Consumer Informedness: A Simple Way to Explain Maximal or Minimal Differentiation

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Abstract

The principle of Minimum Differentiation arises as the equilibrium outcome in spatial models when price is fixed (Hotelling 1929). Conversely, when firms set prices independently, models suggest that firms will differentiate in order to reduce the intensity of price competition (d’Aspremont et al. 1979). Yet in many competitive industries where firms compete on price, firms are observed to collocate. This puzzle has been referred to as the Hotelling paradox. We propose an explanation for this puzzle based on two simple observations. The first is that markets are imperfectly informed of the alternatives that are available within a category. That is, despite the marketing investments of competing firms, there is a significant fraction of the market that is uninformed about a firm’s offer. The second observation is that the awareness of local products tends to be marginally higher than the awareness of non-local products. The magnitude of this effect is a function of the category but this effect permeates all markets where consumer heterogeneity is location (or geographically) based. With these two simple observations, we propose a model that recovers both minimal and maximal differentiation as a function of the degree to which consumers are aware of products. Specifically, maximum differentiation will occur when consumers are well informed about the products in the market, but minimum differentiation will occur when consumers have low awareness of the products. Moreover, the model explains why competing products possess similar characteristics in the introductory stage of the Product Life Cycle but firms naturally move to differentiated offerings as a category matures. In an extension, we make the level of informedness a choice variable for the firms. The extension confirms the intuition of the base model but also demonstrates how media intensity can be used to create endogenous differentiation between firms. The need to differentiate through location is attenuated when firms differentiate through the choice of media intensity, and firms may not choose maximum informedness even if advertising is free.

Keywords: linear city, positioning, differentiation, awareness.
1 Introduction

1.1 Background

The principle of Minimum Differentiation arises as the equilibrium outcome in spatial models when price is fixed (Hotelling 1929). Hotelling’s seminal model is based on consumers being evenly distributed along a linear market where consumers incur linear travel costs between their location and the location of their chosen firm. This principle relies on firms not competing based on price. The reason is that when firms collocate they become almost identical. In such situations, price competition eliminates profits (this is also referred to as Bertrand competition between homogenous goods). It follows then that firms should move away from each other when prices are not fixed. This should reduce the intensity of price competition and give "differentiated firms" a degree of pricing power over consumers that are nearby. However, it is not possible to find equilibrium locations for competing firms that reflect this intuition in a standard Hotelling market with linear travel costs (d’Aspremont, Gabszewicz and Thisse 1979).

In contrast, the principle of Maximum Differentiation has its roots in the idea that firms differentiate to gain pricing power over proximate consumers and to reduce the incentive that the firm has to cut its price in order to capture market share from the competitor. These reasons explain the findings of d’Aspremont et al who solve for the endogenous locations of two firms in a Hotelling linear market. When consumers have quadratic travel costs (the consumer’s cost to travel some distance to purchase at a given firm is proportional to the square of the distance), d’Aspremont et al show that the competing firms maximally differentiate. Despite the predictions of d’Aspremont et al, firms are observed to collocate in many competitive industries where firms compete on price. For example, in the market for autobody repairs, competing firms provide almost identical service and it is common to find competing body shops clustered together.¹ This puzzle has been referred to as the Hotelling paradox. Why would firms locate so closely to each other when the primary effect of co-location is to increase the intensity of price competition?

We propose an explanation for this puzzle based on two simple observations. The first is that markets are imperfectly informed of the alternatives that are available within a category. That is, despite the marketing investments of competing firms, there is a significant fraction of the market that is uninformed about a firm’s offer. The second observation is that the awareness of local products tends to be marginally higher than the awareness of non-local products. The magnitude

¹On Harlech Crescent in Thornhill, Ontario, there are at least 5 autobody shops within 100 meters of each other.
of this effect is a function of the category but this effect permeates all markets where consumer heterogeneity is location (or geographically) based. With these two simple observations, we build a model that recovers both minimal and maximal differentiation as a function of degree to which consumers are aware of products. Moreover, the model explains why competing products possess similar characteristics in the introductory stage of the Product Life Cycle, but firms move to differentiated offerings as a category matures.

Finally, in an extension, we allow different levels of advertising to be chosen by the firms in the second stage of the game. The extension confirms the robustness of base model: higher levels of informedness cause firms to move away from each other in terms of location choice. However, the extension also reveals the ability of media intensity to reduce the intensity of competition between firms. When low levels of advertising are sufficiently attractive, the need of a centrally-located firm to reposition to reduce competitive intensity disappears. As a result, the likelihood of observing maximal differentiation is lower. In fact, the firms may choose less differentiation because it pressures their opponent to reduce their advertising, with the threat of intense price competition acting as the cudgel to enforce softer competition. As part of demonstrating this, we also show that firms may choose not to fully advertise, even if advertising is free.

Next, we review the literature that is relevant to our study.

1.2 Literature Review

A central question in economics is to what extent firms choose to differentiate. We focus on markets characterized by horizontal differentiation where products are of similar quality (Eaton and Lipsy 1996). On the one hand, firms want to locate where the demand is. In a linear city, this is achieved by locating in the middle of the market as suggested by Hotelling (1929). The idea is that a central location allows the firm to maximize its market share. On the other hand, firms want to differentiate in order to soften price competition (Tirole 1988). While Hotelling’s analysis considers price competition, the proposed prices do not form a Nash Equilibrium. In fact, Economides (1984) shows that firms in a linear market have a tendency to move away from each other. This obtains in a model with low reservation prices such that "jumping firms" establish local monopolies. Similarly, Pazgal, Soberman and Thomadsen (2016) show that an equilibrium with moderate differentiation arises in a Hotelling model with linear travel cost if the reservation prices are not too high. When reservation prices are high, d’Aspremont, Gabszewicz and Thisse (1979) show that under linear travel costs a price equilibrium may not exist if firms locate sufficiently close to each other. To
solve this problem, the authors propose a model where firms choose a position along a linear market and consumers have quadratic travel costs, and find that firms will choose to maximally differentiate. However, the assumption of quadratic travel costs is not insignificant. In a spatial model with two firms, the assumption ensures the existence of a pure-strategy pricing equilibrium for all possible locations. However, the assumption may be questionable in many industries; for example, papers studying geographic travel costs typically find that marginal travel costs decrease with greater amounts of travel (e.g., Davis 2006, Tay 2003).

Other authors including Eaton and Lipsey (1975), Novshek (1980) and Ansari et al (1998) show that models which generate equilibria with maximal or minimal levels of differentiation are sensitive to the specific assumptions associated with the model (for example, the number of attributes or the number of firms). Nevertheless, these papers do not provide clear guidance as to when we should expect high or minimal levels of differentiation.

Our approach to study the question of positioning is built on the role played by marketing to inform or activate consumers in a market. Academics have made significant contributions demonstrating how advertising provides information to consumers about product attributes that are valued by some consumers and not by others. This work shows that informative advertising allows consumers to find products that better meet their needs (Butters 1977, Grossman and Shapiro 1984, Robert 1993 and Bester 1995). The structure we adopt is identical to the approach of Grossman and Shapiro (1984) in that advertising creates informedness i.e. consumers who are exposed to advertising from a firm are informed about both the existence of the product and all of its attributes. In addition, we build on the observation that firms have an advantage in terms of awareness in their local markets. As noted by Porter (2006), firms gain by being close to consumers: the knowledge and ability firms have to capitalize on local marketing opportunities allows them to get more for less (Rudd 2008, Carlstone 1997).

Our paper is also related to the literature about how advertising affects competition. Grossman and Shapiro (1984) show that increased advertising can intensify price competition. Further, they find that profits can decrease as advertising costs are decreased due to the introduction of more intense advertising. However, the paper does not then discuss how products should be positioned in the spatial market.\textsuperscript{2} Iyer, Soberman and Villas-Boas (2005) study targeted advertising and find that targeting is an effective tool to reduce price competition. Again, this work does not consider product positioning. Lauga, Ofek and Katona (2018 working paper) study the effect of targeted

\textsuperscript{2}In Grossman and Shapiro (1984), firms exogenously assume equally spaced locations around the circular market.
versus blanket advertising on product quality in a competitive vertically-differentiated market. The authors show that the ability to target advertising provides an incentive for firms to differentiate in terms of quality. In contrast, our analysis focuses on the relationship of advertising intensity to the horizontal positioning of products that are of equal quality.

We now present a model to analyze the impact of informedness on the positioning decisions of firms.

2 The Model

The model is a market of size 1 with three segments of consumers: a Left segment, a Middle segment and a Right segment. The Left and Right segments are of equal size $\epsilon$. The Middle segment is of size $1-2\epsilon$. In the first stage of the game, two ex ante symmetric firms make decisions to position in either the Left, Middle or Right segment. Once positioned, both firms have the ability to produce a product at a marginal cost of $c$. Without loss of generality, we normalize this marginal cost to zero.\(^3\) Consumers purchase at most one product and only consider products that meet their needs. That is, a consumer is willing to consume a product that is located in its segment or in a segment that is immediately adjacent. This implies that consumers in the Left segment will not buy products located in the Right segment and vice versa. However, all informed consumers would consider buying a product that is located in the Middle segment.\(^4\) Next, we present the basis upon which consumers make decisions.

2.1 Consumer Decision Making

Any consumer who is informed about a product knows the product’s location. Consumers costlessly learn the prices of products about which they are informed. This is similar to Meurer and Stahl (1984) and Mayzlin and Shin (2011). Essentially, this implies that a consumer considers a product if and only if she is informed about that product. We assume that if a consumer is informed about a product then she can observe the price of that product easily prior to purchase (within a retailer, for example). As noted earlier, a consumer buys at most one product, and will buy the product that provides her with the maximum utility. The utility $U$ from product $i$ for a consumer in Segment $s$ is given by:

\(^3\) As long as the marginal cost is small compared to the reservation utility of consumers, the findings of the model are unaffected by having a positive marginal cost.

\(^4\) This is a generalization of the market structure developed in Varian (1980).
In this expression, \( r \) is the reservation price for product \( i \) and \( p_i \) is the price for product \( i \). Without loss of generality, we normalize the reservation price to 1. More fully, the reservation price equals 1 if product \( i \) is located in Segment \( s \) or a segment adjacent to Segment \( s \) and zero otherwise. Note that the outside option for consumers provides 0 utility so a consumer declines to buy if the highest utility she knows of is less than 0. We explain the information structure in the following section.

### 2.2 Information Structure

As noted earlier, the first decision taken by firms is positioning. We assume that once both firms have chosen their locations, Firm \( i \) \( (i = 1, 2) \) becomes informed about the location of its competitor before it makes its advertising decision in stage 2. In the second stage, each firm chooses a level of advertising, which we denote as \( \phi_i \), and this determines the increase in awareness that each firm obtains for its product in the marketplace. Similar to Grossman and Shapiro (1984), we assume that the consumers reached by the advertising of each firm are random. In other words, all consumers are equally likely to be exposed to the advertising effort taken by firms. We also assume that when a firm is active, it enjoys a local awareness boost, which reflects the idea that with the same marketing effort, consumers in a firm’s home segment are more likely to be aware of a local firm. We assume that this local awareness boost is equal to \( d \) \( (d > 0) \). Because our objective is to understand the impact that informedness has on positioning, the awareness boosts we consider are significantly less than the level of advertising informedness, i.e., \( d \ll \phi_i \). These assumptions imply that the maximum level of advertising a firm can choose is \( 1 - d \), so a firm benefits from a local boost in awareness even at maximum advertising.

This structure means that after the two firms have advertised, there are up to 4 groups of consumers in each segment. Let the fraction of consumers in segment \( s \) who consider buying product \( i \) be denoted as:

\[
\Omega_{si} = \begin{cases} 
\phi_i + d & \text{if the firm is located in segment } s \\
\phi_i & \text{if the firm is located in a segment adjacent to } s \\
0 & \text{if the firm is located two segments away from } s 
\end{cases}
\]

The 4 groups of consumers in any competitive segment (where the customers are located in the same or adjacent segments to both of the firms) are then as follows: Firm \( i \)'s captive demand in segment \( s \) is then \( \Omega_{si}(1 - \Omega_{sj}) \), while a fraction \( \Omega_{si} \times \Omega_{sj} \) of segment \( s \) represents competitive
customers who consume from the firm with the lowest price. Similarly, firm \( j \)'s captive demand is \( \Omega_{s_j}(1 - \Omega_{s_i}) \), and finally a fraction \( (1 - \Omega_{s_i})(1 - \Omega_{s_j}) \) of the consumers in segment are inactive. Note that when both firms are collocated, they both benefit from the local awareness premium. In contrast, in a segment, where no firm is located, neither firm benefits from a local awareness premium. We now review the decisions that each firm takes.

2.3 Firm Decisions

In our model, the first decision that firms make is where to position themselves. A firm can locate in any of the three segments. Because firms make this decision simultaneously, they cannot coordinate their decisions.

After firms have chosen a location, they then make an advertising decision. In the base model, we consider a game where firms make a discrete decision about advertising. In particular, the firms are assumed to have access to a symmetric technology such that each firm can choose to advertise at a cost \( A \) and inform a fraction \( \phi \) of the market. This form of discrete advertising is similar to the approach of Iyer et al (2005) and Soberman (2009). It can also be thought of as the base awareness level for similar firms within an industry (Dukes and Gal-Or 2003). In an extension, we consider a market where firms make decisions about two different levels of awareness.

The third decision that firms make is pricing. That is, firms choose \( p_i \) \((i = 1, 2) \) simultaneously after the positioning and advertising decisions have been made. In the model, consumers reached by a firm’s advertising are informed about that firm’s product. As noted earlier, consumers costlessly learn the price of such products.

Finally, customers decide which, if any, firm to purchase from.

The extensive form of the game can thus be summarized by the following steps:

1. The firms simultaneously choose a location \((L, M \text{ or } R)\) in the three segment market.

2. The firms simultaneously make advertising decisions.

3. The firms simultaneously set prices for their products.

4. Consumers who have been reached by the advertising of at least one firm make a decision about which, if any, firm to purchase from.

As explained above, the equilibrium we seek is one in which neither firm has an incentive to change its decision taking the competitor’s decisions as given. This implies that if the equilibrium
of the game is asymmetric, the model is agnostic about which of the two firms will be in the more favorable of the two equilibrium positions.

In Section 2.1, we explain the basis that consumers have for making decisions. In the next section, we present the objective functions that firms optimize. The expressions we show summarize the profits of each firm in the pricing subgame (Steps 3 and 4 of the extensive form).

### 2.4 Firm Objective Functions

The above decisions imply that a firm located in segment S optimizes one of three different objective functions depending on whether it is located a) in the same segment as its competitor (co-location), b) in an adjacent segment to its competitor or c) in a segment which is not adjacent to the competitor’s segment (this only happens when one firm is located in the Left segment and the other in the Right segment). After having made the positioning decisions and the advertising decisions, the following are the objective functions for the three cases. Due to the symmetry of the model, we denote positions of $L$ and $R$ as $E$ (an external segment). We utilize $\Pi_i$ to denote the total equilibrium profits earned by Firm $i$ ($i = 1, 2$) as a function of the market primitives (at the equilibrium prices), while $\pi_i$ denotes the profit earned by Firm $i$ as a function of the firm’s price. Profits as a function of prices are then:

**Co-location in $M$ ($p_1 > p_2$)**

$$
\pi_1 = p_1[2\epsilon \phi_1 (1 - \phi_2) + (1 - 2\epsilon)((\phi_1 + d)(1 - (\phi_2 + d)))]
$$

$$
\pi_2 = p_2[2\epsilon \phi_2 + (1 - 2\epsilon)(\phi_2 + d)]
$$

**Co-location in $E$ ($p_1 > p_2$)**

$$
\pi_1 = p_1[\epsilon((\phi_1 + d)(1 - (\phi_2 + d)) + (1 - 2\epsilon)\phi_1(1 - \phi_2)]
$$

$$
\pi_2 = p_2[\epsilon(\phi_2 + d) + (1 - 2\epsilon)\phi_2]
$$

**Adjacent Locations in $E$ and $M$ ($p_E > p_M$)**

$$
\pi_M = p_M \phi_M
$$

$$
\pi_E = p_E[(\phi_E + d)(1 - \phi_M)\epsilon + \phi_E(1 - (\phi_M + d))(1 - 2\epsilon)]
$$

**Adjacent Locations in $E$ and $M$ ($p_M > p_E$)**

---

*5 We will show that in equilibrium, ties in prices occur only with measure 0. However, for completeness, one can assume that the competitive customers are split equally in such a scenario. This division does not affect the analysis.*
Non-Adjacent Locations ($p_1 > p_2$)

$$
\pi_1 = p_1[(\phi_1 + d)\epsilon + \phi_1 (1 - \phi_2) (1 - 2\epsilon)]
$$

$$
\pi_2 = p_2[(\phi_2 + d)\epsilon + \phi_2 (1 - 2\epsilon)]
$$

These profits are generally stated as demand from the extreme segments first and demand from the center segment second. The discrete nature of the segments in this model leads to a situation in which firms undercut to compete for consumers in either the local or adjacent segments who are reached by the advertising of both firms. In the next section, we present the base case in which the advertising decision by firms is discrete.

### 3 Analysis of the Base Case: Fixed Advertising Levels

As noted in Section 2, the base case is one in which firms make discrete advertising decisions. Said differently, the firms choose to advertise at a cost $A$ and inform a fraction $\phi$ of the market or to advertise with level 0 and earn 0 profits. Our equilibrium concept is Subgame Perfect Nash Equilibrium (SPNE) and in accordance, we solve the game going backwards from the final subgame to the first decision that firms make. As a result, we solve the equilibrium outcomes for each of the possible subgames and then work backwards to determine the optimal advertising and positioning decisions for each firm. In the base case, we assume that $A$ is sufficiently low such that firms choose to advertise independent of the positioning decisions they make in the first stage.\(^6\) This simplifies the analysis somewhat and allows us to isolate the impact of informedness on the positioning decision in the first stage of the game.

Because we restrict our attention to values of $A$ where both firms advertise, there is always a fraction of consumers in at least one competitive segment that have seen advertising from both firms. As a result, the equilibrium in the final stage of the game is in mixed strategies because if a firm chooses a price $p$ that is not too low, the competitor would undercut to $p - \epsilon$ (where $\epsilon$ is arbitrarily small) to capture the fraction of competitive consumers who have seen advertising from both firms (these consumers choose the competitor if the price is slightly less). The undercutting continues until one of the firms has an incentive to raise its price back to the reservation price

\(^6\)For any $\phi < 1$, there is a value $\overline{\pi}$ such that both firms advertise as long as $A < \overline{\pi}$.
and then the cycle continues again. This reasoning leads to a mixed strategy pricing equilibrium (Varian, 1980).

As previously mentioned, there are three possible subgames that need to be solved: (1) the firms are located at opposite external positions, (2) the firms are located in adjacent segments and (3) the firms are collocated. We commence our analysis with the firms located at opposite external positions.

### 3.1 Both firms located at external positions

Following the discussion above, we assume that both firms advertise, so $\phi = \phi_1 = \phi_2$. Because the firms are symmetric, we write the objective function for Firm $i$ as a function of the equilibrium cumulative distribution function (the cdf) of prices, denoted as $F(p) = F_i(p) = F_j(p)$ due to symmetry.

$$
\pi_i = p_i (\phi + d) \epsilon + p_i \phi (1 - \phi) (1 - 2\epsilon) + p_i \phi^2 (1 - 2\epsilon) (1 - F(p_i))
$$

(2)

These terms represent profits from firm $i$’s captive customers in the edge and central segments, respectively, and the profits from the competitive customers in the center segment which is realized only when firm $i$’s price is lower than firm $j$’s price. The analysis of this subgame can be solved by noting that this scenario is analogous to the loyals and switchers model of Narasimhan (1988). Following Narasimhan (1988) we apply the guaranteed profit condition to determine a) the equilibrium cdfs of prices, b) the equilibrium profits and c) the minimum prices. These are summarized in Lemma 1 (proofs are provided in the appendix).

**Lemma 1** When the firms are located at external positions, the profit of each firm is

$$
\Pi = (\phi + d) \epsilon + \phi (1 - \phi) (1 - 2\epsilon) - A
$$

and the firms price according to the cdf,

$$
F(p) = \frac{\phi + de - \phi^2 - \phi \epsilon + 2\phi^2 \epsilon - d \phi \epsilon + p \phi \epsilon}{p \phi^2 (2\epsilon - 1)}.
$$

The lowest observed price is $p = \frac{\phi + de - \phi^2 - \phi \epsilon + 2\phi^2 \epsilon}{\phi + de - \phi \epsilon}$.

We now consider the case when firms collocate in the first stage.

### 3.2 Co-location

To simplify our exposition, we only present the co-location case where both firms locate in the middle segment. The rationale for this is that if the firms co-located in either of the external segments then the firms would have a profitable deviation to the other external segment.\footnote{It is straightforward to show – using the same logic as presented in the text – that such a deviation increases profits. Profits if both firms locate at an external segment would be $(\phi + d) (1 - \phi - d) \epsilon + \phi (1 - \phi) (1 - 2\epsilon) - A$, while the profits from maximum differentiation would be $(\phi + d) \epsilon + \phi (1 - \phi) (1 - 2\epsilon) - A$. Thus, deviating to maximum
is not possible for such an outcome to be a pure-strategy Nash Equilibrium. Similar to Section 3.1, we write the objective function for Firm $i$ as a function of the equilibrium cumulative distribution function (the cdf) of prices, which is denoted as $F(p)$ for each firm.

$$
\pi_i = 2\epsilon \left(p_i\phi (1 - \phi) + p_i\phi^2 (1 - F(p_i))\right) + (1 - 2\epsilon) \left(p_i (\phi + d) (1 - (\phi + d)) + p_i (\phi + d)^2 (1 - F(p_i))\right)
$$

The guaranteed profit condition and symmetry imply the outcome in the co-location subgame summarized in Lemma 2.

**Lemma 2** When the firms are collocated in the middle segment, the profit of each firm is

$$\Pi = d + \phi - 2d\phi - 2de - \phi^2 + 2d^2\epsilon - d^2 + 4d\phi\epsilon - A$$

and the firms price according to the cdf,

$$F(p) = \frac{d + \phi - 2d\phi - 2de - \phi^2 + 2d^2\epsilon - dp - d^2 + 2d\phi\epsilon + 4d\phi}{p(-2d\phi - 2d^2\epsilon - d^2 + 4d\phi)}.$$ The lowest observed price is

$$p = \frac{d + \phi - 2d\phi - 2de - \phi^2 + 2d^2\epsilon - dp - d^2 + 2d\phi\epsilon + 4d\phi}{d + \phi - 2d\epsilon}.$$ We now consider the case when firms are located in adjacent segments.

### 3.3 The firms are located in adjacent segments

This scenario can only occur if one firm is located in the external segment while the other firm is located in the middle. Here, we designate the firm located in the middle segment as Firm $M$ and the firm located in the external segment as Firm $E$. Following Section 2, we again write the objective functions as a function of the cdfs of the pricing distribution of each firm. Because the locations of the firms are not symmetric, each firm will have a unique pricing strategy hence the subscript $M$ or $E$ on the cdfs.

$$\pi_M = p_M\phi\epsilon + p_M (\phi + d) (1 - \phi) (1 - 2\epsilon) + p_M (\phi + d)^2 (1 - F_E(p_M))$$

$$+ p_M\phi (1 - (\phi + d))\epsilon + p_M\phi (\phi + d)\epsilon (1 - F_E(p_M))$$

$$\pi_L = p_L\phi (1 - (\phi + d)) (1 - 2\epsilon) + p_L (\phi + d) (1 - (\phi + d)^2 (1 - F_M(p_L))$$

$$+ p_L (\phi + d) (1 - \phi)\epsilon + p_L (\phi + d)\phi\epsilon (1 - F_M(p_L))$$

It is important to recognize that the firm located in the middle segment has a larger captive segment by virtue of its central location. The captive segment of Firm $M$ is $\phi\epsilon + \phi (1 - \phi - d)\epsilon + (\phi + d) (1 - \phi) (1 - 2\epsilon)$ whereas the captive segment of Firm $E$ is $(\phi + d) (1 - \phi)\epsilon + \phi (1 - \phi - d) (1 - 2\epsilon)$. Differentiation increases profits by $(\phi + d)^2\epsilon$. Technically, there can exist a mixed strategy location equilibrium that will lead to co-location at an extreme position happening with a positive probability. We focus on pure strategy location equilibria in this paper.
This asymmetric game can again be solved using the guidance of Narasimhan (1988), which allows us to derive Lemma 3.

**Lemma 3** When the firms are located in adjacent segments, the profit of the firm located in the middle segment is \( \pi_M = d + \phi - d\phi - 2de - \phi^2 + \phi^2\epsilon + d\phi\epsilon \) and the firm located in the external segment is \( \pi_E = (d + \phi - d\phi - 2de - \phi^2 + \phi^2\epsilon + d\phi\epsilon) \frac{\phi + de - \phi\epsilon}{d + \phi - 2de} \). The cdfs for each firm are as follows:

\[
F_M(p) = \begin{cases} 
0 & \text{if } d + \phi - d\phi - 2de - \phi^2 + \phi^2\epsilon + d\phi\epsilon < 0 \\
1 & \text{if } d + \phi - d\phi - 2de - \phi^2 + \phi^2\epsilon + d\phi\epsilon \geq 0 
\end{cases} 
\]

\[
F_E(p) = \begin{cases} 
0 & \text{if } d + \phi - d\phi - 2de - \phi^2 + \phi^2\epsilon + d\phi\epsilon < 0 \\
1 & \text{if } d + \phi - d\phi - 2de - \phi^2 + \phi^2\epsilon + d\phi\epsilon \geq 0 
\end{cases} 
\]

Note that Firm \( M \) has a mass point at 1 which is chosen with probability \( 1 - \frac{\phi + de - \phi\epsilon}{d + \phi - 2de} \). Lemmas 1, 2 and 3 provide the basis to determine the optimal positioning decisions of the firms as a function of the market primitives.

### 3.4 Optimal Positioning for the Firms

The optimal positioning for the firms is determined by solving the normal form for Stage 1 as shown below. For simplicity, the normal form shows the subgame profits but the true profits of the firms in each cell are overstated by \( A \), the cost of advertising.

<table>
<thead>
<tr>
<th>L</th>
<th>M</th>
<th>R</th>
</tr>
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<tbody>
<tr>
<td>L</td>
<td>( \Pi_{LL}, \Pi_{LM} )</td>
<td>( \Pi_{EM}, \Pi_{MM} )</td>
</tr>
<tr>
<td>M</td>
<td>( \Pi_{ML}, \Pi_{MZ} )</td>
<td>( \Pi_{E}^{1*}, \Pi_{M}^{1*} )</td>
</tr>
<tr>
<td>R</td>
<td>( \Pi_{M}^{A}, \Pi_{M}^{2} )</td>
<td>( \Pi_{E}^{1}, \Pi_{M}^{1} )</td>
</tr>
</tbody>
</table>

Note that our analysis does not distinguish between \( (R, L) \) and \( (L, R) \). Nor does it distinguish between \( (L, M) \), \( (R, M) \), \( (M, L) \) or \( (M, R) \). Moreover, as previously noted, we do not present the values of \( \Pi_{LL} \) or \( \Pi_{RR} \) because these outcomes are strictly dominated, although calculating these values is straightforward. However, the analysis will provide a clear direction regarding whether the
equilibrium location outcome consists of maximal differentiation, adjacent locations or co-location. We commence by analyzing the case of low informedness. Analysis of the profit functions leads to Proposition 1.

**Proposition 1** co-location is an equilibrium in the first stage of the game when $\phi \in \left( d, \hat{\phi} \right)$ and $\hat{\phi}$ is a positive root of

\[-6d^2\epsilon^2 + 4d^3\epsilon^2 - d\phi - \phi^3\epsilon^2 + 2d\phi^2 + 3d^2\phi + 5d^2\epsilon - 4d^3\epsilon - \phi^2\epsilon
+ 2\phi^3\epsilon - d^2 + d^3 + 9d^2\phi\epsilon^2 + 2d\phi\epsilon + 2d\phi^2\epsilon - 5d\phi^2\epsilon - 11d^2\phi\epsilon\]

The expression for $\hat{\phi}$ is complex. Accordingly, in Figure 1, we illustrate $\hat{\phi}$ in $(\epsilon, \phi)$ space. Figure 1 also shows the impact that a higher level of $d$ has on $\hat{\phi}$. Essentially, higher levels of $d$ create a stronger incentive for one of the firms to defect to an external segment (the threshold is more restrictive).

We now show that co-location in the middle segment is unique in this parameter range. Proposition 1 shows that the best response to a firm that locates in the middle segment is to collocate when $\phi < \hat{\phi}$. Proposition 2 shows that both firms locating externally is infeasible when informedness is low.
Proposition 2  Firms locating in opposite external segments is not an equilibrium if \( \phi < \tilde{\phi} \) where
\[
\tilde{\phi} = \frac{1}{2\epsilon} \left( -d + \epsilon + de + \sqrt{d^2\epsilon^2 + 2de + \epsilon^2 - 10de^2 - 2d^2\epsilon + d^2} \right).
\]

Proposition 2 is based on the condition which makes locating in the middle segment the best response to a firm that locates in an external segment. The proposition, along with the condition under which co-location is an equilibrium, implies Corollary 1 which explains the equilibrium in Stage 1 when \( \phi > \tilde{\phi} \).

Corollary 1 When \( \tilde{\phi} \in \left( \frac{1}{2\epsilon} \left( -d + \epsilon + de + \sqrt{d^2\epsilon^2 + 2de + \epsilon^2 - 10de^2 - 2d^2\epsilon + d^2} \right), 1 \right) \), the unique pure-strategy-in-locations equilibrium is for firms to locate in opposite external segments.

Using Proposition 1, Proposition 2 and Corollary 1, we fully characterize the parameter space for the allowable range of \( \epsilon \) (the size of the external segments). This characterization is summarized in Theorem 1.

Theorem 1 When the local premium is positive \( (d > 0) \), the pure-strategy-in-locations equilibrium is Minimal Differentiation when \( \phi \in \left( d, \tilde{\phi} \right) \), Partial Differentiation when \( \phi \in \left( \tilde{\phi}, \bar{\phi} \right) \) and Maximal Differentiation when \( \phi \in \left( \bar{\phi}, 1 \right) \).

Theorem 1 shows that informedness has a strong impact on the positioning decisions of firms. The findings show that the need to maximize demand is paramount when the level of informedness is low. In these conditions, both firms maximize their profit by locating centrally and generating demand from all three segments. As the level of informedness increases, the fraction of consumers that see advertising from (both) firms increases and co-location means that profits from these consumers are competed away. This provides an incentive for one of the firms to move away from the middle segment to reduce the competitive intensity between the firms. This leads to partial differentiation (adjacent positioning) when informedness is intermediate, \( \phi \in \left( \tilde{\phi}, \bar{\phi} \right) \). Finally, at high levels of informedness, which reflects the standard conditions for firms in extant studies of positioning using spatial models, the firm in the middle has an incentive to move to the opposite external segment. This obtains because with external positioning both firms fully capitalize on the informed consumers in their home segment: consumers in the external segments do not consider the firm located far away to be a feasible alternative. A complete characterization of the equilibrium is shown in Figure 2.
Figure 2: Fully Characterized Equilibrium Zones ($d = .05$)

Note that in the above analysis we focus on cases where $\phi >> d$. We do this because our goal is to focus on the case where the advertising decisions are the primary source of informedness. However, we briefly note that when $\phi$ gets very small, the local awareness premium becomes the dominant mechanism through which consumers are aware of each firm’s product. In this case, minimum differentiation no longer becomes an equilibrium when $\epsilon$ is large because the firms make almost all of their sales in their own segment. As a result, the firms will locate in a different segment than their competitor in order to extract the maximum value out of the local awareness premium.

We now discuss a simplified version of the model where the local premium is absent.

4 The equilibrium when the local awareness premium is absent

A natural question that readers may ask is what happens when the local awareness premium $d = 0$. We now analyze this case where the conditions are identical to those in Section 3 except that $d = 0$. Proposition 1 leads to Proposition 3 which summarizes the equilibrium in this simplified setting.

**Proposition 3** When $\phi \in \left(0, \frac{1}{2-\epsilon}\right)$, the equilibrium is co-location in the middle segment. When $\phi \in \left(\frac{1}{2-\epsilon}, 1\right)$, the equilibrium is partial differentiation (the firms locate in adjacent segments).
In essence, Proposition 3 implies that at low \( \phi \), the firms collocate in M. At levels of informedness greater than \( \frac{1}{2} \), one firm moves to an external segment to create differentiation. However, the middle firm has no incentive to move except at \( \phi = 1 \), in which case the middle firm is indifferent between the middle and the unoccupied external segment. The modified model without the local premium does not generate maximal differentiation as a unique outcome. However, it demonstrates with clarity the impact of informedness on the incentive to differentiate: Firms will differentiate less when the level of informedness is lower. It also shows that minimal differentiation occurs even when firms compete fiercely on price. In other words, the fixed price assumption, originally used by Hotelling to recover minimum differentiation, is unnecessary for minimum differentiation to occur.

5 Extension with High and Low Advertising

In this section, we examine the robustness of our findings and allow the firms to choose between two levels of advertising: high and low. We focus on a market where the Left, Middle and Right segments are of equal size. The objective is to show how the interaction between product differentiation (in terms of location), advertising and pricing changes when firms are able to choose different levels of advertising. First, we show that firms do not always choose to advertise at a high level, even when doing so is costless, because choosing different levels of advertising intensity can reduce price competition. Second, firms may choose lower levels of differentiation in equilibrium because they are able to reduce the level of price competition through their advertising choices and locational differentiation has a cost: when a firm chooses a non-central location, it forgoes the ability to serve customers in one of the external segments. Finally, the overall effect of having two levels of advertising weakly pushes the firms towards less differentiation.

The game proceeds in the same order as before. In step 1, firms choose their locations. In step 2, firms choose their level of advertising. In step 3, firms set prices. Finally, consumers make their purchases. The difference between the extension and base model is that Firm \( i \) chooses \( \phi_i = \phi_L \) or \( \phi_H \) where \( 0 < \phi_L < \phi_H < 1 \). We assume that advertising has a cost \( A\phi_i \) where \( A \) is a constant. We focus on the special case where \( A = 0 \) because the competitive forces that affect positioning decisions emerge straightforwardly when advertising is costless. In order to keep the extension tractable, we also restrict our analysis to the case of \( \epsilon = \frac{1}{3} \) (i.e., each of the 3 segments have the same size).

Similar to Table 1, we consider all positions for firms in the first stage, and focus on pure-strategies in location and advertising equilibria. (Prices are in mixed strategies as in the previous
sections.) As before, both firms collocating in an external segment is strictly dominated by collocating in the middle segment. In addition, we note that when firms maximally differentiate and locate in opposite external segments, choosing \( \phi_L \) for either firm is dominated: the choice \( \phi_H \) allows for a strictly higher captive market and higher guaranteed profit. When there is partial differentiation, both firms choose to advertise at level \( \phi_H \) or the firm located in an external segment may choose to advertise at \( \phi_L \) while the firm in the center advertises at \( \phi_H \) (in the appendix we show that the best response of the centrally located firm to an externally located competitor is always to choose high advertising). When both firms collocate in the central segment, both firms advertise at \( \phi_H \) or one firm advertises at \( \phi_L \) while the other firm advertises at \( \phi_H \). We present an outline of the proof for the extension in the Appendix.

To simplify the conditions under which different outcomes occur, we limit all our analysis to the case where \( \delta < \frac{9}{100} \). This is follows our assumption that \( d \) represents a small boost to local awareness rather than it being a main driver of awareness. We begin by demonstrating that maximum differentiation requires that \( \phi_H \) is high enough. This finding is summarized in Proposition 4.

**Proposition 4** Any equilibrium involving maximum differentiation will also involve both firms advertising at \( \phi_H \). Such an equilibrium occurs if and only if \( d > 0 \) and

1. \( 1 - 2d < \phi_H < 1 - d \)
2. \( 0 < \phi_L < \frac{2d-d^2+6d\phi_H-6d\phi_H-4\phi_H^2}{2d+4\phi_H} \) or \( \frac{\phi_H-d\phi_H+\phi_H^2}{d+2\phi_H} < \phi_L < \phi_H \).

Proposition 4 demonstrates that maximum differentiation is possible when \( \phi_L \) is very low (in which case, low advertising is not appealing). Here, both firms advertise high if they are partially differentiated. Thus, this case parallels the base case with one level of advertising. Alternatively, when \( \phi_L \) is close to \( \phi_H \) (i.e., the low level of advertising is relatively high), the intuition from the base case again applies and the firms maximally differentiate.

In the proof of Proposition 4, we show that levels of \( \phi_L \) in the interval \( \left( \frac{2d-d^2+6d\phi_H-6d\phi_H-4\phi_H^2}{2d+4\phi_H}, \frac{\phi_H-d\phi_H+\phi_H^2}{d+2\phi_H} \right) \) lead to less differentiation because the ability to soften price competition through lower advertising diminishes the need for product differentiation. Note that the condition on \( \phi_H \) in Proposition 4 matches that of Proposition 2 when \( \epsilon = \frac{1}{3} \). The fact that maximum differentiation does not occur for a range of \( \phi_L \) when \( \phi_H \) is high enough immediately leads to Corollary 2.

**Corollary 2** When \( \phi_L > 0 \), the parameter space where maximum differentiation occurs is smaller.
Corollary 2 demonstrates that the ability that firms have to lessen price competition through the modulation of advertising intensity reduces the tendency of firms to maximally differentiate through location choice.

We next consider the equilibrium when \( \phi_H \) is relatively low (\( \phi_L \) is always restricted to being less \( \phi_H \)). This situation is considered in Proposition 5.

**Proposition 5** If \( \phi_H < \frac{1}{2} \) then the equilibrium is for firms to minimally differentiate and for both firms to choose to advertise at level \( \phi_H \).

Proposition 5 shows that when the highest level of awareness is sufficiently low, minimum differentiation (and high advertising) is the equilibrium similar to the base case. In fact, the level of \( \phi_L \) does not affect this finding. Note that Proposition 5 represents sufficient, but not necessary, conditions for this equilibrium to emerge. In fact, we show that minimum differentiation occurs more frequently when there is more than one level of advertising available to firms. This finding is summarized in Proposition 6.

**Proposition 6** When \( \phi_L > 0 \), the parameter space where co-location is an equilibrium is weakly increased.

Together, Corollary 2 and Proposition 6 imply that having more than one level of advertising available leads to similar or less differentiation compared to the base case where firms can only choose a single level of advertising. Thus, the ability that firms have to reduce competition through the modulation of media intensity reduces the need to differentiate through location.

The findings listed above provide important insight regarding the relationship between \( \phi_H \) and the level of differentiation that emerges in equilibrium. The findings also show that having two levels of advertising often leads to less differentiation in equilibrium compared to the case of a single level of advertising. However, a comprehensive picture of how the locational equilibrium changes as a function of \( \phi_H \) and \( \phi_L \) is complex. Accordingly, we present a complete picture of the equilibrium outcomes graphically. To do this, we define \( N \) such that as \( \phi_L = \frac{\phi_H}{N} \). This allows us to plot the outcomes as a function of \( \phi_H \) and \( N \). We start by examining the equilibrium outcomes when the local awareness premium is zero (\( d = 0 \)). The equilibrium outcomes are shown in Figure 3.

We divide the parameter space into three zones. First we consider Zone 1 where \( N < \frac{3}{2} \). In this case, \( \phi_L \) is relatively large and close to \( \phi_H \); here, low advertising is an effective tool to soften price competition and endogenously create "differentiation" between firms. As a result, there
Figure 3: Equilibrium Zones as Function of $\phi_H$ and $N$ ($d = 0$)
exists a region above $\phi_H = 0.6$ where the firms co-locate even though they would exhibit partial differentiation if the only choice available was to advertise at $\phi_H$. We also observe that when $\phi_H$ is high (close to 1), the equilibrium involves partial differentiation but the external firm chooses to advertise at $\phi_L$. This further relaxes price competition beyond the relief obtained by not being located in the same segment.

Next, we consider Zone 2, where $\frac{3}{2} < N < 3$. In this region, $\phi_L$ is much smaller than $\phi_H$. However, low advertising can still be better than choosing differentiated locations if $\phi_H$ is sufficiently high (close to 1). As a result, firms choose to minimally differentiate at very high levels of $\phi_H$. Conversely, for all $\phi_H < \frac{3}{4}$ the firms differentiate and choose their advertising levels as if $\phi_H$ were the only advertising choice (similar to the base case). In essence, the firms minimally differentiate either when $\phi_H$ is either very high or very low; however, when minimum differentiation is observed and $\phi_H$ is high, the firms reduce the intensity of competition through the choice of advertising intensity (one firm choose $\phi_H$ and the other chooses $\phi_L$). In fact, if one holds $\phi_H$ constant at a level higher than 0.6, we see that firms partially differentiate when $N$ is low, they then switch to minimal differentiation when $N$ is at a moderate level, and then back to partial differentiation as $N$ becomes very high. This is explained by considering the shape of the profit functions of a firm that responds to a centrally located competitor that chooses $\phi_H$. As shown in Figure 4 (for $\phi_H = 0.8$), the profits of being centrally located and choosing low advertising rise and then fall.

Finally, we come to Zone 3, i.e. where $N > 3$. In this case, $\phi_L$ is much smaller than $\phi_H$. In these conditions, a firm never chooses low advertising and the equilibrium is identical to outcomes described in Proposition 3 when $\epsilon = \frac{1}{3}$.

We now illustrate the equilibrium outcomes when the local awareness premium is greater than zero. In Figure 5, we present the equilibrium outcomes when $d = 0.05$. Not unexpectedly, the pattern of outcomes follows similar patterns to those observed in Figure 3 when $d = 0$. First, the level of differentiation is weakly less than the level of differentiation observed when the only advertising choice is $\phi_H$. However, at high levels of $\phi_H$, maximum differentiation obtains for both high and low values of $N$. When $N$ is close to 1, firms cannot soften the intensity of competition through advertising because $\phi_L$ is almost the same as $\phi_H$; as a result, maximum differentiation emerges. Similarly, when $N$ is large, $\phi_L$ is very small. Here, the loss in demand by choosing $\phi_L$ is excessive; this makes $\phi_L$ unattractive as a tool to reduce competition. At the extreme, when $N \gtrsim 2.9$, the equilibrium level of differentiation depends entirely on $\phi_H$: minimal, partial and finally, maximal differentiation occur as $\phi_H$ increases.
Figure 4: Profits of Firm that Responds to Centrally Located Competitor that chooses $\phi_H$ where $\phi_H = 0.8$
Figure 5: Equilibrium Zones as Function of $\phi_H$ and $N$ ($d = 0.05$)
Note that there exists a moderate range of $N$ where both minimum and maximum differentiation emerge simultaneously. In this zone, partial differentiation is unappealing for both firms because the optimal advertising choice when firms are partially differentiated is high and this means intense price competition. On the one hand, if the firms are maximally differentiated, neither firm want to move to the center. On the other hand, if the firms are collocated, one firm chooses $\phi_L$ to soften price competition. Here, the need to differentiate is attenuated because the firms use advertising to create differentiation; hence, neither firm wishes to relocate to the center segment.

When $\phi_L$ is large but not too close to $\phi_H$, (i.e. $N \in (\approx 1.02, \approx 1.7)$), and $\phi_H$ is high, maximum differentiation disappears as an outcome because there is a profit increasing defection to the central segment. In this range, when a firm defects to the central segment, the externally-located competitor’s best response is to choose low advertising; low advertising reduces the intensity of price competition and leads to higher profit for the external firm. However, it also increases the profits of the centrally located firm which is what makes the defection attractive. Thus, having the option to choose low advertising (when $\phi_L$ is still relatively high) leads to less differentiation in equilibrium. This obtains because firms choose advertising strategically to manage price competition in the third stage of the game.

In conclusion, the extension demonstrates four key findings. First, maximal differentiation only occurs at high $\phi_H$ (this echoes the findings of the base model) and reinforces the finding that maximal differentiation is attractive when most consumers in the market are comparison shoppers. Second, minimum differentiation is the unique outcome of the game when the highest level of advertising ($\phi_H$), is low. Here, the extension reinforces the insight of the base model and implies that firms will choose locations to maximize potential demand when the intensity of competition is relatively low. Third, firms sometimes choose lower levels of advertising even when there is no incremental cost to advertise at higher intensity. This underlines the primary insight from the extension, advertising intensity can be used to soften price competition and endogenously creates differentiation between firms. Finally, the availability of choice in terms of advertising intensity leads to less locational differentiation on average compared to situations when only one level of advertising is available. This occurs because the firm using high advertising bullies its competitor into choosing a lower level of advertising: the competitor wants to avoid the intense price competition that would result were it to respond with a high level of advertising and the aggressive high advertiser reaps the benefits.
6 Conclusion

This paper examines how advertising shifts the level of product differentiation chosen by firms. In the traditional product-differentiation literature, we observe that firms differentiate in order to soften price competition, even though such a move sacrifices positioning at a central location which maximizes access to consumers. Using an analysis where firms can choose to advertise at fixed levels, we first demonstrate that if awareness is low then the balance of this trade-off is shifted and firms minimally differentiate. This occurs because both firms want to locate at the optimal central location and the imperfect informedness of consumers is a differentiator that softens price competition. In the extension, we extend the model to allow different levels of advertising to be chosen by the firms in the second stage of the game. The extension confirms the robustness of the base model: higher levels of informedness cause firms to move away from each other in terms of location choice. However, the extension also reveals the power of media intensity to create differentiation between firms all by itself. When low levels of advertising are sufficiently attractive, the need of a centrally-located firm to reposition in order to reduce competitive intensity is reduced. This means the likelihood of observing high levels of differentiation is lower. In a nutshell, the extension shows that lower media intensity endogenously creates differentiation. This finding is unique to the model which allows different levels of advertising to be chosen: it stands in contrast to the results obtained in a framework where the informedness levels are fixed.
References


Appendix

Proof of Lemma 1

The objective function of Firm $i$ is $\pi_i = p(\phi + d)\epsilon + p\phi(1 - \phi)(1 - 2\epsilon) + p\phi^2(1 - 2\epsilon)(1 - F(p))$. At $p = 1$, we know that Firm $i$ captures no consumers from the middle segment who saw advertising from Firm $j$. Hence, the guaranteed profit is $\pi = (\phi + d)\epsilon + \phi(1 - \phi)(1 - 2\epsilon)$. We now set the objective function equal to the guaranteed profit and derive $F(p) = \frac{\phi + \epsilon - \phi^2 - \phi\epsilon + 2\phi^2\epsilon - d\phi + p\phi\epsilon}{p\phi^2(2\epsilon - 1)}$. By setting $F(p) = 0$, we derive $p = \frac{\phi + \epsilon - \phi^2 - \phi\epsilon + 2\phi^2\epsilon}{\phi + \epsilon - \phi\epsilon}$.

Proof of Lemma 2

The objective function of Firm $i$ is $\pi_i = 2\epsilon(p\phi(1 - \phi) + p\phi^2(1 - F(p))) + (1 - 2\epsilon)(p(\phi + d)(1 - (\phi + d)) + p(\phi + d)^2(1 - F(p)))$. At $p = 1$, we know that Firm $i$ captures no consumers from the 3 segments that also saw advertising from Firm $j$. Hence, the guaranteed profit is $\pi = d + \phi - 2d\phi - 2d\epsilon - \phi^2 + 2d^2\epsilon - d^2 + 4d\phi\epsilon$. We now set the objective function equal to the guaranteed profit and derive $F(p) = \frac{d + \phi - 2d\phi - 2d\epsilon - \phi^2 + 2d^2\epsilon - d^2 + 2d\epsilon + 4d\phi\epsilon}{p - 2d\phi - \phi^2 - d^2 + 2d\epsilon + 4d\phi\epsilon}$. By setting $F(p) = 0$, we derive $p = \frac{d + \phi - 2d\phi - 2d\epsilon - \phi^2 + 2d^2\epsilon - d^2 + 4d\phi\epsilon}{d + \phi - 2d\epsilon}$.

Proof of Lemma 3

As noted in Section 3.3, Firm $M$ has a larger captive segment and all consumers have the same reservation price. Following Varian (1980), this implies that Firm $M$ will have a mass point at 1. The objective function of Firm $M$ is $\pi_M = p\phi\epsilon + p(\phi + d)(1 - \phi)(1 - 2\epsilon) + p(\phi + d)^2(1 - F_E(p)) + p\phi(1 - (\phi + d))\epsilon + p\phi(\phi + d)\epsilon(1 - F_E(p))$. We know that at $p = 1$, Firm $M$ captures zero demand from the competitive segments. This implies that $\pi_M = d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon + d\phi\epsilon$. We now set the objective function equal to the guaranteed profit and derive $F_E(p) = \frac{d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon - d\phi + 2d\epsilon + d\phi\epsilon}{p\phi(\epsilon - 1)(d + \phi)}$. By setting $F_E(p) = 0$, we obtain the lowest price in the support $p = \frac{d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon + d\phi\epsilon}{d + \phi - 2d\epsilon}$. We know that at $p = \frac{d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon + d\phi\epsilon}{d + \phi - 2d\epsilon}$, $F_M(p) = 0$. Substituting $p$ into Firm $E$’s profit function, we obtain $\pi_L = (d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon + d\phi\epsilon)\frac{\phi + \epsilon - \phi\epsilon}{d + \phi - 2d\epsilon}$. This expression leads straightforwardly to $F_M(p) = \frac{(d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon + d\phi\epsilon)\frac{\phi + \epsilon - \phi\epsilon}{d + \phi - 2d\epsilon}}{p\phi(\epsilon - 1)(d + \phi)}$. Firm $M$ will not choose a price higher than 1. Hence, $F_M(1) = \frac{\phi + \epsilon - \phi\epsilon}{d + \phi - 2d\epsilon}$ implies that Firm $M$ has a mass point at 1 with probability $1 - \frac{\phi + \epsilon - \phi\epsilon}{d + \phi - 2d\epsilon}$.

Proof of Proposition 1

This proof entails analyzing the difference between the profits when the firms are collocated and the profits that would be earned by a firm that defects to an external segment. $\pi_{co-location} = d + \phi - 2d\phi - 2d\epsilon - \phi^2 + 2d^2\epsilon - d^2 + 4d\phi\epsilon$ and $\pi_{adjacent}^E = (d + \phi - d\phi - 2d\epsilon - \phi^2 + 2\phi^2\epsilon + d\phi\epsilon)\frac{\phi + \epsilon - \phi\epsilon}{d + \phi - 2d\epsilon}$. This implies that $\pi_{co-location} - \pi_{adjacent}^E = \ldots$. 

a
\[-6d^2c^2+4d^2c^2-d\phi^2+2d^2c^2+d^2c^2+6d^2\phi^2+5d^2\phi+5d^2\phi^2-4d^2\phi^2+2d^2\phi^2+d^2+9d^2\phi^2+2d\phi^2=5d^2\phi^2-11d^2\phi=0.\] This is a cubic equation in $\phi$ with three real roots. For $\epsilon < \frac{1-d}{3-2d}$, there is one positive root. This defines $\phi_1$, the threshold above which the difference is negative. The expression is too long for presentation purposes. When $\epsilon > \frac{1-d}{3-2d}$, there is a second positive root $\phi_2 = \frac{1}{2}d \left[ 3d^2+2d-11d+2d^2+9d^2+\sqrt{2d-5d^2+63d^2+31d^2-118d^2-118d^2+31d^2-22d^2+20d^2-16d^2+14d^2+82d^2-14d^2+11d^2+36d^2+d^2+1-1 \right]$. This implies that the difference, $\pi_{co-location} - \pi_{adjacent}^E$, is negative when $d > 0$ and $\phi < \phi_2$. In $(\epsilon, \phi)$ space, $\phi_2$ intersects the x-axis at $\epsilon = \frac{1-d}{3-2d}$ and reaches its maximum at $\epsilon = \frac{1}{2}$ (the maximum allowable value of $\epsilon$). At $\epsilon = \frac{1}{2}$, the second root equals $d$. This implies that when $\epsilon < \frac{1-d}{3-2d}$ and $\phi < \phi_2$, co-location is an equilibrium. When $\epsilon \in \left( \frac{1-d}{3-2d}, \frac{1}{2} \right)$ and $\phi \in \left( d, \phi_2 \right)$, co-location is an equilibrium.

As noted in the model, the objective of the analysis is to study the impact of informedness on positioning i.e. $\phi >> d$.

**Proof of Proposition 2**

To show that collocation is the unique equilibrium when $\phi < \phi_2$, we show that firms locating in opposite external segments is not stable. Let $\Delta = \pi_M - \pi_i^{external} (i = 1, 2)$. Using the subgame equilibrium profits from Lemmas 1 and 3, $\Delta = \left( d + \phi - d\phi - 2d\phi - \phi^2 + d\phi \right) - \left( \left( \phi + d \right) \epsilon + \phi \left( 1 - \phi \right) \left( 1 - 2\epsilon \right) \right) = d - d\phi + 3d\epsilon + \phi - \phi^2 + d\phi$. This is downward facing parabola with 2 roots: $\phi = \frac{1}{2\epsilon} \left( -d + \epsilon + d\epsilon + \sqrt{d^2\epsilon^2 + 2d\epsilon + \epsilon^2 - 10d\epsilon^2 - 2d^2\epsilon + d^2} \right)$ and $\phi = \frac{1}{2\epsilon} \left( -d - \epsilon - d\epsilon + \sqrt{d^2\epsilon^2 + 2d\epsilon + \epsilon^2 - 10d\epsilon^2 - 2d^2\epsilon + d^2} \right)$. The relevant root is the first root implying that $\Delta = \pi_M - \pi_i^{external} > 0$ for all $\phi < \phi_2 = \frac{1}{2\epsilon} \left( -d + \epsilon + d\epsilon + \sqrt{d^2\epsilon^2 + 2d\epsilon + \epsilon^2 - 10d\epsilon^2 - 2d^2\epsilon + d^2} \right)$. Note that when $d = 0$, $\phi = 1$.

**Proof of Corollary 1**

By virtue of Proposition 2, $\Delta = \pi_M - \pi_i^{external} (i = 1, 2) < 0$ when $\phi > \phi_2$. This implies that firms locating in opposite external segments is an equilibrium. Because $\phi > \phi_2$ in this region, locating in opposite external segments is the unique equilibrium.

**Proof of Proposition 4**

We first consider the advertising level the firms will choose when they are maximally differentiated. If both firm advertises high, following Narasimhan (1988), they will earn their guaranteed profit of $(\phi_H - d)/3 + \phi_H (1 - \phi_H)/3 = \frac{d+2\phi_H - \phi_H^2}{3}$. If a firm deviates to low advertising, the other firm will obtain their guaranteed profit of $\frac{d+2\phi_H - \phi_H^2}{3}$. The lowest price that firm would charge would be the price that sets this profit equal to $p_L ((\phi_H + d)/3 + \phi_H / 3)$, which is solved when $p_L = \frac{d+2\phi_H - \phi_H^2}{d+2\phi_H} L \left( (\phi_H + d)/3 + \phi_H / 3 \right) = \frac{d+2\phi_H}{3\phi_H + d} \left( (d+2\phi_H - \phi_H^2) \phi_H \right)$. One can verify that this deviation leads to reduced profits. If both firms advertise low, profits would be $\frac{d+2\phi_L - \phi_L^2}{3}$,
which is always lower than \( \frac{(d+2\phi_f)(d+2\phi_H-\phi_H\phi_f)}{3(2\phi_H+d)} \). Thus, under maximum differentiation \( \phi_H \) is a dominant strategy.

We next consider when firms will choose to maximally differentiate. To analyze that, we need to know what the equilibrium would be if one firm located in the center while the other firm stayed at an external segment. It can be shown that advertising at \( \phi_H \) is a dominant strategy for the firm at the center. Therefore, there are two possible outcomes. Both firms can advertise high, in which case the firm at the center gets it is straightforward to show that the profits earned by the central firm are \( -\frac{1}{3} \left( -d - 3\phi_H + 2d\phi_f + 2\phi_f^2 \right) \) and for the external firm \( -\frac{1}{3} \left( d+2\phi_f \right) \left( -d - 3\phi_H - 2d\phi_f + 2\phi_f^2 \right) \).

When the external firm chooses \( \phi_f \), straightforward calculations show the profits earned by the central firm are \( \frac{3\phi_H-d\phi_f-2d\phi_f\phi_f+d-d\phi_f}{3} \) and for the external firm \( \frac{d+2\phi_f \left( d+3\phi_H - 2d\phi_H - 2\phi_f^2 \right)}{2d+4\phi_f} \). The external firm will choose to advertise at \( \phi_f \) if \( \phi_H > \frac{3-4d+\sqrt{9-8d+8d^2}}{8} \) and \( \phi_f > \frac{2d-d^2+6\phi_H-6d\phi_f-4\phi_f^2}{2d+4\phi_f} \). Note that this assumes that \( d < \frac{9}{10^6} \), as is assumed in this section.

We then compare the profits under each outcome. If \( \phi_H < \frac{3-4d+\sqrt{9-8d+8d^2}}{8} \) or \( \phi_f < \frac{2d-d^2+6\phi_H-6d\phi_f-4\phi_f^2}{2d+4\phi_f} \), both firms will advertise at \( \phi_H \). In this case, we can rely on the results in Corollary 1, which states that maximum differentiation occurs if and only if \( \phi_H > 1 - 2d \) (which itself is greater than \( \frac{3-4d+\sqrt{9-8d+8d^2}}{8} \)), which is the first part of Proposition 4, and occurs whenever \( \phi_f < \frac{2d-d^2+6\phi_H-6d\phi_f-4\phi_f^2}{2d+4\phi_f} \).

On the other hand, when the external firm chooses to advertise at \( \phi_f \), maximum differentiation will occur if and only if \( \frac{3\phi_H-d\phi_f-2d\phi_f\phi_f+d-d\phi_f}{3} < \frac{d+2\phi_f \left( d+3\phi_H - 2d\phi_H - 2\phi_f^2 \right)}{2d+4\phi_f} \), which is true whenever \( \phi_H < 1 - d \) and \( \phi_f > \frac{\phi_H-d\phi_f+\phi_f^2}{d+2\phi_f} \). One can readily confirm that \( \frac{\phi_f-d\phi_f+\phi_f^2}{d+2\phi_f} > \frac{2d-d^2+6\phi_H-6d\phi_f-4\phi_f^2}{2d+4\phi_f} \) for the range of \( \phi_H \) and \( d \) we consider, ensuring that the external firm truly advertises at \( \phi_f \).

**Proof of Proposition 5**

When \( \phi_H < \frac{1}{2} \), firms will always advertise at \( \phi_H \) regardless of their locations. The proof of Proposition 4 above showed that firms will always advertise at \( \phi_H \) under maximum differentiation, and that under partial differentiation both firms will advertise at \( \phi_H \) if \( \phi_H < \frac{3-4d+\sqrt{9-8d+8d^2}}{8} \). It is easy to confirm that \( \frac{3}{8} < \frac{3-4d+\sqrt{9-8d+8d^2}}{8} \). If both firms collocate and both firms locate at the center they will both earn profits of \( \frac{d^2+3\phi_H-3\phi_H\phi_f+d-d\phi_f}{3d+3\phi_f} \). Thus, the profits for the firm with low advertising are \( \frac{d^2+3\phi_H-3\phi_f\phi_f+d-d\phi_f}{3d+3\phi_f} \). If both firms advertise at \( \phi_f \) they both earn their guaranteed profits of \( \frac{d+3\phi_f}{3} \). We then confirm that \( \frac{d^2-2d\phi_f+(1-\phi_f)\phi_f}{3} > \frac{(d+3\phi_f)(d^2+3\phi_H-3\phi_H\phi_f+d-d\phi_f-d\phi_f)}{2(d+3\phi_f)} \). If both firms advertise high in this condition.

Given that firms always advertise at \( \phi_H \) regardless of location, we can use Proposition 1. We set \( \epsilon = \frac{1}{3} \) and determine \( \hat{\phi}_1 = \frac{3}{5} > \frac{1}{2} \). Thus, minimum differentiation is always an equilibrium when \( \phi_H < \frac{1}{2} \).
Proof of Proposition 6

First note that both firms will advertise at $\phi_H$ under partial differentiation if $\phi_H < \frac{3-4d+\sqrt{9-8d+8d^2}}{8}$, where this threshold is always greater than $\frac{3}{5}$ given our restrictions on $d$. Thus, we need to compare the profits of collation with those of the external firm under partial differentiation when both firms advertise high. Using calculations from the proof of Proposition 5 we can confirm that with collation both firms will advertise high when $\frac{d-d^2-2d\phi_H+(1-\phi_H)\phi_H}{3}>\frac{(d+3\phi_H)(-d^2+3\phi_H-3\phi_H\phi_L+d-d\phi_H-d\phi_L)}{2(d+3\phi_H)}$, which happens whenever $\phi_L < \frac{3d-4d^2+9\phi_H-9d\phi_H-9\phi_H^2}{3d+9\phi_H}$. In this case, the tradeoffs are the same as in Proposition 1.

When $\phi_L > \frac{3d-4d^2+9\phi_H-9d\phi_H-9\phi_H^2}{3d+9\phi_H}$, one firm will advertise high and one firm will advertise low when the firms collate. Collation will be an equilibrium as long as that firm does not deviate to the external segment. Using the calculations above, this is a tradeoff of $\frac{(d+3\phi_L)(-d^2+3\phi_H-3\phi_H\phi_L+d-d\phi_H-d\phi_L)}{2(d+3\phi_H)} > \frac{1}{3} \frac{(d+2\phi_H)(d+3\phi_H-2d\phi_H-2\phi_H^2)}{d+3\phi_H}$. This inequality always holds whenever $\phi_H$ is less than the first positive root from Proposition 1. However, it also holds when $\phi_H$ is above the first positive root from low informedness and $\phi_H < \frac{4}{5}$ (this is a sufficient condition to provide nicer bounds) and

$$\phi_L < \frac{3d-4d^2+9\phi_H-6d\phi_H+\sqrt{9d^2-24d^4+4d^4+54d\phi_H-132d^2\phi_H+24d^4\phi_H+81\phi_H^2-252d\phi_H^2+144d^2\phi_H^2-216\phi_H^3+264d\phi_H^3+144d^2\phi_H^3}}{6(d+3\phi_H)}.$$ 

This is a sufficient set of conditions.