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AN ECONOMIC RATIONALE FOR THE PRICING STRUCTURE OF BANK LOAN COMMITMENTS

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An economic rationale is provided for the competitive equilibrium deployment of commitment and usage fees in loan commitment pricing. It is shown that, under perfect information, assessing both fees rather than just one permits optimal risk sharing. When the borrower is privately informed about its probability of future commitment utilization, commitment and usage fees can be used to induce borrowers to identify themselves by self-selection through contract choice. The equilibrium characterized here is dissipative and thus raises the usual existence questions which are addressed in the paper.

1. Introduction

Although bank loan commitments have long been in existence, they have recently assumed a crucial position in bank management and regulation due to an enormous growth in their volume over the last two decades. This has caused a concomitant surge of interest in the theoretical literature. Some writers have emphasized the role that loan commitments may play in the mechanics of monetary policy [for example, Deshmukh, Greenbaum and Kanatas (1982)]. However, most of the recent literature has focused on pricing issues [for example, Hawkins (1982)]. Pricing in turn is linked to the economic motivation for the loan commitment. Many papers model loan commitments as put options [Thakor, Hong and Greenbaum (1981), Thakor (1982) and Ho and Saunders (1982)]; some papers emphasize the role of risk aversion [for example, Campbell (1978)], and others rely on liquidity [Sealey and Heinkel (1985)] and shifts in credit risk [James (1981)]. Except for the paper by James (1981), however, there is no explanation for the pricing *structure* of loan commitments. That is, most of the existing literature essentially assumes that the bank is compensated for the contingent liability imposed by a commitment through a single commitment or facility fee paid by the borrower at the front end of the commitment.[1] In practice, however, pricing is more complex. The purpose of our paper is to explain the equilibrium existence of the multiple fee structure that usually characterizes commitment pricing.

In pricing loan commitments, banks typically assess two fees at two points in time. One fee, based on the total loan commitment, must be paid by the borrower when the commitment is issued by the bank. The other fee, based on how much of the commitment is used, must be paid when borrowing actually takes place. While James (1981) explains a borrower's choice between a commitment fee and a compensating balance, the loan commitment literature offers no satisfactory explanation for the simultaneous deployment of commitment and usage fees.[2] Our principal objective is to provide an economic rationale for the popular utilization of this pricing structure under perfect and imperfect information. In section 2 we develop a model in which loan commitments provide optimal risk sharing between borrowers and lenders (i.e., banks) given interest rate and takedown uncertainty. We demonstrate that a split structure can characterize loan commitment pricing even under perfect information. In section 3 we drop the assumption of perfect information. There are two types of borrowers with different takedown probabilities. Each borrower knows its own takedown probability, but the bank is a priori unable to distinguish between the two borrower types. In this case we show that the 'split' pricing structure has an added role to play; it permits the bank to offer two types of contracts designed to induce each borrower to self-select and reveal its type. Throughout, banks are assumed to be competitive. Under asymmetric information, Cournot–Nash [Rothschild and Stiglitz (1976)] and reactive [Riley (1979)] competitive equilibria are analyzed. A comparison of our work with some other related papers appears in section 4. Section 5 concludes.

2. Pricing under perfect information

Consider an economy that lives for two time periods. The first period begins at $t=1$ and ends at $t=2$, and the second period begins at $t=2$ and ends at $t=3$. The subset of the economy we focus on consists of lenders – whom we shall call 'banks' – and borrowers. Banks are risk neutral and

[1]Even papers such as Bartter and Rendleman (1979), which recognize ex post settling up mechanisms like compensating balances and usage fees based on the actual usage of the commitment, do not attempt to explain *why* such pricing exists.

[2]A direct analogy to this is the pricing of life insurance in the absence of moral hazard. Assuming that both the insurer and the insured are symmetically aware of the probability of death at any instant in time and that all insurers are risk neutral price takers, the insurance premium function can always be designed so that the expected present value of the inflow of the insurance premia equals the expected present value of the face value of the insurance policy. Once this is achieved, it matters little when the insured actually dies, as long as the probability of death is not manipulated.

borrowers are risk averse. Each borrower is assumed to possess a smooth and concave, additively time-separable von Neumann–Morgenstern utility function over wealth of the form

$$U(W_1, W_2, W_3) = \sum_{t=1}^{3} V(W_t), \tag{1}$$

where $V(\cdot)$: $\mathbb{R} \to \mathbb{R}$ is twice continuously differentiable everywhere and satisfies $V'(\cdot) > 0 \, \forall \, W < \infty$, $V'(0) = \infty$, $V'(\infty) = 0$, and $V''(\cdot) < 0$. W_t denotes the borrower's wealth at time t. Throughout, the symbols $\mathbb{R}$ and $\mathbb{R}_+$ are used to denote the real line and the positive real line, respectively. Of course, more generally, if we assume that the single period utility function $V(\cdot)$ is the same intertemporally, we should express the borrower's utility as

$$U(W_1, W_2, W_3) = \sum_{t=1}^{3} \omega_t V(W_t),$$

where $\omega_t \varepsilon (0, 1)$ and the ω_ts form a declining sequence through time. We have ignored the discounting of utility to keep the algebra simple, but it should be noted that, except for minor modifications, all the results go through even if an ω_t is inserted in the borrower's utility.

Each borrower is assumed to have an exogenously determined wealth endowment $\Omega_t \varepsilon \mathbb{R}_+$ at time t. At $t=3$, Ω_3 will be augmented by a cash flow generated by a project initiated at $t=1$. Without loss of generality, the initial investment outlay for the project is subsumed in Ω_1. That is, if no investment is undertaken, the borrower's endowment at $t=1$ would be Ω_1 plus the amount of the initial investment outlay. However, the cash flow at $t=2$ is uncertain. That is, with probability $q \varepsilon (0, 1)$ the project may be self-financing at $t=2$ – in the sense of not requiring an additional outlay – depending on asset turnover. If accounts receivable turnover and/or inventory turnover is quick (or in the case of a real estate development, sale of earlier phases of the project is strong), bank financing is not necessary at $t=2$. Under these conditions the project will yield a net return of K at $t=3$. If, on the other hand, turnover is slow, additional project financing of I will be necessary at $t=2$. Financing must be obtained from a bank. Because this financing is essentially a working capital loan to carry short term assets, net project return at $t=3$ before repayment of the bank loan will be $K+I$. In essence, these conditions imply that the asset turnover at $t=2$ affects not the gross return or liquidation value of the project but only its timing.

We have focused here on the randomness only in the borrower's cash outflows. Cash inflows – such as the project return K – are assumed deterministic. The reason for our specification is that a random cash inflow would not change the analysis. Cash outflows are relevant because they directly affect takedown behavior. Making these outflows random is important because, under asymmetric information, it makes the borrower's (privately known) takedown attribute unobservable ex post to the bank. (This is discussed later.) This prevents the informational asymmetry problem from being trivialized. Cash inflows could, in a different model setting, also affect takedowns, but their randomness would be relevant only to the extent that it introduces further uncertainty in takedowns. We can, however, interpret the takedown uncertainty that we have stipulated as being the *net* takedown uncertainty, reflecting randomness in both cash inflows as well as outflows.

If the borrower takes a loan of I at $t=2$, it must repay an amount RI to the bank at $t=3$. Banks are assumed to be competitive. Consequently, R reflects the bank's one period cost of funds, including operating costs, the deposit interest rate and the cost of fractional reserves. Banks exist as institutions that provide optimal risk sharing – in a sense made precise later – as, for example, in Diamond and Dybvig (1983). Thus, the bank can be viewed as selling loan commitments (sale of spot loans is not precluded) and funding takedowns under these commitments with (elastically supplied) deposits acquired in the spot market at the existing spot rate. Because the bank is risk neutral, it will not keep any deposit reserves in excess of mandatory reserves to fund future takedowns. Rather, it will observe takedown realizations and then proceed to acquire the deposits necessary to satisfy the observed loan demand. The variable R is assumed to be random and binomially distributed such that a high interest rate R^+ occurs with probability s and a low interest rate R^- occurs with probability $1-s$.[3] That is, $R^+ > R^- > 1$. With this notation, a borrower's expected utility, if it decides that it will borrow in the *spot market* if the need arises, will be

$$\mathrm{E}(U)=V(\Omega_1)+V(\Omega_2)+(1-q)V(\Omega_3+K)+sqV(\Omega_3+\hat{K}-R^+I)$$
$$+(1-s)qV(\Omega_3+\hat{K}-R^-I), \tag{2}$$

where $\hat{K}\equiv K+I$ and $\mathrm{E}(\cdot)$ is the expectation operator. To rule out default possibilities, we assume that $\Omega_3+K>R^+I$.

As an alternative to planning to borrow in the spot market, the borrower can purchase a fixed rate loan commitment from a bank at $t=1$. This commitment would obligate the bank to loan the borrower up to an amount I at $t=2$ at a fixed rate of interest, R_F, regardless of the interest rate prevailing then. The borrower would retain the option to use this facility or ignore it. Thus, if at $t=2$ the need to borrow did arise, the borrower would take down an amount I against the commitment if $R=R^+$ and would access

[3]The assumption of binominal interest rates can be viewed as a discrete approximation. As is well known, the limiting case (with the time interval between successive interest rate changes going to zero) of the binomial distribution is the lognormal distribution.

the spot credit market if $R=R^-$ (assuming, for the moment, that $R_F \in [R^-, R^+]$).[4] We assume that the bank can costlessly monitor the use of the funds, so as to prevent the borrower from simply 'arbitraging' when $R=R^+$.

The assumption of bank risk neutrality is adopted for analytical tractability. Our results should be qualitatively sustained even if only differential risk aversion – with the bank less risk averse than the borrower – is assumed. From an empirical standpoint, the assumption that banks exhibit (substantially) less aversion to risk than their borrowers is hard to quarrel with; over 20 percent of long term commercial and industrial loans currently made by U.S. banks are on a fixed rate basis,[5] and recent advertisements in the financial media (Wall Street Journal) indicate an upward trend in fixed rate lending under commitments. From a theoretical standpoint, the contemporary literature on financial intermediary existence has shown that, in equilibrium, an intermediary will be perfectly diversified [Boyd and Prescott (1986)] and thus behave as if it is risk neutral even though each individual agent comprising it is risk averse [Ramakrishnan and Thakor (1984)].

Suppose the bank charges the borrower a commitment fee of mI at $t=1$ and a usage fee of uI at $t=2$. Since the commitment fee is paid when the loan commitment is purchased, it represents an expense for the borrower even if no borrowing takes place. The usage fee, on the other hand, is state contingent and must be paid only if takedown occurs.

This, of couse, is not the only way in which loan commitments are priced. Banks often ask for compensating balances in lieu of or in conjuction with explicit fees. For instance, as described by Mason (1979), the loan commitment price may be expressed as '10 and 10', which means the borrower must keep compensating balances equal to 10 percent of the total commitment plus 10 percent of the average loan balance outstanding. Another popular method is to assess the commitment fee on the total commitment but base the usage fee on the amount of the commitment not taken down rather than on the amount utilized.[6] All these different approaches, however, essentially employ a time additive loan commitment pricing function with two components, one dependent on the total commitment and the other related to the actual usage. We shall, therefore, model the most direct of these arrange-

[4]This ignores the possibility of a partial takedown which would occur if the borrower utilized only a fraction of the total commitment. Greenbaum and Vennezia (1985) have recently provided a rationale for this phenomenon in the context of a dynamic model with Bayesian price revision by the bank.

[5]*Source*: Federal Reserve Statistical Release E.2. If both the bank and the borrower are risk averse, then interest risk is optimally shared by the bank and the borrower [see Arvan and Brueckner (1986)]. Using their model, it is easy to show that a risk neutral bank will absorb all of the interest risk when the borrower is risk averse.

[6]Levying fees against unused portions of commitments effectively riases the borrower's cost of accessing the spot market when the interest rate is lower than the fixed commitment rate, and thus makes the commitment less valuable. In this paper we shall not attempt to explain why such an arrangement may be optimal.

ments, involving explicit fees levied on the total credit line and the actual borrowing.

In a perfectly competitive credit market, each bank must earn zero expected profit. Thus, if the bank and the borrower are symmetrically informed about all pertinent variables, the problem of finding the Pareto optimal loan commitment contract is

$$\max_{m,u} \mathrm{E}(U)=V(\Omega_1-mI)+q[sV(\Omega_2-uI)+(1-s)V(\Omega_2)]$$

$$+(1-q)V(\Omega_2)+q[sV(\Omega_3+\hat{K}-R_{\mathrm{F}}I)+(1-s)V(\Omega_3+R^{-}I)]$$

$$+(1-q)V(\Omega_3+K), \text{ subject to} \tag{3}$$

$$mI+[qsuI](R_0)^{-1}+qsI[R_{\mathrm{F}}-R^{+}](R^{+}R_0)^{-1}=0, \tag{4}$$

$$m\geqq 0, u\geqq 0, \tag{5}$$

where R_0 is the current (riskless) rate of interest. Note that the last term in (4) captures the fact that, since the borrower will not utilize the loan commitment unless $R=R^{+}$, the bank will be paying a funding rate of R^{+}, but will earn a rate of only R_{F} on the loan.[7]

In such a maximization program, the optimal solution can be distorted by large differences in the wealth endowments, Ω_ts, across time. Since these endowments are exogenously specified anyway, we eliminate such distortions by assuming that the borrower's wealth, in the absence of a loan commitment, is constant through time. That is,

$$\Omega_1=\Omega_2=\hat{\Omega}_3\equiv\Omega_3+[1-q]K+sq[\hat{K}-R^{+}I]+(1-s)q[\hat{K}-R^{-}I]. \tag{6}$$

This will allow us to focus on the effects of just the loan commitment itself. Also, for notational convenience, we shall 'normalize' by assuming that $R_0\equiv 1$ henceforth.

The above assumption implies that the borrower's endowment, prior to the

[7]The determination of the current spot rate, the future spot rate and the forward rate are interrelated and must reflect aggregate cash flows. That is, embedded in R^{+} at $t=2$ is the market's knowledge that the banking system has a contingent liability represented by the aggregate amount of fixed loan rate commitments. Thus, the equilibrium R^{+} and R^{-} are the results of a joint process that impounds, on an expectational basis, conditions in the forward market as well as the aggregate supply and demand for funds at $t=2$. This presents obvious difficulties for monetary policy aimed at controlling *aggregate* variables [see Deshmukh, Greenbaum and Kanatas (1982)]. Ours, however, is strictly a partial equilibrium exercise that focuses on bilateral bank-borrower relationships in a competitive market. In such a setting, the *individual* bank is considered an atomistic price taker that views R as being beyond its control (even though aggregate loan commitment activity may affect R).

investment, is declining through time. If I_0 is the investment at $t=1$, then prior to the investment the borrower's endowment at $t=1$ is $\Omega_1 + I_0$, at $t=2$ is Ω_2 and at $t=3$ is Ω_3. Clearly $\Omega_1 + I_0 > \Omega_2 > \Omega_3$.

If the focus of our analysis was on the borrower's investment, then the sensible assumption to make would be that the endowment in every period is equal prior to the investment; that would enable us to (clearly) isolate the effect of the investment on the borrower's behavior. However, since we wish to focus on the effect of the loan commitment, we equate endowments intertemporally prior to the loan commitment.

Note that, with a risk neutral valuation, the borrower's investment will have a positive NPV (be socially optimal) if

$$s\{[1-q]K+q[K+I-R^{+}I]\}\{R^{+}R_0\}^{-1}$$

$$+[1-s]\{[1-q]K+q[K+I-R^{-}I]\}\{R^{-}R_0\}^{-1}>I_0.$$

Our implicit assumption, though, is that the borrower's expected utility with the investment is greater than it is without, regardless of whether a bank loan commitment is purchased.

It is transparent that the borrower will prefer a fixed rate commitment to a state contingent spot market transaction. Since the borrower is risk averse and the bank is risk neutral, a loan commitment contract that maximizes a weighted sum of the expected utilities of the bank and the borrower entails the bank providing complete interest rate insurance to the borrower through a fixed rate commitment. Thus, loan commitment demand (as well as the bank's reason to exist) in this model stems from optimal risk sharing considerations.

Note that the borrower will use the commitment at $t=2$ only if $R_F + u < R^{+}$. In other words, a positive takedown requires that the *total* borrowing cost, rather than just the loan interest itself, should be lower with commitment borrowing. The reason why we did not try to prove $R_F + u < R^{+}$ is that the need for a usage fee has not yet been established. (In Proposition 1 below we will prove that the optimal commitment rate will satisfy $R_F + u < R^{+}$.) For instance, even though a risk averse borrower would like to purchase the put option provided by a fixed rate commitment, it *may* want the usage fee set at zero and the commitment fee appropriately adjusted upward. Thus, we need to show that a 'split' pricing structure is indeed optimal. This is taken up in our first proposition.

Proposition 1. Under perfect information, the Pareto optimal fixed rate loan commitment contract always involves both a commitment fee and a usage fee.

Proof. See the appendix.

The intuition for the above result is that incorporating positive commitment and usage fees facilitates risk sharing by spreading the borrower's payout risk over time. It is also evident that these fees must be functionally related to s, q, R^+ and R_F.

This proposition is another demonstration of the Pareto optimality of two-part tariffs in (symmetrically informed) competitive markets populated by risk neutral sellers and risk averse buyers who face random future consumption needs. Hayes (1984) independently obtained a similar result in her explanation for why monopoly power is inessential for two-part tariffs to exist in markets such as health clubs and bars. As in our analysis, such tariffs are optimal in Hayes' model because they act as a form of insurance. For example, suppose a health club offers a two-part tariff – a fixed fee at the beginning of the year plus a price per visit – to consumers whose consumption of health club visits is random. In a competitive market, a positive fixed fee will result in a per visit charge that is lower than the club's marginal cost. The consumer's income effectively declines by an amount equal to the fixed fee and consumption of health club services in each state will rise relative to other goods due to the reduced marginal cost of consumption. Hayes has shown that there is a positive net utility effect attributable to the reduction in the consumer's utility in low states and the increased utility in high states because of the income and price changes.

Although the Hayes model is different from ours, its intuition parallels the intuition underlying our rationale for commitment and usage fees under symmetric information. In both cases, the consumer faces some uncertainty about future consumption and thus prefers to lower the marginal cost of future consumption by purchasing insurance through the payment of a fixed fee up front.

3. Pricing under asymmetric information

Suppose there are two types of borrowers in the market, with each type distinguished by its takedown probability, q. Type 1 borrowers have a takedown probability of q_1 and type 2 borrowers by a takedown probability of q_2. Let $0<q_1<q_2<1$. In all other respects, the borrowers are identical.[8] Note, however, that as far as endowments are concerned, this assumption is only meant to imply that $\Omega_1=\Omega_2=\Omega_3$. The $\hat{\Omega}_3$s for the borrower types will obviously be different because the qs are different. We make the two-type assumption only for expositional ease. It is not difficult to introduce n borrower types.

[8]This assumption is standard in asymmetric information models. It is made to remove the possibility of detection based on differences in *observable* attributes, because if such a possibility exists, the imperfect information problem is trivialized.

Assume that each borrower knows its own type, but the bank it approaches for a loan commitment does not. This means that, in addition to a borrower's q being unobservable, its $\hat{\Omega}_3$ is also unobservable. If Ω_1, Ω_2, Ω_3 and $\hat{\Omega}_3$ were all observable, one could always deduce a borrower's q using eq. (6). Thus, our specification in this section represents a slight modification of eq. (6). Instead of assuming that $\Omega_1 = \Omega_2 = \hat{\Omega}_3$, we are now assuming that $\Omega_1 = \Omega_2 = \Omega_3 < \hat{\Omega}_3$, with $\hat{\Omega}_3$ not publicly observable. This creates a propensity for borrowers to misrepresent their private information. In the absence of a cost efficient ex post monitor of q, all borrowers will have an incentive to claim that they are type 1, if the bank offers all takers the perfect information contract of Proposition 1 and uses the reported q. Banks will be unable to differentiate borrowers ex post based on either the amount borrowed (because all borrowings will be for the same amount I) or the fact that borrowing took place (a 'dishonest' type 2 customer who borrowed at $t=2$ could continue to claim that its probability of takedown was really q_1; the bank would be powerless to judge the veracity of that claim because a *single* realization of a random variable does not allow a conclusive inference about the underlying probability distribution). This observation and the fact that agents are risk averse means that costless ex post contingent contract (non-dissipative) signaling equilibria of the type discussed by Bhattacharya (1980) are generally not optimal. If the bank naively offers the symmetric information contracts based on the reported q and all type 2 borrowers misrepresent their qs, the bank will lose money (in an expected value sense) on every type 2 borrower and break even on every type 1 borrower. To see this, define Π_1 and Π_2 as the bank's profit on type 1 and type 2 borrowers, respectively. From (4), we get

$$E(\Pi_1) = \Psi_1 + q_1\Psi_2, \quad \text{and} \tag{7}$$

$$E(\Pi_2) = \Psi_1 + q_2\Psi_2, \quad \text{where} \tag{8}$$

$$\Psi_1 \equiv sq_1I[R^+ - R_F]\{R^+[1+q_1s]\}^{-1}, \quad \text{and} \tag{9}$$

$$\Psi_2 \equiv s^2q_1I[R^+ - R_F]\{R^+[1+q_1s]\}^{-1} - sI[R^+ - R_F][R^+]^{-1}. \tag{10}$$

It is apparent that $E(\Pi_1)=0$ and that $\Psi_1 > 0$. Rewriting (8) yields

$$E(\Pi_2) = (q_2 - q_1)\Psi_2 < 0. \tag{11}$$

Under asymmetric information, therefore, the competitive bank must modify the contracts it offers. What types of contracts survive in equilibrium will depend on the notion of competitive equilibrium adopted. We shall initially adopt the Cournot–Nash equilibrium of Rothschild and Stiglitz

(R–S) (1976) to describe equilibrium in a competitive forward market under asymmetric information. As is well known, this equilibrium is separating, involving a pair of loan commitment contracts such that each borrower type self-selects the contract designed for it and every bank earns zero expected profit on each contract (and on each borrower).

Before proceeding further with the formal analysis, it is useful to grasp the intuition behind the subsequent propositions. Our basic point is that when the bank does not know the takedown probabilities of borrowers, incorporating commitment and usage fees into the loan commitment contract enables the bank to induce different types of borrowers to separate themselves through contract choice. Two contracts will be offered; one will have a high commitment fee and a low usage fee whereas the other will have a low commitment fee and a high usage fee. A borrower who has a high takedown probability will want to avoid a high usage fee because the likelihood of actually paying that high fee is greater. On the other hand, a borrower with a low takedown probability is less averse to accepting a contract with a high usage fee because the likelihood of actually paying that high fee is lower. Such a borrower would, however, like to minimize its commitment fee, since it is a 'sunk cost' that is incurred regardless of whether the borrower exercises its commitment option. The borrower with the high takedown probability finds the high commitment fee less onerous because it represents the price of an option that the borrower is, in fact, very likely to exercise. Thus, differences in takedown probabilities fundamentally alter the appeal of varying combinations of commitment and usage fees to different borrowers, inducing each borrower to select the contract most advantageous to it and thereby tacitly reveal its type.

Returning to our analysis, the specific solution to the combination of contracts that forces self-selection depends on the (exogenously determined) values of intertemporal wealth. Because we chose $\Omega_1=\Omega_2=\Omega_3$ as a reasonable assumption, the specific solution to the usage fee and the commitment fee under perfect information is such that the fees are set equal to each other (see the proof of Proposition 1). This contrasts with the solution under asymmetric information which is stated in the following lemma.

Lemma 1. Under asymmetric information, any feasible single loan commitment contract A *that attracts both types of borrowers must have* $u_A > m_A$, *where* u_A *and* m_A *are the usage and commitment fees with contract* A.

Proof. See the appendix.

Setting the commitment and usage fees equal to each other is inefficient under asymmetric information because, with a single contract, type 1 borrowers are subsidizing type 2 borrowers at $m=u$. Under competition, this subsidy will be eliminated as banks attempt to lure away the type 1 borrowers by increasing u and reducing m;[9] such an alteration transfers wealth from the type 2 to the type 1 borrowers.

As shown by Riley (1975), R–S (1976) and Wilson (1977), there may be no (pure strategy) Cournot–Nash equilibria under asymmetric information if, as assumed here, the less informed agents (banks) move first, announcing a schedule of loan commitment offers. Conditions that guarantee the existence of equilibrium, therefore, need to be carefully examined. To do this, we need two simple lemmata which are exploited in the graphical portrayal of equilibrium.

Lemma 2. In the m–u space, the bank's zero profit loci for type 1 and type 2 borrowers do not intersect anywhere except at a single point on the u-axis.

Proof. See the appendix.

Lemma 3. For a type i borrower ($i=1,2$), the indifference curve corresponding to that borrower's expected utility under perfect information is tangential to the bank's zero profit locus for that borrower type, and the point of tangency is the intersection of the zero profit locus with the 45° line.

Proof. See the appendix.

Fig. 1 depicts a situation in which a (separating) Cournot–Nash equilibrium exists. $\{m_2^\circ, u_2^\circ\}$ is the contract awarded type 2 borrowers under perfect information. The Cournot–Nash equilibrium has two contracts, $\{m_1^*, u_1^*\}$ and $\{m_2^\circ, u_2^\circ\}$. Type 1 borrowers take the contract $\{m_1^*, u_1^*\}$ and type 2 borrowers take the contract $\{m_2^\circ, u_2^\circ\}$. Because the pooling zero profit locus lies completely above the type 1 indifference curve passing through the point C, there is no pooling contract that can attract the type 1 borrowers away from $\{m_1^*, u_1^*\}$. Thus, the separating equilibrium cannot be disturbed.

On the other hand, if the type 2 indifference curve through E crosses the type 1 zero profit locus at the point D (as shown in fig. 2) and the type 1 indifference curve through D cuts the pooling zero profit locus from above at B, a Cournot–Nash equilibrium does not exist. To see this, note that the separating allocation that maximizes borrower welfare, subject to incentive compatibility constraints and zero expected profits for the bank, induces type 1 borrowers to take contract $\{\hat{m}_1^*, \hat{u}_1^*\}$ and type 2 borrowers to take contract $\{m_2^\circ, u_2^\circ\}$. But this is not a Cournot–Nash equilibrium because a bank can offer a pooling contract, say Z, that would attract type 1 borrowers

[9] However, the fact that the optimal m_A and u_A are unequal (they may actually be quite far apart) should be viewed as an efficiency loss (in risk sharing terms) due to asymmetric information.

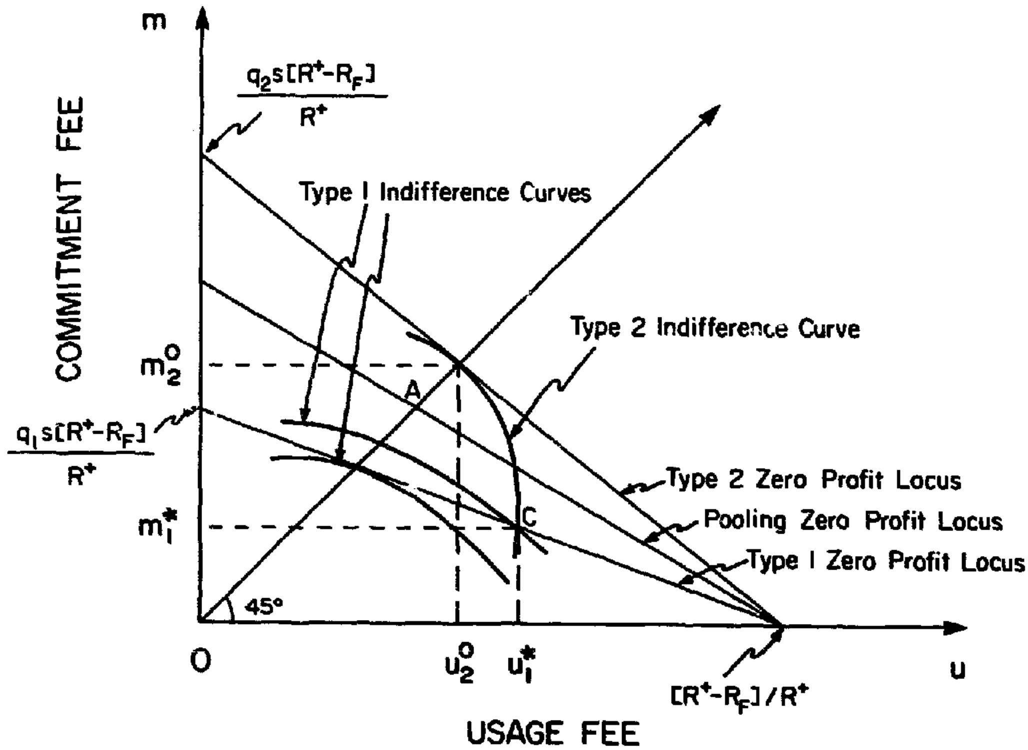

Fig. 1. A Cournot–Nash separating equilibrium exists.

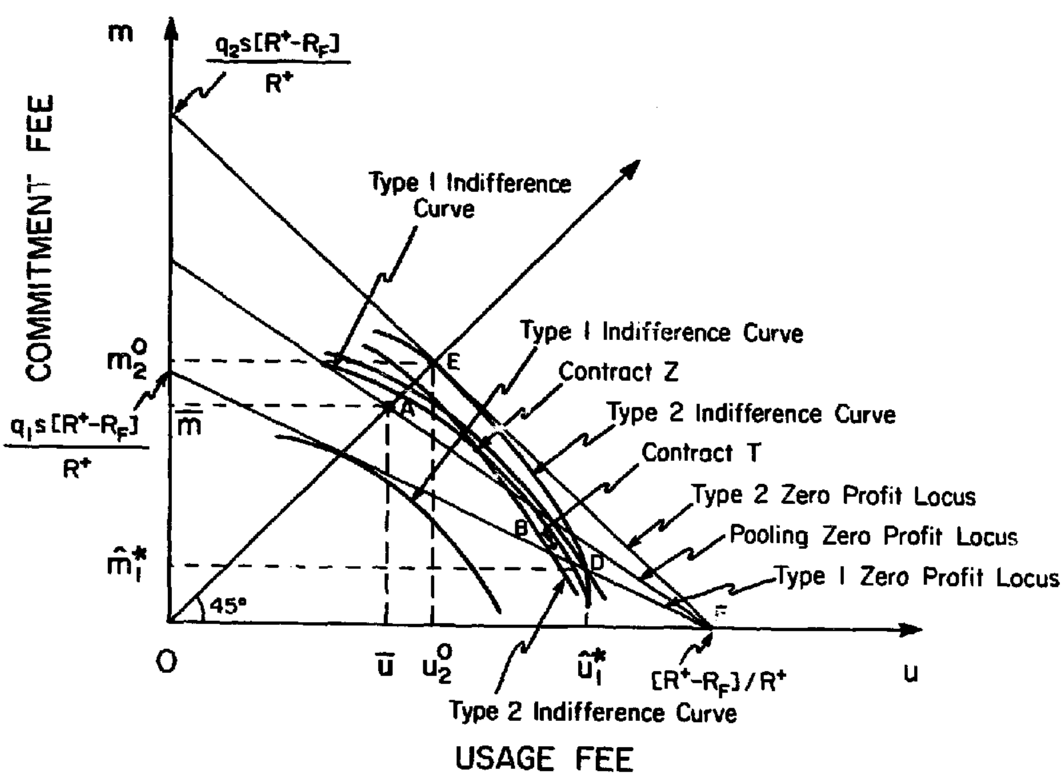

Fig. 2. A (pure strategy) Cournot–Nash equilibrium does not exist but a reactive equilibrium does.

away from $\{\hat{m}_1^*, \hat{u}_1^*\}$ and type 2 borrowers away from $\{m_2^\circ, u_2^\circ\}$. Z can, however, be dominated by any pooling contract in the interior of AB, the pooling zero profit locus. But any pooling contract in the *interior* of AB can be dominated by the contract $\{\bar{m}, \bar{u}\}$ represented by the point A. And this contract cannot be an equilibrium because it lies on the 45° line; recall that Lemma 1 rules out such contracts from the set of feasible pooling contracts. Thus, a Cournot–Nash equilibrium does not exist in this case. This discussion can be summarized as follows.

Proposition 2. Whenever a Cournot–Nash equilibrium exists, it is separating. In equilibrium, borrowers with the low takedown probability take loan commitment contracts with low commitment fees and high usage fees, whereas borrowers with the high takedown probability take contracts with lower usage fees but higher commitment fees. Borrowers with the high takedown probability enjoy the same expected utility as under perfect information but the low takedown probability borrowers suffer a loss in expected utility.

Despite the potential gains from introducing the contract Z – and the consequent non-existence of a Cournot–Nash equilibrium – one can argue that prudent banks are likely to resist defection from the contract set $\{\{m_1^*, u_1^*\}, \{m_2^\circ, u_2^\circ\}\}$. To see this with the help of fig. 2, note that if a bank offers Z, another bank can react with an offer such as T. Contract T 'skims the cream', attracting type 1 borrowers and leaving only the type 2 borrowers selecting contract Z. Consequently, the original defecting bank offering Z suffers losses.

The point here is that the reaction, T, produces profits on *each* applicant that accepts the contract, which means that there can be no further reactions by other banks which result in losses for the bank offering T. The worst outcome for such a bank is that contracts superior to T can bid applicants away, leaving the bank with zero profits. Because the initial defector realizes that there is no risk of loss with a reaction such as T, its incentive to offer Z vanishes. The original pair of contracts $\{\{m_1^*, u_1^*\}, \{m_2^\circ, u_2^\circ\}\}$ is then a 'reactive' equilibrium [Riley (1979)].

As shown in Riley (1979), or under the weaker assumptions developed in Engers and Fernandez (1987), we have

Proposition 3. The set of contracts, A, that is the Pareto efficient set among all loan commitment contracts that at least break even and separate the two borrower types, is the unique reactive equilibrium.[10]

[10]When a Cournot–Nash equilibrium does exist, as in Fig. 1, it is also the unique reactive equilibrium.

Wilson (1977), Miyazaki (1977) and Spence (1978) have proposed alternative non-Nash equilibrium concepts that involve some form of anticipation by banks of the responses to their actions. In our model, there is a unique equilibrium of each type.[11]

4. Comparison with closely related literature

In some ways our model resembles that of Sealey and Heinkel (S–H) (1985). In S–H (1985), a firm with random future cash needs keeps precautionary cash balances because the acquisition of spot liquidity is costly. These precautionary balances are kept as bank deposits because banks are assumed to have a relative advantage in the provision of liquidity. S–H (1985) then show that these deposits can be interpreted as compensating balances since a borrower's deposit level and loan interest rate are linked under symmetric as well as asymmetric information. There are three similarities between the S–H model and ours. First, as in our model, a random future liquidity demand by the borrower plays a key role. In our model, it induces the borrowers to seek a loan commitment; in S–H, it causes the borrower to put up a compensating balance. Second, the first best level of the credit instrument – loan commitment in our model and compensating balance in S–H – is positive in each model. Third, like us, S–H also characterize the fully separating contracts that exist in a Nash equilibrium under asymmetric information.

The key differences between the S–H model and ours are as follows. First differential bank–borrower liquidity costs engender compensating balances as well as a rationale for bank existence in the S–H model in which universal risk neutrality is assumed. We do not assume liquidity costs because they are not needed in our model to rationalize the existence of either banks or a forward market in loan commitments. Rather, loan commitments and banks exist to provide optimal risk sharing between risk averse borrowers and risk neutral banks when interest rates are random. Second, under asymmetric information, the sorting variables in S–H are the loan interest rate and the deposit level; in our model, these are the commitment and usage fees. And third, the primary objective in S–H was to rationalize compensating balances, whereas in our paper it is to explain the complex pricing mechanisms in loan commitments – even when compensating balances are used in lieu of fees. As mentioned earlier, banks frequently require a compensating balance requirement on both the total commitment and the amount actually drawn down. Since a compensating balance requirement imposes a cost on the borrower, such a pricing scheme is logically equivalent

[11]In particular, Miyazaki's (1977) extention of Wilson's (1977) 'anticipatory equilibrium' permits cross-subsidization across contracts. The resulting equilibrium is always fully separating. Thus, even in that equilibrium, banks offer a pair of contracts and borrowers tacitly reveal their types through self-selection.

to the one we study here in the sense that there is a cost attached to the total line as well as the usage. S–H do not explain such 'split pricing' structures in loan commitments, whereas we rationalize them as optimal risk sharing and screening devices.

Also closely related to our work is the research by James (1981). Like us, James also explains the pricing structure of bank loan commitments. He derives a perfectly separating equilibrium under asymmetric information in which high risk borrowers choose to pay (commitment) fees and low risk borrowers choose to keep compensating balances. There are at least three important distinctions between James' research and ours. First, the space of sorting variables differs across the two models. In James, sorting is achieved with a commitment fee and a compensating balance. Sorting is possible in James because borrowers differ in their transition default probabilities which are unknown initially to banks. In our model, however, borrowers have identical default risks but different, ex ante unobservable takedown probabilities. Such borrowers cannot be separated using James' sorting variables. Thus, the James model cannot explain usage fees. On the other hand, our model cannot explain the *choice* between fees and compensating balances that James explains. Second, James implicitly assumes that the borrower's default risk at the time of borrowing (t_1) is revealed to all at t_1; it is the probability of transition from some initial default risk class (at the first point in time t_0) to this risk class at t_1 that is a priori unknown. Given the other assumptions in James, this may permit a non-dissipative, contingent contract signaling equilibrium that would be an alternative to the equilibrium in James. As pointed out earlier, a non-dissipative equilibrium is not possible in our framework. Finally, in the James model, in equilibrium the borrower chooses either a commitment fee or a compensating balance. By contrast, each borrower in our model chooses both of the sorting variables, but in different combinations. Basically, a summary of the distinctions between the James model and ours is that, by assuming different types of informational asymmetries and explaining different aspects of loan commitment pricing, the two models highlight the specific borrower attributes about which asymmetric information must exist in order to observe particular characteristics in loan commitment contracts.

There is a (slight) difference between our model and earlier models that studied Cournot–Nash equilibria in competitive markets, such as R–S and Wilson (1977). In those models, a priori uninformed insurance companies engaged in price–quantity competition, whereas we study two-part tariff competition with quantity (loan size) held fixed. Recent credit market models that study price–quantity (loan size–interest rate) competition among banks include Milde and Riley (1984) and Besanko and Thakor (1987),[12] although

[12]The Besanko and Thakor (1987) model is actually more complex in that it includes *four* sorting variables – collateral and the probability of rationing, in addition to the loan size and loan interest rate.

they focus on non-Nash equilibrium concepts.[13] We build our analysis around commitment and usage fees because, in our context, it would be uninteresting to consider loan size and loan interest rate as sorting variables. The reason is that borrowers differ in the likelihood that they will need a loan, rather than in the payoff characteristics of the projects they intend to fund. Thus, although the two borrower types have different indifference curves in the commitment fee–usage fee space, their indifference curves in the loan size–loan interest rate space coincide. This means that the separation of borrower types through self-selection will be unattainable if banks compete on the basis of loan sizes and loan interest rates.

5. Concluding remarks

We have explored an important and neglected aspect of commercial bank loan commitments, namely the manner in which they are priced. Our model suggests that the deployment of commitment fees in conjunction with usage fees provides optimal risk sharing when banks are risk neutral and borrowers are risk averse. Moreover, if banks do not know commitment takedown probabilities that borrowers know, such a pricing structure also helps resolve the information asymmetry problem by acting as a screening device.

From a positive standpoint, future endeavors could be fruitfully directed at enhancing our understanding of some of the other institutional details of forward credit markets, which may shed light on intertemporal credit allocation processes. Compensating balances are one example, and these have already been rationalized by S–H and James using approaches similar to ours. Other examples are 'own funds' to loan ratios, and the commonly found 'tie-ins' between a customer's consumption of non-credit banking services and the prices of spot credit and loan commitments.

Appendix

Proof of Proposition 1. Substituting (4) into (3), the first order optimality condition for u can be written as

$$V'(\Omega_1+qsI\{R^+u+R_F-R^+\}\{R^+\}^{-1})=V'(\Omega_2-uI), \tag{A.1}$$

which implies

$$u=qs[R^+-R_F]\{R^+(1+qs)\}^{-1}. \tag{A.2}$$

It is clear that the non-negativity constraint, (5), will be satisfied by (A.2) because Proposition 1 assures us that $R^+>R_F$. It is also easy to check that

[13] Milde and Riley (1984) look at both Nash and non-Nash competitive equilibria.

$u<R^+-R_F$, so that the set of states in which takedown occurs is not empty.

Substituting (A.2) in (4), it follows immediately that:

$$m=qs[R^+-R_F]\{R^+(1+qs)\}^{-1}>0. \tag{A.3}$$

Q.E.D.

Proof of Lemma 1. Let $E(U_j^i)$ be the expected utility of a type i borrower with the loan commitment contract j, where $j\varepsilon\Gamma$ and Γ is the feasible contract space. Also let $E(\Pi_j^i)$ be the expected profit of the bank on contract j if the contract is accepted by a type i borrower. Finally, define $\theta\varepsilon(0,1)$ as the fraction of type 1 borrowers in the population and $1-\theta$ as the fraction of type 2 borrowers.

Using (4) and setting $q=\theta q_1+[1-\theta]q_2$, we have

$$m_A=s[\theta q_1+(1-\theta)q_2][(R^+-R_F)(R^+)^{-1}-u_A]. \tag{A.4}$$

Now set $m_A=u_A$, so that

$$m_A=u_A=s[\theta q_1+(1-\theta)q_2](R^+-R_F)(R^+)^{-1}\{1+s[\theta q_1+(1-\theta)q_2]\}^{-1}. \tag{A.5}$$

Thus, the bank's expected profit on a type 1 borrower will be

$$E(\Pi_A^1)=\Psi_3+q_1\Psi_4-sq_1I(R^+-R_F)(R^+)^{-1}, \tag{A.6}$$

and on a type 2 borrower it will be

$$E(\Pi_A^2)=\Psi_3+q_2\Psi_4-sq_2I(R^+-R_F)(R^+)^{-1}, \quad \text{where} \tag{A.7}$$

$$\Psi_3\equiv sI\Psi_5(R^+-R_F)(R^+)^{-1}[1+s\Psi_5]^{-1}, \tag{A.8}$$

$$\Psi_4\equiv sI[s\Psi_5(R^+-R_F)(R^+)^{-1}][1+s\Psi_5], \tag{A.9}$$

$$\Psi_5\equiv\theta q_1+(1-\theta)q_2. \tag{A.10}$$

Quite transparently, we can see that

$$E(\Pi_A^1)>0 \quad \text{and} \quad E(\Pi_A^2)<0,$$

which means that every type 1 borrower is subsidizing type 2 borrowers at $m_A=u_A$.

Next substitute (A.4) in a type 1 borrower's utility function [given by (3)] and differentiate partially with respect to u_A to obtain

$$\partial \mathrm{E}(U_A^1)/\partial u_A\big|_{m_A=u_A} = Is\Psi_5 V'(\Omega_1 - m_A I) - Isq_1 V'(\Omega_2 - u_A I) > 0 \quad \text{since}$$

$$\theta \in (0,1) \quad \text{and} \quad q_2 > q_1.$$

Consequently, if a bank were to offer a single contract with $m \geqq u$, a second bank would announce another contract more attractive to type 1 borrowers (i.e., with $u > m$). This will leave the first bank with only type 2 borrowers on whom it earns negative expected profits. Q.E.D.

Proof of Lemma 2. The bank's zero profit curve for a type i borrower is obtained by differentiating (4) totally. It is described by

$$\mathrm{d}m/\mathrm{d}u = -q_i s; \quad i = 1, 2. \tag{A.11}$$

Moreover, from (4) we see that when $u = 0$,

$$m = \begin{cases} q_1 s[R^+ - R_F][R^+]^{-1} & \text{for} \quad i = 1 \\ q_2 s[R^+ - R_F][R^+]^{-1} & \text{for} \quad i = 2 \end{cases} \tag{A.12}$$

and when $m = 0$,

$$u = [R^+ - R_F][R^+]^{-1} \quad \text{for} \quad i = 1, 2. \tag{A.13}$$

The desired result follows from combining (A.11), (A.12) and (A.13). Q.E.D.

Proof of Lemma 3. The indifference curve for a type i borrower is obtained by totally differentiating (3) and is described by

$$\mathrm{d}m/\mathrm{d}u = -q_i s V'(\Omega_2 - uI)[V'(\Omega_1 - mI)]^{-1} < 0. \tag{A.14}$$

And $$\mathrm{d}^2 m/\mathrm{d}u^2 = q_i Is V''(\Omega_2 - uI)[V'(\Omega_1 - mI)]^{-1} < 0. \tag{A.15}$$

Thus, the borrower's indifference curves are decreasing and concave in the m–u space. At $m = u$, we have $\mathrm{d}m/\mathrm{d}u = -q_i s$ since $\Omega_2 = \Omega_1$ by assumption. This means that a type i borrower's indifference curve has the same slope as the bank's zero profit line for that type at the point $m = u$. Q.E.D.

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