

Regulatory Pricing and Capital Investment under Asymmetric Information about Cost

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I. Introduction

The regulation of natural monopolies involves setting an output price for the regulated firm such that given factor input prices (operating costs) as well as production technology and market demand conditions, the firm's privately chosen optimal level of resource use allows it to earn a rate of return consistent with social welfare maximization.¹ This price setting process is usually complicated by the fact that the regulated firm has better information about its own operating costs than the regulator. To overcome its informational disadvantage, the regulator could try to estimate these costs from actual and reported costs over some "test period."² Unfortunately, historical costs may be poor indicators of actual costs, and reported costs may be exaggerated by the firm to secure higher prices.³ Hence, it is useful to establish pricing rules under asymmetric information which are *ex ante* efficient in the sense that they motivate regulated firms to charge economically efficient prices. Our objective in this paper is to develop such a pricing schedule in a regulatory environment in which the firm's variable production cost is known to the firm but unknown to the regulator.

Our research is part of a growing literature on regulatory pricing under asymmetric information.⁴ Baumol, Bailey and Willig [5] as well as Panzar and Willig [12] have argued

1. To this effect, Bower of the New York State Public Service Commission states: "The agency objective is to maximize the weighted sum of consumer and producer surplus through time, and the weight accorded producer surplus will be small if it is a state regulatory agency" [7].

2. Although two types of test years are used—historical and forecast—in practice, historical test periods predominate. In its 1977 West Iowa Telephone Company decision rejecting a forecast test period, the Iowa State Commission stated: "We have always rejected attempts at using projected data in lieu of known and measurable facts. Speculation is the anathema of regulation." (18 PUR 4th 227).

3. In addition to the moral hazard problem, there are serious problems of cost measurement. For example, operating expenses, capital costs, and depreciation over the test period must be associated with the average investment in the test period. Such estimates require recognition that during the test period, plant and equipment may be added to meet augmented demand or to take advantage of operating efficiencies. Further, not all utilities are alike. Telephone and water utilities, which are not covered by the Public Utility Holding Act, commonly have non-utility unregulated operations such as merchandising, manufacturing, or real estate investment. Others, such as gas and electric, may operate in several states, or may be subject to various state and federal commission jurisdictions. The diversity of interests makes the allocation of joint costs, such as general overhead expenses and capital charges, an extremely difficult task.

4. See, for example, [2], [6] and [16].

that, by permitting the entry of rival firms, regulators may induce existing firms to implement welfare maximizing prices without knowing their costs, because doing so would constitute a limit entry pricing strategy for these firms. While this is an interesting hypothesis, its empirical significance remains largely untested. In like vein, Loeb and Magat [11] propose giving the firm title to the entire social surplus and allowing the firm to choose a price (vector) to maximize this surplus. The right to the monopoly franchise can then be auctioned among competing firms to transfer surplus from producers to consumers. But an auction will be ineffective if only one firm can supply the service efficiently.

In contrast, Baron and Myerson [3] suggest that a revelation game can be employed to coax the firm to report its private information truthfully. Contingent upon the report, the regulator determines the output price, whether to grant the firm a license to operate, and the size of the subsidy to be given the firm. There are two difficulties with this approach. First, regulators in many regulated industries do not have the power to grant subsidies. Moreover, even when subsidization is feasible, the financing of the subsidy—for example, through differential taxation—can itself cause relative price distortions. Second, the optimal regulatory policy could involve closing down a firm that reports truthfully and whose continued operation, from an *ex post* perspective, could have benefited the firm as well as consumers.

Although similar in spirit, our approach differs substantially from this literature. In our framework the firm engages in Spencian signaling [14] of its *a priori* unknown variable cost to the regulator through its choice of capital investment. Specifically, the regulator designs a price schedule in which the output price is a function of the firm's (observable) capital investment. The price schedule has the following feature: when the firm responds to the price schedule and selects a profit maximizing level of investment, it earns an allowed rate of return no less than its cost of capital and the resulting allocation maximizes social surplus. We obtain two principal results. First, in the presence of asymmetric information the firm must be allowed to earn a rate of return exceeding its cost of capital, but the level of excess profits enjoyed by the firm is a decreasing function of its variable cost. Thus, our paper provides a possible explanation for empirically documented positive differences between rates of return and costs of capital for regulated firms. Second, the optimal regulatory price schedule is a decreasing function of the firm's level of capital investment.

Our approach is sufficiently robust to deal with most economically significant production technology and market demand specifications. And as far as possible, the analysis is carried out with general functions. However, in order to obtain sharper insights specific assumptions about functional forms are made in certain parts of the analysis. The assumptions are that production is Cobb–Douglas and market demand for the firm's output is homogeneous (of some arbitrary degree) in price.

The remainder of the paper is organized in four sections. Section II contains the development of the regulatory pricing problem in the full information case. Section III develops the asymmetric information case. The paper is concluded in section IV.

II. Regulatory Pricing under Full Information

To provide a perspective for the more complex but interesting problem of regulatory pricing under asymmetric information, we first analyze the problem in the case where the firm and the regulator are equally informed about the cost of the firm's variable input factor. Consider a regulated firm which is a natural monopoly. Further, assume a single-period certainty

framework wherein all capital is financed through equity and the firm's objective is to maximize the present value of its profits net of invested capital.

Let p denote the price per unit of output that the regulator allows the firm to charge. Define $q(p)$ as the market demand function, which is assumed to be at least continuously differentiable, bounded, and monotone decreasing in price. The firm's production function, represented by $f(K, L)$, is also assumed to be at least twice continuously differentiable, bounded and strictly quasi-concave in its arguments, where K denotes the firm's capital investment and L , the number of units of variable input (labor) employed. K and L are assumed to be divisible, continuous and non-negative. The cost of capital, $r - 1 (> 0)$, is the riskless rate of interest, and the wage rate per unit of labor, w , is a positive scalar. For now we assume that the wage rate is known to the regulator as well as the firm. We shall relax this assumption in the next section.

Then the firm's profit, Π , can be written as

$$\Pi = pq(p) - wL \quad (1)$$

and its discounted present value as

$$V = \Pi/r. \quad (2)$$

In a Walrasian equilibrium in the product market, demand must equal production (supply). Thus,

$$q(p) = f(K, L). \quad (3)$$

Utilizing the implicit function theorem, the units of labor required can be expressed in terms of demand and the level of capital investment as follows:

$$L = L(q, K). \quad (4)$$

Thus, given a regulated price p and a chosen investment level K , the net present value of the firm, Z , is

$$Z(K; p) = V(K; p) - K. \quad (5)$$

The firm will choose an optimal investment level K to maximize its net present value, Z , and it will elect to supply service at the regulated price only if the maximized Z is non-negative. Since the regulated price, $p(s)$, depends upon (one plus) the rate of return, s , that the regulator wants the firm to earn on invested capital,⁵ s must be set at least as high as r to ensure the firm's willingness to provide service. The precise value of s is established by the regulator to maximize social surplus, which is a weighted sum of consumer and producer surplus.

Problem Statements

Formally then, the regulatory pricing problem can be described as the regulator selecting an allowed return $\hat{s}(w)$, to maximize

$$(1 - u) \int_{\hat{p}}^{\infty} q(p) dp + u(s - r) \hat{K} \quad (6)$$

5. Note that in this model, price, p , is a function of the allowed rate of return, s , through constraint (8), which follows.

subject to the constraint that

$$\bar{S} \geq s \geq r, \quad (7)$$

such that, given the price, $\hat{p}(\hat{s})$, the firm earns the allowed return

$$\hat{p}\hat{q} - w\hat{L} = \hat{s}\hat{K}, \quad (8)$$

at the optimal level of investment, $\hat{K}(w; \hat{s})$, given by

$$\hat{K} \in \operatorname{argmax} Z(K; \hat{p}). \quad (9)$$

In (6)–(9), the arguments of p, q, K, L have been suppressed (where no confusion results) to minimize notation, and hats are used to denote optimal values. The scalar $u \in [0, 1]$ is a weighting factor to determine how consumer surplus, $\int_{\hat{p}}^{\infty} q(p) dp$, and producer surplus, $(s - r)\hat{K}$, enter into the social welfare objective and are jointly maximized.⁶ \bar{S}, r are the upper and lower bounds for s . Henceforth, we shall refer to s and r as the allowed rate of return and the riskless rate, respectively, even though they are to be understood as one plus the respective rates. The optimal level of capital investment is given by the first-order condition corresponding to (9)

$$-\hat{L}_K = r/w, \quad (10)$$

and by differentiating (3) we obtain

$$-\hat{L}_K = f_K/f_L. \quad (11)$$

Combining (10) and (11) we see that the marginal rate of factor substitution equals the factor price ratio. This leads to our first observation.

PROPOSITION 1 *Under full information, the regulated firm achieves a technologically efficient use of production inputs for any given regulatory price, p .*

This result is well known and found in [4] and [10]. The key is that the firm views the output price as beyond its control. Technological efficiency can thus be achieved at any price, as long as that price permits the firm to earn a return at least equal to its cost of capital. However, as we shall see in the next section, this result is crucially dependent on the assumption that the regulator and the firm are equally informed about costs. Under asymmetric information, technological efficiency is unattainable.

Optimal Pricing Policy

To characterize the optimal regulatory pricing policy, we need the following lemma, whose proof appears in the Appendix.

6. Consumer surplus, the way we have represented it, can also be found in [3] and [15] as well as numerous other papers. To validate it, consider the following argument. Assume that the marginal utility of money income for each household is only a function of income and has constant elasticity. Also, suppose the total expenditure by a household for the service provided by the regulated firm is a very small percentage of total income, and thus, the marginal utility of income is not significantly affected by the price charged by the regulated firm. Then, the social welfare derived from the service of the firm can be equated to a weighted sum of the monetary value of the household consumer surplus, weighting by the marginal utility of money income for each household. Since the marginal utility of income is assumed to be unaffected by the price charged by the regulated firm, consumer surplus can be measured by the expression we have used.

LEMMA. Given any regulatory price, p , the firm's optimal capital investment satisfies the marginal cost condition.

$$\hat{K}_p = (-\hat{L}_K)^{-1} \hat{L}_q \hat{q}_p = (w/r) \hat{L}_q \hat{q}_p. \tag{12}$$

We now wish to examine how the optimal pricing policy changes as the regulator's welfare objective, parameterized by u , changes. Two cases, $u = 0$ and $u = 1$, are examined.

Case 1: $u = 0$. Suppose the social welfare objective of the regulator is to maximize consumer surplus. Then, since consumer surplus always increases as price declines, in the absence of any minimal return considerations, the optimal regulatory pricing strategy is to set a zero price. But since the firm must earn a return at least equal to its cost of capital to ensure service, the optimal regulatory pricing policy will instead be to set price equal to average cost. Moreover, we will also show that, at the optimum, marginal cost exceeds marginal revenue.

To establish these results formally, we form the Lagrangian function associated with the maximization program in (6)–(9):

$$L = [1-u] \int_{\hat{p}}^{\infty} q(p) dp + u[s-r]\hat{K} + t[s-r], \tag{13}$$

where we have used the fact that when consumer surplus is maximized, the upper bound constraint $s \leq \bar{S}$ is not binding. Note that $t \geq 0$ represents a Lagrange multiplier. The first order condition, setting $u = 0$, is

$$-\hat{q}\hat{p}_s + t = 0 \tag{14}$$

and the complementary slackness conditions are

$$t \geq 0, \hat{s} \geq r, t(\hat{s}-r) = 0. \tag{15}$$

Further, differentiating (8), using the Lemma, and rearranging, we have that⁷

$$\hat{p}_s = \hat{K}[\hat{q} + \hat{p}\hat{q}_p - \hat{K}_p\hat{s}]^{-1}. \tag{16}$$

From (14) and (16) we see that $t \neq 0$. From (15) then, $\hat{s} = r$.

Now substituting (16) in (14) and using the Lemma, we have

$$t = \hat{q}\hat{K}\{\hat{q} + \hat{q}_p\hat{p} + \hat{L}_q\hat{q}_p[\hat{L}_K]^{-1}s\}^{-1}. \tag{17}$$

Finally, using (10) and rearranging, we can write (17) as

$$t = \hat{K}\hat{p}\eta^{-1}\{w\hat{L}_q - \hat{p}[1-\eta^{-1}]\}^{-1}, \tag{18}$$

where we have used the fact that $\hat{s} = r$ and have defined

$$\eta \equiv -\hat{q}_p\hat{p}[\hat{q}]^{-1}$$

as the price elasticity of demand. In (18), note that $w\hat{L}_q = \partial[w\hat{L}]/\partial q$ is marginal cost and $\hat{p}[1-\eta^{-1}] = \partial[\hat{p}\hat{q}]/\partial q$ is marginal revenue. Because $t > 0$, we see from (18) that marginal cost must exceed marginal revenue. Thus, if we have increasing returns to scale, then the firm underproduces relative to the profit maximizing output level (which entails marginal

7. To derive (16), express the labor usage L as a function of q and K , and remember that both q and K are functions of p , which is, of course, a function of s . The only constants are u , s , and r .

cost and marginal revenue being equated at the optimum), and if we have decreasing returns to scale, then the firm overproduces relative to the profit maximizing output level.

Case 2: $u = 1$. Now suppose the regulator's social welfare objective is to maximize producer surplus. Assuming that the lower bound constraint on s is non-binding in this case, the first-order condition from (6) and (7), letting $u = 1$, is

$$(\hat{s} - \hat{r})\hat{K}_p\hat{p}_s + \hat{K} - j = 0, \tag{19}$$

where $j \geq 0$ is a Lagrange multiplier and satisfies the complementary slackness condition

$$j \geq 0, \bar{S} \geq \hat{s}, j(\bar{S} - \hat{s}) = 0. \tag{20}$$

Substituting for \hat{p}_s in (19) and for \hat{L}_K from (10), and also utilizing the Lemma and (20), we find that

$$\hat{q} + \hat{p}\hat{q}_p - w\hat{L}_q\hat{q}_p - j(\hat{q} + \hat{p}\hat{q}_p - w\hat{L}_q\hat{q}_p)K^{-1} = 0,$$

which yields the following equation upon rearranging:

$$[1 - j/\hat{K}][\hat{p}(1 - 1/\eta) - w\hat{L}_q]\hat{q} = 0. \tag{21}$$

From (21) it is apparent that if $j = 0$ —so that $s \leq \bar{S}$ —then

$$\hat{p}[1 - \eta^{-1}] = w\hat{L}_q,$$

and hence, marginal revenue equals marginal cost at the optimum. On the other hand, if $j > 0$ —so that $s = \bar{S}$ —then marginal revenue will exceed marginal cost. In this case, the extent of the deviation of marginal revenue from marginal cost depends on the price elasticity of demand.

The above results are summarized in the following proposition:

PROPOSITION 2. *If the regulatory social welfare objective is to maximize consumer surplus, then price is set so that the firm is allowed to earn a rate of return exactly equal to its cost of capital, and marginal cost always exceeds marginal revenue at the optimum. But if the regulatory social welfare objective is to maximize producer surplus, then the output price is set such that the firm either earns a rate of return equal to the highest permissible rate (which exceeds the cost of capital) or marginal revenue and marginal cost are equated at the optimum.*

III. Second Best Regulatory Pricing

Suppose that for a particular regulated firm, the regulator's information about the variable cost of production is inferior to the firm's knowledge about this cost. In such a setting, the regulatory pricing problem becomes more complex because a fixed price policy for the firm is no longer optimal in general. The problem is particularly severe when only one firm supplies the service. A fixed price based upon inadequate information about the variable production cost may either lead to socially suboptimal excess rents for the firm, or may result in the firm refusing to provide the service because its rate of return falls short of its cost of capital. Thus, the informational asymmetry must be resolved, and to do this the regulator should adopt a pricing policy that provides the firm with an incentive to truthfully reveal its private information.

A regulatory pricing schedule, $p(K; s)$, that depends upon the firm's capital investment and allows the firm to earn a socially optimal rate of return, would accomplish this objective. More specifically, the pricing schedule should have the following properties.

1) It should be incentive compatible.⁸ That is, faced with the schedule the firm should have no incentive to "misrepresent" its true cost, w .

2) The schedule should induce the firm to choose an optimal level of investment such that, at that level of investment, the firm earns a rate of return at least as great as its cost of capital.⁹

3) The rate of return earned by the firm should be such that social surplus, a weighted average of consumer and producer surplus, is maximized.

Problem Statement

Formally, then, the regulator's problem is to design a pricing schedule $p(K; s)$ without prior knowledge of the firm's variable cost of production, w , such that the allowed return, $\hat{s}(K)$, maximizes

$$(1 - u) \int_p^\infty q(p) dp + u(s - r)\hat{K} \quad (22)$$

subject to the constraint that

$$\bar{S} \geq s \geq r, \quad (23)$$

and the pricing schedule, $\hat{p}(\hat{K}; \hat{s})$, is such that the firm earns the allowed return

$$\hat{p}\hat{q} - w\hat{L} = \hat{s}\hat{K}, \quad (24)$$

at the optimal capital investment, $\hat{K}(w; s)$, given by

$$\hat{K} \in \arg\max Z(K; \hat{p}). \quad (25)$$

This formulation has appeal from a descriptive point of view since regulators do set prices based upon the firm's capital stock and a rate of return constraint. Furthermore, firms recognize this and choose their capital stocks accordingly. Observe from (25) that investment, K , is chosen by the firm to maximize net present value, given a price, p . What makes the problem interesting and challenging is that price is itself a function of investment, K . The usual "sophistication" requirements in investigations of incentive compatibility such as this dictate that both the regulator and the firm fully take into account these interrelationships and the response of the other party to it.¹⁰

8. A rather subtle point needs to be made here. Although the firm's choice of K conveys information about its w , whatever learning takes place ex post on the regulator's part is redundant. The pricing schedule is ex ante informationally efficient in the sense that the firm's optimizing decision is consistent with the regulator's social welfare objective, and is incentive compatible in the sense that no firm has an incentive to lie by choosing K to masquerade as a firm with a different w . Thus, there is no need for the regulator to attempt to actually compute a firm's w after having observed its K , since any such inference is useless as far as implementation of the pricing policy is concerned.

9. This condition is obviously imposed to insure the provision of service. In many instances, the courts have ruled that certain services, like the provision of heat during winter in the snowbelt areas, are so essential that they cannot be interrupted even if the consumer in question has not paid his bills.

10. The approach taken here is similar to the approach in other self-selection models in which some personal welfare maximizing choice of the informed conveys information about the a priori unknown attribute to the uninformed. See, for example, [14].

Optimal Pricing Policy

From (24) the optimal level of capital investment is given by the first-order condition

$$(\hat{q} + \hat{p}\hat{q}_p)\hat{p}_K - w\hat{L}_q\hat{q}_p\hat{p}_K - w\hat{L}_K - r = 0. \tag{26}$$

Substituting for the variable production cost, w , from (24) into (26), we obtain the following differential equation for price

$$(\hat{q} + \hat{p}\hat{q}_p)\hat{p}_K - (\hat{p}\hat{q} - \hat{s}\hat{K})\hat{L}^{-1}(\hat{L}_q\hat{q}_p\hat{p}_K + \hat{L}_K) - r = 0. \tag{27}$$

The solution to (27) will yield the optimal pricing schedule, $\hat{p}(\hat{K}; \hat{s})$, which determines the price awarded to the firm contingent upon its capital investment. The pricing schedule is such that the firm earns a rate of return, \hat{s} , at the induced level of capital investment, $\hat{K}(w; s)$. Substituting (27) into (26) yields \hat{K} . Further, from (22), the optimal rate of return, \hat{s} , in (27) satisfies the first-order condition¹¹

$$u[\hat{s} - r]\hat{K}_s + \hat{K} - [1 - u]\hat{q}[\hat{p}_s + \hat{p}_K\hat{K}_s] \geq 0, \tag{28}$$

where $\bar{s} > \hat{s} > r$ if the equality holds, $\hat{s} = r$ if the “less than” inequality holds, and $\hat{s} = \bar{s}$, otherwise. Thus, under asymmetric information the regulator observes the firm’s capital investment and then determines the rate of return s and the price p to award the firm by utilizing (27) and (28) simultaneously.¹²

Now define social surplus, M , as

$$M(s; K) = (1 - u) \int_p^\infty q(p) dp + u(s - r)\hat{K}. \tag{29}$$

Ignoring the dependence of s on K , the partial derivative of M with respect to K is

$$M_K = - [1 - u]\hat{q}\hat{p}_K + us. \tag{30}$$

At this point the sign of \hat{p}_K is unknown. Suppose we conjecture that $\hat{p}_K < 0$. Then $\hat{M}_K > 0$. This implies that if we view M as a function of s conditional on \hat{K} , then $M(s; \hat{K}_1) > M(s; \hat{K}_2)$ for all s and $\hat{K}_1 > \hat{K}_2$. Thus, if $\hat{s} \in \text{argmax } M(s; \hat{K})$, then $\hat{s}(\hat{K}_1) > \hat{s}(\hat{K}_2)$. In other words, $\hat{s}_K > 0$.

But totally differentiating (24) yields

$$[(\hat{q} + \hat{p}\hat{q}_p)\hat{p}_K - w\hat{L}_q\hat{q}_p\hat{p}_K - w\hat{L}_K - \hat{s} - \hat{s}_K\hat{K}][d\hat{K}/dw] = L(\hat{q}, \hat{K}). \tag{31}$$

Substituting (26) into (31) produces

$$d\hat{K}/dw = - L(q, \hat{K})/[\hat{s} - r + \hat{s}_K\hat{K}]. \tag{32}$$

Since $\hat{s} \geq r$ and $\hat{s}_K > 0$, we have $d\hat{K}/dw < 0$.

Now, totally differentiating (24) we obtain the expression

$$\hat{Z}_{KK}[d\hat{K}/dw] - \hat{L}_q\hat{q}_p\hat{p}_K - \hat{L}_K = 0. \tag{33}$$

From the second-order sufficiency condition for a maximum, $\hat{Z}_{KK} < 0$, and since $d\hat{K}/dw < 0$, it follows that

11. Explicitly recognized in (28) is the fact that the optimal investment level chosen by the firm depends upon the rate of return, s , that the regulator permits it to earn.

12. The dependence of p on s means that the entire price schedule shifts, depending upon the allowed rate of return. Note that (28) implies that s depends upon u , which means that p depends upon u through its dependence upon s .

$$\hat{L}_q \hat{q}_p \hat{p}_K + \hat{L}_K > 0. \tag{34}$$

Moreover, since $-\hat{L}_K = -\hat{f}_K/\hat{f}_L > 0$, we must have $\hat{L}_q \hat{q}_p \hat{p}_K > 0$. Further, $\hat{L}_q = [1/\hat{f}_L] > 0$ and $\hat{q}_p < 0$ by assumption. Thus, $\hat{p}_K < 0$, as conjectured.

Now suppose $\hat{p}_K = 0$. Then from (30), $\hat{M}_K > 0$, which means that $\hat{s}_K > 0$. This in turn implies $d\hat{K}/dw < 0$, and repeating the arguments just made, we see that $\hat{p}_K < 0$. Thus, we have a contradiction and \hat{p}_K cannot be zero. Finally, suppose $\hat{p}_K > 0$. Then for u sufficiently close to 1, we can see from (30) that $\hat{M}_K > 0$, which implies $\hat{s}_K > 0$. This, in turn, means that $d\hat{K}/dw < 0$ and so $\hat{p}_K < 0$. Hence, we encounter a contradiction again, and conclude that $\hat{p}_K < 0$. This implies $d\hat{K}/dw < 0$. These two observations are stated as propositions below.

PROPOSITION 3. *The optimal pricing schedule under asymmetric information is such that in equilibrium firms that display higher levels of capital investment are granted lower prices.*

PROPOSITION 4. *The optimal pricing schedule under asymmetric information induces the firm to invest more when its variable cost is lower.*

Proposition 3 is intuitive. The regulator wants to set a price based upon the firm’s variable production cost, but is a priori unaware of this cost. Through its assertion that a firm with a high variable cost chooses a low capital investment, Proposition 4 provides a strong indication of the manner in which prices should be determined in such a setting. To ensure that the firm is granted a price consistent with its willingness to provide service, the regulator would like to set a high price if the firm has a high variable cost. Since variable cost cannot be directly observed, the regulator awards a high price when the firm displays a low capital investment, because it knows that the firm in this case is operating with a high variable cost. The attractiveness of this mechanism is that, in spite of the fact that the firm knows exactly how the regulator will behave, the equilibrium is incentive compatible.¹³

Finally, from (26) we can establish the following proposition regarding the firm’s investment in capital.

PROPOSITION 5. *Under asymmetric information, the technologically efficient use of factor inputs is unattainable except possibly in the case where the regulator’s objective is to maximize producer surplus.*

To see this, observe that from the quasi-concavity of the production function, $\hat{L}_{KK} > 0$ (see Appendix), and from the previous discussion, $\hat{p}_K < 0$. Hence, from (26) it is clear that the firm will underinvest in capital if $\hat{q} + \hat{p}\hat{q}_p - w\hat{L}_q > 0$, and overinvest if $\hat{q} + \hat{p}\hat{q}_p - w\hat{L}_q < 0$. We shall now show that a technologically efficient use of factor inputs is possible when the regulator’s objective is to maximize producer surplus. Differentiating (24) totally and substituting (26) in the resulting expression, we obtain

$$[\hat{s} - r]\hat{K}_s + \hat{K} = [\hat{q} + \hat{p}\hat{q}_p - w\hat{L}_q]\hat{p}_s. \tag{35}$$

13. This result can be contrasted with Baron and Taggart’s analysis [4]. They show that the Averch-Johnson [1] overcapitalization bias would be the outcome of “naive” regulation in which the regulator adjusts prices based upon changes in the firm’s capital stock. We quote from their paper:

As long as the regulator follows some predictable price-setting rule such as this, the firm has an incentive to manipulate its capital stock so as to achieve any price consistent with the regulatory constraint . . . , and thus the decision problem of the firm under naive regulation is identical to that in the A-J model” [4,9].

From (28) we see that, for $u = 1$, if we have an interior solution, $\bar{S} > s > r$, then $\{\hat{s} - r\}\hat{K}_s + \hat{K} = 0$. Thus, (35) implies that

$$[\hat{q} + \hat{p}\hat{q}_p - w\hat{L}_q]\hat{p}_s = 0. \tag{36}$$

Since price is not independent of s , (i.e., $\hat{p}_s \neq 0$), this leads to $\hat{p} + \hat{p}\hat{q}_p - w\hat{L}_q = 0$. Hence, there is a technologically efficient use of factor inputs with producer surplus maximization if there is an interior solution to the allowed rate of return.

Proposition 5 may be viewed as the asymmetric information analog of Proposition 1. The finding that there is overinvestment in capital when capital serves as a “signal” of the firm’s unknown variable cost is not surprising, since it has its counterparts in other asymmetric information models. For instance, Spence [14] proves that an individual will overinvest in education when education functions as a signal of his/her a priori unknown native ability.

In practice, of course, the regulator would have to know (or be able to estimate) the specific forms of the firm’s production and demand functions in order to formulate and implement this pricing policy. In the next subsection, we assume specific functional forms to further characterize the optimal pricing policy.

Cobb-Douglas Production and Homogeneous Demand

To illustrate the general approach presented in the previous subsection, we now assume that the demand function is homogeneous in price and the production function is of the Cobb-Douglas form. That is, $q(p) = p^{-c}$, with $c > 1$, and $f(K, L) = L^a K^b$, with $a, b > 0$. The assumption about the demand function seems reasonable when we consider that consumer surplus, as measured by the area under the demand curve, is exact only if the marginal utility of income is constant. This implies utility functions over consumption goods that are homogeneous and demand functions that exhibit constant expenditure elasticity. Moreover, the Cobb-Douglas production function has been used extensively in empirical studies of cost. This is particularly true in cost studies of regulated industries [8]. Moreover, the assumption appears to stand up to empirical testing. For example, in Courville [9] two general production functions belonging to the Constant Elasticity of Substitution family were tested against the Cobb-Douglas production function using a non-linear estimation technique. The Cobb-Douglas production technology could not be rejected.

These particular functional forms lead to a welcome analytical simplification. The determination of the allowed rate of return, s , now becomes independent of the firm’s capital investment. This assertion is proven below.

Since demand, q , is homogeneous of degree c in price, Euler’s theorem permits us to write

$$\hat{p}\hat{q}_p = -c\hat{q}. \tag{37}$$

Moreover, since the production function is Cobb-Douglas, we can use (4) to obtain

$$\hat{q}\hat{L}_q = [1/a]L(\hat{q}, \hat{K}) \tag{38}$$

$$\hat{K}\hat{L}_K = -[b/a]L(\hat{q}, \hat{K}). \tag{39}$$

Substituting (28)–(30) into (27) and rearranging, yields the differential equation

$$[\xi_1 \hat{p}^{1-c} - \xi_2 \hat{K}]\hat{p}_K \hat{K} + [\xi_3 \hat{p}^{1-c} - \xi_4 \hat{K}]\hat{p} = 0, \tag{40}$$

where $\xi_1 = 1 - c + [c/a]$, $\xi_2 = [c/a]\hat{s}$, $\xi_3 = [b/a]$, and $\xi_4 = [b/a]\hat{s} + r$.

The solution to (40) is a power function,

$$\hat{p}(K; s) = (\lambda K)^{1/(1-c)}, \tag{41}$$

where

$$\lambda = r + \theta_1 \theta_2^{-1} [\hat{s} - r], \tag{42}$$

and

$$\begin{aligned} \theta_1 &= [1/a][c - \{c - 1\}b] \\ \theta_2 &= \theta_1 - [c - 1] = [1/a][c - \{c - 1\}\{a + b\}]. \end{aligned}$$

From the mean value theorem, there exists a price, $\bar{p} \in (p, \infty)$, such that

$$\bar{p}q(\bar{p}) = \int_p^\infty q(p) dp. \tag{43}$$

Because $q(p)$ is homogeneous of degree c in price and the optimal price function is given by (41), the function $\bar{p}q(\bar{p})$ is linear in K . Thus, the \hat{s} that satisfies (22) will be independent of K .

To examine the properties of the optimal price schedule, we differentiate the optimal regulatory pricing schedule in (41) with respect to capital investment,

$$\hat{p}_K = -[c-1]^{-1} \hat{p} \hat{K}^{-1} < 0, \tag{44}$$

which confirms Proposition 3. Further, differentiating the optimal regulatory pricing schedule with respect to rate of return, s , we have

$$\hat{p}_s = -[c-1]^{-1} [\theta_1/\theta_2]^{-1} \hat{p} \lambda^{-1} < 0. \tag{45}$$

This leads to the following proposition.

PROPOSITION 6. *Assume a Cobb-Douglas production function and a market demand function homogeneous in price. When there is asymmetric information about variable cost, the optimal regulatory price is a decreasing function of the rate of return on capital the regulator allows the firm to earn.*

Comparing this to the analysis in section II, we see that the presence of an informational gap between the regulator and the firm causes a complete reversal in the direction of the effect of allowed return on price. From (44) it is apparent that $\hat{p}_{KK} > 0$, so that varying the allowed return, \hat{s} , produces a whole family of decreasing convex curves in the price-capital investment space, with movement in the southwest direction corresponding to higher regulatory rates of return. By contrast, the full information relationship between price and capital investment is represented by horizontal straight lines, with higher lines associated with higher rates of return. These relationships are graphically displayed in Figure 1.

The independence of allowed rate of return from the firm's optimal level of capital investment that was established earlier is significant. It implies that the firm will be allowed to earn the same rate of return regardless of its variable cost.¹⁴ The allowed return is constrained only by the firm's cost of capital and the upper bound \bar{S} . However, as we shall show in the subsequent analysis, since the optimal level of capital investment, \hat{K} , varies inversely

14. Some may argue that casual empiricism would indicate that not all regulated firms earn the same rate of return ex post. This, however, cannot be a valid basis for challenging our assumption that the allowed rate of return for firms regulated by the same regulatory agency is the same. This is because cross-sectional variations in realized rates of return may simply be due to these firms facing (non-identical) idiosyncratic risks in a world of uncertainty.

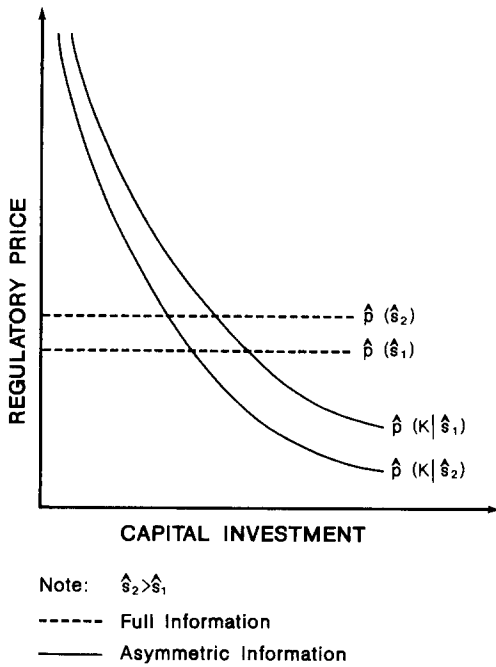


Figure 1. Regulatory Price As a Function of Capital Investment

with variable production cost, w , the firm will earn a larger profit if its variable production cost is lower.

To assess the impact of the optimal regulatory pricing schedule on the firm’s capital investment, substitute the optimal pricing schedule (41) into the expression for net present value, Z , given by (5). This produces

$$Z = r^{-1} \{ [\lambda - r] \hat{K} - w [\lambda^{c/[c-1]a}] \hat{K} - [ba^{-1} + c \{ a[1-c] \}^{-1}] \}. \tag{46}$$

Differentiating net present value first once, and then twice with respect to capital investment, K , yields the expressions

$$Z_K = \{ [\lambda - r] - [\theta_1 / \{c-1\}] w [\lambda^{c/[c-1]a}] K^{\theta_2/[c-1]} \} r^{-1} \tag{47}$$

and

$$Z_{KK} = - \{ [\{ \theta_1 \theta_2 \} / \{c-1\}^2] w^{c/(c-1)a} [K^{\theta_2/c(c-1)}] \} r^{-1}. \tag{48}$$

From (48) it is clear that the second-order sufficient condition for a maximum requires that

$$\theta_1 \theta_2 > 0. \tag{49}$$

With this restriction, the feasibility condition $\hat{s} > r$ is also satisfied. Moreover, from (45), setting $Z_K = 0$, and substituting for λ from (42), we obtain the optimal capital investment as

$$\hat{K}(w; \hat{s}) = [\lambda]^{-c[a\theta_2]^{-1}} [\hat{s} - r]^{[c-1]\theta_2^{-1}} [w\theta_2 [c-1]^{-1}]^{[1-c]\theta_2^{-1}}. \tag{50}$$

Differentiating the optimal capital investment with respect to w , we see that $\partial \hat{K} / \partial w < 0$ since $\hat{s} > r$.

Now consider how the regulator will set \hat{s} . Substitute for \hat{K} from (50) into (46) to obtain

$$\begin{aligned} Z(w; s) &= r^{-1} [\lambda - r] [\theta_2 / \theta_1] \hat{K}(w; \hat{s}) \\ &= r^{-1} [\hat{s} - r] \hat{K}(w; \hat{s}), \end{aligned} \tag{51}$$

where $\hat{K}(w; \hat{s})$ is given by (50). Hence, from (51), we can write producer surplus, PS , as

$$PS = (\hat{s} - r) \hat{K}(w; \hat{s}). \tag{52}$$

Consumer surplus, CS , is $\int_{\hat{p}}^{\infty} q(p) dp$. With the assumed demand function, we have

$$\begin{aligned} CS &= (c-1)^{-1} p^{1-c} \\ &= (c-1)^{-1} \{ \theta_1 / \theta_2 \} [\hat{s} - r] + \{ \theta_2 / \theta_1 \} r \hat{K}(w; \hat{s}). \end{aligned} \tag{53}$$

We now consider the two polar cases, $u = 0$ and $u = 1$. If the regulator's objective is to maximize consumer surplus, then from (53), the first-order condition is

$$[\{ c-1 \} / \theta_2] [\hat{s} - r] + \{ \theta_2 / \theta_1 \} r + [1 - \{ c / a \theta_2 \}] [\hat{s} - r] = 0.$$

Solving for \hat{s} , we have

$$\hat{s} = r + [a / b] [\theta_2 / \theta_1] r > r, \tag{54}$$

since $\theta_2 / \theta_1 > 0$. And if the regulator's objective is to maximize producer surplus, then from (52) we find that

$$s = r + [a / b] [\theta_2 / \{ c-1 \}] > r \text{ if } \theta_2 > 0. \tag{55}$$

The solution is not defined for $\theta_2 < 0$. Thus, we must have $\theta_1, \theta_2 > 0$.

The results established above can be summarized in the following proposition.

PROPOSITION 7. *Assume a Cobb-Douglas production function and a market demand function homogeneous in price. With asymmetric information about variable cost, the regulator must allow the firm to earn a rate of return in excess of its cost of capital whenever either consumer or producer surplus is maximized.*

Thus, if the regulator wishes to use investment-based price regulation to induce a separating equilibrium in which investment levels "convey information," firms must be allowed to earn excess returns, at least for the polar welfare cases. This is in contrast to the full information solution characterized in Proposition 2. This difference arises because, in the latter case, the regulator is compelled to use the promise of superior returns as an instrument to entice the firm to choose an optimal investment level that truly reflects its private knowledge about its variable cost. Clearly, how high the allowed return is set above the firm's cost of capital will depend upon the regulator's social welfare objective. However, given the social welfare objective, the deviation of the allowed return in the information asymmetry case from the allowed return in the full information case will be the key determinant of the welfare loss due to asymmetric information.

This result can be viewed as a possible explanation for empirically documented positive differentials between the rates of return and costs of capital of regulated firms. For example, Roberts, Maddala and Enholm report, ". . . with an imbedded cost of capital of 8 percent, the firm . . . would be granted 8.95 percent" [13,620].

IV. Concluding Remarks

We have explored the regulatory price setting process in an environment in which the monopolistic firm’s variable cost is a priori unknown to the regulator. The only informational requirements we impose are that the regulator is aware of market demand conditions, the firm’s production technology, and its cost of capital. The principal advantages of our approach are its easy implementability, and the fact that it calls neither for randomized auditing strategies nor the threat of service disruption.

The optimal regulatory pricing policy we have characterized is ex ante efficient in the sense that no other feasible policy (a policy that yields the firm a return at least equal to its cost of capital) yields a higher value for the regulator’s social welfare objective. It is, however, not ex post efficient. Once the firm has chosen a capital investment, the regulator can infer its variable cost. The regulator could then adopt other pricing policies which would generate a higher level of social welfare than the ex ante optimal pricing policy and also induce the firm to produce. But if the firm recognizes this possibility, it may not signal truthfully. That is, although the ex ante efficient pricing policy may be dominated ex post, none of these ex post superior pricing policies can do better ex ante.

Although we have examined the case where the factor price is unknown to the regulator, it is clear that equivalent results can be obtained if a parameter that enters linearly in the labor requirements function is a priori unknown to the regulator. For example, the production function could be specified as

$$f(K, L) = AL^a K^b,$$

where the positive constant A can be viewed as a technological efficiency (returns to scale) parameter. One can then assume that A is unknown to the regulator. In such a setting, the form of the results developed in this paper would remain the same. In fact, some may argue that this is a potentially more interesting application than the case of asymmetric cost, since regulators are perhaps likely to be better informed about factor prices than about the firm’s technology.

Appendix

Proof of Lemma. Let \hat{F} denote the bordered Hessian of the production function. By the strict quasi-concavity of the production function, $\hat{F} > 0$. Differentiating (11) with respect to K yields

$$\hat{L}_{KK} = (\hat{f}_L)^{-3} \hat{F}. \tag{A-1}$$

Since $\hat{f}_L, \hat{F} > 0$, (A-1) implies that $\hat{L}_{KK} > 0$.

Now using (3) and the implicit function theorem, we get

$$\hat{K}_L = -\hat{f}_L/\hat{f}_K. \tag{A-2}$$

Differentiating (11) with respect to q and using (A-2), one obtains

$$\hat{L}_{Kq} = -\hat{F}\hat{L}_q(\hat{f}_K\hat{f}_L^2)^{-1}. \tag{A-3}$$

Further, differentiating (10) with respect to p (recognizing that the optimal investment, K , is a function of the regulatory price), yields

$$\hat{K}_p = -(\hat{L}_{KK})^{-1} \hat{L}_{Kq} \hat{q}_p. \quad (\text{A-4})$$

Substituting (A-1) and (A-3) into (A-4) and using (10) and (11) now leads us to the desired result. *Q.E.D.*

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