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Equilibrium Forward Contracts on Nonstorable Commodities in the Presence of Market Power

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Bilateral supply contracts are widely used despite the presence of spot markets. In this paper, we provide a potential explanation for this prevalence of supply contracts even when spot markets are liquid and without delivery lag. Specifically, we consider the determination of an equilibrium forward contract on a nonstorable commodity between two firms that have mean-variance preferences over their risky profits and negotiate the forward contract through a Nash bargaining process. We derive the unique equilibrium forward contract in closed form and provide an extensive analysis. We show that it is the risk-hedging benefit from a forward that justifies its prevalence in spite of liquid spot markets. In addition, while a forward does not affect production decisions due to the presence of spot markets, it does affect inventory decisions of the storable input factor due to its hedging effect against the inventory risk. We also show that price volatilities and correlations are important determinants of the equilibrium contract. In particular, the equilibrium forward price can be nonmonotonic in the spot price volatility and can decrease as the initial spot price increases.

Subject classifications: finance: futures price, hedging, spot market; inventory/production: dual sourcing, supply contract.
Area of review: Manufacturing, Service, and Supply Chain Operations.
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1. Introduction

As information technology advances and business-to-business (B2B) commerce shifts to the Internet, spot commodity markets have grown significantly, not only in contract variety and trading volume, but also in liquidity (IntercontinentalExchange 2004). Today, many industries have more than one online marketplace where commodity-type raw materials or components can be traded widely among suppliers and manufacturers. Still, major buyers and suppliers conduct the majority of their transactions through negotiated bilateral contracts (Grey et al. 2005). For example, in the Electronics Business Network's 2002 poll of 150 original equipment manufacturers and electronics manufacturing services providers, respondents reported that 72% of their procurement spending in 2001 was through bilateral contracts and forecasted the same level for the coming year. In the electric power market covered by PJM interconnection (a major power exchange in the United States) (Laughlin 2003), 54% of the trading was through bilateral transactions.

Given that manufacturers and suppliers already have access to relatively liquid spot markets, why do they still need to negotiate bilateral supply contracts? How should managers determine the optimal contracting rules in the presence of spot markets? How should managers adjust their supply contracts according to market conditions? These are all examples of economically important, but largely unanswered, questions that we address in this paper.

Specifically, we examine the equilibrium forward contract on a nonstorable commodity between a supplier and a manufacturer in the presence of a liquid spot market for the commodity.[1] To capture the essence of a bilateral contract, we assume that both the supplier and the manufacturer have significant market powers in the forward market and negotiate the forward contract through a Nash bargaining process at a fixed negotiation cost, where their market powers are represented by their bargaining powers. The supplier produces the commodity and faces uncertainties in input factor cost and commodity sale price in the spot markets. The manufacturer uses the commodity to produce a final product at some future time and faces uncertainties in commodity cost, the sale price of the final product, and the demand for the final product. In addition, we assume that both the supplier and the manufacturer are risk averse and have mean-variance preferences over their future risky profits.

We show that there exists a unique equilibrium for the Nash bargaining game and derive the equilibrium forward contract in closed form. The equilibrium forward contract reduces commodity price risk for at least one party through the fixed forward price and fixed forward quantity. In our model, it is exactly this risk reduction benefit that explains why bilateral supply contracts are still widely used even in the presence of a liquid spot market. Our model thus suggests that risk hedging alone can be a potential cause of the existence of bilateral supply contracts. In addition, our model also suggests that because of the negotiation costs, both parties in a bilateral contract can lose money in expectation from the contract and thus they enter into such a contract only for risk-hedging in this case. For the

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optimal contracting rule, our numerical study demonstrates that price uncertainties in various markets in the supply chain have different impacts on the equilibrium forward contract. Specifically, both the forward price and the forward quantity should be set higher when the volatility of the final product sale price decreases. However, the forward price should be set higher and the forward quantity should be set lower when the volatility of the input factor price decreases or the commodity production cost increases. Both the forward price and the quantity can change nonmonotonically with the volatility of the nonstorable commodity spot price, which implies that the equilibrium forward price can decrease as the initial spot price of the underlying asset increases, if a higher initial spot price can lead to higher future spot-price volatility. In addition, increases in price correlations between various markets decrease the risk-hedging benefit of the forward contract and lead to a decrease of the forward quantity.

This paper offers a coherent framework for studying procurement contracts in the presence of spot markets, market power, and risk-averse decision makers. It contributes to the dual/multisourcing and supply contracts research in the operations management literature. The contributions to the dual/multisourcing literature are twofold: (1) We capture the bilateral nature of a supply contract negotiation where the market powers of the buyer and the seller play a critical role in determining the equilibrium contract, and (2) we introduce the notion of risk hedging into this literature by modeling firms as risk-averse decision makers and show that in the presence of a spot market, the motive for the second sourcing through the bilateral contract is not mainly profit maximization, but rather risk reduction. In contrast to the existing supply contract literature, we consider optimal contracting for risk-averse participants and focus on the risk-sharing role of the supply contracts instead of the profit-sharing role. We conduct an extensive analysis of the optimal contracting rule and provide managerial guidelines for setting the optimal contract price and quantity according to fundamental characteristics of various markets in the supply chain.

2. Literature Review

In the operations literature, this paper is related to the dual/multiple sourcing research in which the main focus is to derive the optimal sourcing portfolio given a set of supply sources with known lead-time distribution and deterministic costs. We refer readers to Ramasesh et al. (1991), Lau and Zhao (1994), Fong et al. (2000), Chen et al. (2001), and references therein for research in this area. Research on online marketplaces grows rapidly and adds new dimensions (e.g., spot market and associated spot price uncertainty) to the dual-sourcing literature (for example, see Li and Kouvelis 1999, Yi and Scheller-Wolf 2001, Kleinknecht et al. 2002, Lee and Whang 2002, Milner and Kouvelis 2002, Goel and Gutierrez 2004, Martínez-de-Albéniz and Simchi-Levi 2005, and Araman and Özer 2005). Most of the above research focuses on the buyer's side optimal sourcing portfolio and treats the supplier's price as exogenous. Papers that model the supplier(s) as the Stackelberg game leader(s) but assume illiquid spot markets include Dong and Durbin (2005), which assumes a finite number of players in the spot market and endogenizes the spot market clearing price; Wu et al. (2002), which assumes that access to the spot market is limited and thus creates incentives for signing contracts; and Wu and Kleindorfer (2005), which characterizes the Bertrand-Nash equilibrium for a multiseller one-buyer contract market. A common assumption in all of the above literature is that decision makers are risk neutral. Thus, results and insights are driven by the expected profit maximization or expected cost minimization. A risk-hedging perspective, which is missing in this literature and considered in our model, offers an alternative explanation for the frequent usage of bilateral contracts in various industries, even when spot markets are liquid and available to both suppliers and buyers.

This work is also related to the supply-contracting literature. In a world without spot markets for the interested commodity, suppliers and buyers form contractual relationships to create and share profits in the supply chain (for representative work, see Lariviere and Porteus 2001, Cachon 2004, Cachon and Lariviere 2005, and references therein). Contract terms can also be set to extract forecast information (Tsay 1999, Cachon and Lariviere 2001) and sales effort (Taylor 2002). When a liquid commodity spot market exists, firms have the option of using the spot market as if it were an actual supplier, warehouse, or customer. Thus, it changes how businesses perceive their opportunities and offers critical information for the negotiation and valuation of supply contracts. In this paper, we take a simple but important contract, the forward contract, as an example to show that in the presence of a spot market the fundamental value created by a supply contract may be risk reduction instead of profit generation.

In addition to the vast finance literature, risk aversion has also been used by researchers in operations. Risk aversion generally arises if the firm's risk is not completely diversifiable (see Van Mieghem 2003 and Pindyck and Rubinfeld 1995 for more discussions). Some works that assume risk aversion to study one-period inventory models include Eeckhoudt et al. (1995), Agrawal and Seshadri (2000), Chen and Federgruen (2000), Gaur and Seshadri (2005), Caldentey and Haugh (2004), and Van Mieghem (2004). Risk consideration and management in more complex operational settings has become an active research area in operations. As Van Mieghem (2003) notes, mitigating risks in business involves counterbalancing actions that make the future payoff vary less over the possible state of nature. The nature of a counterbalancing action can be financial or operational. Financial hedging involves financial instruments such as derivative securities like forward and option contracts, while operational hedging involves operational

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strategies such as capacity investment, inventory policies, and postponement decisions. Works that explore financial and/or operational risk-hedging opportunities in various operational settings include Van Mieghem (2003) (capacity portfolio investment), Van Mieghem (2004) (newsvendor network), Ding et al. (2004) (production allocation), and Seifert et al. (2004) (dual sourcing of storable components). In the context of our dual-sourcing setting, a forward contract between the supplier and the manufacturer can be viewed as a financial instrument due to the presence of a liquid spot market; the supplier's and manufacturer's production and inventory flexibility can be viewed as operational instruments. This paper examines how these two different types of instruments interact to change the manufacturer's and the supplier's risk and profitability.

A vast finance literature exists on pricing derivatives for storable commodities (see Litzenberger and Rabinowitz 1995, Brennan and Crew 1996, Chambers and Bailey 1996, Ng and Pirrong 1994, Routledge et al. 2001, and references therein). In contrast, even though the literature on pricing derivatives on nonstorable commodities has grown considerably since the deregulation of the electricity market (see Kawai 1983, Bessembinder and Lemmon 2002, Pirrong and Jermakyan 1999, and Eydeland and Geman 1999), it is still relatively limited because the well-known no-arbitrage argument no longer applies. In addition, in this limited literature, derivative markets are usually assumed to be perfectly competitive. For example, both Kawai (1983) and Bessembinder and Lemmon (2002) assume that both the producers and the retailers are price takers in the derivative markets and thus consider only the competitive equilibrium. In this paper, we assume a liquid spot market but incorporate market power in the derivation of the equilibrium contract to accommodate the existence of a large number of bilateral contracts observed in the industries mentioned earlier.

The remainder of this paper is organized as follows. Section 3 sets up the main model. In §4, we derive the unique equilibrium contract and provide a set of general analytical results. In §5, we conduct a numerical study to examine important properties of the equilibrium forward contract. In §6, we summarize our findings and discuss possible future extensions. In the appendix, we provide all the proofs.

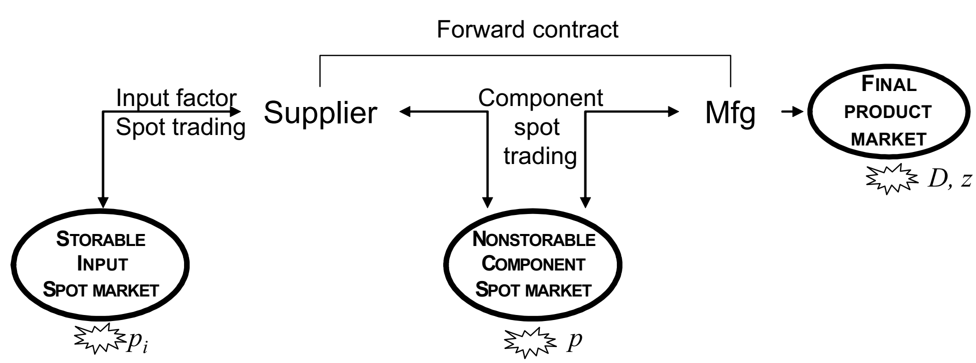
3. The Model

We consider a single-period model running from time 0 to time 1. At time 1, a manufacturer uses a certain component (also referred to as "commodity") to produce a final product and sells it in a final product market. The component is assumed to be economically or physically nonstorable to capture the essence of large storage costs or fast depreciation typically associated with many types of components (e.g., electricity). To model the bilateral nature of the widely used supply contracts, we assume that at time 0 the manufacturer can negotiate a supply contract that matures at time 1 with only one supplier. The negotiation is through a Nash bargaining game, where the bargaining power represents the market power of a participant.[2] The supplier and the manufacturer incur fixed negotiation costs of F_s and F_m, respectively. Because forward contracts are commonly used in practice, we assume that the supply contract is in the form of a forward contract (f, Q), where a positive Q means that the manufacturer (supplier) agrees to buy (sell) Q units of the components at time 1 from (to) the supplier (the manufacturer) at a price of $\$f$ per unit. In addition to the forward contract, both the manufacturer and the supplier can also trade in a spot market for the component.

It is typically the case that some input factors for producing a nonstorable component are themselves storable. For example, although electricity is nonstorable, some inputs used to generate electricity, such as coal and gas, can be, and are, frequently stored. To model this empirical fact, we assume that one of the input factors for producing the component is storable and can be purchased or short sold (i.e., borrow to sell at time 0 and buy to return at time 1) in an input spot market.[3] The supplier chooses the optimal inventory level for the input factor immediately after the forward agreement at time 0. A graphical illustration of the supply chain is provided in Figure 1.

We assume that in all three markets both the supplier and the manufacturer are small relative to other firms in these markets and are thus only price takers. The final product

Figure 1. Supply chain interactions.



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demand D, the final product sale price z, the component spot price p, and the input factor spot price p_i, are all possibly correlated random variables with bounded supports $[0, \bar{D}]$, $[0, \bar{z}]$, $[0, \bar{p}]$, and $[0, \bar{p}_i]$ and with means and variances (μ_D, σ_D^2), (μ_s, σ_z^2), (μ_p, σ_p^2), and $(\mu_{p_i}, \sigma_{p_i}^2)$, respectively. Note that in theory, one can derive a relationship between p and p_i, using a general equilibrium model. However, because the same input factor can also be used to produce products other than the component, a reasonable general equilibrium model would be complicated and carry us away from our main focus. Therefore, we use the correlation to capture their relationship instead. All random variables are realized and observed at time 1.

For simplicity, we set the time discount rate to be zero and assume that one unit of the final product can be produced from one unit of the component, which in turn requires one unit of the input factor to produce. The supplier's total cost of producing k units of the components at time 1 is $C(k) = w(k) + p_i k$, where $w(k)$ is strictly increasing and strictly convex[4] in k with $w'(0^+) = 0$ and $w'(\infty) = \infty$. The magnitude of $w(k)$ can be used as a measure of the economic storability of the component. As $w(k)$ decreases, a larger fraction of the production costs comes from purchasing the input factor (such as gas or coal for generating electricity), and thus the economic storability of the component increases in our model. Therefore, our model applies to components that have different degrees of economic storability.

In addition, we assume that the expected value of p_i is equal to its unit cost of carry c_0 (which is equal to time 0 spot price plus storage cost), i.e., $\mu_{p_i} = c_0$, so that no firm can make money in expectation by trading solely in the input spot market.

After the resolution of uncertainties at time 1, the manufacturer and the supplier choose optimal production levels to maximize their time 1 profits. Back to time 0, we assume that both the manufacturer and the supplier are risk averse and have mean-variance preferences over their own time 1 risky profits. Let π be a firm's profit at time 1. Then, the firm's utility at time 0 is

$$U_j(\pi) = \mathrm{E}\pi - \lambda_j \operatorname{var} \pi,$$

where $j \in \{m, s\}$ is the subscript representing either the manufacturer or the supplier, λ_j is the corresponding risk-aversion coefficient, and E and var denote, respectively, the expectation and variance operators over the time 1 distributions of random variables p_i, p, s, and D. We only consider the region where the utility functions are increasing in profits, i.e., $\lambda_j < 1/\bar{\pi}_j$, where $\bar{\pi}_j$ is firm j's maximum possible profits for $j \in \{m, s\}$. The expressions of $\bar{\pi}_m$ and $\bar{\pi}_s$ will be given in §4, where details of the profit functions are specified. Let π_j be firm j's time 1 profit with forward contract (f, Q), and π_{j0} be his (her) time 1 profit without a forward contract for $j \in \{m, s\}$. Then, we define $\widetilde{U}_j(f, Q)$ as firm j's *utility gain* from the forward contract (f, Q),

$$\widetilde{U}_j(f, Q) = U_j(\pi_j) - U_j(\pi_{j0})$$

for $j \in \{m, s\}$. Then, the Nash bargaining game can be written as

$$\max_{(f, Q)} \widetilde{U}_m(f, Q)^\theta \widetilde{U}_s(f, Q)^{1-\theta}, \tag{1}$$

subject to the participation constraints (individual rationality conditions)

$$\widetilde{U}_m(f, Q) \geqslant 0, \qquad \widetilde{U}_s(f, Q) \geqslant 0,$$

where $\theta \in [0, 1]$ and $1 - \theta$ represent the manufacturer's and the supplier's relative bargaining power, respectively. The expressions of $\widetilde{U}_m(f, Q)$ and $\widetilde{U}_s(f, Q)$ will be given in §4.

To summarize, we now provide an outline of the event and choice sequence described above:

1. At time 0, through a Nash bargaining game over their utility gains, the supplier and the manufacturer negotiate an equilibrium forward contract that matures at time 1, subject to fixed negotiation costs of F_s and F_m, respectively.
2. Immediately after the forward agreement, the supplier chooses the optimal inventory level for the input factor at the unit cost of carry c_0 to maximize her time 0 utility.
3. At time 1, the final product demand D, final product price z, component spot price p, and input factor spot price p_i are realized and observed.
4. The supplier then chooses the optimal component production level to maximize her time 1 profit and trades in the input spot market.[5]
5. The forward quantity is settled by physical delivery and the corresponding payment. Both the supplier and the manufacturer can trade in the component spot market.
6. The manufacturer chooses the optimal final product production level to maximize his time 1 profit and sells to the final product market in which the unit goodwill cost of not meeting the realized demand D at time 1 is g.[6]

4. Equilibrium Contract and Comparative Statics

In this section, we first calculate both firms' time 1 profits and time 0 utility gains from the forward contract (f, Q). We then derive and discuss the equilibrium forward contract (f^*, Q^*) as the solution to the Nash bargaining game (1). Finally, we provide a set of analytical results for a better understanding of the equilibrium forward contract.

We first consider the manufacturer's utility gain from a forward contract. Let $U_m(f, Q)$ be the manufacturer's utility with forward contract (f, Q) and U_{m0} be his utility without a forward contract. Because the manufacturer's time 1 profit without a forward contract can be considered as a special case of that with a forward contract by setting the forward quantity Q to be zero, we will present the time 1 profit for the with-forward case and then derive the counterpart for the without-forward case and highlight the impact of the forward contract.

Let q be the manufacturer's time 1 production level of the final product (and thus also the required amount of

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the component for production). The manufacturer's time 1 profit is equal to the revenue from final product market $z\min(q, D)$, minus the cost of forward contract fQ, minus the cost of spot trading $p(q-Q)$, minus the goodwill cost $g(D-q)^+$ from not satisfying demand D, and minus the negotiation cost $F_m 1_{\{Q\neq 0\}}$, i.e.,

$$\pi_m(f, Q, D, p, z) = \max_{q\geqslant 0}\{z\min(q, D) - fQ - p(q-Q) - g(D-q)^+\} - F_m 1_{\{Q\neq 0\}}. \quad (2)$$

The optimal production level q^* is

$$q^* = \begin{cases} D & \text{for } z+g > p, \\ 0 & \text{for } z+g \leqslant p. \end{cases} \quad (3)$$

From now on, we drop the profit function arguments for notational simplicity wherever confusion is unlikely. The optimal profit π_m can then be rewritten as

$$\pi_m = \pi_{m0} + \big[(p-f)Q - F_m 1_{\{Q\neq 0\}}\big], \quad (4)$$

where $\pi_{m0} \equiv (z+g-p)^+D - gD$ represents the manufacturer's time 1 profit without a forward contract (hereafter referred to as *manufacturer's operational profit*), and the term in the brackets represents the manufacturer's time 1 realized profit from the forward contract (hereafter referred to as *manufacturer's financial profit* for the reasons stated below). The decomposition in Equation (4) shows that in the presence of the spot market, physical delivery of the component to fulfill the forward contract is not essential at all. To the manufacturer (and also to the supplier), cash settlement (i.e., the supplier pays a cash amount equal to $(p-f)Q$ to the manufacturer) serves the purpose equally well because any quantity from a forward can always be reversed in the spot market. As a result, as shown in (3), the optimal production level q^* is always independent of the forward contract (f, Q), and thus in the presence of a spot market a forward contract does not affect time 1 production decisions. This demonstrates that one of the important functions of the component spot market is to change the physical forward contract (i.e., involving physical delivery) into a purely financial contract so that production decisions at time 1 are independent of the supply contract. This result remains valid for more general supply contracts whose payoffs only depend on time 1 exogenous variables. Note that the fact that the manufacturer makes his time 1 production decision after the realization of the market uncertainties provides him with a real option $(z+g-p)^+D$ of satisfying demand only when prices are favorable to him. This observation will be helpful in the numerical analysis in §5.

It follows that, given a forward contract (f, Q), the manufacturer's utility $U_m(f, Q)$ is given by

$$U_m(f, Q) = \mathrm{E}[\pi_m] - \lambda_m \operatorname{var}[\pi_m], \quad \text{where} \quad (5)$$

$$\mathrm{E}[\pi_m] = \mathrm{E}[\pi_{m0}] + (\mu_p - f)Q - F_m 1_{\{Q\neq 0\}},$$

$$\operatorname{var}[\pi_m] = \operatorname{var}[\pi_{m0}] + \sigma_p^2 Q^2 - 2Q\mathrm{COV}_m, \quad \text{and} \quad (6)$$

$$\mathrm{COV}_m \equiv -\operatorname{cov}(\pi_{m0}, (p-f)) = \operatorname{cov}((z+g-p)^+D - gD, -p), \quad (7)$$

and cov is the covariance operator given time 1 distributions of random variables p_i, p, z, D.

Setting $Q = 0$ in expressions (4)–(6), we can easily obtain the manufacturer's utility without a forward contract, denoted as U_{m0}. The manufacturer's utility gain from the forward contract (f, Q) can then be written as

$$\widetilde{U}_m(f, Q) = U_m(f, Q) - U_{m0} = (\mu_p - f)Q - F_m 1_{\{Q\neq 0\}} - \lambda_m \Delta\operatorname{var}_m, \quad (8)$$

where

$$\Delta\operatorname{var}_m \equiv \operatorname{var}[\pi_m] - \operatorname{var}[\pi_{m0}] = \sigma_p^2 Q^2 - 2Q\,\mathrm{COV}_m. \quad (9)$$

Equation (8) suggests that the financial profit $(p-f)Q - F_m 1_{\{Q\neq 0\}}$ from a forward contract affects the manufacturer's utility in two ways: (1) It provides an expected financial profit of $(\mu_p - f)Q - F_m 1_{\{Q\neq 0\}}$, and (2) it changes the profit variance by $\Delta\operatorname{var}_m$. Equation (9) implies that a forward contract serves as a hedge against the operational profit and thus reduces the manufacturer's total profit risk if the covariance $(-Q\,\mathrm{COV}_m)$ between the operational profit π_{m0} and the financial profit is negative enough to offset the financial profit variance $\sigma_p^2 Q^2$. Because the hedging benefit of a forward per unit of the component comes only from COV_m (which, for the convenience of exposition, is defined as the negative of the covariance between the manufacturer's operational profit and the per-unit financial profit), we will refer to COV_m as the forward's *marginal hedging benefit* to the manufacturer. Because the forward quantity $Q^*_{mv} \equiv \mathrm{COV}_m/\sigma_p^2$ maximizes the manufacturer's profit variance reduction to the level of $\Delta\operatorname{var}_m(Q^*_{mv}) = -\mathrm{COV}_m^2/\sigma_p^2$, Q^*_{mv} can be interpreted as the manufacturer's *financial hedging demand* (from a forward contract). It can easily be seen that if the component spot price p were not random, then the manufacturer's financial hedging demand Q^*_{mv} would be zero.

Now we consider the supplier's utility gain from a forward. Again, we start with the case of with-forward contract (f, Q). Let π_s be the supplier's time 1 profit and i_0 be the amount of the input factor purchased at time 0. Then at time 1, the supplier chooses the component production level k to maximize her total profit, which is equal to the revenue from selling k units to the spot market pk, minus the total production cost $p_i k + w(k)$, plus the revenue from trading time 0 inventory in the input spot market $(p_i - c_0)i_0$, plus the realized profit from the forward contract $(f-p)Q - F_s 1_{\{Q\neq 0\}}$. That is,

$$\pi_s = \max_{k\geqslant 0}\{pk - p_i k - w(k)\} + (p_i - c_0)i_0 + \big[(f-p)Q - F_s 1_{\{Q\neq 0\}}\big]. \quad (10)$$

The strict convexity of $w(k)$ implies the strict concavity of $pk - p_i k - w(k)$ in k, and thus we have the optimal

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component production

$$k^* = w'^{-1}((p-p_i)^+),$$

where $w'^{-1}(\cdot)$ is the inverse function of the first derivative of $w(\cdot)$. The fact that the supplier makes a time 1 production decision after the realization of spot prices provides her with a real option $(p-p_i)^+$ of not producing in an unfavorable price environment. This observation is helpful for understanding the effects of price volatility and cross-commodity price correlation to be studied in §5.

Let $\pi_{s00}(p, p_i) \equiv (p-p_i)^+k^* - w(k^*)$ be the supplier's time 1 profit from producing the component. Then,

$$\pi_s = \pi_{s00} + (p_i - c_0)i_0 + \big[(f-p)Q - F_s 1_{\{Q\neq 0\}}\big]. \tag{11}$$

Similar to the manufacturer, Equation (11) suggests that the supplier's time 1 total profit is equal to the operational profit from producing the component and clearing the input factor inventory plus the financial profit from the forward. In addition, the optimal production decision k^* is independent of both the input factor inventory decision and the forward contract.

Back to time 0, the supplier chooses i_0 to maximize utility

$$U_s(f, Q) = \max_{i_0}\{\mathrm{E}[\pi_s] - \lambda_s \operatorname{var}[\pi_s]\},$$

where

$$\mathrm{E}[\pi_s] = \mathrm{E}[\pi_{s00}] + (f-\mu_p)Q - F_s 1_{\{Q\neq 0\}},$$

$$\begin{aligned}\operatorname{var}[\pi_s] &= \operatorname{var}[\pi_{s00}] + \sigma_{p_i}^2 i_0^2 + \sigma_p^2 Q^2 - 2\,\mathrm{COV}_i\, i_0\\ &\quad - 2Q\,\mathrm{COV}_s - 2Q i_0 \rho\sigma_p\sigma_{p_i},\end{aligned} \tag{12}$$

$$\begin{aligned}\mathrm{COV}_s &\equiv -\operatorname{cov}(\pi_{s00}, (f-p))\\ &= \operatorname{cov}((p-p_i)^+k^* - w(k^*), p),\end{aligned} \tag{13}$$

$$\begin{aligned}\mathrm{COV}_i &\equiv -\operatorname{cov}(\pi_{s00}, (p_i-c_0))\\ &= \operatorname{cov}((p-p_i)^+k^* - w(k^*), -p_i),\end{aligned} \tag{14}$$

and ρ represents the correlation between p and p_i.

Straightforward derivation yields the time 0 optimal input factor inventory level as

$$i_0^*(Q) = \frac{\mathrm{COV}_i + Q\rho\sigma_p\sigma_{p_i}}{\sigma_{p_i}^2} = i_{00}^* + Q\rho\sigma_p/\sigma_{p_i}, \tag{15}$$

where $i_{00}^* \equiv \mathrm{COV}_i/\sigma_{p_i}^2$ represents the optimal input inventory level in the absence of a forward contract. This shows that although a forward contract does not affect the supplier's production decision, it does change her input inventory decision at time 0 if the component spot price p and the input spot price p_i are correlated. In addition, when $Q>0$ and $\rho>0$ (see Corollary 1), having a forward contract increases input inventory because the inventory risk is reduced by the forward.

We now briefly discuss the role of the input spot market without a forward contract. Let $\pi_{s0} \equiv \pi_{s00} + (p_i - c_0)i_{00}^*$ denote the supplier's time 1 profit without a forward contract. Recall that the assumption $\mu_{p_i} = c_0$ implies that the supplier does not make a positive profit in expectation from any input inventory. This assumption allows us to focus on the hedging benefit offered by input inventory. In particular, it can be shown that $\operatorname{var}[\pi_{s0}] - \operatorname{var}[\pi_{s00}] = -\mathrm{COV}_i^2/\sigma_{p_i}^2$. This implies that input inventory can serve as a hedge against the production profit π_{s00}. We will refer to COV_i as the supplier's *marginal hedging benefit from input inventory*.

We now turn to the impact of the forward contract on the supplier's utility. Let U_{s0} be the supplier's utility in the without-forward case. The supplier's utility gain from forward (f, Q) is then

$$\begin{aligned}\widetilde{U}_s(f, Q) &= U_s(f, Q) - U_{s0}\\ &= (f-\mu_p)Q - F_s 1_{\{Q\neq 0\}} - \lambda_s \Delta \operatorname{var}_s,\end{aligned} \tag{16}$$

where

$$\begin{aligned}\Delta \operatorname{var}_s &\equiv \operatorname{var}[\pi_s] - \operatorname{var}[\pi_{s0}]\\ &= \sigma_{p_i}^2(i_0^{*2} - i_{00}^{*2}) + \sigma_p^2 Q^2 - 2\,\mathrm{COV}_i(i_0^* - i_{00}^*)\\ &\quad - 2Q\mathrm{COV}_s - 2Q i_0^* \rho\sigma_p\sigma_{p_i} \qquad (17)\\ &= (1-\rho^2)\left[\sigma_p^2 Q^2 - 2Q\frac{\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}}{1-\rho^2}\right]. \qquad (18)\end{aligned}$$

Equation (16) suggests that a forward affects the supplier's utility by providing an expected financial profit and a change in the profit variance, as for the manufacturer. Equation (18) implies that a forward can serve as a hedge for the supplier against the production profit if the covariance $(-Q\,\mathrm{COV}_s)$ between the production profit and the financial profit is negative enough. In addition, the forward can also serve as a hedge against the profit from the input factor inventory if the correlation between the component spot price and the input spot price ρ is nonzero. This latter hedging benefit comes from two sources. First, the covariance between the financial profit and input inventory profit reduces the inventory profit variance by the last term in Equation (17). Second, given the forward contract, the supplier also optimally adjusts her input inventory level (as indicated by (15)), which in turn results in a greater hedging benefit of the input inventory against the production profit. Therefore, for the supplier, a forward can serve as a hedge against both the production profit and the input inventory profit.

Because the forward quantity

$$Q_{sv}^* \equiv \frac{\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}}{(1-\rho^2)\sigma_p^2} \quad (\text{for } \rho^2 \neq 1)$$

maximizes the supplier's profit risk reduction to the level of

$$-\Delta \operatorname{var}_s(Q_{sv}^*) = \frac{(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})^2}{(1-\rho^2)\sigma_p^2},$$

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Q^*_{sv} will be referred to as the supplier's *financial hedging demand*.[7] Clearly, the term $(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})/(1-\rho^2)$ represents the sum of the forward's hedging benefits against the production profit and the input inventory profit and will therefore be referred to as the forward's *marginal hedging benefit* to the supplier. Similar to the manufacturer, the driving factor in the supplier's financial hedging demand is the component price volatility because a nonrandom component spot price would imply $Q^*_{sv} = 0$.

We next provide the solution to the Nash bargaining game (1). Let $\widetilde{U}^*_m$, $\widetilde{U}^*_s$, and $\widetilde{U}^*$ denote the manufacturer's, the supplier's, and the total utility gains at the equilibrium, respectively. Then, we have the following result.

Theorem 1. *There exists a unique equilibrium* (f^*, Q^*) *for the Nash bargaining game* (1), *where*

$$Q^* = \begin{cases} \dfrac{\lambda_m \mathrm{COV}_m + \lambda_s(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})}{(\lambda_m + \lambda_s(1-\rho^2))\sigma_p^2} \\ \quad \text{if } |\lambda_m \mathrm{COV}_m + \lambda_s(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})| \\ \quad > \sqrt{(\lambda_m + \lambda_s(1-\rho^2))\sigma_p^2(F_m + F_s)}, \\ 0 \quad \text{otherwise}, \end{cases} \tag{19}$$

and

$$f^* = (1-\theta)[\mu_p - \lambda_m(\sigma_p^2 Q^* - 2\mathrm{COV}_m) - F_m/Q^*] + \theta[\mu_p + \lambda_s((1-\rho^2)\sigma_p^2 Q^* - 2(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})) + F_s/Q^*] \tag{20}$$

if $Q^* \neq 0$. *Moreover,*

$$\widetilde{U}^*_m = \theta\widetilde{U}^*, \qquad \widetilde{U}^*_s = (1-\theta)\widetilde{U}^*, \tag{21}$$

where

$$\widetilde{U}^* = (\lambda_m + \lambda_s(1-\rho^2))\sigma_p^2 Q^{*2} - (F_m + F_s)1_{\{Q^* \neq 0\}}.$$

We provide rigorous proofs of all analytical results (including Theorem 1) in the appendix. Here we only outline the basic idea for the proof of Theorem 1, which is helpful for understanding the main results. Consider the following Nash bargaining game,

$$\max_{(f,Q)} \widetilde{U}_m(f,Q)^\theta \widetilde{U}_s(f,Q)^{1-\theta},$$

where $\widetilde{U}_m$ and $\widetilde{U}_s$ are general utility gain functions. It can be easily verified that the solution is characterized by the first-order conditions

$$\theta\widetilde{U}_s(f,Q)\widetilde{U}_{mf}(f,Q) + (1-\theta)\widetilde{U}_m(f,Q)\widetilde{U}_{sf}(f,Q) = 0 \tag{22}$$

and

$$\theta\widetilde{U}_s(f,Q)\widetilde{U}_{mQ}(f,Q) + (1-\theta)\widetilde{U}_m(f,Q)\widetilde{U}_{sQ}(f,Q) = 0, \tag{23}$$

where $\widetilde{U}_{jf} \equiv \partial\widetilde{U}_j/\partial f$ and $\widetilde{U}_{jQ} \equiv \partial\widetilde{U}_j/\partial Q$ for $j \in \{m, s\}$. Because in our model $\widetilde{U}_{mf}(f,Q) = -\widetilde{U}_{sf}(f,Q)$, condition (22) then yields the equilibrium-sharing rule (21), which, combined with (23), implies that the forward quantity Q is chosen to maximize the total utility gain $\widetilde{U} \equiv \widetilde{U}_m + \widetilde{U}_s$ and f is then chosen according to the optimal sharing rule (21). Because of the negotiation costs, it is optimal to have a forward contract only if the total risk-adjusted benefit exceeds the negotiation costs. These considerations yield the optimal forward contract in expressions (19) and (20).

We now provide several key insights from our Nash bargaining model. First, the equilibrium sharing rule (21) holds for any Nash bargaining game as long as the sum of the utility gains is independent of one of the choice variables. This conclusion is implied by the multiplicative nature of the Nash bargaining game, as illustrated above.

Second, because a forward contract is a zero-sum game (i.e., the sum of the realized forward profits of the two parties is always zero) and the forward quantity is chosen to maximize the total utility gain from the variance reduction, it must be the variance reduction benefit of the forward that justifies the wide usage of forward contracts. This conclusion applies to more general supply contracts that are zero-sum games in the presence of spot markets. In contrast, the main role of supply contracts in the absence of spot markets is to make the physical delivery so that production is feasible.

Third, although a forward contract is defined by both the forward quantity and the forward price, the negotiation is only over the forward price and not over the forward quantity. As shown in Theorem 1, regardless of their bargaining power, both agree on the same optimal forward quantity, and bargaining power only affects the optimal forward price. Conditions (22) and (23) imply that this result holds as long as the ratio $\widetilde{U}_{mf}(f,Q)/\widetilde{U}_{sf}(f,Q)$ is independent of the bargaining power θ. This separation of the roles of the contract quantity and contract price simplifies managers' negotiation strategy.

Finally, it can be shown that the equilibrium forward quantity Q^* (if nonzero) is a risk-aversion weighted sum of the manufacturer's and supplier's financial hedging demands, i.e.,

$$Q^* = \frac{\lambda_m}{\lambda_m + (1-\rho^2)\lambda_s} Q^*_{mv} + \frac{(1-\rho^2)\lambda_s}{\lambda_m + (1-\rho^2)\lambda_s} Q^*_{sv}. \tag{24}$$

This observation is helpful for the subsequent hedging and speculation analysis and can serve as a guideline to managers in determining the optimal supply contract quantity. Note that in (24) the supplier's effective risk-aversion coefficient is $(1-\rho^2)\lambda_s$. This is because the supplier's variance increase of σ_p^2 is reduced to $(1-\rho^2)\sigma_p^2$ through the operational hedge from the input inventory.[8] Recall that if the component spot price were not random, the financial hedging demands for a forward from the supplier and the manufacturer would be zero. Expression (24) implies a stronger

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result that there would be no forward contract in equilibrium if the component spot price were not random. This shows that the uncertainty in the component spot price p is the driving force for the viability of a forward contract. The effect of the final product demand and price volatility will be studied in the numerical study in §5.

Because the manufacturer needs to buy the component to produce the final product and faces the risk of a high time 1 spot price, typically to reduce the price risk he should buy a forward, i.e., $Q^* > 0$. However, the following corollary shows that it may be optimal for the manufacturer to sell a forward instead if his operational profit is positively correlated with his financial profit (i.e., $\mathrm{COV}_m < 0$), which can happen when the component spot has significant impact on the final product demand and price.

Corollary 1.

$$Q^* \begin{cases} > 0 & \text{if } \lambda_m \mathrm{COV}_m + \lambda_s(\mathrm{COV}_s + \mathrm{COV}_i \rho\sigma_p/\sigma_{p_i}) \\ & \quad > \sqrt{(\lambda_m + \lambda_s(1-\rho^2))\sigma_p^2(F_m+F_s)} > 0, \\ < 0 & \text{if } \lambda_m \mathrm{COV}_m + \lambda_s(\mathrm{COV}_s + \mathrm{COV}_i \rho\sigma_p/\sigma_{p_i}) \\ & \quad < -\sqrt{(\lambda_m + \lambda_s(1-\rho^2))\sigma_p^2(F_m+F_s)} < 0, \\ = 0 & \text{otherwise.} \end{cases}$$

To help better understand the benefits gained from a forward, we next examine under what conditions a firm's profit variance is reduced and under what conditions a firm's expected profit is increased. Let $\Delta\,\mathrm{var}_m^*$ and $\Delta\,\mathrm{var}_s^*$ denote, respectively, the changes of profit variances $\Delta\,\mathrm{var}_m$ and $\Delta\,\mathrm{var}_s$ in equilibrium. Define

$$\alpha_m \equiv \frac{\mathrm{COV}_m}{(\mathrm{COV}_s + \mathrm{COV}_i \rho\sigma_p/\sigma_{p_i})/(1-\rho^2)}$$

as a measure of the manufacturer's relative marginal hedging benefit (to that of the supplier). It follows that $\alpha_m/(1-\alpha_m)$ (respectively, $1/(1-\alpha_m)$) measures the manufacturer's (respectively, supplier's) marginal hedging benefit relative to the difference of two firms' marginal hedging benefits.

Proposition 1. *Suppose that $Q^* \neq 0$ and let $\widetilde{U}^*$ be as defined in Theorem 1. Then, forward contract (f^*, Q^*):*

(1) *reduces the manufacturer's equilibrium profit variance (i.e., $\Delta\,\mathrm{var}_m^* \leqslant 0$) iff*

$$\frac{\lambda_s(1-\rho^2)}{\lambda_m + \lambda_s(1-\rho^2)} \leqslant \left|\frac{\alpha_m}{1-\alpha_m}\right|;$$

(2) *reduces the supplier's equilibrium profit variance (i.e., $\Delta\,\mathrm{var}_s^* \leqslant 0$) iff*

$$\frac{\lambda_m}{\lambda_m + \lambda_s(1-\rho^2)} \leqslant \left|\frac{1}{1-\alpha_m}\right|;$$

(3) *increases the manufacturer's expected profit (i.e., $(\mu_p - f^*)Q^* - F_m > 0$) iff*

$$\theta > \theta_1 \equiv \frac{-\lambda_m \Delta\,\mathrm{var}_m^*}{\widetilde{U}^*}; \quad \textit{and}$$

(4) *increases the supplier's expected profit (i.e., $(f^* - \mu_p)Q^* - F_s > 0$) iff*

$$1-\theta > 1-\theta_2 \equiv \frac{-\lambda_s \Delta\,\mathrm{var}_s^*}{\widetilde{U}^*}.$$

Parts (1) and (2) of Proposition 1 provide necessary and sufficient conditions under which the equilibrium forward contract reduces profit variance for a participant. Intuitively, to maximize the total variance reduction, it is more efficient to reduce the profit variance for the party who has higher risk aversion or receives a higher marginal hedging benefit from the forward. Accordingly, Proposition 1 suggests that when the marginal hedging benefit or the risk aversion of the manufacturer (supplier) is high, the equilibrium forward indeed reduces the profit variance of the manufacturer (supplier). Although unlikely, it is possible, as will be seen in the discussion of Table 1, that the total variance minimization leads to one party's profit variance reduction at the cost of the other party's variance increase.

Parts (1) and (2) of Proposition 1 also imply that whether the equilibrium forward contract reduces profit risk for a participant is independent of the bargaining power. This is because the bargaining power, as we have discussed in Theorem 1, only affects the equilibrium forward price and the forward price does not affect profit risk.

Parts (3) and (4) of Proposition 1 state the intuitive result that the party who has a market power above a certain threshold earns a positive expected profit (also called a "speculation benefit") from the forward. It is worthwhile to note the following somewhat obvious implication of Proposition 1: If a participant's profit risk is not reduced in equilibrium (e.g., $\Delta\,\mathrm{var}_m^* > 0$), then this participant has to earn a speculation benefit, i.e., make a positive expected profit from the forward.

Table 1 provides a complete characterization of the hedging and speculation analysis of the forward contract implied by Proposition 1. Case 1 represents an equilibrium in which the forward reduces the manufacturer's profit variance at the cost of increasing the supplier's profit variance, and thus the supplier is compensated by a positive expected payoff from the forward contract. In this case, the manufacturer uses the forward contract to hedge and the supplier uses it to speculate. This case occurs when both the magnitude of the marginal hedging benefit and risk aversion are relatively large for the manufacturer. The opposite scenario, but similar intuition, is represented by Case 2. In these two cases, some firms use supply contracts to speculate on the spot price risk rather than smooth out payoffs. This is a manifestation of the well-known role of derivative contracts such as forwards for risk sharing

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Table 1. Hedging and speculation benefits analysis for $Q^* \neq 0$.

	Manufacturer			Supplier		
Case	$\Delta\,\mathrm{var}_m^*$	$(\mu_p - f^*)Q^* - F_m$	H and/or S	$\Delta\,\mathrm{var}_s^*$	$(f^* - \mu_p)Q^* - F_s$	H and/or S
1	−	−	H	+	+	S
2	+	+	S	−	−	H
3	−	+	HS	−	−	H
4	−	−	H	−	+	HS
5	−	−	H	−	−	H

Note. H (S) means that the forward provides hedging (speculation) benefit to the participant. Case 1:

$$\frac{\lambda_s(1-\rho^2)}{\lambda_m+\lambda_s(1-\rho^2)} < \left|\frac{\alpha_m}{1-\alpha_m}\right| \quad \text{and} \quad \frac{\lambda_m}{\lambda_m+\lambda_s(1-\rho^2)} > \left|\frac{1}{1-\alpha_m}\right|;$$

Case 2:

$$\frac{\lambda_s(1-\rho^2)}{\lambda_m+\lambda_s(1-\rho^2)} > \left|\frac{\alpha_m}{1-\alpha_m}\right| \quad \text{and} \quad \frac{\lambda_m}{\lambda_m+\lambda_s(1-\rho^2)} < \left|\frac{1}{1-\alpha_m}\right|;$$

Case 3:

$$\frac{\lambda_s(1-\rho^2)}{\lambda_m+\lambda_s(1-\rho^2)} < \left|\frac{\alpha_m}{1-\alpha_m}\right|, \quad \frac{\lambda_m}{\lambda_m+\lambda_s(1-\rho^2)} < \left|\frac{1}{1-\alpha_m}\right|, \quad \text{and} \quad \theta > \theta_1;$$

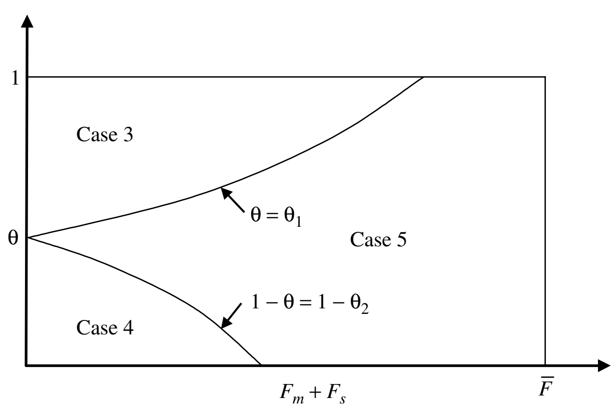
Case 4:

$$\frac{\lambda_s(1-\rho^2)}{\lambda_m+\lambda_s(1-\rho^2)} < \left|\frac{\alpha_m}{1-\alpha_m}\right|, \quad \frac{\lambda_m}{\lambda_m+\lambda_s(1-\rho^2)} < \left|\frac{1}{1-\alpha_m}\right|, \quad \text{and} \quad 1-\theta > 1-\theta_2;$$

Case 5: otherwise.

among risk-averse participants. However, this is in contrast to the fact that most firms, in practice, do not use supply contracts solely for speculation. It shows a limitation of the model with mean-variance risk-averse agents trading derivative contracts. On the other hand, Cases 1 and 2 are extreme cases that happen only when the marginal hedging benefits and the risk aversions differ greatly between the two negotiating firms. Therefore, this model is better suited

Figure 2. Illustration of Cases 3, 4, and 5 in Table 1, where $\theta_1 = \lambda_m\Delta\,\mathrm{var}_m^*/(\lambda_m\Delta\,\mathrm{var}_m^* + \lambda_s\Delta\,\mathrm{var}_s^* + F_m + F_s)$, $\theta_2 = (\lambda_m\Delta\,\mathrm{var}_m^* + F_m + F_s)/(\lambda_m\Delta\,\mathrm{var}_m^* + \lambda_s\Delta\,\mathrm{var}_s^* + F_m + F_s)$, and $\bar{F} = -(\lambda_m\Delta\,\mathrm{var}_m^* + \lambda_s\Delta\,\mathrm{var}_s^*)$.



to those instances where the marginal hedging benefits and the risk aversions are not drastically different across the two negotiating firms, and thus speculation does not play a major role in the negotiation. Cases 3, 4, and 5 fall into this category. In these cases, both parties enjoy the hedging benefits from the forward. Whether a party earns a positive expected profit from the forward depends on whether the party's market power is large enough and whether the negotiation costs are small enough (as illustrated in Figure 2). When the negotiation cost is very high, regardless of the market power, both participants lose in expectation from the forward and both use the forward contract solely for the hedging purposes (Case 5 in Table 1).

Next, we provide comparative statics with respect to the relative market power θ and risk-aversion coefficients λ_m and λ_s on the equilibrium contract. These comparative statics are independent of distribution assumptions. The comparative statics with respect to the volatilities of the spot market price, the final product sale price, and the final product demand; and with respect to the correlations between those random variables, are distribution dependent and will be discussed through a numerical study in the next section.

Proposition 2. (1) *Q^* is independent of θ; f^* decreases in θ iff $Q^* > 0$; $\widetilde{U}_m^*$ increases in θ; and $\widetilde{U}_s^*$ decreases in θ.*

(2) *Q^* increases in λ_m iff*

$$\frac{\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}}{1-\rho^2} < \mathrm{COV}_m;$$

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$\widetilde{U}_m^*$ and $\widetilde{U}_s^*$ increase in λ_m iff $\Delta\operatorname{var}_m^* < 0$; if $F_m = F_s = 0$,[9] *then f^* increases in λ_m iff*

$$\lambda_s^2(1-\rho^2)^2 \cdot \frac{\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}}{1-\rho^2} < \mathrm{COV}_m((1-\theta)(\lambda_m + \lambda_s(1-\rho^2))^2 + \lambda_s^2(1-\rho^2)^2).$$

(3) *Q^* increases in λ_s iff*

$$\frac{\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}}{1-\rho^2} > \mathrm{COV}_m;$$

$\widetilde{U}_m^*$ and $\widetilde{U}_s^*$ increase in λ_s iff $\Delta\operatorname{var}_s^* < 0$; if $F_m = F_s = 0$, then f^* increases in λ_s iff

$$(\theta(\lambda_m + \lambda_s(1-\rho^2))^2 + \lambda_m^2) \cdot \frac{\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}}{1-\rho^2} < \lambda_m^2 \mathrm{COV}_m.$$

As the manufacturer's relative market power increases, the equilibrium-forward price moves in the direction that favors the manufacturer, and as a result the manufacturer's equilibrium utility gain from the forward contract increases. As the manufacturer's risk aversion increases, he is willing to accept a less favorable forward price in exchange for profit risk reduction. Thus, if the manufacturer's marginal hedging benefit is larger than that of the supplier, then the forward quantity will increase to further decrease the manufacturer's profit risk. Both the manufacturer's and the supplier's utility gains from the forward contract increase with the manufacturer's risk aversion if he hedges with the forward contract. This is because as the manufacturer's risk aversion increases, hedging improves his utility more and the supplier shares this additional improvement through negotiation. For the forward price, if the manufacturer's marginal hedging benefit COV_m is large, then he is willing to buy the forward at a higher price as his risk aversion increases. The intuitions for the comparative statics on the supplier's risk-aversion coefficient are similar to those for the manufacturer.

5. Numerical Analysis

In the previous section, we presented analytical results on the properties of the equilibrium forward contracts for general distributions of the market uncertainties. As shown in Theorem 1, equilibrium forward contracts also depend on distribution-specific parameters, such as price and demand volatilities and correlations. Understanding how these parameters affect the optimal forward contract would provide useful guidelines for managers to determine optimal supply contracts and optimal hedging strategies in practice. Unfortunately, given the high nonlinearity of reasonable distribution functions (such as the log-normal distributions used below), analytical comparative statics seem infeasible. We therefore conduct an extensive numerical analysis in this section. In particular, we will focus on the impact on the equilibrium forward contract (f^*, Q^*), the effectiveness of the financial hedge $-\Delta\operatorname{var}_m^*/\operatorname{var}[\pi_{m0}]$ and $-\Delta\operatorname{var}_s^*/\operatorname{var}[\pi_{s0}]$, and the hedge ratios Q^*/μ_D and $Q^*/E[k^*]$.

For all the numerical analysis in this section, we assume that at time 1 the input price for the supplier p_i, the component spot price p, the quantity demanded D, and the price z for the manufacturer's final product, are all log-normally distributed. These assumptions on the distributions guarantee the nonnegativeness of the prices and the demand. Let ρ, ρ_{pD}, ρ_{pz}, and ρ_{Dz} be the respective correlations among p_i, p, D, and z. We then truncate these log-normally distributed random variables at the 97.5% quantile to ensure that utility satiation is not reached for any participant.

Similar to Bessembinder and Lemmon (2002), we assume the following cost function $C(k)$ for the supplier:

$$C(k) = w(k) + p_i k = a1_{\{k>0\}} + \frac{b}{c}k^c + p_i k, \quad c \geqslant 2.$$

This cost function, as Bessembinder and Lemmon argue, allows flexibility to account for complexities that are not formally modeled, such as the use of an inefficient plant to meet high level of demand, as well as the capacity constraint.

We first provide a base case for subsequent analysis. In the base case, we take $\mu_{p_i} = 0.2$, $\sigma_{p_i} = 0.05$, $\mu_p = \mu_D = 1.0$, $\sigma_p = \sigma_D = 0.3$, $\mu_z = 1.5$, $\sigma_z = 0.1$, $g = 0.1$, $a = 0$, $b = 1$, and $c = 2$. We assume zero correlations among all uncertainties (i.e., $\rho = \rho_{pD} = \rho_{Dz} = \rho_{pz} = 0$), which allows us to best isolate the effect of a particular factor by avoiding the convoluted effects of the correlations among these uncertainties. We also assume identical risk-aversion coefficients ($\lambda_m = \lambda_s = 0.02$) for the manufacturer and the supplier to prevent any distorted effect from asymmetric parameter values. Because the fixed cost of negotiation only determines the hedging benefit threshold above which a forward contract is negotiated, but does not affect the qualitative insights for sensitivity analysis, we assume zero fixed cost of negotiation ($F_m = F_s = 0$) throughout the numerical study in this section for simplicity.

The base case belongs to Case 3 described in Table 1, i.e., participating in the forward contract reduces the profit variances of both the manufacturer and the supplier, and in addition it increases the expected profit of the manufacturer. A wide range of parameter values around the base case imply that both the manufacturer and the supplier gain hedging benefits from the equilibrium forward, and thus suggests that except for extreme values, we normally have Cases 3, 4, or 5 in Table 1. In the base case, both the manufacturer's and the supplier's marginal hedging benefits are positive (i.e., $\mathrm{COV}_m > 0$ and $\mathrm{COV}_s > 0$), which implies that both the manufacturer and the supplier have positive financial hedging demand, i.e., $Q_{mv}^* > 0$ and $Q_{sv}^* > 0$. By (24), we have $Q_{mv}^* < Q^* < Q_{sv}^*$, and the manufacturer is the buyer and the supplier is the seller of the forward. The

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financial hedge from the forward is effective, with variance reductions of 62.7% ($-\Delta\,\mathrm{var}^*_m\,/\,\mathrm{var}[\pi_{m0}]$) for the manufacturer and 93.2% ($-\Delta\,\mathrm{var}^*_s/\,\mathrm{var}[\pi_{s0}]$) for the supplier. Furthermore, the hedge ratio at equilibrium is less than one for the manufacturer ($Q^*/\mu_D = 0.918$), but greater than one for the supplier ($Q^*/E[k^*] = 1.148$). This is because the embedded option $(z+g-p)^+$ in the manufacturer's operational profit function already hedges away some downside spot-price risk for the manufacturer, and thus he needs less of the financial hedge from the forward; but the supplier's production level k^* is positively correlated with her profit margin $(p-p_i)^+$, which makes her operational profit highly risky, and thus she needs more of the financial hedge from the forward.

The sequence of the comparative static study in the remainder of the section is as follows: the impact of the price volatilities in three markets, the impact of cross-market price correlations, and finally, the impact of the supplier's production cost. Qualitative results from these analyses are summarized in Table 2.

5.1. Changes in Risks

Given that the base case assumes zero correlation between market prices, one might expect that the changes of price volatilities in the input factor spot market and the final product market do not affect the equilibrium forward contract. We find, to the contrary, that price volatilities in both of these markets do affect the equilibrium contract, and their effects are different. This is mainly due to the interplay between the forward payoff and the existing embedded options in the manufacturer's and the supplier's operational profits. Recall that the manufacturer has an embedded option $(z+g-p)^+$ in his operational profit that can be viewed either as a call option on the final product price z, or as a put option on the component spot price p. Similarly, the supplier has an embedded option $(p-p_i)^+$ in her operational profit that can be viewed either as a call option on the component spot price p or as a put option on the input factor spot price p_i. As the price volatilities in spot markets and the final product market increase, these options become more valuable. To understand the difference in their effects, we provide figures on the equilibrium forward contract f^* and Q^*, on the effectiveness of the financial hedge $-\Delta\,\mathrm{var}^*_m/\mathrm{var}[\pi_{m0}]$ and $-\Delta\,\mathrm{var}^*_s/\mathrm{var}[\pi_{s0}]$, and on the hedge ratios Q^*/μ_D and $Q^*/E[k^*]$. These results are also summarized in Table 2 together with results on the equilibrium utility gain $\widetilde{U}^*_m$, the profit variances without a forward contract $\mathrm{var}[\pi_{m0}]$ and $\mathrm{var}[\pi_{s0}]$, and the magnitude of the financial hedge $-\Delta\,\mathrm{var}^*_m$ and $-\Delta\,\mathrm{var}^*_s$.

Figure 3 plots the changes of equilibrium as functions of the final product price risk σ_z. As σ_z increases, the value of the call option $(z+g-p)^+$ increases for the manufacturer, resulting in a less negative covariance between the call payoff and the unit forward payoff $p-f$. Thus, the increase of σ_z decreases the forward's marginal hedging benefit for the manufacturer. The manufacturer requires less forward quantity and prefers lower forward price. As a result, both the equilibrium forward quantity and price decrease. This leads to a decrease in the hedge ratios and the effectiveness of the financial hedge for both the manufacturer and the supplier.

Figure 4 plots the equilibrium impact of the component spot-price risk σ_p. It shows that both the forward price and the forward quantity are nonmonotonically affected by this price risk. For the manufacturer, as σ_p^2 increases, initially the hedging benefit increases because of the increase in the uncertainty. However, as σ_p^2 increases, the value of the put option $(z+g-p)^+$ also increases, which can reduce the forward's marginal hedging benefit. Thus, the manufacturer's financial hedging demand Q^*_{mv} increases initially and then decreases. For the supplier, her optimal production level k^* is positively related to the realized profit marginal $(p-p_i)^+$, and her profit increases faster than the component price p. Thus, the supplier's financial hedging demand

Table 2. Impact of price risks, cross-market price correlations, and the magnitude of supplier's production cost.

	Contract			Profit variance without forward		Financial hedge		Effectiveness of financial hedge		Hedge ratio	
	f^*	Q^*	$\widetilde{U}^*_m$	$\mathrm{var}[\pi_{m0}]$	$\mathrm{var}[\pi_{s0}]$	$-\Delta\,\mathrm{var}^*_m$	$-\Delta\,\mathrm{var}^*_s$	$-\Delta\,\mathrm{var}^*_m\,/\,\mathrm{var}[\pi_{m0}]$	$-\Delta\,\mathrm{var}^*_s\,/\,\mathrm{var}[\pi_{s0}]$	Q^*/μ_D	$Q^*/E[k^*]$
Price risks											
σ_z	↓	↓	↓	↑	–	↓	↓	↓	↓	↓	↓
σ_p	∼	∼	↑	↑	↑	↑	↑	∼	↓	∼	∼
σ_{p_i}	↓	↑	↑	–	↑	↓	↑	↓	↓	↑	↓
Correlation coefficients											
ρ	↑	↓	↓	–	↓	↑	↓	↑	↓	↓	↓
ρ_{pz}	↓	↓	↓	↓	–	↓	↓	↓	↓	↓	↓
ρ_{Dz}	↓	↓	↓	↓	–	↓	↓	↑	↓	↓	↓
Magnitude of supplier's production cost											
b	↑	↓	↓	–	↓	∼	↓	↑	∼	↓	↑

Note. ↑, ↓ –, ∼ represent, respectively, the increasing, decreasing, independent, and nonmonotonic trend observed at the equilibrium as the parameter increases. ρ_{Ds} decreases from zero.

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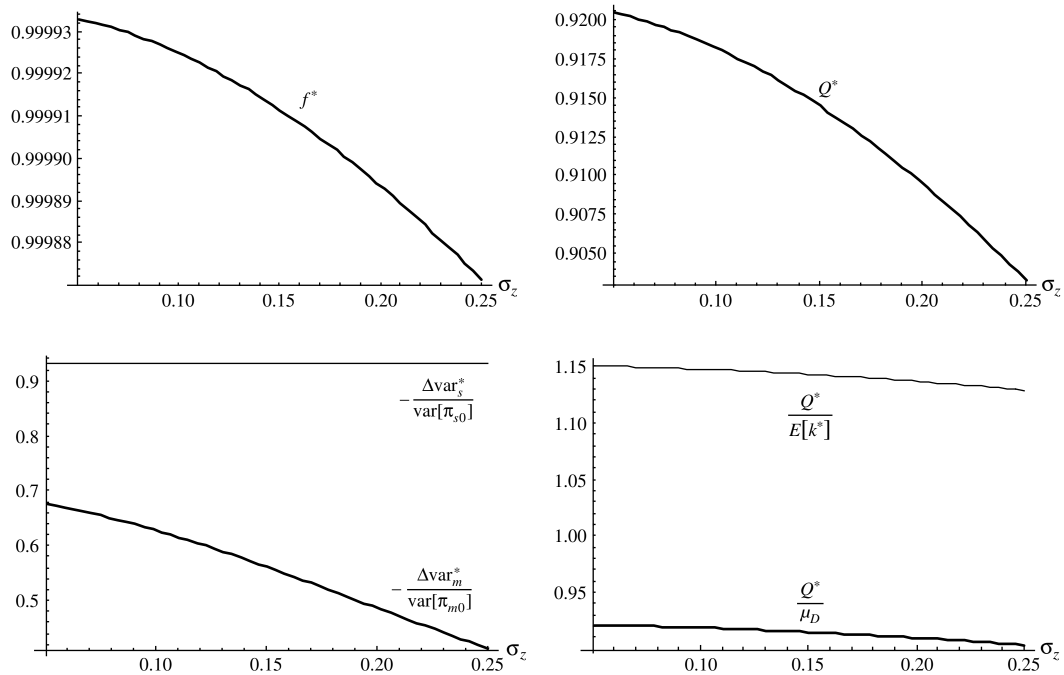
Figure 3. Change in σ_z.



Q_{sv}^* increases in σ_p^2. It is this difference in the hedging-demand sensitivity pattern that drives the nonmonotonicity of the forward price and the forward quantity in the component price risk.

Figure 5 plots the equilibrium impact of the input spot-price risk σ_{p_i}. As σ_{p_i} increases, the value of the put option $(p-p_i)^+$ increases for the supplier, which makes her operational profit more positively correlated with the component spot price p, resulting in a higher marginal hedging benefit to the supplier. Thus, the increase of σ_{p_i} increases the supplier's hedging demand Q_{sv}^*. Through his market power, the manufacturer can then negotiate the equilibrium forward price downward. On the other hand, because the increase of σ_{p_i} increases the variance of operational profit for the supplier, eventually the effectiveness of the financial hedge decreases for the supplier. The increase of the option value of $(p-p_i)^+$ makes the supplier produce more components on average, i.e., leading to an increase of $E[k^*]$.

Figure 4. Change in σ_p.

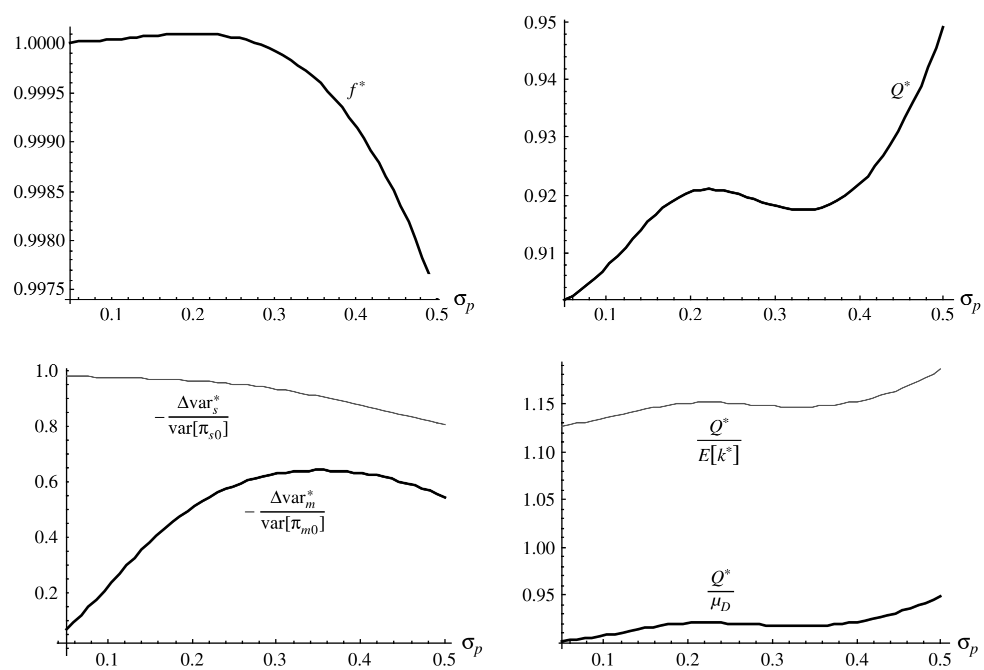


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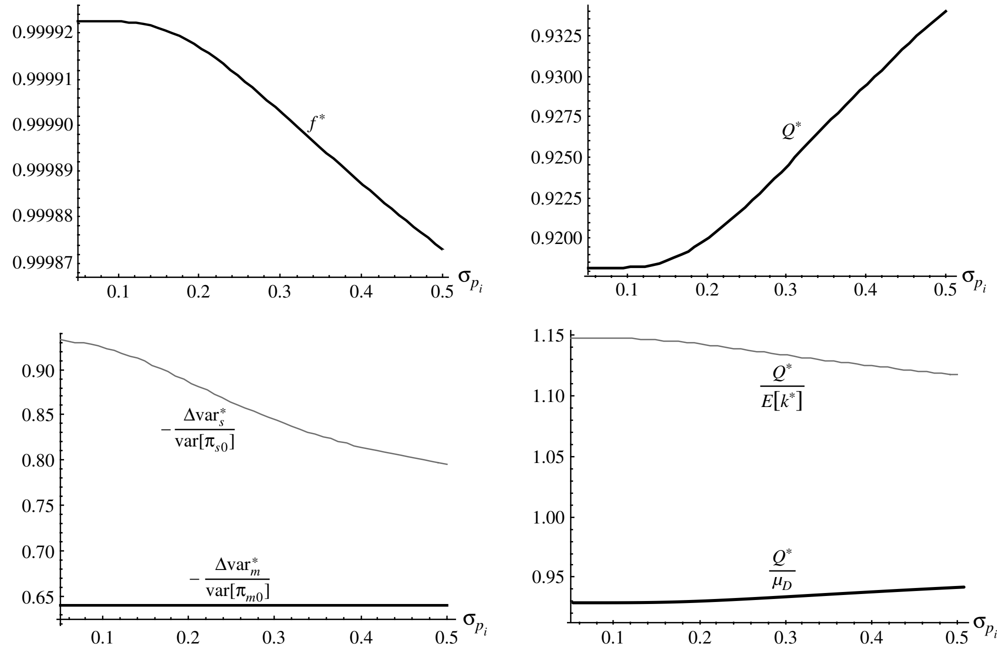
Figure 5. Change in σ_{p_i}.



Thus, although Q^* increases, the supplier's hedge ratio decreases.

5.2. Changes in Correlations

We next examine how changes in price correlations between markets and correlation between price and demand in the final product market affect the equilibrium forward contract. We will see that as prices become more positively correlated and demand becomes more negatively correlated with price, natural hedges (a natural hedge is the reduction in risk that can arise from an institution's normal operating procedures) arise to reduce the manufacturer's and the supplier's needs for financial hedge.

As the correlation ρ between the spot prices of the input factor and the component increases, the variation in the supplier's profit margin $(p-p_i)^+$ decreases. In other words, the supplier has a natural hedge in place whose hedging effect increases as the correlation between her storable input spot price and the nonstorable output (i.e., the component) spot price increases. The increase in the effectiveness of the natural hedge decreases the marginal hedging benefit of a forward contract for the supplier, and thus she requires a smaller forward quantity and a higher forward price, which leads to a decrease of the equilibrium forward quantity and an increase of the equilibrium forward price.

The manufacturer's profit depends on the product of the demand D and the profit margin $(z+g-p)^+$. Thus, similar to the supplier, he also has a natural hedge in operation if the correlation ρ_{pz} between p and z is positive or the correlation ρ_{Dz} between D and z is negative. In addition, the effectiveness of the natural hedge for the manufacturer increases as ρ_{pz} increases from zero and as ρ_{Dz} decreases from zero, which implies a decrease in the marginal hedging benefit from a forward for the manufacturer. Therefore, his financial hedging demand decreases, which in turn causes a decrease in the equilibrium-forward quantity. Moreover, the forward price must be lowered to entice the manufacturer to participate in the forward contract. It is worth noting that the effectiveness of the financial hedge from the forward actually increases for the manufacturer as ρ_{Dz} decreases from zero. This is due to the reduction of his operational profit variance from the natural hedge.

Our model can shed some light on the causes of the California electricity crisis in 2000–2001. The then-effective regulatory rules discouraged long-term contracts between utility companies and independent suppliers, and the retail price was largely fixed. As a result, utility companies had to rely on the volatile spot wholesale market to supply the electricity to retail customers and customers were completely insulated from the movement in wholesale prices (see Congressional Budget Office 2001 and Joskow 2001 for more detailed accounts of the California electricity crisis). Our model suggests that when final product demand and price are unaffected by the component spot price and demand always has to be fulfilled, the ideal forward quantity for the manufacturer to reduce his profit variance is equal to the expected demand, i.e., he needs full hedge (i.e., $Q^*_{mv}/\mu_D = 1$), whereas utility companies did not hedge much during that period. The lack of hedging and the extremely high electricity and fuel spot prices in summer 2000 then led directly to major utility companies' insolvency in the beginning of 2001. Our model also indicates that if the freeze on the retail price was removed, the more negative correlation between the retail price and demand and the more positive correlation between the wholesale

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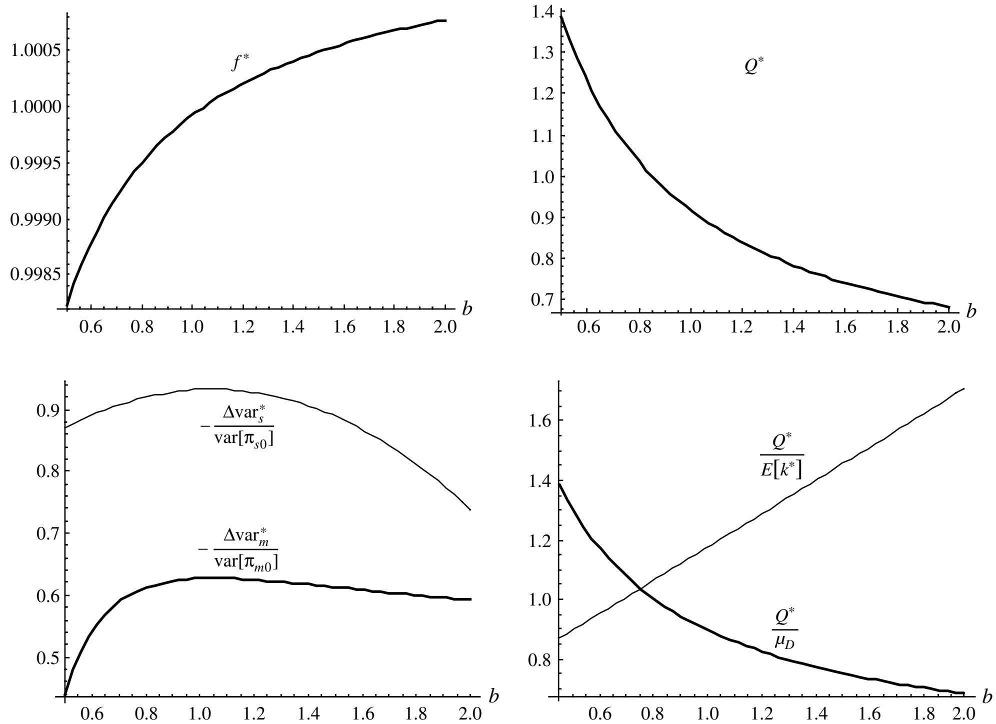
Figure 6. Change in b.



spot price and the retail price would create better natural hedges against profit risks for the utility companies. Consistent with what would be suggested by our analysis, California's government then changed the rules to encourage bilateral long-term contracts between electricity suppliers and distributors, and increased the price cap on the retail price.

5.3. Change in the Supplier's Production Cost

An increase in the supplier's production cost $w(k)$ implies a decrease in the economic storability of the component. Figure 6 plots the equilibrium impact of the supplier's production variable cost parameter b. An increase of production cost, or equivalently, a decrease in the economic storability of the component, decreases the supplier's production and thus reduces the forward quantity needed for hedging. However, the optimal hedge ratio for the supplier increases due to a faster decrease in the production than the equilibrium forward quantity Q^*. Thus, given the same expected production level, a supplier of a component that is less economically storable relies more on a financial (forward) contract to hedge risk.

6. Concluding Remarks

We consider the equilibrium forward contract on a nonstorable commodity in the presence of a spot market. The forward contract is negotiated through a Nash bargaining process between two risk-averse participants who both have market powers. We show the existence and uniqueness of an equilibrium and derive the equilibrium forward contract in closed form. We provide an extensive analysis of the equilibrium forward contract. We find that one potential fundamental driving force for a supply contract in the presence of a spot market is a firm's need for risk hedging. We also show how risk can be reduced, shared, or shifted between firms through supply contracts such as forwards.

For some extreme cases where firms differ greatly in risk aversion and financial hedging demand, our model predicts that some firms would participate in bilateral supply contracts solely for speculation. Although it is a direct result from the mean-variance utility assumption and the nature of derivative contracts, it is in contrast to the fact that most manufacturing firms, in practice, do not use supply contracts solely for speculation. Therefore, our model is better suited to the cases where firms are not drastically different in risk attitude or financial hedging demand, and thus speculation does not play a major role in the negotiation of the supply contracts.

Several extensions can be built upon this model. First, the framework used in the paper can also be applied to study other bilateral contracts such as call and put options. Second, one can combine the framework in our model and the dynamic programming principle to study the negotiation of multiple forward contracts before a given time T. An interesting and related question is how to find the right time for the forward contract negotiation. Our one-period model provides the value function for any choice of the negotiation time. It implies that for the optimal choice of the negotiation time, all qualitative results derived in this

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paper regarding the determination of equilibrium and the optimal producing, hedging, and trading strategies remain the same. To rigorously examine the optimal timing problem, one needs to set up a continuous-time model, explicitly specify the entire stochastic processes for spot prices and the final product demand, and formulate an optimal stopping-time problem, which is beyond the focus of this paper.

Finally, this model only considers bilateral supply contracts in the presence of a liquid spot market and abstracts from the selection process for the particular counterparty. Therefore, it is obviously a reduced form of the actual forward market. An interesting but challenging extension would be to allow multiple sellers and multiple buyers to trade strategically in both the forward and the spot markets (see, for example, Allaz and Vila 1993). In that setting, the introduction of a forward market will have an impact on the market clearing price in the spot market and thus have an impact on each player's profit and risk.

Appendix

In this appendix, we collect the proofs for the analytical results.

Proof of Theorem 1. Taking the partial derivative of $\widetilde{U}_m^{\theta}\widetilde{U}_s^{1-\theta}$ with respect to f, we have

$$\frac{\partial \widetilde{U}_m^{\theta}\widetilde{U}_s^{1-\theta}}{\partial f}=\widetilde{U}_m^{\theta-1}\widetilde{U}_s^{-\theta}\left(\theta\widetilde{U}_s\frac{\partial\widetilde{U}_m}{\partial f}+(1-\theta)\widetilde{U}_m\frac{\partial\widetilde{U}_s}{\partial f}\right). \quad (25)$$

Because

$$\frac{\partial\widetilde{U}_m}{\partial f}=-Q \quad\text{and}\quad \frac{\partial\widetilde{U}_s}{\partial f}=Q,$$

(25) can be rewritten as

$$\widetilde{U}_m^{\theta-1}\widetilde{U}_s^{-\theta}\left(\theta\widetilde{U}_s\frac{\partial\widetilde{U}_m}{\partial f}+(1-\theta)\widetilde{U}_m\frac{\partial\widetilde{U}_s}{\partial f}\right)$$
$$=\widetilde{U}_m^{\theta-1}\widetilde{U}_s^{-\theta}Q(-\theta\widetilde{U}_s+(1-\theta)\widetilde{U}_m).$$

Hence, the first-order condition (FOC) with respect to f is given by

$$\theta\widetilde{U}_s=(1-\theta)\widetilde{U}_m. \quad (26)$$

Because

$$\left.\frac{\partial^2\widetilde{U}_m^{\theta}\widetilde{U}_s^{1-\theta}}{\partial f}\right|_{f=f^*}=-\widetilde{U}_m^{\theta-1}\widetilde{U}_s^{-\theta}Q^2<0,$$

f^* that solves (26) is the unique global maximizer of $\widetilde{U}_m^{\theta}\widetilde{U}_s^{1-\theta}$ for any given Q.

By (26), (8), and (16), for a given Q, the optimal forward price

$$\begin{aligned}f^*=&(1-\theta)\big(\mu_p-\lambda_m(\sigma_p^2Q-2\mathrm{COV}_m)-F_m1_{\{Q\neq0\}}\big)\\&+\theta\big(\mu_p+\lambda_s(\sigma_p^2(1-\rho^2)Q\\&-2(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))+F_s1_{\{Q\neq0\}}\big). \quad (27)\end{aligned}$$

By (26), (1) can be simplified to

$$\max_{f,Q}\ \widetilde{U}_m^{\theta}\widetilde{U}_s^{1-\theta}=\left(\frac{1-\theta}{\theta}\right)^{1-\theta}\max_{Q}\ \widetilde{U}_m(Q).$$

Substituting (27) into (8), we can write $\widetilde{U}_m$ as a single variable function of Q:

$$\begin{aligned}\widetilde{U}_m(Q)=\theta\{&-\lambda_m(\sigma_p^2Q^2-2Q\mathrm{COV}_m)\\&-\lambda_s(\sigma_p^2(1-\rho^2)Q^2-2Q(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))\\&-(F_m+F_s)1_{\{Q\neq0\}}\}. \quad (28)\end{aligned}$$

Because the optimal $\widetilde{U}_m$ can be rewritten as

$$\widetilde{U}_m^*=\max\{0,\tilde{u}_m(Q)\},$$

where

$$\begin{aligned}\tilde{u}_m(Q)=\theta\{&-\lambda_m(\sigma_p^2Q^2-2Q\mathrm{COV}_m)-\lambda_s(\sigma_p^2(1-\rho^2)Q^2\\&-2Q(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))-(F_m+F_s)\},\end{aligned}$$

we take the FOC of $\tilde{u}_m(Q)$,

$$\begin{aligned}\tilde{u}_m'(Q)=\theta\big(&-\lambda_m(2Q\sigma_p^2-2\mathrm{COV}_m)-\lambda_s(2Q(1-\rho^2)\sigma_p^2\\&-2(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))\big)=0.\end{aligned}$$

$\tilde{u}_m'(Q)$ is a decreasing function of Q. Therefore, $\tilde{u}_m'(Q)$ is concave in Q. Clearly,

$$\widehat{Q}=\frac{\lambda_m\mathrm{COV}_m+\lambda_s(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})}{(\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2}$$

satisfies the above FOC. The necessary and sufficient condition for $\widetilde{U}_m^*=\tilde{u}_m(\widehat{Q})$ is $\tilde{u}_m(\widehat{Q})>0$, i.e.,

$$\begin{aligned}&|\lambda_m\mathrm{COV}_m+\lambda_s(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})|\\&\quad>\sqrt{(\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2(F_m+F_s)}.\end{aligned}$$

Equation (27) then implies (20). Plugging (19) into (28) yields the expression for $\widetilde{U}_m$, and then (26) implies the expression for $\widetilde{U}_s$. □

Proof of Corollary 1. Straightforward from (19). □

Proof of Proposition 1. Part (1):

$$\begin{aligned}\Delta\mathrm{var}_m^*&=\sigma_p^2Q^{*2}-2Q^*\mathrm{COV}_m\\&=\big[\sigma_p^2(\lambda_m\mathrm{COV}_m+\lambda_s(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))^2\\&\quad-(2((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)(\lambda_m\mathrm{COV}_m+\lambda_s(\mathrm{COV}_s\\&\quad+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))\mathrm{COV}_m)\big]\\&\quad\cdot(((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)^2)^{-1}\\&=\big[(-\lambda_m^2\mathrm{COV}_m^2\sigma_p^2+\lambda_s^2\sigma_p^2(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})^2\\&\quad-2\lambda_m\lambda_s\mathrm{COV}_m^2(1-\rho^2)\sigma_p^2)\\&\quad-2\lambda_s^2\mathrm{COV}_m(1-\rho^2)\sigma_p^2(\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})\big]\\&\quad\cdot(((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)^2)^{-1}\\&=(\lambda_s^2((\mathrm{COV}_s+\mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})-\mathrm{COV}_m(1-\rho^2))^2\\&\quad-\mathrm{COV}_m^2(\lambda_m+\lambda_s(1-\rho^2))^2)\\&\quad\cdot((\lambda_m+\lambda_s(1-\rho^2))^2\sigma_p^2)^{-1}.\end{aligned}$$

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Part (2):

$$\begin{aligned}\Delta\,\mathrm{var}_s^* &= (1-\rho^2)\sigma_p^2 Q^{*2} - 2Q^*(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})\\ &= (\lambda_m^2((\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}) - \mathrm{COV}_m(1-\rho^2))^2\\ &\quad - (\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})^2(\lambda_m + \lambda_s(1-\rho^2))^2)\\ &\quad \cdot ((\lambda_m + \lambda_s(1-\rho^2))^2(1-\rho^2)\sigma_p^2)^{-1}.\end{aligned}$$

The desired results follow.

Part (3): Because

$$\widetilde{U}_m^* = (\mu_p - f^*)Q^* - F_m - \lambda_m\Delta\,\mathrm{var}_m^* = \theta\widetilde{U}^*,$$

it follows that $(\mu_p - f^*)Q^* - F_m \geqslant 0$ iff $\theta \geqslant -\lambda_m\Delta\,\mathrm{var}_m^*/\widetilde{U}^*$. Part (4) can be proved similarly. □

Proof of Proposition 2. The independence of Q^* w.r.t. θ is obvious. The monotonicity is straightforward from the expressions of Q^*, $\widetilde{U}_m^*$, and $\widetilde{U}_s^*$:

$$\begin{aligned}\frac{df^*}{d\theta} &= \lambda_m(\sigma_p^2Q^* - 2\mathrm{COV}_m) + F_m/Q^* + \lambda_s((1-\rho^2)\sigma_p^2Q^*\\ &\quad - 2(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})) + F_s/Q^* = -\widetilde{U}^*/Q^*.\end{aligned}$$

Comparative statics for Q^*:

$$\frac{dQ^*}{d\lambda_m} = \frac{-\lambda_s(-\mathrm{COV}_m(1-\rho^2)\sigma_p^2 + \sigma_p^2(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))}{((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)^2},$$

$$\frac{dQ^*}{d\lambda_s} = \frac{\lambda_m(-\mathrm{COV}_m(1-\rho^2)\sigma_p^2 + \sigma_p^2(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))}{((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)^2}.$$

When $\mu_{p_i} - c_0 = 0$,

$$\frac{dQ^*}{d\lambda_m} = \frac{-\lambda_s(-\mathrm{COV}_m(1-\rho^2)\sigma_p^2 + \sigma_p^2(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))}{((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)^2},$$

$$\frac{dQ^*}{d\lambda_s} = \frac{\lambda_m(-\mathrm{COV}_m(1-\rho^2)\sigma_p^2 + \sigma_p^2(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))}{((\lambda_m+\lambda_s(1-\rho^2))\sigma_p^2)^2}.$$

Comparative statics for $\widetilde{U}_m^*$: Because $\widetilde{U}_m^* = \theta\widetilde{U}^* = (-\lambda_m\Delta\,\mathrm{var}_m^* - \lambda_s\Delta\,\mathrm{var}_s^* - (F_m+F_s)1_{\{Q^*\neq 0\}})$, it follows that

$$\begin{aligned}\frac{d\widetilde{U}_m^*}{d\lambda_m} &= \theta\frac{\partial\widetilde{U}^*}{\partial\lambda_m} + \theta\frac{\partial\widetilde{U}^*}{\partial Q^*}\frac{\partial Q^*}{\partial\lambda_m}\\ &= -\theta\Delta\,\mathrm{var}_m^* + 0\end{aligned}$$

and

$$\frac{d\widetilde{U}_m^*}{d\lambda_s} = -\theta\Delta\,\mathrm{var}_s^*.$$

Comparative statics for f^*: For $F_m = F_s = 0$,

$$\begin{aligned}\frac{df^*}{d\lambda_m} &= (1-\theta)\left(-(Q^*\sigma_p^2 - 2\mathrm{COV}_m) - \lambda_m\sigma_p^2\frac{dQ^*}{d\lambda_m}\right)\\ &\quad + \theta\lambda_s(1-\rho^2)\sigma_p^2\frac{dQ^*}{d\lambda_m}\\ &= (1-\theta)\Bigg(-(Q^*\sigma_p^2 - 2\mathrm{COV}_m)\\ &\qquad - ((\lambda_m + \lambda_s(1-\rho^2))\sigma_p^2)\frac{dQ^*}{d\lambda_m}\Bigg)\\ &\quad + \lambda_s(1-\rho^2)\sigma_p^2\frac{dQ^*}{d\lambda_m}\\ &= (1-\theta)\mathrm{COV}_m + \lambda_s(1-\rho^2)\sigma_p^2\frac{dQ^*}{d\lambda_m}\\ &= -(-(1-\theta)\mathrm{COV}_m(\lambda_m + \lambda_s(1-\rho^2))^2\\ &\quad + \lambda_s^2(1-\rho^2)(-\mathrm{COV}_m(1-\rho^2) + (\mathrm{COV}_s\\ &\quad + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})))((\lambda_m + \lambda_s(1-\rho^2))^2)^{-1},\end{aligned}$$

$$\begin{aligned}\frac{df^*}{d\lambda_s} &= -\big(\theta(\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i})(\lambda_m + \lambda_s(1-\rho^2))^2\\ &\quad + \lambda_m^2(-\mathrm{COV}_m(1-\rho^2) + (\mathrm{COV}_s\\ &\quad + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i}))\big)((\lambda_m + \lambda_s(1-\rho^2))^2)^{-1}.\end{aligned}$$

The desired results follow. □

Endnotes

1. We use nonstorability to model significant storage costs commonly seen in industries (the electricity industry and semiconductor industry, for example). Our model also allows different degrees of storability, as will be shown later. An illiquid spot market would strengthen our main results on the importance of bilateral supply contracts.

2. As Nash (1950) shows, the Nash bargaining solution is the only bargaining solution that is independent of utility units, Paretian, symmetric, and independent of irrelevant alternatives (see also Proposition 22.E.1 of Mas-Colell et al. 1995).

3. Allowing short sale is consistent with the existence of a spot market for the input factor at time 1. The main results for the case with no short sale remain the same and are available in the electronic companion at http://or.journal.informs.org/.

4. A linear or concave production cost would be inconsistent with the assumption of the supplier being a price taker in the component spot market.

5. The nonstorability of the component implies that events 4, 5, and 6 happen almost simultaneously. Because both the supplier and the manufacturer can trade in the spot market before or after the delivery of the forward contract, any order of events 4, 5, and 6 will result in the same modeling of time 1 decisions.

6. Our model can be easily extended to allow a stochastic goodwill cost. However, because this extension does not change the main qualitative results, we assume a constant goodwill cost for expositional simplicity.

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7. When $\rho^2 = 1$, we have $p = a \pm (\sigma_p/\sigma_{p_i})p_i$ almost surely, where a is a constant, which implies $\mathrm{COV}_s + \mathrm{COV}_i\rho\sigma_p/\sigma_{p_i} = 0$. In this case, $\Delta\,\mathrm{var}_s(Q) = 0$ for any value of Q, and without loss of generality, we define $Q^*_{sv} = 0$.
8. An upper bound of π_m is given by $\bar{\pi}_m = \bar{z}\bar{D} + 2\bar{p}\bar{Q}$, and an upper bound of π_s is given by $\bar{\pi}_s = \bar{\pi}_{s00} + \bar{p}_i\bar{\iota}^*_{00} + \bar{p}\bar{Q}$, where $\bar{Q} = \max(\bar{Q}^*_{mv}, \bar{Q}^*_{sv})$, $\bar{\pi}_{s00} = \bar{p}\bar{k}^* - w(\bar{k}^*)$, $\bar{k}^* = w'^{-1}(\bar{p})$, $\bar{\iota}^*_{00} = \overline{\mathrm{COV}}_i/\sigma_p^2$, $\bar{Q}^*_{mv} = \overline{\mathrm{COV}}_m/\sigma_p^2$, $\bar{Q}^*_{sv} = (\overline{\mathrm{COV}}_s + \overline{\mathrm{COV}}_i\rho\sigma_p/\sigma_{p_i})/((1-\rho^2)\sigma_p^2)$, $\overline{\mathrm{COV}}_i = \bar{\pi}_{s00}\mu_{p_i}$, $\overline{\mathrm{COV}}_m = (\bar{z}\bar{D} + g\mu_D)\mu_p$, and $\overline{\mathrm{COV}}_s = \bar{\pi}_{s00}\mu_p$. When $\lambda_m < 1/\bar{\pi}_m$ and $\lambda_s < 1/\bar{\pi}_s$, the satiation will not occur at the equilibrium.
9. The corresponding results for the case with positive negotiation costs can also be easily derived. However, because the expressions are much longer and the intuition is similar, we do not report them to save space.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

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