

# Estimating the Value of Information

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## **Abstract**

We derive a general expression for the value of information to a price-taking investor in a dynamic environment and provide a framework for its estimation. We study the value of both private and public information and break it down into its instrumental and psychic parts. To illustrate, we estimate values of leading macroeconomic indicators (GDP, employment, etc.) and rank them. Using variations in option prices we find that a consumer-investor with conventional preference parameters would pay 3 to 5 basis points of her wealth for a one-time private peek into these indicators. Such signals provide substantial instrumental value but only a minor psychic value. Estimated values of information increase with the time discount factor, decrease with risk aversion, and increase with the elasticity of intertemporal substitution.

JEL classification: G10, G12, G13, D80, D82, D83

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How much would investors pay to receive investment-relevant information? This question is far from hypothetical. Legions of analysts and computer programs are constantly collecting millions of pieces of information and producing thousands of reports on various aspects of the economy. The incentives to collect information are central to the study of the informational efficiency of capital markets (Fama 1970; Grossman and Stiglitz 1980). Information providers can improve their services with a better understanding of the value for information and its sources. In this paper we present a general framework for evaluating informative signals and analyzing the source of this value from the point of view of a utility maximizing investor. We then illustrate our framework by estimating values of key macroeconomic indicators, ranking them, and providing comparative statics for the determinants of the value of information.

Our thought exercise is as follows. We empathize with a risk-averse investor who can trade contingent claims that pay a dollar if a particular state realizes. Without additional information, the investor would use her prior probabilities about each state to optimally choose her consumption and investments. We offer this investor a peek into a posterior distribution updated to reflect an additional signal from a particular information source (e.g. GDP report), allowing an improved consumption-investment choice. We estimate the share of wealth she is willing to forgo to observe such information, by proxying for the prior and posterior using probabilities recovered from option prices observed just before and after informational releases.

The value of information derives from two distinct sources. The first is the *instrumental* value of information, which reflects the improvement in consumption and investment associated with better informed decisions. The second is the *psychic* value of information, due to a preference for early resolution of uncertainty (Kreps and Porteus 1978). Such preference is entirely about the attitude of the agent toward uncertainty, even when she cannot alter her consumption plan. Following Ai (2007), we decompose the value of information into its instrumental and psychic channels, allowing us to evaluate the importance of each for various signals and under various parameterizations.<sup>1</sup>

Our framework allows us to estimate the value of both private and public information. Private

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<sup>1</sup>The “instrumental” versus “psychic” terminology follows Epstein, Farhi, and Strzalecki (2014). Ai (2007) refers to the psychic value as the “value from early resolution of uncertainty” and to the instrumental value as the “value from resource allocation.”

information allows the investor to trade on it at stale prices, yet to reflect this information, whereas public information restricts the investor to trade only after prices adjust. The estimation approach for both is similar with the sole difference being the prices faced by the investor. Both cases are of interest, and different types of informational signals may lend themselves naturally to either the private or public setting. For example, quantifying the value of private information is key for penalizing insider trading (DeMarzo, Fishman, and Hagerty 1998) and information leakages (Bernile, Hu, and Tang 2016), and for compensating analysts and money managers (Savov 2014). Understanding the demand for public information can assist government agencies charged with producing it.

One may wonder why not simply use announcement returns as a measure of the value of information (Fama, Fisher, Jensen, and Roll 1969). Such a statistic may provide an intuitive test whether additional price-relevant information is revealed on announcement dates. However, the economic value of information depends on how consumption and investment would change in response. For example, the value of information is small for a highly risk-averse investor if despite an additional signal she would remain invested in a fairly riskless portfolio. Similarly, an agent that strongly dislikes substituting her current consumption for future consumption would have little use for information that improves her investments. Moreover, the optimal financial instrument or trading strategy for utilizing each signal is not obvious. Quantifying the value of information instead requires a model of preferences, probabilities and investment opportunity sets.

To this end, we build on the dynamic stochastic recursive preferences setup of Epstein and Zin (1989) who generalize the Merton-Samuelson consumption and investment problem. We augment this setup by allowing a small price-taking agent to purchase a subscription to a signal, which she can observe just before it becomes available to the public. To estimate the value of private information, the small agent is allowed to consume and invest based on her private information without affecting market prices. We define the value of an information source as the fraction of wealth the agent is willing to give up in order to obtain signals generated by this source. To estimate the value of public information we follow a similar route, but only allow the agent to trade after the signal has been revealed publicly and prices have adjusted.

We derive an estimable expression for the value of information, which is based on its expected effect on the value-to-consumption ratio, where the expectation takes into account the different signals generated by the information source. In some cases it is useful to consider a stream of signals released periodically, while in others the value of a one-time signal is of interest. We show that the value of a one-time signal can be derived as a special case of the repeated signals model, and present results for both cases.

The expression for the value of information as an expectation over signals generated by the information source leads naturally to the application of the generalized method of moments (GMM) to estimate mean increases in value-to-consumption ratios, which, in turn, determine the value of information. By conditioning on dates of particular information releases, we estimate the value of information associated with different information sources, building on the standard approach that uses large sample means to estimate population moments (Hansen and Singleton 1982). We also provide a simpler closed-form first-order approximation for the value of information, which does not require numerical optimization and proves to be quite accurate.

We illustrate this framework by estimating values of information for each of a set of key macroeconomic indicators including FOMC decisions, GDP and employment reports. In going to data, we need to make several assumptions on the parameters of the utility function (discount rate, risk aversion, and elasticity of intertemporal substitution), the state space, the investment opportunity set, and the trading frequency. The value of information we obtain naturally depends on these choices. For example, the value of an early peek into an FOMC rate decision would presumably be higher if the agent can trade Fed funds futures than if she is restricted to trading S&P 500 options.

Beyond these, we require two sets of inputs: state prices and their “physical” probabilities. The estimated value of information derives from the average change in option-implied probabilities on announcement days. Intuitively, a more informative signal is associated with a greater concentration of the state probability distribution. State prices are relatively straightforward to extract from option prices following Breeden and Litzenberger (1978). Recovering forward-looking physical probabilities from state prices is an active topic of research, which is not the focus of the current paper, but a necessary step nonetheless. We therefore take a simple parametric approach, common

in empirical options studies (e.g. Bakshi, Kapadia, and Madan 2003; Bliss and Panigirtzoglou 2004), which assumes that physical probabilities are an exponentially-tilted version of contemporaneous risk-neutral probabilities, calibrated to match the equity premium.

In our benchmark estimation we consider the case of a private signal and set the agent’s preference parameters to values used in standard asset pricing calibrations (Bansal and Yaron 2004). We estimate that this agent is willing to pay between 3.5 and 5.4 basis points of her wealth for a one-time private signal of the leading macroeconomic indicators. Standard errors are typically small, allowing us to rank the informational sources by the value they provide. For example, we find that employment reports are both economically and statistically more valuable than GDP reports.

As Epstein, Farhi, and Strzalecki (2014) emphasize, the utility parameters we use feature substantial preference for early resolution of uncertainty, giving rise to a potentially important psychic value of information.<sup>2</sup> Their analysis focuses solely on this psychic channel. Our exercise is quite different, as we allow the agent to alter her consumption process based on the information, without constraining it to match aggregate income in the economy. Thus, the value of information that we estimate captures both its instrumental and psychic parts. Moreover, we break down the value of information into these two channels and quantify their relative importance. We find that the value of information for these macroeconomic signals derives almost entirely from their instrumental value. The psychic values we estimate from the reduction in uncertainty associated with these macroeconomic events are considerably smaller than the upper bounds calibrated in Epstein, Farhi, and Strzalecki (2014), which uses similar preferences but completely eliminates all future uncertainty (see Section 5.3 for details).

Turning to the case of public information, we estimate values of public signals that are uniformly smaller than values of private signals. For example, under our benchmark parameterization, trading on a single employment report at stale prices is worth 5.4 basis points of wealth, but trading after prices adjust is only worth 0.2 basis points. By construction, the psychic values are identical to the private information case, as they derive from the mean reduction in uncertainty, holding consumption and investment fixed, and do not depend on prices. Thus, the lower values of public

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<sup>2</sup>See also Ai (2007) and Croce, Marchuk, and Schlag (2016).

versus private information are due to lower instrumental values.

When allowing the agent to receive a signal every period, we find that the values of both private and public information are larger in magnitude for all of the macroeconomic indicators. For the benchmark parameterization, we estimate the agent is willing to pay between 13 and 23 percent of her wealth for a signal every period. As in the one-time signal case, the instrumental component accounts for the bulk of the value. Beyond the increase in the number of signals, a stream of signals contributes to a reduction in long-run risk, resulting in a higher value of information.

We next conduct a series of numerical comparative statics exercises and robustness tests by solving the model for different values of the underlying parameters, utilizing the flexibility of Epstein-Zin preferences in separating between risk-aversion and the elasticity of intertemporal substitution (EIS). First, we find that the value of information is increasing with the time discount factor as the agent assigns more value to future periods. This can be interpreted as a higher value of information for higher frequency trading. Second, we find that the value of information declines with risk aversion. Higher risk aversion increases the value of uncertainty-reducing information, but simultaneously also reduces the investor's demand for risky assets, which makes information less valuable. We find that this second "demand" effect dominates. Comparative statics in the EIS are also intuitive: a higher EIS is associated with a higher willingness of the agent to defer consumption and save. Accordingly, we find that an increase in the EIS leads to a higher value of information.

Related work by [Cabrales, Gossner, and Serrano \(2013\)](#) shows that for a log utility agent, faced with a static investment problem, the value of information equals the mean reduction in entropy generated by the information source. We establish that in our dynamic consumption-investment model, the value of information for an agent with log utility generalizes to the present value of expected future reductions in entropy. [Cabrales, Gossner, and Serrano \(2013\)](#) focus on the theoretical aspects of ranking different information sources, but do not provide a practical method for its estimation. Thus, our framework is useful for estimating such "reduction in entropy" rankings. In general, however, our approach to calculating the value of information differs from [Cabrales, Gossner, and Serrano \(2013\)](#), who consider the log utility case as an upper bound on the value of information across all agent with "ruin-averse" preferences. Instead, we use results from

dynamic asset-pricing that point to a set of benchmark parameters for Epstein-Zin preferences, which we use to calculate the value of information (e.g. [Bansal and Yaron 2004](#)). This difference is key for practical purposes as our estimates show that the economic magnitude of the value of information differs substantially between the log utility case and our benchmark parameters.

Our framework allows for the estimation of the value of private information for investment in addition to the value of public information. By considering a better-informed price-taking agent, we depart from the literature focusing solely on the public/social value of information.<sup>3</sup> Private information is pervasive yet hardly reflected by equity prices ([Collin-Dufresne and Fos 2015](#); [Kacperczyk and Pagnotta 2016](#)).<sup>4</sup>

Quantitative work on the value of private information is rare and has thus far relied on stronger assumptions. [Savov \(2014\)](#) is an exception, which calibrates a dynamic dispersed information model with constant relative risk aversion (CRRA) agents to study mutual fund performance. [Manela \(2014\)](#) estimates the value of diffusing information in a dynamic competitive noisy rational expectations equilibrium with constant absolute risk averse (CARA) investors. In that model, the value of information takes into account the equilibrium fraction of informed investors and the resulting partial informativeness of prices. [Avdis \(2016\)](#) shows that in such dynamic equilibrium models, the stochastic supply of the risky asset plays a significant informational role. [Breon-Drish \(2015\)](#) and [Malamud \(2015\)](#) suggest that moving beyond CARA utility can be important. Here, we generalize to a neoclassical infinite horizon recursive preferences investor, with a more general state space and trading opportunities, but at the cost of abstracting from such information externalities and equilibrium effects.<sup>5</sup>

The paper proceeds as follows. We describe the theoretical setup in Section 1. We derive

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<sup>3</sup>The seminal contributions are [Hirshleifer \(1971\)](#) and [Spence and Zeckhauser \(1972\)](#). See [Maurer and Tran \(2015\)](#) for recent work on the social value of information.

<sup>4</sup>Our work also relates to a literature studying information acquisition and markets for financial information. Recent such work studies media frenzies ([Veldkamp 2006](#)), large price movements ([Barlevy and Veronesi 2000](#); [Dow, Goldstein, and Guembel forthcoming](#); [García and Strobl 2011](#); [Mele and Sangiorgi 2015](#)), portfolio choice ([Peress 2004](#); [Van Nieuwerburgh and Veldkamp 2010](#)), mutual funds behavior ([García and Vanden 2009](#); [Kacperczyk, Van Nieuwerburgh, and Veldkamp 2016](#)), the home bias puzzle ([Van Nieuwerburgh and Veldkamp 2009](#)), sell-side equity analysts ([Kelly and Ljungqvist 2012](#); [Kadan, Michaely, and Moulton 2016](#)), and bank runs ([He and Manela 2016](#)). See [Veldkamp \(2011\)](#) for a good review of this literature.

<sup>5</sup>Recent work by [Borovička \(2015\)](#) shows that in models with recursive preferences agents with heterogeneous beliefs may survive in the long run, which suggests that equilibrium effects may be important for the value of information. Quantifying such effects would be an interesting avenue for future work.

a general expression for the value of information in Section 2 and provide a framework for its estimation in Section 3. We apply this methodology to estimate the values of key macroeconomic indicators from options in Section 4 and report the results in Section 5. We conclude in Section 6. Estimation details, the value of delayed information, limiting cases and additional comparative statics are in the online appendix.

## 1 Theoretical framework

Our starting point is the standard [Merton-Samuelson](#) dynamic consumption and investment choice setup with recursive preferences ([Epstein and Zin 1989](#)). In this setup, the agent solves for her optimal consumption and investment taking prices as given, and aggregate consumption is not restricted to match aggregate resources. We augment this setup by allowing a small price-taking agent to obtain a private signal, which improves her consumption plan. We use this tool to study the agent’s willingness to pay for this signal—the value of private information. In [Section 2.3](#) we modify this setup to study the value of public information.

### 1.1 State space and preferences

Time is discrete with an infinite horizon. At each date  $t > 0$ , a random state  $z_t$  is drawn from a finite set  $\{1, \dots, n\}$ , with the initial state  $z_0$  given. We assume that the state evolution is Markovian and denote the transition probabilities by  $p(z_{t+1}|z_t)$ , where  $\sum_{z_{t+1}} p(z_{t+1}|z_t) = 1$ . To prevent pathologies we assume that any state can be reached from any other state, i.e,  $0 < p(z_{t+1}|z_t) < 1$  for all  $z_t, z_{t+1} \in \{1, \dots, n\}$ . In particular, the Markov matrix is ergodic.

When state  $z_t$  is realized there is trade in  $n$  Arrow-Debreu securities corresponding to the  $n$  states at time  $t + 1$ . The time- $t$  price of a security paying 1 when state  $z_{t+1}$  is realized and zero otherwise given that the current state is  $z_t$  is denoted by  $q(z_{t+1}|z_t) > 0$  (the state price). We assume no arbitrage so that such positive state prices do exist.

Consider a small price-taking agent whose preferences over consumption  $c_t$  are represented by



a recursive utility function as in [Epstein and Zin \(1989\)](#),

$$U_t = J [c_t, \mu_t (U_{t+1})], \tag{1}$$

where  $U_t$  is short-hand for utility starting at some date- $t$  history,  $J$  is a time aggregator, and  $\mu_t$  is a certainty equivalent function based on conditional probabilities as of time  $t$ . The time aggregator  $J$  is the only determinant of consumption dynamics in a deterministic environment, while the certainty equivalent function  $\mu$  is all that matters in a static problem with uncertainty.

We specialize to the widely-used constant elasticity of substitution (CES) time aggregator

$$J [c, \mu] = \left\{ (1 - \beta) c^{1-\rho} + \beta \mu^{1-\rho} \right\}^{\frac{1}{1-\rho}}, \tag{2}$$

where  $\rho \geq 0$  can be interpreted as the inverse of the EIS, and the certainty equivalent function takes the homogeneous form

$$\mu [U] = \left( E_t U^{1-\gamma} \right)^{\frac{1}{1-\gamma}}, \tag{3}$$

where  $\gamma > 0$  is the coefficient of relative risk aversion and  $E_t$  is the conditional expectation operator. Expected utility with constant relative risk aversion is the special case with  $\rho = \gamma$ .

## 1.2 Information and timing

For a fixed date  $t$  we assume the agent is allowed to purchase a stream of informative signals about future states of nature in dates  $t + 1, t + 2, \dots$ . The standard way to model such signals is using the notion of an information structure (e.g. [Cabrales, Gossner, and Serrano 2013](#)). An information structure  $\alpha$  is a finite set of  $S_\alpha$  signals. If an agent buys information structure  $\alpha$ , then in each period starting from date  $t$  she gets to observe a signal  $s_t \in \{1, \dots, S_\alpha\}$ , which may be informative about the state realization in the next period  $z_{t+1}$ .

The question we are asking is how much would the agent be willing to pay to privately observe such a stream of signals. We also consider the value of a one-time signal, which may be more relevant in some settings. We derive our main results for the infinite stream of signals, and then

show that the value of a one-time signal can be obtained as a special case by essentially zeroing out the value of all future signals.

In general, the time- $t$  signal distribution may depend on the entire history of states and signals. Thus, in our setup, an information structure  $\alpha$  is represented by time-homogeneous conditional probabilities  $\alpha(s_t|z_1, \dots, z_t, z_{t+1}, s_1, \dots, s_{t-1})$  of observing signal  $s_t$  given that next period's signal is  $z_{t+1}$ , the history of states is  $z_1, \dots, z_t$ , and the history of signals is  $s_1, \dots, s_{t-1}$ . To reduce the dimensionality of the problem, we assume that the signal at time  $t$  is independent of all state realizations and signals up to time  $t$ , conditional on the state in time  $t + 1$ , i.e.,

$$\alpha(s_t|z_1, \dots, z_{t+1}, s_1, \dots, s_{t-1}) = \alpha(s_t|z_{t+1}). \quad (\text{A1})$$

This assumption means that the future state already encapsulates all prior information contained in previous signals and states, even though the signal may be correlated with them.

Given this assumption, and similar to [Cabrales, Gossner, and Serrano \(2013\)](#), we can consider an information structure as a matrix of conditional probabilities  $\alpha(s_t|z_{t+1})$  for all  $s_t \in \{1, \dots, S_\alpha\}$  and  $z_{t+1} \in \{1, \dots, n\}$  representing the probability of observing signal  $s_t$  given that the next period's state is  $z_{t+1}$ . We restrict attention to imperfect information structures. That is, any signal may be observed with positive probability given any realized state:  $\alpha(s_t|z_{t+1}) > 0$  for all  $s_t$  and  $z_{t+1}$ . Imperfect information structures are natural to consider because typical signals observed in reality likely provide useful but imperfect predictions on future states.

We also assume that conditional on the state  $z_t$ ,  $z_{t+1}$  is independent of the history of states and signals up to time  $t - 1$ . Specifically, the probability of observing state  $z_{t+1}$  conditional on the history of states and signals drawn from information structure  $\alpha$  is

$$p_\alpha(z_{t+1}|z_1, \dots, z_t, s_1, \dots, s_{t-1}) = p(z_{t+1}|z_t). \quad (\text{A2})$$

Assumptions (A1) and (A2) arise naturally in a variety of settings. For example, if the signal is a noisy version of the next-period state,  $s_t = z_{t+1} + \varepsilon_t$ , where  $\varepsilon_t$  is independent of all signals and states, then these two assumptions are satisfied. These two assumption reduce the dimensionality

of our problem considerably as established by the following result.

*Claim 1.* Under assumptions (A1) and (A2), for all histories  $z_1, \dots, z_t$  and  $s_1, \dots, s_t$

1. The total probability of observing signal  $s_t$  conditional on the history of states and signals depends only on  $z_t$ :

$$\alpha(s_t | z_1, \dots, z_t, s_1, \dots, s_{t-1}) = \alpha(s_t | z_t). \quad (4)$$

2. The posterior probability of state  $z_{t+1}$  given the history of states and signals is strictly positive and depends only on the most recent signal and state:

$$p_\alpha(z_{t+1} | z_1, \dots, z_t, s_1, \dots, s_t) = p_\alpha(z_{t+1} | z_t, s_t) > 0. \quad (5)$$

*Proof.* By the law of total probability

$$\alpha(s_t | z_1, \dots, z_t, s_1, \dots, s_{t-1}) = \sum_{z_{t+1}} \alpha(s_t | z_1, \dots, z_{t+1}, s_1, \dots, s_{t-1}) p(z_{t+1} | z_1, \dots, z_t, s_1, \dots, s_{t-1}).$$

Using assumptions (A1) and (A2) this expression simplifies to

$$\alpha(s_t | z_1, \dots, z_t, s_1, \dots, s_{t-1}) = \sum_{z_{t+1}} \alpha(s_t | z_{t+1}) p(z_{t+1} | z_t) = \alpha(s_t | z_t),$$

establishing the first part of the claim. For the second part, note that

$$p_\alpha(z_{t+1} | z_1, \dots, z_t, s_1, \dots, s_t) = \frac{\alpha(s_t | z_1, \dots, z_{t+1}, s_1, \dots, s_{t-1}) p_\alpha(z_{t+1} | z_1, \dots, z_t, s_1, \dots, s_{t-1})}{\sum_{z_{t+1}} \alpha(s_t | z_1, \dots, z_{t+1}, s_1, \dots, s_{t-1}) p_\alpha(z_{t+1} | z_1, \dots, z_t, s_1, \dots, s_{t-1})}. \quad (6)$$

By assumptions (A1) and (A2) this expression simplifies to

$$p_\alpha(z_{t+1} | z_1, \dots, z_t, s_1, \dots, s_t) = \frac{\alpha(s_t | z_{t+1}) p(z_{t+1} | z_t)}{\sum_{z_{t+1}} \alpha(s_t | z_{t+1}) p(z_{t+1} | z_t)} = p_\alpha(z_{t+1} | z_t, s_t).$$

Since the state Markov chain is ergodic and the information structure  $\alpha$  is imperfect,  $p_\alpha(z_{t+1} | z_t, s_t) > 0$  for all  $s_t$  and  $z_t$ . □

As a special case, let  $\alpha_0$  denote an information structure that consists of one signal only, i.e.,

$S_\alpha = 1$ . Then,  $\alpha_0$  is completely uninformative since regardless of  $z_{t+1}$  the same signal is always observed by the agent. In particular,  $p_{\alpha_0}(z_{t+1}|s_t, z_t) = p(z_{t+1}|z_t)$ . Thus, obtaining the information structure  $\alpha_0$  is equivalent to obtaining no information.

The timing of events in each period is as described in Figure 1a. At the beginning of time  $t$  the state  $z_t$  is realized and revealed publicly. Next, the signal  $s_t$  (taken from information structure  $\alpha$ ) is realized and revealed privately to the agent who bought the information structure. The agent then makes a decision based on  $z_t$  and  $s_t$  choosing how to allocate her wealth  $a_t$  between consumption  $c_t$  and a vector of investments in Arrow-Debreu securities corresponding to the  $n$  states of nature, denoted by  $\mathbf{w}_{t+1}$ .<sup>6</sup> Here  $\mathbf{w}_{t+1} = (w_{1t+1}, \dots, w_{nt+1})$ , where  $w_{it+1}$  is the fraction of remaining wealth,  $a_t - c_t$ , invested in the Arrow-Debreu security that pays 1 dollar when  $z_{t+1} = i \in \{1, \dots, n\}$  is realized and zero otherwise. Similar to Grossman and Stiglitz (1980), Admati (1985), Ausubel (1990), and Admati and Pfleiderer (1991), we assume the informed trader is essentially infinitesimal, so that her trades do not affect prices. Thus the agent who purchased information structure  $\alpha$ , gets to consume and trade at stale prices using the realized signal  $s_t$ , giving her an informational advantage compared to the public information available at time  $t$ .

### 1.3 Discussion

We pause to discuss several model features: market incompleteness, noise, and dynamic learning.

#### 1.3.1 Market incompleteness

The market we model is incomplete. Indeed, given an information structure  $\alpha$ , we have  $n$  states and  $S_\alpha$  signals for a total of  $n \times S_\alpha$  possible combinations. However, we only have  $n$  Arrow-Debreu securities—one for each state. Because the signals are given privately to the agent, additional assets that allow the agent to directly trade on signal realizations would introduce (riskless) arbitrage opportunities. Beyond its theoretical appeal in preventing arbitrage, we find such incompleteness a good description of the real world where such assets are not available for many signals. For example, there is currently no asset that allows one to trade directly on employment figures. On

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<sup>6</sup>Throughout the paper we denote vectors in bold.

the other hand, FOMC rate decisions could be traded relatively closely using Fed funds futures. This incompleteness also affects our estimation as we discuss in Section 4.

### 1.3.2 Noise and prices

It is common in trading models to introduce noise into the price process to prevent arbitrage and facilitate trade (e.g. [Grossman and Stiglitz 1980](#)). For tractability, we take a different approach. We prevent arbitrage by allowing the agent to trade only once each period after privately observing the signal  $s_t$ . The next time she can trade, prices would adjust, but also a new state  $z_{t+1}$  would realize. Thus, the state plays a dual role as both the fundamental value and a source of noise which prevents arbitrage.

Potentially, one may consider a model in which the agent could trade again, after prices adjust to the signal, but before a new state is realized. In that situation, an arbitrage opportunity would theoretically arise. In the real world, however, new information about the fundamental value or alternative investment opportunities continuously arrives into markets, providing a natural source of noise ([Grossman 1995](#)). Our model captures this realistic feature by allowing the agent to trade only after such “noise” is realized in the next period.

As we shrink the period length, the restriction to trade only once each period loses its bite. In our benchmark empirical analysis we set a period to one month. In the comparative statics analysis (Section 5.4), we gradually shrink the trading horizon down to two days, which captures the value of information to more active traders. Our daily options data does not allow us to study higher frequencies, though our framework is general enough to accommodate them.

We do not explicitly account for price impact as in [Kyle \(1985\)](#), or for partially reflective prices due to equilibrium learning as in [Grossman and Stiglitz \(1980\)](#). The degree to which prices reflect this private information depends on the size of the trade, the noise in the market and the liquidity of the security more generally. However, the value of information in such settings would be a convex combination of the two extreme cases we study: fully private (no price impact) and fully public information (full price impact).

### 1.3.3 Dynamic learning

The agent in our model learns from both prior states and signals. Much of the previous literature on dynamic learning in asset pricing relies on a different setup (e.g. Pástor and Veronesi 2009b; Ai 2010).<sup>7</sup> They assume that the state variable is subject to normally-distributed shocks, in which case the mean and variance of the historical signals form sufficient statistics for the entire time-series of signals. This allows them to consider the entire history of signals while keeping the dimensionality of the state space manageable. Instead, we do not impose an a priori parametric structure on the joint distribution of states and signals. We do, however, impose assumptions (A1) and (A2), which make the learning problem Markovian—the only relevant history is in the most recent period.

## 1.4 Optimization

Using Claim 1, the agent’s problem is characterized by the Bellman equation

$$V(a_t, z_t, s_t) = \max_{c_t, \mathbf{w}_{t+1}} \left\{ (1 - \beta) c_t^{1-\rho} + \beta E_t \left[ V(a_{t+1}, z_{t+1}, s_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}, \quad (7)$$

subject to the wealth constraint,  $a_{t+1} = (a_t - c_t) \sum_{i=1}^n w_{it+1} R_{it+1} = (a_t - c_t) R_{pt+1}$ , where  $a_t$  denotes wealth,  $R_{it+1}$  is the gross random return of the  $i$ th Arrow-Debreu security,  $R_{it+1} = \frac{1_{\{z_{t+1}=i\}}}{q(z_{t+1}=i|z_t)}$ , and  $R_{pt+1}$  is the gross return on a portfolio  $\mathbf{w}_{t+1}$ , of such securities. Note that the expectation here is taken over both states and signals.

The linearity of the budget constraint and the homogeneity of the time and risk aggregators imply wealth homogeneity of the value function:

$$V(a_t, z_t, s_t) = a_t L(z_t, s_t), \quad (8)$$

for some scaled value function  $L$ . This useful separation implies that  $L(z_t, s_t)$  is the marginal utility

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<sup>7</sup>See Veronesi (1999), Veronesi (2000), and Pástor and Veronesi (2003) for earlier related work and Pástor and Veronesi (2009a) for a survey.

of wealth (Backus, Routledge, and Zin 2005). The first-order condition for consumption is

$$\beta E_t \left[ R_{pt+1}^{1-\gamma} L(z_{t+1}, s_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} = (1-\beta) \left( b_t^{-1} - 1 \right)^\rho, \quad (9)$$

where  $b_t = b(z_t, s_t) \equiv \frac{c_t}{a_t}$  is the consumption-to-wealth ratio. Plugging into (7) and simplifying gives a relationship between  $V$ ,  $L$  and  $b_t$ :

$$V(a_t, z_t, s_t) = a_t L(z_t, s_t) = a_t (1-\beta)^{\frac{1}{1-\rho}} b_t^{\frac{-\rho}{1-\rho}}. \quad (10)$$

This familiar “solution” (Backus, Routledge, and Zin 2005) of the value function is still implicit because it remains to solve for  $b_t$ . Isolating  $b_t$  in (9) and plugging the result into (7) gives a recursion for the value-to-wealth ratio  $L(z_t, s_t)$ ,

$$L(z_t, s_t) = \left\{ (1-\beta)^{\frac{1}{\rho}} + \beta^{\frac{1}{\rho}} E_t \left[ R_{pt+1}^{1-\gamma} L(z_{t+1}, s_{t+1})^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{\rho}{1-\rho}}, \quad (11)$$

which depends on its next-period values  $L(z_{t+1}, s_{t+1})$ , the return-on-wealth  $R_{pt+1}$ , and the parameters of the model. Using the first-order condition for portfolio weight  $w_{it+1}$  on the Arrow-Debreu security for each future state  $i$ ,

$$w_{it+1} = \frac{\left\{ q(z_{t+1} = i | z_t)^{\gamma-1} E \left[ L(z_{t+1}, s_{t+1})^{1-\gamma} | z_{t+1} = i \right] p_\alpha(z_{t+1} = i | z_t, s_t) \right\}^{\frac{1}{\gamma}}}{\sum_j \left\{ q(z_{t+1} = j | z_t)^{\gamma-1} E \left[ L(z_{t+1}, s_{t+1})^{1-\gamma} | z_{t+1} = j \right] p_\alpha(z_{t+1} = j | z_t, s_t) \right\}^{\frac{1}{\gamma}}}, \quad (12)$$

which pins down the optimal return on wealth in state  $z_{t+1}$  conditional on the current state  $z_t$  and signal  $s_t$ ,  $R_p(z_{t+1} = i | z_t) = \frac{w_{it+1}}{q(z_{t+1}=i|z_t)}$ .

Plugging the optimal return on wealth into (11), and rearranging we get the following recursion

$$L(z_t, s_t) = \left\{ (1-\beta)^{\frac{1}{\rho}} + \beta^{\frac{1}{\rho}} \left[ \sum_{z_{t+1}} \left\{ q(z_{t+1} | z_t)^{\gamma-1} E \left[ L(z_{t+1}, s_{t+1})^{1-\gamma} | z_{t+1} \right] p_\alpha(z_{t+1} | z_t, s_t) \right\}^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{\rho} \frac{1-\rho}{1-\gamma}} \right\}^{\frac{\rho}{1-\rho}}, \quad (13)$$

where the expectation on the right is over next period’s signal  $s_{t+1}$  conditional on a state  $z_{t+1}$  realizing. Beyond the posterior distribution needed to evaluate this expectation, this is a func-

tional equation in  $L(\cdot)$  that only depends on state-prices  $q$ , physical probabilities  $p$ , and preference parameters.

Denote by  $v \equiv \log(V/c)$  the log value-to-consumption ratio. Plugging the identity  $b_t = L_t e^{-v(z_t, s_t)}$  into (10) and rearranging gives the following relationship between the wealth-scaled value function and the log value-to-consumption ratio:

$$L(z_t, s_t) = (1 - \beta) e^{\rho v(z_t, s_t)}. \quad (14)$$

Recursion (13) can therefore be expressed in terms of the log value-to-consumption ratio:

$$v(z_t, s_t; \alpha) = \frac{1}{1 - \rho} \log \left\{ 1 - \beta + \beta^{\frac{1}{\rho}} \left[ \sum_{z_{t+1}} \left\{ q(z_{t+1}|z_t)^{\gamma-1} E \left[ e^{(1-\gamma)\rho v(z_{t+1}, s_{t+1}; \alpha)} | z_{t+1} \right] p_\alpha(z_{t+1}|z_t, s_t) \right\}^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{\rho} \frac{1-\rho}{1-\gamma}} \right\}. \quad (15)$$

In the case in which the signal is drawn from the uninformative information structure,  $\alpha_0$ , this recursion becomes

$$v(z_t; \alpha_0) = \frac{1}{1 - \rho} \log \left\{ 1 - \beta + \beta^{\frac{1}{\rho}} \left[ \sum_{z_{t+1}} \left\{ q(z_{t+1}|z_t)^{\gamma-1} e^{(1-\gamma)\rho v(z_{t+1}; \alpha_0)} p(z_{t+1}|z_t) \right\}^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{\rho} \frac{1-\rho}{1-\gamma}} \right\}. \quad (16)$$

The value of information, which we define next, derives from the expected wedge between the value-to-consumption ratio in the informed and uninformative cases in equations (15) and (16).

## 2 The value of information

An information structure is potentially valuable for the agent because it allows her to make better informed investment and consumption decisions and because it reduces uncertainty earlier. We define the value of information structure  $\alpha$  in state  $z_t \in \{1, \dots, n\}$  as the fraction of current wealth that the agent is willing to give up to observe a stream of signals  $s_t, s_{t+1}, \dots$ , each generated by  $\alpha$ .

**Definition 1.** The *value of information structure*  $\alpha$  in state  $z_t$  is the fraction of wealth  $\Omega$  such that

$$\mu[V(a_t(1 - \Omega), z_t, s_t; \alpha) | z_t; \alpha] = V(a_t, z_t; \alpha_0), \quad (17)$$



where  $\mu[\cdot]$  is the certainty equivalent defined in (3) and its expectation is taken over the signal  $s_t$ .

The left hand side of (17) is the certainty equivalent of lifetime consumption to the agent if she gives up a fraction  $\Omega$  of her current wealth to pay for information structure  $\alpha$ . The right hand side is the value of lifetime consumption resulting from resorting to the uninformative information structure  $\alpha_0$  (at no cost). The fraction of wealth  $\Omega$  can also be interpreted as the fraction of a consumption perpetuity the agent is willing to give up, which is commonly used to define welfare gains in related settings, though this distinction is not important for our purposes.

In what follows, it is useful to work with the transformed value of information  $\omega(z_t; \alpha) \equiv -\log(1 - \Omega(z_t; \alpha))$ , which can potentially take values on the entire real line. A first-order expansion of  $\Omega(\omega) = 1 - e^{-\omega}$ , reveals that  $\Omega \approx \omega$  when these are close to zero.

Since the value function is homogeneous, (17) can be written using (14) in terms of value-to-consumption ratios

$$\omega(z_t; \alpha) = \log \mu \left[ e^{\rho\{v(z_t, s_t; \alpha) - v(z_t; \alpha_0)\}} | z_t; \alpha \right]. \quad (18)$$

Intuitively, the value of information  $\omega$  is the (nonlinear) average increase in the log value-to-consumption ratio due to the information source  $\alpha$  relative to the uninformed benchmark  $\alpha_0$ . Rearranging (18) using the definition of the certainty equivalent (3),

$$E \left[ e^{(1-\gamma)\rho v(z_t, s_t; \alpha)} | z_t; \alpha \right] = e^{(1-\gamma)[\rho v(z_t; \alpha_0) + \omega(z_t; \alpha)]}. \quad (19)$$

Plugging (15) into (19), and using next period's version of (19) we get

$$E \left[ e^{(\gamma-1)[\rho v(z_t; \alpha_0) + \omega(z_t; \alpha)]} \left\{ 1 - \beta + \beta^{\frac{1}{\rho}} \Gamma(z_t, s_t; \alpha)^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}} \right\}^{\frac{\rho(1-\gamma)}{1-\rho}} - 1 | z_t \right] = 0, \quad (20)$$

with

$$\Gamma(z_t, s_t; \alpha) \equiv \sum_{z_{t+1}} p_\alpha(z_{t+1} | z_t, s_t) e^{\frac{1-\gamma}{\gamma} [\rho v(z_{t+1}; \alpha_0) + \omega(z_{t+1}; \alpha) + \log(p_\alpha(z_{t+1} | z_t, s_t) / q(z_{t+1} | z_t))]}, \quad (21)$$

which given  $v(z_t; \alpha_0)$ ,  $q$ ,  $p$ , and preference parameters  $\beta$ ,  $\gamma$ , and  $\rho$ , is a set of  $n$  equations in the  $n$  unknown  $\omega(z_t; \alpha)$ 's, one for each state  $z_t$ . Here,  $\Gamma(z_t, s_t; \alpha)$  is the expectation of a non-linear

function of (gross) asset returns  $q(z_{t+1}|z_t)^{-1}$ , future log value-to-consumption ratios  $v(z_{t+1}; \alpha_0)$ , and future values of information  $\omega(z_{t+1}; \alpha)$ . The agent values high payoffs not only in high future log value-to-consumption ratios states, but also in high value of information states.

The conditional population moments (20) provide the theoretical basis for the GMM estimation approach we develop below. This representation is particularly useful because the outer expectation over signals can be estimated using sample averages without taking a stance on the exact signal space and the conditional distribution of the signals. In the online appendix we provide expressions for some limiting cases (e.g. log utility).

## 2.1 One-time signals

Our analysis thus far has considered an infinite stream of signals, one in each period. In many settings the value of a one-time signal may be more relevant. The optimization problem in this case consists of a Bellman equation for the initial period with the signal,

$$V(a_t, z_t, s_t; \alpha) = \max_{c_t, \mathbf{w}_{t+1}} \left\{ (1 - \beta) c_t^{1-\rho} + \beta E_t \left[ V(a_{t+1}, z_{t+1}; \alpha_0)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}, \quad (22)$$

and a second Bellman equation for all subsequent periods without the signal,

$$V(a_t, z_t; \alpha_0) = \max_{c_t, \mathbf{w}_{t+1}} \left\{ (1 - \beta) c_t^{1-\rho} + \beta E_t \left[ V(a_{t+1}, z_{t+1}; \alpha_0)^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}, \quad (23)$$

subject to the same wealth constraint as before. Note that unlike in (7), the continuation value in (22) has no future signal.

A derivation similar to the one provided in the previous section shows that the moments that identify the value of information are special cases of (20), where the continuation values of information  $\omega(z_{t+1}; \alpha)$  in (21) are set to zero. That is,

$$\Gamma(z_t, s_t; \alpha) \equiv \sum_{z_{t+1}} p_\alpha(z_{t+1}|z_t, s_t) e^{\frac{1-\gamma}{\gamma} [\rho v(z_{t+1}; \alpha_0) + \log(p_\alpha(z_{t+1}|z_t, s_t)/q(z_{t+1}|z_t))]} \quad (24)$$

## 2.2 Psychic vs. instrumental values of information

The value of information generally comes from two sources. The first is the instrumental value of information, due to the improvement in consumption and investment choices. An additional psychic value of information, due to a preference for early resolution of uncertainty arises if  $\gamma > \rho$  (Kreps and Porteus 1978; Epstein and Zin 1989). Such preference is entirely about the attitude of the agent toward uncertainty, even when she cannot do anything to alter her future consumption. We find it instructive to decompose the value of information into these two channels.

Similar to Ai (2007), we define the psychic value of information structure  $\alpha$  in state  $z_t \in \{1, \dots, n\}$  as the fraction of current wealth that the agent is willing to give up in order to obtain the same stream of signals considered above, with one critical difference. Specifically, the agent is not allowed to change her consumption-investment plan relative to the uninformed  $\alpha_0$  benchmark case. Instead, the only benefit from the signals comes from early resolution of uncertainty.

Denote by  $V^P$  the present value of the uninformed consumption and investment plans evaluated using informed posterior probabilities derived from  $\alpha$ , which is given recursively by

$$V^P(a_t, z_t, s_t) = \left\{ (1 - \beta) (b(z_t; \alpha_0) a_t)^{1-\rho} + \beta E \left[ V^P(a_{t+1}, z_{t+1}, s_{t+1})^{1-\gamma} \mid z_t, s_t; \alpha \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1}{1-\rho}}, \quad (25)$$

and wealth follows  $a_{t+1} = a_t (1 - b(z_t; \alpha_0)) R_p(z_{t+1} \mid z_t; \alpha_0)$ .

**Definition 2.** The *psychic value of information structure*  $\alpha$  in state  $z_t$  is the fraction of wealth  $\Omega^P$  such that

$$\mu \left[ V^P \left( a_t (1 - \Omega^P), z_t, s_t; \alpha \right) \mid z_t; \alpha \right] = V(a_t, z_t; \alpha_0), \quad (26)$$

where  $\mu[\cdot]$  is the certainty equivalent whose expectation is taken over the signal  $s_t$ .

Because we fix consumption  $b(z_t; \alpha_0) a_t$  and investment  $R_p(z_{t+1} \mid z_t; \alpha_0)$  to their uninformed benchmark values, the psychic value of information arises only because the expectation over the future  $V^P$  in (26) conditions on the signal  $s_t$ .

Similarly we can define the instrumental value of information as the fraction of wealth  $\Omega^I$  that an agent who acquired the stream of signals is willing to give up to be able to optimize her

consumption-investment plan according to the signals.

**Definition 3.** The *instrumental value of information structure*  $\alpha$  in state  $z_t$  is the fraction of wealth  $\Omega^I$  such that

$$\mu \left[ V \left( a_t \left( 1 - \Omega^I \right), z_t, s_t; \alpha \right) | z_t; \alpha \right] = \mu \left[ V^P \left( a_t, z_t, s_t; \alpha \right) | z_t; \alpha \right], \quad (27)$$

where  $\mu[\cdot]$  is the certainty equivalent whose expectation is taken over the signal  $s_t$ .

Certainty equivalents are used on both sides because the value is determined before the signals are observed. Due to the homogeneity of the value function,

$$1 - \Omega = \left( 1 - \Omega^P \right) \left( 1 - \Omega^I \right). \quad (28)$$

Thus, the total value of information is approximately the sum of the psychic and instrumental values. The decomposition is exact for the transformed values of information,

$$\omega = \omega^P + \omega^I. \quad (29)$$

Below we use decomposition (29) to evaluate the relative importance of each component.

It is straightforward to see from (27) that the instrumental value of private information must be nonnegative.<sup>8</sup> By contrast, nothing restricts the psychic value to be nonnegative, even if preferences favor early resolution of uncertainty ( $\gamma > \rho$ ). Intuitively, holding the consumption plan fixed, a signal that skews probabilities toward worse states could have a negative psychic value. As a result, if the psychic value dominates, the total value of private information may also be negative.

### 2.3 Private vs. public information

The discussion thus far focused on the value of private information as illustrated in Figure 1a, which shows that prices adjust only after the informed agent consumes and trades. Prior literature

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<sup>8</sup>Note that  $1 - \Omega^I$  can be factored out of  $\mu(\cdot)$  and that for any signal realization  $s_t$ ,  $V(a_t, z_t, s_t; \alpha) \geq V^P(a_t, z_t, s_t; \alpha)$  because while both use the same probabilities, the first additionally allows the agent to optimize her consumption/investment plan.

has emphasized the value of public information due to its reduction in uncertainty (Lucas 1987) or its early resolution (Epstein, Farhi, and Strzalecki 2014). This distinction between the value of private versus public information should not be confused with the private versus public/social value of information (Hirshleifer 1971; Spence and Zeckhauser 1972).

We can quantify the value of public information with a minor modification to our framework, which allows us to contrast it with the value of private information. The idea is illustrated in Figure 1b. The key difference between the private and public cases is that in the private case the agent can use the information before market prices react. It is essential for this assumption that the agent is small and therefore has no price impact. This restriction is relaxed once considering public information as the agent can only trade on the signal at prices that reflect this new information. The value of public information can be quantified by replacing the state prices  $q(z_{t+1}|z_t)$  in (21) with the updated ones  $q(z_{t+1}|z_t, s_t)$ .

The value of public information could come from either the instrumental or psychic channels. By construction, the psychic value of public information is identical to the private information case because it holds consumption and investment at their uninformed levels, which are based on pre-signal prices. The instrumental value of public information, however, can differ substantially from the private information counterpart. Intuitively, public information would have no instrumental value if price adjustments offset the potential gains from improved investment returns. Importantly, the psychic value is a pure gain in social welfare as opposed to a transfer among agents. The instrumental value of private information is a transfer from other investors in an exchange economy, though a social value could arise from improved capital allocation to production (Ai 2007).

## 2.4 The log utility special case

To clarify the economic intuition we consider the case of log utility,  $\gamma = \rho = 1$ , for which the value of information is particularly tractable. Cabrales, Gossner, and Serrano (2013) have shown that in a one-period investment choice model with log utility, the value of information coincides with the expected reduction in entropy. Our dynamic consumption-investment setup leads to a dynamic version of this result in the log utility special case.

Denote by  $H(z_t, s_t; \alpha) \equiv -\sum_{z_{t+1}} p_\alpha(z_{t+1}|z_t, s_t) \log p_\alpha(z_{t+1}|z_t, s_t)$  the entropy of the future state  $z_{t+1}$  distribution given the current state  $z_t$  and signal  $s_t$ . Similarly,  $H(z_t; \alpha_0) \equiv -\sum_{z_{t+1}} p(z_{t+1}|z_t) \log p(z_{t+1}|z_t)$  is the unconditional entropy in state  $z_t$ . Recall that entropy is a measure of the dispersion of the probability distribution. Thus, the reduction in entropy associated with signal  $s_t$ ,  $H(z_t, \alpha_0) - H(z_t, s_t; \alpha)$ , is a measure of the information in this signal.

In the log utility case, the value of information boils down to four additively-separable terms:

$$\omega(z_t; \alpha) = \beta [I^p(z_t; \alpha) + I^q(z_t; \alpha) + I^v(z_t; \alpha)] + \beta \sum_{z_{t+1}} \omega(z_{t+1}; \alpha) \sum_{s_t} p_\alpha(z_{t+1}|z_t, s_t) \alpha(s_t|z_t), \quad (30)$$

where

$$I^p(z_t; \alpha) \equiv \sum_{s_t} [H(z_t; \alpha_0) - H(z_t, s_t; \alpha)] \alpha(s_t|z_t) \quad (31)$$

$$= E[E[\log w(z_{t+1}|z_t, s_t; \alpha) | z_t, s_t; \alpha] - E[\log w(z_{t+1}|z_t; \alpha_0) | z_t; \alpha_0] | z_t; \alpha],$$

$$I^q(z_t; \alpha) \equiv -\sum_{z_{t+1}} \left[ \sum_{s_t} p_\alpha(z_{t+1}|z_t, s_t) \alpha(s_t|z_t) - p(z_{t+1}|z_t) \right] \log q(z_{t+1}|z_t) \quad (32)$$

$$= E[E[\log R(z_{t+1}|z_t) | z_t, s_t; \alpha] - E[\log R(z_{t+1}|z_t) | z_t; \alpha_0] | z_t; \alpha],$$

$$I^v(z_t; \alpha) \equiv \sum_{z_{t+1}} \left[ \sum_{s_t} p_\alpha(z_{t+1}|z_t, s_t) \alpha(s_t|z_t) - p(z_{t+1}|z_t) \right] v(z_{t+1}; \alpha_0) \quad (33)$$

$$= E[E[v(z_{t+1}; \alpha_0) | z_t, s_t; \alpha] - E[v(z_{t+1}; \alpha_0) | z_t; \alpha_0] | z_t; \alpha].$$

Here,  $I^p(z_t; \alpha)$  is the expected reduction in entropy from the signal. To understand the economic meaning of this expression note that in the log utility case the optimal portfolio weight is  $w(z_{t+1}|z_t, s_t) = p_\alpha(z_{t+1}|z_t, s_t)$ , and the log return on the Arrow-Debreu security that pays in state  $z_{t+1}$  is  $\log R(z_{t+1}|z_t) = -\log q(z_{t+1}|z_t)$  (see Equation (12)). Thus,  $I^p(z_t; \alpha)$  measures the expected change (over signals) in the expected (log) portfolio weight assigned to state  $z_{t+1}$ .  $I^q(z_t; \alpha)$  measures the expected change in the expected log return. The term  $I^v(z_t; \alpha)$  measures a similar expected change but in the continuation log value-to-consumption ratio.

By the law of total probability,

$$p(z_{t+1}|z_t) = \sum_{s_t} p_\alpha(z_{t+1}|z_t, s_t) \alpha(s_t|z_t), \quad (34)$$

both  $I^q$  and  $I^v$  are zero and equation (30) becomes

$$\omega(z_t; \alpha) = \beta I^P(z_t; \alpha) + \beta \sum_{z_{t+1}} \omega(z_{t+1}; \alpha) p(z_{t+1}|z_t). \quad (35)$$

Intuitively, the value of information is large when the expected reduction in entropy due to the signal  $I^P(z_t, \alpha)$  is large. This channel operates by concentrating the distribution of portfolio weights on more likely states. This is a similar result to the one obtained in [Cabrales, Gossner, and Serrano \(2013\)](#). Unlike in their static investment model, here the entire future path of entropy reductions matters. The asset pricing literature, however, typically considers risk aversion and EIS parameters that deviate significantly from this case. In those cases, entropy does not capture the value of information and our estimation approach could prove more applicable.

### 3 Estimation framework

In this section we develop a procedure for estimating the value of information from (20). To this end, we follow prior literature and take the parameters  $\beta$ ,  $\gamma$ , and  $\rho$  as given at values shown to fit important asset pricing facts (e.g. the equity premium). We then estimate the state-dependent value of information  $\omega(z_t, \alpha)$ ,  $z_t = 1, \dots, n$  using variation in state prices  $q$  and state probabilities  $p$ , both estimated from options, as we elaborate below. To distinguish between an informative information structure  $\alpha$  and an uninformative information structure  $\alpha_0$ , we focus on informational events indexed by  $m = 1 \dots M$  (e.g. employment releases) and observe price behavior prior to the information release and shortly after the information release. We consider the information structure just before the information release as  $\alpha_0$  and the one just after as  $\alpha$ .

The crux of our estimation approach builds on a long tradition that uses large sample means to estimate population moments ([Hansen and Singleton 1982](#)). The idea is to set the starting date

of our theoretical model one day prior to each observation date  $m$ , and then estimate a GMM residual in (20) from the one-day change in the log value to consumption ratio associated with this particular signal realization. We then estimate the value of information by minimizing the GMM objective associated with these residuals over all  $M$  events.

For every observation  $m$  where a particular signal  $s_m$  occurs, we form three  $n \times n$  data matrices: state prices  $Q_m = [q_{ijm}]_{i,j=1,\dots,n}$ , where  $q_{ijm}$  is a shorthand for  $q_m(z_{t+1} = j | z_t = i)$ , prior physical probabilities  $P_m = [p_{ijm}]_{i,j=1,\dots,n}$ , where  $p_{ijm} = p_m(z_{t+1} = j | z_t = i)$ , and posterior physical probabilities  $P_m^*(s_m) = [p_{ijm}^*(s_m)]_{i,j=1,\dots,n}$ , where  $p_{ijm}^*(s_m) = p_{\alpha m}(z_{t+1} = j | s_m, z_t = i)$ . For any given event  $m$ , we follow the model by keeping the transition matrix stationary. We do, however, allow for a different transition matrix in different event dates, as reflected in their  $m$  subscripts, to account for unobserved variation between event dates. We then use GMM to estimate the information values  $\omega$  in (20). By conditioning on dates of particular information releases (e.g. Employment report), we estimate the value of information released on those dates, from the (non-linear) average change in state transition probabilities.

Intuitively, we consider a small agent who can trade state contingent securities on each date, but can purchase information that markets will have upon the release of the signal, at stale prices (in the private information case) or at updated prices (in the public information case). By observing many realized updates to her information set, we back out her willingness to pay for the average signal. We provide more details of the data construction and methodology next, before moving to the results in Section 5.

### 3.1 GMM approach

To estimate the value of information  $\omega = (\omega_1, \dots, \omega_n)$  for states  $1, \dots, n$ , let  $f_i(\omega; x_m)$  denote the state  $i$  residual associated with (20) of the  $m$ -th observation, i.e.,

$$f_i(\omega; x_m) = e^{(\gamma-1)[\rho v_{im}(\alpha_0) + \omega_i]} \left\{ 1 - \beta + \beta^{\frac{1}{\rho}} \Gamma_{im}^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}} \right\}^{\frac{\rho(1-\gamma)}{1-\rho}} - 1, \quad (36)$$



where

$$\Gamma_{im} = \sum_{j=1}^n p_{ijm}^* e^{\left(\frac{1-\gamma}{\gamma}\right) [\rho v_{jm}(\alpha_0) + \omega_j + \log(p_{ijm}^*/q_{ijm})]}. \quad (37)$$

Here  $x_m$  denotes time varying data consisting of  $Q_m, P_m^*$ , and the no-information case log value-to-consumption ratio  $v_m(\alpha_0)$  (which depends on the prior probabilities  $P_m$ ).<sup>9</sup> We estimate the vector  $v_m(\alpha_0)$  by numerically finding a fixed point to (16) separately on each event day  $m$ . As discussed in Section 2.1, the one-time signal special case is estimated by setting  $\omega_j = 0$ , for  $j = 1 \dots n$  in (37). Stacking these we obtain a sample moments vector

$$g_M(\boldsymbol{\omega}) = \frac{1}{M} \sum_{m=1}^M f(\boldsymbol{\omega}, x_m). \quad (38)$$

The GMM estimator of  $\boldsymbol{\omega}$  for each subsample of  $M$  observations minimizes the quadratic objective

$$\boldsymbol{\omega}_M = \arg \min_{\boldsymbol{\omega}} \frac{1}{2} g_M(\boldsymbol{\omega})' W g_M(\boldsymbol{\omega}), \quad (39)$$

for some positive weighting matrix  $W$ .

We report second-stage (efficient) GMM estimates, where the first stage uses an identity weighting matrix  $W$ , and the second stage uses the inverse of the Newey-West estimate of the covariance matrix with two lags  $\Sigma_M$ . The standard errors we report are the square-root of the entries along the diagonal of

$$\text{var}(\boldsymbol{\omega}_M) = \frac{1}{M} \left( Dg_M' \Sigma_M^{-1} Dg_M \right)^{-1} \quad (40)$$

where  $Dg_M(\boldsymbol{\omega}) \equiv \frac{\partial g_M'}{\partial \boldsymbol{\omega}_M} = \frac{1}{M} \sum_{m=1}^M Df(\boldsymbol{\omega}, x_m)$  is the mean Jacobian.<sup>10</sup>

We estimate  $\boldsymbol{\omega}_M$  for different subsamples associated with announcement days of the macroeco-

<sup>9</sup>For brevity we use here  $v_{im}(\alpha_0)$  as a shorthand for  $v_m(z_t = i; \alpha_0)$ .

<sup>10</sup>Calculation shows that the  $ij$ -th entry of the Jacobian of  $f$  is

$$\begin{aligned} Df_{ij}(\boldsymbol{\omega}; x_m) &= (1-\gamma) e^{(\gamma-1)[\rho v_{im}(\alpha_0) + \omega_i]} \\ &\times \left\{ \left[ 1 - \beta + \beta^{\frac{1}{\rho}} \Gamma_{im}^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}} \right]^{\frac{\rho(1-\gamma)}{1-\rho} - 1} \beta^{\frac{1}{\rho}} \Gamma_{im}^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)} - 1} p_{ijm}^* e^{\left(\frac{1-\gamma}{\gamma}\right) [\rho v_{jm}(\alpha_0) + \omega_j + \log(p_{ijm}^*/q_{ijm})]} \right. \\ &\left. - 1_{\{i=j\}} \left[ 1 - \beta + \beta^{\frac{1}{\rho}} \Gamma_{im}^{\frac{\gamma(1-\rho)}{\rho(1-\gamma)}} \right]^{\frac{\rho(1-\gamma)}{1-\rho}} \right\}. \end{aligned} \quad (41)$$

conomic indicators discussed in Section 4.3. We report the value of information as fraction of wealth  $\Omega_M = 1 - e^{-\omega_M}$ , for the middle (current) state (estimates for other states are similar in magnitude given a set of parameters). By the Delta method, because  $\Omega'(\omega) = e^{-\omega} = 1 - \Omega(\omega)$ , the standard errors for  $\Omega(\omega_M)$  are simply those of  $\omega_M$  multiplied by  $1 - \Omega(\omega_M)$ .

### 3.2 A useful approximation

A first-order Taylor expansion of the moment equation (38) around  $\omega = 0$ , which is a reasonable guess for many events and parameter values gives

$$0 = g_M(\omega) \approx g_M(0) + Dg_M(0)\omega. \quad (42)$$

Solving for  $\omega$  we get a first-order approximation for small values of information

$$\tilde{\omega}_M = -Dg_M(0)^{-1}g_M(0), \quad (43)$$

which requires no optimization and turns out to be quite accurate. Thus, beyond being a good initial guess for numerical optimization, in some applications this approximation could suffice.

## 4 Application: Estimating the value of macro announcements

Our estimation framework is fairly general and could be applied in a variety of settings. To illustrate, we quantify values of key macro indicators to an agent who can trade in contingent claims on the S&P 500. For example, what is the value of an advance look at employment reports? In going to data we must make several choices about preferences, probabilities, trading frequency, and investment opportunity sets. The value of information we obtain is naturally tied to these choices. We assume that the state  $z_t$  is captured by the level of the S&P 500 and that the agent can trade monthly in a full set of state-contingent claims expiring one month from signal arrival. We use S&P 500 index options to estimate the state-price matrix  $Q_m$  at market close one day before event  $m$ , and then recover the physical probabilities  $P_m$  using a convenient parameterization of the stochastic

discount factor. We recover posterior physical probabilities  $P_m^*$  from re-estimated state prices on the announcement day, right after the information associated with signal  $s_m$  is released.

#### 4.1 Estimating the state-price matrix

Following [Breedon and Litzenberger \(1978\)](#), we estimate state prices from S&P 500 index (SPX) European options, where the state is captured by values of the index. Options data is from OptionMetrics, which provides for each trading day from January 4, 1996 to August 31, 2015, a panel of options prices with strike prices on each side of the current index value, and at various terms (maturities). This data also includes the closing price of the SPX, a term structure of zero coupon risk-free rates, and an estimate of the dividend yield. Moneyness as usual is defined as the option's strike price over the SPX spot price, and terms are measured in years.

We apply several standard filters to this data. We only use at or out of the money calls (moneyness at least 1) and puts (moneyness at most 1), which have a strictly positive trading volume. We include only relatively liquid options with 7 days to one quarter terms because our focus is on monthly investment horizons.

Since the [Breedon and Litzenberger \(1978\)](#) method relies on second-order-differences, the smoothness of the option price in its strike is essential. We therefore use the [Carr and Wu \(2010\)](#) approach to construct an implied volatility surface on each day, which parameterizes the implied variance dynamics as a mean-reverting lognormal process (LNV). Carr and Wu show that, under their assumptions, implied variance as a function of log moneyness  $k$  and term  $\tau$  is determined by an easy-to-solve quadratic equation. The implied variance surface is then determined by six parameters, which are related to the stock price and implied variance dynamics. By relying on the Carr-Wu model we avoid most forms of arbitrage by construction (see online appendix for details).

Unlike Carr and Wu, who use a Kalman filter and assume these parameters are highly persistent (a random walk), we estimate the volatility surface at each point in time with only contemporaneous option prices, using a daily nonlinear least squares regression of implied volatility on log moneyness and term. We do so because our value of information estimates come from changes in the implied volatility surface upon news arrival, and we would not want an arbitrary assumption

about parameter persistence to drive our results. The volatility surface fit is quite good, with a mean R-squared over the entire sample of 0.93 and a median of 0.94.

We provide further details in the online appendix, but to get an intuitive sense of this process, Figure 2 shows an example volatility surface broken into slices. As expected, we have more observations (dots) close to the spot price with log moneyness around 0 that expire within a month, than farther out-of-the-money or with longer time to expiration. The figure shows that implied volatility, especially at-the-money, decreases after the employment report is released.

Following Ross (2015), for each event  $m$  we discretize the state relative to the current spot price of the SPX, so that log monthly returns on the SPX  $r_j^{mkt}$  take one of  $n = 11$  possible equally-spaced values in  $[-0.24, 0.24]$ . We focus on one-month transition matrices, so that, for example, each entry  $q_{ijm}$  in the state price transition matrix is the price in state  $i$  of a security paying \$1 if state  $j$  realizes within a month of event  $m$ . The posterior probabilities  $p_{ijm}^*(s_m)$  are estimated at the end of the day on which the signal is publicly revealed. The prior probabilities,  $p_{ijm}$  are estimated one trading day before that. This choice is guided by the daily frequency of our options data.

Note that the state-price matrix  $Q_m$  consists of prices of Arrow-Debreu securities maturing in one month for the current state of the  $m$ -th event, as well as other potential states that did not realize. However, as pointed out by Ross (2015), the volatility surface, instead encodes the state prices corresponding only to the state that was realized for various horizons. Ross overcomes this problem using a least squares projection utilizing the fact that the moneyness-term matrix can be constructed from powers of  $Q_m$ . We take a different route by assuming that, consistent with the Carr-Wu model, the implied volatility surface is a function of moneyness and term, but does not depend on the spot price. This assumption allows us to calculate implied volatility for states other than the current one and construct  $Q_m$  directly. Because we identify the value of information from day-to-day changes in  $Q_m$ , we only keep observations where all elements of  $Q_m$  are positive on consecutive trading days, or zero on both, and therefore admit no arbitrage.

## 4.2 Recovering the physical probability matrix

Physical probabilities are related to state prices by a stochastic discount factor. It is well known that in an incomplete market such as ours (see Section 1.3.1), absence of arbitrage implies the existence of a large number of stochastic discount factors. The projection of each of these discount factors on the space of traded assets is a unique asset, which is itself a stochastic discount factor (see [Cochrane 2005](#), Ch. 4). Furthermore, the pricing implications over the space of traded assets are identical for any choice of a stochastic discount factor. Thus, as discussed by [Chaudhuri and Schroder \(2015\)](#), in the space of tradable assets available in our setting—the  $n$  Arrow-Debreu securities defined by the discretization of S&P 500 returns—there is a unique stochastic discount factor  $\mathbb{M}_{ijm}$ . We therefore focus on this  $\mathbb{M}_{ijm}$  to recover physical probabilities associated with the  $n$  states from their state prices via the linear map

$$q_{ijm} = \mathbb{M}_{ijm} p_{ijm}. \tag{44}$$

To value a signal we require both a prior probability distribution that does not condition on the signal  $s_m$ , and a posterior that does condition on  $s_m$ . As econometricians, we observe state prices before and after the event, which we use to recover prior probabilities  $p_m$  and posterior probabilities  $p_m^*$ . To do so we need a mapping from  $q$  to  $p$  that applies even between events. We therefore assume that the mapping (44) applies at any time including these intermediate periods.

The recovery of forward-looking physical probabilities from options is an active topic of research.<sup>11</sup> Empirically, [Malamud \(2015\)](#) finds that probabilities recovered from index options are informative about expected returns. Traditional approaches to this problem use historical realizations to proxy for future probabilities (e.g. [Jackwerth 2000](#); [Ait-Sahalia and Lo 2000](#)). Such an approach, however, results in a slow-moving probability distribution, which is inappropriate for our purposes because we estimate the value of information from relatively high-frequency changes in this distribution. Another approach, which we follow here, has been to place sufficient parametric assumptions on the stochastic discount factor.

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<sup>11</sup>The “recovery” nomenclature is due to [Dybvig and Rogers \(1997\)](#) who study the recoverability (or identifiability) of preferences in a neoclassical investment problem.

Specifically, we assume that  $M_{ijm}$  is proportional to  $e^{-\epsilon r_j^{mkt}}$ , so that physical probabilities are an exponentially-tilted version of state prices, governed only by a tilt parameter  $\epsilon$ :

$$p_{ijm} = \frac{e^{\epsilon r_j^{mkt}} q_{ijm}}{\sum_k e^{\epsilon r_k^{mkt}} q_{ikm}}. \quad (45)$$

Intuitively, the prices of securities paying in good states with high returns are relatively cheap, with the size of the wedge determined by  $\epsilon$ . This mapping results if market returns ( $r^{mkt}$ ) are a leveraged return to a claim on aggregate consumption and the representative agent has recursive (or expected) utility with constant relative risk aversion, and is commonly invoked in empirical options studies (e.g. Bakshi, Kapadia, and Madan 2003; Bliss and Panigirtzoglou 2004). We calibrate  $\epsilon$  to 1.5 to match the equity premium over our sample of 6 percent (see Table 1).<sup>12</sup>

An alternative recovery approach recently suggested by Ross (2015) allows one to estimate the physical probability of asset returns from the state-price transition process alone. Ross’s theoretical result has been extended substantially (Carr and Yu 2012; Walden 2014), but also criticized (Dubynskiy and Goldstein 2013; Tran and Xia 2014). As pointed out by Borovička, Hansen, and Scheinkman (2016), in a more general setting, Ross’s approach recovers a probability measure that reflects long-term pricing, as opposed to transition probabilities, which may be quantitatively important (Alvarez and Jermann 2005; Bakshi and Chabi-Yo 2012).

Recovery is not the focus of the current paper, but a necessary step nonetheless. We therefore take the simple parametric approach (45). Future work could potentially use other ways of recovering the physical transition probabilities, which can then be plugged into our framework.

### 4.3 Informational events and timing

We estimate values of key macroeconomic indicators. Stock returns realized around pre-scheduled macroeconomic announcements, such as the employment report and the FOMC statements, account for virtually all of the market equity premium during our sample period (Ai and Bansal 2016). Our

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<sup>12</sup>In the robustness of the online appendix we perturb  $\epsilon$ . Significantly higher values of  $\epsilon$  result in equity premia so high that agent’s utility often diverges, as discussed in Epstein and Zin (1989). Because the equity premium varies with option prices over our sample, we only keep observations where the necessary and sufficient conditions for existence and uniqueness of recursive utilities hold (Borovička and Stachurski 2017). For example, an  $\epsilon$  of 10 would leave only 193 valid daily observations out of 4931.

data on macroeconomic indicators release dates comes from Bloomberg’s Economic Calendar. We focus on the following key indicators:

1. **FOMC Decision.** The Federal Open Market Committee (FOMC) of the Federal Reserve Board of Governors Fed funds rate decision, which usually occurs eight times a year. Because [Lucca and Moench \(2015\)](#) document large average excess returns on U.S. equities in anticipation of FOMC decisions (a pre-FOMC announcement drift), we also report results for the trading day period to FOMC decisions. We restrict our sample to scheduled releases.<sup>13</sup>
2. **Employment.** The employment situation is reported monthly by the Bureau of Labor Statistics (BLS), usually on the first Friday of the month. The two main figures released in this report are the change in nonfarm payroll employment and the unemployment rate.
3. **GDP.** Gross domestic product is reported by the Bureau of Economic Analysis (BEA). Initial reports are for the previous quarter, which are then revised in each of the following two months.
4. **Jobless Claims.** Initial jobless claims are reported each Thursday by the U.S. Department of Labor.
5. **Mortgage Apps.** Mortgage Bankers’ Association purchase applications index of applications at mortgage lenders. This leading indicator for single-family home sales and housing construction is reported each Wednesday.
6. **Consumer Comfort.** The Bloomberg Consumer Comfort Index is a weekly, random-sample survey tracking Americans’ views on the condition of the U.S. economy, their personal finances and the buying climate. The survey is formerly sponsored by ABC News since 1985. It is reported each Thursday.

When events occur after trading hours, we designate the following trading day as the event day.

Table 1 reports mean levels on event days and changes from the previous day of the expectation, volatility, and Sharpe ratio of S&P 500 excess returns implied by physical probabilities we recover

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<sup>13</sup>By following the procedure of [Lucca and Moench \(2015\)](#) we exclude 6 unscheduled FOMC announcements, which seem to occur after sharp market drops (e.g. October 8, 2008). Including these hardly changes our estimates.

from SPX options. The first line shows that unconditionally, options-implied market excess returns are about 6 percent on average with 20 percent volatility and a Sharpe ratio of 27 percent. As mentioned above, we have calibrated the tilt parameter  $\epsilon$  to match the equity premium of 6 percent over our sample, which is shown on the last line of Table 1. Even though we control only a single parameter, we match the volatility and Sharpe ratio of the S&P 500 quite well. Conditioning on particular events, results in minor differences across events in event-day levels. However, the table also shows that FOMC and employment report days are associated with the largest average reduction in uncertainty measured as either volatility or entropy.

To get a better sense of the variation used to estimate the value of information, Figure 3 plots the probabilities of the future state (S&P 500 return in one month), estimated from options on the day before and on the day of the same example employment report of Figure 2. We use these to recover the corresponding physical prior and posterior return distributions. As expected, both the risk-neutral and the physical distribution concentrate around their mean when such information arrives due to a reduction in uncertainty. The risk-neutral distribution is relatively more pessimistic than the physical one because it accounts for a risk premium. The log value-to-consumption ratio depends nonlinearly on state prices, preferences, and state transition probabilities. A tighter posterior distribution means both a better consumption-investment choice (the instrumental value), and a higher certainty equivalent of the continuation value.

An advantage of the semi-parametric form of our estimator is that it does not require us to specify the likelihood of observing a particular signal realization given the state. For tractability, our model assumes that this likelihood  $\alpha(s_t|z_{t+1})$  and the state transition probabilities  $p(z_{t+1}|z_t)$  do not depend on the history of the state or signals (Assumptions A1 and A2). The first assumption essentially requires one to choose a state space that is rich enough so that conditioning on the state, the signal is independent of past history, which we cannot directly test. This assumption is plausible, for example, when applied to employment reports, assuming the level of the S&P 500 depends on the contemporaneous level of employment and the report provides a noisy signal of future employment. The second assumption requires that conditioning on the level of the S&P 500 at time  $t$ , its next period level (and therefore return) does not depend on the history of prices and



signals. We find it quite plausible given the large body of evidence on market efficiency.

#### 4.4 Imposing rational expectations

We estimate posterior probabilities  $p_{ijm}^*$  for event  $m$  by applying the same exponential tilting SDF to state prices  $q_{ijm}^*$  estimated at market close on the announcement date. These probabilities may or may not satisfy the law of total probability with respect to realized signals (34). We therefore impose that (34) holds, by tilting the posterior probabilities by a constant per state  $\pi_{ij}$ ,

$$p_{ijm}^*(\boldsymbol{\pi}) = \frac{e^{\pi_{ij} + \epsilon r_j^{mkt}} q_{ijm}^*}{\sum_k e^{\pi_{ik} + \epsilon r_k^{mkt}} q_{ikm}^*}, \quad (46)$$

so that the following  $(n-1)n$  moments, in addition to those already in (38), are set to zero:

$$h_{ij}(\boldsymbol{\omega}, \boldsymbol{\pi}; x_m) = p_{ijm}^*(\boldsymbol{\pi}) - p_{ijm}, \quad i = 1 \dots n, j = 1 \dots n - 1. \quad (47)$$

Because the probabilities add up to one, we set  $\pi_{in} = 0$  for all  $i$ , and the  $n$  moments not included in (47) are redundant. The exactly identified intercepts  $\pi_{ij}$  are important when the stochastic discount factor changes in a predictable manner on event days.

## 5 Estimation results

### 5.1 The value of private information

Our benchmark parameters are commonly-used in the asset pricing literature: time discount rate  $\beta = 0.998$ , relative risk aversion  $\gamma = 10$ , EIS  $1/\rho = 1.5$ , and a monthly horizon  $\tau = 1/12$ . [Bansal and Yaron \(2004\)](#) and subsequent literature calibrate these parameters to match key asset pricing moments such as the equity premium and volatility of the risk free rate.

The first three columns of Table 2 report benchmark estimates of the value of information for the middle (current) state. Panel A reports estimates for a one-time signal. We find that the agent is willing to pay between 3.4 and 5.4 basis points of her wealth for a one-time peek into these macroeconomic indicators. Standard errors are small, which allows us to reject zero at the one

percent significance level, and to distinguish between the informativeness of certain signals. For example, we find that employment reports are both economically and statistically more valuable than GDP reports (we provide formal difference tests in the online appendix). Table 1 already hints at this ordering because the reduction in uncertainty associated with employment reports is greater. We emphasize, however, that the reduction in entropy is sufficient for ordering values of information only in the log utility special case (see Section 2.4), as evidenced by the fact that GDP reports yield a higher entropy reduction but a lower value of information than jobless-claims reports. More generally, equation 18 shows that a signal is more valuable if it is associated with a greater mean increase in the value-to-consumption ratio, which depends on price changes and other moments of the probability distribution. The third column reports the first-order approximation (43) for the value of information, which yields similar estimates.

We contrast these estimates with two interesting special cases: expected utility ( $\gamma = \rho = 10$ ) and log utility ( $\gamma = \rho = 1$ ), in which there is no preference or aversion to early resolution of uncertainty. Columns 4–6 show that an agent with expected utility and relative risk aversion of 10 is willing to pay about the same for these signals. Because this agent has no preference for early resolution of uncertainty, these results provide a first indication that the instrumental value dominates the value of these one-time signals.

We next consider the log utility case. This case offers an interesting point of view since Cabrales, Gossner, and Serrano (2013) show that it sets an upper bound for the value of information in a static setup with expected utility and relative risk aversion above one. Table 2 shows that such an agent is willing to pay a much greater share of her wealth for the same information. For example, this agent is willing to pay about half a percent of her wealth for an early look at employment. Thus, while the log utility case offers a tractable upper bound on the value of information, this bound is somewhat lax when considering levels of risk aversion required to fit asset pricing facts.

Panel B of Table 2 reports estimates for a signal received every period. For the benchmark parameters, we estimate the agent is willing to pay between 13 and 23 percent of her wealth for a signal every period. Such signals are valuable not only for the obvious reason that there are more of them over time, but also because of long run risk. An agent with recursive utility cares about

covariation of returns and the continuation value of information (see equation (21)).

Interestingly, an agent with expected utility and relative risk aversion of 10 is willing to pay about half as much. The higher EIS of our benchmark recursive utility agent makes her more willing to substitute current for future consumption, which makes the informational investment returns more valuable. An agent with log utility, however, would be willing to pay upward of 70 percent of her wealth for the same repeated signals.

## 5.2 Psychic vs. instrumental value of private information

Table 3 decomposes the (transformed) fraction of wealth the agent is willing to pay for private information  $\omega$  under the benchmark parameterization into two additive terms: one due only to her preference for early resolution of uncertainty ( $\omega^P$ ) and another due to the instrumental value of information ( $\omega^I$ ), as defined in Section 2.2.

From Panel A, we learn that when considering a one-time signal, the values of all studied information sources derives entirely from their instrumental value as they allow the agent to improve her consumption-investment plan, and that their psychic value is essentially zero.

Panel B shows that for an information source generating a signal every period, the bulk of the value still comes from the instrumental value of information. For example, for FOMC decisions, over 99 percent of the value of the stream of signals is due to the instrumental value of information. The psychic value is sometimes negative, though economically still zero, which is not surprising because, especially in finite samples, there is nothing restricting it to be positive (see Section 2.2).

## 5.3 Comparison with Epstein, Farhi, and Strzalecki (2014)

The economically small psychic values that we estimate are potentially surprising given that Epstein, Farhi, and Strzalecki (2014) report a psychic value of 29% associated with the same preferences. But the reductions in uncertainty considered here are much smaller than the one in their thought experiment. Epstein, Farhi, and Strzalecki (2014)'s timing premium is the fraction of wealth (or consumption perpetuity) the agent is willing to pay to resolve *all* future consumption uncertainty—an upper bound on the psychic value. By contrast, our estimates depend on the mean

reductions in uncertainty about the S&P 500 associated with the macro announcements we study, which are more modest.

To facilitate comparison, we abandon for the moment the option-implied probability changes in our data. Instead, we estimate only the prior from options, and simulate mean-preserving uncertainty reductions by setting the posterior probabilities on each day in the sample to a convex combination of the option-implied prior and a mass point that gives the same expected return on the S&P 500. We calculate the value of information for these simulated reductions in uncertainty and compare them to those we find for real data.

Denote by  $\bar{r}_i \equiv Er_j^{mkt} = \sum_j r_j^{mkt} p_{ij}$  the expected market return under the prior distribution  $p_{ij}$ , and let the perfect information posterior  $\bar{p}_{ij}$  assign probability 1 to the state  $j$ , where  $r_j^{mkt} = \bar{r}_i$ , and zero to other states.<sup>14</sup> We can then control the reduction in uncertainty by parameterizing with a constant  $\eta \in [0, 1]$  the posterior probability of state  $z_{t+1} = j$  given current state  $z_t = i$  as

$$p_{ij}(\eta) = (1 - \eta) p_{ij} + \eta \bar{p}_{ij}. \quad (48)$$

By construction, expected returns are the same for all  $\eta$ , but higher  $\eta$  generally yields lower posterior variance and entropy (a mean-preserving spread). This construct allows us to synthetically manufacture posteriors that become ever more informative as  $\eta$  tends to 1.

The first four columns of Table 4 show that as we increase  $\eta$  from 0 to 100 percent, we simulate larger reductions in uncertainty measured by return volatility, Sharpe ratios or entropy. While none of these is exactly the right measure of the value of information with recursive preferences, they facilitate comparison with the uncertainty reductions that the macro events generate.

The next four columns show that for a one-time signal, the psychic value of information is increasing in  $\eta$  but never exceeds 3 basis points. By contrast, the instrumental value is much larger and becomes infinite when all uncertainty is resolved, because positive state prices on zero probability states generate arbitrage opportunities. The last four columns show that the psychic value of a signal every period, is much larger and peaks at 18 percent when all future uncertainty is

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<sup>14</sup>An approximation is needed when the number of states is finite. We set  $\bar{p}_{ij} = 0$  for all  $j$  except the two that bracket the expected return, say  $j = b$  and  $j = a$  (below and above), with  $\bar{p}_{ia} = \frac{\bar{r}_i - r_b^{mkt}}{r_a^{mkt} - r_b^{mkt}}$  and  $\bar{p}_{ib} = \frac{r_a^{mkt} - \bar{r}_i}{r_a^{mkt} - r_b^{mkt}}$ .

resolved. This exercise is the closest to the [Epstein, Farhi, and Strzalecki \(2014\)](#) experiment, and indeed generates estimates of the same magnitude.<sup>15</sup>

Compared with the mean uncertainty reductions reported in [Table 1](#), the macro announcements we study correspond to an  $\eta$  of less than 5%. The simulations in [Table 4](#) show that such modest reductions in uncertainty generate a psychic value of less than 1 basis point for a one time signal and of 73 basis points for a signal every period, which is quite similar to our GMM estimates that rely on actual changes in option prices on macro announcement days. Evidently, in both actual options data around macro announcements, and in simulations that generate similar reductions in uncertainty, such signals provide substantial instrumental value but only a minor psychic value.

#### 5.4 Comparative statics

We next consider several comparative statics exercises around these benchmark parameters for jobless claims reports. The results for other events are qualitatively the same and therefore relegated to the online appendix. For each parameter that we vary in [Figure 4](#) we plot the value of a one-time signal on the left and that of a signal every period on the right.

An advantage of the recursive utility framework is the separation between risk aversion and intertemporal substitution. [Figure 4a](#) shows that increasing the coefficient of relative risk aversion  $\gamma$  while keeping the EIS  $1/\rho$  constant, reduces the value of information. Risk aversion has two opposing effects on the value of information. On the one hand, higher risk aversion increases the value of uncertainty-reducing information. On the other hand, a more risk averse investor demands less risky assets, which makes information less useful. Evidently, this second “demand” effect dominates here. Intuitively, investment-relevant information is less useful to a more risk averse agent because her willingness to change her portfolio to take into account the information is limited. An agent with an extreme level of risk aversion would choose a near-riskless portfolio regardless of the signal. This effect is more pronounced in the case of a signal every month. With recursive utility, as we increase  $\gamma$  there is a counter effect coming from a stronger preference for early

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<sup>15</sup>Our estimates could also differ from [Epstein, Farhi, and Strzalecki \(2014\)](#) because we rely on the empirical prior distributions we recover from options as opposed to the ones implied by the consumption process they assume, and because we provide a perfect signal every period about only the next period, while they provide a perfect signal about the entire future consumption path.

resolution of uncertainty, even if the agent’s actual consumption and investment are less sensitive to the information. But because the psychic value is modest relative to the instrumental value, the first effect dominates.

Figure 4b shows that the effect on the value of information of increasing the EIS while holding risk aversion fixed is positive in the case of a signal every period. As the EIS increases, the agent is more willing to substitute intertemporally and use the information to increase her future consumption. Preference for early resolution of uncertainty also works in the same direction.

Figure 4c shows that, as expected, when the time discount factor  $\beta$  increases, and the agent attaches more value to future periods, the value of information increases. The horizon in our model is the time between periods implied by this  $\beta$ . A pure change in horizon preference can therefore be seen from 4c, which shows that more frequent signals (higher  $\beta$ ) are more valuable.

This horizon choice also dictates the term of the options used to construct the state transition matrices. We chose to focus on the monthly horizon in our benchmark tests because options are highly-traded around this expiration. We consider this additional aspect in Figure 4d where in addition to changing  $\beta$ , we change the horizon of recovered state prices and transition probabilities.<sup>16</sup>

By shrinking the horizon we better capture the value of information to a more active trader. Due to the daily frequency of our data, the shortest horizon we consider is 2 days. We find that shorter maturity options are less sensitive to the macro announcements, and therefore the value of one-time information is mostly increasing on net in the investment horizon. By contrast, the value of a signal every period increases substantially as we shrink  $\tau$ . Intuitively, more frequent signals mean more frequent gains from portfolio adjustments.

## 5.5 The value of public information

As discussed in Section 2.3, we can estimate values of public information by solving the problem of an agent who faces updated prices which do condition on the signals. Estimating this value requires a small modification to our estimation procedure, where we replace in equation (37) the state prices estimated just before each event  $q_m$ , with those estimated just after  $q_m^*$ .

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<sup>16</sup>As we change the horizon we simultaneously change  $\beta$  so that the annual discount rate remains the same (i.e.  $\beta(\tau) = e^{-r \times \tau}$  for a fixed  $r$ , and also proportionally change the spacing between states to  $\tau \times dk$ ).

Table 5 shows that, as expected, values of public information are uniformly smaller than private values of information reported in Table 2 and turn slightly negative for the less informative events. For example, under our benchmark parameterization (first column), trading on a single employment report at stale prices is worth 5.4 basis points of wealth, but trading after prices adjust is only worth 0.2 basis points.

By construction, the psychic values are identical to the private information case. Thus, as shown in Table 6, the lower values of public versus private information are due to lower instrumental values. We find that instrumental values of one-time signals are essentially zero when the agent is faced with prices updated to reflect the new information. However, subscriptions to monthly employment reports and FOMC rate decisions are worth an economically significant fraction of wealth—0.9 and 1.9 percent respectively—even when these are announced publicly.

## 5.6 The log utility special case

The linearity of the log utility case allows us to provide better insight into the estimation. The moment conditions in this case (30) can be stacked into vectors and solved in closed-form

$$\boldsymbol{\omega} = B [I^p(\alpha) + I^q(\alpha) + I^v(\alpha)] = \boldsymbol{\omega}^p + \boldsymbol{\omega}^q + \boldsymbol{\omega}^v \quad (49)$$

where  $\boldsymbol{\omega}^p \equiv BI^p(\alpha)$ ,  $\boldsymbol{\omega}^q \equiv BI^q(\alpha)$ ,  $\boldsymbol{\omega}^v \equiv BI^v(\alpha)$ , and  $B \equiv \beta (Id - \beta E[P^*])^{-1}$ .

As discussed in Section 2.4, equation (34) reflects rational expectations which satisfy time-consistency on probabilities. In population, when the number of signal observations grows large, imposing rational expectations would leave only the expected reduction in entropy of the physical transition probabilities  $I^p(\alpha)$ . Without imposing (34), the value of information is higher if the expected increase in expected returns  $I^q(\alpha)$  or the expected increase in the future value-to-consumption ratio  $I^v(\alpha)$  are large.

Even though we impose rational expectations on average for each of the events in our sample, a finite sample bias can keep  $\boldsymbol{\omega}^q$  and to a lesser extent  $\boldsymbol{\omega}^v$  away from zero. The bias in  $\boldsymbol{\omega}^q$  arises from a mechanical covariance between event  $m$  probability changes and log state prices. To see

why, we can write our estimator as

$$\hat{\omega}_i^q(\alpha) = B \times \left[ \sum_j \hat{h}_{ij} \hat{r}_{ij} + \sum_j \frac{1}{M} \sum_m (h_{ijm} - \hat{h}_{ij}) (r_{ijm} - \hat{r}_{ij}) \right] \quad (50)$$

where  $h_{ijm} = p_{ijm}^* - p_{ijm}$  are probability changes,  $r_{ijm} = -\log q_{ijm}$  are log returns on Arrow securities given time  $m$  options, and  $\hat{h}_{ij} = \frac{1}{M} \sum_m h_{ijm}$  and  $\hat{r}_{ij} = \frac{1}{M} \sum_m r_{ijm}$  are their sample means. Imposing rational expectations implies  $\hat{h}_{ij} = 0$  and the first term in the brackets vanishes. But unless probability changes  $h_{ijm}$  are orthogonal to state prices  $q_{ijm}$ , the second term in brackets can be nonzero. We expect this covariance to be positive because by construction,  $p_{ijm} \propto e^{\epsilon r_j^{mkt}} q_{ijm}$ , where  $r_j^{mkt}$  is the market return.

Table 7 decomposes the estimates of the value of information for the log utility case based on (49). The empirical decomposition into the three terms of  $\omega$  shows that for some events like jobless claims this sampling error increases the estimated value of information, while for others like employment this error is small relative to the mean reduction in physical probability entropy  $\omega^p$ . A useful avenue for future research would be to find estimators that are less sensitive to measurement error in the general recursive utility case, where eliminating it is more challenging.

## 6 Conclusion

We derive a general expression for the value of information to an investor in a dynamic environment with recursive utility and provide a framework for its estimation. To illustrate, we estimate values of key macroeconomic indicators from changes in index option prices. We consider both private and public information and break down these values of information into their psychic and instrumental parts. Comparative statics exercises show that time discounting, risk aversion and attitudes toward intertemporal substitution play an important role in determining values of information.

Our approach facilitates structural estimation of the value of informational signals. Thus, we contribute to the literature by shifting from the traditional view that information is beneficial to a more practical level of quantifying the benefit of an informational signal and shedding light on its sources. While we apply this approach to macroeconomic indicators, future applications may use



our methodology to rank information sources at the firm level, such as in the context of mergers and acquisitions, earnings releases, or analyst forecasts. Estimates of the value of information could be applied to the pricing of information services and for the enforcement and litigation of insider trading.<sup>17</sup>

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<sup>17</sup>US and UK securities laws tie penalties to the profit gained or loss avoided as a result of insider trading and communication. For example, the Financial Conduct Authority’s handbook stipulates it “will seek to deprive an individual of the financial benefit derived as a direct result of the market abuse (which may include the profit made or loss avoided) where it is practicable to quantify this.” We estimate an *ex ante* measure of benefit from market abuse, which depends on the individual investor, her wealth, preferences, and investment opportunity set. Unlike commonly-used *ex post* measures of profit, our estimates do not depend on the random nature of securities markets or on the actions of other market participants.

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Order of Events During Time $t$			
State $z_t$ realized	Investor observes signal $s_t$	Investor chooses consumption $c_t$ and investment portfolio weights $\mathbf{w}_{t+1}$	Signal $s_t$ becomes public and prices adjust

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(a) Private information setup

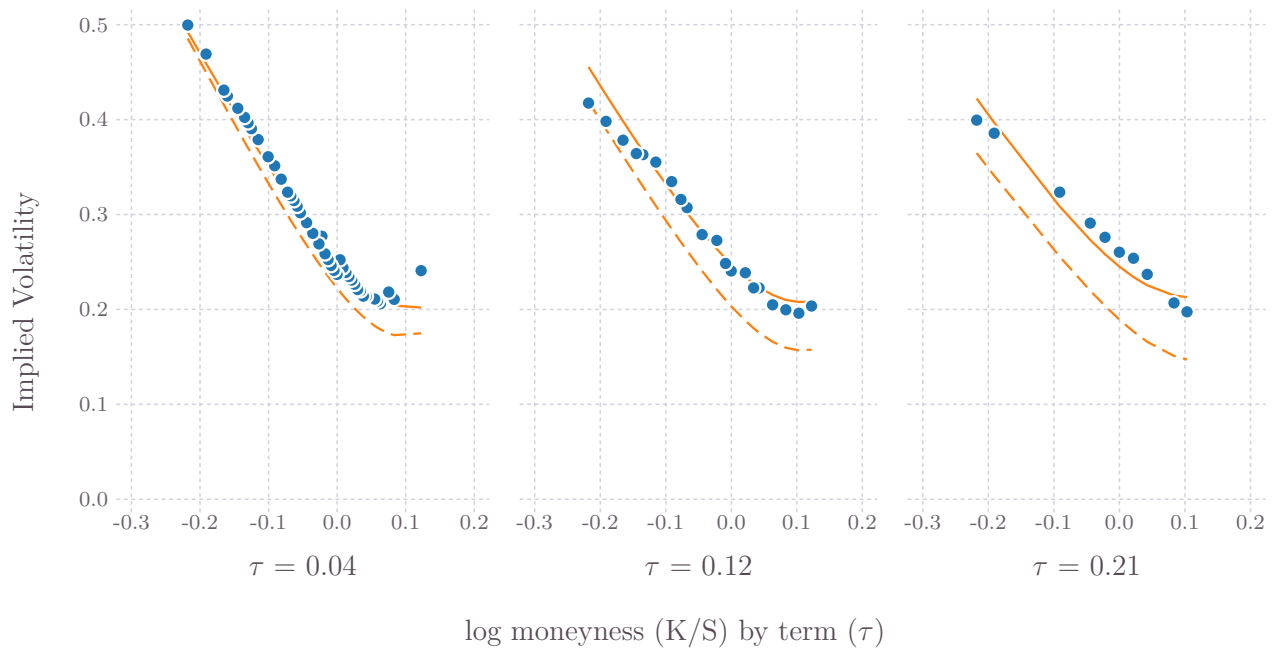
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Order of Events During Time $t$			
State $z_t$ realized	Investor observes signal $s_t$	Signal $s_t$ becomes public and prices adjust	Investor chooses consumption $c_t$ and investment portfolio weights $\mathbf{w}_{t+1}$

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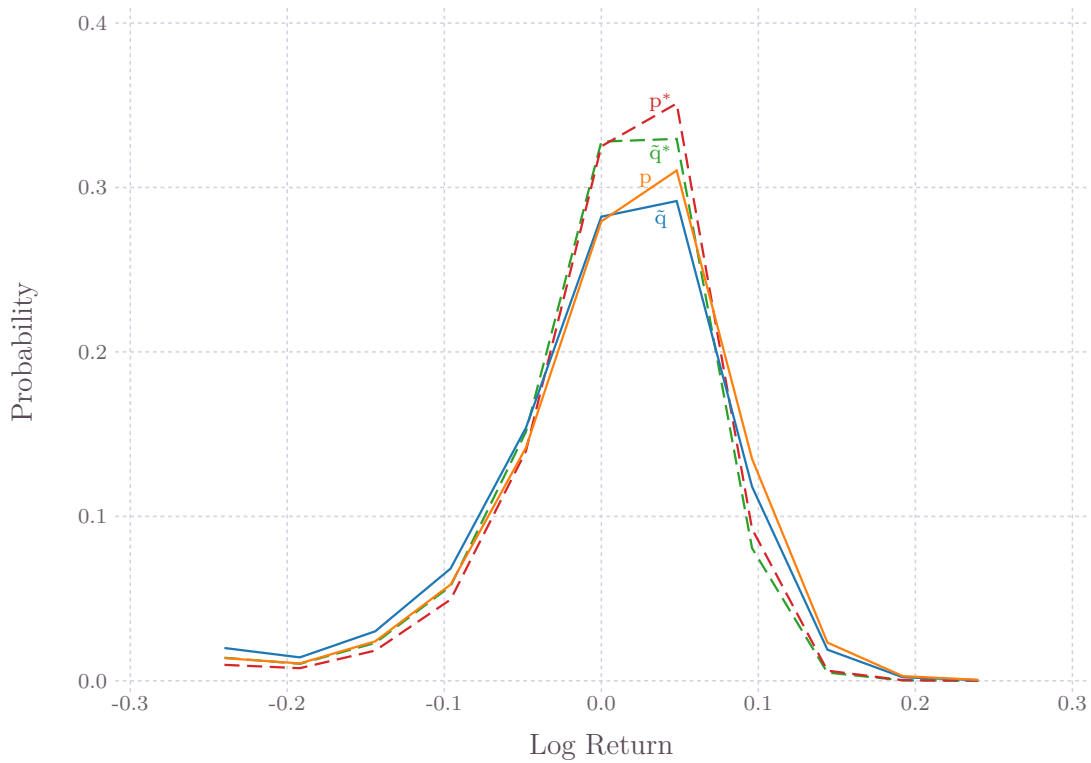
(b) Public information setup

Figure 1: **Model Timeline**



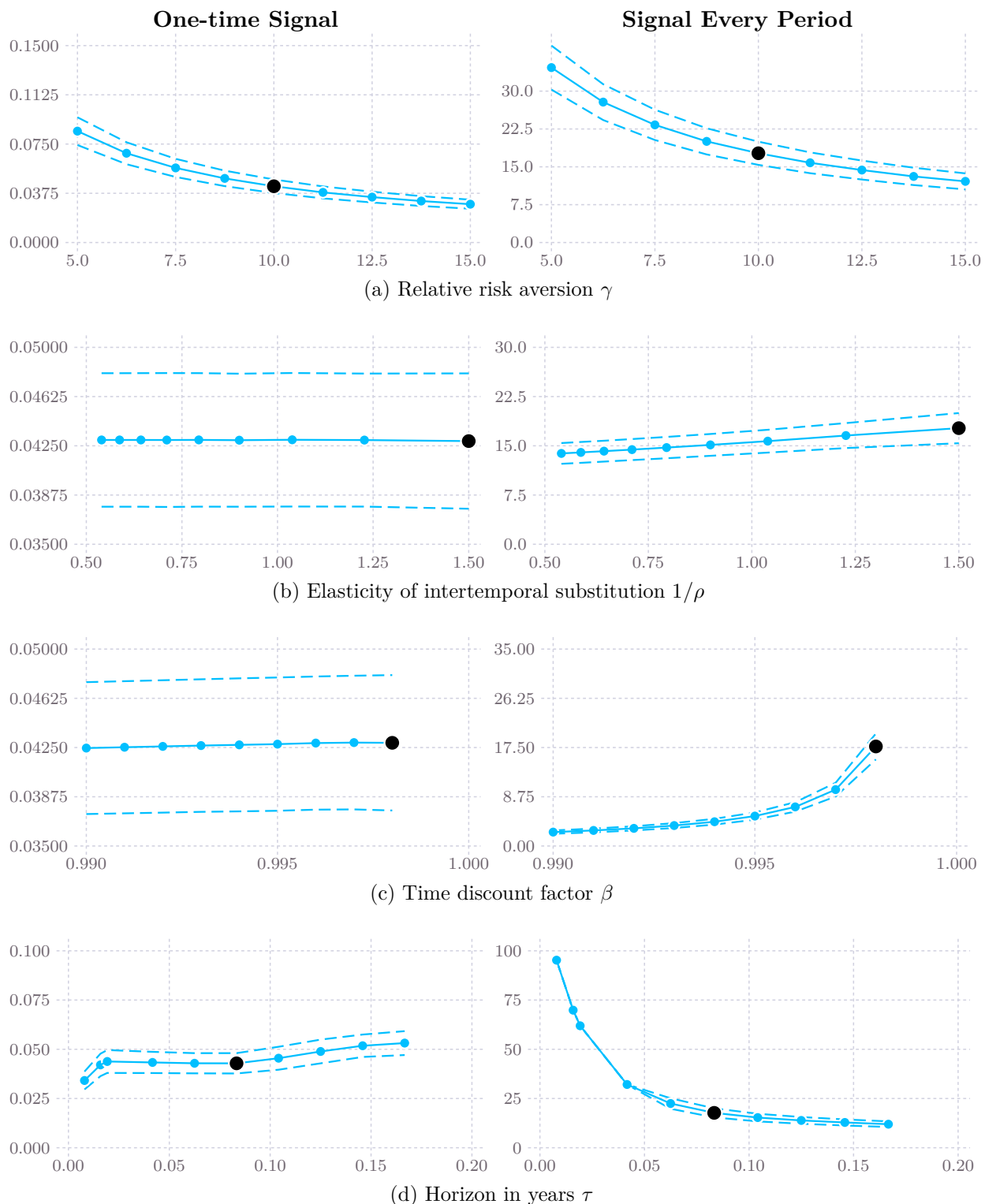
**Figure 2: Implied Volatility Surface: December 4, 1998 Employment Report**

An example of a fitted implied volatility surface using the Carr and Wu (2010) LNV parameterization one day prior to the release (solid line) and on the release day (dashed line). Dots represent observed implied volatilities on the prior day. Each panel shows a cross-section of option-implied volatility as a function of log moneyness ( $K/S$ ) fixing their term  $\tau$ .



**Figure 3: Prior vs. Posterior Probabilities: December 4, 1998 Employment Report**  
 Plotted are the prior  $\tilde{q}_j = q_j / \sum_i q_i$  and corresponding posterior  $\tilde{q}_j^*$  risk-neutral probabilities of the future state (S&P 500 return in one month), estimated from options on the day before and on the day of an example Employment report. We use these to recover the corresponding physical prior  $p_j$  and posterior  $p_j^*$  return distributions.





**Figure 4: Comparative Statics: Value of Information on Jobless Claims**

Plotted are comparative statics of the value of private information as percent of wealth around our benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). The benchmark estimate is circled in each plot. Dashed lines are the 95 percent confidence interval using Newey-West standard errors with two lags. The value of a one-time signal is on the left and that of a signal every period is on the right.

Table 1: **Event Summary Statistics**

Event	Levels on event day				Changes from previous day				Obs
	$E[r^e]$	$\sigma[r^e]$	$SR$	$H^p$	$\Delta E[r^e]$	$\Delta \sigma[r^e]$	$\Delta SR$	$\Delta H^p$	
All	5.93 (0.07)	19.89 (0.09)	26.69 (0.16)	149.50 (0.41)	0.00 (0.02)	-0.00 (0.02)	0.02 (0.05)	0.00 (0.08)	4931
Consumer Comf.	5.43 (0.24)	18.34 (0.27)	25.36 (0.51)	140.13 (1.24)	-0.06 (0.05)	-0.07 (0.05)	-0.39 (0.13)	-0.25 (0.23)	606
Employment	5.96 (0.34)	19.83 (0.43)	26.61 (0.78)	149.00 (2.04)	-0.30 (0.08)	-0.25 (0.09)	-1.16 (0.26)	-1.19 (0.41)	219
FOMC	6.11 (0.44)	19.94 (0.50)	27.47 (0.97)	149.63 (2.37)	-0.33 (0.13)	-0.60 (0.10)	-0.61 (0.30)	-2.76 (0.44)	147
Pre-FOMC	6.44 (0.45)	20.55 (0.52)	28.06 (1.00)	152.38 (2.40)	0.19 (0.16)	0.08 (0.10)	0.82 (0.36)	0.21 (0.43)	147
GDP	5.84 (0.33)	19.97 (0.41)	26.20 (0.73)	149.75 (1.93)	-0.11 (0.08)	-0.13 (0.07)	-0.29 (0.22)	-0.61 (0.33)	222
Jobless Claims	6.14 (0.17)	19.94 (0.20)	27.48 (0.38)	149.50 (0.95)	0.00 (0.04)	-0.05 (0.04)	-0.02 (0.11)	-0.18 (0.18)	948
Mortgage App.	5.51 (0.24)	18.31 (0.28)	25.86 (0.51)	139.75 (1.27)	0.07 (0.05)	-0.15 (0.05)	0.45 (0.13)	-0.89 (0.22)	601
Historical	6.00 (0.28)	19.49 (0.20)	30.81 (22.61)						4931

Reported are mean levels on event days and changes from the previous day multiplied by 100 of the expectation ( $E[r^e]$ ), volatility ( $\sigma[r^e]$ ), and Sharpe ratio (SR) of the annualized excess return on the S&P 500 implied by physical probabilities recovered from SPX options. We also report the mean physical probability entropy ( $H^p$ ) and its mean change on the event day. Each row is estimated separately using only announcement days of Bloomberg’s consumer comfort index, employment, FOMC rate decision (FOMC), the day before (Pre-FOMC), GDP, initial jobless claims, and mortgage applications. The first line shows “All” sample days without conditioning on any particular event. The last line shows “Historical” moments for annualized daily excess returns on the S&P 500 that realized over the same period. Standard errors are in parentheses.

Table 2: **Estimated Value of Private Information**

Panel A: One-time Signal

Event	$RRA = 10, EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			Obs
	$\Omega$	$se(\Omega)$	$\tilde{\Omega}$	$\Omega$	$se(\Omega)$	$\tilde{\Omega}$	$\Omega$	$se(\Omega)$	$\tilde{\Omega}$	
Consumer Comf.	0.039	(0.003)	0.039	0.034	(0.004)	0.033	0.417	(0.040)	0.413	574
Employment	0.054	(0.005)	0.053	0.061	(0.007)	0.060	0.565	(0.052)	0.554	207
FOMC	0.035	(0.005)	0.030	0.037	(0.007)	0.030	0.341	(0.045)	0.283	133
Pre-FOMC	0.038	(0.005)	0.037	0.032	(0.005)	0.032	0.406	(0.046)	0.395	134
GDP	0.034	(0.003)	0.034	0.033	(0.004)	0.033	0.364	(0.035)	0.360	206
Jobless Claims	0.043	(0.003)	0.043	0.041	(0.003)	0.040	0.445	(0.030)	0.441	887
Mortgage App.	0.035	(0.003)	0.034	0.034	(0.003)	0.032	0.357	(0.030)	0.346	570

Panel B: Signal Every Period

Event	$RRA = 10, EIS = 1.5$			$RRA = 10 = 1/EIS$			$RRA = 1 = 1/EIS$			Obs
	$\Omega$	$se(\Omega)$	$\tilde{\Omega}$	$\Omega$	$se(\Omega)$	$\tilde{\Omega}$	$\Omega$	$se(\Omega)$	$\tilde{\Omega}$	
Consumer Comf.	14.13	(1.22)	13.57	7.90	(0.76)	7.87	78.19	(0.12)	77.99	574
Employment	23.37	(2.01)	21.73	12.38	(1.12)	12.83	86.47	(0.02)	85.98	207
FOMC	14.64	(0.05)	12.12	7.42	(0.22)	6.15	69.76	(0.00)	62.57	133
Pre-FOMC	15.41	(0.66)	14.48	7.03	(1.22)	7.21	75.43	(0.03)	74.27	134
GDP	14.44	(0.95)	13.82	7.02	(0.66)	7.18	71.55	(0.01)	71.18	206
Jobless Claims	17.69	(1.17)	16.84	8.59	(0.53)	8.82	78.70	(0.05)	78.38	887
Mortgage App.	12.52	(0.93)	11.78	7.90	(0.76)	7.86	72.69	(0.10)	71.41	570

Reported are GMM estimates for the state-dependent value of information as percent of wealth  $\Omega$  for the middle (current) state of the 11 states. The first set of results uses the benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). The second set is expected utility with  $\gamma = \rho = 10$ . The third set is the log utility limiting case. Each row is estimated separately using only announcement days of Bloomberg’s consumer comfort index, employment, FOMC rate decision (FOMC), the day before (Pre-FOMC), GDP, initial jobless claims, and mortgage applications. Newey-West standard errors in parentheses correct for autocorrelation in errors with two lags.  $\tilde{\Omega}$  is a first-order approximation around zero that requires no numerical optimization.

Table 3: **Psychic vs. Instrumental Value of Private Information**

Panel A: One-time Signal

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	$\omega^P$	$\omega^I$	$ \omega^P / \omega $	$ \omega^I / \omega $	Obs
Consumer Conf.	0.039	0.039	0.000	0.039	0.014	99.986	574
Employment	0.054	0.054	-0.000	0.054	0.002	100.002	206
FOMC	0.035	0.035	0.000	0.035	0.006	99.994	130
Pre-FOMC	0.038	0.038	-0.000	0.038	0.004	100.004	131
GDP	0.034	0.034	0.000	0.034	0.014	99.986	202
Jobless Claims	0.043	0.043	0.000	0.043	0.021	99.979	875
Mortgage App.	0.035	0.035	-0.000	0.035	0.010	100.010	570

Panel B: Signal Every Period

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	$\omega^P$	$\omega^I$	$ \omega^P / \omega $	$ \omega^I / \omega $	Obs
Consumer Conf.	14.129	15.233	-0.002	15.234	0.011	100.011	574
Employment	23.368	26.616	0.007	26.608	0.028	99.972	206
FOMC	14.636	15.824	0.005	15.819	0.034	99.966	130
Pre-FOMC	15.415	16.741	-0.006	16.747	0.034	100.034	131
GDP	14.435	15.590	0.002	15.588	0.012	99.988	202
Jobless Claims	17.689	19.467	-0.000	19.467	0.002	100.002	875
Mortgage App.	12.523	13.379	-0.004	13.383	0.032	100.032	570

Reported are GMM estimates for the state-dependent value of information as percent of wealth  $\Omega = 1 - e^{-\omega}$  for the middle (current) state of the 11 states. We decompose  $\omega$  into its psychic ( $\omega^P$ ) and instrumental ( $\omega^I$ ) parts. Results are based on the benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). Each row is estimated separately using only announcement days of Bloomberg's consumer comfort index, employment, FOMC rate decision (FOMC), the day before (Pre-FOMC), GDP, initial jobless claims, and mortgage applications.

Table 4: Value of Simulated Mean-preserving Uncertainty Reductions

$\eta$	Changes from previous day			One-time signal				Signal every period			
	$\Delta\sigma[r^e]$	$\Delta SR$	$\Delta H^P$	$\Omega$	$\omega$	$\omega^P$	$\omega^I$	$\Omega$	$\omega$	$\omega^P$	$\omega^I$
0	0.00	0.02	0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00
5	-0.45	0.63	-2.65	0.02	0.02	0.00	0.01	7.54	7.83	0.73	7.11
10	-0.92	1.28	-5.59	0.06	0.06	0.00	0.06	26.75	31.13	1.47	29.65
20	-1.89	2.76	-12.27	0.23	0.23	0.01	0.22	79.73	159.59	3.03	156.56
30	-2.91	4.53	-20.02	0.50	0.50	0.01	0.49	100.00	5205.00	4.66	5200.34
40	-4.00	6.67	-28.88	0.89	0.90	0.01	0.89	100.00	5525.72	6.38	5519.35
50	-5.18	9.36	-38.88	1.43	1.44	0.02	1.42	100.00	5693.14	8.18	5684.95
60	-6.47	12.88	-50.09	2.13	2.16	0.02	2.14	100.00	5798.69	10.07	5788.62
70	-7.91	17.77	-62.72	3.09	3.14	0.02	3.11	100.00	5664.59	12.03	5652.56
80	-9.56	25.22	-77.06	4.44	4.54	0.03	4.52	100.00	2807.44	14.01	2793.43
90	-11.57	38.94	-93.69	6.65	6.88	0.03	6.86	100.00	4853.38	15.94	4837.44
100	-14.45	88.75	-115.44	$\infty$	$\infty$	0.03	$\infty$	$\infty$	$\infty$	17.66	$\infty$

Reported are GMM estimates for the state-dependent value of information as percent of wealth  $\Omega = 1 - e^{-\omega}$  for the middle (current) state of the 11 states. We decompose  $\omega$  into its psychic ( $\omega^P$ ) and instrumental ( $\omega^I$ ) parts. Results are based on the benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). Rather than rely on probability changes in the data, for this table we simulate mean-preserving uncertainty reductions by setting the posterior probabilities on each day in the sample to a convex combination of the prior and a mass point that gives the same expected return on the S&P 500. The parameter  $\eta$ , reported in percent, controls the distance of the posterior from the prior. At  $\eta = 0$ , the posterior is the same as the prior, while at  $\eta = 100\%$  there is approximately zero posterior uncertainty. We report three measures of the reduction in uncertainty: the mean changes in volatility ( $\Delta\sigma[r^e]$ ) and Sharpe ratio ( $\Delta SR$ ) of the annualized excess return on the S&P 500, and the mean change in physical probability entropy ( $\Delta H^P$ ).

Table 5: **Estimated Value of Public Information**

Panel A: One-time Signal

Event	<i>RRA</i> = 10, <i>EIS</i> = 1.5			<i>RRA</i> = 10 = 1/ <i>EIS</i>			<i>RRA</i> = 1 = 1/ <i>EIS</i>			Obs
	$\Omega$	<i>se</i> ( $\Omega$ )	$\tilde{\Omega}$	$\Omega$	<i>se</i> ( $\Omega$ )	$\tilde{\Omega}$	$\Omega$	<i>se</i> ( $\Omega$ )	$\tilde{\Omega}$	
Consumer Comf.	0.000	(0.000)	0.000	0.000	(0.000)	-0.001	0.010	(0.003)	0.007	574
Employment	0.002	(0.000)	0.001	0.003	(0.000)	0.002	0.032	(0.004)	0.021	207
FOMC	0.005	(0.000)	-0.000	0.007	(0.000)	-0.000	0.068	(0.006)	0.009	133
Pre-FOMC	-0.002	(0.000)	-0.003	-0.002	(0.000)	-0.002	-0.019	(0.004)	-0.030	134
GDP	0.001	(0.000)	0.000	0.000	(0.000)	-0.000	0.011	(0.001)	0.007	206
Jobless Claims	-0.000	(0.000)	-0.001	-0.001	(0.000)	-0.002	0.004	(0.004)	-0.000	887
Mortgage App.	-0.001	(0.000)	-0.002	0.000	(0.000)	-0.001	0.004	(0.004)	-0.008	570

Panel B: Signal Every Period

Event	<i>RRA</i> = 10, <i>EIS</i> = 1.5			<i>RRA</i> = 10 = 1/ <i>EIS</i>			<i>RRA</i> = 1 = 1/ <i>EIS</i>			Obs
	$\Omega$	<i>se</i> ( $\Omega$ )	$\tilde{\Omega}$	$\Omega$	<i>se</i> ( $\Omega$ )	$\tilde{\Omega}$	$\Omega$	<i>se</i> ( $\Omega$ )	$\tilde{\Omega}$	
Consumer Comf.	0.12	(0.06)	0.01	-0.02	(0.07)	-0.37	2.24	(0.66)	1.14	574
Employment	0.94	(0.10)	0.43	0.53	(0.04)	0.21	6.55	(0.52)	2.60	207
FOMC	1.92	(0.03)	-0.35	1.13	(0.02)	-0.45	12.66	(0.10)	-8.33	133
Pre-FOMC	-0.75	(0.05)	-1.24	-0.39	(0.01)	-0.45	-4.58	(0.38)	-8.49	134
GDP	0.23	(0.01)	0.08	-0.02	(0.01)	-0.10	1.02	(0.30)	-0.40	206
Jobless Claims	-0.28	(0.08)	-0.44	-0.44	(0.06)	-0.58	-1.09	(1.02)	-2.53	887
Mortgage App.	-0.28	(0.07)	-0.72	-0.00	(0.07)	-0.37	-1.17	(1.24)	-5.81	570

Reported are GMM estimates for the state-dependent value of information as percent of wealth  $\Omega = 1 - e^{-\omega}$  for the middle (current) state of the 11 states. The first set of results uses the benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). The second set is expected utility with  $\gamma = \rho = 10$ . The third set is the log utility limiting case. Each row is estimated separately using only announcement days of Bloomberg’s consumer comfort index, employment, FOMC rate decision (FOMC), the day before (Pre-FOMC), GDP, initial jobless claims, and mortgage applications. Newey-West standard errors in parentheses correct for autocorrelation in errors with two lags.  $\tilde{\Omega}$  is a first-order approximation around zero that requires no numerical optimization.

Table 6: **Psychic vs. Instrumental Value of Public Information**

Panel A: One-time Signal

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	$\omega^P$	$\omega^I$	$ \omega^P / \omega $	$ \omega^I / \omega $	Obs
Consumer Conf.	0.000	0.000	0.000	0.000	1.233	98.767	574
Employment	0.002	0.002	-0.000	0.002	0.039	100.039	206
FOMC	0.005	0.005	0.000	0.005	0.042	99.958	130
Pre-FOMC	-0.002	-0.002	-0.000	-0.002	0.074	99.926	131
GDP	0.001	0.001	0.000	0.001	0.728	99.272	202
Jobless Claims	-0.000	-0.000	0.000	-0.000	3.578	103.578	875
Mortgage App.	-0.001	-0.001	-0.000	-0.001	0.477	99.523	570

Panel B: Signal Every Period

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^P + \omega^I$	$\omega^P$	$\omega^I$	$ \omega^P / \omega $	$ \omega^I / \omega $	Obs
Consumer Conf.	0.123	0.124	-0.002	0.125	1.375	101.375	574
Employment	0.938	0.942	0.007	0.935	0.783	99.217	206
FOMC	1.925	1.944	0.005	1.938	0.276	99.724	130
Pre-FOMC	-0.752	-0.749	-0.006	-0.743	0.757	99.243	131
GDP	0.234	0.234	0.002	0.233	0.820	99.180	202
Jobless Claims	-0.278	-0.278	-0.000	-0.278	0.127	99.873	875
Mortgage App.	-0.280	-0.279	-0.004	-0.275	1.531	98.469	570

Reported are GMM estimates for the state-dependent value of information as percent of wealth  $\Omega = 1 - e^{-\omega}$  for the middle (current) state of the 11 states. We decompose  $\omega$  into its psychic ( $\omega^P$ ) and instrumental ( $\omega^I$ ) parts. Results are based on the benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). Each row is estimated separately using only announcement days of Bloomberg's consumer comfort index, employment, FOMC rate decision (FOMC), the day before (Pre-FOMC), GDP, initial jobless claims, and mortgage applications.

Table 7: **Decomposition of the Value of Private Information for Log Utility**

Panel A: One-time Signal

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^p + \omega^q + \omega^v$	$\omega^p$	$\omega^q$	$\omega^v$	Obs
Consumer Conf.	0.417	0.417	-0.070	0.487	0.000	574
Employment	0.565	0.567	0.479	0.088	-0.000	207
FOMC	0.341	0.342	0.211	0.131	-0.000	133
Pre-FOMC	0.406	0.407	0.233	0.173	0.000	134
GDP	0.364	0.365	0.124	0.241	-0.000	206
Jobless Claims	0.445	0.446	-0.062	0.509	0.000	887
Mortgage App.	0.357	0.358	0.057	0.301	0.000	570

Panel B: Signal Every Period

Event	$\Omega = 1 - e^{-\omega}$	$\omega = \omega^p + \omega^q + \omega^v$	$\omega^p$	$\omega^q$	$\omega^v$	Obs
Consumer Conf.	78.19	152.28	-36.47	188.77	-0.02	574
Employment	86.47	200.04	172.32	27.69	0.02	207
FOMC	69.76	119.59	82.19	37.46	-0.06	133
Pre-FOMC	75.43	140.35	105.83	34.47	0.06	134
GDP	71.55	125.70	36.20	89.52	-0.02	206
Jobless Claims	78.70	154.64	-39.83	194.51	-0.04	887
Mortgage App.	72.69	129.78	36.52	93.26	-0.00	570

Reported are GMM estimates for the state-dependent value of information as percent of wealth  $\Omega = 1 - e^{-\omega}$  for the middle (current) state of the 11 states. We focus on the log utility limiting case because  $\omega$  in this case can be decomposed into three additive terms: the mean reduction in entropy ( $\omega^p$ ), the mean increase in risk-neutral entropy ( $\omega^q$ ), the mean increase in the expected value-to-consumption ratio ( $\omega^v$ ). Results are based on the benchmark parameters:  $\beta = 0.998$ ,  $\gamma = 10$ ,  $\rho = 1/1.5$ , and  $\tau = 1/12$  (Bansal and Yaron, 2004). Each row is estimated separately using only announcement days of Bloomberg's consumer comfort index, employment, FOMC rate decision (FOMC), the day before (Pre-FOMC), GDP, initial jobless claims, and mortgage applications.