}{\partial \alpha} = -\frac{1}{18}(1-\theta)(2-\alpha-\theta+\alpha\theta) < 0$. Therefore, Π_S^{oo} increases in α when $c \le c_2^{oo}$, which is equivalent to $\alpha \ge \frac{2c-1}{1-\theta}$.

(b) If $c \le c_1^{oo}$, $\frac{\partial \Pi_W^{oo}}{\partial \alpha} = -\frac{1}{18}(1-\theta)(4\theta-2-2c-\alpha+\alpha\theta) < 0$. If $c_1^{oo} < c \le c_2^{oo}$, $\frac{\partial \Pi_W^{oo}}{\partial \alpha} = \frac{1}{9}(1-\theta)(2\theta-1+2\alpha-2\alpha\theta) > 0$. If $c > c_2^{oo}$, $\frac{\partial \Pi_W^{oo}}{\partial \alpha} = \frac{1}{9}(1-\theta)(2\theta-1+2\alpha-2\alpha\theta) > 0$. Therefore, Π_W^{oo} increases in α for $c > c_1^{oo}$, which is equivalent to $\alpha > \frac{2\theta-2c-1}{3(1-\theta)}$.

(c) If $c \le c_1^{oo}$, then $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} = \frac{1}{6}(1-\theta)(5\theta+4\alpha\theta-4-4\alpha)$, so in this case $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ if and only if $\alpha < \frac{5\theta-4}{4(1-\theta)}$. If $c_1^{oo} < c \le c_2^{oo}$, then $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} = \frac{1}{12}(1-\theta)(\alpha-\alpha\theta+4\theta+6c-5)$, so in this case $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ if and only if $\alpha > \frac{5-6c-4\theta}{1-\theta}$. If $c > c_2^{oo}$, $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} = \frac{1}{6}(1-\theta)(2\alpha+2\theta-\alpha\theta-1) > 0$. Combining the effect of α on the CM's profit in these three cases gives that $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ if and only if (1) $\alpha < \min\left\{\frac{5\theta-4}{4(1-\theta)}, \frac{2\theta-2c-1}{3(1-\theta)}\right\}$, (2) $\alpha \ge \max\left\{\frac{5-6c-4\theta}{1-\theta}, \frac{2\theta-2c-1}{3(1-\theta)}, \frac{2c-1}{1-\theta}\right\}$, or (3) $\alpha < \frac{2c-1}{1-\theta}$. Note that, if $c > \frac{1}{2}$, then we have $\frac{2\theta-2c-1}{3(1-\theta)} < 0$ and $\frac{5-6c-4\theta}{1-\theta} < 0$ ($\theta \ge \underline{\theta} = \max\left\{\frac{2+7c}{7}, \frac{1+c}{2}\right\}$ has to hold so that Firm W will not be driven out of the market). Thus, the conditions for $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ can be simplified as $\alpha \ge \frac{2c-1}{1-\theta}$ or $\alpha < \frac{2c-1}{1-\theta}$, i.e., the CM's profit is always increasing in α if $c > \frac{1}{2}$. If $c \le \frac{1}{2}$, then $\frac{2c-1}{1-\theta} \le 0$; thus, the conditions for $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ to hold can be simplified as $\alpha < \min\left\{\frac{5\theta-4}{4(1-\theta)}, \frac{2\theta-2c-1}{3(1-\theta)}\right\}$, or $\alpha \ge \max\left\{\frac{5-6c-4\theta}{1-\theta}, \frac{2\theta-2c-1}{3(1-\theta)}\right\}$; moreover, if $c > \frac{8-7\theta}{8}$, then we have $\frac{5\theta-4}{4(1-\theta)} > \frac{2\theta-2c-1}{3(1-\theta)}$ and $\frac{5-6c-4\theta}{1-\theta} < \frac{2\theta-2c-1}{3(1-\theta)}$. Thus, if $\frac{8-7\theta}{8} < c \le \frac{1}{2}$, the conditions for $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ to hold can be further simplified as $\alpha < \frac{2\theta-2c-1}{3(1-\theta)}$, or $\alpha \ge \frac{2\theta-2c-1}{3(1-\theta)}$; that is, the CM's profit is always increasing in α if $\frac{8-7\theta}{8} < c \le \frac{1}{2}$. If $c \le \min\left\{\frac{8-7\theta}{8}, \frac{1}{2}\right\}$, the conditions for $\frac{\partial \Pi_{CM}^{oo}}{\partial \alpha} > 0$ to hold can be further simplified as $\alpha < \frac{5\theta-4}{4(1-\theta)}$, or $\alpha \ge \frac{5-6c-4\theta}{1-\theta}$, i.e., the CM's profit is increasing in α if α is either sufficiently small or sufficiently large. ■

Proof of Proposition 3

In order to identify the CM's preferred sourcing structure, we compare the CM's profits under (O, I) and (O, Ob). Let $\Delta_{CM} = \Pi_{CM}^{oo} - \Pi_{CM}^{oi}$.

Scenario 1: $c < \min\{c^{oi}, c_1^{oo}\}$. We have $\frac{\partial^2 \Delta_{CM}}{\partial c^2} = -\frac{2}{3} < 0$, i.e., in this scenario, Δ_{CM} is concave in c. Letting $\Delta_{CM} = 0$, we have $c = \theta - \frac{1}{2} \pm \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2}$. Therefore, $\Delta_{CM} > 0$ if and only if $\theta - \frac{1}{2} - \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2} < c < \theta - \frac{1}{2} + \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2}$. In addition, we have

$\theta - \frac{1}{2} + \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2} - c_1^{oo} =$

$\frac{3}{2}\alpha(1-\theta) + \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2} > 0$. That is, in this scenario, the condition for $\Delta_{CM} > 0$ can be written as $\theta - \frac{1}{2} - \frac{1}{2}\sqrt{(2\theta-1)^2 - 4\alpha^2(1+\theta)^2 - 8\alpha + 18\alpha\theta - 10\alpha\theta^2} < c < \min\{c^{oi}, c_1^{oo}\}$.

Scenaro 2: $c_1^{oo} \le c < c^{oi}$. We have $\frac{\partial^2 \Delta_{CM}}{\partial c^2} = -\frac{1}{3} < 0$. Letting $\Delta_{CM} = 0$, we have $c = \theta - \frac{1}{2} + \frac{3}{2}\alpha - \frac{3}{2}\alpha\theta \pm \frac{1}{2}\sqrt{10\alpha^2(1-\theta)^2 + 2 - 8\theta(1-\theta) - 16\alpha + 36\alpha\theta - 20\alpha\theta^2}$. Therefore, $\Delta_{CM} > 0$ if and only if $\max\left\{c_1^{oo}, \theta - \frac{1}{2} + \frac{3}{2}\alpha - \frac{3}{2}\alpha\theta - \frac{1}{2}\sqrt{10\alpha^2(1-\theta)^2 + 2 - 8\theta(1-\theta) - 16\alpha + 36\alpha\theta - 20\alpha\theta^2}\right\} < c < \min\left\{c^{oi}, \theta - \frac{1}{2} + \frac{3}{2}\alpha - \frac{3}{2}\alpha\theta + \frac{1}{2}\sqrt{10\alpha^2(1-\theta)^2 + 2 - 8\theta(1-\theta) - 16\alpha + 36\alpha\theta - 20\alpha\theta^2}\right\}$.

Scenario 3: $c^{oi} \le c < c_1^{oo}$. We have $\frac{\partial^2 \Delta_{CM}}{\partial c^2} = -\frac{17}{12} < 0$. Letting $\Delta_{CM} = 0$, we have $c = \frac{2}{17} + \frac{5}{17}\theta \pm \frac{2}{17}\sqrt{2\theta^2 + 22\theta - 16 + 153\alpha\theta - 68\alpha - 85\alpha\theta^2 - 34\alpha^2(1-\theta)^2}$. It is worth noting that the constraint $c^{oi} \le c < c_1^{oo}$ implies $\theta > \frac{7+9\alpha}{8+9\alpha}$. Moreover, we have $c^{oi} - \left(\frac{2}{17} + \frac{5}{17}\theta - \frac{2}{17}\sqrt{2\theta^2 + 22\theta - 16 + 153\alpha\theta - 68\alpha - 85\alpha\theta^2 - 34\alpha^2(1-\theta)^2}\right)$ $= \frac{2}{17}\sqrt{2\theta^2 + 22\theta - 16 + 153\alpha\theta - 68\alpha - 85\alpha\theta^2 - 34\alpha^2(1-\theta)^2} - \left(\frac{32\theta-28}{51}\right)$. Given $\theta > \frac{7+9\alpha}{8+9\alpha}$, we have $\frac{32\theta-28}{51} > 0$. Let $\Upsilon = \left(\frac{2}{17}\right)^2\left(2\theta^2 + 22\theta - 16 + 153\alpha\theta - 68\alpha - 85\alpha\theta^2 - 34\alpha^2(1-\theta)^2\right) - \left(\frac{32\theta-28}{51}\right)^2 = -(\frac{8}{17}\alpha^2 + \frac{20}{17}\alpha + \frac{56}{153})\theta^2 + \left(\frac{16}{17}\alpha^2 + \frac{36}{17}\alpha + \frac{152}{153}\right)\theta - \left(\frac{8}{17}\alpha^2 + \frac{16}{17}\alpha + \frac{80}{153}\right)$, which is concave in θ. For $\theta = \frac{7+9\alpha}{8+9\alpha}$, $\Upsilon = \frac{4\left(43\alpha^2+57\alpha+18\right)}{17(8+9\alpha)^2} > 0$; for $\theta = 1$, $\Upsilon = \frac{16}{153} > 0$; that is, given $\theta > \frac{7+9\alpha}{8+9\alpha}$, we have $\Upsilon > 0$, which is equivalent to $c^{oi} - \left(\frac{2}{17} + \frac{5}{17}\theta - \frac{2}{17}\sqrt{2\theta^2 + 22\theta - 16 + 153\alpha\theta - 68\alpha - 85\alpha\theta^2 - 34\alpha^2(1-\theta)^2}\right) > 0$. Similarly, given $\theta > \frac{7+9\alpha}{8+9\alpha}$, we have $\frac{2}{17} + \frac{5}{17}\theta + \frac{2}{17}\sqrt{2\theta^2 + 22\theta - 16 + 153\alpha\theta - 68\alpha - 85\alpha\theta^2 - 34\alpha^2(1-\theta)^2} - c_1^{oo} > 0$. Therefore, we always have $\Delta_{CM} > 0$ in this scenario.

Scenario 4: $c \ge \max\{c^{oi}, c_1^{oo}\}$. Similar to Scenario 3, we always have $\Delta_{CM} > 0$ in this scenario. ■

Proof of Proposition 4

If Firm W commits to not using brand spillover, it is optimal for the CM to induce the sourcing structure (O, O). Then by Lemma 3 and setting $\alpha = 0$, Firm W's profit is $\Pi_W^{oo}|_{\alpha=0} = \left(\frac{2\theta-1-c}{3}\right)^2$ if $c \le c_1^{oo}|_{\alpha=0} = \frac{1}{2}(2\theta-1)$, and $\Pi_W^{oo}|_{\alpha=0} = \left(\frac{2\theta-1}{6}\right)^2$ if $c_1^{oo}|_{\alpha=0} < c \le c_2^{oo}|_{\alpha=0}$.

If Firm W does not make any commitment, then the equilibrium sourcing structure is derived in Proposition 3. There are two cases:

(a) If the equilibrium structure is (O, I), we know $c < c^{oi}$; then by Lemma 2 we can derive $\Pi_W^{oi} = \left(\frac{2\theta-1-c}{3}\right)^2$. Clearly, if $c \le \frac{1}{2}(2\theta-1)$, then $\Pi_W^{oi} = \Pi_W^{oo}|_{\alpha=0}$, and committing to not using brand spillover has no impact on Firm W's profit. But if $c > \frac{1}{2}(2\theta-1)$, we have $\Pi_W^{oo}|_{\alpha=0} - \Pi_W^{oi} = \left(\frac{2\theta-1}{6}\right)^2 - \left(\frac{2\theta-1-c}{3}\right)^2 = \frac{(6\theta-3-2c)(2c-2\theta+1)}{36} > 0$, and then Firm W is better off committing to not using brand spillover.

(b) If the equilibrium structure is (O, O), then we need to compare Firm W's profit under (O, O) for a non-zero α (i.e., $\Pi_W^{oo}|_{\alpha>0}$) with its profit when committing to not using brand spillover (i.e., $\Pi_W^{oo}|_{\alpha=0}$). From Proposition 2, Firm W's profit is increasing in α if and only if $\alpha > \frac{2\theta-2c-1}{3(1-\theta)}$, which is equivalent to $c > c_1^{oo} = \frac{1}{2}(2\theta - 3\alpha + 3\alpha\theta - 1)$. If $c > \frac{1}{2}(2\theta-1)$, then $\frac{2\theta-2c-1}{3(1-\theta)} < 0$ and $\alpha > \frac{2\theta-2c-1}{3(1-\theta)}$ always hold, and hence $\Pi_W^{oo}|_{\alpha>0} > \Pi_W^{oo}|_{\alpha=0}$ regardless of the value of α. If $c \le \frac{1}{2}(2\theta-1)$ and $\alpha \le \frac{2\theta-2c-1}{3(1-\theta)}$, then $\Pi_W^{oo}|_{\alpha>0} < \Pi_W^{oo}|_{\alpha=0}$ always holds. If $c \le \frac{1}{2}(2\theta-1)$ and $\alpha > \frac{2\theta-2c-1}{3(1-\theta)}$, which are equivalent to $c_1^{oo} < c \le \frac{1}{2}(2\theta-1) \le c_2^{oo}$, by Lemma 3, we have $\Pi_W^{oo}|_{\alpha>0} = \left(\frac{2\theta-1+2\alpha-2\alpha\theta}{6}\right)^2$ and $\Pi_W^{oo}|_{\alpha=0} = \left(\frac{2\theta-1-c}{3}\right)^2$. Then setting $\Pi_W^{oo}|_{\alpha>0} \ge \Pi_W^{oo}|_{\alpha=0}$ leads to $\alpha \ge \frac{2\theta-1-2c}{2(1-\theta)}$. Therefore, Firm W should commit to not using brand spillover if $\alpha < \frac{2\theta-1-2c}{2(1-\theta)}$. ■

Proof of Proposition 5

In Region 3 of Figure 2, where Firm W adopts brand spillover strategy, if $c > c_2^{oo} = \frac{1}{2}(1+\alpha-\alpha\theta)$, $\Pi_S^{oo} = \left(\frac{2-\alpha-\theta+\alpha\theta}{6}\right)^2$; otherwise, $\Pi_S^{oo} = \left(\frac{7-\alpha\theta+\alpha-6c-2\theta}{12}\right)^2$. In the absence of brand spillover, if $c > \frac{1}{2}$, $\Pi_S^{oo} = \left(\frac{2-\theta}{6}\right)^2$; if $\frac{1}{2}(2\theta-1) < c \le \frac{1}{2}$, $\Pi_S^{oo} = \left(\frac{7-6c-2\theta}{12}\right)^2$; otherwise, $\Pi_S^{oo} = \left(\frac{2-c-\theta}{3}\right)^2$. We examine the impact of brand spillover on Firm S's profit as follows. Let Λ_S denote the difference of Firm S's profits with and without brand spillover.

If $c > \frac{1}{2}(1+\alpha-\alpha\theta)$, $\Lambda_S = -\frac{1}{36}\alpha(1-\theta)(4-2\theta-\alpha+\alpha\theta) < 0$.

If $\frac{1}{2} < c \le \frac{1}{2}(1+\alpha-\alpha\theta)$, $\frac{\partial^2\Lambda_S}{\partial c^2} = \frac{1}{2} > 0$. Setting $\Lambda_S = 0$ leads to $c = \frac{3+\alpha-\alpha\theta}{6}$ and $c = \frac{11-4\theta+\alpha-\alpha\theta}{6} > \frac{1}{2}(1+\alpha-\alpha\theta)$; thus, in this case $\Lambda_S > 0$ if $c < \frac{3+\alpha-\alpha\theta}{6}$; otherwise, $\Lambda_S \le 0$.

If $\frac{1}{2}(2\theta-1) < c \le \frac{1}{2}$, $\frac{\partial^{\Lambda_S}}{\partial c} = -\frac{\alpha-\alpha\theta}{12} < 0$. Setting $\Lambda_S = 0$ leads to $c = \frac{14-4\theta+\alpha-\alpha\theta}{12} > \frac{1}{2}$; thus, in this case $\Lambda_S > 0$.

If $c \le \frac{1}{2}(2\theta-1)$, $\frac{\partial^2\Lambda_S}{\partial c^2} = \frac{5}{18} > 0$. Setting $\Lambda_S = 0$ leads to $c = \frac{2\theta-1+\alpha-\alpha\theta}{2} > \frac{1}{2}(2\theta-1)$ and $c = \frac{15-6\theta+\alpha-\alpha\theta}{10} > \frac{1}{2}(2\theta-1)$; thus, in this case, $\Lambda_S > 0$.

Combining these results, we find that brand spillover benefits Firm S if and only if $c < \frac{3+\alpha-\alpha\theta}{6}$.

Similarly, we examine the impact of brand spillover on the CM's profit and find that brand spillover benefits the CM if and only if c is large enough. ■

Proof of Proposition 6

By Proposition 5, within Region 3 of Figure 4, Firm S is strictly better off with brand spillover if $c < t_S$ (i.e., $\Pi_S^{oo}|_{\alpha>0} > \Pi_S^{oo}|_{\alpha=0}$). Along with the increase of θ, if the optimal strategy for Firm W switches from not using brand spillover (Region 1 in Figure 2) to using brand spillover (Region 3 in Figure 2), Firm S's profit jumps from $\Pi_S^{oo}|_{\alpha=0}$ to $\Pi_S^{oo}|_{\alpha>0}$ across the boundary of Region 1 and Region 3. ■

Proof of Proposition 7

Define $c^{oiV} = \frac{1}{3}(2-\theta)$,

$$\begin{aligned} c_1^{ooV} =& \Bigg(8(4-\theta)(4+\alpha+(2+\alpha)\theta)\sqrt{\theta(\alpha+(1-\alpha)\theta)}\left(16+4\alpha-\alpha^2-2\theta\left(2+3\alpha-\alpha^2\right)+\alpha(2-\alpha)\theta^2\right) \\ &+4\alpha\theta(1-\theta)(\alpha+(1-\alpha)\theta)\left(32+8\alpha+2\alpha^2-\left(8+12\alpha+5\alpha^2\right)\theta+4\alpha(1+\alpha)\theta^2-\alpha^2\theta^3\right)\Bigg)\Bigg/ \\ &\Bigg(\theta(16(4-\theta)(16-4\theta+2\alpha\left(2-3\theta+\theta^2\right)-\alpha^2(1-\theta)^2)\sqrt{\theta(\alpha+(1-\alpha)\theta)}-(\theta+(1-\theta)\alpha)(\alpha^4 \\ &\theta(1-\theta)^4-2\alpha^3(8-\theta)(1-\theta)^3-8\alpha^2(1-\theta)^2\left(4-2\theta+\theta^2\right)+64\alpha\left(4-5\theta+\theta^2\right)+32(4-\theta)^2))\Bigg), \end{aligned}$$

$$c_2^{ooV} = \frac{8-2\theta+\alpha\left(4-5\theta+\theta^2\right)-\alpha^2(1-\theta)^2}{4(4-\alpha-\theta+\alpha\theta)}, T_2^V = \frac{4\theta(4-\theta)+2\left(8-8\theta-\theta^2+\theta^3\right)\alpha-4(1-\theta)^2\alpha^2-\sqrt{\Psi}}{4\left(2\left(8-6\theta+\theta^2\right)-4\left(3-4\theta+\theta^2\right)\alpha+(1-\theta)^2\alpha^2\right)},$$

$$T_3^V = \frac{4\theta(4-\theta)+2\left(8-8\theta-\theta^2+\theta^3\right)\alpha-4(1-\theta)^2\alpha^2+\sqrt{\Psi}}{4\left(2\left(8-6\theta+\theta^2\right)-4\left(3-4\theta+\theta^2\right)\alpha+(1-\theta)^2\alpha^2\right)}, \text{where}$$

$$\begin{aligned} \Psi =& 2\left(4(4-\theta)+2\left(2-3\theta+\theta^2\right)\alpha-(1-\theta)^2\alpha^2\right)\Big(4\theta(4-\theta)-4\left(12-25\theta+16\theta^2-3\theta^3\right)\alpha+(40-24\theta \\ &+3\theta^2)(1-\theta)^2\alpha^2-(4-\theta)(1-\theta)^3\alpha^3\Big), \end{aligned}$$

$$t_W^V = \Big(64\theta\left(32-32\theta+10\theta^2-\theta^3\right)+8\theta\left(128-272\theta+200\theta^2-63\theta^3+7\theta^4\right)\alpha-2\theta^2\left(1-\theta\right)^2\left(16-28\theta+7\theta^2\right)\alpha^2-\theta\left(1-\theta\right)^3\left(2-\theta\right)\left(16-8\theta+\theta^2\right)\alpha^3-\theta\left(1-\theta\right)^4\left(8-4\theta+\theta^2\right)\alpha^4-\left(4-\theta\right)\left(4\left(4-\theta\right)+2(2-3\theta+\theta^2)\alpha-\left(1-\theta\right)^2\alpha^2\right)\left(8\left(2-\theta\right)+2\left(2-\theta-\theta^2\right)\alpha-\left(1-\theta\right)^2\left(2-\theta\right)\alpha^2\right)\sqrt{\theta\left(\theta+\alpha\left(1-\theta\right)\right)}\Big)\Big/\Big(64(2-\theta)^2\left(4-\theta\right)^2-64\left(1-\theta\right)\left(2-\theta\right)^3\left(4-\theta\right)\alpha-4\left(64-160\theta+112\theta^2+16\theta^3-51\theta^4+22\theta^5-3\theta^6\right)\alpha^2-4\left(1-\theta\right)^3\left(32-32\theta+16\theta^2-\theta^3\right)\alpha^3+4\left(1-\theta\right)^4\left(2-\theta\right)^2\alpha^4\Big),$$

$$t_S^V = \frac{2\left(4-\theta\right)^2+\left(16-26\theta+11\theta^2-\theta^3\right)\alpha-2\left(1-\theta\right)^2\alpha^2}{4\left(4-\theta\right)^2},$$

$$t_{CM}^V = \begin{cases} \Big(2\theta\left(8-6\theta+\theta^2\right)-\theta\left(1-\theta\right)\left(4+2\theta-\theta^2\right)\alpha+\theta\left(1-\theta\right)^2\alpha^2+\left(\alpha\theta\left(1-\theta\right)\left(4\left(4-\theta\right)+2(2-3\theta+\theta^2)\alpha-\left(1-\theta\right)^2\alpha^2\right)\left(2\left(24-44\theta+21\theta^2-3\theta^3\right)+\left(16-9\theta-14\theta^2+8\theta^3-\theta^4\right)\alpha-\left(1-\theta\right)^2(4-\theta)\alpha^2\right)\right)^{1/2}\Big)\Big/\Big(4\left(2-\theta\right)^2\left(4-\theta\right)+4\left(4-2\theta-3\theta^2+\theta^3\right)\alpha-4\left(1-\theta\right)^2\alpha^2\Big), & \text{if } c<c_1^{ooV}, \\ \Big(\sqrt{\left(4-\theta\right)\left(28-12\theta+\theta^2-\left(4-5\theta+\theta^2\right)\alpha\right)\left(4\left(4-\theta\right)+2\left(2-3\theta+\theta^2\right)\alpha-\left(1-\theta\right)^2\alpha^2\right)}-\left(4-\theta\right)\left(4-\left(1-\theta\right)\alpha\right)\Big)\Big/\Big(\left(4-\theta\right)\left(8-2\theta-\left(1-\theta\right)\alpha\right)\Big), & \text{otherwise.} \end{cases}$$

Define T_1^V as the cost c that satisfies $\Pi_{CM}^{ioV}=\Pi_{CM}^{ooV}$ in the interval $c<\min\{c^{oiV},c_1^{ooV}\}$, which exists and is unique. The expression of T_1^V is tedious and thus omitted.

Suboptimal Sourcing Structures

In the vertical differentiation model, we first show that the CM never prefers the sourcing structures (I, I) and (I, O).

Under the sourcing structure (I, I), the two firms' optimal quantities are $q_S^{iiV}=\frac{2-\theta-c}{4-\theta}$ and $q_W^{iiV}=\frac{(1+c)\theta-2c}{\theta(4-\theta)}$. The profits of the CM and the two firms are $\Pi_{CM}^{iiV}=0$, $\Pi_S^{iiV}=\left(\frac{2-\theta-c}{4-\theta}\right)^2$, and $\Pi_W^{iiV}=\frac{((1+c)\theta-2c)^2}{\theta(4-\theta)^2}$, respectively.

Under (I, I), the CM obtains zero profit. Under other structures, the CM obtains a non-negative profit. Thus, the sourcing structure (I, I) is never preferred by the CM.

Under (I, O), the two firms' optimal quantities are $q_S^{ioV}\left(w_W^{ioV}\right)=\frac{2-\theta-2c+w_W^{ioV}}{4-\theta}$ and $q_W^{ioV}\left(w_W^{ioV}\right)=\frac{(1+c)\theta-2w_W^{ioV}}{\theta(4-\theta)}$. Then, their profits are $\Pi_S^{ioV}=\left(\frac{2-\theta-2c+w_W^{io}}{4-\theta}\right)^2$ and $\Pi_W^{ioV}=\frac{\left((1+c)\theta-2w_W^{io}\right)^2}{\theta(4-\theta)^2}$.

Given the two firms' quantity responses for a given w_W^{ioV}, the CM's optimization problem is

$$\begin{gathered}\max_{w_W^{ioV}}\Pi_{CM}^{ioV}=w_W^{ioV}q_W^{ioV},\\ \text{s.t. }\Pi_W^{ioV}\left(w_W^{ioV}\right)\geq\Pi_W^{iiV}.\end{gathered}$$

The constraint guarantees that Firm W will accept w_W^{ioV} and outsource, and it is equivalent to $w_W^{ioV}\leq c$. Thus, the CM's optimal wholesale price to Firm W is $w_W^{ioV}=\min\left\{c,\frac{(1+c)\theta}{4}\right\}$. That is, there exists a threshold $c^{ioV}=\frac{\theta}{4-\theta}$ such that for $c<c^{ioV}$, $w_W^{ioV}=c$ and $\Pi_{CM}^{ioV}=\frac{c((1+c)\theta-2c)}{\theta(4-\theta)}$, and for $c\geq c^{ioV}$, $w_W^{ioV}=\frac{(1+c)\theta}{4}$ and $\Pi_{CM}^{ioV}=\frac{\theta(1+c)^2}{8(4-\theta)}$.

Similarly, under (O, I), there exists a threshold $c^{oiV}=\frac{1}{3}\left(2-\theta\right)$ such that for $c<c^{oiV}$, $\Pi_{CM}^{oiV}=\frac{c(2-\theta-c)}{4-\theta}$, and for $c\geq c^{oiV}$, $\Pi_{CM}^{oiV}=\frac{(2-\theta+c)^2}{8(4-\theta)}$.

Now we compare the CM's profits under (I, O) and (O, I) to identify its preference. Let $\Delta_1^V=\Pi_{CM}^{ioV}-\Pi_{CM}^{oiV}$.

If $c < c^{ioV}$, then $\Delta_1^V = -\frac{c(1-\theta)(\theta+2c)}{\theta(4-\theta)} < 0$.

If $c^{ioV} \le c < c^{oiV}$, then $\frac{\partial^2 \Delta_1^V}{\partial c^2} = \frac{8+\theta}{4(4-\theta)} > 0$. Thus, Δ_1^V is convex in c. In addition, when $c = c^{ioV}$, $\Delta_1^V = -\frac{\theta(1-\theta)(6-\theta)}{(4-\theta)^3} < 0$, and when $c = c^{oiV}$, $\Delta_1^V = -\frac{(1-\theta)\left(64-25\theta+\theta^2\right)}{4(72-\theta)} < 0$. Therefore, $\Delta_1^V < 0$ in this case.

If $c \ge c^{oiV}$, then $\Delta_1^V = -\frac{(1-\theta)\left((2+c)^2-\theta\right)}{8(4-\theta)} < 0$.

In conclusion, the CM prefers the sourcing structure (O, I) to (I, O). That is, given that only one firm outsources, the CM is better off by inducing Firm S rather than Firm W to outsource.

Results under (O, I)

Next, we derive the results under the sourcing structure (O, I).

Under the sourcing structure (O, I), for a given wholesale price w_S^{oiV}, each firm's profit is concave in its production quantity. From the first-order conditions, we derive the quantity decisions $q_S^{oiV} = \frac{2-\theta+c-2w_S^{oiV}}{4-\theta}$ and $q_W^{oiV} = \frac{\left(1+w_S^{oiV}\right)\theta-2c}{\theta(4-\theta)}$. Then, the CM's profit is $\Pi_{CM}^{oiV} = \frac{\left(2-\theta+c-2w_S^{oiV}\right)w_S^{oiV}}{4-\theta}$, which is concave in w_S^{oiV}. The first order condition leads to $w_S^{oiV} = \frac{1}{4}\left(2-\theta+c\right)$.

For (O, I) to be the equilibrium sourcing structure, the optimal wholesale price $w_S^{oiV} \le c$ must hold so that Firm S chooses outsourcing to the CM. Therefore, the optimal $w_S^{oiV} = \min\left\{c, \frac{1}{4}\left(2-\theta+c\right)\right\}$. Solving $c = \frac{1}{4}\left(2-\theta+c\right)$, we have $c = \frac{1}{3}\left(2-\theta\right)$. That is, there exists a threshold $c^{oiV} = \frac{1}{3}\left(2-\theta\right)$ such that under (O, I) $w_S^{oiV} = c$ if $c \le c^{oiV}$, and $w_S^{oiV} = \frac{1}{4}\left(2-\theta+c\right)$ otherwise.

Results under (O, O)

Similarly, we derive the results under the sourcing structure (O, O).

Under (O, Ob), given w_S^{ooV} and w_W^{ooV}, the optimal quantity responses from the first-order conditions are $q_S^{ooV} = \frac{2-\alpha-\theta+\alpha\theta-2w_S^{ooV}+w_W^{ooV}}{4-\alpha-\theta+\alpha\theta}$ and $q_W^{ooV} = \frac{(\alpha+\theta-\alpha\theta)\left(1+w_S^{ooV}\right)-2w_W^{ooV}}{(1-\alpha)\theta(4-2\alpha-\theta+\alpha\theta)+\alpha(4-\alpha)}$. Then, we have that the CM's profit is jointly concave in w_S^{ooV} and w_W^{ooV}. The first-order conditions lead to $w_S^{ooV} = \frac{1}{2}$ and $w_W^{ooV} = \frac{1}{2}\left(\theta+\alpha\left(1-\theta\right)\right)$.

However, the optimal wholesale prices must satisfy the two firms' participation constraints under (O, Ob). Note that, due to brand spillover, Firm W is more likely to outsource, whereas Firm S is less likely to outsource. Given w_S^{ooV} and w_W^{ooV}, by comparing Firm S's profits under different sourcing structures, we can rewrite Firm S's participation constraint under (O, Ob) as $w_S^{ooV} \le \frac{1}{2(4-\theta)}\left(2c\left(4-\theta\right)-\alpha\left(1-\theta\right)\left(2+2c-w_W^{oo}\right)\right)$. Next, we consider three possible cases.

(i) We first consider the case with a low insourcing cost c such that the optimal wholesale prices to both firms are bounded. Let $w_S^{ooV} = \frac{1}{2(4-\theta)}\left(2c\left(4-\theta\right)-\alpha\left(1-\theta\right)\left(2+2c-w_W^{oo}\right)\right)$. By comparing Firm W's profits under (O, Ob) and (O, I), we derive the bounded wholesale price to Firm W

$$\begin{aligned}w_W^{ooV} = &\Big(4\left(4-\theta\right)^2\left(\left(4+\alpha\right)c-\left(2+\left(2+\alpha\right)c\right)\theta\right)\sqrt{\theta\left(\alpha+\left(1-\alpha\right)\theta\right)}+2\theta\left(\alpha+\left(1-\alpha\right)\theta\right)\left(64\left(1+c\right)\right.\\&+8(2+c)\alpha-\left(32\left(1+c\right)+10\left(2+c\right)\alpha+\left(1+c\right)\alpha^2\right)\theta+\left(4\left(1+c\right)+2\left(2+c\right)\alpha+2\left(1+c\right)\alpha^2\right)\theta^2\\&\left.-\left(1+c\right)\alpha^2\theta^3\right)\Big)\Big/\Big(16\left(4-\theta\right)^2+8\alpha\left(1-\theta\right)\left(4-\theta\right)^2+8\alpha^2\left(1-\theta\right)^2\left(2-\theta\right)+\alpha^3\left(1-\theta\right)^3\theta\Big),\end{aligned}$$

and similarly, the bounded wholesale price to Firm S:

$$w_S^{ooV}=\Big(2\left(1-\theta\right)\left(4-\theta\right)\left(\left(4+\alpha\right)c-\left(2+\left(2+\alpha\right)c\right)\theta\right)\alpha\sqrt{\theta\left(\alpha+\left(1-\alpha\right)\theta\right)}+(16c\left(4-\theta\right)^2-4\left(1-c\right)(1$$
$$-\theta)\left(4-\theta\right)^2\alpha-2\left(8+\left(4-c\right)\theta\right)\left(1-\theta\right)^2\alpha^2-\left(2c-\theta\right)\left(1-\theta\right)^3\alpha^3)\theta\Big)\Big/\Big((16(4-\theta)^2+8\alpha\left(1-\theta\right)$$
$$\times\left(4-\theta\right)^2+8\alpha^2\left(1-\theta\right)^2\left(2-\theta\right)+\alpha^3\left(1-\theta\right)^3\theta)\theta\Big).$$

(ii) We then consider the case with an intermediate c such that the optimal wholesale price to Firm W is not bounded.

Substituting $w_S^{ooV}=\frac{1}{2(4-\theta)}\left(2c\left(4-\theta\right)-\alpha\left(1-\theta\right)\left(2+2c-w_W^{ooV}\right)\right)$ into the CM's profit function, we derive the optimal wholesale price to Firm W from the first-order condition

$$w_W^{ooV}=\Big(\left(\theta+\alpha\left(1-\theta\right)\right)\left(8\left(1+2c\right)-4\alpha c-\left(2\left(1+2c\right)-\left(4c-1\right)\alpha\right)\theta+\alpha\theta^2\right)\Big)\Big/\Big(8\left(4-\theta\right)+4\alpha(2-3\theta$$
$$+\theta^2)-2\alpha^2\left(1-\theta\right)^2\Big),$$

and similarly, the bounded wholesale price to Firm S:

$$w_S^{ooV}=\Big(16\left(4-\theta\right)c-2\left(1-\theta\right)\left(8+\left(2c-1\right)\theta\right)\alpha-\left(1-\theta\right)^2\left(2-\theta+4c\right)\alpha^2+\left(1-\theta\right)^3\alpha^3\Big)\Big/\Big(16\left(4-\theta\right)$$
$$+8\alpha\left(2-3\theta+\theta^2\right)-4\alpha^2\left(1-\theta\right)^2\Big).$$

We define the threshold c_1^{ooV} that separates the bounded and unbounded wholesale prices to Firm W as follows:

$$c_1^{ooV}=\Big(8\left(4-\theta\right)\left(4+\alpha+\left(2+\alpha\right)\theta\right)\sqrt{\theta\left(\alpha+\left(1-\alpha\right)\theta\right)}\left(16+4\alpha-\alpha^2-2\theta\left(2+3\alpha-\alpha^2\right)+\alpha\left(2-\alpha\right)\theta^2\right)$$
$$+4\alpha\theta\left(1-\theta\right)\left(\alpha+\left(1-\alpha\right)\theta\right)\left(32+8\alpha+2\alpha^2-\left(8+12\alpha+5\alpha^2\right)\theta+4\alpha\left(1+\alpha\right)\theta^2-\alpha^2\theta^3\right)\Big)\Big/$$
$$\Big(\theta(16\left(4-\theta\right)\left(16-4\theta+2\alpha\left(2-3\theta+\theta^2\right)-\alpha^2\left(1-\theta\right)^2\right)\sqrt{\theta\left(\alpha+\left(1-\alpha\right)\theta\right)}-\left(\theta+\left(1-\theta\right)\alpha\right)(\alpha^4$$
$$\theta\left(1-\theta\right)^4-2\alpha^3\left(8-\theta\right)\left(1-\theta\right)^3-8\alpha^2\left(1-\theta\right)^2\left(4-2\theta+\theta^2\right)+64\alpha\left(4-5\theta+\theta^2\right)+32\left(4-\theta\right)^2))\Big).$$

(iii) We now consider the case with a large c such that the optimal wholesale prices to both firms are not bounded. That is, $w_S^{ooV}=\frac{1}{2}$ and $w_W^{ooV}=\frac{1}{2}\left(\theta+\alpha\left(1-\theta\right)\right)$.

We define the threshold c_2^{ooV} that separates the bounded and unbounded wholesale prices to Firm S as $c_2^{ooV}=\Big(8-2\theta+\alpha\left(4-5\theta+\theta^2\right)-\alpha^2\left(1-\theta\right)^2\Big)\Big/\Big(4(4-\alpha-\theta+\alpha\theta)\Big)$.

Finally, given the optimal decisions of all firms under (O, I) and (O, O), we can derive their corresponding profits. By comparing those profits, we can determine the CM's preference over the sourcing structure and the impacts of brand spillover, as shown in the proposition. ■

Proof of Proposition 8

Define $c^{ioP}=\frac{1}{3}\left(2\theta-1\right)$,

$T_1^P=4\theta-\alpha(1-\theta)-\frac{7}{2}+\frac{1}{2}\sqrt{51+42\alpha-6\left(19+15\alpha\right)+12\left(5+4\alpha\right)\theta^2}$,

$T_2^P=\frac{6}{13}-\frac{3}{26}\theta+\frac{7}{26}\alpha\left(1-\theta\right)-\frac{3}{26}\sqrt{\left(4-\theta\right)^2-2\alpha\left(8-23\theta+15\theta^2\right)+17\alpha^2\left(1-\theta\right)^2}$,

$t_W^P = \frac{1}{10} + \frac{2}{5}\theta - \frac{3}{5}\alpha(1-\theta) - \frac{1}{10}\sqrt{11 - 2\theta - 12\alpha(1-\theta)^2 - 4\theta^2 - 4\alpha^2(1-\theta)^2}$, and

$$t_S^P = \begin{cases} \theta - \frac{1}{2} - \frac{1}{12}\sqrt{18 - 288\theta + 288\theta^2 - 72\alpha(1-3\theta+2\theta^2) + 24\alpha^2(1-\theta)^2}, & \text{if } c < c^{ioP}, \\ 1 - 2\theta + \frac{1}{3}\sqrt{9 + 72\theta - 72\theta^2 + 36\alpha(1-3\theta+2\theta^2) - 12\alpha^2(1-\theta)^2}, & \text{otherwise.} \end{cases}$$

It is worth noting that given the wholesale prices, the two firms' optimal quantities in this extension are the same as those in the main model.

There are two possible contracting outcomes between Firm S and the CM: Firm S insources or outsources. For each outcome, we analyze the contracting between the CM and Firm W.

Contracting between the CM and Firm W

First, provided that Firm S insources at a unit cost of c, the CM will induce Firm W to outsource because the CM's profit under (I, I) is zero. Consistent with the main model, under (I, O), the CM's optimal wholesale price to Firm W is $w_W^{oiP} = \min\left\{c, \frac{2\theta-1+c}{4}\right\}$. That is, there exists a threshold $c^{ioP} = \frac{1}{3}(2\theta-1)$ such that $w_W^{oiP} = c$ if $c < c^{ioP}$ and $w_W^{oiP} = \frac{2\theta-1+c}{4}$ otherwise.

Next, provided that Firm S outsources at a wholesale price of w_S^P, we identify the CM's preferred sourcing structure by comparing the CM's profits under the structures (O, I) and (O, O).

Under (O, I), the two firms' quantity decisions are $q_S^{oiP} = \frac{1}{3}(2-\theta+c-2w_S^P)$ and $q_W^{oiP} = \frac{1}{3}(2\theta-1-2c+w_S^P)$. Based on these optimal quantity responses, we can obtain the CM's profit for a given w_S^P.

Under (O, O), the two firms' quantity decisions are $q_S^{ooP} = \frac{1}{3}(2-\theta-\alpha+\alpha\theta-2w_S^P+w_W^{ooP})$ and $q_W^{ooP} = \frac{1}{3}(2\theta+2\alpha-2\alpha\theta-1+w_S^P-2w_W^{ooP})$. The CM's optimization problem over the wholesale price is

$$\max_{w_W^{ooP}} \Pi_{CM}^{ooP} = w_S q_S^{ooP} + w_W^{ooP} q_W^{ooP},$$
$$\text{s.t. } \Pi_W^{ooP}\left(w_S^P, w_W^{ooP}\right) \geq \Pi_W^{oiP}\left(w_S^P\right).$$

Solving the optimization problem, we obtain the CM's optimal price for Firm W $w_W^{ooP} = \min\left\{c+\alpha(1-\theta), \frac{1}{4}(2w_S+2\theta-1+2\alpha-2\alpha\theta)\right\}$. That is, there exists a threshold $h_{CM}^{ooP} = \frac{1}{2}+2c-\theta+\alpha(1-\theta)$ such that $w_W^{ooP} = c+\alpha(1-\theta)$ if $w_S > h_{CM}^{ooP}$ and $w_W^{ooP} = \frac{1}{4}(2w_S+2\theta-1+2\alpha-2\alpha\theta)$ otherwise. Based on the optimal w_W^{ooP} and the two firms' quantity responses, we can obtain the CM's profit under (O, O) for a given w_S^P.

Let $\Delta_1^P = \Pi_{CM}^{ooP} - \Pi_{CM}^{oiP}$. If $w_S > h_{CM}^{ooP}$, $\Delta_1^P = \frac{1}{3}(c+\alpha-\alpha\theta)(w_S-1-2c+2\theta)$; Since $w_S > h_{CM}^{ooP} > 1+2c-2\theta$, we have $\Delta_1^P > 0$ in this scenario. Similarly, if $w_S \leq h_{CM}^{ooP}$, we also have $\Delta_1^P > 0$. Thus, taking w_S^P as given, the CM always prefers the structure (O, O) over (O, I).

The above analysis shows that regardless of Firm S's outsourcing decision, the CM should always induce Firm W to outsource. Finally, we identify Firm S's preferred sourcing structure by comparing its profits under the structures (I, O) and (O, O).

Firm S's preferred sourcing structure

If Firm S chooses to insource, we can obtain Firm S's profit based on the CM's optimal wholesale price under (I, O).

If Firm S chooses outsourcing, Firm S's optimization problem over the wholesale price is

$$\begin{aligned}&\max_{w_S^{ooP}} \Pi_S^{ooP} = \left(p_S - w_S^{ooP}\right) q_S^{ooP},\\ &\text{s.t. } \Pi_{CM}^{ooP}\left(w_S^{ooP}\right) \geq \Pi_{CM}^{ioP}.\end{aligned}$$

Substituting the CM's optimal w_W^{ooP} into Firm S's profit function, we have $\Pi_S^{ooP} = \frac{1}{9}\left(2+c-\theta-2w_S^{ooP}\right)^2$ if $w_S^{ooP} > h_{CM}^{ooP}$ and $\Pi_S^{ooP} = \frac{1}{144}\left(7-2\alpha+2\alpha\theta-2\theta-6w_S^{ooP}\right)^2$ otherwise. Note that, w_S^{ooP} cannot be prohibitively high to guarantee a positive production quantity. We know that Firm S's profit is always decreasing in w_S^{ooP}. Thus, Firm S will set a lowest possible wholesale price that just satisfies the CM's participation constraint.

The CM's profit under (I, O) is $\Pi_{CM}^{ioP} = \frac{1}{3}c\left(2\theta-1-c\right)$ if $c < c^{ioP}$ and $\Pi_{CM}^{ioP} = \frac{1}{24}\left(2\theta-1+c\right)^2$ otherwise.

Therefore, we have four scenarios to examine the CM's participation constraint.

Scenario 1: $c < c^{ioP}$ and $w_S^{ooP} \leq h_{CM}^{ooP}$. Then the CM's participation constraint requires $w_S^{ooP} \geq \frac{1}{2} - \frac{1}{3}\sqrt{3+6c-3\alpha+6c^2+3\alpha^2-3\theta\left(1+4c-3\alpha+2\alpha^2\right)+3\theta^2\left(1-\alpha\right)^2}$. Moreover, the condition $w_S^{ooP} \leq h_{CM}^{ooP}$ leads to $c \geq \frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$ or $c \geq \frac{1}{10}+\frac{2}{5}\theta-\frac{3}{5}\alpha\left(1-\theta\right)-\frac{1}{10}\sqrt{11-2\theta-4\theta^2-2\alpha\left(11-27\theta+16\theta^2\right)+16\alpha^2\left(1-\theta\right)^2}$. We find that $\frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$ is always greater than
$\frac{1}{10}+\frac{2}{5}\theta-\frac{3}{5}\alpha\left(1-\theta\right)-\frac{1}{10}\sqrt{11-2\theta-4\theta^2-2\alpha\left(11-27\theta+16\theta^2\right)+16\alpha^2\left(1-\theta\right)^2}$. Thus, if $c < c^{ioP}$ and $c \geq \frac{1}{10}+\frac{2}{5}\theta-\frac{3}{5}\alpha\left(1-\theta\right)-\frac{1}{10}\sqrt{11-2\theta-4\theta^2-2\alpha\left(11-27\theta+16\theta^2\right)+16\alpha^2\left(1-\theta\right)^2}$, it is optimal for Firm S to set $w_S^{ooP} = \frac{1}{2}-\frac{1}{3}\sqrt{3+6c-3\alpha+6c^2+3\alpha^2-3\theta\left(1+4c-3\alpha+2\alpha^2\right)+3\theta^2\left(1-\alpha\right)^2}$.

Scenario 2: $c < c^{ioP}$ and $w_S^{ooP} > h_{CM}^{ooP}$. Then the CM's participation constraint requires $w_S^{ooP} \geq \frac{1}{2}+\frac{1}{2}c-\frac{1}{4}\theta-\frac{1}{4}\alpha\left(1-\theta\right)-\frac{1}{4}\sqrt{4+8c-4c^2-4\theta\left(1+c\right)+\theta^2-2\alpha\left(1-\theta\right)\left(2+6c-7\theta\right)+\alpha^2\left(1-\theta\right)^2}$. Moreover, the condition $w_S^{ooP} > h_{CM}^{ooP}$ leads to $c < \frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$ and $c < \frac{1}{10}+\frac{2}{5}\theta-\frac{3}{5}\alpha\left(1-\theta\right)-\frac{1}{10}\sqrt{11-2\theta-4\theta^2-2\alpha\left(11-27\theta+16\theta^2\right)+16\alpha^2\left(1-\theta\right)^2}$. Thus, we have if $c < \frac{1}{10}+\frac{2}{5}\theta-\frac{3}{5}\alpha\left(1-\theta\right)-\frac{1}{10}\sqrt{11-2\theta-4\theta^2-2\alpha\left(11-27\theta+16\theta^2\right)+16\alpha^2\left(1-\theta\right)^2}$, it is optimal for Firm S to set $w_S^{ooP} = \frac{1}{2}+\frac{1}{2}c-\frac{1}{4}\theta-\frac{1}{4}\alpha\left(1-\theta\right)-\frac{1}{4}\sqrt{4+8c-4c^2-4\theta\left(1+c\right)+\theta^2-2\alpha\left(1-\theta\right)\left(2+6c-7\theta\right)+\alpha^2\left(1-\theta\right)^2}$.

Scenario 3: $c \geq c^{ioP}$ and $w_S^{ooP} \leq h_{CM}^{ooP}$. Then the CM's participation constraint requires $w_S^{ooP} \geq \frac{1}{2} - \frac{1}{6}\sqrt{9+6c-3c^2-12\theta c-12\alpha\left(1-3\theta+2\theta^2\right)+12\alpha^2\left(1-\theta\right)^2}$. Moreover, the condition $w_S^{ooP} \leq h_{CM}^{ooP}$ leads to $c \geq \frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$ or
$c \leq \frac{1}{49}\left(1+22\theta-24\alpha+24\alpha\theta+2\sqrt{37+11\theta-61\alpha+189\alpha\theta-26\theta^2+46\alpha^2-128\alpha\theta^2-92\alpha^2\theta+46\alpha^2\theta^2}\right)$. We find that $\frac{1}{49}\left(1+22\theta-24\alpha+24\alpha\theta+2\sqrt{37+11\theta-61\alpha+189\alpha\theta-26\theta^2+46\alpha^2-128\alpha\theta^2-92\alpha^2\theta+46\alpha^2\theta^2}\right)$ is always greater than $\frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$. Thus, we have if $c \geq c^{ioP}$, it is optimal for Firm S to set $w_S^{ooP} = \frac{1}{2} - \frac{1}{6}\sqrt{9+6c-3c^2-12\theta c-12\alpha\left(1-3\theta+2\theta^2\right)+12\alpha^2\left(1-\theta\right)^2}$.

Scenario 4: $c \geq c^{ioP}$ and $w_S^{ooP} > h_{CM}^{ooP}$. Then the CM's participation constraint requires $w_S^{ooP} \geq \frac{1}{2}+\frac{1}{2}c-\frac{1}{4}\theta-\frac{1}{4}\alpha\left(1-\theta\right)-\frac{1}{4}\sqrt{3+2c+8\theta c-13c^2-3\theta^2-2\alpha\left(1-\theta\right)\left(2+6c-7\theta\right)+\alpha^2\left(1-\theta\right)^2}$. Moreover, the condition $w_S^{ooP} > h_{CM}^{ooP}$ leads to $c < \frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$ and
$c > \frac{1}{49}\left(1+22\theta-24\alpha+24\alpha\theta+2\sqrt{37+11\theta-61\alpha+189\alpha\theta-26\theta^2+46\alpha^2-128\alpha\theta^2-92\alpha^2\theta+46\alpha^2\theta^2}\right)$. Since $\frac{1}{49}\left(1+22\theta-24\alpha+24\alpha\theta+2\sqrt{37+11\theta-61\alpha+189\alpha\theta-26\theta^2+46\alpha^2-128\alpha\theta^2-92\alpha^2\theta+46\alpha^2\theta^2}\right)$ is always greater than $\frac{1}{2}\left(\theta-\alpha+\alpha\theta\right)$, scenario 4 can never emerge as an equilibrium.

With these optimal wholesale prices under (I, O) and (O, O), we can obtain all firms' profits and then derive Firm S's preferred sourcing structure and the impacts of brand spillover by comparing these profits, as shown in the proposition. ■

Appendix B: The derivation of $\underline{\theta}$

As long as the optimal q_W is higher than zero, Firm W will not be driven out of the market. Under the sourcing structure (O, I), substituting the CM's optimal wholesale prices into the optimal quantity responses, we have:

If $c \le c^{oi}$, then $q_S^{oi} = \frac{2-\theta-c}{3}$ and $q_W^{oi} = \frac{2\theta-1-c}{3}$; clearly, q_S^{oi} is always higher than zero, and $q_W^{oi} > 0$ requires $\theta > \frac{1+c}{2}$.

If $c > c^{oi}$, then $q_S^{oi} = \frac{2-\theta+c}{6}$ and $q_W^{oi} = \frac{7\theta-2-7c}{12}$; clearly, q_S^{oi} is always higher than zero, and $q_W^{oi} > 0$ requires $\theta > \frac{2+7c}{7}$.

Therefore, under (O, I), Firm W will not be driven out of the market if $\theta > \max\left\{\frac{1+c}{2}, \frac{2+7c}{7}\right\}$.

Under the sourcing structure (O, Ob), we have:

If $c \le c_1^{oo}$, $q_S^{oo} = \frac{2-\theta-c+\alpha-\alpha\theta}{3}$ and $q_W^{oo} = \frac{4\theta-2-2c-\alpha+\alpha\theta}{6}$; clearly, q_S^{oo} is always higher than zero; substituting $c = c_1^{oo}$ into q_W^{oo} gives $\frac{2\theta-1+2\alpha-2\alpha\theta}{6}$, which is higher than zero if $\theta > \frac{1-2\alpha}{2(1-\alpha)}$. Since q_W^{oo} decreases in c, $q_W^{oo} \ge \frac{2\theta-1+2\alpha-2\alpha\theta}{6}$. Furthermore, $\theta \ge \underline{\theta} = \max\left\{\frac{2+7c}{7}, \frac{1+c}{2}\right\}$ so that Firm W is not driven out of the market, so $\theta > \frac{1+c}{2} > \frac{1-2\alpha}{2(1-\alpha)}$. Then $\frac{2\theta-1+2\alpha-2\alpha\theta}{6} > 0$ always holds and $q_W^{oo} > 0$ in this case.

If $c_1^{oo} < c \le c_2^{oo}$, $q_S^{oo} = \frac{7-6c-2\theta+\alpha-\alpha\theta}{12}$ and $q_W^{oo} = \frac{2\theta-1+2\alpha-2\alpha\theta}{6}$; here, q_S^{oo} is decreasing in θ and is higher than zero even if $\theta = 1$; $q_W^{oo} > 0$ requires $\theta > \frac{1-2\alpha}{2(1-\alpha)}$. Thus, given $\theta > \frac{1+c}{2}$, we always have $q_W^{oo} > 0$.

If $c > c_2^{oo}$, $q_S^{oo} = \frac{2-\alpha-\theta+\alpha\theta}{6}$ and $q_W^{oo} = \frac{2\theta-1+2\alpha-2\alpha\theta}{6}$; clearly, q_S^{oo} is always higher than zero; given $\theta > \frac{1+c}{2}$, we always have $q_W^{oo} > 0$.

That is, under (O, O), given $\theta > \max\left\{\frac{1+c}{2}, \frac{2+7c}{7}\right\}$, no firm will be driven out of the market. Therefore, we define $\underline{\theta} = \max\left\{\frac{1+c}{2}, \frac{2+7c}{7}\right\}$. ■