Whence GARCH?

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Abstract

We develop an equilibrium asset-pricing model that provides an explanation for the new empirical facts in finance, particularly the observance of volatility clustering. Agents in our model care about both consumption and wealth changes, similar to Barberis, Huang, and Santos (2001). Sensitivity to past investment performance influences stock prices in such a way that, even when news about dividends is an IID process, return volatility exhibits properties similar to what is observed in monthly return data. Investors in our model use a mental scorecard to measure how far ahead (past gains) or behind (past losses) their portfolio is. As shocks move the scorecard from its expected level, investors change their level of risk aversion and become temporarily more sensitive to news. Gradually, investors become accustomed to the new level of wealth, restoring the levels of risk aversion and news sensitivity. The state-dependent sensitivity to news creates volatility clustering. We test specific and refutable implications of our model with monthly U.S. stock data. We find support for our model’s predictions that relate the scorecard to conditional volatility and skewness. We find limited support for our predictions regarding excess conditional returns.

1 Introduction

Over a decade ago, stock prices were commonly characterized as a “random walk with a drift.” This description not only seemed to empirically capture stock price movements, but also was backed by solid theory. Competition among investors should make prices efficient, eliminating any predictability. The last decade, however, has produced some new empirical facts; for example, returns seem to predictably vary over time, to be higher than consumption models predict, and to vary more than cash flow models can justify.\(^1\) Perhaps the most robust new fact is that stock return volatility is serially correlated.

Nearly all empirical work in finance published this decade acknowledges and accounts for volatility clustering in returns. Volatility autocorrelation is typically modeled using ARCH (Engle (1982)), GARCH (Bollerslev (1986)), Markov switching (Hamilton (1988, 1989)), nonparametrics (Pagan and Ullah (1988)) or some extension of these statistical models.\(^2\) The motivation for using

\(^1\)See Cochrane (1999) for a review of the “new facts in finance.”

\(^2\)Pagan and Schwert (1990) compare, contrast, and test the GARCH, Markov switching, and nonparametric representations of stock return volatility.
conditional volatility models is their utility—they test the data. Whereas our statistical knowledge of the degree and nature of volatility clustering is impressive, our theoretical knowledge of why volatility clusters seems paltry. The default explanation for time-dependent volatility is that news about economic fundamentals itself is clustered, as if an exogenous news supplier works overtime in some periods and takes vacations in others. This default explanation is problematic given the regular nature of macroeconomic news releases and Schwert’s (1989) and Pagan and Schwert’s (1990) finding that stock return volatility is only weakly related with economic fundamentals.\(^3\)

Several recent papers develop endogenous explanations for volatility clustering using heterogeneous traders, where the trading process, not the fundamental news, induces volatility autocorrelation. Brock and LeBaron (1996) show that if diverse traders adapt their trading strategies on a slower time scale than trading takes place, volatility will be autocorrelated. Kelly and Steigertwald (1999), Coval and Hirshleifer (2000), and Gaunersdorfer and Hommes (2000) show how the entrance of sidelined investors can induce time-dependent volatility. Peng and Xiong (2001) show how changes in the rate at which financial analysts digest news can result in GARCH-like returns. In these models, the generation of fundamental news follows an IID random process but the arrival of news in stock prices through trading is clustered. These explanations are based on market frictions and feedback trading; thus, seem suitable for high-frequency returns; yet, clustering is observed in monthly as well as high-frequency returns. The leverage models of Black (1972 and 1976) and Christie (1982) predict higher volatility after bad news, as firms operate with higher leverage. However, these leverage models also predict lower levels of volatility after good news, a prediction inconsistent with empirical evidence.\(^4\) Timmermann’s (2001) structural breaks model, Veronesi’s (1999) fundamental regimes model generate conditional heteroskedasticity through gradually updated beliefs; however, Timmermann’s model identifies only one or two post-war structural breaks and cannot fully account for the relatively common episodes of clustering.\(^5\) Heterogeneous traders, leverage, and structural breaks models can capture some, but not all, of the characteristics of volatility clustering.

We propose an asset pricing model capable of explaining many of the new empirical facts in finance, including the robust finding of volatility clustering. Our explanation can be viewed as an extension of two seemingly different extant models. On one hand, we extend the behavioral model of Barberis, Huang, and Santos (2001) by allowing investors who are away from their customary level of wealth to be more sensitive to news. On the other hand, we extend the volatility feedback model of Campbell and Hentschel (1992) by endogenizing the cause of volatility autocorrelation.

Following Barberis, Huang, and Santos’ (2001), we develop an equilibrium asset pricing model that ties investors’ utility to volatility autocorrelation. The model allows investors to derive utility from fluctuations in financial wealth as well as from consumption. The level of utility derived from portfolio fluctuations depends on a slow moving measure of prior investment performance (a mental scorecard). When the portfolio is below the benchmark (a recent series of negative return shocks) investors become more risk averse. Additionally, the utility increases from gains is smaller than

\(^3\) For example, money supply and initial jobless claims data are announced every week; inflation, employment, production, trade, and deficit data are announced every month; and GDP data is announced each quarter. Jones, Lamont, and Lumsdaine (1998) find some evidence, using New York Times headlines, that “major” news events are positively autocorrelated.

\(^4\) Specifically, Engle and Ng (1993) show that returns are negatively correlated with the conditional variance of next period’s returns. See Wu (2000) for more recent evidence of and an explanation for asymmetric volatility. The asymmetry of volatility clustering is depicted in ‘News impact Curves’, introduced by Pagan and Schwert (1990) and used extensively by Hentschel (1995) and Campbell and Hentschel (1992).\(^5\)

\(^5\) David’s (1997) probability regimes model also illustrates how Bayesian learning can lead to serial correlation in volatility.
the utility decreases from losses (loss aversion). We modify Barberis, Huang, and Santos’s (2001) model, and the resulting new model explains volatility clustering. Whereas Barberis, Huang, and Santos’ investors use the mental scorecard to measure how far ahead or behind their portfolio is, our investors use the benchmark to measure how far away from the expected, customary, or habit level the portfolio is. We allow investors to become more sensitive or attentive to news when the portfolio is far away from the benchmark. Gradually, investors become accustomed to the new level of wealth restoring the normal sensitivity to news. Our time-varying sensitivity to news generates volatility clustering even when news itself follows an IID random process.

Campbell and Hentschel’s (1992) volatility feedback model exogenously imposes symmetrically-clustered news. A large piece of bad news has a direct and indirect effect. Directly, the bad news lowers stock prices. Indirectly, the news signals volatile times, causing discount rates to increase and prices to fall even further. For good news, the indirect (volatility feedback) effect acts, rather than compounds, the direct effect. Thus, volatility feedback implies an asymmetry in the correlation between stock price changes and future volatility. In our model, the clustering is endogenized. Rather than presupposing that news is clustered, our agents’ cause returns to be clustered because they become temporarily news sensitive or attentive when perturbed from their customary level of wealth.

Thus, like Barberis, Huang, and Santos (2001) we start at the very beginning by writing down a utility function. We then numerically solve the resulting asset pricing model and show that solutions to and simulations of our model are internally consistent with the empirical facts. However, as Shefrin and Statman (1994) imply, behavioral models are capable of explaining nearly anything and to be useful must be structured so that they can be falsified. We structure our model so that it makes specific refutable predictions that go beyond the new empirical facts that motivated our model in the first place. Thus, like Campbell and Hentschel (1992), we are able to end at the finish by taking our refutable predictions to the data. We need support for our model’s predictions regarding conditional volatility and skewness, and limited support for its predictions about excess returns.

Our paper is organized as follows. In Section 2 we modify the standard consumption-based model to include wealth-varying degrees of risk aversion and sensitivity to news. In Section 3 we develop our model’s implications for equilibrium asset prices. In Section 4 we numerically solve the model and illustrate the solutions. In this section we also report simulations of the model and show how our simulated data captures many of the new empirical facts, including GARCH. In Section 5, we develop our model’s new refutable predictions with respect to volatility, excess returns, and skewness and test the predictions on U.S. stock data. Section 6 concludes.

2 The Model

2.1 A Lucas-Type Economy

Following Lucas (1978), identical, in...initely-lived agents with a total mass of one maximize the following time-additive utility function by allocating wealth between consumption, $C_t$, and investment, $S_t$, in each period $t$. The amount consumed and invested is subject to a series of feasibility constraints $C_t + S_t = Y_t + S_{t-1} R_t$; where $Y_t$ is income from a non-...ncial source and $S_{t-1} R_t$ is...
the value of the portfolio carried over from \( t \) to \( t+1 \). Following Barberis, Huang, and Santos, our agents derive utility not only from consumption, \( U(C_t) \), but also from unexpected fluctuations in the value of their financial wealth, \( V(X_{t+1}) \):

\[
\max_{X_t} \mathbb{E}_{t=0} \left[ \pm U(C_t) + b_t C_t \pm^{t+1} V(X_{t+1}) \right],
\]

where \( \pm \) is the time or subjective discount rate capturing investors’ exogenous preference for immediacy and \( b_t C_t \) scales wealth utility to consumption utility. \( C_t \) is aggregate per-capita consumption and keeps wealth utility in line with consumption utility, even if aggregate wealth increases over time. Higher levels of \( b_t > 0 \) make wealth utility more important relative to consumption utility, whereas \( b_t = 0 \) makes wealth utility irrelevant reducing to the standard consumption utility model.

Agents can invest in a riskfree (bond) and risky (stock) asset at time \( t \). The riskfree asset is in zero net supply and the supply of the risky asset is normalized to one. We denote the gross return on the riskfree and the risky asset from \( t \) to \( t+1 \) as \( R_{f,t+1} = 1 + r_{f,t+1} \), and \( R_{t+1} = 1 + r_{t+1} = (P_{t+1} + D_{t+1})/P_t \), respectively, where \( P_{t+1} \) and \( D_{t+1} \) are the price and dividend at time \( t+1 \). We choose the standard two-income economy since the historical correlation between aggregate consumption and the dividend growth rates is weak.

Following Barberis, Huang, and Santos (2001), we assume equal expected growth rates, \( g \), for financial and non-financial income. Importantly, our two sources of news, \( v_t \) and \( "_t \), are not serially correlated. That is, new information or news is not clustered in time.7

We define financial wealth gains and losses relative to an expected level as follows:

\[
X_{t+1} = S_t[R_{t+1} - E_t(R_{t+1})];
\]

Investors experience wealth utility, \( V(X_{t+1}) \), when the return on the risky asset is greater than expected and disutility when it is less than expected. Thus, \( V(X_{t+1}) \) takes its sign from \( X_{t+1} \): In contrast to Barberis, Huang, and Santos (2001), our investors measure performance monthly rather than annually and compare their portfolio’s performance to expectations rather than to the riskfree rate.

2.2 Utility Functions

With regards to consumption, we make the standard power utility assumption, \( U(C) = C^{1-\theta} \): Power utility leads to a constant relative risk aversion parameter, \( \theta \), that measures the local curvature of the utility function and also defines the rate of intertemporal substitution. By itself, utility from the traditional consumption model is unable to justify historical risk premiums without extreme levels of risk aversion; levels that are inconsistent with observed low riskfree rates. Furthermore, the standard model does not predict any of the new empirical facts in finance, such

7Our assumption of IID news contrasts with Campbell and Hentschel (1992) and Bansal and Yaron (2000) who assume news follows a GARCH process.
as time-varying expected returns, high ex-post volatilities of the market return, and, relevant for our paper, the clustering of volatility.\footnote{For early work on the equity-premium puzzle, see Mehra and Prescott (1985), Weil (1989), and Hansen and Jagannathan (1991) and for a review of the overall shortcomings of the standard consumption-based asset pricing model see Cochrane (1997 and 2001).} By adding a second source of utility, Barberis, Huang, and Santos’ model overcomes the consumption model’s deficiency by allowing time- or wealth-varying degrees of risk aversion.\footnote{Another approach, introduced by Campbell and Cochrane’s (1999), to overcoming the standard model’s deficiencies is to allow investors to derive utility from consumption relative to a slowly developing habit level of consumption along the lines of Abel’s (1990) “keeping up with the Joneses” model. The addition of a time-varying “surplus consumption ratio” drives a wedge between risk aversion and the rate of intertemporal substitution, producing the high risk premiums and low interest rates found in practice. Other solutions to the equity-premium puzzle include Constantinides and Duffe (1996) who incorporate uninsured idiosyncratic risks, Epstein and Zin (1989) who separate risk aversion from intertemporal substitution, and Bansal and Yaron (2000) who allow for shocks to the growth rate (not just the levels) of dividends.}

With regards to wealth fluctuations, we make the following utility assumptions:

\[
V(X_{t+1}) = \phi(z_t) X_{t+1}; \quad (3)
\]

\[
\phi(z_t) = k(a_0 + a_1 z_t) \text{ for } X_{t+1} < 0; \quad (4)
\]

\[
= a_0 + a_1 z_t \text{ for } X_{t+1} > 0;
\]

where \( k > 0; a_0 > 0 \) and \( a_1 > 0 \) and where \( z_t \sim \mathcal{N}(1, 1) \) is the mental scorecard of prior investment performance, the only state variable in our economy. \( \phi(z_t) \) controls how much utility our investors receive from a gain (greater than expected return) and how much disutility from a loss; consequently, \( \phi(z_t) \) measures our investors’ level of risk aversion.

Two characteristics of our wealth-varying risk aversion are important: loss aversion and scorecard dependence. First, \( k \) measures the degree of loss aversion. For any level of the scorecard, the “hurt” felt from a loss is greater than the “help” felt from an equivalently-sized gain. Second, \( a_1 \) measures how the past performance affects the magnitude of the utility derived from gains and losses. After a sequence of poor returns the mental scorecard, \( z_t \), is negative and large, and gains and losses are felt more intensely. Thus, investors are more risk averse after past losses. In contrast, after past gains, investors feel gains and losses less intensely and are therefore less risk averse. \( a_0 \) is the investors’ baseline-level of utility derived from gains. The baseline level is intensified by \( k \) for a loss and by \( a_1 z_t \) after past losses and is mitigated by \( a_2 z_t \) after past gains. The notion of loss aversion in (4) is consistent with Kahneman and Tversky’s (1979) prospect theory: the unconditional impact on utility of losses is greater than gains. The idea that investors are more (less) risk averse after losses (gains) is consistent with Thaler and Johnson’s (1988) house-economy: the impact of both losses and gains on utility is conditional on the sequence of prior gains and losses. Loss aversion and scorecard dependence are illustrated in Figure 1.

Figure 1 shows the financial utility derived from gains and losses for three different levels of the scorecard. The kinks in the three \( \phi(z_t) \) functions shown in Figure 1 are driven by loss aversion. With \( k = 1.5 \); a loss’s impact on utility is 50 percent greater than a gain’s impact regardless of past performance. The relative slopes of the three \( \phi(z_t) \) functions are driven by scorecard dependence. After past losses, say \( z_t = 1.5 \) illustrated by the dotted line, shocks have a much bigger impact on utility than after past gains, say \( z_t = 0.5 \) illustrated by the dashed line.

We define the law of motion for \( z_t \); the investors scorecard of prior performance, as:

\[
z_{t+1} = A z_t + h(z_t) X_{t+1}; \quad (5)
\]

\[
h(z_t) = a_2 j z_t + a_3; \quad (6)
\]
where $0 < \lambda < 1$ is the decay or memory parameter, $h(z_t)$ measures the scorecard’s sensitivity to wealth shocks, $a_2 > 0$ is scorecard’s impact on sensitivity to shocks, and $a_3 > 0$ is the state-independent or base level of the scorecard’s sensitivity to shocks. Three characteristics of the scorecard are important; the first two follow Barberis, Huang, and Santos (2001). First, investors only harken back so far when remembering past wealth shocks. Investors gradually become accustomed to new levels of wealth; thus $z_t$ decays toward a steady state equal to 0. For example, levels of $\lambda$ close to 1 indicate that investors have long memories (or persistent habits) and, once perturbed, investors remain so for a long time. Second, changes in the scorecard are driven by wealth shocks. Positive surprises increase and negative surprises decrease the scorecard.

The third scorecard characteristic is unique to our model. In the spirit of Campbell and Cochrane (1999) and Constantinides and Ducce (1996) we reverse engineer our model to explain volatility clustering (see Cochrane (2001) page 476). In our model, the amount that a positive shock increases and a negative shock decreases the scorecard varies with the level of the scorecard. Investors’ responses to news varies over time or, more specifically, varies over levels of past performance. After a large shock or several shocks of the same sign, investors become perturbed or unaccustomed to their level of wealth. When perturbed, the investors are more sensitive or attentive to news. Our assumption in (6), has parallels to the assumptions in Campbell and Cochrane (1999). Whereas Campbell and Cochrane’s investors become accustomed to a level of consumption, our investors become accustomed to a level of wealth. As in Campbell and Cochrane (1999), our investors become more risk averse after negative shocks and less risk averse after positive shocks. When away from their habit-level of wealth, our investors are also away from their habit-level of sensitivity to news. After unexpected declines in the market, our investors pay more attention and have a greater reaction to news than otherwise.

The behavioral assumptions in our model have roots in Helson’s (1964) adaptation-level theory from psychology. Helson showed how people respond to changes in stimuli as well as the level of stimuli. For example, his theory explains why a professor from the Midwest (say Michigan) presenting a paper in the sun belt (say Arizona State University) in January wears short sleeves to lunch while the hosts wear jackets or sweaters. The response to temperature depends on the deviation from a reference temperature or temperature scorecard based on recent experience. Adaptation-level theory also explains why the same professor, while on a nine-month visit to ASU, would wear a jacket to lunch in January. The professor has become accustomed or adapted to warmer temperatures and, on a relative basis, January now seems cool.

The behavioral underpinnings of adaptation-level theory are not new to economics. For example, Duesenberry (1949) argued that the psychological reward or utility derived from consumption depends on the context surrounding the consumption. The relative nature of utility also lies at the heart of the recent “habit” and “keeping up with the Jones” models in finance. In Veronesi’s (1999) and David’s (1997) models, investors’ response to news is also contextual, depending on the degree of perceived uncertainty about the economic state and the degree of profitability, respectively. In a similar manner, we argue that the financial utility derived from a gain or loss depends on the context (scorecard of past gains and losses) surrounding the gain or loss.

Another implication of adaptation-level theory is that a large stimulus captures one’s attention since it is a distinct departure from the norm. In contrast, a small stimulus receives little

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10 This relative utility concept led Duesenberry to predict a preference for a rising consumption pattern in contrast to the smoothed consumption pattern generated by Friedman’s permanent income hypothesis. Using adaptation-level theory, Scitovsky (1976) posits that expenditures for “pleasure” (deviations from the norm such as vacations, holidays, and presents) generate more utility than expenditures on “comfort” (increases in the norm such as new carpet, car, stereo).
notice, the primary effect being a movement in the reference point. In our model, the attention and response to news depends on the context and, once aroused, the investors remain attentive until they adapt to the new level of financial wealth. As such, our model has parallels to Odean and Barber’s (2001) attention model. Odean and Barber show that investors, confronted with a large cross-section of stocks, ration their attention (and consequently purchases) to those stocks in the news and those with recent large price swings. In contrast, our investors ration attention across time, giving more attention to their portfolio after large price swings.

3 Equilibrium

Since $z_t$ is the single state variable, we construct a one-factor Markov equilibrium. Our investors choose consumption, $C_t$; and investment, $S_t$; allocations to maximize expected lifetime utility as in (1) restated below:

$$\max_{C_t} \sum_{t=0}^{\infty} \beta^t U(C_t) + b_0 C_t + b_1 V(X_{t+1})$$

where

$$V(X_{t+1}) = \gamma_0(z_t) X_{t+1};$$

$$\gamma_0(z_t) = \gamma_0(a_0 + a_1 z_t) \quad \text{for} \quad X_{t+1} < 0;$$

$$\gamma_0(z_t) = \gamma_0(a_0 - a_1 z_t) \quad \text{for} \quad X_{t+1} > 0;$$

The transition of the ‘scorecard’ state variable, $z_t$, follows

$$z_{t+1} = A z_t + a_2 j z_t + a_3 X_{t+1}.$$ 

The first-order conditions of (1) for an optimal consumption and investment choice produce the following Euler Equation:

$$1 = \pm E_t [U_t(C_{t+1}) / U_t(C_t)] R_{t+1} + V_t(X_{t+1}). \quad (7)$$

Equation (7) has the standard interpretation: the marginal utility cost of consuming one less unit and investing that unit at time $t$ should equal the expected marginal utility benefit from the proceeds of selling that investment at time $t + 1$. However, in our model the proceeds generate both marginal consumption and financial utility. Substituting (2) into (3) and taking the derivative with respect to $S_t$ yields the marginal utility of financial wealth $V_t(X_{t+1}) = \gamma_0(z_t) [R_{t+1} E_t(R_{t+1})]$; which, along with the marginal utility of consumption, can be substituted into (7) to yield:

$$1 = \pm E_t \left\{ \frac{\mu}{C_t} \right\} \left[ \frac{C_{t+1}}{C_t} \right] R_{t+1} + \gamma_0(z_t) E_t(R_{t+1}) \right\} \quad (8)$$

where $m_{t+1} = \beta E_t(m_{t+1} R_{t+1}) + \gamma_0(z_t)$ and $m_t = \gamma_0(z_t) E_t(R_{t+1})$, and $m_{t+1}$ is the stochastic discount factor. Since we make the standard power utility assumptions on consumption and the standard assumption about lognormal consumption growth, the conditional expectation of $m_{t+1}$ is
\[
E_t[m_{t+1}] = \cdot tE_t \left( \mu \frac{C_{t+1}}{C_t} \right) + \cdot (z_t) \\
= \cdot t E_t \left( \mu \frac{C_{t+1}}{C_t} \right) + \cdot (z_t) \\
= \cdot t E_t \left( \mu \frac{C_{t+1}}{C_t} \right) j^t + \cdot (z_t) \\
= \cdot t E_t G^\frac{1}{2} e^{\frac{1}{2} \frac{1}{\sigma^2} \frac{\mu}{\sigma} + \frac{1}{2} \left( 1 - \frac{1}{2} \right) \frac{\mu^2}{\sigma^2} + \cdot (z_t)},
\]

where \( G = e^\theta \). Equation (9) highlights two components of the stochastic discount factor: consumption risk, terms found inside the brackets, and financial utility risk, \( \cdot (z_t) \). The financial utility risk fluctuates with the sensitivity of the investor. Figure 2 shows how \( E_t[m_{t+1}] \) varies with the state variable \( z_t \). Intuitively, future cash flows are penalized more heavily when \( z_t \) is negative, consistent with higher risk aversion and sensitivity. In contrast, future cash flows receive a much smaller discount when \( z_t \) is positive. Our model's ability to give more worth to a future cash flow in some states than others and its ability to have state autocorrelation (memory) allows the model to be consistent with the level and persistence of return volatility and the size and predictability of equity premiums found in stock returns.

For the risk-free asset, \( R_{t+1} = E_t(R_{t+1}) \) always equals zero so (8) reduces to a similar Euler equation as found in Mehra and Prescott (1985),

\[
1 = \pm R_{f,t+1} E_t \frac{1}{\mu \frac{C_{t+1}}{C_t}}.
\]

Solving Equation (10) for the risk-free rate yields:

\[
R_{f,t+1} = \frac{1}{\pm E_t \frac{C_{t+1}}{C_t}}.
\]

which is constant. Furthermore, since our model generates the risk premium via financial utility and loss aversion rather than high levels of the risk aversion coefficient, the model can have both a low constant risk-free rate and a high equity premium, a combination that is puzzling for the standard model.

Multiplying (8) by \( P_t \) and dividing by \( D_t \) yields the following pricing kernel form of the Euler Equation:

\[
\frac{P_t}{D_t}(z_t) = E_t m_{t+1} \frac{\mu P_{t+1}}{D_{t+1}} + 1 \frac{D_{t+1}}{D_t}.
\]

We provide numerical solutions to the model in the next section, but before moving to the solutions we discuss some of the qualitative intuition of the model. For example, assume that an investor receives a negative dividend shock which leads to a negative return shock, \( X_{t+1} \). The bad news will drive up risk aversion through \( \cdot (z_t) \); resulting in an additional price decline. Also, assuming that the investor's scorecard was at its average level of 0, the negative dividend shock would increase the investor's sensitivity to subsequent shocks and increase expected volatility. The higher expected
volatility would feed back, causing a further negative impact on prices. That is, bad news about dividends poses a triple threat to prices: the bad news itself, the resulting increase in risk aversion, and the resulting increase in sensitivity to news. The second and third threat allow returns to be more volatile than the growth shocks on fundamentals. Furthermore, the impact lingers because the investor's heightened sensitivity decays slowly over time and reaction to future dividend shocks will be greater than if the investor was not perturbed.

If instead the investor experiences a positive shock, the contemporaneous shock itself will be exacerbated by the accompanying reduction in risk aversion; a double threat. However, as in Campbell and Hentschel (1992), this increase in price will be tempered by the increase in expected volatility. Noting that the sensitivity of the scorecard is symmetric to both positive and negative shocks, the behaviorally induced increase in prices after positive shocks will be smaller than the behaviorally-induced price decline after a negative shock. Consequently, although our model has symmetric sensitivity, it will produce asymmetric responses to news, a result consistent with empirical evidence [see Bollerslev (1986)].

Our model will also generate skewness and kurtosis in stock return distributions, features similar to those found in the data. Skewness of stock returns is a result of the asymmetric response (triple vs. double threat) to news described above. The return distributions will also be leptokurtotic since the variance of stock returns moves overtime, and the mixing of conditional distributions induces higher unconditional fourth moments relative to the normal distribution.

4 Solving the Model

In this section we first discuss our choices for the parameter values of our base-case model and then discuss the results of a numerical solution to this model including some robustness tests. We also report the results of our simulations.

4.1 Parameter Values

The parameter values we assume when solving the model numerically are given in Table 1. The first six rows of Table 1 report parameter values that are associated with the traditional consumption model and the next six rows report parameters that are unique to our model. We set the values of observable parameters, $g$, $\pm$, $\kappa^2$, and $\kappa^2$ equal to their historical U.S. values taken from either Campbell and Cochrane (1999) or Barberis, Huang, and Santos (2001). We also set $\kappa$ equal to 2 following Campbell and Cochrane (1999). The loss-aversion parameter, $k$, is set equal to 1.5, which is lower than values used in Barberis, Huang, and Santos (2001). A smaller $k$ makes our model less dependent on the behavioral explanation of loss aversion and makes our model more stable.

In our base-case solution, we choose the model-specific parameters, $A$, $a_0$, $a_1$, $a_2$, $a_3$, and $b_0$ to meet two objectives: 1) to match certain moments of the postwar data (such as the equity premium and clustered volatility) and 2) to achieve stable solutions to the model.\footnote{We add an additional restriction to the model which reduces the number of parameters we need to choose. Specifically, we impose the restriction that $(a_i) = 1$ for all $i$, that the investor becomes risk-neutral as the scorecard increases. Given this condition and a choice of $a_1$, the implication is that $a_i = 1 + a_1$. For this reason we drop any discussion of the parameter choice of $a_2$.} The second objective is important because of the explosive nature of our model. After an exogenous shock, our investors become sensitive to news, exacerbating the response to the next exogenous shock.
4.2 Numerical Solution

Solving our model requires a solution to

\[
\frac{P_t}{D_t} (z_t) = E_t \cdot \mu \frac{P_{t+1}}{D_{t+1}} + 1 \frac{\frac{D_{t+1}}{D_t}}{D_{t+1}}
\]

as well as the law of motion for the scorecard, (5), which is also endogenous. Our numerical solution requires that we simultaneously solve equations (12) and (5) due to their self-referential nature. In order to obtain conditional price-dividend ratios (12), we need to know the law of motion for \(z_t\); (5), which itself requires a knowledge of conditional price-dividend ratios. Since both price-dividend ratios and the scorecard are endogenous, we use an iterative method similar to Barberis, Huang, and Santos (2001) to obtain solutions for the model. Specifically, we guess a solution for the pricing equation, (12), and use this solution to construct a transition relationship for \(z_t\), which satisfies the intermediate pricing solution. Given the new transition relationship for \(z_t\), we update the pricing solution and iterate until both the pricing solution and the transition relationship each converge.

Price dividend ratio: Since the stochastic discount factor varies, so too does the price/dividend ratio. We can restate (12), conditional on levels of \(z_t\) as

\[
\frac{P_t}{D_t}(z_t) = E_t \cdot \mu \frac{C_{t+1}}{C_t} + \gamma(z_t) \frac{\frac{D_{t+1}}{D_t}}{D_{t+1}} \gamma 1 + \frac{\frac{D_{t+1}}{D_t}}{D_{t+1}} (z_{t+1})
\]

Solving (13) numerically using parameter values from Table 1 yields the relationship between \(\frac{P_t}{D_t}\) and \(z_t\), illustrated in Figure 3. After a sequence of good returns \(z_t > 0\); our investors “feel” changes in financial wealth less intensely, are less risk averse, and consequently discount future dividends at a lower rate and are willing to pay more for stocks than otherwise. The concavity of the price-dividend function is a result of heightened sensitivity to future news when the scorecard is away from zero. The sensitivity feedback moves against lower risk aversion when \(z_t\) is positive and with higher risk aversion when \(z_t\) is negative. After an unusual run of good returns, the state variable could reach 1.0 and the price dividend ratio approach 13.7. Figure 3 also indicates that price-dividend ratio could fall to about 12.3.\(^{12}\)

Stock return: The resulting conditional expected stock return is

\[
E_t(r_{t+1}) = E_t \left[ \frac{\mu \frac{P_{t+1}}{D_{t+1}}}{P_t} \right] = E_t \left[ \frac{\mu \frac{P_{t+1}}{D_{t+1}}}{P_t} (z_{t+1}) + 1 \frac{\frac{D_{t+1}}{D_t}}{D_{t+1}} \right] : (14)
\]

\(^{12}\)We have solved a similar version of this model where the sensitivity and risk aversion functions are more general and nonlinear. In this unreported model the spread of price/dividend ratio can vary from 4 to 23 while maintaining similar qualitative features to our reported model (volatility dynamics, expected returns). The choice to report this model was motivated by parsimony.
After obtaining a solution to (13), we use numerical integration to solve for the conditional expectation in (14). Figure 4 illustrates the negative relationship between conditional expected returns and $z_t$. When our investors’ scorecards are low ($z_t < 0$), resulting from a sequence of negative return shocks, the investors “feel” changes in financial wealth more intensely, exhibit high levels of risk aversion, and consequently require a high expected return on stocks. Figure 4 shows that the model allows the expected return to vary from a high of about 18 percent to a low of about 7 percent annually.

An implication of our model is that sometimes investors will have contrarian expectations while other time their expectations will be positively autocorrelated. If investors receive a positive wealth shock, they may continue to believe that expected returns will be high if their current scorecard is negative. However, if the positive return is sufficient to drive the scorecard positive, expected returns will be expected to decline below unconditional expectations. Consequently, on occasion returns will follow momentum patterns, while at other times contrarian patterns.

**Risk Premium:** From (11) and (9) the risk-free rate is
\[
R_{f,t+1} = \mathbb{E}_t \left( G^{1/2} e^{\sigma Z_{t+1}^2 t + 1/2 (1 - 1/2) Z_{t+1}^2} \right).
\] (15)

Since (15) shows that $R_{f,t+1}$ is independent of the scorecard, $z_t$, the risk-free rate in Figure 4 is constant. As our investors realize a sequence of positive shocks, their aversion to risk declines. Figure 4 shows that at very high levels of $z_t$, our investors approach risk neutrality and the risk premium approaches zero as the expected return on stocks approaches the risk-free rate. In general, however, our investors’ concern for financial wealth and fear of losses is capable of generating a risk premium without increasing the coefficient of risk aversion, $\gamma$; to puzzling levels. Figure 4 indicates that the annualized conditional risk premium varies between 1 and 12 percent.

**Standard deviation:** The standard deviation of returns, $\mathbb{E}_t (\Delta R_{t+1}) = \mathbb{E}_t (r_{t+1} - \bar{r})^2$, conditional on the scorecard, $z_t$, can be calculated using numerical integration techniques. Figure 5 graphs the relationship between the standard deviation of returns, $\mathbb{E}_t (\Delta R_{t+1})$, and $z_t$. Figure 5 shows that as our investors become perturbed in either direction, their sensitivity to shocks and consequently the expected standard deviation of stock returns both increase. However, negative shocks lead to greater volatility than do positive shocks. This asymmetry results from the fact that shocks (positive or negative) increase sensitivity and have an effect of reducing prices which exacerbates price declines after negative shocks but tempers price increases after positive shocks. Figure 5 shows how the annualized expected standard deviation of stock returns can vary from 12.5 percent to 24 percent.

**Skewness:** Skewness, conditional upon the scorecard, follows the relationship as seen in Figure 6, where $\gamma_3$ is the skewness coefficient and $\gamma_3 = \mathbb{E}_t ((r_{t+1} - \bar{r})^3)$. Intuitively, the negative slope of the conditional skewness curve in Figure 6 is related to the law of motion for the scorecard as in (5). When the scorecard is negative, we expect a reversion of the scorecard towards zero as the

---

13Contrarian expectations are also predicted by models such as Barberis, Huang, and Santos (2001) and Campbell and Cochrane (1999) and are empirically supported by papers such as Campbell and Shiller (1988) and Fama and French (1988). See Jegadeesh and Titman (1993) for empirical evidence of momentum.

14We also .nd, but do not present, negative skewness in the unconditional return distributions.
investor becomes accustomed to the new level of wealth. This movement towards zero will have a positive impact on prices and returns independent of the news regarding dividends, thus inducing a positive skew or more accurately less negative. On the other hand, when the scorecard is positive, (5) shows that the expected movement in the scorecard is a decline towards zero that will again cause a decline in prices and returns independent of any dividend news, increasing the negative skewness.

4.3 Simulations

After investors observe the new information \((v_{t+1} \text{ and } r_{t+1}; z_{t+1}, P_{t+1}, D_{t+1})\), and \(r_{t+1}\) can be simultaneously determined. For each simulation, we create a sample path of 10,000 monthly observations of income and consumption shocks, \(v_{t+1} \text{ and } r_{t+1}\); based on their historical standard deviations reported in Table 1. Then we use these two series of exogenous shocks to develop a time series of returns. For each simulation we find the average level of variables such as the risk-premium and price-dividend ratio as well as the average parameter estimates from a GARCH(1,1) model on the simulated stock returns using the traditional GARCH formulation,

\[
\begin{align*}
    r_t &= c + e_t \\
    \text{Var}(e_t) &= h_t = c_0 + c_1 h_{t-1} + c_2 e_{t-1}^2.
\end{align*}
\]

We perform 10 simulations for the base case and, for comparison purposes, 10 simulation for the traditional case. In the traditional case \(b_0 = 0\), which renders financial utility meaningless and collapses our model to the “traditional” consumption-only model. The first row of Table 2 reports the average values from all ten simulation of our base case and the second row reports the average values from all 10 simulations of the traditional case.

The first four columns of Table 2 show the degree and significance of our model’s ability to capture clustered volatility. By adding a behavioral response to dividend shocks, our investors’ scorecards can move. Once moved, the sensitivity function, (6) indicates that the scorecard is likely to move by an even larger amount in the next period. The \(a_2\) parameter controls the extent that one movement in the scorecard increases the potential size of the next movement. Thus, the sensitivity function, in particular the \(a_2\) parameter, generates ARCH-like behavior in returns. The base-case ARCH parameter of 0.045 (p-value of 0.001) documents the ability of our model to generate autoregressive conditional heteroskedasticity that is not significant in the traditional model. The \(\Delta\) parameter controls the speed of the scorecard’s decay and consequently the speed at which sensitivity to news returns to normal. Thus, the scorecard’s law of motion function, in particular the \(\Delta\) parameter, generates GARCH-like behavior in returns. The simulation of our model generates a GARCH parameter of 0.806 (p-value less than 0.001). A similar simulation of the traditional case yield an insignificant GARCH coefficient. The first four columns of Table 2 show that our model can generate clustered volatility, even when fundamental news itself is not clustered.

The Barberis, Huang, and Santos (2001) model is designed to capture some of the new empirical findings in finance. Since our model is based on theirs, our base case also captures these unconditional moments of the return distribution. Relative to the traditional consumption-only model, our model has higher levels of return volatility (annualized return standard deviation of 13.82 rather than 11.19 percent) and a reasonable equity premium (annualized premium of 4.24 rather than -0.38 percent). Because of the fear of losses, stocks are less attractive to investors in our base case than in the traditional case. Our model yields annualized price dividend ratio of 13.13 rather than 30.23 and, not reported, a Sharpe ratio of 0.199 rather than 0.133. However, the
The attractiveness of stocks in our base case varies over time whereas the only source of variation in the traditional case is from the exogenous dividend stream. Also, not reported, the standard deviation of the price-dividend ratio is 0.755 in our base case and 0.000, by definition, in the traditional case. The standard deviation of the scorecard in our base case is 0.329. Although the scorecard is zero on average, scores of 1.0 or -1.0 (three standard deviations away from zero) are possible. In our simulations we observe rare occurrences of scorecards above 1.0 and below -1.0.\(^\text{15}\)

The last six rows of Table 2 illustrate the sensitivity of our results to changes in the six parameter values. In each row, we change one of the original parameters then run ten simulations and report the average values of the results. In each case we perturb the base-case parameter in a direction that decreases the importance of behavioral responses. That is, we explore the model as we move parameters in the direction of the traditional, consumption-only model. As is shown in Table 2, as our investors become less behavioral motivated, the degree of volatility clustering and the ability of the model to explain the new empirical facts is diminished.

5 Testing the Scorecard's Correlation with Conditional Volatility, Excess Returns, and Skewness

Prior to this section, we reverse-engineered a model to generate return moments, such as clustered volatility found in historical returns, and then solved and simulated the model. We have shown that our model is consistent with the well-documented conditional second moment features of returns that motivated the model. We now turn our focus to testing the theoretical model using new predictions about conditional moments. In this section we test our scorecard's ability to predict not only conditional volatility, but also conditional excess returns and skewness. Using our model to predict conditional returns and skewness provide powerful tests because these moments did not influence our model and solutions as did conditional volatility. Hence, we cannot search over model parameters or "mine" the data to provide better forecasts.

5.1 The Scorecard and Conditional Volatility

An obvious prediction of our theoretical model is that return volatility is clustered. However, the clustering of returns motivated our model in the first place; consequently, a test for clustering itself cannot be used as a test of our model. To this end, new and refutable predictions are needed. Our model makes such predictions. Our theoretical model links past volatility to future volatility in a very specific way—through the investors' scorecard as illustrated in Figure 5.

First, our theoretical model predicts that the further from zero a weighted sum of past shocks, \(z_t\), becomes, the greater the next period's volatility. Prior statistical descriptions of clustered volatility generically relate large shocks to future large shocks. In contrast, our model specifically predicts a cumulative effect, for example, two successive large shocks of the same sign will have a much bigger impact on future volatility than two successive shocks with opposite signs. Second, our theoretical model predicts that negative levels of \(z_t\) will be associated with news having a greater impact on future volatility than positive levels of \(z_t\). Note that our second prediction is not just a restatement of the GARCH asymmetry that helped motivate our model. GARCH asymmetry suggests that a single piece of good news will be followed by high levels of volatility, but a single piece of bad news will be followed by even higher levels of volatility. In contrast,

\(^{15}\) This empirical distribution of the scorecard informs the x-axis scale of Figures 2 to 5.
our model conditions volatility, asymmetrically, on the scorecard. For example, when \( z_t \) is close to zero, bad news will have little effect on future volatility, whereas after a string of good news, another piece of good news will be followed by very high levels of volatility.\(^{16}\)

We test our two volatility predictions using two empirical models. Our first empirical model, following Pagan and Schwert (1990), regresses an estimate of conditional return volatility \( \sigma_t^2 \); denoted \( \Phi_t \), on an estimate of the lagged scorecard, \( \bar{z}_{t-1} \), allowing for the nonlinearity:

\[
\Phi_t = -0 + \hat{\beta}_1 \bar{z}_{t-1} + \hat{\beta}_2 \bar{N} \bar{z}_{t-1} + \nu_t;
\]

where \( N \) is a binary variable equal to 1 when \( \bar{z}_{t-1} \) is negative and zero otherwise. Our model predicts \( \hat{\beta}_1 > 0; \hat{\beta}_2 < 0; \) with \( \hat{\beta}_1 \hat{\beta}_2 > \hat{\beta}_2 \hat{\beta}_1 \). We test these predictions against null hypothesis 1, \( H_1: \hat{\beta}_1 = -\hat{\beta}_2 = 0 \). In our second empirical model, we explore whether our measure of the scorecard has predictive power beyond Nelson's (1991) EGARCH model. The EGARCH (1,1) model allows returns to be asymmetrically correlated with future volatility. Specifically, empirical model 2 regresses an estimate of conditional return volatility, \( \Phi_t \); on an estimate of the lagged scorecard, \( \bar{z}_{t-1} \); and on predictions of conditional volatility, \( \Phi^2_t \), from an EGARCH (1,1) model:

\[
\Phi_t = -0 + \hat{\beta}_1 \bar{z}_{t-1} + \hat{\beta}_2 \bar{N} \bar{z}_{t-1} + \hat{\beta}_3 \Phi^2_{t-1} + \nu_t;
\]

where \( \hat{\beta}_3 \) is reserved for Tables 5 and 6. Our estimate \( \Phi_t \) is the squared residuals from a simple regression of monthly index returns on a constant, \( r_t = c + \epsilon_t \). Our estimate of \( \bar{z}_t \) is from \( \bar{z}_t = \bar{z}_{t-1} + (a_2 \bar{z}_{t-1} + a_3) X_t \). We start our scorecard at zero in December 1925 and using the parameter estimates in Table 1 (\( A = 0.92; a_2 = 5.0; \) and \( a_3 = 2.3 \)) to recalculate the scorecard each subsequent month, and let \( X_t \) be the deviation of the portfolio return from its unconditional average.\(^{17}\) Following Nelson (1991), our forecast of \( \Phi^2_t \) is from the EGARCH (1,1) model, \( \log(\Phi^2_t) = \hat{\alpha}_0 + \hat{\alpha}_1 \log(\Phi^2_{t-1}) + \hat{\alpha}_2 \Phi^2_{t-1} \hat{\nu}_{t-1} + \hat{\alpha}_3 \Phi^2_{t-1} \hat{\nu}_{t-1} \).

The first two rows of Table 3 report the results of estimating empirical models 1 and 2, respectively, using post-war (1952 to 2000) data.\(^{18}\) We test our model's predictions on monthly U.S. value-weighted CRSP returns. After reporting the base case, we also report results from alternative time periods and the results of sensitivity analysis with respect to our proxies for \( \Phi_t \) and \( \bar{z}_{t-1} \). All our tests use Newey-West's (1987) t-statistics, a correction for generalized serial correlation.

In the 1952 to 2000 regression, \( \hat{\beta}_1 \) is positive, \( \hat{\beta}_2 \) is negative, and \( \hat{\beta}_2 \) \( \Phi^2_t \) is greater than \( \hat{\beta}_2 \Phi^2_{t-1} \) as predicted. However, only \( \hat{\beta}_2 \) is significant different from zero (p-value less than 0.001). The adjusted \( R^2 \) is 0.095; for comparison purposes, the \( R^2 \) for an EGARCH (1,1) model on the same data is just over 2 percent. Null hypothesis 1, \( H_1: \hat{\beta}_1 = -\hat{\beta}_2 = 0 \), is rejected with a p-value of less than 0.001. Figure 7, panel (a), illustrates the combined effect of \( \bar{z}_{t-1} \) on \( \Phi_t \) over ranges of the scorecard found in historical return data and closely resembles the effect, illustrated in Figure 5, found in our theoretical model. Historically, conditional volatility is lowest when the scorecard is near zero and increases as investors are perturbed. Furthermore, the historical relationship in Figure 7 shows that negative perturbations have a bigger influence than positive perturbations.

When volatility forecasts from the EGARCH model are included, empirical model 2, the \( R^2 \) increases slightly to 0.098, the \( H_1 \) is still significantly rejected and \( \hat{\beta}_4 \) is significant. Thus, both

\(^{16}\)A third prediction is that as \( z_t \) moves away from zero future volatility increases exponentially. The curvature in Figure 5, however, is not pronounced. Unreported tests of this third prediction result in coefficients that have the predicted sign (convexity) but are not significant.

\(^{17}\)Starting the scorecard in December 1925 at 1 or -1 has no material effect on the results reported in Table 3.

\(^{18}\)Our 1952 break point corresponds with a new Federal Reserve/Treasury accord and is the post-war separation date used by Campbell (1991) and Campbell and Hentschel (1992).
the EGARCH and scorecard are significant, indicating that our behavioral explanation and other explanations (clustered news, for example) may both be causing volatility autocorrelation. Figure 7, panel (b), illustrates the incremental effect (above that of the EGARCH(1,1) predictions) of the scorecard on conditional volatility. The scorecard's value in predicting volatility is not diminished when competing with an EGARCH(1,1) model. In addition to the Table 3 regressions, we also estimated an EGARCH (1,1) model augmented by the scorecard,

$$\log(\sigma_t^2) = \beta_0 + \beta_1 \log(\sigma_{t-1}^2) + \beta_2 \epsilon_{t-1} + \beta_3 z_{t-1} + \beta_4 N z_{t-1}.$$  

In this unreported regression, $\beta_4$ and $\beta_5$ are both statistically significant and the $R^2$ of the regression increases from 0.023 to 0.139 when the two scorecard terms are included.

We check the robustness of the 1952 to 2000 results by looking at two alternative time periods; the first alternative time period is 1927 to 2000. Whereas the CRSP data start in January of 1926, our regression starts in January of 1927 to make the first value of $z_t$ meaningful. As with the post-war data, the scorecard can be used to predict conditional volatility; $H_1$ is rejected, the $R^2$ is over 20 percent, and both $\beta_1$ and $\beta_2$ are significant. Our second alternative time period is based on the 1802 to 1925 pre-CRSP data of Schwert (1990); we start the scorecard at zero in December of 1801 and begin our tests in January 1803 to allow the scorecard to move away from zero. For Schwert’s pre-CRSP data, both beta coefficients are statistically significant with and without the EGARCH estimates of conditional volatility.

In the first three time periods of Table 3, our proxy for the scorecard is built up using $A = 0.92$; $a_2 = 5.0$; and $a_3 = 2.3$. These behavioral parameter estimates were found when calibrating the theoretical model but are not directly observable. We check for robustness by examining alternative measures of $A$, $a_2$, and $a_3$. In each scenario, we decrease one behavioral parameter, making the scorecard and financial wealth utility less important. The changes to the behavioral parameters have little effect on the scorecard’s ability to predict conditional volatility.

We also use Pagan and Schwert’s (1990) more robust method for constructing a measure of conditional volatility, $\sigma^2_t$. The Pagan and Schwert method first regresses returns on 12 monthly dummy variables, second regresses these first-pass residuals, $u_t$, on lagged values, $u_t = \sum_{i=1}^{10} \beta_i u_{t-i} + \epsilon_t$; then squares the second-pass residuals. The two enhancements control for seasonality and simple autoregression in the residuals. Test results using the enhanced measure for conditional volatility are reported in the last two rows of Table 3. In both empirical models, $H_1$ is rejected and both coefficients have the predicted sign.

5.2 The Scorecard and Conditional Excess Returns

The solution and simulation sections of our paper show that our theoretical model is consistent with a high unconditional risk premium and that the premium is obtained without resorting to unreasonable degrees of consumption risk aversion. However, finding evidence of a high excess returns does not constitute a test of our model since the premium puzzle was one of the motivating forces for the model. As seen in Figure 4, our model makes two new refutable predictions about expected excess returns conditional on the scorecard. First, $z_t$ is negatively correlated with future conditional excess returns. When $z_t$ is negative (positive), a large (small) reward is needed to induce investors to bear the risk of stocks; consequently, expected returns should be high (low). Second, this negative correlation is stronger when $z_t$ is negative than when it is positive. When the scorecard is negative, investors demand a high premium for two reinforcing reasons: the investors themselves are unusually risk averse and stocks (due to investor sensitivity to news) are extra volatile. When the scorecard is positive, the later force cancels some of the former’s effect on the conditional excess return, $r_t - r_{f,t}$. We test these two predictions using empirical model 3:
\[ r_{t_i} r_{f,t} = -0 - 1 \bar{R}_{i} 1 + -2 N \bar{R}_{i} 1 + v_t; \]

Regarding excess returns, our model's two new predictions are that both \(-1\) and \(-2\) are negative. We compare these two predictions against null hypothesis two: \(H_2: -1 = -2 = 0\).

We also explore whether our measure of the scorecard has predictive power beyond that of financial and macroeconomic indicators previously found to predict excess returns. Predicting excess returns is difficult and these additional dependent variables, despite the finance profession's potential data mining abuses, typically have significance limited to specific sample periods or return horizons. Empirical model 4 tests for a relationship between the scorecard and excess returns in the expanded regression, empirical model 4:

\[ r_{t_i} r_{f,t} = -0 + -1 \bar{R}_{i} 1 + -2 N \bar{R}_{i} 1 + 4(r_{t_i} 1) + -5DY_{t_i} 1 + -6DEF_{t_i} 1 + -7RREL_{t_i} 1 + -8TRM_{t_i} 1 + v_t; \]

where \(r_{t_i} 1\) is the lagged excess return, \(DY\) is the dividend-yield ratio, \(DEF\) is the default spread (difference between BAA and AAA corporate bond yields), \(RREL\) is the relative short-term interest rate (3-month Treasury Bill yield less its 12-month moving average), and \(TRM\) is the term spread (difference between the long-term Treasury Bond yield and the 3-month Treasury Bill yield). Data sources and definitions for the additional dependent variables in empirical model 4 are given in the Appendix.

The first two rows of Table 4 report the results of estimating empirical models 3 and 4 using post-war (1920 to 2000) data. For empirical model 3, although \(-1\) has the predicted sign, neither scorecard coefficient is significant and \(H_2\) cannot be rejected (p-value = 0.616). In contrast, after including the additional dependent variables, the predictive negative relationship between the scorecard and excess returns becomes significant \((-1 = 1: 0.875; \) p-value = 0.085) although \(-2\) remains insignificantly positive. The combined influence of the scorecard on next month's excess return is illustrated in panel (a) of Figure 8. The figure highlights both our scorecard's ability to predict low excess returns when our investors are not very risk adverse and also highlights our scorecard's inability to predict high excess returns when our investors should be very risk adverse.

Both scorecard coefficients retain their signs and reduce their significance in the 1927 to 2000 sample period. In the pre-CRSP sample, the scorecard/excess return relationship is negative only when the scorecard is less than zero. Alternative measures of \(a_2\); \(a_3\); and \(A\) have little effect on the two scorecard coefficients. Decreasing \(a_2\) makes \(-1\) slightly more negative and decreasing \(A\) makes \(-1\) slightly more positive. In absolute terms, the scorecard is a poor predictor of excess returns. However, relative to a common set of so-called predictors, the scorecard's performance is noteworthy. For example \(-1\)'s p-value is typically lower than the p-values of \(-4\) to \(-8\).

In an attempt to understand why low scorecards are not predicting subsequent high excess return, we examined the few data points in sharpest contrast to our prediction of \(-2 < 0\). Interesting, the most theory-conflicting points were associated with one event, the oil embargo of 1974. In response to the United States supplying arms to Israel during the war in late 1973, Arab countries stopped selling oil to the U.S. December 1973 was the first full month when no Arab oil was supposed to enter the U.S. The last two rows of Table 4 report the results of empirical models 3 and 4 without the 1974 data, and panel (b) of Figure 8 graphs the estimated conditional return relationship. In both models \(-1\) and \(-2\) are negative as predicted and in model 4 null hypothesis.

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\(^{19}\) Initial evidence on momentum of returns are found in Jegadeesh and Titman (1993). Initial evidence of the predictive power of the dividend yield are found in Shiller (1981). Initial evidence for the default and term spreads come from Fama and French (1989) and initial evidence for the relative short rate are from Campbell (1991).
two is rejected with a p-value of 0.015. On one hand, not too much should be made about the results without 1974; all models look better if one deletes selective data. On the other hand, our behavior model seems to have some predictive power with regards to excess returns, just not enough to overcome a shock the size of the oil embargo.

5.3 The Scorecard and Conditional Skewness

Given the triple effect of bad news but double effect of good news, our theoretical model is clearly consistent with returns that are unconditionally negatively skewed. Finding unconditional negative skewness in-and-of-itself does not constitute a powerful test of our model. Such a powerful test would need new refutable implications relating $z_t$ with conditional skewness. Our theoretical model makes such predictions as is evidenced by Figure 6. The first prediction is a negative relationship between the scorecard and conditional skewness. From (5), when $z_t$ is large and negative, investors expect that in the next period the scorecard will take a large step back toward zero since $\hat{A}$ is less than 1. This expected large step toward less risk aversion and less sensitivity to news conditionally dampens the effect of bad news making large negative returns less likely. On the other hand, when the scorecard is large and positive, the $\hat{A}$-induced large step back toward zero exacerbates bad news causing conditionally negative skewness. The second prediction is that the scorecard/skewness relationships will be concave because the size of the expected $\hat{A}$-induced step toward a scorecard of zero is larger when the scorecard is further away from zero. We capture these two predictions using empirical model 5:

$$r_{3,t} = \bar{z}_{t} + \zeta_{t} + \bar{z}_{3,t} + \zeta_{t} + \nu_{t}$$

where our estimate of each month's skewness coefficient, $r_{3,t}$, is created from within-month daily returns in the spirit of French, Schwert, and Stambaugh (1987). That is $r_{3,t} = \frac{1}{N} \sum_{n=1}^{N} (r_{n,t} - \bar{r}_{t})$, where $r_{n,t}$ is the market return on day $n$ of month $t$, $\bar{r}_{t}$ is the average daily return in month $t$, $N$ is the number of trading days in month $t$, and $\bar{z}_{3,t} = \frac{1}{N} \sum_{n=1}^{N} r_{n,t}^2 + \frac{2}{N} \sum_{n=1}^{N} r_{n,t}r_{n,t+1}$. When calculating this variance, the small daily mean is ignored and the cross-product accounts for non-synchronous trading. The definitions and sources of our daily return data are in Appendix A.

Regarding skewness, our model's two new predictions are that both $\bar{z}_{t}$ and $\bar{z}_{3,t}$ are negative. We compare these two predictions against null alternative three: H3: $\bar{z}_{t} = \bar{z}_{3,t} = 0$.

In an effort to test the scorecard's ability to predict skewness beyond simple autocorrelation we also test H3 after including lagged skewness and returns using empirical model 6:

$$r_{3,t} = \bar{z}_{t} + \bar{z}_{3,t} + \bar{z}_{4,t} + \bar{z}_{5,t} + \nu_{t}$$

Table 5 reports the result of empirical models 5 and 6 with the base case results (1952 to 2000) reported in the second two rows. For both empirical models, $\bar{z}_{1}$ and $\bar{z}_{3}$ are negative and significant as predicted by our empirical model. Although the overall predictability of the scorecard is low ($R^2 = 0.039$ and 0.050 for models 5 and 6, respectively) the conciseness level for the predictability that does exist is very high. H3 is strongly rejected (F-test of 13.080, p-value less than 0.001). The sensitivity analysis reported in the remaining rows of Table 5 indicated that the first prediction, $\bar{z}_{1} < 0$; is very robust. The higher the scorecard, the more negatively skewed the return distribution. In the two earlier samples, $\bar{z}_{1}$ remains negative and significant. When the three behavioral parameters, $\hat{A}$; $a_2$; and $a_3$; are relaxed, $\bar{z}_{1}$ maintains its the predicted sign, although statistical significance is limited to empirical model 5. Support for the conciseness prediction is not robust and is only statistically significant in the base case regression.
We have empirically tested three sets of predictions relating our scorecard to clustered volatility, excess returns, and skewness. However, our theoretical model potentially captures other stylized facts about stock returns. We cite several examples. First, Figure 4 suggests that our model is capable of explaining periods of momentum. Specifically, when \( z_t \) is large and negative (positive) we expect returns to be above (below) unconditional averages until investors become accustomed to their new levels of wealth. Second, Figures 4 and 5 together suggest that our theoretical model is capable of explaining the “Sharpe ratio volatility puzzle” documented by Lettau and Ludvigson (2002). Many of the newer asset pricing models such as Campbell and Cochrane (1999) and Barberis, Huang, and Santos (2001), unlike the traditional CAPM, are capable of generating time-varying equity risk premium. However, these newer theoretical models require the equity risk premium to vary positively with stock market volatility. Evidence of the co-movement of time-varying excess returns and volatility is mixed. For example, Lettau and Ludvigson (2002) and Li (2001) document a negative relationship whereas Campbell and Hentschel (1992) report a positive relationship. Our model predicts a positive relationship half the time (when scorecards are negative) and allows for a negative relationship the other half of the time. Third, by allowing for returns to be drawn from different distributions conditional on the scorecard, our model is capable of generating leptokurtic unconditional return distributions. An obvious path for future research is testing the historical relationship between our scorecard and conditional return properties such as momentum, time-varying Sharp’s ratios, and kurtosis.

5.4 Competing Scorecards

Lettau and Ludvigson (2001) posit that investors, in an effort to smooth their consumption path, invest less and consume more (out of labor income and existing investments) when expected returns are high. Consequently, the log consumption-aggregate wealth ratio should be able to forecast stock returns. One can think of deviations from the consumption-aggregate wealth ratio trend as a competing scorecard. Lettau and Ludvigson (2001) develop a proxy for the log consumption-aggregate wealth ratio, \( \hat{d}_{cay} \), that is significantly correlated with future short and intermediate-horizon excess stock returns. They find that the proxy has more predictive ability than extant covariates, such as the dividend yield. Furthermore, in a follow-on study, Lettau and Ludvigson (2002) show that \( \hat{d}_{cay} \) not only predicts excess returns, but also the volatility of excess returns and hence Sharpe ratios.

Campbell and Cochrane (1999), too, have a competing scorecard. In their model, investors’ degree of risk aversion is influenced by a scorecard they call the surplus consumption ratio, \( \hat{scr}_t \), where \( X_t \) is a slow-moving, time-varying habit level of consumption. For example, in a boom, consumption is well above the habit level, investors are less risk averse, and future returns are expected to be low. Campbell and Cochrane (1999) do not formally test their scorecard but do provide a graph (their Figure 9) that visually relates their scorecard’s prediction to historical levels of the price-dividend ratio.

We now compare and contrast the predictive ability of \( \hat{z} \), \( \hat{d}_{cay} \), and \( \hat{scr}_t \) to predict conditional volatility, excess return, and skewness. Our data on \( \hat{d}_{cay} \) comes directly from Lettau and Ludvigson and is available monthly starting in January 1959 and quarterly starting in the first quarter of 1952. Data on consumption (nondurable plus service) is available quarterly. We obtain consumption data from Lettau and Ludvigson and population data from the Census Bureau to create per-capita consumption. We create a quarterly measure of \( \hat{scr}_t \) using Campbell and Cochrane’s equations (3) and (10) plus the parameter values in their Table 1, and assume the ratio begins at the steady state in December 1951.
Speciﬁcally, using monthly and quarterly data and the following empirical models, 7 and 8, we retest empirical models 1, 3, and 5 both without and with the two competing scorecards:

\[
\text{Dependent Variable} = -0 + -1z_{t-1} + -2N z_{t-1} + -3z_{t-1} + v_t;
\]

\[
\text{Dependent Variable} = -0 + -1z_{t-1} + -2N z_{t-1} + -3z_{t-1} + -4d_{ayt}, 1 + -5sc_{rt}, 1 + v_t;
\]

The results of our competing scorecards tests are reported in Table 6 with monthly results in Panel (A) and quarterly results in Panel (B). In Panel (A), the addition of day_{t, 1} has little effect on our scorecard’s relationship with conditional volatility. -1 and -2 change only slightly from model 7 to 8 and collectively remain signiﬁcant (H1 is rejected). Furthermore, -4 is signiﬁcantly negative (p-value = 0.001), conﬁrming Lettau and Ludvigson’s (2001) ﬁnding that when consumption is temporarily below its trend relationship with assets and labor, stocks tend to be volatile.

In contrast to our scorecard’s limited ability to predict excess returns, day_{t, 1} is positively and signiﬁcantly related to next month’s excess return (4 = 0.5343; p-value less than 0.001). Speciﬁcally, hypothesis two, H2: -1 = -2 = 0, cannot be rejected in either empirical model 7 or 8, although -1 is signiﬁcantly negative (p-value = 0.093) in model 8.

With regards to skewness within a month, our scorecard’s predictive ability is signiﬁcantly related to the consumption-aggregate wealth ratio’s trend deviations. In both empirical models 7 and 8, -1 and -3 are negative and signiﬁcantly related to hypothesis three, H3: -1 = -3 = 0, is strongly rejected (p-value less than 0.001). In contrast, day_{t, 1} is unrelated to skewness.

Panel B of Table 6 reports the quarterly results of model 7 followed by model 8 which includes the surplus consumption ratio. Regarding quarterly volatility, the combination of -1 the -2, both signiﬁcant, support our model’s two predictions. The addition of the two competing scorecards does little to change these ﬁndings. The quarterly relationship between day and r^2 remains signiﬁcant; whereas, the coeﬃcient on the surplus consumption ratio is not.

With regards to quarterly excess return, both -1 the -2 have the sign predicted by our model but are not signiﬁcant. The same is true of -5; the higher the surplus consumption ratio, the lower the subsequent quarter’s excess return, however, the p-value is 0.112. Trend deviations in the consumption-wealth ratio are signiﬁcantly related to quarterly excess returns as in the monthly regression. Furthermore, the R^2 in the quarterly regression increase from the monthly level of 0.029 to 0.089 indicate that long-horizon returns are more predictable than short-horizon returns.

Neither day nor scr can be used to predict quarterly skewness; -4 and -5 are not signiﬁcant. In contrast, z does have predictive power. Null hypothesis three, H3: -1 = -3 = 0 is strongly rejected in both models 7 and 8; although, individually only -1 is signiﬁcant.

In conclusion, z_{t, 1} seems to be better at predicting conditional volatility and skewness whereas Lettau and Ludvigson’s day_{t, 1} measure is better at predicting excess returns. Campbell and Cochrane’s scr_{t, 1} is not signiﬁcantly related to quarterly conditional volatility, excess returns, or skewness.20

20Tallarini and Zhang (2000) ﬁnd that the surplus consumption ratio “matches reasonably well the mean stock returns but it fails to match the higher moments such as variance, skewness, and kurtosis.” Our tests suggest that even the ability to match the conditional mean is questionable when competing scorecards are included. Li (2001) ﬁnds that the surplus consumption ratio can predict a small portion on the changes in excess return; however, this ability is found after searching over three memory parameters, four horizons, linear and non-linear models, and monthly, quarterly, and annual data aggregation levels. Menzly, Santos, and Veronesi (2001) ﬁnd that the surplus consumption ratio does well in cross-section tests.
6 Conclusion

In contrast to the historical description of stock prices as a “random walk with a drift,” Cochrane (1999) summarizes many of the “new facts in finance.” Stocks exhibit time-varying risk premia that are on average higher than models allow, with excess ex-post volatility, and, most relevant to our paper, serial correlation in volatility. We develop an equilibrium model that provides an explanation for the empirical observance of volatility clustering as well as many of the other new facts. Agents in our theoretical model derive utility from both consumption and financial wealth changes, similar to Barberis, Huang, and Santos (2001). Utility from financial wealth changes depend on a mental scorecard of past investment performance. After past unexpected gains (losses) the scorecard is positive (negative), investors are less (more) risk adverse, and risk premium required to entice them to hold stocks is small (large). Thus the scorecard can explain time or wealth-varying excess returns. As investors expected wealth is perturbed in either direction (positive or negative scorecard), they become more sensitive to news, and stock prices react more to news than when the investors are not sensitive (or returns are as expected). Because investors’ sensitivity to news is persistent in our model, return shocks lead to greater return volatility and this heightened volatility decays slowly over time.

We numerically solve a model based on our investors’ utility functions. The model is consistent with time-varying excess returns, high risk premia, skewness, and clustered volatility. We generate artificial data from our model and find that these data display many of the characteristics found in the new empirical literature. We also structure our model so that it not only generates known stock returns characteristics but also generates new and specific refutable predictions.

We test several of these new predictions on monthly U.S. stock data. Specifically, we feed our model actual return data and create a time series of the scorecard then test whether the scorecard can predict volatility, excess returns, and skewness. First, we find our scorecard is significantly correlated with future volatility in ways predicted by our model. As the scorecard deviates from zero, next period’s volatility increases. Negative deviations (past losses) predict higher volatility than positive deviations, and the effect of shocks on volatility is exponential. Second, high levels of our scorecard are negatively correlated with low excess returns (past gains induce less risk aversion) as predicted; however, negative scorecards are not correlated with high excess returns. Third, our scorecard is significantly and negatively correlated with return skewness as predicted by our theoretical model.
Appendix A

7.1 Data

We provide a description of the variables used in our empirical analysis as well as our sources for each data series below. In general, we use the variable definitions and sources common in the literature. Unlike many recent empirical papers, we are not just interested in testing our model on post-war data; consequently, we occasionally deviate from normal sources in order to test our model over longer horizons.

**Monthly Returns (1802 - 2000),** $r_t$

We use the return (with dividends) on the CRSP value-weighted portfolio for our measure of monthly return over the period 1926 - 2000. In addition, we use Schwert's (1990) index return data for our measure of return over the period 1802 - 1925.

**Daily Returns (1885 - 2000),** $r_t$

We use the return (with dividends) on the S&P500 index portfolio for our measure of daily return over the period July 1962 - 2000. Our source of this data is CRSP. In addition, we use Schwert's (1990) index return data over the period 1885 - 1962. Schwert's data contains daily stock returns for the Dow Jones composite portfolio from February 16, 1885 through January 3, 1928, and for the Standard & Poor's composite portfolio from January 4, 1928 through July 2, 1962.

**Risk-Free Rate (1926 - 2000),** $r_{f,t}$

We use the return on 30-day Treasury Bill as our measure of the risk-free rate. Our source is the Ibbotson SBBI database.

**Excess Return (1802 - 2000),** $r_t - r_{f,t}$

We use the difference between monthly or quarterly return and the risk-free rate as our measure of excess returns. Due to the lack of interest rate data over the period 1802 - 1925 we use actual, and not excess, over this period.

**Dividend Yield (1926 - 2000),** $DY_t$

We use the trailing 12 months' dividends divided by the price at the end of month $t$. The data are created from the CRSP value-weighted total returns and the monthly CRSP value-weighted capital gains returns.

**Default Premium (1926 - 2000),** $DEF_t$

We use the difference between the end of month yield-to-maturity on BAA bonds and AAA bonds as provided by Moody's Corporate Bond Indices. Our source is the FRED® database.

**Short Rate (1926 - 2000),** $RREL_t$

We use the return on the 30-day Treasury Bill, $r_{f,t}$, minus its own four quarter moving average as our measure of the short rate.

**Term Spread (1926 - 2000),** $TRM_t$

We use the difference between the yield-to-maturity on Ibbotson's Long-Term Government bond portfolio and the yield-to-maturity on 3-month Treasury Bills as our measure of the term spread. Our sources are the Ibbotson SBBI data base and the FRED® database.
Consumption Wealth Ratio (1959 - 2000), $c_{d_t}$

We use the Lettau and Ludvigson (2001) monthly and quarterly log consumption-aggregate wealth ratio that is a linear function of aggregate consumption, labor income, and asset wealth. See Lettau and Ludvigson (2001) for a more detailed description of the data.

Consumption (1959 - 2000)

We use a quarterly consumption measure that include nondurables and services. Our source is Lettau and Ludvigson (2001) who obtain the data from the U. S. Department of Commerce, Bureau of Economic Analysis.

Population (1959 - 2000)

We use a measure defined as the total population, all ages including armed forces overseas. This series is used to create per capita consumption and in the creation of our surplus consumption state variable [see Campbell and Cochrane (1999)]. Our source is the U.S. Department of the Commerce, Census Bureau.
References


Wealth Utility Function, $V(X_{t+1})$

$\sigma_1 = 0.40$ and $k = 1.50$

Figure 1
Expected Conditional Stochastic Discount Factor

$E_t(m_{t+1})$

Current State of scorecard, $z_t$

Figure 2
Annualized Conditional Price–Dividend Ratio
\( \frac{P_t}{D_t} \)

Figure 3

Current State of scorecard, \( z_t \)
Figure 4

Annualized Conditional Expected Return Stock, $E_t(r_{t+1})$
and Risk-Free Rate, $r_{t+1}$
Annualized Conditional Expected Standard Deviation of Stock Returns, $E_t(\sigma_{t+1})$
Conditional Monthly Expected Skewness Coefficient

$E_t(\mu_{3,t+1})$

Current State of scorecard, $z_t$

Figure 6
Conditional Variance and Scorecard ($z_t$)

(a) Model 1, 1952 - 2000

Scorecard's ($z_t$) Incremental Effect on Conditional Variance Beyond EGARCH(1,1)

(b) Model 2, 1952 - 2000

Figure 7
Conditional Risk Premium and the Scorecard ($z_t$)

(a) 1952 - 2000, Model 3

(b) 1952 - 2000 (Excluding 1974), Model 3

Figure 8
Conditional Skewness Coefficient and the Scorecard ($z_t$)

(a) Model 5, 1952 - 2000

Scorecard's ($z_t$) Incremental Effect on the Conditional Skewness Coefficient

(b) Model 6, 1952 - 2000

Figure 9
### Table 1
Model Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traditional:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean consumption growth</td>
<td>$g$</td>
<td>1.89%*</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$d$</td>
<td>0.98%*</td>
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<tr>
<td>Standard deviation of non-financial income growth</td>
<td>$s_v$</td>
<td>3.5%*</td>
</tr>
<tr>
<td>Standard deviation of dividend growth</td>
<td>$s_e$</td>
<td>11.2%*</td>
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<tr>
<td>Correlation coefficient between $\Delta d$ and $\Delta y$</td>
<td>$\rho$</td>
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</tr>
<tr>
<td>Consumption utility power</td>
<td>$\gamma$</td>
<td>2</td>
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<tr>
<td><strong>Model Specific:</strong></td>
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<td></td>
</tr>
<tr>
<td>Memory or persistence</td>
<td>$f$</td>
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</tr>
<tr>
<td>State-dependent degree of risk aversion</td>
<td>$a_1$</td>
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</tr>
<tr>
<td>State-induced score card sensitivity to shocks</td>
<td>$a_2$</td>
<td>5</td>
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<tr>
<td>Base-level score card sensitivity to shocks</td>
<td>$a_3$</td>
<td>2.3</td>
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<tr>
<td>Relative importance of financial wealth</td>
<td>$b_0$</td>
<td>1</td>
</tr>
<tr>
<td>Loss aversion</td>
<td>$k$</td>
<td>1.5</td>
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</table>

* corresponds to annualized numbers. Traditional parameters are from Campbell and Cochrane (2000) or Barberis, Huang, and Santos (2001). Model specific parameters are chosen to calibrate the model to fit the post war U.S. stock market data.
Table 2
Simulation Results and Sensitivity Analysis

\[ r_t = c + \epsilon_t \]

\[ \text{Var}(\epsilon_t) = h_t = c_0 + c_1 h_{t-1} + c_2 \epsilon_{t-1}^2 \]

<table>
<thead>
<tr>
<th></th>
<th>Average $c_1$</th>
<th>Average p-value</th>
<th>Average $c_2$</th>
<th>Average p-value</th>
<th>Average $\sigma$</th>
<th>Average $\bar{r}$</th>
<th>Risk Premium</th>
<th>Average P/D</th>
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<tr>
<td>Traditional Case</td>
<td>0.606</td>
<td>0.268</td>
<td>0.003</td>
<td>0.385</td>
<td>11.19</td>
<td>5.28</td>
<td>-0.38</td>
<td>30.23</td>
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<td>Base Case</td>
<td>0.806</td>
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<td>0.001</td>
<td>13.82</td>
<td>9.90</td>
<td>4.24</td>
<td>13.13</td>
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</table>

Sensitivity Analysis

<table>
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<tr>
<th>Parameter</th>
<th>Average $c_1$</th>
<th>Average p-value</th>
<th>Average $c_2$</th>
<th>Average p-value</th>
<th>Average $\sigma$</th>
<th>Average $\bar{r}$</th>
<th>Risk Premium</th>
<th>Average P/D</th>
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<tr>
<td>$k = 1.25$</td>
<td>0.613</td>
<td>0.292</td>
<td>0.004</td>
<td>0.407</td>
<td>11.85</td>
<td>8.08</td>
<td>2.42</td>
<td>21.42</td>
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<tr>
<td>$N = 0.90$</td>
<td>0.765</td>
<td>0.205</td>
<td>0.001</td>
<td>0.522</td>
<td>12.37</td>
<td>8.47</td>
<td>2.81</td>
<td>15.41</td>
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<tr>
<td>$a_1 = 0.38$</td>
<td>0.759</td>
<td>0.050</td>
<td>0.016</td>
<td>0.152</td>
<td>12.75</td>
<td>8.34</td>
<td>2.68</td>
<td>16.11</td>
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<tr>
<td>$a_2 = 4.75$</td>
<td>0.675</td>
<td>0.119</td>
<td>0.016</td>
<td>0.139</td>
<td>12.71</td>
<td>9.25</td>
<td>3.59</td>
<td>13.75</td>
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<tr>
<td>$a_3 = 2.2$</td>
<td>0.731</td>
<td>0.029</td>
<td>0.013</td>
<td>0.241</td>
<td>12.85</td>
<td>9.12</td>
<td>3.46</td>
<td>13.90</td>
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<tr>
<td>$b_0 = 0.98$</td>
<td>0.752</td>
<td>0.024</td>
<td>0.013</td>
<td>0.192</td>
<td>12.89</td>
<td>8.47</td>
<td>2.81</td>
<td>15.45</td>
</tr>
</tbody>
</table>

Results from simulating sample paths for given parameter values and estimating a GARCH (1,1) model on the simulated returns. All numbers reported above are average values from 10 simulations for each set of parameter values where in each simulation we construct a sample of 10,000 observations. The columns labeled “Average p-value” report the results of a t-test for the $c_1$ and $c_2$ parameters being equal to zero. The standard deviations and return results are in percentages and the last four columns have been annualized. Traditional case refers to consumption utility only model, where $b_0 = 0$, similar to Mehra and Prescott (1985). Base case refers to model where parameters are defined in Table 1.
Table 3

Tests of the Scorecard’s Ability to Predict Conditional Volatility, $\hat{\epsilon}^2$

\[
Model 1: \hat{\epsilon}_t^2 = \beta_0 + \beta_1 \hat{\epsilon}_{t-1} + \beta_2 N\hat{\epsilon}_{t-1} + \nu_t \\
Model 2: \hat{\epsilon}_t^2 = \beta_0 + \beta_1 \hat{\epsilon}_{t-1} + \beta_2 N\hat{\epsilon}_{t-1} + \beta_4 \sigma_t^2 + \nu_t
\]

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<th>Scenario</th>
<th>Model</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_4$</th>
<th>$\bar{R}^2$</th>
<th>F-test</th>
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<td>Enhanced $\hat{\epsilon}^2$</td>
<td>Model 1</td>
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<td>(0.127)</td>
<td>(0.004)</td>
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<td>(0.001)</td>
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$p$-values are in parentheses and are based on Newey-West (1987) standard errors. Our estimate $\hat{\epsilon}^2$ is the squared residuals from a simple regression, $r_t = c + \epsilon_t$ where $r_t$ is the CRSP value-weighted stock index return in month $t$. Our estimate of $\hat{\epsilon}_{t-1}$ is from $z_t = \phi z_{t-1} + (a_1 |x_{t-1}| + a_2) x_{t-1}$, $X_{t-1}$ is the deviation of the return from its unconditional average, and $N$ is a binary variable equal to 1 when $\hat{\epsilon}_{t-1}$ is negative and zero otherwise. Our estimate of $\sigma_t^2$ is from Nelson’s (1991) EGARCH (1,1) model of returns. Pre-1926 data are from Schwert (1990). F-tests are for the null hypothesis one: $\beta_1 = \beta_2 = 0$. 

38
Table 4
Tests of the Scorecard’s Ability to Predict Conditional Excess Return, \( r - r_f \)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Model</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( \beta_6 )</th>
<th>( \beta_7 )</th>
<th>( \beta_8 )</th>
<th>( R^2 )</th>
<th>F-test</th>
</tr>
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<tbody>
<tr>
<td>1952-2000</td>
<td>Model 3</td>
<td>-0.0483</td>
<td>0.0530</td>
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<td></td>
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<td>(0.524)</td>
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<td></td>
<td></td>
<td></td>
<td>(0.616)</td>
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</tr>
<tr>
<td></td>
<td>Model 4</td>
<td>-0.0838</td>
<td>0.0731</td>
<td>0.0747</td>
<td>0.0056</td>
<td>-0.1602</td>
<td>-3.9740</td>
<td>0.1418</td>
<td>0.013</td>
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<tr>
<td></td>
<td></td>
<td>(0.098)</td>
<td>(0.371)</td>
<td>(0.126)</td>
<td>(0.283)</td>
<td>(0.754)</td>
<td>(0.093)</td>
<td>(0.380)</td>
<td>(0.224)</td>
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</tr>
<tr>
<td>1927-2000</td>
<td>Model 3</td>
<td>-0.0036</td>
<td>0.0151</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>0.000</td>
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<td></td>
<td></td>
<td>(0.890)</td>
<td>(0.781)</td>
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<td></td>
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<td>(0.759)</td>
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<td>Model 4</td>
<td>-0.0276</td>
<td>0.0420</td>
<td>0.0905</td>
<td>0.0057</td>
<td>0.2309</td>
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<td>(0.477)</td>
<td>(0.250)</td>
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<td>(0.727)</td>
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<td></td>
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<td>(0.031)</td>
<td>(0.002)</td>
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<td>(0.009)</td>
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</tr>
<tr>
<td>( a_2 = 4.5 )</td>
<td>Model 3</td>
<td>-0.0545</td>
<td>0.0617</td>
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<td>1952-2000</td>
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<td>(0.285)</td>
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<td>(0.581)</td>
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<td>Model 4</td>
<td>-0.0903</td>
<td>0.0813</td>
<td>0.0775</td>
<td>0.0060</td>
<td>-0.1822</td>
<td>-4.0250</td>
<td>0.1424</td>
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<td>(0.350)</td>
<td>(0.114)</td>
<td>(0.281)</td>
<td>(0.722)</td>
<td>(0.088)</td>
<td>(0.379)</td>
<td>(0.228)</td>
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</tr>
<tr>
<td>( a_3 = 2.0 )</td>
<td>Model 3</td>
<td>-0.0540</td>
<td>0.0575</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>1952-2000</td>
<td></td>
<td>(0.323)</td>
<td>(0.537)</td>
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<td>(0.630)</td>
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<td>Model 4</td>
<td>-0.0956</td>
<td>0.0831</td>
<td>0.0776</td>
<td>0.0061</td>
<td>-0.1797</td>
<td>-4.0041</td>
<td>0.1455</td>
<td>0.013</td>
<td>1.477</td>
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<td></td>
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<td>(0.100)</td>
<td>(0.363)</td>
<td>(0.111)</td>
<td>(0.270)</td>
<td>(0.726)</td>
<td>(0.091)</td>
<td>(0.369)</td>
<td>(0.228)</td>
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<tr>
<td>( \phi = 0.85 )</td>
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<td>-0.0039</td>
<td>0.0032</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>0.002</td>
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<tr>
<td>1952-2000</td>
<td></td>
<td>(0.948)</td>
<td>(0.976)</td>
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<td></td>
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<td>(0.998)</td>
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<tr>
<td></td>
<td>Model 4</td>
<td>-0.0931</td>
<td>0.0642</td>
<td>0.0861</td>
<td>0.0061</td>
<td>-0.1251</td>
<td>-4.1823</td>
<td>0.1461</td>
<td>0.012</td>
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<td></td>
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<td>(0.180)</td>
<td>(0.554)</td>
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<td>(0.257)</td>
<td>(0.811)</td>
<td>(0.090)</td>
<td>(0.377)</td>
<td>(0.347)</td>
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</tr>
<tr>
<td>1952-2000 w/o 1974</td>
<td>Model 3</td>
<td>-0.0357</td>
<td>-0.0399</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.004</td>
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<tr>
<td></td>
<td></td>
<td>(0.440)</td>
<td>(0.571)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.104)</td>
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<td></td>
<td>Model 4</td>
<td>-0.0726</td>
<td>-0.0273</td>
<td>0.0973</td>
<td>0.0056</td>
<td>-0.3033</td>
<td>-3.7493</td>
<td>0.1162</td>
<td>0.019</td>
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<td>(0.139)</td>
<td>(0.695)</td>
<td>(0.056)</td>
<td>(0.283)</td>
<td>(0.556)</td>
<td>(0.115)</td>
<td>(0.481)</td>
<td>(0.015)</td>
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</tbody>
</table>

See notes to Table 3. \( p \)-values are in parentheses and are based on Newey-West (1987) standard errors. \( r_{3m} \) is the 3-month Treasury Bill yield, \( DY_t \) is the CRSP Dividend Yield, \( DEF_t \) is the default spread (BAA - AAA corporate bond rates), \( RREL_t \) is the relative Bill rate (3-month Treasury Bill yield less its four-quarter moving average), and \( TRM_t \) is the term spread (10-year Treasury Bond yield less the 3-month Treasury Bill yield). F-tests are for the null hypothesis two: \( \beta_1 = \beta_2 = 0 \).
Table 5
Tests of the Scorecard’s Ability to Predict Conditional Skewness, \( \mu_3 \)

\[
\text{Model 5} : \hat{\mu}_{3t} = \beta_0 + \beta_1 \tilde{r}_{t-1} + \beta_2 \tilde{r}_{t-1} + \beta_4 \mu_{3t-1} + \beta_5 r_{t-1} + \nu_t,
\]

\[
\text{Model 6} : \hat{\mu}_{3t} = \beta_0 + \beta_1 \tilde{r}_{t-1} + \beta_2 \tilde{r}_{t-1} + \beta_4 \mu_{3t-1} + \beta_5 r_{t-1} + \nu_t,
\]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Model 5</th>
<th>( \beta_1 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( R^2 )</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952-2000</td>
<td>Model 5</td>
<td>-1.7467 (0.000)</td>
<td>-1.8776 (0.0555)</td>
<td>-</td>
<td>-</td>
<td>0.039</td>
<td>13.080 (0.000)</td>
</tr>
<tr>
<td></td>
<td>Model 6</td>
<td>-1.3391 (0.001)</td>
<td>-1.6503 (0.074)</td>
<td>0.0864 (0.067)</td>
<td>-1.3034 (0.026)</td>
<td>0.050</td>
<td>5.682 (0.003)</td>
</tr>
<tr>
<td>1927-2000</td>
<td>Model 5</td>
<td>-0.9473 (0.001)</td>
<td>-0.1610 (0.736)</td>
<td>-</td>
<td>-</td>
<td>0.037</td>
<td>18.194 (0.000)</td>
</tr>
<tr>
<td></td>
<td>Model 6</td>
<td>-0.7057 (0.010)</td>
<td>-0.1444 (0.737)</td>
<td>0.0774 (0.062)</td>
<td>-1.1899 (0.001)</td>
<td>0.048</td>
<td>5.903 (0.003)</td>
</tr>
<tr>
<td>1885-1925</td>
<td>Model 5</td>
<td>-1.8955 (0.000)</td>
<td>3.2567 (0.327)</td>
<td>-</td>
<td>-</td>
<td>0.039</td>
<td>10.786 (0.000)</td>
</tr>
<tr>
<td></td>
<td>Model 6</td>
<td>-1.7947 (0.001)</td>
<td>3.2071 (0.333)</td>
<td>-0.0427 (0.295)</td>
<td>-0.4492 (0.441)</td>
<td>0.038</td>
<td>8.793 (0.000)</td>
</tr>
<tr>
<td>( a_3 = 4.5 )</td>
<td>Model 5</td>
<td>-1.1767 (0.004)</td>
<td>0.3111 (0.754)</td>
<td>-</td>
<td>-</td>
<td>0.020</td>
<td>6.973 (0.001)</td>
</tr>
<tr>
<td>1952-2000</td>
<td>Model 6</td>
<td>-0.6302 (0.143)</td>
<td>0.6334 (0.500)</td>
<td>0.0954 (0.045)</td>
<td>-1.6464 (0.004)</td>
<td>0.036</td>
<td>4.071 (0.017)</td>
</tr>
<tr>
<td>( a_3 = 2.0 )</td>
<td>Model 5</td>
<td>-1.0842 (0.003)</td>
<td>0.5302 (0.577)</td>
<td>-</td>
<td>-</td>
<td>0.021</td>
<td>7.214 (0.001)</td>
</tr>
<tr>
<td>1952-2000</td>
<td>Model 6</td>
<td>-0.5886 (0.135)</td>
<td>0.7416 (0.404)</td>
<td>0.0947 (0.047)</td>
<td>-1.6298 (0.004)</td>
<td>0.036</td>
<td>4.064 (0.017)</td>
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<td>( \phi = 0.85 )</td>
<td>Model 5</td>
<td>-1.1648 (0.009)</td>
<td>1.7383 (0.162)</td>
<td>-</td>
<td>-</td>
<td>0.017</td>
<td>6.227 (0.002)</td>
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<tr>
<td>1952-2000</td>
<td>Model 6</td>
<td>-0.4153 (0.413)</td>
<td>2.0918 (0.086)</td>
<td>0.0996 (0.037)</td>
<td>-1.7098 (0.004)</td>
<td>0.033</td>
<td>4.962 (0.007)</td>
</tr>
</tbody>
</table>

See notes to Table 3. \( p \)-values are in parentheses and are based on Newey-West (1987) standard errors.

\[
\hat{\mu}_{3t} = \left[ \frac{1}{\sqrt{N}} \sum_{n=1}^{N} (r_{n,t} - \bar{r}_t) \right] / \sigma_{3t}^{1/2}, \quad \text{where } r_{n,t} \text{ is the market return on day } n \text{ of month } t, \quad \bar{r}_t \text{ is the average daily return in month } t, N \text{ is the number of trading days in month } t, \text{ and where } \sigma_{3t}^2 = \sum_{n=1}^{N} r_{n,t}^2 + 2 \sum_{n=0}^{N} \tilde{r}_n \tilde{r}_{n+1}. \quad \text{When calculating the variance, the small daily mean is ignored and the cross-product accounts for non-synchronous trading. Between January 1928 and December 2000 the daily returns used to create our skewness estimate are based on the Standard and Poor's composite portfolio. The daily Standard and Poor's return data is from French, Schwert, and Stambaugh (1987) for the 1928 to 1961 period and from CRSP for the 1962 to 2000 period. Daily returns from 16 February 1885 through December 1927 are for the Dow Jones composite portfolio and are supplied by Schwert (1989). F-tests are for the null hypothesis three: } \beta_1 = \beta_4 = 0.
### Table 6
Competing Scorecards’ ($z, cay,$ and $scr$) Ability to Predict Conditional Volatility, Excess Return, and Skewness

Model 7: Dependent Variable = $\beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N_{z_{t-1}}^\hat{z} + \beta_3 \hat{z}_{t-1}^2 + \nu_t$

Model 8: Dependent Variable = $\beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N_{z_{t-1}}^\hat{z} + \beta_3 \hat{z}_{t-1}^2 + \beta_4 cay_{t-1} + \beta_5 scr_{t-1} + \nu_t$


<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Model</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$R^2$</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{e}_t^2$</td>
<td>Model 7</td>
<td>0.0060 (0.243)</td>
<td>-0.0271 (0.001)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.096</td>
<td>27.837 (0.000)</td>
</tr>
<tr>
<td></td>
<td>Model 8</td>
<td>0.0059 (0.235)</td>
<td>-0.0272 (0.000)</td>
<td>-</td>
<td>-0.0169 (0.062)</td>
<td>-</td>
<td>0.099</td>
<td>11.69 (0.000)</td>
</tr>
<tr>
<td>$r_t - r_{ft}$</td>
<td>Model 7</td>
<td>-0.0859 (0.156)</td>
<td>0.0929 (0.322)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.000</td>
<td>1.051 (0.350)</td>
</tr>
<tr>
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<td>Model 8</td>
<td>-0.0970 (0.0935)</td>
<td>0.0942 (0.285)</td>
<td>-</td>
<td>0.5343 (0.000)</td>
<td>-</td>
<td>0.029</td>
<td>1.545 (0.213)</td>
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<tr>
<td>$\mu_3$</td>
<td>Model 7</td>
<td>-1.6910 (0.000)</td>
<td>-</td>
<td>-1.8070 (0.0659)</td>
<td>-</td>
<td>-</td>
<td>0.038</td>
<td>11.043 (0.000)</td>
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<tr>
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<td>Model 8</td>
<td>-1.6633 (0.000)</td>
<td>-</td>
<td>-1.7497 (0.071)</td>
<td>0.0887 (0.956)</td>
<td>-</td>
<td>0.035</td>
<td>9.022 (0.000)</td>
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</table>

#### Panel B: Quarterly Data (1952 – 2000)

<table>
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<tr>
<th>Dependent Variable</th>
<th>Model</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$R^2$</th>
<th>F-test</th>
</tr>
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<tbody>
<tr>
<td>$\hat{e}_t^2$</td>
<td>Model 7</td>
<td>0.0263 (0.045)</td>
<td>-0.0579 (0.000)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.079</td>
<td>9.208 (0.000)</td>
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<tr>
<td></td>
<td>Model 8</td>
<td>0.0263 (0.045)</td>
<td>-0.0558 (0.000)</td>
<td>-</td>
<td>-0.1252 (0.000)</td>
<td>0.0301 (0.217)</td>
<td>0.138</td>
<td>25.923 (0.000)</td>
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<td>$r_t - r_{ft}$</td>
<td>Model 7</td>
<td>-0.0573 (0.665)</td>
<td>-0.0717 (0.692)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.002</td>
<td>1.206 (0.301)</td>
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<td>Model 8</td>
<td>-0.0608 (0.623)</td>
<td>-0.0993 (0.569)</td>
<td>-</td>
<td>1.7549 (0.007)</td>
<td>-0.4532 (0.112)</td>
<td>0.089</td>
<td>1.479 (0.230)</td>
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<tr>
<td>$\mu_3$</td>
<td>Model 7</td>
<td>-1.7985 (0.013)</td>
<td>-</td>
<td>-1.6031 (0.266)</td>
<td>-</td>
<td>-</td>
<td>0.042</td>
<td>5.262 (0.005)</td>
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<td>Model 8</td>
<td>-1.8710 (0.012)</td>
<td>-</td>
<td>-1.7187 (0.239)</td>
<td>-3.4957 (0.259)</td>
<td>-1.2493 (0.525)</td>
<td>0.038</td>
<td>11.899 (0.000)</td>
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</table>

See notes to Table 3. $p$-values are in parentheses and are based on Newey-West (1987) standard errors. $cay_t$ is Lettau and Ludvigson’s (2001b) proxy for the log consumption-aggregate wealth ratio. $scr_{t-1}$ is the Campbell and Cochrane (1999) surplus consumption ratio. $\hat{e}_t^2$ is our estimate of volatility; $r_t - r_{ft}$ is our estimate of excess return; and $\hat{\mu}_3$ is our estimate of skewness. F-tests in the first four rows of each panel are for null hypotheses one and two: $\beta_1 = \beta_2 = 0$. F-tests in the fifth and sixth rows of each panel are for the null hypothesis three: $\beta_1 = \beta_3 = 0$. 