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**MORAL HAZARD AND SECURED LENDING IN AN INFINITELY
REPEATED CREDIT MARKET GAME***

BY ARNOUD W. A. BOOT AND ANJAN V. THAKOR¹

We analyze repeated moral hazard with discounting in a competitive credit market with risk neutrality. Even without learning or risk aversion, long-term bank-borrower relationships are welfare enhancing. The main result is that the borrower obtains an infinite sequence of unsecured loans at below spot market cost following the first good project realization. This contract produces first-best action choices. Prior to this stage, the borrower gets secured loans with above-market borrowing cost. The optimal contract thus displays a "selective memory" feature, taking only one of two forms at any given point in time, depending on prior history.

1. INTRODUCTION

We consider an infinitely repeated bank-borrower relationship with moral hazard and universal risk neutrality, and explore the ramifications of credit contract duration when the two well-known benefits of long-term contracting—learning and improved risk sharing—are absent. Our objectives are twofold. The first is to analyze an infinitely repeated game with discounting in a credit market context. We thus relate the theory of repeated games to financial intermediation theory, particularly to issues of moral hazard and collateral-related distortions. The second objective is to understand noteworthy stylized facts of credit markets, such as durability in credit relationships, lower borrowing costs for established borrowers, and unsecured loans for established borrowers versus secured loans for newer borrowers. While these practices may be partly due to learning, we show that learning is *not* essential to rationalize them. Even if information decays rapidly and banks learn nothing about borrowers through time, these aspects of credit contracting arise in a repeated-game context.

Our main results are as follows. First, despite universal risk neutrality and the absence of learning, a durable bank relationship benefits the borrower.² The intuition is that the long-term contracting permitted by a durable relationship enables the bank to efficiently tax and subsidize the borrower through time to reduce the use of (costly) collateral. Second, the average welfare loss (due to moral hazard) per period is positive even as the time horizon goes to infinity. Third, the optimal infinite-horizon contract has the striking property that, following the period

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² With a risk-averse agent, the obvious benefit of a long-term contract is that it expands the agent's ability to smooth consumption intertemporally. However, as Fudenberg, Holmstrom, and Milgrom (1990) note, it seems empirically untenable to suppose that consumption smoothing is a major reason for the use of long-term contracts.

in which the borrower encounters the first project success, he is awarded an unsecured loan with a below spot market interest rate in every subsequent period perpetually; this eliminates welfare losses thereafter. Prior to this first success, the borrower must accept a secured loan with an above spot market borrowing cost in every period. It takes a new borrower a finite number of periods to "become established," i.e., achieve its first project success. In this sense, memory plays a role in the Pareto optimal contract (see Rogerson 1985). However, the selectivity of this memory feature leads to a particularly simple dependence of the optimal contract on prior history. The optimal contract asks only whether the borrower has experienced at least one project success. If the answer is yes, the optimal contract always takes one form, and if the answer is no, it always takes another form. As in Abreu (1988), the assumption that payoffs are discounted is important.³

Our research is related to the credit contracting literature as well as the literature on repeated moral hazard with discounting. Moral hazard has long been recognized as a key problem in loan contracting (Stiglitz and Weiss 1981, 1983, Bizer and DeMarzo 1992, Chiesa 1992, and Bhattacharya and Thakor 1993) and this has led to explorations of how collateral influences loan interest rates (Barro 1976), reduces rationing (Bester 1985, and Besanko and Thakor 1987), and resolves incentive problems (Boot, Thakor, and Udell 1991). However, these are static models and none examines how collateral can ameliorate the dynamic incentive problems analyzed, for example, in Stiglitz and Weiss (1983).

The literature on repeated moral hazard (for example, Lambert 1983, Rogerson 1985, and Spear and Srivastava 1987) examines optimal contracts with discounting where multiperiod contracting improves risk sharing. There are four key differences between these papers and ours. First, unlike these papers, ours is concerned with *credit* contracts, specifically the intertemporal role of collateral. Second, because of stronger assumptions and a different model structure, we have a novel characterization of the optimal contract: after a finite number of time periods, the borrower is awarded an infinite sequence of contracts that result in *no* moral hazard. Third, in these models the agent is not allowed to save for future consumption. In our model, this restriction is unnecessary because in equilibrium the agent has no incentive to save. Finally, the gains from long-term contracting in our model do *not* come from improved risk sharing, so that a repeated game is valuable despite agent risk neutrality.

The rest is organized as follows. Section 2 presents the basic model and the single-period solution. Section 3 analyzes the infinitely repeated game. Section 4 presents a Markovian analysis of the infinitely repeated game. Section 5 discusses

³ Moreover, the strikingly simple dependence of the optimal scheme on prior history is reminiscent of Abreu's "simple strategy profile." His result on "optimal penal codes" implies that a player contemplating deviation from an optimal scheme can be deterred by threatening him with the restarting of a punishment already in place since the early stages of the optimal punishment are more undesirable than the remainder. In our model, any project failure *not* preceded by project success essentially "restarts" the "punishment" of requiring the borrower to take a secured loan with an above-market cost of borrowing. The analogy is far from exact, however, since the borrower is "punished" in our model even though he is following the action choice prescribed in equilibrium. That is, we do not have punishments for "deviant" borrowers. Moreover, Abreu's optimal penal codes are not "competitively sustainable" in our model.

model robustness and generalizations. Section 6 concludes. All proofs are in the Appendix.

2. THE SINGLE PERIOD CREDIT MARKET GAME

A. Market Structure and Investment Technology. Consider a competitive credit market with risk neutral agents. Banks compete for loans and can raise elastically supplied deposits at an exogenously given riskless interest factor (one plus the riskless interest rate) $r > 1$. Interbank loan competition leads to credit contracts that maximize borrowers' project surplus subject to incentive constraints and participation constraints (banks must earn at least zero expected profits). The borrower has limited liability; the bank has no claim to any borrower asset other than the project cash flow and collateral.

A borrower takes a \$1 loan to invest in a point-input, point-output project. If successful, the project yields $\$R$ and if unsuccessful, it yields zero. All borrowers possess identical projects, but the probability of project success, $p(\omega)$, is increasing in the action, ω , chosen by the borrower. The private cost of ω for the borrower is $V(\omega) > 0$, with $V' > 0$, $V'' > 0$. Let $p(0) = V(0) = 0$. At the start of the period, each borrower has collateral-eligible wealth, W_0 , that could be used for securing a bank loan. However, liquidating any part of W_0 to self-finance the project is costly. Further, each borrower's project has a strictly positive NPV, no alternative investments are available to borrowers, and taking a loan from a bank is the best way to finance the project. These assumptions guarantee that all borrowers enter the competitive banking system at time $t = 0$.

We assume that collateral is costly in that the bank's valuation of the collateral is a fraction $\beta \in (0, 1)$ of the borrower's valuation. That is, if the borrower posts collateral C and defaults, the bank's net payoff is βC . The difference, $[1 - \beta]C$, is the repossession cost of collateral, stemming from two sources. First, assets acquired as collateral from a delinquent borrower are often worth less piecemeal to the bank than they are to the borrower as components of a productive whole. Second, transferring control of these assets from the borrower to the bank involves legal and other administrative costs, an important reason why bankers value collateral more for its incentive effect than for its intrinsic worth to the bank.

A *credit contract* is an interest factor (one plus the interest rate) α , and a collateral C .⁴ After accepting a credit contract, the borrower chooses a privately observed action ω . However, since there is no other informational uncertainty in the model, the bank can compute ex ante the borrower's action choice for any contract. Thus, the competitive bank's problem is to

$$(1) \quad \max_{\{\alpha, C\}} H = M(\omega^*) - p(\omega^*)\alpha - [1 - p(\omega^*)]C$$

where $M(\omega) \equiv p(\omega)R - V(\omega)$ subject to

$$(2) \quad p(\omega^*)\alpha + [1 - p(\omega^*)]\beta C \geq r$$

⁴ We assume that unless a loan is secured, the bank gets nothing when the borrower's project fails. This is an extreme characterization of the notion that unsecured creditors' claims are highly uncertain in the event of bankruptcy and depend on the outcome of often protracted legal proceedings.

$$(3) \quad \omega^* \in \operatorname{argmax}_{\omega^* \in \Omega} \{M(\omega) - p(\omega)\alpha - [1 - p(\omega)]C\}$$

$$(4) \quad \text{and } \alpha \geq 1, C \geq 0.$$

The objective, (1), is to maximize the borrower's expected net return on the project minus the cost, $V(\omega^*)$, of action ω^* . The participation constraint for the bank is stated in (2); one for the borrower is redundant (all projects have sufficiently high positive NPV, net of action costs, given equilibrium credit contracts). The optimal action, ω^* , is determined by the contract $\{\alpha, C\}$ through the Nash constraint (3), where the borrower's feasible set of actions is $\Omega \equiv \{0, \omega_0, \omega_1\}$ with $0 < \omega_0 < \omega_1$. Finally, (4) represents feasibility constraints on the contracting variables.

Since $p(0) = V(0) = 0$, we assume that the borrower's choice of $\omega = 0$ is always dominated by the choice of participating in the credit market and choosing $\omega > 0$. For notational ease, let $p(\omega_0) \equiv p_0$, $p(\omega_1) \equiv p_1 > p_0$, $V(\omega_0) \equiv V_0$, $V(\omega_1) \equiv V_1 > V_0$.

B. The Solution. Despite universal risk neutrality, a first-best outcome is unattainable. This is counter to the standard result in principal-agent models that the first-best outcome can be achieved through a "pure rental" contract in which the agent gives the principal a fixed amount in exchange for title to all of the random project return, i.e., a solution that is equivalent to the agent holding all of the equity and issuing riskless debt to the principal. Given our assumptions, however, the borrower is only able to issue *risky* debt, unless it secures its debt with collateral large enough to make the bank's payoff state-independent. Even if doing this is optimal, there is a distortion away from first best since collateral involves a dissipative cost.

For a first-best action choice of ω_i , the borrower's equilibrium expected utility (obtained by substituting $C = 0$ and $\alpha = rp_i^{-1}$ in (1)) is

$$(5) \quad p_i R - V_i - r,$$

so that if ω_1 is the first-best action choice, it must be true that $p_1 R - V_1 > p_0 R - V_0$. We will impose the stronger restriction that

$$(R1) \quad p_1[R - 1] - V_1 > p_0[R - 1] - V_0$$

to ensure that the constraint $\alpha > 1$ is always satisfied.

The first-best outcome can be attained if the borrower's action choice can be observed by the bank, so that the incentive constraint (3) can be dropped from the program in (1)–(4). The solution then entails an unsecured contract with $C = 0$ and $\alpha = rp_1^{-1}$, conditional on an action choice of ω_1 . A contract which stipulates that the borrower must pay R in the successful state if it chooses $\omega \neq \omega_1$ will ensure that ω_1 is chosen. Now, consider the second-best case in which ω cannot be observed by the bank. Suppose initially this loan is unsecured. Since (2) can be easily shown to hold tightly in a competitive equilibrium, the bank will set $\alpha(\omega_1) = rp_i^{-1}$ if it anticipates that the borrower will choose the first-best action ω_1 . However, we assume that

$$(R2) \quad p_1[R - rp_1^{-1}] - V_1 < p_0[R - rp_1^{-1}] - V_0$$

so that the first-best action choice can never be enforced with an unsecured loan contract. The bank may, therefore, ask for collateral. Let $\{\alpha^*, C^*\}$ denote the equilibrium secured credit contract involving $C^* > 0$. For $\{\alpha^*, C^*\}$ to be an equilibrium, it must be true that

$$(R3) \quad p_1[R - \alpha^*] - [1 - p_1]C^* - V_1 \geq p_0[R - \alpha(\omega_0)] - V_0 > 0$$

holds. That is, the borrower must be at least as well off with a secured loan and an action choice ω_1 as it is with an unsecured loan and an action choice ω_0 . Note that both alternatives strictly dominate autarky. The (second-best) solution to (1) through (4) is presented below.

THEOREM 1. *The solution, $\{\alpha^*, C^*\}$, to (1) through (4) is*

$$(6) \quad \alpha^* = rp_1^{-1} - [1 - p_1]\beta C^* p_1^{-1}$$

$$(7) \quad C^* = p_1 A_1^{-1} \{-R + rp_1^{-1} + A_2\},$$

where $A_1 \equiv [1 - p_1]\beta + p_1$, $A_2 \equiv [V_1 - V_0][p_1 - p_0]^{-1}$, provided that (R3) holds with (6) and (7) and $\alpha(\omega_0) = rp_0^{-1}$.

Theorem 1 shows that securing a loan can reduce moral hazard. The intuition is that an unsecured loan has a relatively high interest rate to satisfy the bank's participation constraint. Since the borrower pays this interest rate in the successful state, it reduces his marginal return to effort. A secured loan contract results in a smaller reduction in this return to effort because it smooths the borrower's repayment cost over the successful and unsuccessful states.

Could the borrower do better than the second best by liquidating sufficient collateral to raise \$1 and then self-financing? Assuming that the borrower faces the same liquidation costs as the bank, $1/\beta$ of collateral must be liquidated to raise the necessary funds. We now have the following.

COROLLARY 1. *Even if its action choice is unobservable to the bank, it is better for the borrower to take a bank loan rather than self-finance.*

This result implies that the borrower will seek a bank loan in each period. The intuition is as follows. With self-financing, costly liquidation of collateral must occur ex ante, regardless of the project outcome. With a bank loan, such costly liquidation occurs only if the project fails; this economizes on liquidation costs.⁵

⁵ The dominance of a bank loan over self-financing is *not* because actions are discrete. If actions lie in a continuum, a bank loan can mimic the self-financing outcome by specifying a state-independent fee for external funds ($\alpha = C$). Generally, however, the borrower optimally demands a bank loan with $\alpha \neq C$. Since this is optimal for the borrower, the borrower is better off with a bank loan. The bank's participation constraint will again hold tightly at the optimum, and hence the bank loan Pareto dominates self-financing. What does change, however, is that the optimal action choice of the borrower is no longer (necessarily) identical for the bank loan and self financing cases.

3. THE INFINITELY REPEATED CREDIT MARKET GAME

A. *The Nature of Repeated Contracting and the Potential Gains.* Suppose now that the borrower enters the credit market repeatedly for infinitely many discrete periods. Every period the borrower has a one-period, strictly positive NPV project which requires \$1 bank financing. Although the borrower's ability to save is not restricted, the borrower chooses not to save. The reason is as follows. If the borrower's project fails in any period, he lacks investment capital in the next period, necessitating a bank loan. If the project succeeds, the borrower *could* save to reduce the next bank loan. But the optimal long-term credit contract awards the borrower a "better-than-first-best" allocation in perpetuity following a project success. Hence, the borrower finds it privately optimal to *not* save, and needs a loan every period.

The single-period spot credit contract available at any time t , $\{\alpha^*, C^*\}$, is *time-independent* because project payoffs in different periods, conditional on the same borrower action each period, are independent and identically distributed (i.i.d.), and the action choice in a particular period is unaffected, *ceteris paribus*, by past project realizations. This is not true for the long-term contract which specifies a credit contract for each period that may depend on the borrower's credit history. We interpret this as a situation in which, if the borrower defaults in any period, he declares bankruptcy, collateral is transferred to the bank, and the borrower reenters the credit market the next period. Although the borrower's credit history may affect the long-term credit contract available upon reentry, the borrower need not repay debts incurred prior to bankruptcy. In each period the borrower is assumed capable of posting the necessary collateral. These assumptions ensure that the borrower enters the credit market every period and that the Markovian nature of the (discounted) game is preserved. The discount factor per period is $D = 1/r$.

We can now see the motivation for studying a long-term contract in this setting. With single-period contracting, the borrower works hard because it reduces the probability of surrendering collateral to the bank. With long-term contracting the bank has an additional incentive device, namely the promise of below-market financing in the future if the borrower works hard at present. Since the borrower's action is unobservable, the contract cannot be conditioned directly on action. But the bank can condition its promised subsidy on project success, the probability of which is increasing in action. This additional incentive device permits a lower collateral requirement, thereby improving welfare since collateral is dissipative. The bank can recover the present value of future subsidies by charging above-market prices prior to the triggering of the subsidies. From the borrower's standpoint, paying these high prices early in its relationship can be viewed as an investment in establishing creditworthiness, with the returns to this investment expected to be realized in the future when below market cost financing becomes available. This is similar to the reputation mechanism in Shapiro (1983) where high-quality producers initially sell at below cost because they view these losses as investments in reputation, with the returns being realized after reputation is established and the product can be sold at more than cost. In what follows, we make these ideas more precise.

B. *The Key Assumptions.* The competitive bank is constrained to earn zero expected profit over its infinite-horizon relationship with the borrower, which means it could subsidize the borrower in some periods and tax it in some others, subject to two conditions: *competitive sustainability* and *renegotiation-proofness*. Competitive sustainability means that the bank must offer contracts that are at least as attractive to the borrower as spot contracts. Since the borrower's past repayment behavior and the history of credit contracts offered by the bank are common knowledge, the bank has no proprietary information. Thus, at any time t , the entire sequence of present and future (intertemporal) credit contracts offered by the bank must yield the borrower an expected utility at least as great as the expected utility with the sequence of spot credit contracts available then.

Following Fudenberg and Tirole (1990), we define a credit contract as renegotiation proof if it does not provide *both* the bank and the borrower with an incentive to renegotiate the remainder of the contract at a future time.⁶ To see how this constraint can bind, suppose the collateral required in a particular state under the original contract (agreed upon at $t = 0$) is higher than that required in a spot market contract in that state. Assume as well that the interest factor is sufficiently low to preserve competitive sustainability. Such a contract is not renegotiation proof because a lowering of collateral and a raising of the interest rate can reduce collateral-related costs, the benefit of which can be shared by the bank and the borrower.

We do not permit penalties on the borrower for changing banks. This precludes renegotiation of the terms of settlement of a previous contract after the contingency specified in that contract has been realized. For example, let the long-term contract stipulate a particular transfer of collateral from the borrower to the bank if there is project failure (default) in a given period. Suppose failure is observed in that period, and the borrower wants to renegotiate the settlement terms so that no collateral is transferred, in exchange for the borrower's promise to pay that bank higher interest rates on subsequent loans. Our analysis indicates that if the bank agrees to this, the borrower will subsequently switch to a spot loan from another bank, thereby rendering moot his promise to pay the original bank higher future rates, unless such behavior is penalized. The exclusion of such penalties implies that the bank cannot hold the borrower "hostage" and that commitments to future contract terms bind only the bank.⁷ The bank is a *credible institution*. Thus, even though the borrower has the option to walk away from a previously negotiated intertemporal contract, the bank will always honor its commitment.

C. *The Structure of the Repeated Game.* The structure of the infinite-horizon game is shown in Figure 1. The states are numbered from the top down, where x_{t+T}^j is the j th state at time $t + T$. In the following, we distinguish between "upper-half"

⁶ See also van Damme (1987). Because our model differs from Fudenberg and Tirole (1988), renegotiation proofness does not call for mixed strategies.

⁷ The assumption that one party—the employer in labor market games or the bank in credit market games—makes a binding long-term commitment while the other party—the employee or the borrower—has the option to walk away is fairly common. See, for example, Holmstrom and Ricart I Costa (1986). In our context, the effect of credible commitment by the borrower and the effect of penalties on the borrower for leaving the bank are similar: they both serve to reduce dissipative contracting costs if they can be successfully implemented.

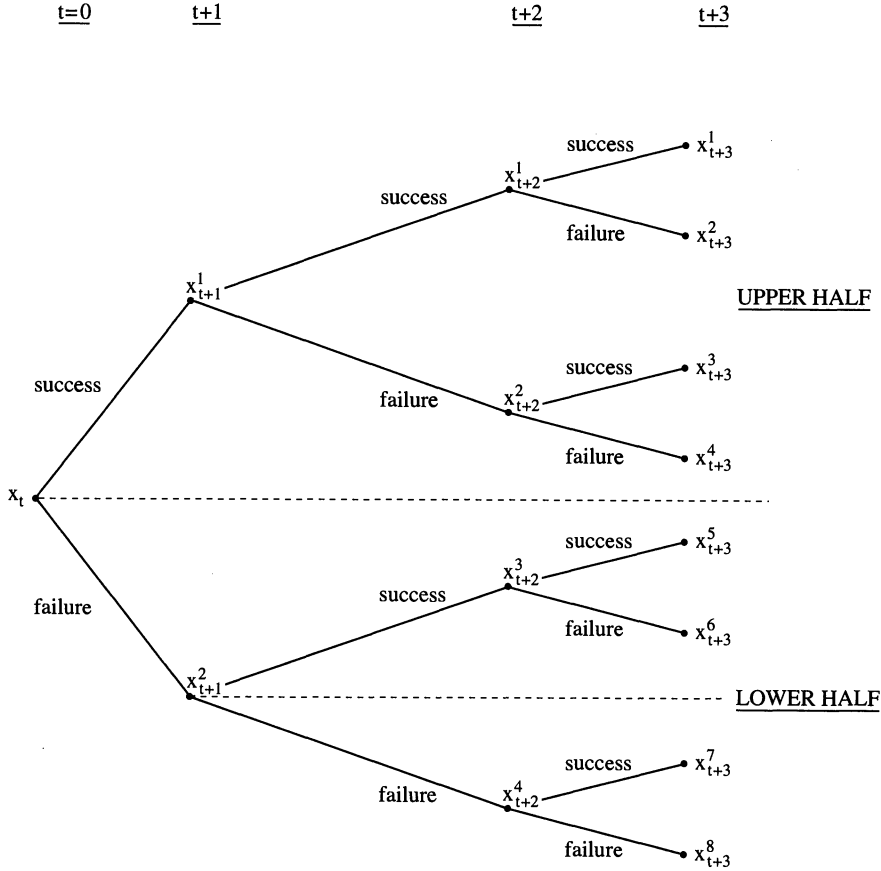


FIGURE 1

THE INFINITE-HORIZON MODEL (THIS ASSUMES THERE WAS NO PROJECT SUCCESS PRIOR TO t)

and “lower-half” states. Suppose that there was no project success prior to time t . Viewed at time t , upper-half states are all future states following project success during $[t, t + 1]$, i.e., the states $UH_t \equiv \{x_{t+1}^1, x_{t+2}^1, x_{t+2}^2, x_{t+3}^1, \dots\}$ in Figure 1; lower-half states are all states following project failure during $[t, t + 1]$, i.e., the states $LH_t \equiv \{x_{t+1}^2, x_{t+2}^3, x_{t+2}^4, x_{t+3}^5, \dots\}$. It is crucial that there was no project success prior to t . Once a project success is encountered, our later results show that the borrower gets the same contract perpetually, so that the notion of upper- and lower-half states becomes meaningless. Thus, if at time $t + 1$, we are in state x_{t+1}^2 , the upper-half states in Figure 1 are $UH_{t+1} \equiv \{x_{t+2}^3, x_{t+3}^5, x_{t+3}^6, \dots\}$ and the lower-half states in Figure 1 are $LH_{t+1} \equiv \{x_{t+2}^4, x_{t+3}^7, x_{t+3}^8, \dots\}$. But if we are in state x_{t+1}^1 at time $t + 1$, such a distinction is vacuous and upper-half and lower-half states are not defined. Thus, when we write UH_{t+1} and LH_{t+1} , it is to be understood that we are in state x_{t+1}^2 at time $t + 1$, not in state x_{t+1}^1 . This motivates some related notation. Suppose we are in state x_t at time t , with no project success prior to t . Then define $\Xi_1 \equiv x_t \cup \{LH_t \setminus \cup_{T=1}^{\infty} UH_{t+T}\}$ and Ξ_2 as the set of all

the other possible states occurring after x_t is realized, i.e., states that are *not* contained in Ξ_1 .⁸ Note that $\Xi_1 = \{\langle x_{t+T}^2 \rangle | T = 0, 1, 2, 3, \dots\} = \{x_t, x_{t+1}^2, x_{t+2}^4, x_{t+3}^8, \dots\}$, where we have normalized $x_t^1 = x_t$, and $\Xi_2 = \{x_{t+1}^1, x_{t+2}^1, x_{t+2}^2, x_{t+2}^3, x_{t+3}^1, \dots, x_{t+3}^7, x_{t+4}^1, \dots\}$. In words, every state in Ξ_1 is preceded in time by a sequence of states *all* involving failure while every state in Ξ_2 is preceded by at least one successful state.

D. Analysis of the Long-Term Contract. To obtain a benchmark, we will first analyze the long-term contract when the bank is constrained to earn nonnegative expected profit in each period. It turns out that in this case the bank offers the spot contract, $\{\alpha^*, C^*\}$ in every state.⁹

THEOREM 2: *If banks are constrained to earn nonnegative expected profits in each period, the infinite-horizon long-term contract involves state-independent contracts that repeat the single period solution period by period, i.e., in every state contained in Ξ_1 and Ξ_2 the contract is $\{\alpha^*, C^*\}$.*

We now discuss the bank's optimal intertemporal allocation of subsidies and taxes when it is not constrained to break even each period. A subsidy in a period means negative expected profit for the bank in that period, whereas a tax means positive expected profit. We now establish our main result.

THEOREM 3. *Suppose that $[A_1]^{-1} - B_3 B_4 [B_5]^{-1} \geq 0$ (this ensures that the feasibility constraint $C_a \geq 0$ is slack at the optimum), where B_3 , B_4 and B_5 are defined below. Then the solution to the infinite-horizon model is state dependent. In every state contained in Ξ_1 the contract is*

$$(8) \quad \alpha_a^* = r[p_1]^{-1} + p_1 D[1 - D]^{-1} B_2 - [1 - p_1] \beta C_a^* [p_1]^{-1}$$

$$(9) \quad C_a^* = \{[A_1]^{-1} - B_3 B_4 [B_5]^{-1}\} p_1 B_2$$

where

$$B_2 \equiv -R + r[p_1]^{-1} + A_2$$

$$B_3 \equiv p_1[1 - [1 - p_1]D]^{-1} - p_0[1 - [1 - p_0]D]^{-1}$$

$$B_4 \equiv D[1 - p_1][1 - \beta]\{A_1[1 - D]\}^{-1}$$

$$B_5 \equiv \{p_1[1 - p_0] - \beta p_0[1 - p_1]\}\{p_1 - p_1[1 - p_0]D\}^{-1} \\ - [1 - \beta][1 - p_1][1 - [1 - p_1]D]^{-1},$$

⁸ For two nonempty sets A and B , $A \setminus B = \{x | x \in A, x \notin B\}$. Note that we could have just as well defined $\Xi_1 = x_t \cup \{\text{LH}_t \setminus \cup_{T=0}^{\infty} \text{UH}_{t+T}\}$. This would have made no difference since the intersection of LH_t and UH_t is empty. The set Ξ_1 only contains those states that will qualify as lower-half states at *any* point in time in the future.

⁹ Nonnegative expected profits together in each period and the competitive sustainability condition imply nonnegative expected profits in each state, i.e., inter-state (but *not* intertemporal) taxes and subsidies are precluded.

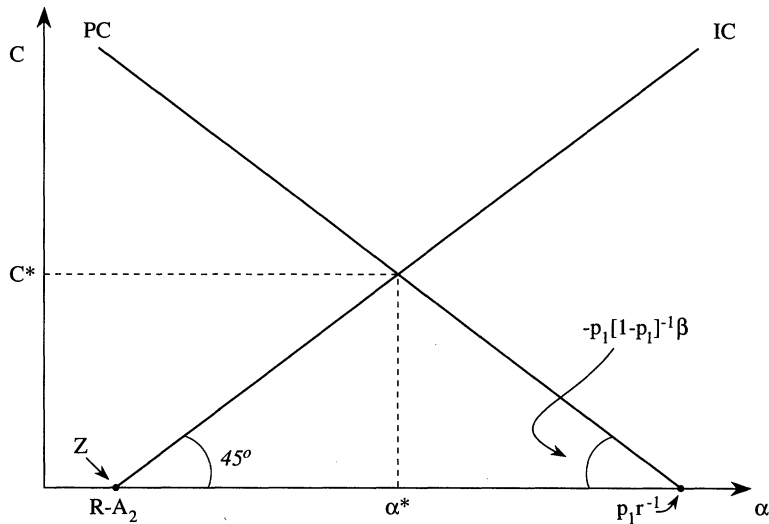


FIGURE 2

EQUILIBRIUM IN THE SINGLE-PERIOD GAME

and we assume that $\alpha_a^* < R$. In every state contained in Ξ_2 the contract is

$$(10) \quad \alpha_b^* = R - A_2 > 1$$

$$(11) \quad C_b^* = 0.$$

To understand the intuition, consider first the one-period problem with the incentive compatibility constraint (IC) given by (3) and the participation constraint (PC) given by (2). These constraints are depicted in Figure 2. To maximize the borrower's welfare, the PC should bind. The IC dictates that eliciting α^* requires C^* ; a higher α would require a higher C and create an additional loss. Hence, $\{\alpha^*, C^*\}$ is the optimal one-period contract (Theorem 1).

Now, when the bank-borrower relationship is repeated, the solution depends on whether the PC needs to be satisfied period by period or only across the relationship time horizon. If the PC is to be satisfied in every period, $\{\alpha^*, C^*\}$ is still the optimal contract (Theorem 2). But if the PC needs to be satisfied only in present value terms across the relationship horizon, then the single-period outcome can be improved upon because intertemporal taxes and subsidies can replace collateral to motivate the borrower. The question is: how should these taxes and subsidies be designed? Since the borrower can walk away from the relationship at any time (the competitive sustainability constraint), the tax should be imposed early in the relationship. Indeed, Theorem 3 asserts that the tax should be paid out of the *first* successful realization of the project, after which C is reduced to zero and α is determined at point Z in Figure 2.

The contract at point Z involves no deadweight loss, and it is incentive compatible because it lies on the IC line. But the bank is losing money since Z lies

below the PC line. It recovers the present value of its losses by charging sufficiently high taxes prior to the granting of subsidies to the borrower, so that the PC is satisfied over the contracting horizon. Since the borrower's payoff is greater the earlier it is awarded the contract at point Z, it now perceives a larger benefit from increasing the project success probability through a higher action choice. This means that less collateral can be used to motivate higher action. The attractiveness of this scheme, therefore, is that incentive compatibility is achieved *without* collateral after the contract at point Z is triggered and with *less* collateral (than in the single-period contract; compare (7) and (9)) before the contract at point Z is triggered.

Theorem 3 shows that extending the contracting period to infinity reduces welfare losses but it does not eliminate them. This is familiar from earlier infinite-horizon models with discounting (e.g., Spear and Srivastava 1987). The surprising aspect of Theorem 3 is that upon one successful performance, the borrower enters an indefinite cycle of states (Ξ_2) with nondissipative contracts. Before that cycle is reached, the credit market has a memory in that future contracts depend on past realizations. But in all periods following a successful state, contracts reflect a selective memory in remembering only the borrower's first project success.¹⁰

4. A MARKOVIAN ANALYSIS OF THE EQUILIBRIUM

The infinite-horizon solution of Theorem 3 can be represented as a Markov chain. We have megastate Ξ_1 where the contract in (8) and (9) is relevant and megastate Ξ_2 where the contract in (10) and (11) is relevant. Theorem 3 specifies a stochastic process $\{Z_t; t = 0, 1, 2, \dots\}$, where t signifies time. The process has a countable state space, $[\Xi_1, \Xi_2]$, and Z_t , the state of the process at time t , is a collection of random variables representing a particular history of project realizations up to time t . The process is Markov because the conditional distribution of any future state Z_{t+1} , given Z_t , is independent of Z_0, \dots, Z_{t-1} .

The Markov chain in this model is reducible and aperiodic. The transition matrix, given the state space $[\Xi_1, \Xi_2]$, is

$$(12) \quad \|P\| = \begin{bmatrix} 1 - p_1 & p_1 \\ 0 & 1 \end{bmatrix}.$$

¹⁰ The infinite-horizon structure of the model serves to guarantee the simplicity of the optimal contract presented in Theorem 3. A finite-horizon structure will *not* unravel the (finite-duration) long-term contract through the "usual" backward induction argument. To see this, consider the last period of some finite horizon. If the project succeeds in the penultimate period, then the contract for the last period in the long-term contract is better than the static first-best contract, so the borrower will prefer the long-term contract. On the other hand, if the project fails in the penultimate period, then the competitive sustainability constraint precludes the final-period contract in the long-term contract being any worse for the borrower than the single-period contract available then. Thus, the long-term contract can not be unraveled.

Note that once a borrower succeeds, subsequent project outcomes are irrelevant for contracting. If the borrower fails in some period after experiencing success, his debt repayment for that period is forgiven and he continues to receive the contract described in (10) and (11) for each of the remaining periods.

From (12) it follows that state Ξ_1 is transient and state Ξ_2 is recurrent (and absorbing). Define $g(i)$ as the expected (one-period) welfare loss for the borrower over the next period when it is in state $i \in \{\Xi_1, \Xi_2\}$. Thus, $g(\Xi_1) = [1 - p_1] \times [1 - \beta]C_a^*$, and $g(\Xi_2) = [1 - p_1][1 - \beta]C_b^*$, where C_a^* and C_b^* are defined in (9) and (11) respectively. Further, define

$$g \equiv \begin{bmatrix} g(\Xi_1) \\ g(\Xi_2) \end{bmatrix},$$

and let g' be the transpose of g . We can now prove the following theorem.

THEOREM 4. *The expected number of time periods until state Ξ_2 is reached is p_1^{-1} . The expected (average) welfare loss per period in the repeated, infinite-horizon game is*

$$(13) \quad L(\infty) = f(\Xi_1)[1 - D], \text{ where}$$

$$(14) \quad f(\Xi_1) \equiv \{[1 - D]g(\Xi_1) + Dp_1g(\Xi_2)\}[1 - D + Dp_1]^{-1}[1 - D]^{-1}.$$

This theorem answers two important questions. First, how long should a borrower expect to wait before being perpetually awarded an unsecured contract with a below-market cost of borrowing? Second, are there any welfare losses in the infinite-horizon game and if so, what is their magnitude? We confirm the well-known result that the welfare losses per period in the infinite-horizon game are lower than in the single-period game. To see this, note that the welfare loss in the single-period game is $L(1) = [1 - p_1][1 - \beta]C^*$. Since $C^* > \max[C_a^*, C_b^*]$, it follows that $L(1) > L(\infty)$, where (13) defines $L(\infty)$.

5. MODEL ROBUSTNESS AND GENERALIZATIONS

In this section we discuss three robustness issues: risk aversion, learning, and discount factor and project payoff variations. Risk aversion and learning are likely to qualitatively strengthen our key result about the value of long-term contracting, whereas the discount factor and the project payoff in the successful state could be made to take values that would alter our results.

(i) *Risk Aversion.* An intended contribution of our paper is to show that long-term contracting is valuable despite universal risk neutrality. Consider, however, the effect of risk aversion. Since collateral imposes losses when a *bad* state is realized, a risk-averse borrower will find secured lending more undesirable than will a risk-neutral borrower. Hence, collateral reduction through long-term contracting will be of even greater value with risk aversion. Moreover, as shown in the previous literature, long-term contracting permits better risk sharing than single-shot transactions.

(ii) *Learning.* The assumption that project payoffs through time, conditional on identical action choices, are i.i.d. means that there is no role for learning in our model. In a sense this implies that the scheme characterized in Theorem 3 is a promising contracting mechanism for transitory disturbances in cash flow, as

opposed to autocorrelated shocks with permanent components. In a setting with autocorrelated shocks, learning would be important.

Recently, Sharpe (1990) has shown that (Bayesian) learning by lenders about borrowers' repayment attributes affects long-term contracts, and long-term contracting with learning is valuable despite universal risk neutrality. Thus, the effect of learning in *our* context will be to reinforce the value of repeated contracting, although it will raise issues related to ex post monopoly rents arising for banks due to the intertemporal accumulation of proprietary information about the borrower. To the extent that interbank competition transfers some of these rents to the borrowers ex ante, learning will also heighten the value of a long-term contract to the borrower, but it is unlikely that the optimal long-term contract will continue to be as simple as our contract.

(iii) *High Discount Factor or Low Payoff in Successful State.* We have assumed thus far that the feasibility constraints $\alpha_a \leq R$ and $C_a \geq 0$ are not violated. The constraint $\alpha_a \leq R$ requires that $R - r$ be sufficiently large, i.e., the project should be sufficiently profitable. Since a higher $R - r$ augments the borrower's marginal return to effort, it lowers moral hazard, so that this restriction implies that moral hazard not be too great. On the other hand, the constraint $C_a \geq 0$ requires that $R - r$ not be too high. A sufficiently high $R - r$ would permit intertemporal contracting to resolve moral hazard without collateral, even with a finite-horizon contract, and would lead to a more complicated contract than our infinite-horizon contract. $C_a \geq 0$ precludes this possibility and establishes a lower bound for the permissible moral hazard.

The two feasibility constraints therefore impose lower and upper bounds on the exogenous parameters. To see how jointly restrictive these constraints are, we conducted a numerical analysis. It reveals that the restrictions on R and D (the inverse of r) are quite reasonable. In Table 1 we show the range of values of R for different constellations of values of the other exogenous parameters such that the feasibility constraints are honored. We also provide the values of the contract parameters endogenously determined in Theorem 3. Even when D is as high as 0.9434, the value of R can be as low as 2.9. In Table 2 we provide a different look at the problem by varying r , holding R fixed. With $R = 2.95$, we see that r can be as low as 1.03 or D as high as 0.9709. Note that β is one measure of the moral hazard cost in that a lower β implies a greater cost to resolving moral hazard. The numerical analysis indicates that the model can accommodate a wide range (0.75 to 0.99) of β values.

Nonetheless, it is interesting to ask: what happens when the feasibility constraints are violated? To examine this, consider $D = 1$ so that the solution in (8) and (9) violates both feasibility constraints. Since these constraints are binding at the optimum, intertemporal subsidies will now be unable to eliminate *all* collateral from all the upper-half states. Thus, if the contract is of infinite duration, then there will remain at least two connected upper-half states (where one is an upper-half state for the other) in which some collateral remains.¹¹ But such a contract cannot be

¹¹ Two states x_{t+T}^i and x_{t+T}^j are connected upper-half states if state x_{t+T}^j is an upper-half state, given that we are in state x_{t+T}^i , i.e., x_{t+T}^j can be reached from x_{t+T}^i and is in the set of upper-half states evaluated from state x_{t+T}^i .

TABLE 1
THE OPTIMAL LONG-TERM CONTRACT PARAMETERS FOR VARYING VALUES OF R

Nonvarying Exogenous Parameters	R	α_a^*	C_a^*	α_b^*
$p_1 = 0.6, p_0 = 0.4, r = 1.06$ (or $D = 0.9434$), $V_1 - V_0 = 0.25, \beta = 0.99$	3.010	1.831	0.004	1.760
	3.000	1.927	0.010	1.750
	2.990	2.023	0.016	1.740
	2.980	2.119	0.022	1.730
	2.970	2.215	0.028	1.720
	2.960	2.311	0.034	1.710
	2.950	2.407	0.040	1.700
	2.940	2.503	0.046	1.690
	2.930	2.599	0.052	1.680
	2.920	2.695	0.058	1.670
	2.910	2.791	0.064	1.660
	2.900	2.887	0.070	1.650
$p_1 = .06, p_0 = 0.4, r = 1.06$ (or $D = 0.9434$), $V_1 - V_0 = 0.25, \beta = 0.75$	3.010	1.831	0.004	1.760
	3.000	1.928	0.010	1.750
	2.990	2.025	0.016	1.740
	2.980	2.122	0.022	1.730
	2.970	2.219	0.028	1.720
	2.960	2.316	0.034	1.710
	2.950	2.413	0.040	1.700
	2.940	2.510	0.046	1.690
	2.930	2.607	0.052	1.680
	2.920	2.704	0.059	1.670
	2.910	2.801	0.065	1.660
	2.900	2.898	0.071	1.650

renegotiation proof. For example, positive collateral requirements in states x_{t+2}^1 and x_{t+3}^1 (where x_{t+3}^1 is an upper-half state for x_{t+2}^1) will trigger renegotiations in state x_{t+2}^1 aimed at reducing collateral in both states. Hence, the original (infinite-duration) solution at time t cannot be an equilibrium. The threat of renegotiations means that optimal contract length will now be finite. In particular, in designing long-term contracts, the optimal contract length will be such that collateral is completely eliminated from all upper-half states for that finite-duration contract; adding more periods to the contract would introduce collateral in connected upper-half states and trigger renegotiations. This will happen when the time- t collateral requirement becomes zero.¹² Thus, the infinite contracting horizon will involve a sequence of finite-duration contracts. At time t , a finite-duration contract will be negotiated for say T periods. Then, at time $t + T$, a new finite duration contract will be negotiated. In each of the finite-duration contracts in the sequence, welfare losses will be eliminated because of the absence of collateral.

6. CONCLUSION

We have analyzed an infinite-horizon model of a competitive credit market in which borrowers can choose unobservable actions that affect payoff distributions. The purpose is to study the potential gains from durable bank-borrower relation-

¹² Recall that removing collateral from upper-half states reduces the time- t collateral requirement.

TABLE 2
THE OPTIMAL LONG-TERM CONTRACT PARAMETERS FOR VARYING VALUES OF r

Nonvarying Exogenous Parameters	r	α_a^*	C_a^*	α_b^*
$p_1 = 0.6, p_0 = 0.4, R = 2.95,$ $V_1 - V_0 = 0.25, \beta = 0.99$	1.320	2.939	0.300	1.700
	1.310	2.927	0.290	1.700
	1.300	2.915	0.280	1.700
	1.290	2.903	0.270	1.700
	1.280	2.890	0.260	1.700
	1.160	2.716	0.140	1.700
	1.150	2.697	0.130	1.700
	1.140	2.678	0.120	1.700
	1.130	2.657	0.110	1.700
	1.120	2.634	0.100	1.700
	1.110	2.609	0.090	1.700
	1.100	2.581	0.080	1.700
	1.090	2.548	0.070	1.700
	1.080	2.510	0.060	1.700
	1.070	2.465	0.050	1.700
	1.060	2.407	0.040	1.700
	1.050	2.330	0.030	1.700
	1.040	2.220	0.020	1.700
	1.030	2.043	0.010	1.700
$p_1 = 0.6, p_0 = 0.4, R = 2.95,$ $V_1 - V_0 = 0.25, \beta = 0.75$	1.290	2.941	0.279	1.700
	1.280	2.928	0.269	1.700
	1.270	2.914	0.258	1.700
	1.260	2.899	0.247	1.700
	1.250	2.885	0.237	1.700
	1.160	2.737	0.143	1.700
	1.150	2.717	0.133	1.700
	1.140	2.696	0.122	1.700
	1.130	2.673	0.112	1.700
	1.120	2.649	0.102	1.700
	1.110	2.622	0.091	1.700
	1.100	2.593	0.081	1.700
	1.090	2.559	0.071	1.700
	1.080	2.520	0.061	1.700
	1.070	2.472	0.051	1.700
	1.060	2.413	0.040	1.700
	1.050	2.335	0.030	1.700
	1.040	2.223	0.020	1.700
	1.030	2.045	0.010	1.700

ships which are commonly observed. We find that it pays for borrowers to negotiate long-term credit contracts with banks even though there is universal risk neutrality and no learning or reputation effects. Our most striking result is that just one successful project realization is sufficient to guarantee the borrower an unsecured loan contract devoid of welfare losses over the rest of its (infinite) planning horizon.¹³ The average welfare loss per period with discounting in our model cannot be eliminated even in the limit. While this is familiar from the previous literature,¹⁴ in our model welfare losses are exaggerated by the competitive sustainability and

¹³ The assumption that the bank can credibly commit to honor the terms of the contract is obviously important for this result.

¹⁴ See, for example, Radner (1981) and Rubinstein and Yaari (1983).

renegotiation proofness constraints on credit contracts. Moreover, it takes a finite number of periods on average to achieve the first project success. Until then the borrower must accept a secured loan with a higher total cost of borrowing than it will have after its first project success. Thus, we explain why banks make secured loans to borrowers without "track records," reserving the privilege of unsecured debt for established borrowers. Moreover, we can rationalize, in a model without learning, why the costs of borrowing in the later stages of the bank-borrower relationship are lower than in the earlier stages.

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APPENDIX

PROOF OF THEOREM 1. Note that we can restate (2) and (3) as

$$(2') \quad p_1 \alpha + [1 - p_1] \beta C \geq r \quad \text{and}$$

$$(3') \quad p_1 [R - \alpha] - [1 - p_1] C - V_1 \geq p_0 [R - \alpha] - [1 - p_0] C - V_0,$$

respectively. From (2), (6) and (7) it follows that (4) is satisfied. Thus, we may formulate the Lagrangian by taking into account only (2') and (3'),

$$L = p_1 [R - \alpha] - [1 - p_1] C - V_1 - \lambda \{r - p_1 \alpha - [1 - p_1] \beta C\} \\ - \mu \{-[p_1 - p_0] [R - \alpha] - [p_1 - p_0] C + V_1 - V_0\},$$

where λ and μ are the Lagrange multipliers associated with (2') and (3') respectively. The first-order conditions are

$$(A.1) \quad \frac{\partial L}{\partial \alpha} = -p_1 + \lambda p_1 - \mu [p_1 - p_0] = 0$$

$$(A.2) \quad \frac{\partial L}{\partial C} = -[1 - p_1] + \lambda [1 - p_1] \beta + \mu [p_1 - p_0] = 0.$$

From (A.1) and (A.2) we get

$$(A.3) \quad \lambda = A_1^{-1} > 0.$$

(A.3) shows that (2') is always binding. Now substitute (A.3) in (A.1) to obtain

$$(A.4) \quad \mu = p_1 [p_1 - p_0]^{-1} \{-1 + A_1^{-1}\} > 0.$$

Since the expression in (A.4) implies $\mu > 0$ for all $\beta \in [0, 1]$, (3') is always binding. We now solve (2') and (3') as equalities to obtain the α^* and C^* given in (6) and (7). (R3) holds by assumption, and a bank anticipating ω_0 will set $\alpha(\omega_0) = r p_0^{-1}$ in a competitive equilibrium. \square

PROOF OF COROLLARY 1. Since the borrower's action choice is identical with self-financing as it is with a second-best (secured) bank loan, we just need to compare expected financing costs in the two cases. With a bank loan, the cost is $p_1 \alpha^* + [1 - p_1]C^* = r + [1 - p_1][1 - \beta]C^*$ (using (6)). With self-financing, the expected financing cost is $r\beta^{-1}$. Thus, we need to show that $r + [1 - p_1][1 - \beta]C^* < r\beta^{-1}$. To see that this inequality holds, write $r\beta^{-1}$ as $r + [1 - \beta]r\beta^{-1}$, use (7) to substitute for C^* and note that $\{-R + rp_1^{-1} + A_2\} < rp_1^{-1}$ (from (R1)). \square

PROOF OF THEOREM 2. We will first prove that the optimal contract is state independent, and then prove that it involves the single period solution $\{\alpha^*, C^*\}$ in each state.

(i) *State-independence.* The proof is by contradiction. Define the vector $\Psi_t \equiv \{\bar{\alpha}, \bar{C}_t\}, \{\bar{\alpha}_{t+T}^j, \bar{C}_{t+T}^j \forall T, j\}$ as the optimal contract (the pair (T, j) refers to the j th state at time $t + T$; see Figure 1). Suppose, counterfactually, that the contract is state dependent, i.e., $\exists \hat{T}, j^0 \ni \{\bar{\alpha}_{t+\hat{T}}^{j^0}, \bar{C}_{t+\hat{T}}^{j^0}\} \neq \{\bar{\alpha}_t, \bar{C}_t\}$. Once the state j^0 occurs at time $t + \hat{T}$, competing banks in the spot market can offer the contract $\Psi_{t+\hat{T}}^{j^0}$, which is identical to Ψ_t except that the initial state is now $x_{t+\hat{T}}^{j^0}$ instead of x_t (the Markov nature of our problem is important here). Given the bank's nonnegative expected profit condition in each state, the original bank's contract Ψ_t from state $x_{t+\hat{T}}^{j^0}$ onwards cannot do better than the (spot) contract $\Psi_{t+\hat{T}}^{j^0}$. Also, starting from state $x_{t+\hat{T}}^{j^0}$, contract Ψ_t cannot yield the borrower less utility than $\Psi_{t+\hat{T}}^{j^0}$ (competitive sustainability). Hence, starting from any state in the future, the remaining part of Ψ_t should do as well for the borrower as the best available (spot) contract, and the (spot) contract (as observed before) is identical in every state. But this can only be true if $\{\bar{\alpha}_{t+\hat{T}}^{j^0}, \bar{C}_{t+\hat{T}}^{j^0}\} = \{\bar{\alpha}_t, \bar{C}_t\} \forall T, j$. That is, we were wrong in supposing $\exists \hat{T}, j^0 \ni \{\bar{\alpha}_{t+\hat{T}}^{j^0}, \bar{C}_{t+\hat{T}}^{j^0}\} \neq \{\bar{\alpha}_t, \bar{C}_t\}$. This proves state independence.

(ii) *Repeated Single Period Solution $\{\alpha^*, C^*\}$.* Given (i), we can write the optimal intertemporal contract as $\{\bar{\alpha}, \bar{C}\}$, which is the contract in every state. The intertemporal maximization program now becomes

$$\max_{\{\bar{\alpha}, \bar{C}\}} [1 - D]^{-1} \{M(\omega^*) - p(\omega^*)\bar{\alpha} - [1 - p(\omega^*)]\bar{C}\}$$

$$\text{subject to } p(\omega^*)\bar{\alpha} + [1 - p(\omega^*)]\beta\bar{C} \geq r,$$

$$\omega^* \in \operatorname{argmax}_{\omega^* \in \Omega} \{[1 - D]^{-1} [M(\omega) - p(\omega)\bar{\alpha} - [1 - p(\omega)]\bar{C}]\} \quad \text{and} \quad \bar{\alpha}, \bar{C} \geq 0.$$

The solution is as in (1) through (4), implying that $\{\bar{\alpha}, \bar{C}\} = \{\alpha^*, C^*\}$. \square

PROOF OF THEOREM 3. Before proving Theorem 3, it would be useful to prove three lemmas. In what follows, a convenient notation for the states is as follows. Viewed at time t , let $x(t, T, k)$ represent an upper-half state at time $t + T$ following k failures during $[t + 1, t + T]$. This parsimonious representation is made possible by the Markovian nature of the problem. For instance, in Figure 1, $x(t, 3, 1)$

corresponds to x_{t+3}^2 as well as x_{t+3}^3 , both of which have the same equilibrium probability of occurrence.

LEMMA 1. *Any intertemporal subsidy to an upper-half state is welfare improving as long as the level of collateral is positive in that state.*

PROOF. Evaluated at time t , the probability of state $x(t, T, k)$ is $p_1[p_1^{T-k-1} \times \{1 - p_1\}^k]$, $0 \leq k < t - 1$. A net transfer from state x_t (see Figure 1) to the upper half state $x(t, T, k)$, evaluated at t , affects the bank's expected profit and the incentive compatibility constraint. As a reference, take the single period solution, $\{\alpha^*, C^*\}$ (see Theorem 2). The linearity of the problem permits the evaluation to be made starting from any incentive compatible, zero profit contract. Let $\{\alpha^1, C^1\}$ represent the state x_t contract and $\{\alpha^x, C^x\}$ the state $x(t, T, k)$ contract. Now, with a transfer of ϕ from state x_t to state $x(t, T, k)$, the perturbation in the bank's zero profit condition for state $x(t, T, k)$ is

$$(A.5) \quad p_1 \Delta \alpha^x + [1 - p_1] \beta \Delta C^x = -\phi.$$

For the perturbation $\{\Delta \alpha^x, \Delta C^x\}$ to preserve incentive compatibility of the state $x(t, T, k)$ contract, we have $p_1 \Delta \alpha^x + [1 - p_1] \Delta C^x = p_0 \Delta \alpha^x + [1 - p_0] \Delta C^0$, and using (A.5) gives

$$(A.6) \quad \Delta \alpha^x = \Delta C^x = -\phi A_1^{-1}.$$

To ensure that the bank's participation constraint is honored, we need

$$(A.7) \quad p_1 \Delta \alpha^1 + [1 - p_1] \beta \Delta C^1 = B_1,$$

where $B_1 \equiv D^T p_1 [p_1^{T-k-1} \{1 - p_1\}^k] \phi$ is the (discounted) expected present value of the transfer at time T . (Recall that $D^T = (1/r)^T$.) Now, rearranging (A.7) yields

$$(A.8) \quad \Delta \alpha^1 = B_1 p_1^{-1} - [1 - p_1] \beta \Delta C^1 p_1^{-1}.$$

Since the perturbation $\{\Delta \alpha^x, \Delta C^x, \Delta \alpha^1, \Delta C^1\}$ is designed to not affect incentive compatibility with respect to action choice, we have

$$(A.9) \quad -p_1 \Delta \alpha^1 - [1 - p_1] \Delta C^1 - [B_1 / \phi] [p_1 \Delta \alpha^x + (1 - p_1) \Delta C^x] \\ = -p_0 \Delta \alpha^1 - [1 - p_0] \Delta C^1 - [p_0 p_1^{-1}] [B_1 / \phi] [p_1 \Delta \alpha^x + (1 - p_1) \Delta C^x].$$

On the left-hand side (right-hand side) we have the effect of the perturbation on expected utility if action ω_1 (ω_0) is chosen. Upon substitution of (A.6) and (A.8) in (A.9), we have

$$(A.10) \quad \Delta C^1 = -(1 - p_1)(1 - \beta) [A_1^{-1}]^2 B_1.$$

(A.6) and (A.10) indicate that, for any net transfer, the use of collateral decreases in both states, and thus welfare losses are reduced. Once the use of collateral in state $x(t, T, k)$ has been reduced to zero, any future transfer only reduces the interest rate in that state which does *not* have any welfare effects (to see this, repeat the analysis with $\Delta C^x = 0$ in (A.5) and see that ΔC^1 is also unaffected). In

establishing the optimal intertemporal contract (Theorem 3), we explicitly recognize that a subsidy between state x_t and state $x(t, T, k)$ alters the borrower's incentives in any state that falls between these two states. \square

LEMMA 2. *Any intertemporal subsidy to a lower-half state is welfare reducing.*

PROOF. Follow the steps taken in proving Lemma 1 but now consider a net transfer from state x_t (see Figure 1) to the lower-half state $y(t, T, k)$, defined as follows. Viewed at time t , $y(t, T, k)$ represents a lower-half state at time $t + T$ following k failures during $[t + 1, t + T]$. Evaluated at time t , the probability of state $y(t, T, k)$ is $[1 - p_1] p_1^{T-k-1} [1 - p_1]^k$, $0 \leq k \leq t - 1$. \square

LEMMA 3. *The net effectiveness—the wealth gain per unit of transfer—of a transfer from state x_t to an upper-half state is equal across states.*

PROOF. It is sufficient to prove that the gain per unit of transfer is independent of T and k . The expected value at time t of the wealth gains associated with the perturbations ΔC^1 and ΔC^x are $-(1 - \beta)(1 - p_1)\Delta C^1$ and $-(1 - \beta)[B_1/\phi][1 - p_1]\Delta C^x$, respectively. Therefore, the total wealth gain is

$$(A.11) \quad -(1 - \beta)\{[1 - p_1]\Delta C^1 + [B_1/\phi][1 - p_1]\Delta C^x\}.$$

A net transfer ϕ to state $x(t, T, k)$ implies a current time t transfer of B_1 . So, the effectiveness of a transfer (\equiv NET) is the wealth gain in (A.11) divided by B_1 , i.e.,

$$(A.12) \quad \text{NET} = -(1 - \beta)\{[1 - p_1]\Delta C^1 + [B_1/\phi][1 - p_1]\Delta C^x\}[B_1]^{-1}.$$

Substitute (A.6) and (A.10) into (A.12) and simplify to get,

$$(A.13) \quad \text{NET} = (1 - \beta)[1 - p_1][A_1]^{-1}\{[1 - p_1][1 - \beta][A_1]^{-1} + 1\}.$$

The resulting expression in (A.13) is independent of T and k . \square

PROOF OF THEOREM 3. We start with the contract $\{\alpha^*, C^*\}$ in all states (see Theorem 2). By Lemma 1, if feasible, all upper-half states get a maximum subsidy such that collateral is removed from the contract for those states. Define $\{\alpha^x, C^x\} = \{\alpha^* + \Delta\alpha^x, C^* + \Delta C^x\}$ as the contract for any upper-half state $x(t, T, k)$ after subsidy. Define ϕ^* as the subsidy received in state $x(t, T, k)$ such that C^* becomes zero. As stated in the proof of Lemma 1, intertemporal subsidies that *only* affect interest rates do not have welfare effects. Thus, these are not considered. Recall from (A.6) and (A.7) that $\Delta\alpha^x = \Delta C^x = -\phi A_1^{-1}$ and $C^* = p_1 A_1^{-1} B_2$. Note that $C^x = C^* + \Delta C^x$ becomes zero (see Lemma 1) for

$$(A.14) \quad \phi^* = p_1 B_2.$$

The subsidy ϕ^* is identical for each state $x(t, T, k)$, and gives (10) and (11). The subscript b (like the superscript x) indicates upper-half states. We now show that (10) and (11) apply not only to upper-half states viewed at time t (identified by the subscript x) but to *all* upper-half states viewed at any time $t + T \forall T \geq 0$ (identified by the subscript b) namely Ξ_2 .

By competitive sustainability and Lemma 2, the contract in state x_{t+1}^2 equals the then available (intertemporal) spot market contract. This is the same as the (spot) contract in state x_t (both states x_t and x_{t+1}^2 entail the same decision horizon). Renegotiation-proofness precludes differences in collateral requirements across states. Thus, the contracts in state x_{t+1}^2 and x_t are the same. This logic applies to any period. Start now in state x_{t+1}^2 (do *not* start in state x_{t+1}^1 , since by Lemma 1 the contract for this state has been subsidized at time t). By Lemma 1, all upper-half states following state x_{t+1}^2 (the states x_{t+2}^3 , x_{t+3}^5 , x_{t+3}^6 , ...) will be subsidized. Again the subsidy in (A.14) is optimal. Therefore, these states will also belong to Ξ_2 . We can now repeat these arguments; this logic identifies equality between the states within the sets Ξ_1 and Ξ_2 , and proves the optimality of $\{\alpha_b^*, C_b^*\}$ in (10) and (11), for the set of states Ξ_2 .

To show the optimality of the contract given in (8) and (9) for the set of states Ξ_1 , recall that the perturbations should not affect the borrower's incentive compatibility constraint and the bank's participation constraint. The total present value to the bank at time t of all intertemporal subsidies ϕ^* (see (A.14)) to upper-half states viewed at time t is

$$(A.15) \quad p_1 D \phi^* + p_1 p_1 D^2 \phi^* + p_1 [1 - p_1] D^2 \phi^* \\ + p_1 p_1 p_1 D^3 \phi^* + \dots = p_1 D [1 - D]^{-1} \phi^*.$$

The bank is compensated for this in the contract for state x_t . The same is true for all the other states in Ξ_1 . Thus, $p_1 \Delta \alpha_a + [1 - p_1] \beta \Delta C_a = p_1 D [1 - D]^{-1} \phi^*$, which implies

$$(A.16) \quad \Delta \alpha_a = D [1 - D]^{-1} \phi^* - [1 - p_1] \beta \Delta C_a [p_1]^{-1}.$$

From (A.6), we have $\Delta C_b = \Delta \alpha_b = -\phi^* A_1^{-1}$. Thus, a subsidized state increases the borrower's utility in that state by $-p_1 \Delta \alpha_b - [1 - p_1] \Delta C_b = \phi^* A_1^{-1}$. Use Figure 1 to see that this has a neutral effect on the borrower's incentive compatibility condition at time t if

$$\begin{aligned} & -p_1 \Delta \alpha_a^* - [1 - p_1] \Delta C_a^* + p_1 D [1 - D]^{-1} \phi^* A_1^{-1} \\ & \quad \text{(state } x_t \text{ and upper-half states following } x_t) \\ & + [1 - p_1] D \{-p_1 \Delta \alpha_a^* - [1 - p_1] \Delta C_a^* + p_1 D [1 - D]^{-1} \phi^* A_1^{-1}\} \\ & \quad \text{(state } x_{t+1}^2 \text{ and upper-half states following } x_{t+1}^2) \\ & + [1 - p_1]^2 D^2 \{-p_1 \Delta \alpha_a^* - [1 - p_1] \Delta C_a^* + D [1 - D]^{-1} \phi^* A_1^{-1}\} + \dots \\ & = -p_0 \Delta \alpha_a^* - [1 - p_0] \Delta C_a^* + p_0 D [1 - D]^{-1} \phi^* A_1^{-1} \\ & + [1 - p_0] D \{-p_0 \Delta \alpha_a^* - [1 - p_0] \Delta C_a^* + p_0 D [1 - D]^{-1} \phi^* A_1^{-1}\} \\ & + [1 - p_0]^2 D^2 \{-p_0 \Delta \alpha_a^* - [1 - p_0] \Delta C_a^* + p_0 D [1 - D]^{-1} \phi^* A_1^{-1}\} + \dots \end{aligned}$$

Some algebra leads to the following simplification

$$(A.17) \quad \{1 - [1 - p_1] D\}^{-1} \{-p_1 \Delta \alpha_a^* - [1 - p_1] \Delta C_a^* + p_1 D [1 - D]^{-1} \phi^* A_1^{-1}\} \\ = \{1 - [1 - p_0] D\}^{-1} \{-p_0 \Delta \alpha_a^* - [1 - p_0] \Delta C_a^* + p_0 D [1 - D]^{-1} \phi^* A_1^{-1}\}.$$

Substitute (A.16) in (A.17) and solve for ΔC_a^* to get

$$(A.18) \quad \Delta C_a^* = -B_3 B_4 [B_5]^{-1} \phi^*.$$

From (A.18) we can derive (9) by substituting (A.14) and adding C^* (since $C_a^* = C^* + \Delta C_a^*$), where C^* is given in (7). Since $[A_1]^{-1} - B_3 B_4 [B_5]^{-1} \geq 0$, we have $C_a^* > 0$. To obtain (8), we substitute (A.18) in (A.16), and use $\alpha_a^* = \alpha^* + \Delta \alpha_a^*$, (6) and (A.14).

Finally, we need to check renegotiation proofness. Note that the contract for the set Ξ_1 equals the best available intertemporal spot market contract; thus, renegotiation proofness is guaranteed. Renegotiation proofness for all states belonging to Ξ_2 holds because welfare losses are absent in the contract $\{\alpha_b^*, C_b^*\}$ and the contract is better than first best. \square

PROOF OF THEOREM 4. The expected time to reach Ξ_2 is the expected number of periods spent in Ξ_1 . From (12) this is $1 + [1 - p_1] + [1 - p_1]^2 + \dots = p_1^{-1}$.

The total discounted welfare losses, starting from state i , are

$$(A.19) \quad F^D g(i) = E_i \left(\sum_{t=0}^{\infty} D^t g(Z_t) \right), \quad i \in \{\Xi_1, \Xi_2\},$$

where $E(\cdot)$ is the expectation operator. Define $f \equiv F^D g(i)$ as the D -potential of the function g . Since g is bounded and nonnegative, $f = F^D g$ is the unique bounded solution of the system of linear equations, $[I - D \parallel p \parallel] f = g$. Thus $f = [I - D \parallel p \parallel]^{-1} g$, and substituting (12) yields

$$(A.20) \quad f = \{[1 - D + Dp_1][1 - D]\}^{-1} \times \begin{bmatrix} 1 - D & -Dp_1 \\ 0 & 1 - D + Dp_1 \end{bmatrix} \times g.$$

(A.20) gives the welfare losses starting from state Ξ_1 . Note that the "discounted" time horizon is $1 + D + D^2 + \dots = [1 - D]^{-1}$. Thus, the expected welfare losses per period are

$$(A.21) \quad L(\infty) = f(\Xi_1)[1 - D] = \{[1 - D]g(\Xi_1) + Dp_1 g(\Xi_2)\}[1 - D + Dp_1]^{-1}.$$

\square

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