Competitive Equilibrium with Type Convergence in an Asymmetrically Informed Market

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This article studies an asymmetric information game with "type convergence," in which, under some realizations of a common uncertainty, inducing informed agents to reveal their types through self-selection by contract choice is either costly or impossible. Under other realizations, self-selection permits costless distinctions between informed agents. I obtain sufficient conditions under which contracting with options prior to the realization of the common uncertainty leads to the existence of a perfectly separating, costless Nash equilibrium. Applications to variable rate loan commitments and life insurance contracting are discussed.

We analyze competitive, asymmetrically informed markets with "type convergence," that is, the ability of the uninformed to discern differences between observationally identical but heterogeneous privately informed agents through a self-selection game is weakened through time for some specific realizations of exogenous uncertainty. In such situations, the earlier the uninformed contract with the informed, the more efficient is the information extraction. We apply this idea to show how forward contracting with *options* can improve welfare under asymmetric information. The setting is that of a competitive credit market. This application generates a rationale for the existence of bank

loan commitments—*credit options* that look like (partial) interest rate insurance policies—in an economy with universal risk neutrality. Moreover, it produces a simple explanation for why formal loan commitments are normally never given to the bank's marginal customers. An application in life insurance is also discussed.

We examine a market with agents privately informed about their types and uninformed agents who will contract with them for the provision of a service/good. The actual compensation of the uninformed at a subsequent date will depend on some random outcome of a venture undertaken by the informed agent. The probability distribution of this random outcome depends on the informed agent's "type," which is private information. The market for the provision of the service/good is competitive and, thus, in equilibrium the uninformed must break even on each transaction. This setting captures the essence of many observed markets. The standard approach to this problem is for the uninformed to offer a menu of contracts that induces a tacit revelation of types through self-selection by contract choice. I call this spot contracting.

Suppose now that instead of analyzing the market game at the time of contracting, we analyze it some time *prior* to contracting. I call this forward contracting. Viewed at that earlier time, suppose there are many possible realizations—at the time of contracting—of some common state uncertainty that will affect the *probability distributions* of random outcomes for all informed agents, but *differentially* across types. Thus, relationships among types will depend on the state at the time of contracting. In some states, types may be costlessly separated through self-selection. In others, this separation may either be costly or totally frustrated because of the lack of a sufficiently rich vector of sorting instruments. In these latter states, there will be welfare losses. The concept of type convergence refers to the presence of such states. To recapitulate, there are three points in time— the point at which forward contracting can occur, the next point at which a common state uncertainty is realized and there is spot contracting, and a third point at which the random outcome for each type is realized.

These welfare losses can be avoided by contracting *prior* to the realization of the state uncertainty. This is because the states in which costless separation is feasible with spot contracting may offer so much flexibility that information extraction with forward contracting could be more efficient. Since the relationship among types changes from state to state, vectors of state-contingent contracts including state-specific subsidies will be differently valued by different types. Thus, at the first point in time, we can promise an agent of a given type a subsidized contract in a particular state at the next point in time and demand a fee at the first point in time in exchange for this promise. This fee-subsidy combination can be designed to be the most attractive for that agent type. In principle, we can design as many such fee-subsidy combinations as there are types, provided that certain (regularity) conditions hold. In all states other than the one in which the subsidy is provided, we can give the recipient of the subsidy its

first best allocation. This enables complete separation with forward contracting that is not possible with spot contracting.

An important feature of the ex ante separation with forward contracting is that it is *nondissipative,* although the possibility of weakening this non-dissipative structure is discussed later. Thus, this analysis has roots in the pioneering work of Bhattacharya (1980) and is related to Heinkel (1982), Brennan (1986), Brennan and Kraus (1987) and Constantinides and Grundy (1986). However, a key distinction is that they provide conditions under which costless equilibria are possible with spot contracting. We derive conditions under which forward contracting with options enables costless separation despite the presence of states in which costless separation is *not* possible with spot contracting. Clearly, these conditions are restrictive, so in some circumstances even sorting with options involves deadweight losses, though this cost is often lower than the spot sorting cost. Thus, this article suggests that options-based forward contracting will *reduce,* rather than always eliminate, informational distortions, by permitting more efficient sorting. In this sense, the analysis reinforces the Ross (1976) observation that options facilitate market completeness by allowing trades over a larger set of states.

Our research is part of the emerging literature on multiperiod contracting.[1] The basic theme of this literature is that multiperiod contracting enables the achievement of allocations that cannot be reached with single-shot arrangements. This is either because multiperiod contracting expands risk-sharing opportunities as in Palfrey and Spatt (1985) or permits incentive constraints to be satisfied with lower welfare dissipation because future allocations can be made contingent on current choices as in Townsend (1982), Boot and Thakor (1988), and others. In these latter models, private information restricts contractual opportunities, thereby leading to a role for contract length. This notion has recently been used by Dunn and Spatt (1988) to rationalize feature heterogeneity and contract length in mortgages.

Increasing contract length beyond one period is one method to weaken incentive constraints under asymmetric information and obtain superior allocations. Alternatively, Kumar (1987) suggests using a dissipative non-pecuniary penalty on the agent in some states to mitigate the binding constraint on ex post pecuniary transfers from the agent to the principal; this lessens surplus extraction by the privately informed agent.[2]

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[1] In our analysis the informed agent purchases an *option* and thus has the ability to abandon the contract. In this regard, the option contract considered here is similar to that in Boot and Thakor (1988) where dynamic credit contracting eliminates the credit rationing that arises endogenously with spot contracting under asymmetric information. The focus of that article, however, is different as it is concerned with dynamic incentive compatibility in a dissipative setting. An article in which contracts do *not* involve constraining commitments on *either* side is Harris and Holmstrom (1987) which explains deterministic boundedness in contract length. Numerous other articles have also examined the contract length issue. Examples are Cantor (1985), Dye (1985a, 1985b), and Gray (1978).

[2] Thus, in Kumar a dissipative cost helps ameliorate distributional inequities, in contrast to our analysis in which this is often achieved costlessly.

The rest is organized as follows. Section 2 discusses type convergence. In Section 3, the concept is applied to the credit market. To get to the intuition as directly as possible, this model is specialized to two borrower types, a two-spike project return distribution for each type, and two possible future spot riskless interest rates, although a generalization is discussed later. In this section, we also analyze the symmetric information equilibrium and the spot market equilibrium under asymmetric information. It is shown that no (separating) equilibrium exists in the spot market under asymmetric information but a costless separating equilibrium exists with a loan commitment. It is also indicated that alternative institutional arrangements may achieve similar allocative results. Section 4 discusses other possible applications. Section 5 concludes. All formal proofs are in the Appendix.

1. Type Convergence and Costless Sorting

Consider a three-date model in a market in which there are $n > 1$ types of informed agents at time zero. Assume all agents—informed and uninformed—are risk neutral. Suppose $t \in \{1, 2, \ldots, n\}$ represents an informed agent's type. If an informed agent transacts at time 1 with an uninformed agent, then there is an outcome at time 2 that is a random variable with a probability distribution conditional on t as well as θ, an exogenous uncertainty which is realized at time 1. Viewed at time 0, θ is a random variable that can take values $\theta_1, \ldots, \theta_m$ with associated probabilities $\beta_1, \ldots, \beta_m$. Let $m > n$. The parameter t is private knowledge for each informed agent at time 0; all other parameters are common knowledge.

I assume away moral hazard by letting t be unalterable. To undertake the endeavor at time 1, each informed agent must acquire some resource from an uninformed agent. The uninformed agent's compensation is a share of the (random) outcome at time 2. The sharing rule is specified in a contract C negotiated at time 1. Let $\mathbf{v}(\theta_i)$ be a vector of sorting variables available at time 1 to the uninformed agents in state θ_i such that all the informed agents can be *costlessly* separated at time 1. That is, if state θ_i occurs, then n distinct contracts can be offered, one for each informed agent, such that a perfect separation is achieved through self-selection by contract choice. Brennan and Kraus (1987) call this a "strongly revealing equilibrium" in state θ_i.

Let $V(\theta_i)$ be the set of all *feasible* contracts, using the sorting variables available in $\mathbf{v}(\theta_i)$, that the uninformed can offer to the informed agents in state θ_i. Feasibility here is taken to mean two things. First, every contract in $V(\theta_i)$ must be generated from the elements of $\mathbf{v}(\theta_i)$. And second, a contract designed for a type-t agent must be such that, if accepted by a type-t agent, it yields the uninformed agent at least his reservation utility. Let $C^\circ(\theta_i, t) \in V(\theta_i)$. Let $U(C^\circ(\theta_i, t), t')$ be the expected utility of a type-t' agent, evaluated at time 1, when he takes a contract designed for a type-t

agent. Since separation is costless in these states, $C^\circ(\theta_i, t)$ is the first best contract for agent type t in state θ_i.

It will *not* always be possible to achieve perfect costless separation for every θ_i. Let Θ be the state space of the random variable, θ, and define

$$\Theta_S \equiv \left\{ \begin{array}{l} \theta_i \in \Theta \mid \exists \ \mathbf{v}(\theta_i) \ni U(C^\circ(\theta_i, t), t) \\ \geq U(C^\circ(\theta_i, t'), t) \ \forall \ t \in \{1, \ldots, n\} \\ \text{and } C^\circ(\theta_i, t) \in V(\theta_i), \\ \text{with } C^\circ(\theta_i, t) \neq C^\circ(\theta_i, t') \ \forall \ t \neq t' \end{array} \right\}$$

$$\Theta_N \equiv \Theta \backslash \Theta_S$$

where $C^\circ(\theta_i, t) \neq C^\circ(\theta_i, t')$ means that the contracts are *nonidentical*, and $\Theta \backslash \Theta_S$ means the set of elements that belong to Θ but not to Θ_S. Note that Θ_N is the set of states in which costless separation is not possible.

To analyze this problem, we adopt the standard competitive Nash equilibrium concept [see, e.g., Rothschild and Stiglitz (1976)]. This precludes pooling allocations and implies that, if an equilibrium exists, the uninformed earn exactly their reservation utility on every contract and the informed agents capture all the surplus. There are two possibilities. One is that, for at least some $\theta_i \in \Theta_N$, $\mathbf{v}(\theta_i)$ can be augmented with *costly* sorting variables. For these θ_i, we would then have a *dissipative* Nash equilibrium. Of course, as is well known from the analysis of Rothschild and Stiglitz (1976) and others, such Nash equilibria sometimes do not exist. In those cases, however, we may appeal to non-Nash equilibrium concepts such as Riley's (1979) reactive equilibrium. On the other hand, it may be impossible to augment $\mathbf{v}(\theta_i)$ with costly sorting variables. In this case, an equilibrium will not exist.

If contracting occurs at time 0 rather than at time 1, we establish conditions to ensure the existence of an equilibrium in which costless separation is achieved for all $\theta_i \in \Theta$. To see this, suppose the number of elements of Θ_S is greater than or equal to the number of types of informed agents, n. For *each* $t \in \{1, \ldots, n\}$, suppose there is at least one $\theta_{i(t)} \in \Theta_S$ such that[3]

$$U(C^\circ(\theta_{i(t)}, t'), t') - U(C^\circ(\theta_{i(t)}, t), t') \geq \tau(\theta_{i(t)}, t)$$

for all $t' \neq t$ and a sufficiently large, real-valued, positive scalar, $\tau(\theta_{i(t)}, t)$, which may vary with t. This assumption means that for each type t, there is a state such that every *other* type strictly prefers his own contract in that state to the contract for type t in *that* state by a sufficiently large amount. We can now define $\hat{\Theta}_S$ as the subset of Θ_S containing those states for which the incentive compatibility constraints are sufficiently slack in the above sense. Moreover, define

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[3] The state $\theta_{i(t)}$ need not be unique. Having more than one state in which the inequalities below hold for a specific t only makes it easier to implement the forward contracting mechanism since subsidies can be offered to the type-t agent in more than one state.

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$$b \equiv \max_{\substack{\theta \in \Theta \\ t' \in \{1,\dots,n\}}} \left\{ \max_{t \in \{1,\dots,n\}} [U(C^\circ(\theta, t), t') - U(C^\circ(\theta, t'), t')] \right\} < \infty$$

as an upper bound on the gain in any state to any type-t' agent from misrepresentation in order to obtain the first-best contract for a type-t agent in that state instead of his own first-best contract. We now have the following proposition.

Proposition 1. *Suppose $\hat{\Theta}_S$ exists and contains at least n states, one for each t. Then, there exists at time 0 a positive number $\hat{b}$ such that, for $b \leq \hat{b}$, contracting with options at time 0 leads to a competitive Nash equilibrium in which every borrower type is costlessly separated at time 0 even though such perfect costless separation is not possible in every state in Θ if contracting is restricted to time 1. Given the conditions that permit this separation at time 0, a Nash equilibrium always exists at time 0.*

This result should be contrasted with the Brennan and Kraus (1987) analysis, which focuses upon spot contracting. Their message is that commonly observed financial securities permit a rich variety of possible combinations such that firms with privately known payoff attributes can often self-select from among these combinations in raising financing and be correctly and costlessly identified in equilibrium. Their mechanism involves investors identifying those types of firms for which an observed financing combination is feasible and pricing that combination as if it were issued by the worst type of firm within the feasible set. They then show that in their "strongly revealing equilibrium," one can construct as many financing combinations as there are firm types and incentive compatibility is guaranteed as each type uniquely selects the combination it most prefers. We start with the premise that the number of states in which a costless strongly revealing equilibrium is possible is at least as great as the number of types. However, there are also other states in which such a costless equilibrium is unattainable. In the Brennan and Kraus framework, these are states in which the risk-adjusted terminal payoff density functions of the informed agents fail to satisfy "strong K-admissibility." We show that, despite the presence of these states, forward contracting with options enables costless separation.

The intuition is as follows. The condition that the incentive compatibility constraints in a given state $\theta_{i(t)}$ are sufficiently slack implies that the allocation received by the type-t agent in that state is sufficiently unattractive to agents of other types. Consequently, the uninformed agents can give the type-t agent a positive subsidy in that state without precipitating a violation of incentive compatibility. This produces negative profits for the uninformed in state $\theta_{i(t)}$ on the contract offered to the type-t agent. To compensate for this, the uninformed can charge a fixed fee at time 0. In each of the states $\theta_{i(t)} \in \hat{\Theta}_S$, there will be a different subsidy associated with the contract offered at time 1 to the agent type whose allocation is sufficiently unattractive to other types in that state. Self-selection, however,

takes place at time 0. Each informed agent can purchase at time 0 an option contract. This contract stipulates a fee (the price of the option) that must be paid at time 0 and the contract that will be given to that informed agent in every possible state at time 1. The actual resources the informed desire from the uninformed will still be provided at time 1 and payoffs will be divided at time 2. However, associated with each type, there is (at least) one state in which that type receives a subsidy; in all other states, that type receives his first-best allocation. Also associated with each subsidy is a distinct fee charged at time 0. A type-t agent will pay the fee that gives him a subsidy in (its most desired) state $\theta_{i(t)}$. Other agents will not covet that contract because they do not value highly an allocation in that state. This achieves complete separation of types at time 0 as every agent selects his preferred contract from among n fee-subsidy pairs. Note that a type t', in deciding not to mimic another type $t \neq t'$, trades off the possible gains from mimicry in states other than $\theta_{i(t)}$ against the disadvantage of accepting the contract designed for type t in state $\theta_{i(t)}$.

Although in the proof of Proposition 1 a distinct fee is charged at $t = 0$ for the purchase of each contract, it is not always necessary to do so. It is sometimes possible to adjust allocations across the realizations of the common uncertainty so that complete separation is achieved costlessly at $t = 0$ without an up-front fee. This is taken up further in Section 4.

Although the proof of Proposition 1 relies on slackness in the incentive compatibility conditions, the argument extends to a continuum of types. Suppose an informed agent's type t belongs to a compact subset, $[t^-, t^+]$, of the real line and that informed agents' preferences are cross-sectionally smooth in type. Let $\beta(\theta)$ be the density function for the random variable θ. Assume $\beta(\theta)$ is continuous and strictly positive on its support, Θ, a compact subset of the real line. Define the subsets Θ_S and Θ_N as done previously. Let $\mu(\cdot)$ be the Lebesgue measure on the real line and assume $\mu(\Theta_S) > \mu([t^-, t^+])$. For *each* $t \in [t^-, t^+]$ suppose there is at least one *subset* $\Theta_{i(t)} \subset \Theta_S$ such that, for $t' \geq t$, $\partial\{U(C^\circ(\theta_{i(t)}, t'), t') - U(C^\circ(\theta_{i(t)}, t), t')\}/\partial t' \geq \tau(\theta_{i(t)}, t) \, \forall \, \theta_{i(t)} \in \Theta_{i(t)}$. Assume $\mu(\Theta_{i(t)}) > 0 \, \forall \, t \in [t^-, t^+]$. This is the continuum analog of the slackness condition involving τ stated earlier for the discrete type case. As before, define b as the finite upper bound on the gain in any state to any type-t' agent from misrepresentation in order to obtain the first-best contract for a type-t agent in that state instead of his own first-best contract. Given our assumptions about $\beta(\theta)$ and $\Theta_{i(t)}$, we have

$$\int_{\theta_{i(t)}} \beta(\theta) \, d\theta > 0 \, \forall \, \Theta_{i(t)} \subset \Theta_S$$

With this structure, we can prove Proposition 1 when informed agents' types lie in a continuum. The derivative condition stated earlier ensures that the *loss* from mimicking in states belonging to Θ_S increases at some minimum rate as the type of the mimicking agent moves a greater distance from the type of the agent being mimicked. Thus, for any two sufficiently disparate types, the incentive compatibility conditions are sufficiently slack,

whereas for two sufficiently similar types, misrepresentation incentives are so small as to require little slackness in the incentive compatibility conditions.

The fact that our results are sustained with a continuum of types suggests potential improvements in the sorting capabilities of even the costless signaling models developed earlier, if these models are modified to conform to our essential structure. For example, in Brennan and Kraus (1987), suppose the project payoff depends on the realization of a common uncertainty—such as the state of the economy—as well as an idiosyncratic random variable whose distribution is private information. Then, costless separation can be achieved with weaker restrictions on the class of possible firm types than those in Brennan and Kraus by allowing the firm, prior to the realization of the common uncertainty, to purchase an option on how to raise financing later.

Two points are worth noting. First, the (forward) contract sold to the informed agent at time 0 is an *option*. At time 1, if the contract for a given state is less attractive than the spot contract available to the informed agent in that state, then the spot contract will be taken. Thus, the time 0 contract is binding only on the uninformed agent. I assume that there is an enforceable contract provision that guarantees that the uninformed agent will honor the time 1 contract. Note that it is mutually beneficial for both the informed and the uninformed to agree on such a provision at time 0. Given this, the equilibrium here is renegotiation-proof [Fudenberg and Tirole (1988)] in the sense that the informed and the uninformed do not *both* have an incentive to renegotiate the contract. Second, to the extent that sorting in the spot market must involve the use of instruments with deadweight costs, contracting with options, whenever feasible, yields unambiguous welfare improvements by eliminating these costs. In the credit market application in the next section, sorting with options has only redistributive effects. However, in Section 4 we discuss how options can also lead to unambiguous welfare improvements by eliminating deadweight losses that would otherwise be incurred.

2. The Credit Market Model and Loan Commitments

Consider a perfectly competitive credit market in which all agents are risk neutral. There are many firms that wish to borrow and many banks. Banks have access to elastically supplied deposits at the spot riskless rate.[4] Competitivity implies that banks compete among themselves to offer borrowers the most attractive contracts, subject to informational constraints (if any) and the constraint that the bank earns zero expected profit. Taxes are assumed to be zero throughout.

The basic idea developed in this section is an application of type con-

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[4] We may view deposit insurance as de facto complete. However, little changes if we relax this assumption.

vergence and is as follows. In an economy in which the qualities of borrowers' investment options are differentially correlated with some economywide state variable, the realization of this state variable will determine the value of each firm's investment option. But the state of the economy will affect *differentially* the values of investment options in the cross section of firms in the economy. When the state of the economy is adverse, some firms will be affected more adversely than others. The levels of investment across otherwise observationally identical firms of heterogeneous qualities will then reflect the manner in which values of the investment options of these firms are influenced by the realized state of the economy. This will permit postinvestment inferences about the qualities of these firms that were otherwise unavailable. However, these inferences may not be possible in every state. In some states, actual investments will *not* reveal firms' types. In these states, losses will manifest themselves either through heterogeneous firms being pooled in the credit allocations that they receive or through firms signaling their types with attribute-related costs.

For concreteness, there are two types of borrowers; call them "good" and "bad." There are three points in time. At time 0, each borrower knows that he will have a single-period investment opportunity available at time 1. Viewed at time 0, there are two economywide states of nature at time 1. In the "favorable" state, the spot riskless rate is low. The current (time 0) spot riskless interest factor (1 plus the spot riskless rate) is R_f. Conditional on R_f, the time 1 spot riskless interest factor will be θ_h with probability (w.p.) $\beta \in (0, 1)$ and θ_l w.p. $1 - \beta$, where $\infty > \theta_h > \theta_l > 1$. At time 1, the type-$i$ borrower will have available a single-period project requiring a \$1 investment and yielding at time 2 a return $R_i(\theta_j)$ w.p. $\delta_i \in (0, 1)$ and 0 w.p. $1 - \delta_i$, where $i \in \{g, b\}$ and $j \in \{h, l\}$. I assume that $R_g(\theta_j) > R_b(\theta_j) \ \forall \ j \in \{h, l\}$, $R_i(\theta_h) < R_i(\theta_l) \ \forall \ i \in \{g, b\}$, $\delta_g > \delta_b$. Thus, both borrower types have project payoffs that are correlated with the economywide state uncertainty (the riskless spot rate) realized at time 1. Each borrower type gets a higher return in the successful state at time 2 if the time 1 spot rate is low than if it is high. However, regardless of the time 1 spot rate, the good borrower has both a higher success probability and a higher return in the successful state than the bad borrower.

At time 0, borrowers are observationally indistinguishable. It is common knowledge that, in the population of borrowers, a fraction $\gamma \in (0, 1)$ consists of good borrowers and a fraction $1 - \gamma$ of bad borrowers. Each borrower has available a liquidity L at time 0. L can be saved at the riskless rate for a period, to obtain $LR_f \in (0, 1)$ at time 1. Thus, even if the borrower saves L entirely for a period, he cannot use it to completely self-finance at time 1. The borrower has two choices. One is to approach a bank at time 0 and purchase a commitment for a loan at time 1. A commitment fee F must be paid out of L for this commitment. The other choice is to carry L over to time 1 and use it as equity in conjunction with a loan from the bank in the spot market.

We first analyze the *first-best equilibrium,* attainable when each borrow-

er's type is common knowledge. In this case, borrowing under loan commitments is Pareto equivalent to spot loans, so I focus on the latter. With first-best spot borrowing, it is irrelevant whether the borrower uses his liquidity as an equity input or consumes it and borrows the entire $1 at time 1. The good borrower's equilibrium expected utility, evaluated at time 0, is

$$\beta\delta_g[R_g(\theta_b) - \theta_b\delta_g^{-1}]R_f^{-1}\theta_b^{-1} + (1 - \beta)\delta_g[R_g(\theta_l) - \theta_l\delta_g^{-1}]R_f^{-1}\theta_l^{-1} + L \qquad (1)$$

where it is assumed that all of L is consumed at time 0 and that

$$R_g(\theta_j) > \theta_j\delta_g^{-1} \qquad \forall \ j \in \{b, \ l\} \qquad (1a)$$

so that the good borrower always wants to borrow at competitive prices. Note that implicit in Equation (1) is the equilibrium break-even condition for the bank. In each state the loan is priced to yield the bank exactly zero expected profit, given a cost of funding equal to the prevailing riskless spot rate. Moreover, discounting from time 1 to time 0 is at the current riskless interest factor and discounting from time 2 to time 1 is at the spot riskless interest factor prevailing then. For the bad borrower, suppose

$$R_b(\theta_b) \leq 1 \qquad (1b)$$

This means that, if the unfavorable state occurs at time 1, the bad borrower has no incentive to participate in the credit market. However,

$$R_b(\theta_l) > \theta_l\delta_b^{-1} \qquad (1c)$$

which implies that the bad borrower will borrow in the favorable state even when his type is correctly identified. With these parametric assumptions, the bad borrower's equilibrium expected utility, evaluated at time 0, is

$$(1 - \beta)\delta_b[R_b(\theta_l) - \theta_l\delta_b^{-1}]R_f^{-1}\theta_l^{-1} + L \qquad (2)$$

Let us now examine the *spot market Nash equilibrium under asymmetric information*. In equilibrium, the contracts offered by banks must maximize each borrower's expected utility subject to the constraints that (1) the bank earns zero expected profit on each borrower and (2) incentive compatibility is guaranteed. The strict break-even requirement on *each* borrower in (1) precludes pooling equilibria.[5] In our context, any pooling allocation subsidizes the bad borrowers at the expense of the good borrowers and is, therefore, susceptible to destruction by a defecting bank that lures away only the good borrowers. Later, I will show that a loan commitment achieves

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[5] Our focus on the Pareto-dominant separating equilibrium results from our use of the Rothschild and Stiglitz (1976) Nash equilibrium concept for a game in which the uninformed agent moves first. However, this separating outcome has received support in the literature on games in which the informed agent moves first. For example, the Cho and Kreps (1987) intuitive criterion rules out pooling equilibria for two types. For more than two types, this criterion admits pooling equilibria, but the universal divinity refinement of Banks and Sobel (1987) often rules them out. For certain values of the bargaining weights, the equilibrium I characterize can also be derived as Myerson's (1984) ''neutral bargaining solution'' in the context of a cooperative (bargaining) game under asymmetric information.

perfect separation without dissipative loss. Thus, the good borrower can never be enticed away from his equilibrium allocation by a pooling contract; this makes the separating allocation robust. The constraints in (2) guarantee that, given his own equilibrium contract, no borrower has an incentive to strictly prefer an equilibrium contract designed for another type of borrower. Such constraints are standard and may be justified by the "revelation principle" of Myerson (1979) and others.[6]

The only way for banks to separate borrowers in the spot market is by offering contracts with differing debt/equity ratios. That is, the bank can offer two credit contracts, each with a distinct loan interest factor and debt/equity ratio, and let borrowers select their preferred contract. If these contracts are designed to be incentive compatible and satisfy the break-even constraint, then each borrower will select the contract that correctly reveals his type and the bank will earn zero expected profit on every contract. Suppose the bank asks a borrower of type i to put up equity $E^i \in [0, LR_f]$ and accept an interest factor $\alpha^i \in (1, \infty)$. Such a borrower will need to borrow $1 - E^i$ from the bank and its contractual repayment obligation will be $\alpha^i[1 - E^i]$.[7] However, our first result establishes that a separating equilibrium is unattainable in the spot market in one state.

Proposition 2. *There exists no (separating) Nash equilibrium in the spot credit market when the riskless spot rate is low.*

The intuition is as follows. In a separating equilibrium in which equity is a "signal" of borrower type, we should find that the "equity cum loan" contract for the good borrower satisfies two conditions: (1) It allows the bank to break even when taken by the good borrower and (2) it is not coveted by the bad borrower. Because of the linearity of the bank's expected payoff in loan size, the break-even loan interest factor is independent of loan size and hence independent of the borrower's equity input. (That is, the borrower's repayment obligation is his loan size times a constant interest factor.) This means that the interest factor must be pegged at $\alpha_l^g = \theta_l\{\delta_g\}^{-1}$ in the low spot riskless rate state. This interest factor in the good borrower's contract is attractive to the bad borrower because it is lower than that in his own contract, $\alpha_l^b = \theta_l\{\delta_b\}^{-1}$. The value of this subsidy in loan interest rate to the bad borrower is an increasing function of its loan size. The key is that, because of borrower risk neutrality, the subsidy value is linearly increasing (decreasing) in loan size (equity). Thus, as long as the loan size is positive, the value of the subsidy is positive. The only way to eliminate it and restore incentive compatibility is to make the loan size

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[6] The revelation principle says that, under asymmetric information, the uninformed can never do better than to design a mechanism that induces each informed agent to directly and truthfully reveal its private information.

[7] Note that it has been assumed here that the borrower carries over its liquidity for a period at the current riskless interest factor R_f.

zero. But in this case, the amount of equity demanded of the good borrower exceeds his available liquidity. This is infeasible.

The "problem" here should now be apparent. Borrower types are different both because they have different investment policies in the high spot rate state, and they have different return distributions when each invests. With spot contracting, only one of these two distinctions can be exploited to learn borrower types. Once the low spot rate state is realized, no information about different investment policies in the high spot rate state is available to lenders. Thus, the opportunity to generate inferences based on that distinction is irrevocably lost ex post. The return distributions distinction still remains. However, the sorting variables available to the bank are ineffective in separating borrowers based on this distinction because both borrowers are willing to put up all their equity to obtain a lower loan interest factor.[8] That is, the indifference curves of both borrower types are horizontal straight lines in E-α space, with expected utility increasing as one moves down.

This intuition opens the door for loan commitments. Because the loan commitment contract is negotiated at time 0, rather than at time 1, it can effectively utilize differences in investment policies across types in the high spot rate state. It is, therefore, more efficient than spot contracting in using all the information about possible differences between types that exists at time 0. This suggests that a useful approach is to think of this as a "convergence" effect with information. At time 0, distinctions between the two types are the greatest. Conditional on the low spot riskless rate state being realized at time 1, these distinctions are smaller. Finally, conditional on the (possibly identical) realized project returns at time 2, these distinctions are not possible to discern. Thus, the earlier the bank contracts with borrowers, the more effective it is in distinguishing between borrowers.

In what follows, the optimal resolution comes in the form of a *variable-rate* loan commitment. This instrument is interesting for two reasons. First, it illustrates the type convergence concept. Second, loan commitments are widely used—commitments at selected large commercial banks in 1985 amounted to $385 billion[9]—and yet our understanding of why they are demanded by risk-neutral agents, such as corporations owned by diversified shareholders, remains incomplete. The interest rate insurance argument— the borrower is partially insuring against a random future borrowing rate by getting the bank to commit to how this rate will be computed—is

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[8] This result obtains because equity is costless. Separation in the spot market may be possible if the setting was similar to that in Besanko and Thakor (1987), for example. They assume dissipatively costly collateral is available. The analog of this in our model would be costly equity. Apart from the fact that a dissipative cost associated with equity is hard to justify without taxes or agency costs, another reason for assuming the absence of a dissipative cost with equity is that *nondissipative* separation is possible in this model with a loan commitment. We have also precluded complex combinations of financial securities with spot contracting as in Brennan and Kraus (1987). This is due to our focus on banks, which are not allowed to hold equity and other "speculative" securities.

[9] From New York Federal Reserve Bank.

unconvincing with risk-neutral borrowers. Also of little help is the Mayers and Smith (1982) rationale for insurance demand by corporations, namely that it ameliorates incentive problems due to delegated monitoring by the insurance company. That logic only explains why the shareholders would want their firm to borrow from a bank, not why a loan commitment would be demanded from the bank.

To resolve the informational asymmetry with loan commitments, the bank can offer two loan commitment contracts $\{(F^i, \alpha_b^i, \alpha_l^i) \mid i \in \{g, b\}\}$, one designed for the good borrowers and one for the bad borrowers. Each contract is a triplet, specifying the commitment fee F^i to be paid at time 0, and the loan interest factors α_b^i and α_l^i applicable on the loan taken under commitment at time 1 in the high and low riskless spot rate states, respectively.

Following Spence (1978),[10] we can formulate the competitive bank's problem as the following constrained optimization program ($i \in \{g, b\}$):

$$\underset{F^i, \alpha_b^i, \alpha_l^i}{\text{Max}} \ U_g(F^g, \alpha_b^g, \alpha_l^g) \tag{3}$$

subject to

$$U_b(F^b, \alpha_b^b, \alpha_l^b) \geq U_b^0 \tag{4}$$

$$U_g(F^g, \alpha_b^g, \alpha_l^g) \geq U_g(F^b, \alpha_b^b, \alpha_l^b) \tag{5}$$

$$U_b(F^b, \alpha_b^b, \alpha_l^b) \geq U_b(F^g, \alpha_b^g, \alpha_l^g) \tag{6}$$

$$F^i \in [0, LR_f) \ \forall \ i \in \{g, b\} \tag{7}$$

$$\alpha_j^i \geq 1 \ \forall \ i \in \{g, b\} \qquad j \in \{h, l\} \tag{8}$$

$$\pi^i(F^i, \alpha_b^i, \alpha_l^i) = 0 \ \forall \ i \in \{g, b\} \tag{9}$$

where $U_j(F^i, \alpha_b^i, \alpha_l^i)$ is the time 0 expected utility of a type-j borrower with the loan commitment contract designed for a type-i borrower, U_b^0 is the first-best expected utility of the bad borrower, defined in Equation (2), and $\pi^i(F^i, \alpha^i, \alpha^i)$ is the expected profit of the bank when a type-i borrower takes the contract designed for him. Thus, equilibrium contracts must maximize the good borrower's expected utility subjects to constraint (4) that the bad borrower gets at least his first-best utility, the incentive compatibility constraints (5) and (6), the feasibility constraints (7) and (8), and the bank's zero expected profit condition (9) for each borrower type. If it is optimal not to use a loan commitment for a type-i borrower, then this program will yield $F^i = 0$ for that type. Also, implicit in the definition of a type-i borrower's expected utility, $U_i(F^i, \alpha_b^i, \alpha_l^i)$, is the notion that the loan commitment is an *option* [see Thakor, Hong, and Greenbaum (1981)

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[10] Spence (1978) provides this mathematical formulation for the Wilson (1977) anticipatory equilibrium. However, his approach is applicable here since our (Nash) equilibrium, when it exists, is also anticipatory.

and Thakor (1982)] and that the borrower will borrow under the commitment only if the commitment rate is no greater than the spot rate the borrower can avail of at time 1; otherwise, the commitment will expire unexercised in that state. The outcome of this maximization program is described next.

Proposition 3. *There exists a $\hat{\beta} \in (0, 1)$ such that a sufficient condition for the existence of a (separating) competitive Nash equilibrium with a loan commitment is that $\beta \geq \hat{\beta}$. In this equilibrium, all good borrowers purchase loan commitments and all bad borrowers borrow exclusively in the spot market. All borrowers enjoy their first-best levels of expected utility.*

This proposition serves three purposes. First, it provides a rationale for loan commitments with universal risk neutrality.[11] Second, it sheds light on the observed link between loan commitment use and borrower quality—loan commitments are usually not available to the bank's weakest borrowers—that was previously unexplained. Thus, an equilibrium is characterized such that *both* spot lending and loan commitments are observed. And finally, it shows that costless separation is attainable with loan commitments.

The intuition should be clear. Good borrowers like to invest in both states, whereas bad borrowers only invest in the low interest rate state. Thus, only good borrowers are willing to pay a fee at time 0 for an interest rate subsidy on borrowing in the high interest rate state at time 1. With risk neutrality, such lump-sum tax-subsidy transfers across states leave borrowers' expected utilities unchanged as long as the bank's expected profit remains anchored at zero. Thus, borrowers are costlessly separated in equilibrium. In general, with many borrower types and many states, we will have borrowers of different types displaying investment preferences that vary differentially across subsets of possible future states. This will permit separation using a spectrum of loan commitment contracts involving distinct tax-subsidy transfers. If the measure of the subset of states in which the borrower finds it profitable to invest is monotonically increasing in borrower quality, then the higher-quality borrowers select loan commitment contracts with larger commitment fees and greater interest rate protection. Moreover, all but the lowest-quality borrowers obtain loan commitments.[12] Thus, we can explain the rich variety of observed loan commitment contracts [see Melnick and Plaut (1986b)].

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[11] Earlier explanations rely on risk aversion or transaction costs. For example, Campbell (1978) and Thakor and Udell (1987) assume risk-averse borrowers, whereas Melnick and Plaut (1986a) assume risk-averse banks. Mason (1979) and Greenbaum, Kanatas, and Venezia (1987) assume commitments help reduce transaction costs. More recent articles such as Boot, Thakor, and Udell (1987) and Kanatas (1987) do not assume either risk aversion or transaction costs. But they do not explain either the observed heterogeneity in loan commitment terms or the borrower quality-linked dichotomy between commitment borrowing and spot market borrowing. Note also that a random future spot rate is inessential to rationalize commitments in our model, but this randomness does permit an explanation for variable-rate commitments.

[12] An alternative story for why loan commitments are not given to weak borrowers is that the lender is concerned about potential deterioration in their creditworthiness and thus does not want to offer a contract

The reason why β, the probability of occurrence of the high spot interest rate state, should be sufficiently high is that the feasibility restrictions [Equations (7) and (8)] may otherwise be violated.[13] If β is relatively low, the *expected* (present) value at time 0 of any interest rate subsidy in the high spot rate state will also be relatively low. This implies, through Equation (9), that F will also be low. Thus, it may not be high enough to deter bad borrowers from purchasing commitments and mimicking good borrowers who are given a very favorable loan rate in the low spot riskless rate state. To prevent this, F must be raised. But this causes α_g^s to be lowered. Unfortunately, Equation (8) puts a lower bound on α_g^s, which means that a sufficiently low β could eliminate any solution to the maximization program in Equations (3) to (9).

It is useful to emphasize that the parameter restriction in Proposition 3 merely guarantees that a (separating) solution exists to the program in Equations (3) to (9). The existence of such a solution guarantees the existence of a Nash equilibrium. Because of the nondissipative separation, the usual unraveling argument to upset the equilibrium does not apply (see the proof of Proposition 1). Thus, our findings are not predicated on any additional parametric restrictions needed to ensure existence of a Nash equilibrium.

I do *not* wish to claim that a loan commitment is the *only* efficient resolution of the informational problem considered here. Another simple arrangement that permits a costless revealing equilibrium is as follows.[14] In the two-type case, suppose borrowers have two choices: either take a fixed-rate, two-period loan at time 0, collateralized by the borrower's equity and involving a prepayment penalty, or take a one-period spot loan at time 1.[15] The two-period loan is priced to break even if taken by the good borrower and the spot loan is priced to break even if taken by the bad borrower. Note that the prepayment penalty on the two-period loan can be made arbitrarily high since it will never be incurred by the good borrower. Under reasonable conditions, the bad borrower will not desire the two-period loan because the occurrence of state h will imply that he must

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that may impose overall losses on the bank in the face of additional private information accumulation by the borrower. However, in such instances the bank may use its priors about the possible future deterioration in the borrower's credit risk and determine a market-clearing commitment price for such borrowers, unless a rationing argument such as that in Thakor (1982) is relevant. Moreover, if the borrower's future private information is correlated with an observable common uncertainty—as in our model—the lender can invoke the ubiquitous "general nervous clause" to deny credit to a customer whose credit risk has "materially deteriorated" since the granting of the commitment. This provides a "legally supported" upper bound on the bank's ex post loss on the commitment, making it feasible to design a commitment contract that breaks even ex ante.

[13] This should be contrasted with Proposition 1, where the sufficiency condition is a restriction on the severity of the incentive compatibility problem. In the two-type case considered here, this would be equivalent to a restriction on the set $\{\{\delta_i, R_i(\theta_j)\} \mid i \in \{g, b\}, j \in \{h, l\}\}$. However, given any such *unrestricted* set, we get a restriction on β.

[14] I thank the referee for pointing out this alternative resolution.

[15] Our explanation for a prepayment penalty can be compared with that in Dunn and Spatt (1985) who observe that the due-on-sale clause in home mortgage contracts is equivalent to a prepayment penalty. In their model, however, the due-on-sale clause is optimal for risk-sharing reasons.

either bear the exorbitantly high prepayment penalty or lose all his equity at time 2 when he can only partially repay the loan from project proceeds. This will make it optimal for such a borrower to wait until state ℓ is revealed and then borrow. Thus, loan maturity is an alternative costless sorting instrument in this type convergence model. Note, however, that loan maturity will not provide complete costless sorting when the common uncertainty is resolved p periods from now and the number of informed agents' types, n, exceeds $p + 1$; there are not as many combinations of the available sorting instruments as there are types. By contrast, the loan commitment solution accommodates even a continuum of types. More generally, though, there may exist mechanisms other than loan commitments that lead to costless sorting in type convergence settings. The loan commitment solution is merely an illustration of one possible mechanism.

3. Other Applications

In this section, I informally discuss an application of the type convergence equilibrium to life insurance[16] and also indicate the usefulness of the concept in other contexts. Two commonly used life insurance policies are term insurance and whole life insurance. The term insurance contract has a short duration and renewal is at rates that depend on future contingencies. With whole life insurance, the same contract applies over the entire lifetime of the insured and the premia remain constant over the duration of the contract. Whole life insurance is considered "more expensive" than term insurance and many have recommended that consumers buy only term insurance.[17] However, unless consumers are irrational, it is hard to explain the existence of whole life insurance in a competitive industry if it is a dominated contract for consumers.[18]

The type convergence idea can be used to explain the purchase of whole life insurance by rational consumers. Suppose we have consumers who become progressively more risk-averse through time, starting out as being risk-neutral, and for each of whom the probability of death depends on his fixed "type" ("smoker" or "nonsmoker") as well as his random state of health. Costly medical exams can reveal the state of health (correlated with the consumer's type) but not a consumer's type or death probability. Smokers are a priori indistinguishable from nonsmokers. The idea is that, early in their life, consumers may not demand insurance because they are risk-neutral. Much later in their life, they may demand whole life insurance

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[16] I thank Michael Brennan for suggesting a life insurance application.

[17] In *Consumer Reports* (1986), Consumer Union recommends the purchase of term insurance over whole life insurance. For a survey of the term versus whole life insurance debate, see Babbel and Ohtsuka (1985).

[18] Berkovitch and Venezia (1986) assume symmetric information to propose that whole life insurance costs more because of the adverse selection created by the implicit option in the contract—consumers whose health deteriorates relative to their cohorts continue with the contract whereas those whose health improves relatively abandon the contract and buy cheaper term insurance. In their model, *all* consumers buy whole life insurance initially, although some switch to term insurance later.

regardless of their type because they are extremely risk-averse. In intervening periods, the consumers' demands for specific insurance contracts—term or whole life—may depend on their type. However, sorting consumers by type during these periods may involve both costly medical exams and some consumers choosing term insurance with possibly suboptimal risk sharing relative to whole life insurance. Contracting in the early stages of the consumer's life may improve welfare. Insurance companies may offer consumers a choice between (1) term insurance without a medical exam early in the consumer's life with the option to convert to whole life without an exam later and (2) term insurance or whole life (with or without an exam) at a later point in time. Consumers who purchase (1) are buying an option because they do not need insurance that early in their life. Depending on their type, they may be willing to pay for this option to distinguish themselves from consumers of another type who may choose (2). The potential benefits of this scheme (relative to a spot contracting outcome in which all consumers buy insurance at an intermediate stage in their lives) are lower medical examination costs and improved risk-sharing.

Two related points are worth noting. First, in contrast to loan commitments, strict welfare improvements may be achieved here. Second, it is easy to relax the ex ante nondissipative structure. As this application suggests, in the dissipative sorting case, type convergence facilitates a reduction in welfare dissipation through options contracting.

Another application of the type convergence notion with strict welfare improvements, rather than redistributive effects only, is to the labor market. Workers with heterogeneous innate abilities and productivities that depend on these abilities as well as the realization of a common uncertainty[19] may be offered wage options that costlessly sort them. With only spot contracting, sorting may require costly verification of abilities; such verification could be obviated by utilizing wage options. Formal work on this application indicates that the optimal scheme requires no up-front fee for the wage option, which is in contrast to the loan commitment example. The implementability of this scheme requires the employer to make a binding commitment while the worker has the option not to exercise the wage option and take a spot contract instead. Harris and Holmstrom (1982) assume this to explain wage rigidity in a dynamic labor market model under *symmetric* information.

4. Conclusion

This article has discussed the concept of type convergence and applied it to the credit market. The basic idea in type convergence is that payoff-relevant heterogeneity among observationally identical, informed agents

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[19] A simple example of a setting in which the worker's productivity depends both on ability and a realized common uncertainty is sharecropping. A farmer's productivity is a function of his skill as well as rainfall.

declines through time if certain states of nature—in which informed agents of different types would have made disparate choices—fail to be realized. Thus, the temporal resolution of an exogenous uncertainty may cause a loss of valuable information that could have permitted the ex ante sorting of informed agents through self-selection by contract choice. In our model, this effect implies that options enable *costless* sorting that spot credit contracts do not. To the extent that sorting in the spot market would have involved dissipative costs, options directly improve efficiency by eliminating these costs. Thus, this paper shows that the presence of asymmetric information magnifies the welfare gains from the creation of options.

Our credit market application of this idea enables us to rationalize variable-rate loan commitments in a risk-neutral environment and explain why such commitments are usually not available to the bank's marginal customers. An informal discussion of life insurance and labor markets indicates the richness of potential applications of forward contracting in asymmetric information settings satisfying the type convergence property.

The analysis in this article has been carried out with fairly simple models, but its substantive conclusions are robust. Extensions of the model have indicated that introducing a continuous probability distribution for the exogenous uncertainty and a continuum of privately informed agent types do not alter the main results. In like vein, no parametric restrictions are needed for the existence of equilibrium, beyond the mild sufficiency condition that delivers the existence of a solution to the maximization program used to generate equilibrium allocations. Given these observations on robustness and the earlier remarks on the potential scope of other applications, it is hoped that the type convergence idea can be fruitfully applied to other aspects of financial markets as well. It should enable us to better understand the welfare implications of forward markets in general.

Appendix

Proof of Proposition 1

For each state $\theta_{i(t)} \in \hat{\Theta}_s$, let $\beta_{i(t)}$ be the probability of occurrence. Focus on a particular $t' \in \{1, \ldots, n\}$. (The general notation for the probability of a state $\theta_i \in \Theta$ is β_i; this includes all θ's in $\hat{\Theta}_s$ and those that do not belong to $\hat{\Theta}_s$.) The first-best contract offered to this agent in the spot market is $C^\circ(\theta_{i(t')}, t')$. Perturb this contract now and obtain a new contract, $\tilde{C}(\theta_{i(t')}, t')$, that is even more attractive for the type-t' agent and satisfies a condition that will be stated later. Define the gain, relative to the first best, to the type-t' agent from the perturbed contract

$$Z(t') \equiv U(\tilde{C}(\theta_{i(t')}, t'), t') - U(C^\circ(\theta_{i(t')}, t'), t') > 0 \qquad \text{(A1)}$$

In every other state $\theta_i \in \Theta$, give the type-t' agent his first-best contract, $C^\circ(\theta_i, t')$. Now, let $d(t')$ be the (discounted) loss in expected profit for the uninformed agent from offering the type-t' agent contract $\tilde{C}(\theta_{i(t')}, t')$

instead of $C°(\theta_{i(t')}, t')$ in state $\theta_{i(t')}$ and this contract is taken by the type-t' agent. Note that $d(t') > 0$ or else $C°(\theta_{i(t')}, t')$ was not a first-best contract. The uninformed agent then asks for a fee $F(t') = \beta_{i(t')} d(t')$ of any agent who wants the contract vector

$$\{C°(1, t'), C°(2, t'), \ldots, \tilde{C}(\theta_{i(t')}, t'), \ldots, C°(m, t')\}$$

This fee must be paid at time 0. It enables the uninformed agent to break even in view of the subsidy in the perturbed contract. The condition, alluded to earlier, that the perturbation $Z(t')$ must satisfy is that

$$\min_{\substack{t \in \{1,\ldots,n\} \\ t \neq t'}} \{U(C°(\theta_{i(t')}, t), t) - U(\tilde{C}(\theta_{i(t')}, t'), t)\} = \tau(\theta_{i(t')}, t') \quad \text{(A2)}$$

We do the above for every $t \in \{1, n\}$. The idea is to fully exhaust all available slackness in the incentive compatibility (IC) constraints. Now, the IC constraints are, for every $t, t' \in \{1, \ldots, n\}$,

$$-F(t') + \delta\beta_{i(t')}U(\tilde{C}(\theta_{i(t')}, t'), t') + \delta \sum_{\substack{j=1 \\ j \neq i(t')}}^{n} \beta_j U(C°(\theta_j, t'), t')$$

$$\geq -F(t) + \delta\beta_{i(t)}U(\tilde{C}(\theta_{i(t)}, t), t') + \delta \sum_{\substack{j=1 \\ j \neq i(t)}}^{n} \beta_j U(C°(\theta_j, t), t') \quad \text{(A3)}$$

where δ is the single-period utility discount factor. Now, by definition of first best, and the fact that U is linear, we have

$$-F(t') + \delta\beta_{i(t')}U(\tilde{C}(\theta_{i(t')}, t'), t') = \delta\beta_{i(t')}U(C°(\theta_{i(t')}, t'), t') \quad \text{(A4)}$$

Substituting Equation (A4) in Equation (A3) yields

$$\delta \sum_{j=1}^{n} \beta_j U(C°(\theta_j, t'), t') \geq -F(t) + \delta\beta_{i(t)}U(\tilde{C}(\theta_{i(t)}, t), t')$$

$$+ \delta \sum_{\substack{j=1 \\ j \neq i(t)}}^{n} \beta_j U(C°(\theta_j, t), t') \quad \text{(A5)}$$

If we replace $U(\tilde{C}(\theta_{i(t)}, t), t')$ by $U(C°(\theta_j, t), t')$ in Equation (A5), then we need to show that

$$\delta \sum_{\substack{j=1 \\ j \neq i(t)}}^{n} \beta_j U(C°(\theta_j, t'), t') \geq -F(t) + \delta \sum_{\substack{j=1 \\ j \neq i(t)}}^{n} \beta_j U(C°(\theta_j, t), t') \quad \text{(A5a)}$$

Note that, since $U(C°(\theta_j, t), t') \geq U(\tilde{C}(\theta_{i(t)}, t), t')$, Equation (A5a) is a stronger restriction than Equation (A5). Thus, if Equation (A5a) is shown to hold, Equation (A5) will have also been shown to hold. Rearranging Equation (A5a) we get

$$\delta \sum_{\substack{j=1 \\ j \neq i(t)}}^{n} \beta_j[U(C^\circ(\theta_j, t), t') - U(C^\circ(\theta_j, t'), t')] \leq F(t) \qquad (A6)$$

Because b is at least as large as any $U(C^\circ(\theta_j, t), t') - U(C^\circ(\theta_j, t'), t')$, to show that Equation (A6) holds, it is sufficient to show that

$$\delta \sum_{\substack{j=1 \\ j \neq i(t)}}^{n} \beta_j b \leq F(t)$$

which is true for every t if we define $\hat{b}$ through the relationship

$$\min_{t \in \{1,\dots,n\}} F(t) = \hat{b}$$

We have thus shown that the incentive compatibility constraints needed for complete separation at time 0 are satisfied. What remains is to prove that this is a (competitive) Nash equilibrium. To see this, note that every informed agent gets his first-best allocation at time 0 under this scheme. Thus, there does not exist any pooling contract at time 0 that pools two types, say t' and t, gives *each* a higher expected utility than his equilibrium allocation, *and* allows the uninformed agent offering the pooling contract to earn nonnegative expected profit on the contract. Thus, there is no viable defection from the equilibrium, and a perfectly separating, competitive (zero profit for the uninformed) Nash equilibrium exists. ∎

Proof of Proposition 2
In the two-type case, we know that the good borrower (the type that is "jeopardized") will use the signal (equity here) and the bad borrower will not signal but will receive his first-best allocation. We will focus here only on the low interest rate state since incentive compatibility is trivially satisfied in the high interest rate state. Suppose a good borrower's contract requires him to put up equity $E \in [0, LR_l]$ and borrow $1 - E$ from the bank. The net expected utility of the good borrower with this contract is

$$\delta_g[R_g(\theta_l) - \{1 - E\}\theta_l\{\delta_g\}^{-1}] - E\theta_l \qquad (A7)$$

where we recognize that the competitive interest factor—the one that gives the bank zero expected profit—is $\alpha_l^g = \theta_l\{\delta_g\}^{-1}$. If a bad borrower misrepresents his type and takes the good borrower's contract, then its expected utility will be

$$\delta_b[R_b(\theta_l) - \{1 - E\}\theta_l\{\delta_g\}^{-1}] - E\theta_l \qquad (A8)$$

The bad borrower's expected utility with his own first-best contract is

$$\delta_b[R_b(\theta_l) - \theta_l\{\delta_b\}^{-1}] \qquad (A9)$$

where we take the competitive interest factor $\alpha_l^b = \theta_l\{\delta_b\}^{-1}$. In equilibrium, the incentive compatibility condition guaranteeing that the bad borrower will not envy the good borrower always holds tightly. That is, we have

$$\delta_b[R_b(\theta_l) - \{1 - E\}\theta_l\{\delta_g\}^{-1}] - E\theta_l = \delta_b[R_b(\theta_l) - \theta_l\{\delta_b\}^{-1}]$$

Solving the above equation gives $E = 1$. That is, if a separating equilibrium is to exist, it must involve the good borrower being totally excluded from the credit market. However, it is not feasible for the good borrower to completely self-finance the project because his available liquidity is less than \$1. ∎

Proof of Proposition 3

The expected utility of the bad borrower with a loan commitment (evaluated at time 0) is

$$[1 - \beta][\delta_b\{R_b(\theta_l) - \theta_l\delta_g^{-1}\}]R_f^{-1}\theta_l^{-1} - F + L \tag{A10}$$

and without a loan commitment it is

$$[1 - \beta][\delta_b\{R_b(\theta_l) - \theta_l\delta_b^{-1}\}]R_f^{-1}\theta_l^{-1} + L \tag{A11}$$

For incentive compatibility, the equilibrium F equates Equations (A10) and (A11) and is

$$F = R_f^{-1}[1 - \delta_b\delta_g^{-1}][1 - \beta] \tag{A12}$$

Note that $F < 1$. I will assume also that $F \le LR_f$. Now, for the good borrower, $\alpha_l^g = \theta_l\{\delta_g\}^{-1}$. Thus, we must set his interest factor α_b^g in the high riskless spot interest rate state to satisfy the following zero expected profit condition for the bank:

$$F + \delta_g \alpha_b^g \beta\{\theta_b R_f\}^{-1} = \beta\{R_f\}^{-1}$$

Solving this equation yields

$$\alpha_b^g = [\beta - \{1 - \beta\}\{1 - \delta_b\delta_g^{-1}\}][\delta_g\beta\{\theta_b\}^{-1}]^{-1} \tag{A13}$$

Feasibility requires that $\alpha_b^g \ge 1$ (negative interest rates are not allowed). From Equation (A13), this means that

$$\beta[1 - \beta]^{-1} \ge [\delta_g - \delta_b]\theta_b\{[\theta_b - \delta_g]\delta_g\}^{-1} \tag{A14}$$

This will hold if β is large enough, say greater than some $\hat{\beta} \in (0, 1)$.

All that needs to be verified is that the good borrower's expected utility with the loan commitment is equal to first best. The good borrower's expected utility with the loan commitment is

$$\beta R_f^{-1}\theta_b^{-1}[\delta_g\{R_g(\theta_b) - \alpha_b^g\}] + [1 - \beta]R_f^{-1}\theta_l^{-1}[\delta_g\{R_g(\theta_l) - \theta_l\delta_g^{-1}\}] + L - F \tag{A15}$$

The good borrower's expected utility in the first-best case is

$$\beta R_f^{-1}\theta_b^{-1}[\delta_g\{R_g(\theta_b) - \theta_b\delta_g^{-1}\}] + [1 - \beta]R_f^{-1}\theta_l^{-1}[\delta_g\{R_g(\theta_l) - \theta_l\delta_g^{-1}\}] + L \tag{A16}$$

It is straightforward to verify that the quantities in Equations (A15) and

(A16) are equal. It is also easy to check that the good borrower *strictly* prefers his own loan commitment contract to the (first-best) contract of the bad borrower. Finally, the existence of a Nash equilibrium can be argued along the same lines as in the proof of Proposition 1. ∎

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