

## TOWARD A THEORY OF BANK LOAN COMMITMENTS\*

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The characteristics of fixed and variable rate bank loan commitments are analyzed in a contingent-claims framework, and valuation expressions are derived for these commitments. The valuation expressions are used to present estimates of the impact of interest rate uncertainty on the liability assumed by a bank issuing loan commitments. Finally, a simple, two-period, asymmetric information model is employed to explain the recent trend among bankers to substitute variable rate commitments for their fixed rate counterparts.

### 1. Introduction

The unprecedented volatility of interest rates in the past decade has established a highly uncertain environment for financial institutions [see Greenbaum (1975)] and has induced significant changes in borrowing and lending practices. Notable among these changes was the striking reduction in the volume of fixed rate loans and loan commitments offered by commercial banks. This movement of commercial banks from fixed rate to variable rate loans and commitments has been documented by Crane and White (1972) and Miller (1975).

This paper focuses on the optimal adaptive behavior of banks, during periods of volatile interest rates, with respect to the loan commitments they sell. Two principal issues are addressed. First, theoretical valuation formulae for fixed and variable rate loan commitments are developed. Although these commitments are an integral part of commercial banking in the U.S., their valuation is only vaguely understood and has received scant attention in the literature. Second, a partial equilibrium model is constructed in order to derive sufficient conditions under which it is optimal for a profit-maximizing bank to substitute variable rate loan commitments for fixed rate loan commitments.

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The paper is organized in five sections. Section 2 describes commercial bank loan commitments and briefly reviews the related literature. The section also sketches a distribution-free model of loan commitment valuation which shows that the value of a loan commitment is an increasing function of the uncertainty in the interest rate environment. In section 3, distributional assumptions are invoked in order to solve the valuation problem. Valuation expressions are then used to estimate loan commitment values for a range of prime rate variances. To provide a basis for comparison, monthly data are used to compute the actual instantaneous variance in the prime over the periods 1933–1947, 1955, 1967 and 1974. The penultimate section presents a two-period, asymmetric-information model to explain the eschewal of fixed rate loan commitments by banks. This latter analysis owes a great deal to Stiglitz and Weiss (1981) who justify endogenous price rigidities, leading to credit rationing by banks. Section 5 concludes.

## 2. Commercial bank loan commitments

To satisfy uncertain future borrowing needs, potential borrowers often seek forward commitments. These loan commitments, sometimes referred to as credit lines or authorizations, are agreements which stipulate a maximum borrowing amount for the customer, the length of time over which the agreement remains in force, the terms of borrowing, and a consideration for the commitment (fee).<sup>1</sup> Bank loan commitment can be classified as either *fixed rate* or *variable rate*. A fixed rate loan commitment obligates the bank to lend a stipulated amount of funds during a predetermined time interval at a fixed rate of interest at the option of the borrower. This type of commitment provides the borrower insurance against variations in its specific default (credit) risk premium as well as the market (prime) rate of interest.<sup>2</sup> A variable rate (or fixed formula) loan commitment is the same except that the rate of interest charged is typically quoted as the prime rate prevailing at the time the customer actually borrows plus a fixed add-on.<sup>3</sup> This commitment provides the customer with the more limited protection against variation in the mark-up, i.e., insurance against deterioration in the borrower's credit rating, but leaves the borrower exposed to market interest rate variations.

<sup>1</sup>Most bank loan commitments include clauses whereby the bank can refuse to loan under the commitment if the borrower's financial condition suffers material deterioration. These clauses bring into question the legal force of most commitments. However, for the remainder of the paper it will be assumed that the bank always honors its commitment.

<sup>2</sup>Credit risk is reflected in the add-on to the prime rate of interest that banks employ to arrive at the customer's borrowing rate.

<sup>3</sup>This is not the only type of variable rate commitment issued by commercial banks. For example, the borrowing rate is sometimes specified as the future (random) prime *times* a *fixed multiple*. Although the formal analysis in subsequent sections considers only the prime-plus-a-fixed-add-on type variable rate commitment, all the results can also be obtained with the fixed-multiple type commitment.

Since both fixed and variable rate commitments guarantee availability of funds, they also guarantee against changes in the bank's credit policy that might result in the potential borrower being rationed [see Deshmukh, Greenbaum and Kanatas (1981a) for a discussion of bank lending policy].

The literature on loan commitments is not extensive. Campbell (1978) analyzes the supply and demand characteristics of the loan commitments market in an expected-utility-maximization framework but does not address the problem of valuing commitments. Lintner (1976) appears to be the first attempt to develop a rigorous theory of forward credit markets. Lintner explains how expected values and variances of future market rates affect the volume of and rates on commitments. The analysis proceeds within the standard mean-variance model of portfolio theory. Some of Lintner's hypotheses have been tested empirically by Lintner, Piper and Fortune (1978) using life insurance company data. Horan (1980) develops an alternative model by modifying Lintner's (1976) approach. Thakor, Greenbaum and Hong (1981) develop a model for valuing bank loan commitments within the framework of the now-familiar Black and Scholes (1973) methodology. However, their model applies only to variable rate loan commitments.

With a loan commitment, the customer has the option of borrowing any amount up to the maximum committed by the bank. This implies an isomorphic correspondence between loan commitments and common stock put options. This property of loan commitments is now exploited to construct the valuation model.

Consider first a fixed rate bank loan commitment made at time  $t=0$  to lend up to an amount  $L$  at time  $t=T$ .<sup>4</sup> The loan, if taken, will mature at time  $\tau$  [the loan's term-to-maturity is  $(\tau - T)$ ] and the instantaneous interest rate on the loan will be a fixed number,  $R_F$ , mutually agreed upon by the bank and the borrower at  $t=0$ . Let  $R_{pt} + K_t$  be the instantaneous rate of interest the bank would charge on this same loan in the absence of a commitment, where  $R_{pt}$  is the prevailing prime interest rate at time  $t$  and  $K_t$  is an add-on (expressed in the same units as  $R_{pt}$ ) which reflects the perceived risk of default (the borrower's credit rating) and perhaps other borrower attributes, such as demand elasticity.

Temporal uncertainty in  $R_{pt}$  and  $K_t$  implies that  $R_{pt} + K_t$  is a stochastic variable. If at  $t=T$ ,  $R_{pT} + K_T$  exceeds  $R_F$ , the customer will exercise the loan commitment. If, on the other hand,  $R_{pT} + K_T$  is lower than  $R_F$ , the customer will utilize alternative means of financing available.<sup>5</sup> If the bank's customer exercises its option, the bank is required to purchase a claim against the

<sup>4</sup>Given the option to borrow any amount up to  $L$ , borrowers often exercise only a fraction of the committed funds. This is called the partial take-down phenomenon. For simplicity, however, this complication is ignored here by assuming that the borrower either borrows  $L$  or nothing.

<sup>5</sup>It is assumed that in the absence of a commitment the customer can borrow at  $R_{pT} + K_T$  from any bank.

customer for the agreed upon price of  $L > X_T$ , where

$$X_T = L \exp[(R_F - R_{pT} - K_T)(\tau - T)] \quad (1)$$

is the value of the claim at  $t = T$ . The value of the fixed rate commitment at  $t = T$  is

$$\begin{aligned} &0 && \text{if } X_T > L \text{ and the option is not exercised,} \\ &L - X_T && \text{if } X_T \leq L \text{ and the option is exercised.} \end{aligned} \quad (2)$$

Hence a loan commitment permits the borrower to sell a risky security (the borrower's indebtedness) to the option writer (bank) at some specified future date(s) and price. The commitment may be viewed as a put option with a striking price equal to the face value of the commitment,  $L$ . If we denote  $F(X_t, t)$  as the value of the fixed rate commitment at time  $t$ , then in a competitive loan commitment market the option will be sold by the bank at  $t = 0$  for  $F(X_0, 0)$  and this option will mature at  $t = T$ . The underlying state variable here ( $X_t$ ) is the market value of a debt contract issued by the borrower. The risk of the commitment seller (the bank) arises primarily from the stochastic nature of  $R_{pT}$  and  $K_T$  — if  $R_{pT} + K_T$  exceeds  $R_F$ , the bank sustains a loss of  $L - X_T$ . Clearly, the borrower is fully insured against those states of nature in which  $R_{pT} + K_T$  is higher than  $R_F$ , but is not precluded from accessing lower cost sources of financing if  $R_{pT} + K_T$  is lower than  $R_F$ .

In general, using arbitrage considerations alone (without invoking preferences) the value of the fixed rate commitment can be expressed as

$$F(X_0, 0) = \int F(X_T, T) dP^*, \quad (3)$$

where  $P^*$  is an equivalent martingale measure obtained as a unique extension of the probability measure  $P$  defined on an appropriate probability space [see Harrison and Kreps (1979)]. Of course, the valuation is considerably simplified if one assumes universal risk neutrality. In that case

$$F(X_0, 0) = \exp[-rT] \int F(X_T, T) dS^*(X_T | X_0), \quad (3')$$

where  $S^*(\cdot | \cdot)$  is the distribution function of  $X_T$  conditional on  $X_0$ , and  $r$  is the instantaneous riskless rate of interest.

The valuation model discussed above employs numerous analytical simplifications. The most important are listed below.

- (1) The guaranteed-source-of-funds value of the commitment has not been explicitly taken into account, i.e., the value of the insurance that the commitment owner will not be rationed is ignored. It appears difficult to treat this problem analytically.

- (2) Transactions costs are ignored. This is not a significant deviation if the transactions costs involved in searching for a commitment seller (bank) now are equal (or deterministically related) to the costs involved in searching for a lender in future periods. The problem arises when a commitment permits reduced searching. For instance, for commitments with relatively short maturities (related to predictably frequent or seasonal credit needs), savings in transactions costs may be the major motivation for purchasing a commitment. Incorporating these costs into the valuation expression is not particularly difficult — the case of deterministic transactions costs is obviously simple; stochastic transactions costs can also be accommodated, but at the expense of adding further complexity to the valuation expressions.
- (3) The partial takedown phenomenon has been assumed away. If the takedown fraction is known for sure ex-ante, there is no problem. But unpredictable takedown behavior introduces intractable complications for valuation.
- (4) Finally, loan commitments are typically exercised over a *pre-specified* time period rather than at a single point in time. This makes the commitment a complicated combination of European and American put options. This aspect is ignored by assuming that all takedowns occur at the end of the commitment period.

As a consequence of the simplifying assumptions, any valuation expression derived from this model will understate the value of the fixed rate commitment.

Consider now a variable rate loan commitment made by the bank at  $t=0$  to lend an amount  $L$  at  $t=T$ . The term-to-maturity of the loan is again  $\tau-T$ , but the instantaneous rate of interest on the loan is now  $R_{pT} + \hat{K}$ , where  $R_{pT}$  is the prime rate prevailing at  $t=T$  and  $\hat{K}$  is a *fixed* add-on determined at  $t=0$ . The risk exposure of the bank (and the value of the commitment to the borrower) inheres in the possibility that between times  $t=0$  and  $t=T$ , the borrower's credit rating might deteriorate or the lender's credit standards may tighten and  $K_T$  might exceed  $\hat{K}$ . Let

$$\begin{aligned}\hat{X}_T &\equiv L \exp [(R_{pT} + \hat{K} - R_{pT} - K_T)(\tau - T)] \\ &= L \exp [(\hat{K} - K_T)(\tau - T)].\end{aligned}\tag{4}$$

Then the value of the variable rate commitment at  $t=T$  is

$$\begin{aligned}0 &\quad \text{if } \hat{X}_T > L \text{ and the option is not exercised,} \\ L - \hat{X}_T &\quad \text{if } \hat{X}_T \leq L \text{ and the option is exercised.}\end{aligned}\tag{5}$$

An expression similar to eq. (3) can now be used to value the variable rate commitment.

Since the fixed rate commitment offers greater insurance than the variable rate commitment, one might surmise that the value of the former should always exceed that of the latter. The following proposition, whose formal proof is available upon request, indicates the contrary.

*Proposition 1. A fixed rate commitment by a bank to lend an amount  $L$  at time  $T$  need not be more valuable than a variable rate commitment to lend the same amount to the same borrower at the same time.*

To see the intuition behind Proposition 1, consider a customer who obtains a fixed rate commitment from bank A and a variable rate commitment from bank B. Suppose the current prime is 18% and the current add-on for this customer is 2%. Further, assume bank A sets  $R_F = 18\% + 2\% = 20\%$  and bank B sets  $\hat{K} = 2\%$ . Then, if at  $t = T$ ,  $R_{pT} = 16\%$  and  $K_T = 3\%$ , the customer's borrowing rate will be 19% in the absence of a commitment, 20% under the fixed rate commitment and 18% under the variable rate commitment. Thus only the variable rate commitment will be exercised. A fixed rate commitment cannot always dominate a variable rate commitment because with a fixed rate commitment the customer faces a *single* constant interest rate,  $R_F$ , and is therefore unable to fully exploit situations in which the market interest rate (the prime) has dropped and the customer's own credit rating has worsened. This suggests the following corollary:

*Corollary C.1. A commitment by a bank to loan an amount  $L$  at  $t = T$  at an interest rate  $R^0 = \min(\hat{R}_p, R_{pT}) + \min(\hat{K}, K_T)$  is more valuable than either a fixed rate commitment with  $R_F = \hat{R}_p + \hat{K}$  or a variable rate commitment with a fixed add-on of  $\hat{K}$ .*

The provision of interest rate insurance is an important economic function of bank loan commitments. This suggests that as future interest rates become more uncertain, the value of loan commitments should rise. This conjecture is formalized in the following proposition with proof in the appendix:<sup>6</sup>

*Proposition 2. As future interest rates become more uncertain and make the value of the customer's indebtedness under the commitment ( $X_T$ ) more risky [in the Rothschild and Stiglitz (1970) sense], the value of the loan commitment increases.*

In proving the above proposition, increased uncertainty in the prime rate is introduced by adding a random variable  $\omega$  with the property,  $E[\exp(-\omega)]$

<sup>6</sup>The proof is an adaptation of Merton's (1973) Theorem 8 proof.

= 1. Since the exponential function is convex, Jensen's inequality implies that  $E(\omega) > 0$ . This means that the increased uncertainty surrounding future interest rates is not mean-preserving. Clearly such an assumption weakens the proposition, but I conjecture that a similar result can be established for the case in which the prime rate becomes more unpredictable through the addition of (mean-preserving) pure white noise. Incidentally, from an empirical standpoint the assumption of an increasing mean and variance for interest rates is consistent with the behavior of interest rates over the last few decades. Of course, since a loan commitment is essentially a put option, Proposition 2 should not come as a surprise — Merton (1973) has proved that as the underlying security becomes more risky, the value of a put option increases. However, as the proof of Proposition 2 indicates, extending Merton's analysis to loan commitments is not obvious. Unlike common stock put options, the values of loan commitments have to be related to the volatility in *interest rates* rather than directly to the uncertainty in the value of the underlying security.

Although, as Proposition 1 indicates, a fixed rate commitment need not be more valuable than a variable rate commitment, Proposition 2 demonstrates that as the prime rate becomes more volatile (and the add-on does not), the value of the fixed rate commitment should rise relative to that of the variable rate commitment. This observation is illustrated in the following section, after the necessary valuation expressions have been derived, with a wide range of values for the variance in the prime rate.

### 3. Fixed and variable rate loan commitment valuation

Although some interesting insights have been gleaned from the general loan commitment valuation model in the previous section, additional distributional assumptions are needed to obtain further results. Throughout it is assumed that the riskless rate of interest, interpretable as the instantaneous rate on a default-free discount bond, is constant.

Suppose, for the moment, that variations in the prime rate are described by the stochastic differential equation

$$dR_p = \mu_1 dt + \sigma_1 d\phi_1 \quad (6)$$

and movements in the add-on,  $K$ , are described by

$$dK = \mu_2 dt + \sigma_2 d\phi_2, \quad (7)$$

where  $\mu_1$ ,  $\mu_2$ ,  $\sigma_1$  and  $\sigma_2$  are constants,  $dt$  is an infinitesimal time increment and  $d\phi_1$  and  $d\phi_2$  are the increments of two Wiener processes. It can then be shown that at the end of any finite time interval  $[0, T]$ , the distribution of

$R_{pT} + K_T$  will be normal with mean  $R_{p_0} + K_0 + (\mu_1 + \mu_2)T$  and variance  $(\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2)T$ , where  $R_{p_t}$  is the value of the prime rate at  $t \in [0, T]$ ,  $K_t$  is the value of the add-on at  $t \in [0, T]$  and  $\rho_{12}$  = instantaneous correlation between the Wiener processes  $d\phi_1$  and  $d\phi_2$ .

Unfortunately, normality is not an appropriate distributional assumption for interest rates since (nominal) rates cannot be negative. To avoid this difficulty and to explicitly recognize that the riskless rate of interest,  $r$ , forms a natural lower bound for  $R_{pT} + K_T$ , I truncate the normal distribution on the left at  $r$ . The density function of this truncated random variable is given in the appendix.<sup>7</sup> [See eq. (A.6).]

This density function is now used, in Propositions 3 and 4, to derive the value of a fixed rate loan commitment.<sup>8</sup>

*Proposition 3.* Define  $Z \equiv X_T/X_0$  and  $\ln \bar{C} \equiv \ln(L/X_0) + (R_F - r)(\tau - T)$ . Then the distribution of  $Z$  is truncated-lognormal [density function given by eq. (A.7) in the appendix], with the truncation on the right at  $\bar{C}$ .

Proposition 4 uses the above result to derive the value of a fixed rate commitment. For analytical convenience universal risk neutrality is assumed.<sup>9</sup> Note however that this assumption can be relaxed (even though neither the customer's indebtedness nor the loan commitment are traded in a secondary market) at the expense of adding further technical details. Valuation of untraded options written on untraded variables is discussed in Garman (1977) and Harrison and Kreps (1979).

*Proposition 4.* The initial (time 0) value of a fixed rate commitment is given by

$$F(X_0, 0) = \left[ L \exp[-rT] N[\Omega] - X_0 \exp\left[\frac{2A + B^2}{2} - rT\right] N[\Omega - v(\tau - T)] \right] / N\left[\frac{\alpha}{v}\right], \quad (8)$$

<sup>7</sup>Truncation is not a commonly used technique. The usual approach in the finance literature is to employ reflecting or absorbing barriers when some variable has to be prevented from taking certain values. This enables one to derive the appropriate first-passage time distributions. However, I am merely interested in stipulating a reasonable terminal distribution with certain properties (like time-increasing variance) for *illustrative* purposes. Thus, truncation is appropriate in this context, although there are some fine technical points (mostly related to probability-mass concentration and differentiability) which I shall ignore in this paper.

<sup>8</sup>Propositions 3 through 6 are stated without proofs, which are available from the author on request.

<sup>9</sup>If one views the loan commitment as nothing more than an insurance policy, the risk neutrality assumption looks awkward, because in a risk neutral world the demand for

where

$$\Omega \equiv (\alpha - R_F + r)/v,$$

$$A \equiv \ln \bar{C} - \alpha(\tau - T),$$

$$B \equiv v(\tau - T),$$

$N[\cdot] \equiv$  standard normal cumulative distribution function,

$$v \equiv \sqrt{(\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2)T}.$$

Consider next the problem of valuing a variable rate loan commitment. Recall that

$$\hat{X}_t \equiv L \exp[(\hat{K} - K_t)(\tau - T)],$$

where  $\hat{X}_t$  is the value of  $\hat{X}$  at  $t \in [0, T]$ ,  $K_t$  is the value of  $K$  at  $t \in [0, T]$ , and  $\hat{K}$  is the fixed add-on.

Since at  $t = T$ , the value of the add-on (a random variable at  $t = 0$ ) cannot be negative, I assume that the density function of  $K_T$  is truncated-normal, with the truncation on the left at zero. Note that eq. (7) implies that at the end of a finite time interval  $[0, T]$  the distribution of  $K_T$  is normal with mean  $K_0 + \mu_2 T$  and variance  $\sigma_2^2 T$ . It is this normal density function that is truncated on the left at zero to yield the appropriate density for  $K_T$ , which is given in the appendix [see eq. (A.8)].

*Proposition 5.* Define  $\hat{Z} \equiv \hat{X}_T/\hat{X}_0$  and  $\ln \hat{C} \equiv \ln(L/\hat{X}_0) + \hat{K}(\tau - T)$ . Then  $\hat{Z}$  follows a truncated-lognormal distribution [density function given by eq. (A.9) in the appendix] with the truncation occurring on the right at  $\hat{C}$ .

This terminal distribution for  $X_T$  is now used to derive the initial value of the variable rate commitment.

*Proposition 6.* The initial value of a variable rate commitment is given by

$$V(\hat{X}_0, 0) = \frac{L \exp[-rT] N[(\hat{\alpha} - \hat{K})/\sigma_2 \sqrt{T}] - \hat{X}_0 \exp[(2\hat{A} + \hat{B}^2)/2 - rT] N[\frac{\hat{C}}{\hat{\alpha}}]}{N(\hat{\alpha}/\sigma_2 \sqrt{T})}, \tag{9}$$

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competitively priced insurance is zero. However, since the commitment is an option, that objection is irrelevant.

where

$$\hat{A} \equiv \ln \hat{C} - \hat{\alpha}(\tau - T),$$

$$\hat{B} \equiv (\tau - T)\sigma_2\sqrt{T},$$

$$\xi \equiv (\hat{\alpha} - \hat{K})/\sigma_2\sqrt{T} - (\tau - T)\sigma_2\sqrt{T},$$

$$\hat{\alpha} \equiv \mu T + K_0.$$

With explicit valuation expressions for fixed and variable rate loan commitments, it is possible to present some estimates of the effect of increased uncertainty about interest rates on the values of loan commitments. Fixing every parameter except  $\sigma_1$ , we will examine the behavior of the value of the fixed rate commitment (relative to the value of the variable rate commitment) as  $\sigma_1$  takes on a range of values. Note that for a *given* truncation, the variance of the truncated random variable is a monotone increasing function of the variance of the corresponding untruncated random variable. Thus, although  $v^2$  is not the variance of (the truncated random variable)  $R_{pT} + K_T$ , the variance of  $R_{pT} + K_T$  increases as  $v^2$  is increased,<sup>10</sup> and higher values of  $\sigma_1$  imply higher prime rate variances.

Consider a variable rate loan commitment with  $L = \$1$ ,  $T = 1$ ,  $\tau = T = 2$ ,  $\mu_2 = 0.01$ ,  $K_0 = 0.02$ ,  $\sigma_2 = 0.01$ , and  $\hat{K} = \hat{\alpha} = \mu_2 T + K_0 = 0.03$ . Let  $r = 0.05$ . With these parameters eq. (9) simplifies to

$$V(\hat{X}_0, 0) = \frac{Le^{-rt}[N[0] - \exp(\hat{B}^2/2)\{1 - N[\tau - T)\sigma_2\sqrt{T}\}]}{N[\hat{K}/\sigma_2\sqrt{T}]}, \quad (10)$$

and  $\hat{V}(X_0, 0) = 0.00752602$ .

Consider now a fixed rate commitment with  $r = 0.05$ ,  $R_{p0} = 0.06$ ,  $\mu_1 = 0.1$ ,  $\rho_{12} = 0.5$ ,  $R_F = \alpha + r = 0.15$  and the remaining parameters the same as for the variable rate commitment. Eq. (8) then simplifies to

$$F(X_0, 0) = \frac{Le^{-rt}[N[0] - \exp(B^2/2)\{1 - N[v(\tau - T)]\}]}{N[\alpha/v]}. \quad (11)$$

Table 1 displays the values of  $F(X_0, 0)$  for  $\sigma_1$  ranging from 0.001 to 0.20. The table also indicates the difference between  $F(X_0, 0)$  and  $V(X_0, 0)$  for each value of  $\sigma_1$ .

<sup>10</sup>Numerical techniques can be easily employed to obtain the mean and variance of a truncated random variable from the mean and the variance of the corresponding untruncated variable and vice-versa. In presenting loan commitment value estimates, only the assumed values of  $\sigma_1$ ,  $\sigma_2$ ,  $\rho_{12}$  etc. will be stated — the interested reader can compute the implied values for the corresponding truncated variables.

Table 1  
Loan commitment values.

$\sigma_1$	$v$	$F(X_0, 0)$	$F(X_0, 0) - V(\hat{X}_0, 0)$
0.001	0.0105357	0.00791368	0.00038766
0.005	0.0132288	0.00992164	0.00239562
0.01	0.01732051	0.01290307	0.00537705
0.05	0.05567764	0.04102406	0.03349804
0.10	0.10535654	0.08518662	0.0776606
0.15	0.1552417	0.13262263	0.12509661
0.20	0.2051828	0.17924003	0.17171401

The differential impact of an increasingly volatile prime interest rate on the bank's liability under fixed and variable rate commitments is clear. For example, with  $L = \$10$  million and  $\sigma_1 = 0.20$ , a fixed rate commitment is \$1.7 million more valuable (or more costly) than a variable rate commitment! The reason is simple — since the fixed rate commitment provides the customer with insurance against prime rate risk and the variable rate commitment does not, heightened volatility in the prime rate causes the value of the former to escalate but leaves the latter unaffected. The following proposition and the accompanying corollary establish upper and lower bounds on the values of fixed and variable rate commitments.

*Proposition 7.* Let  $F(X_0, 0)$  be given by (8). Then,

$$\lim_{v \uparrow \infty} F(X_0, 0) = Le^{-rT}, \tag{12}$$

and

$$\begin{aligned} \lim_{v \downarrow 0} F(X_0, 0) &= 0 \quad \text{if } R_F \geq \alpha + r, \\ &= Le^{-rT}[1 - \exp\{(R_F - r - \alpha)(\tau - T)\}] \quad \text{if } R_F < \alpha + r, \end{aligned} \tag{13}$$

where  $Le^{-rT}[1 - \exp\{(R_F - r - \alpha)(\tau - T)\}] \in (0, Le^{-rT})$  when  $R_F < \alpha + r$ .

*Proof.* See appendix.

*Corollary C.2.* Let  $V(\hat{X}_0, 0)$  be given by (9). Then,

$$\lim_{\sigma_2 \uparrow \infty} V(\hat{X}_0, 0) = Le^{-rT}, \tag{14}$$

and

$$\begin{aligned} \lim_{\sigma_2 \downarrow 0} V(\hat{X}_0, 0) &= 0 \quad \text{if } \hat{K} \geq \hat{\alpha}, \\ &= Le^{-rT}[1 - \exp\{(\hat{K} - \hat{\alpha})(\tau - T)\}] \quad \text{if } \hat{K} < \hat{\alpha}, \end{aligned} \quad (15)$$

where  $Le^{-rT}[1 - \exp\{(\hat{K} - \hat{\alpha})(\tau - T)\}] \in (0, Le^{-rT})$  when  $\hat{K} < \hat{\alpha}$ .

Clearly, as the volatility of future interest rates increases to infinity, the customer's gain at  $t=T$  approaches  $L$  with probability one because the value of its indebtedness ( $X_T$  or  $\hat{X}_T$ ) is (almost surely) driven to zero.<sup>11</sup> The initial value of the loan commitment is then simply the discounted present value of this 'almost-sure' terminal gain,  $L$ . When the variance approaches zero all uncertainty about future interest rates is being eliminated, implying perfect foresight (at  $t=0$ ) as to whether the commitment will be exercised (at  $t=T$ ). The actual value of the loan commitment then depends solely on how high  $R_F$  (or  $\hat{K}$ ) is set in relation to the known interest rate that will prevail at  $t=T$ . If the fixed rate is set too high, the commitment will not be exercised (with probability one) and its value is zero; if the fixed rate is set relatively low, the commitment has a positive value and this value depends only on the level of the fixed rate. In either case, probabilities do not enter the valuation expression and the terminal value of the loan commitment is known with certainty  $t=0$ .

During periods of rising interest rates the credit ratings of borrowers often deteriorate. This suggests a positive instantaneous correlation between  $R_{pt}$  and  $K_t$ . As  $R_{pt}$  and  $K_t$  become more correlated, the value of a fixed rate commitment to the borrower increases because  $v$  increases. While this directly affects the risk exposure of a bank issuing fixed rate commitments, there is also another source of 'aggregate risk' that is related to diversification possibilities. If the credit risks of individual borrowers are statistically independent, the bank could conceivably achieve a substantial reduction in its aggregate risk through diversification. However, if these risks are (positively) correlated with general market interest rate movements, they will not be statistically independent and the problem of diversification will be exacerbated as this correlation rises.

To establish the reasonableness of the range of values over which  $\sigma_1$  is varied in table 1, the instantaneous standard deviation in the prime rate has

<sup>11</sup>As the variance of future interest rates goes up, very large and very small values of future interest rates become increasingly probable. However, interest rates are bounded from below ( $r$  in the case of  $R_{pT} + K_T$  and 0 in the case of  $K_T$ ). The high extremal values of terminal interest rates made possible by the rising volatility will contribute to increasingly lower values of  $X_T$  (or  $\hat{X}_T$ ), because these rates enter  $X_T$  (or  $\hat{X}_T$ ) with a negative sign in the exponent. This results in  $X_T$  (or  $\hat{X}_T$ ) being driven to zero as the variance tends to infinity.

been computed using monthly data for one selected year in each of the last five decades.<sup>12</sup> The actual values of the prime rate as well as the calculated instantaneous standard deviations are listed in table 2. The prime rate has clearly become more volatile in recent years. Consequently, the liability imposed (on banks) by fixed rate loan commitments has risen significantly. This fact, coupled with secularly declining capital-to-assets ratios in commercial banks, appears to have led to a general movement toward substituting variable rate loan commitments for fixed rate commitments. The purpose of the next section is to develop a theoretical argument to explain why variable rate commitments have generally replaced fixed rate commitments in commercial bank portfolios.

Table 2  
Prime rate data and instantaneous variances.<sup>a</sup>

Month	Prime rate in percentage			
	1933-1947	1955	1967	1974
January	1-1/2	3	6	9-1/2
February	1-1/2	3	5-3/4	9
March	1-1/2	3	5-3/4	9-1/4
April	1-1/2	3	5-1/2	10.0
May	1-1/2	3	5-1/2	11.0
June	1-1/2	3	5-1/2	11-3/4
July	1-1/2	3	5-1/2	12
August	1-1/2	3-1/4	5-1/2	12
September	1-1/2	3-1/4	5-1/2	11-3/4
October	1-1/2	3-1/2	5-1/2	11-1/4
November	1-1/2	3-1/2	6	10-1/2
December	1-1/2	3-1/2	6	10-1/2
Instantaneous standard deviation	0.0000	0.0697283	0.03213361	0.24101655

<sup>a</sup>Source of raw prime rate data: Prime Rate Series, Loan and Credit Department, Federal Reserve Bank of Boston.

#### 4. Asymmetric information and the bank credit market

The retreat of commercial banks from the fixed rate loan commitment market is well-documented [see Crane and White (1972) and Miller (1975)]. The preferred explanation for this phenomenon is that bankers have been averse to absorbing the interest rate risk associated with unanticipated prime rate variations (caused, no doubt, by sharp fluctuations in the banks' own

<sup>12</sup>Note that the instantaneous standard deviation values computed in table 2 correspond to the actual truncated random variable and thus the  $\sigma_1$  implied by each value will be (slightly) higher than the value itself. This means that the values of  $\sigma_1$  used in table 1 are not unrealistically high.

cost of funds) and have therefore chosen to transfer some of the risk to their borrowing customers.<sup>13</sup> The apparent plausibility of this explanation stems from the fact that bankers' risk aversion can be invoked to theoretically justify the concomitant convergence of asset and liability maturities at commercial banks.<sup>14</sup> However, this explanation for the abandonment of fixed rate commitments is flawed — if these loan commitments are correctly priced to reflect the higher risk imposed on the commitment-seller by an increasingly volatile prime rate, it is not clear that the risk aversion of bankers is sufficient to explain the banks' withdrawal from this market. In other words, if the fixed rate loan commitment market does not clear at a certain price ( $F(X_0, 0)$ ), we would expect the price to adjust upward till an equality between demand and supply was achieved in equilibrium. In fact, since the commitment fee is typically paid to the bank at the time the commitment is issued (at  $t=0$ ), the bank can eliminate at least part of its risk exposure by increasing the fee and augmenting its net worth, which suggests that we should never observe any permanent excess demand for commitments. To see this more clearly, consider the extreme case in which the commitment fee ( $F(X_0, 0)$ ) charged by the bank for a fixed rate commitment is  $Le^{-rT}$ . In this case, the bank is indifferent to both the interest rate risk associated with the commitment and the default risk associated with the loan made under the commitment.

One possible explanation assumes some form of intertemporal rigidity in the commitment price. Clearly, if loan commitment fees are 'sticky' and do not respond expeditiously to an increase in either the mean or the variance of the market interest rate, the demand for fixed rate commitments could exceed the supply in equilibrium. This line of reasoning is reminiscent of some of the earlier justifications for credit rationing in the loan market. [See Baltensperger (1978).] Unfortunately such an approach provides no clues about why commitment fees are 'sticky', and in the absence of a rationale for price rigidity one is forced to accept the implausible conjecture that the rigidity is due to systematic irrationality on the part of banks.

A more satisfying explanation for the passage of fixed rate commitments is provided if one recognizes that this phenomenon is a manifestation of credit rationing and exploits some of the insights provided by the more recent work on credit rationing. [See Deshmukh, Greenbaum and Kanatas (1981a,b), Jaffee and Modigliani (1969), Jaffee and Russell (1976), and particularly,

<sup>13</sup>Although in considerably diminished amounts, banks still issue fixed rate loan commitments. However, in these cases their response to heightened interest rate volatility has been to shorten the commitment period,  $T$ . It is easy to check from the valuation expression (8) that smaller values of  $T$  imply lower values of  $F(X_0, 0)$ .

<sup>14</sup>Niehans and Hewson (1976) have used a mean-variance framework to correctly show that increased uncertainty about future interest rates will lead banks to adopt more closely matched asset and liability maturity structures. Their paper does not address the loan commitment issue however.

Stiglitz and Weiss (1981).] Before developing the formal model, a few preliminary motivating comments will be useful. Note that in the preceding discussion of the loan commitment market, a dichotomy has been assumed between the bank's loan customers and its loan commitment customers. When the spot market and the forward market for bank loans are thus viewed as disjoint subsets of the overall asset markets in which the bank operates, it is difficult to rationalize fixed rate commitment rationing. However, in practice the customers who purchase fixed and variable rate loan commitments from banks also simultaneously borrow (at  $t=0$ ) without the facility of a commitment. Thus the total amount a customer borrows from the bank at  $t=0$  consists of the amount it needs to finance its current cash needs plus the amount required to pay the bank's fee for committing credit at some future date. This implies that with a fixed rate commitment the borrower's demand for funds at  $t=0$  will be an increasing function of the variance in the prime rate of interest. Now if it is assumed that informational asymmetries between the borrower and the bank permit the bank to have only partial control over the investment project the borrower finances with the loan at  $t=0$ , then the price charged by the bank on its initial loan (at  $t=0$ ) could act as an incentive device — a higher interest rate could induce the borrower to adopt a riskier project to the bank's detriment. In such situations, with two projects (one risky and one safe) there will be a critical interest rate at which the borrower switches from the safe to the risky project. Interestingly, as the borrower's credit need (at  $t=0$ ) rises (but its feasible set of investment projects remains unaltered) due to an increase in  $F(X_0, 0)$ , the critical interest rate falls, causing a decline in the bank's expected return. Of course, this decline is partially offset by the higher loan commitment price, but the model developed below demonstrates that there are situations in which the bank is better off issuing a variable rather than a fixed rate commitment.

### *The model*

Consider a discrete time model with three points in time,  $t=0, 1, 2$ . Focus on a bank and a borrowing customer, both risk neutral. At the initial time ( $t=0$ ) the borrower approaches the bank for credit to finance an investment project. The borrower faces a choice between two projects — a risky project ( $D$ ) with a gross return of  $\gamma^D$  if successful (and zero if unsuccessful) and a safe project ( $S$ ) with a gross return of  $\gamma^S$  if successful (and zero if unsuccessful). Each project requires an initial outlay of  $L_0$  (at  $t=0$ ) and an additional financing of  $L_1$  at  $t=1$ . Gross returns are realized only at  $t=2$  (there are no intermediate cash flows available at  $t=1$ ). Let  $p^S$  and  $p^D$  be the success probabilities associated with projects  $S$  and  $D$  respectively and assume  $\gamma^D > \gamma^S$ ,  $p^D < p^S$ , and  $\gamma^D p^D < \gamma^S p^S$ . Information asymmetry is now introduced

through the simple assumption that although the bank knows the characteristics of projects  $S$  and  $D$ , it has no (cost-effective) means of enforcing a particular choice of project by the borrower after the loan is made.

At  $t=0$  the borrower requests a loan of  $L_0$  and a commitment to lend  $L_1$  at  $t=1$ . Both borrowings are to be repaid with interest at  $t=2$  and repayment occurs only if the chosen project is successful. If the bank issues a variable rate commitment at  $t=0$ , the customer can borrow an amount  $L_1$  at  $t=1$  at a rate of interest  $\zeta_1 + K$ , where  $\zeta_1$  is the bank's (random) cost of funds at  $t=1$  and  $K$  is a fixed add-on which subsumes the bank's operating costs, profit margin, etc.<sup>15</sup> Unlike in the previous section, it is assumed that  $K$  is a (non-stochastic) constant. Moreover, no credit rationing possibilities are admitted, which implies that the value of (and thus the fee for) the variable rate commitment,  $V$ , is zero.<sup>16</sup> Alternatively, the bank also could issue a fixed rate commitment which would permit the borrower to borrow  $L_1$  at a fixed rate  $R_F \equiv \zeta_1^F + K$ . The bank's fee for this commitment,  $F$ , must be paid at  $t=0$ . The borrower therefore needs to borrow only  $L_0$  at  $t=0$  with a variable rate commitment, but must borrow  $L_0 + F$  with a fixed rate commitment.

Given  $\zeta_0$ , the bank's known cost of funds at  $t=0$ ,  $\zeta_1$  can take one of two possible values,  $\zeta_1^+$  and  $\zeta_1^-$ , with probabilities  $\beta$  and  $(1-\beta)$ , respectively. Let  $\zeta_1^+ > \zeta_1^-$  and assume that  $\zeta_1^F \in (\zeta_1^-, \zeta_1^+)$ . Since with a fixed rate commitment the borrower will exercise its option only if  $\zeta_1 > \zeta_1^F$ , forcing  $\zeta_1^F$  to lie between  $\zeta_1^-$  and  $\zeta_1^+$  eliminates the extreme situations in which the borrower either always exercises its option or never does so. If the borrower chooses not to exercise the fixed rate commitment at  $t=1$ , it presumably can obtain credit from an alternative source that will provide the necessary funds at a rate  $\zeta_1 + \hat{K}$ . For the moment, assume that there is some market valuation mechanism [perhaps the discrete time analog of eq. (8) with appropriate modifications for differences in the probability distributions of interest rates] that prices fixed rate commitments so that the only decision variables for the bank are (i) the (two-period) rate of interest,  $R_0$ , to set on the amount borrowed at  $t=0$ , and (ii) whether to issue a fixed rate or a variable rate loan commitment.

The bank's decisions will be influenced by its inability to *force* the borrower to choose a specific project. As I will shortly demonstrate, this informational asymmetry permits the rate of interest,  $R_0$ , to act as an

<sup>15</sup>The definition of  $K$  here differs from that in the previous section, in that it subsumes the previous  $K$  and in addition, also includes the difference between the prime and the bank's cost of funds.

<sup>16</sup>Actually, in this model the variable rate commitment is redundant because it will be assumed that in the absence of a loan commitment the customer can borrow  $L_1$  at  $\zeta_1 + K$  from any bank at  $t=1$ . However, its explicit introduction is a necessity. This stylized setting is assumed merely to keep the algebra tractable — the major conclusions remain unaltered with sufficient conditions imposed to make  $V$  positive.

incentive device — the rate charged by the bank will determine the borrower's choice of project.<sup>17</sup> If the bank does indeed prefer that project *S* be adopted, the incentive effect of the bank's pricing policy will impose an implicit upper bound on  $R_0$  [see Stiglitz and Weiss (1981)]. In addition however, the bank's decision regarding the type of commitment to offer will also have an incentive effect — under some conditions, this incentive effect could induce the bank to prefer a variable rate commitment. This line of reasoning is now formalized with the following propositions with proofs provided in the appendix.

*Proposition 8.* *If the bank issues a variable rate commitment at  $t=0$ , and  $\Phi \equiv (p^S)^2\gamma^S - 2p^D p^S \gamma^D + (p^D)^2\gamma^D > 0$ , then the bank prefers that project *S* be selected and the optimal rate of interest is given by*

$$R_0^* = \left[ \frac{p^S \gamma^S - p^D \gamma^D - (p^S - p^D)[\beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K)] - L_0(p^S - p^D)}{L_0(p^S - p^D)} \right] - \varepsilon,$$

where  $\varepsilon$  is an arbitrarily small positive scalar.

Since project *S* has a higher *expected* cash flow, one might expect the bank to unconditionally prefer its adoption. However, project *D* yields a higher cash flow,  $\gamma^D$ , if successful. Since the maximum interest rate the bank can charge (the rate at which the borrower is indifferent between borrowing and not borrowing) is an increasing function of the project cash flow in the 'good state', the higher the  $\gamma^D$  the higher is the bank's expected profit *conditional* on project *D* being preferred investment. Whether this expected profit exceeds the bank's highest expected profit associated with project *S* depends on the extent to which the difference between  $\gamma^D$  and  $\gamma^S$  is offset by the difference between the success probabilities of the two 'projects. This reasoning is formalized in Proposition 8 which identifies the condition needed to guarantee the bank's preference for project *S*, namely that  $\Phi$  should be positive. By defining  $\gamma^S \equiv \gamma^D - \Delta_0$ , where  $\Delta_0 > 0$ , it follows that this condition is equivalent to  $(p^S - p^D)^2 > (\gamma_D)^{-1} \Delta_0 (p^S)^2$ . This implies that the bank prefers the safer project if the spread between its success probability and that of the riskier project is sufficiently high in relation to  $\Delta_0$ . Further, once the safer project is identified as the desirable investment from the bank's perspective, the bank must set the interest rate lower than the rate at which the borrower switches from project *S* to project *D*. The first term in  $R_0^*$  is the 'switching'

<sup>17</sup>It is assumed that no further information about the borrower's choice of project is available to the bank at  $t=1$ , which means no punitive action can be taken by the bank ex-post.

rate. And, since the bank's expected profit is monotonically increasing in the interest rate it charges, the optimal interest rate is lower than the 'switching' rate by an arbitrarily small positive scalar,  $\varepsilon$ .

The results in Proposition 8 are predicated upon the assumption that the bank issues a variable rate commitment. The next proposition presents similar results for a fixed rate commitment.

*Proposition 9.* Suppose the bank decides to issue a fixed rate commitment at  $t=0$  and that  $\Phi > (p^S - p^D)^2 [(1 - \beta)L_1(1 + \zeta_1^- + K)]$ . Then, the bank prefers that project  $S$  be selected and the optimal rate of interest is given by

$$R_0^{**} = \left[ \frac{p^S \gamma^S - p^D \gamma^D - (p^S - p^D) [\beta L_1 (1 + \zeta_1^F + K) + (1 - \beta) L_1 (1 + \zeta_1^- + K)]}{(p^S - p^D)(L_0 + F)} \right] - \varepsilon.$$

Propositions 8 and 9 have helped to identify the optimality conditions associated with fixed and variable rate commitments. In the next proposition these results are joined in order to examine the relationship between increased uncertainty in the bank's future cost of funds and the bank's preference for the type of commitment it issues.

*Proposition 10.* Suppose  $\Phi > (p^S - p^D)^2 [(1 - \beta)L_1(1 + \zeta_1^- + K)] \equiv \Phi_0$  and at an initial equilibrium the bank prefers to issue a fixed rate loan commitment. Then,  $\{\Pi_B^{F,S}(\hat{R}_0) - \Pi_B^{V,S}(\hat{R}_0)\}$  is a declining function of the variance in  $\zeta_1$  and there exists a critically high variance (in  $\zeta_1$ ) beyond which the bank prefers to issue a variable rate commitment.<sup>18</sup>

Proposition 10 also provides formal support to the conjecture that commercial banks have abandoned fixed rate commitments because 'unusually high interest rate uncertainty has imposed an unacceptably high liability on banks issuing fixed rate commitments' — the inequality in (A.29) (refer to the appendix) indicates that the bank will prefer a fixed rate commitment only as long as the value (or liability) of the commitment is below some 'acceptable' number. However, the rationale for this phenomenon is quite different from the usual one — risk aversion is neither sufficient nor necessary for the eschewal of fixed rate commitments. What is crucial is the incentive effect of forward contracts like fixed rate commitments

<sup>18</sup>Those interested in being strictly faithful to Proposition 2 may assume that the rate at which  $\zeta_1^-$  falls is slower than the rate at which  $\zeta_1^+$  increases (i.e.,  $|\partial \zeta_1^- / \partial \zeta_1^+| < 1$ ), so that increased variance in  $\zeta_1$  is accompanied by a higher mean. Also note that although Proposition 2 has been proved for the continuous time model, it is also true in discrete time — modifying the proof in accordance with the discrete time model is trivial.

and the interaction between interest rate uncertainty, loan commitment values and borrower behavior.<sup>19</sup>

## 5. Conclusions

An important economic function of financial intermediaries is the creation of state-contingent financial claims. Our understanding of financial intermediaries is therefore enhanced by analyzing the characteristics of these claims and the conditions under which the creation of these claims is optimal for financial intermediaries. I have focused on two special types of claims — fixed and variable rate commercial bank loan commitments — and have explored the valuation aspects of these commitments.

There is considerable scope for future research on this subject. Four areas appear particularly promising. First, much may be learned from a model that satisfactorily explains the partial takedown phenomenon. Second, the interrelationships between credit rationing and forward commitments need to be better understood. Third, a valuation model that relaxes some of the constraining assumptions used here could yield substantial insights. Finally, the potential impact of the rapidly burgeoning financial futures market on the demand for and supply of bank loan commitments is surely a topic that merits study.

## Appendix

*Proof of Proposition 2.* The proposition is proved for a fixed rate commitment, but extension to a variable rate commitment is straightforward.

Suppose a bank has  $n$  borrowers and let  $R_F$  be the common fixed rate on all commitments, and  $R_T^i \equiv R_{pT} + K_T^i$  be the rate, at  $t=T$ , at which the  $i$ th borrower could borrow in the absence of a commitment. Without loss of generality, assume  $L$  and  $(\tau - T)$  are unity for all commitments. Then, the terminal value of the commitment to the  $i$ th customer is

$$F_T^i = \max [0, 1 - X_T^i] \quad \text{where} \quad X_T^i = \exp [(R_F - R_T^i)].$$

The value of a portfolio of such commitments, with  $\lambda_i$  ( $\sum_{i=1}^n \lambda_i = 1$ ,  $\lambda_i \in [0, 1]$ ) invested in the  $i$ th commitment is  $\sum_{i=1}^n \lambda_i F_T^i$ .

Now suppose there exists an  $n+1$ st customer whose indebtedness is worth

$$X_T^{n+1} = \sum_{i=1}^n \lambda_i X_T^i \quad \text{at time } t=T$$

(i.e., this customer's characteristics are a weighted average of the

<sup>19</sup>The asymmetric information arguments employed in this section increase in importance with the maturity of loan commitments because interest rate uncertainty increases with the maturity of the commitment.

characteristics of the  $n$  customers). The terminal value of a commitment made to this customer is

$$F_T^{n+1} = \max \left[ 0, 1 - \sum_{i=1}^n \lambda_i X_T^i \right].$$

By the convexity of the 'max' function it is clear that  $F_T^{n+1} \leq \sum_{i=1}^n \lambda_i F_T^i$ . This, in turn, implies that

$$F_0^{n+1} \leq \sum_{i=1}^n \lambda_i F_0^i, \quad (\text{A.1})$$

where  $F_0^{n+1}$  and  $F_0^i$  are the initial (time zero) values of the commitments to the  $n+1$ st and the  $i$ th customers respectively. Further, if the  $X_T^i$ 's are identically distributed, the values of all the ( $i=1, \dots, n$ ) commitments will be equal, i.e.,  $F_0^i = F_0 \forall i \in [1, \dots, n]$ . This means

$$F_0^{n+1} \leq F_0^i = F_0 \quad \text{since} \quad \sum_{i=1}^n \lambda_i = 1. \quad (\text{A.2})$$

Now drop the superscript (subscript)  $i$  and focus on one customer. The time  $T$  value of this customer's indebtedness is

$$X_T = \exp [(R_F - R_T)],$$

where  $R_T$  is the rate at which the customer can borrow at  $t=T$  in the absence of a commitment when the prime rate is  $R_{pT}$  and the customer's add-on is  $K_T$ . Our purpose is to examine the changes in the value of the commitment as the distribution of  $R_{pT}$  is varied, holding  $K_T$  fixed. Parameterize increased uncertainty in the prime rate by defining

$$R_T^j \equiv R_T + \omega_j, \quad j = 1, \dots, n, \quad (\text{A.3})$$

where the  $\omega_j$ 's are independent and identically distributed random variables with  $\text{cov}(\omega_j, \omega_i) = 0$  for  $j \neq i$ , and  $\text{cov}[R_T, \omega_j] = 0$ ,  $E[\exp(-\omega_j)] = 1 \forall j \in [1, \dots, n]$ . Thus, the interest rate  $R_T^j$  is drawn from a noisier (higher variance) distribution than the one from which  $R_T$  is drawn. Let

$$X_T^j = \exp [(R_F - R_T^j)]. \quad (\text{A.4})$$

From eqs. (A.3) and (A.4) it follows that

$$X_T^j = X_T + \theta_j, \quad j = 1, \dots, n, \quad (\text{A.5})$$

where the  $\theta_j$ 's are independent and identically distributed random variables with the property  $E(\theta_j | X_T) = 0 \forall j \in [1, \dots, n]$ . According to the Rothschild and Stiglitz (1970) definition of risk,  $X_T^j$  is a riskier security than  $X_T$ . Now

define

$$\hat{X}_T^{n+1} \equiv \frac{1}{n} \sum_{j=1}^n X_T^j = X_T + \frac{1}{n} \sum_{j=1}^n \theta_j.$$

Let  $F_T = \max [0, 1 - X_T]$ ,  $F_T^j = \max [0, 1 - X_T^j]$  and  $\hat{F}_T^{n+1} = \max [0, 1 - \hat{X}_T^{n+1}]$  be the terminal values of the commitment to the customer when the distributions of the interest rate are such that the terminal values of the customer's indebtedness are  $X_T, X_T^j$  and  $\hat{X}_T^{n+1}$ , respectively. Since the  $X_T^j$ 's are identically distributed (by construction), it follows from (A.2) (by letting  $\lambda_i = 1/n$ ) that

$$\hat{F}_0^{n+1} \leq F_0^j \quad \forall j \in [1, \dots, n].$$

By the law of large numbers,  $\hat{X}_T^{n+1}$  converges in probability to  $X_T$  as  $n \rightarrow \infty$ . Thus,

$$\lim_{n \rightarrow \infty} \hat{F}_0^{n+1} = F_0^*,$$

where  $F_0^*$  is the initial (time zero) value of the commitment when future interest rates are drawn from the  $R_T$ -distribution. This means

$$F_0^* \leq F_0^j \quad \forall j \in [1, \dots, n],$$

which proves that increased uncertainty about future interest rates leads to higher loan commitment values. Q.E.D.

*Density function of  $R_{pT} + K_T$*

$$f(R_{pT} + K_T | R_{p0} + K_0) = \frac{\frac{1}{\sqrt{2\pi}v} \exp \left[ -\frac{1}{2} \left( \frac{R_{pT} + K_T - \alpha}{v} \right)^2 \right] I_{[r, \infty)}(R_{pT} + K_T)}{\int_r^\infty \frac{1}{\sqrt{2\pi}v} \exp \left[ -\frac{1}{2} \left( \frac{R_{pT} + K_T - \alpha}{v} \right)^2 \right] d(R_{pT} + K_T)}, \tag{A.6}$$

where

$$v \equiv \sqrt{(\sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2)T},$$

$$\alpha \equiv (\mu_1 + \mu_2)T + R_{p0} + K_0,$$

and

$$I_{[r, \infty)}(R_{pT} + K_T) \equiv 1 \quad \text{if } R_{pT} + K_T \in [r, \infty), \\ = 0 \quad \text{otherwise.}$$

Density function of  $Z$

$$q(Z) = \frac{\frac{1}{\sqrt{2\Pi v(\tau-T)Z}} \exp\left[-\frac{1}{2}\left(\frac{\ln Z - \{\ln \bar{C} - \alpha(\tau-T)\}}{v(T-T)}\right)^2\right] I_{(0, c)}(Z)}{\int_0^c \frac{1}{\sqrt{2\Pi v(\tau-T)Z}} \exp\left[-\frac{1}{2}\left(\frac{\ln Z - \{\ln \bar{C} - \alpha(\tau-T)\}}{v(\tau-T)}\right)^2\right] dZ}. \quad (\text{A.7})$$

Density function of  $K_T$

$$h(K_T | K_0) = \frac{\frac{1}{\sqrt{2\Pi\sigma_2\sqrt{T}}} \exp\left[-\frac{1}{2}\left(\frac{K_T - \hat{\alpha}}{\sigma_2\sqrt{T}}\right)^2\right] I_{(0, \infty)}(K_T)}{\int_0^{\infty} \frac{1}{\sqrt{2\Pi\sigma_2\sqrt{T}}} \exp\left[-\frac{1}{2}\left(\frac{K_T - \hat{\alpha}}{\sigma_2\sqrt{T}}\right)^2\right] dK_T}, \quad (\text{A.8})$$

where  $\hat{\alpha} \equiv \mu T + K_0$ .

Density function of  $\hat{Z}$

$$g(\hat{Z}) = \frac{\frac{1}{\sqrt{2\Pi\sigma_2\sqrt{T}(\tau-T)\hat{Z}}} \exp\left[-\frac{1}{2}\left(\frac{\ln \hat{Z} - \{\ln \hat{Z} - \hat{\alpha}(\tau-T)\}}{(\tau-T)\sigma_2\sqrt{T}}\right)^2\right] I_{(0, c)}(\hat{Z})}{\int_0^c \frac{1}{\sqrt{2\Pi\sigma_2\sqrt{T}(\tau-T)\hat{Z}}} \exp\left[-\frac{1}{2}\left(\frac{\ln \hat{Z} - \{\ln \bar{C} - \hat{\alpha}(\tau-T)\}}{(\tau-T)\sigma_2\sqrt{T}}\right)^2\right] d\hat{Z}}. \quad (\text{A.9})$$

*Proof of Proposition 7.* Consider eq. (8). Let  $R_F \geq \alpha + r$ . Then,

$$\lim_{v \downarrow 0} F(X_0, 0) = \frac{L \exp[-rT] \{N[-\infty] - \exp[(R_F - r - \alpha)(\tau - T)] N[-\infty]\}}{N[\infty]} = 0.$$

Now let  $R_F < \alpha + r$ . In this case

$$\begin{aligned} \lim_{v \downarrow 0} F(X_0, 0) &= \frac{L \exp[-rT] \{N[\infty] - \exp[(R_F - r - \alpha)(\tau - T)] N[\infty]\}}{N[\infty]} \\ &= L \exp[-rT] \{1 - \exp[(R_F - r - \alpha)(\tau - T)]\} \\ &> 0 \quad \text{since } \exp[(R_F - r - \alpha)(\tau - T)] < 1 \\ &< L \exp[-rT] \quad \text{since } \exp[(R_F - r - \alpha)(\tau - T)] > 0. \end{aligned}$$

Define

$$Q_1 \equiv \frac{L \exp[-rT] N[(\alpha - R_F + r)/v]}{N[\alpha/v]}, \quad \text{and}$$

$$Q_2 \equiv \frac{L \exp[-rT] \exp[(R_F - r - \alpha)(\tau - T) + B^2/2] N[(\alpha - R_F + r)/v - v(\tau - T)]}{N[\alpha/v]}.$$

Obviously,

$$F(X_0, 0) = Q_1 - Q_2. \quad (\text{A.10})$$

Now,

$$\begin{aligned} \lim_{v \uparrow \infty} Q_1 &= \lim_{v \uparrow \infty} \left[ \frac{L \exp[-rT] N[(\alpha - R_F + r)/v]}{N[\alpha/v]} \right] \\ &= \frac{L \exp[-rT] N[0]}{N[0]} = L \exp[-rT], \end{aligned} \quad (\text{A.11})$$

and

$$\lim_{v \uparrow \infty} Q_2 = \lim_{v \uparrow \infty} \left[ \frac{L \exp[-rT] \exp[(R_F - r - \alpha)(\tau - T) + B^2/2] \times N[(\alpha - R_F + r)/v - v(\tau - T)]}{N[\alpha/v]} \right]$$

By L'Hospital's rule,

$$\lim_{v \uparrow \infty} Q_2 = \lim_{v \uparrow \infty} \left[ \frac{L \{(R_F - \alpha - r)/v^2 - (\tau - T)\} \times \exp[(R_F - \alpha - r)(\tau - T) - rT] N'[\alpha - v(\tau - T)]}{\{-v(\tau - T)^2 \exp[-v^2(\tau - T)^2/2] N[\alpha/v]\} - (\alpha/v^2) \exp[-v^2(\tau - T)^2/2] N'[\alpha/v]} \right]$$

where  $N'[\cdot]$  denotes the normal density function. Therefore,

$$\lim_{v \uparrow \infty} Q_2 = \lim_{v \uparrow \infty} \left[ \frac{L\{\exp[(R_F - \alpha - r)(\tau - T) - rT]\} \times \left( \frac{R_F - \alpha - r - v^2(\tau - T)}{\sqrt{2\pi}} \right) \exp[-\frac{1}{2}(\Omega - v(\tau - T))^2]}{v^2 \left\{ -v(\tau - T)^2 \exp\left[\frac{-v^2(\tau - T)^2}{2}\right] N[\alpha/v] - \frac{\alpha}{v^2} \exp\left[\frac{-v^2(\tau - T)^2}{2}\right] \left\{ \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\alpha^2}{2v^2}\right] \right\} \right\}} \right].$$

But,

$$\begin{aligned} \exp[-\frac{1}{2}(\Omega - v(\tau - T))^2] &= \exp[-\frac{1}{2}\{\Omega^2 - 2(\alpha - R_F + r)(\tau - T) + v^2(\tau - T)^2\}] \\ &= \exp\left[\frac{-v^2(\tau - T)^2}{2}\right] \exp[-\frac{1}{2}\{\Omega^2 - 2(\alpha - R_F + r)(\tau - T)\}]. \end{aligned}$$

Making this substitution above yields

$$\begin{aligned} \lim_{v \uparrow \infty} Q_2 &= \lim_{v \uparrow \infty} \left[ \frac{L\{\exp[(R_F - \alpha - r)(\tau - T) - rT]\} \left( \frac{R_F - \alpha - r}{v^2} - (\tau - T) \right) \times \exp[-\frac{1}{2}\{\Omega^2 - 2(\alpha - R_F + r)(\tau - T)\}^2]}{-v(\tau - T)^2 N[\alpha/v] - (\alpha/v^2) \sqrt{2\pi} \exp[-\frac{1}{2}(\alpha/v)^2]} \right] \\ &= L(\tau - T) \exp[(R_F - \alpha - r)(\tau - T) - rT] \exp[(\alpha - R_F + r)(\tau - T)] \\ &= 0. \end{aligned} \tag{A.12}$$

Combining (A.10), (A.11) and (A.12) gives

$$\begin{aligned} \lim_{v \uparrow \infty} F(X_0, 0) &= \lim_{v \uparrow \infty} Q_1 - \lim_{v \uparrow \infty} Q_2 \quad (\text{since the individual limits exist}) \\ &= L \exp[-rT]. \end{aligned}$$

Q.E.D.

*Proof of Proposition 8.* First note that since the bank issues a variable rate commitment the borrower needs  $L_0$  at  $t=0$  and  $L_1$  at  $t=1$ . With a variable rate commitment, the borrower's expected discounted profit, if it adopts project  $D$ , is (with a given  $R_0$ )

$$\Pi_C^{V,D} = \delta_2 p^D \{ [\gamma^D - (1 + R_0)L_0] - [\beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K)] \}, \quad (\text{A.13})$$

where  $\delta_2$  is the two-period discount rate used by the customer (assumed to be the same for the bank). If the customer adopts project  $S$ , its expected discounted profit is

$$\Pi_C^{V,S} = \delta_2 p^S \{ [\gamma^S - (1 + R_0)L_0] - [\beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K)] \}. \quad (\text{A.14})$$

Suppose at some rate of interest  $R_0 = \hat{R}_0$ , the borrower is indifferent between projects  $S$  and  $D$ . This means

$$\Pi_C^{V,D}(\hat{R}_0) = \Pi_C^{V,S}(\hat{R}_0),$$

which implies

$$\hat{R}_0 = \left[ \frac{p^S \gamma^S - p^D \gamma^D - (p^S - p^D) [\beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K)]}{L_0(p^S - p^D)} \right] \quad (\text{A.15})$$

It can be shown that if  $R_0 > \hat{R}_0$ , the borrower adopts project  $D$  and if  $R_0 < \hat{R}_0$ , the borrower adopts project  $S$ . Thus for a given  $R_0$ , the bank's expected discounted profit,  $\Pi_B^V$ , is

$$\Pi_B^{V,S}(R_0) \quad \text{if } R_0 < \hat{R}_0 \quad \text{and project } S \text{ is adopted,}$$

$$\Pi_B^{V,D}(R_0) \quad \text{if } R_0 > \hat{R}_0 \quad \text{and project } D \text{ is adopted,}$$

where

$$\Pi_B^{V,S}(R_0) = \left\{ \begin{array}{l} \delta_2 p^S [(1 + R_0)L_0 + \beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K)] \\ - (1 + \zeta_0)L_0 - \delta_1 [\beta L_1(1 + \zeta_1^+) + (1 - \beta)L_1(1 + \zeta_1^-)] \end{array} \right\} \quad (\text{A.16})$$

and

$$\Pi_B^{V,D}(R_0) = \left\{ \begin{array}{l} \delta_2 p^D [(1 + R_0)L_0 + \beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K)] \\ - (1 + \zeta_0)L_0 - \delta_1 [\beta L_1(1 + \zeta_1^+) + (1 - \beta)L_1(1 + \zeta_1^-)] \end{array} \right\}. \quad (\text{A.17})$$

Note that  $\delta_1$  is defined as the bank's single-period discount rate. As long as the bank can ensure that the borrower will adopt project  $S$ , the bank's expected discounted profit is a monotonically increasing function of  $R_0$ . This monotonicity is violated the moment  $R_0$  exceeds  $\hat{R}_0$  and the borrower switches to project  $D$ , i.e. at  $R_0 = \hat{R}_0$ , there is a drop in the bank's expected profit. However, beyond  $\hat{R}_0$  the bank's expected discounted profit once again is a monotonically increasing function of  $R_0$ , because further increases in  $R_0$  do not affect the borrower's project selection. At  $R_0 = R_0^\circ$ , such that

$$\gamma^D = (1 + R_0^\circ)L_0 + \beta L_1(1 + \zeta_1^+ + K) + (1 - \beta)L_1(1 + \zeta_1^- + K),$$

the borrower's expected profit becomes zero and it is indifferent between borrowing and not borrowing from the bank. Thus the maximum rate the bank can possibly charge is given by

$$R_0^\circ = (\gamma^D - \beta L_1(1 + \zeta_1^+ + K) - (1 - \beta)L_1(1 + \zeta_1^- + K) - L_0)/L_0. \quad (\text{A.18})$$

This implies that

$$\sup_{R_0} \Pi_B^Y(R_0) = \max \{ \Pi_B^{Y,S}(\hat{R}_0), \Pi_B^{Y,D}(R_0^\circ) \}, \quad (\text{A.19})$$

where [by substituting eq. (A.15) in eq. (A.16)]

$$\begin{aligned} \Pi_B^{Y,S}(\hat{R}_0) &= \delta_2 p^S \left[ \frac{p^S \gamma^S - p^D \gamma^D}{p^S - p^D} \right] - (1 + \zeta_0)L_0 \\ &\quad - \delta_1 [\beta L_1(1 + \zeta_1^+) + (1 - \beta)L_1(1 + \zeta_1^-)], \end{aligned} \quad (\text{A.20})$$

and [by substituting eq. (A.18) in eq. (A.17)]

$$\Pi_B^{Y,D}(R_0^\circ) = \delta_2 p^D \gamma^D - (1 + \zeta_0)L_0 - \delta_1 [\beta L_1(1 + \zeta_1^+) + (1 - \beta)L_1(1 + \zeta_1^-)]. \quad (\text{A.21})$$

If the bank prefers that project  $S$  be selected, it must be true that  $\Pi_B^{Y,S}(\hat{R}_0) > \Pi_B^{Y,D}(R_0^\circ)$ . A comparison of eqs. (37) and (38) reveals that this condition is equivalent to  $\Phi > 0$ . To complete the proof note that to ensure the selection of project  $S$  the bank must set  $R_0$  below  $\hat{R}_0$ , and since  $\Pi_B^{Y,S}(R_0)$  increases monotonically with  $R_0 \forall R_0 < \hat{R}_0$  the bank's optimal interest rate is obviously  $R_0^* = \hat{R}_0 - \varepsilon$ , where  $\varepsilon (> 0)$  is arbitrarily small. Q.E.D.

*Proof of Proposition 9.* Since the bank issues a fixed rate commitment, the borrower needs an amount  $L_0 + F$ . At  $t = 1$ , if  $\zeta_1 = \zeta_1^+ > \zeta_1^F$ , the borrower exercises

its option and borrows the additional amount  $L_1$ ; if  $\zeta_1 = \zeta_1^- < \zeta_1^F$ , the borrower does not exercise its option.

Let  $\hat{R}_0$  be the rate of interest at which the borrower is indifferent between projects  $S$  and  $D$ . By following a procedure similar to that in the previous proof, it can be shown that

$$\hat{R} = \left[ \frac{p^S \gamma^S - p^D \gamma^D - (p^S - p^D) [\beta L_1 (1 + \zeta_1^F + K) + (1 - \beta) L_1 (1 + \zeta_1^- + K)] + (p^S + p^D) (L_0 + F)}{(p^S - p^D) (L_0 + F)} \right]. \quad (\text{A.22})$$

Let  $\bar{R}_0^\circ$  be the maximum rate of interest the bank can possibly charge (the rate that makes the borrower's expected profit zero). The analog to eq. (A.18) is

$$\bar{R}_0^\circ = (\gamma^D - \beta L_1 (1 + \zeta_1^F + K) - (1 - \beta) L_1 (1 + \zeta_1^- + K) - (L_0 + F)) / (L_0 + F). \quad (\text{A.23})$$

For a given  $R_0$ , the bank's expected discounted profit (with the fixed rate commitment), is

$$\begin{aligned} \Pi_B^E &= \Pi_B^{E,S}(R_0) \quad \text{if } R_0 < \hat{R}_0 \quad (\text{customer adopts project } S), \\ &= \Pi_B^{E,D}(R_0) \quad \text{if } R_0 > \hat{R}_0 \quad (\text{customer adopts project } D), \end{aligned}$$

where

$$\Pi_B^{E,S}(R_0) = \left[ \frac{\delta_2 p^S [(1 + R_0)(L_0 + F) + \beta L_1 (1 + \zeta_1^F + K)]}{-(1 + \zeta_0)(L_0 + F) - \delta_1 L_1 (1 + \zeta_1^+) \beta} \right], \quad (\text{A.24})$$

and

$$\Pi_B^{E,D}(R_0) = \left[ \frac{\delta_2 p^D [(1 + R_0)L_0 + F] + \beta L_1 (1 + \zeta_1^F + K)}{-(1 + \zeta_0)(L_0 + F) - \delta_1 \beta L_1 (1 + \zeta_1^+)} \right]. \quad (\text{A.25})$$

Again, by direct analogy to eq. (A.19),

$$\sup_{R_0} \Pi_B^E(R_0) = \max \{ \Pi_B^{E,S}(\hat{R}_0), \Pi_B^{E,D}(\bar{R}_0^\circ) \}. \quad (\text{A.26})$$

where [by substituting eq. (A.22) in eq. (A.24)]

$$\Pi_B^{E,S}(\hat{R}_0) = \left[ \frac{\delta_2 p^S [p^S \gamma^S - p^D \gamma^D - (p^S - p^D) (1 - \beta) L_1 (1 + \zeta_1^- + K)] (p^S - p^D)^{-1}}{-(1 + \zeta_0)(L_0 + F) - \delta_1 L_1 (1 + \zeta_1^+) \beta} \right], \quad (\text{A.27})$$

and [by substituting eq. (A.23) in eq. (A.25)]

$$\Pi_B^{F,D}(\bar{R}_0^\circ) = \left[ \begin{array}{c} \delta_2 p^D [\gamma^D - (1-\beta)L_1(1+\zeta_1^- + K)] - (1+\zeta_0)(L_0 + F) \\ - \delta_1 L_1 \beta (1+\zeta_1^+) \end{array} \right] \quad (\text{A.28})$$

Obviously, the bank prefers that project  $S$  be adopted if  $\Pi_B^{F,S}(\hat{R}_0) > \Pi_B^{F,D}(\bar{R}_0^\circ)$ . A comparison of eqs. (44) and (45) indicates that this condition is equivalent to

$$\Phi > (p^S - p^D)^2 [(1-\beta)L_1(1+\zeta_1^- + K)].$$

To ensure the selection of project  $S$  the bank must set  $R_0 < \hat{R}_0$  and since  $\Pi_B^{F,S}(R_0)$  is monotonically increasing in  $R_0 \forall R_0 < \hat{R}_0$ , the bank's optimal rate of interest,  $R_0^{**}$ , is obviously  $\hat{R}_0 - \varepsilon$ . Q.E.D.

*Proof of Proposition 10.* Note that since  $\Phi > (p^S - p^D)^2 [(1-\beta)L_1(1+\zeta_1^- + K)] > 0$ , the bank prefers project  $S$  irrespective of whether a fixed rate or variable rate commitment is issued. Further, since the bank prefers to issue a fixed rate commitment,  $\Pi_B^{F,S}(\hat{R}_0) > \Pi_B^{V,S}(\hat{R}_0)$ . Using eqs. (A.20) and (Q.27) it can be shown that this inequality implies

$$\delta_2 p^S (1-\beta)L_1(1+\zeta_1^-) + \delta_2 p^S (1-\beta)L_1 K + (1+\zeta_0)F < \delta_1 L_1 (1-\beta)(1+\zeta_1^-) \quad (\text{A.29})$$

or equivalently,

$$F < L_1 (1-\beta) [\delta_1 (1+\zeta_1^-) - \delta_2 p^S (1+\zeta_1^-) - \delta_2 p^S K] / (1+\zeta_0). \quad (\text{A.30})$$

By hypothesis, this condition is satisfied at an initial equilibrium. To introduce increased variance in  $\zeta_1$ , assume that  $\zeta_1^+$  increases and  $\zeta_1^-$  decreases without altering  $\beta$ . Proposition 2 asserts that  $F$  will go up as  $\zeta_1^+$  increases (and  $\zeta_1^-$  decreases). I want to examine the effect of this on the inequality in (A.26). Let  $M \equiv \delta_2 p^S (1-\beta)L_1(1+\zeta_1^-) + \delta_2 p^S (1-\beta)L_1 K + (1+\zeta_0)F$  and  $N \equiv \{\delta_1 L_1 (1-\beta) \times (1+\zeta_1^-)\}$ . Then,

$$\partial M / \partial \zeta_1^+ = \delta_2 p^S (1-\beta)L_1 (\partial \zeta_1^- / \partial \zeta_1^+) + (1+\zeta_0) (\partial F / \partial \zeta_1^+), \quad (\text{A.31})$$

and

$$\partial N / \partial \zeta_1^+ = \delta_1 L_1 (1-\beta) (\partial \zeta_1^- / \partial \zeta_1^+), \quad (\text{A.32})$$

where  $\partial \zeta_1^- / \partial \zeta_1^+ < 0$ . In passing, note that a decline in  $\zeta_1^-$  (which has a positive lower bound) reduces  $\Phi_0$ , leaving the inequality  $\Phi > \Phi_0 > 0$ , undisturbed. This means that heightened uncertainty in  $\zeta_1$  (of the type considered here) will not change the bank's preference for project  $S$ . Now, the first term in eq. (A.31) is negative and the second term is positive, making the sign of  $\partial M / \partial \zeta_1^+$  ambiguous. Further,  $\partial N / \partial \zeta_1^+ < 0$ . But, since  $\delta_2 < \delta_1$  and  $p^S < 1$ ,

$$\delta_2 p^S (1-\beta)L_1 \partial \zeta_1^- / \partial \zeta_1^+ > \delta_1 L_1 (1-\beta) \partial \zeta_1^- / \partial \zeta_1^+$$

which implies  $\partial M/\partial \zeta_1^+ > \partial N/\partial \zeta_1^+$ . This means that as the variance in  $\zeta_1$  is increased there will be some point beyond which the inequality in (A.29) is reversed and the bank will prefer to issue a variable rate commitment. Q.E.D.

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