

# Market Making with Asymmetric Information and Inventory Risk\*

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## Abstract

Market makers in over-the-counter markets often make offsetting trades and have significant market power. We develop a market making model that captures these market features as well as other important characteristics such as information asymmetry and inventory risk. In contrast to the existing literature, a market maker in our model can optimally shift some trade with the informed to other discretionary investors by adjusting bid or ask. As a result, we find that consistent with empirical evidence, expected bid-ask spreads may *decrease* with information asymmetry and bid-ask spreads can be positively correlated with trading volume.

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As shown by the existing empirical literature,<sup>1</sup> market makers in over-the-counter markets tend to make offsetting trades and have significant market power. In this paper, we develop a market making model that captures these market features as well as other important characteristics such as information asymmetry and inventory risk. In contrast to the existing rational expectations models and microstructure models with information asymmetry,<sup>2</sup> this model introduces an alternative equilibrium setting where some uninformed investor with market power (e.g., a market maker) can optimally shift some trades with the informed to other discretionary investors by adjusting bid or ask. As a result, this model can help explain the puzzle that bid-ask spreads may decrease with information asymmetry, as shown by empirical studies.<sup>3</sup> Moreover, we show that consistent with empirical evidence,<sup>4</sup> bid-ask spreads can be positively correlated with trading volume. In contrast to double auction models (e.g., Kyle (1989), Rostek and Weretka (2012)), supply and demand function competition models (e.g., Vives (2011)), and Nash Bargaining models (e.g., Atkeson, Eisfeldt, and Weill (2014)), some agent in our model (i.e., the market maker) serves a dual role: a buyer in one market and simultaneously a seller in another. Our solution shows how this dual role of some participants affects the equilibrium outcome in these markets.

Specifically, we consider a setting with two trading dates and three types of risk averse investors: informed investors, uninformed investors, and an uninformed market maker. On date 0, all investors optimally choose how to trade a risk-free asset and a risky security (e.g., an OTC stock, a corporate bond, or a derivative security) to maximize their expected constant absolute risk averse (CARA) utility from the terminal wealth on date 1. All may be endowed with some shares of the risky security. The security payoff becomes public just before trading on date 1. Informed investors observe a private signal about the date 1 payoff of the security just before trading on date 0 and thus have trading demand motivated by private information. Informed investors also have non-information-based incentives to trade, which we term as a liquidity shock and model as a random endowment of a nontradable asset

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<sup>1</sup>E.g., Sofianos (1993), Shachar (2012), Garman (1976), Lyons (1995), Ang, Shtauber, and Tetlock (2011).

<sup>2</sup>E.g., Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), Glosten and Milgrom (1985), Kyle (1989), Admati and Pfleiderer (1988), Boulatov, Hendershott, and Livdan (2010), Goldstein, Li, and Yang (2013).

<sup>3</sup>E.g., Brooks (1996), Huang and Stoll (1997), Acker, Stalker and Tonks (2002), Acharya and Johnson (2007).

<sup>4</sup>See, for example, Lin, Sanger and Booth (1995), Chordia, Roll, and Subrahmanyam (2001).

whose payoff is correlated with that of the risky security.<sup>5</sup> It follows that informed investors also have trading demand motivated by the liquidity needs for hedging.

Due to high search costs, informed and uninformed investors must trade through the market maker. We assume that the market maker posts bid and ask price schedules first as in a Stackelberg game, taking into account their impact on other investors' trading demand, other investors then trade optimally taking the posted price schedules as given.<sup>6</sup> The equilibrium bid and ask depths are determined by the market clearing conditions at the bid and at the ask, i.e., the total amount the market maker buys (sells) at the bid (ask) is equal to the total amount other investors sell (buy). In equilibrium, the risk-free asset market also clears.

Although this model incorporates many important features in these markets, such as asymmetric information, inventory risk, imperfect competition, and risk aversion, and allows both bid/ask prices and depths as well as all demand schedules to be endogenous, the model is still tractable. Indeed, we solve the equilibrium bid and ask prices, bid and ask depths, trading volume, and inventory levels in closed-form even when investors have different risk aversion, different inventory levels, different liquidity shocks, different resale values of the risky asset and heterogeneous private information.<sup>7</sup>

We find that in equilibrium, both bid-ask spread and market trading volume are proportional to the absolute value of the reservation price difference between the informed and the uninformed.<sup>8</sup> The key intuition is that because the market maker can buy from some investors at the bid and sell to other investors at the ask, what matters for the spread and the

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<sup>5</sup>An alternative approach is to have three types of investors whom the market maker trades with: the informed who trade only on private information, the discretionary uninformed, and the noise traders some of whom buy and some of whom sell an exogenously given amount. This alternative model is much less tractable because of the non-Gaussian filtering problem and more importantly, would not yield different qualitative results, because the liquidity motivated trade of the informed in our model can be reinterpreted as noise traders' trade and the main intuition for our results still applies.

<sup>6</sup>This is equivalent to a setting where other investors submit demand schedules to the market maker (as in Kyle (1989) and Cespa and Vives (2012)), who then chooses bid and ask prices. The order size dependence of price schedules is consistent with the bargaining feature in less liquid markets. Indeed, we show that the equilibrium outcome is equivalent to the solution to a Nash bargaining game between investors and the market maker where the market maker has all the bargaining power.

<sup>7</sup>In the generalized model (in Section 5), there are eight types of equilibria characterized by the trading directions of investors, e.g., some investors may choose not to trade in equilibrium, and both the informed and the uninformed can trade in the same directions.

<sup>8</sup>The reservation price is the critical price such that an investor buys (sells) the security if and only if the ask (bid) is lower (higher) than this critical price.

trading volume is the reservation price difference between these investors. As in Goldstein, Li, and Yang (2013), investors have different motives to trade and thus the reservation prices can change differently in response to the same information and liquidity shocks. The greater the reservation price difference, the greater the total gain from trading, the more other investors trade, and because of the market maker's market power, the higher the spread. This also implies that in contrast to the literature on portfolio selection with transaction costs (e.g., Davis and Norman (1990), Liu (2004)), bid-ask spreads can be positively correlated with trading volume. Clearly, a market maker's market power and the feasibility of making offsetting trades, which are missing in most of the existing literature, are critical for this result. Therefore, our model predicts that in markets where dealers have significant market power and can make offsetting trades, trading volume is positively correlated with bid-ask spreads.

Empirical studies have shown that bid-ask spreads can decrease with information asymmetry. For example, among studies that decompose the components of bid-ask spreads, Huang and Stoll (1997) find that the asymmetric information component of the bid-ask spread can be *negative* and statistically significant. Acharya and Johnson (2007) show that in the credit default swap (CDS) market, spreads can also be lower with greater information asymmetry. In contrast, extant asymmetric information models predict that as information asymmetry increases, bid-ask spreads also increase. We show that our model can help explain this puzzle. Intuitively, as information asymmetry increases, to reduce exposure to the greater adverse selection effect, the market maker prefers to shift part of the trades with the informed to the uninformed. Unlike "noise traders" who have to trade the same amount at whatever quoted prices, in our paper the uninformed's trading amount depends on the trading price and the adverse selection effect they face. Therefore, with a greater adverse select effect, the market maker needs to offer a better price (lower ask or higher bid) to the uninformed in order to induce them to take some of the market maker's trades with the informed. This results in a narrower spread. On the other hand, if the uninformed have an initial endowment of the asset, then there is an opposing force: as information asymmetry increases, the uncertainty about the value of the initial endowment increases, and thus the uninformed are willing to sell at a lower bid price. This opposing force drives down the bid

price when the informed buy and thus can drive up the spread. Accordingly, our model predicts that in markets where market makers have significant market power and can make frequent offsetting trades and the current holdings of the uninformed are small, the average spread decreases with information asymmetry.

In most existing models on the determination of bid-ask spread or price impact in the presence of information asymmetry (e.g., Glosten and Milgrom (1985), Kyle (1989)), a market maker deals with the adverse selection problem by lowering the bid and/or increasing the ask or increasing the price impact per unit of trade. Our model demonstrates a second approach a market maker can use to control the adverse selection effect: shifting part of her trade with the informed to other investors.<sup>9</sup> We show that it is optimal for the market maker to combine these two approaches to best manage the adverse selection effect. When a market maker only uses the first approach, bid-ask spread is higher with asymmetric information, trading volume is negatively correlated with bid-ask spread and market breaks down (i.e., no trade) when bid-ask spread is infinity. In contrast, when a market maker also uses the second approach, not only bid-ask spread can be lower with asymmetric information, trading volume can be positively correlated with bid-ask spreads, but also market breaks down when the bid-ask spread is zero.

The critical driving forces behind our main results are: (1) the market maker has market power; (2) investors trade through the market maker (possibly due to high search costs); and (3) the uninformed are discretionary. Because of (1) and (2), the spread is proportional to the absolute value of the reservation price difference between the informed and the uninformed. Because of (3), the adverse selection effect of information asymmetry drives down the expected spread as explained above. Our main results are robust to changes in other ingredients of the model. For example, a market maker's risk aversion is irrelevant because it does not affect the reservation price difference between the informed and the uninformed. As we show in an earlier version and report in Section A.4 in the Appendix, having multiple market makers engaging in oligopolistic Cournot competition lowers the spread but does not change the main qualitative results. Similarly, the assumption that all the informed

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<sup>9</sup>In contrast, in the existing literature, while noise traders pay worse prices as information asymmetry increases, a market maker cannot choose to transfer more of the trade with the informed to noise traders by adjusting prices.

are price takers and have the same information is only for simplicity. For example, suppose the informed have heterogeneous information and are strategic, but all have significantly higher reservation prices than the uninformed. Then in equilibrium all the informed will be buyers and the uninformed will be the sellers, as in our simplified model. The spread will be proportional to the absolute value of the difference between some weighted average of the reservation prices of the informed and the reservation price of the uninformed by similar intuition.

While as cited before, there are findings where bid-ask spreads can decrease with information asymmetry, and trading volume and bid-ask spreads can be positively correlated, there are also findings where the opposite is true (e.g., Green, Hollifield and Schürhoff (2007), Edwards, Harris and Piwowar (2007)). The existing literature cannot reconcile these seemingly contradictory empirical evidence. While many factors may drive these opposite findings and it is beyond the scope of this paper to pinpoint the key drivers for these results through a thorough empirical analysis, our model provides conditions under which these opposite empirical findings can arise and can shed some light on possible sources. For example, our model might help explain the negative relationship between spread and information asymmetry found by Acharya and Johnson (2007), because they focus on more active CDS markets where dealers with significant market power can make relatively frequent offsetting trades and most customers have small initial holdings. In addition, consistent with the prediction of positive correlation between spreads and trading volume, Li and Schürhoff (2011) find that in municipal bond markets central dealers, who likely have greater market power and can make offsetting trades more easily than peripheral dealers, charge higher bid-ask spreads and also experience greater trading volume.

As our paper, Hall and Rust (2003) also consider a monopolist market maker who posts bid and ask prices. Different from our model, however, their model abstracts away from asymmetric information, which is the main focus of our analysis. Although our model focuses on OTC financial markets, minor modifications of the model can be applied to other markets such as a market where producers have market power in both input and output markets and buy from input producers and sell to output buyers.<sup>10</sup>

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<sup>10</sup>For example, if the output is proportional to input due to a linear production technology, then our model

The remainder of the paper proceeds as follows. In Section 1 we briefly describe the OTC markets and discuss additional related literature. We present the model in Section 2. In Section 3 we derive the equilibrium. In Section 4 we provide some comparative statics on asset prices and bid-ask spreads. We present, solve and discuss a generalized model in Section 5. We conclude in Section 6. All proofs are provided in Appendix A. In Appendix B, we present the rest of the results of Theorem 2 for the generalized model.

## 1. Over-the-Counter Markets and Related Literature

Most types of government and corporate bonds, a wide range of derivatives (e.g., CDS and interest rate swaps), securities lending and repurchase agreements, currencies, and penny stocks are traded in the OTC markets.<sup>11</sup> In almost all of these markets, investors only trade with designated dealers (market makers) who typically quote a pair of bid and ask prices that are explicitly or implicitly contingent on order sizes.<sup>12</sup> Nash bargaining has been widely used by the existing literature to model bilateral negotiation in OTC markets (e.g., Duffie, Garleanu, and Pedersen (2005), Gofman (2011), Atkeson, Eisfeldt, and Weill (2014)). As shown in Section A.2 in Appendix A, our modeling approach where the market maker chooses prices to maximize her utility taking into account the impact on other investors' trading demand is equivalent to the solution to a Nash bargaining game between investors and the market maker where the market maker has all the bargaining power.

Dealers in OTC markets face significant information asymmetry and inventory risk, and therefore, they frequently engage in offsetting trades within a short period of time with other customers or with other dealers when their inventory level deviates significantly from desired targets (e.g., Acharya and Johnson (2007), Shachar (2012)). The cost of searching for a counterparty can be significant in some OTC markets for some investors, which motivates many studies to use search-based or network-based models for OTC markets (e.g., Duffie,

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only needs almost trivial changes.

<sup>11</sup>The Nasdaq stock market was traditionally also a dealer market before the regulatory reforms implemented in 1997. Some of the empirical studies we cite (e.g., Huang and Stoll (1997)) use before-1997 Nasdaq data.

<sup>12</sup>For example, Li and Schürhoff (2011) show that dealers intermediate 94% of the trades in the municipal bond market, with most of the intermediated trades representing customer-dealer-customer transactions.

Gârleanu, and Pedersen (2005), Vayanos and Weill (2008)). While we do not explicitly model the search cost, the assumption that investors can only trade through a market maker can be viewed as a result of significant costs of searching for other counterparty. In addition, the generalized model can indirectly capture some additional costs for liquidation of inventory on date 1. For example, the effect of high search cost, long search time, and highly uncertain resale value of the security is qualitatively similar to the effect of a low mean and high volatility distribution for the resale value of the security acquired by a market maker in the generalized model. This is clearly just a reduced form, but likely indirectly captures the first order effect of these features. More importantly, explicitly modeling searching would not change our main results such as bid-ask spreads and trading volume increase with the magnitude of the reservation price difference and bid-ask spreads can decrease with information asymmetry, because after a successful search of a counterparty, traders face the same optimization problems as what we model.

In contrast to this model, existing market making literature either ignores information asymmetry (e.g., Garman (1976), Stoll (1978), and Ho and Stoll (1981)) or abstracts away a market maker's inventory risk (e.g., Kyle (1989), Glosten and Milgrom (1985), Admati and Pfleiderer (1988)). However, both information asymmetry and inventory risk are important determinants of market prices and market liquidity for many over-the-counter (OTC) markets. Different from inventory-based models, our model takes into account the impact of information asymmetry on bid and ask prices and inventory levels. In contrast to most information-based (rational expectations) models (e.g., Grossman and Stiglitz (1980), Glosten and Milgrom (1985)), in our model a market maker faces discretionary uninformed investors, has significant market power, profits from bid-ask spreads, and can face significant inventory risk. In particular, the market maker in our model may be willing to lose money from a particular trade in expectation in equilibrium especially when she has high initial inventory.<sup>13</sup>

Different from most of the existing literature on dealership markets,<sup>14</sup> a market maker in

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<sup>13</sup>On the other hand, when the market maker has low initial inventory she makes positive expected profit from inventory carried to date 1 because of the required inventory risk premium, consistent with the findings of Hendershott, Moulton, and Seasholes (2007).

<sup>14</sup>E.g., Leach and Madhavan (1992, 1993), Lyons (1997), Biais, Foucault and Salanié (1998), Viswanathan and Wang (2004), and Duffie, Gârleanu, and Pedersen (2005).



our model can shift part of the trade with the informed to other customers by adjusting bid or ask and as a result, expected spread can decrease with information asymmetry.

## 2. The model

We consider a one period setting with trading dates 0 and 1. There are a continuum of identical informed investors with mass  $N_I$ , a continuum of identical uninformed investors with mass  $N_U$ , and  $N_M = 1$  designated market maker who is also uninformed. They can trade one risk-free asset and one risky security on date 0 and date 1 to maximize their expected constant absolute risk aversion (CARA) utility from their wealth on date 1. There is a zero net supply of the risk-free asset, which also serves as the numeraire and thus the risk-free interest rate is normalized to 0. The total supply of the security is  $N \times \bar{\theta} \geq 0$  shares where  $N = N_I + N_U + N_M$  and the date 1 payoff of each share  $\tilde{V} \sim \mathbf{N}(\bar{V}, \sigma_V^2)$  becomes public on date 1, where  $\bar{V}$  is a constant,  $\sigma_V > 0$ , and  $\mathbf{N}$  denotes the normal distribution. The aggregate risky asset endowment is  $N_i \bar{\theta}$  shares for type  $i \in \{I, U, M\}$  investors. No investor is endowed with any risk-free asset.

On date 0, informed investors observe a private signal

$$\hat{s} = \tilde{V} - \bar{V} + \tilde{\varepsilon} \quad (1)$$

about the payoff  $\tilde{V}$ , where  $\tilde{\varepsilon}$  is independently normally distributed with mean zero and variance  $\sigma_\varepsilon^2$ .<sup>15</sup> To prevent the informed's private information from being fully revealed in equilibrium, following Wang (1994), O'Hara (1997), and Vayanos and Wang (2012), we assume that the informed also have non-information based trading demand. Specifically, we assume that an informed investor is also subject to a liquidity shock that is modeled as a random endowment of  $\hat{X}_I \sim \mathbf{N}(0, \sigma_X^2)$  units of a non-tradable risky asset on date 0, with  $\hat{X}_I$  realized on date 0 and only known to informed investors.<sup>16</sup> The non-tradable asset has a per-unit payoff of  $\tilde{N} \sim \mathbf{N}(0, \sigma_N^2)$  that has a covariance of  $\sigma_{VN}$  with  $\tilde{V}$  and is realized and

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<sup>15</sup>Throughout this paper, “bar” variables are constants, “tilde” random variables are realized on date 1 and “hat” random variables are realized on date 0.

<sup>16</sup>The random endowment can represent any shock in the demand for the security, such as a liquidity shock or a change in the needs for rebalancing an existing portfolio or a change in a highly illiquid asset.

becomes public on date 1. The correlation between the non-tradable asset and the security results in a liquidity demand for the risky asset to hedge the non-tradable asset payoff.

In addition, to provide a good measure of information asymmetry, we assume that there is a public signal

$$\hat{S}_s = \hat{s} + \hat{\eta} \quad (2)$$

about the informed's private signal  $\hat{s}$  that all investors (i.e., the uninformed, the market maker, and the informed) can observe, where  $\hat{\eta}$  is independently normally distributed with mean zero and variance  $\sigma_\eta^2$ .<sup>17</sup> This public signal represents public news about the asset payoff determinants, such as macroeconomic conditions, cash flow news and regulation shocks, which is correlated with but less precise than the informed's private signal. As we show later, the noisiness  $\sigma_\eta^2$  of the public signal can serve as a good measure of information asymmetry. In empirical tests, one can use the amount of relevant public news as a proxy for this information asymmetry measure, because the more relevant public news, the better the uninformed can estimate the security payoff.<sup>18</sup>

Due to high search costs, all trades must go through the designated market maker (dealer) whose market making cost is assumed to be 0.<sup>19</sup> Specifically,  $I$  and  $U$  investors sell to the market maker at the bid  $B$  or buy from her at the ask  $A$  or do not trade at all. The market maker posts (commits) her price schedules first. Then informed and uninformed investors decide how much to trade. When deciding on what price schedules to post, the market maker takes into account the best response functions (i.e., the demand schedules) of the informed and the uninformed given the posted price schedules.<sup>20</sup>

For each  $i \in \{I, U, M\}$ , investors of type  $i$  are identical both before and after realizations of signals on date 0 and thus adopt the same trading strategy. Let  $\mathcal{I}_i$  represent a type  $i$

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<sup>17</sup>While this public signal can also be observed by the informed, it is useless to them because they can already perfectly observe it privately.

<sup>18</sup>Information asymmetry proxies such as disclosure level, analyst coverage and transparency commonly used in the empirical literature are clearly some measures of the amount of relevant public news.

<sup>19</sup>Assuming zero market making cost is only for better focus and expositional simplicity. Market making cost is considered in an earlier version, where a potential market maker must pay a fixed market-making utility cost on date 0 to become a market maker. We show that no results in this paper are altered by this fixed cost.

<sup>20</sup>This can be reinterpreted as a Stackelberg game between the market maker and other investors where the market maker moves first by posting bid and ask price schedules (that depend on order sizes), then other players move by trading the optimal amount given the prices.

investor's information set on date 0 for  $i \in \{I, U, M\}$ . For  $i \in \{I, U\}$ , a type  $i$  investor's problem is to choose the (signed) demand schedule  $\theta_i(A, B)$  to solve

$$\max E[-e^{-\delta \tilde{W}_i} | \mathcal{I}_i], \quad (3)$$

where

$$\tilde{W}_i = \theta_i^- B - \theta_i^+ A + (\bar{\theta} + \theta_i) \tilde{V} + \hat{X}_i \tilde{N}, \quad (4)$$

$\hat{X}_U = 0$ ,  $\delta > 0$  is the absolute risk-aversion parameter,  $x^+ := \max(0, x)$ , and  $x^- := \max(0, -x)$ .

Since  $I$  and  $U$  investors buy from the market maker at ask and sell to her at bid, we can view these trades as occurring in two separate markets: the “ask” market and the “bid” market. In the ask market, the market maker is the supplier, other investors are demanders and the opposite is true in the bid market. The monopolist market maker chooses bid and ask prices, taking into account other investors' demand curve in the ask market and other investors' supply curve in the bid market.

Given bid price  $B$  and ask price  $A$ , let the demand schedules of the informed and the uninformed be denoted as  $\theta_I^*(A, B)$  and  $\theta_U^*(A, B)$  respectively. By market clearing conditions, the equilibrium ask depth  $\alpha$  must be equal to the total amount bought by other investors and the equilibrium bid depth  $\beta$  must be equal to the total amount sold by other investors,<sup>21</sup> i.e.,

$$\alpha = \sum_{i=I, U} N_i \theta_i^*(A, B)^+, \quad \beta = \sum_{i=I, U} N_i \theta_i^*(A, B)^-, \quad (5)$$

where the left-hand sides represent the sale and purchase by the market maker respectively and the right-hand sides represent the total purchases and sales by other investors respectively.<sup>22</sup> Note that if an investor decides to buy (sell), then only the ask (bid) price affects how much he buys (sells), i.e.,  $\theta_i^*(A, B)^+$  only depends on  $A$  and  $\theta_i^*(A, B)^-$  only depends on  $B$ . Therefore, the bid depth  $\beta$  only depends on  $B$ , henceforth referred as  $\beta(B)$  and the ask depth  $\alpha$  only depends on  $A$ , hence forth referred as  $\alpha(A)$ .

Then the designated market maker's problem is to choose ask price  $A$  and bid price  $B$

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<sup>21</sup>To help remember, Alpha denotes Ask depth and Beta denotes Bid depth.

<sup>22</sup>The risk-free asset market will be automatically cleared by the Walras' law.

to solve

$$\max E \left[ -e^{-\delta \tilde{W}_M} | \mathcal{I}_M \right], \quad (6)$$

subject to

$$\tilde{W}_M = \alpha(A)A - \beta(B)B + (\bar{\theta} + \beta(B) - \alpha(A))\tilde{V}. \quad (7)$$

This leads to our definition of an equilibrium:

**Definition 1** *An equilibrium  $(\theta_I^*(A, B), \theta_U^*(A, B), A^*, B^*, \alpha^*, \beta^*)$  is such that*

1. *given any  $A$  and  $B$ ,  $\theta_i^*(A, B)$  solves a type  $i$  investor's Problem (3) for  $i \in \{I, U\}$ ;*
2. *given  $\theta_I^*(A, B)$  and  $\theta_U^*(A, B)$ ,  $A^*$  and  $B^*$  solve the market maker's Problem (6);*
3. *market clearing condition (5) is satisfied by  $\theta_I^*(A, B), \theta_U^*(A, B), A^*, B^*, \alpha^*$ , and  $\beta^*$ .*

## 2.1. Discussions on the assumptions of the model

In this subsection, we provide justifications for our main assumptions and discuss whether these assumptions are important for our main results.

The assumption that there is only one market maker is for expositional focus. A model with multiple market makers was solved in an earlier version of this paper, the results of which are reported in Section A.4 in the Appendix. In this model with oligopolistic competition, we show that competition among market makers, while lowering spreads, does not change our main qualitative results (e.g., expected bid-ask spread can decrease with information asymmetry). In illiquid markets such as some OTC markets, it is costly for non-market-makers to find and directly trade with each other. Therefore, most trades are through a market maker.

One important assumption is that the market maker can buy at the bid from some investors and sell at the ask to other investors at the same time. This assumption captures the fact that in many OTC markets, when a dealer receives an inquiry from a client, she commonly contacts other clients (or dealers) to see at which price and by how much she can unload the inquired trade *before* she trades with the initial client. In addition, even

for markets where there is a delay between offsetting trades, using a dynamic model with sequential order arrival would unlikely yield qualitatively different results. For example, in such a dynamic model, spreads can still decrease with information asymmetry, because even when orders arrive sequentially and thus a market maker needs to wait a period of time for the offsetting trades, as long as she has a reasonable estimate of the next order, she will choose qualitatively the same trading strategy.

To keep information from being fully revealed in equilibrium, we assume that informed investors have liquidity shocks in addition to private information. One can interpret this assumption as there are some pure liquidity traders who trade in the same direction as the informed. Alternatively, one can view an informed investor as a broker who combines information motivated trades and liquidity motivated trades. The assumption that all informed traders have the same information and the same liquidity shock is only for simplicity so that there are only two groups of non-market-makers in the model. Our main results still hold when they have different information and different liquidity shocks. Intuitively, no matter how many heterogenous investor groups there are, the equilibrium bid and ask prices would divide these groups into Buy group, Sell group, and No trade group. Therefore, as long as the characteristics of the Buy and Sell group investors are similar to those in our simplified model, our main results still hold. For example, if in equilibrium some informed buy, other informed sell, and the uninformed do not trade, then the reservation price difference between the two informed groups would determine the spread, which can still decrease with information asymmetry (between the two informed groups) by similar intuition.

We also assume that the market maker posts price schedules first (after taking into account what would be the best responses of other investors), then other investors choose their optimal trading strategies taking the posted price schedules as given, and thus other investors are not strategic. As we show, this assumption is equivalent to assuming that in a Nash Bargaining game between the market maker and other investors, the market maker has all the bargaining power. This is consistent with the common practice in OTC markets that a dealer making two-sided markets typically provides a take-it-or-leave-it pair of prices, a bid and an offer, to customers (e.g., Duffie (2012), Chapter 1).

Different from the existing models, we assume there is public signal that is correlated

with the private signal of the informed. This additional signal is not critical for our main results (e.g., spread can be smaller with asymmetric information), but has two main benefits. In addition to providing a good measure of information asymmetry, its introduction also makes our model nest models with different degrees of information asymmetry in one unified setting.<sup>23</sup>

### 3. The equilibrium

In this section, we solve the equilibrium bid and ask prices, bid and ask depths and trading volume in closed form.

Given  $A$  and  $B$ , the optimal demand schedule of a type  $i$  investor ( $i \in \{I, U\}$ ) is

$$\theta_i^*(A, B) = \begin{cases} \frac{P_i^R - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_i]} & A < P_i^R, \\ 0 & B \leq P_i^R \leq A, \\ -\frac{B - P_i^R}{\delta \text{Var}[\tilde{V}|\mathcal{I}_i]} & B > P_i^R, \end{cases} \quad (8)$$

where

$$P_i^R = E[\tilde{V}|\mathcal{I}_i] - \delta \text{Cov}[\tilde{V}, \tilde{N}|\mathcal{I}_i] \hat{X}_i - \delta \text{Var}[\tilde{V}|\mathcal{I}_i] \bar{\theta} \quad (9)$$

is the investor's reservation price (i.e., the critical price such that a non-market-maker buys (sells, respectively) the security if and only if the ask price is lower (the bid price is higher, respectively) than this critical price).

Because the informed know exactly  $\{\hat{s}, \hat{X}_I\}$  while equilibrium prices  $A^*$  and  $B^*$  and the public signal  $\hat{S}_s$  are only noisy signals about  $\{\hat{s}, \hat{X}_I\}$ , the information set of the informed in equilibrium is

$$\mathcal{I}_I = \{\hat{s}, \hat{X}_I\}, \quad (10)$$

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<sup>23</sup>For example, the case where  $\sigma_\eta^2 = 0$  implies that the uninformed and the market maker can perfectly observe  $\hat{s}$  from the public signal and thus represents the symmetric information case. The case where  $\sigma_\eta^2 = \infty$ , on the other hand, implies that the public signal is useless and thus corresponds to the asymmetric information case as modeled in the standard literature, i.e., there is no public signal about the private information.

which implies that

$$E[\tilde{V}|\mathcal{I}_I] = \bar{V} + \rho_I \hat{s}, \quad \text{Var}[\tilde{V}|\mathcal{I}_I] = \rho_I \sigma_\varepsilon^2, \quad \text{Cov}[\tilde{V}, \tilde{N}|\mathcal{I}_I] = (1 - \rho_I) \sigma_{VN}, \quad (11)$$

where

$$\rho_I := \frac{\sigma_V^2}{\sigma_V^2 + \sigma_\varepsilon^2}. \quad (12)$$

Equation (9) then implies that

$$P_I^R = \bar{V} + \hat{S} - \delta \rho_I \sigma_\varepsilon^2 \bar{\theta}, \quad (13)$$

where  $\hat{S} := \rho_I \hat{s} + h \hat{X}_I$  and  $h = -\delta(1 - \rho_I) \sigma_{VN}$  represents the hedging premium per unit of liquidity shock.

While  $\hat{s}$  and  $\hat{X}_I$  both affect the informed investor's demand and thus the equilibrium prices, other investors can only infer the value of  $\hat{S}$  from market prices because the joint impact of  $\hat{s}$  and  $\hat{X}_I$  on market prices is only through  $\hat{S}$ . In addition to  $\hat{S}$ , other investors can also observe the public signal  $\hat{S}_s$  about the private signal  $\hat{s}$ . Thus we conjecture that the equilibrium prices  $A^*$  and  $B^*$  depend on both  $\hat{S}$  and  $\hat{S}_s$ . Accordingly, the information sets for the uninformed investors and the market maker are<sup>24</sup>

$$\mathcal{I}_U = \mathcal{I}_M = \{\hat{S}, \hat{S}_s\}. \quad (14)$$

Then the conditional expectation and conditional variance of  $\tilde{V}$  for the uninformed and the market maker are respectively

$$E[\tilde{V}|\mathcal{I}_U] = \bar{V} + \rho_U(1 - \rho_X) \hat{S} + \rho_U \rho_X \rho_I \hat{S}_s, \quad (15)$$

$$\text{Var}[\tilde{V}|\mathcal{I}_U] = \rho_U \rho_I (\sigma_\varepsilon^2 + \rho_X \sigma_\eta^2), \quad (16)$$

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<sup>24</sup>Note that uninformed only need to observe their own trading price, i.e.,  $A^*$  or  $B^*$ , not both  $A^*$  and  $B^*$ . For OTC markets, investors may not be able to observe trading prices by others, although with improving transparency, this has also become possible in some markets (e.g., TRACE system in the bond market).

where

$$\rho_X := \frac{h^2 \sigma_X^2}{h^2 \sigma_X^2 + \rho_I^2 \sigma_\eta^2}, \quad \rho_U := \frac{\sigma_V^2}{\sigma_V^2 + \rho_X \rho_I \sigma_\eta^2} \leq 1. \quad (17)$$

It follows that the reservation price for a  $U$  investor and the market maker is

$$P_U^R = P_M^R = \bar{V} + \rho_U(1 - \rho_X)\hat{S} + \rho_U \rho_X \rho_I \hat{S}_s - \delta \rho_U \rho_I (\sigma_\varepsilon^2 + \rho_X \sigma_\eta^2) \bar{\theta}. \quad (18)$$

Let  $\Delta$  denote the difference in the reservation prices of  $I$  and  $U$  investors. We then have

$$\Delta := P_I^R - P_U^R = (1 - \rho_U) \left( \left( 1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2} \right) \hat{S} - \frac{\sigma_V^2}{\sigma_\eta^2} \hat{S}_s + \delta \rho_I \sigma_V^2 \bar{\theta} \right). \quad (19)$$

Let

$$\nu := \frac{\text{Var}[\tilde{V}|\mathcal{I}_U]}{\text{Var}[\tilde{V}|\mathcal{I}_I]} = \rho_U + \frac{\rho_U \rho_X \sigma_\eta^2}{\sigma_\varepsilon^2} \geq 1$$

be the ratio of the security payoff conditional variance of the uninformed to that of the informed, and

$$\bar{N} := \nu N_I + N_U + 1 \geq N$$

be the information weighted total population. The following theorem provides the equilibrium bid and ask prices and equilibrium security demand in closed-form.<sup>25</sup>

**Theorem 1** 1. *The equilibrium bid and ask prices are respectively*

$$A^* = P_U^R + \frac{\nu N_I}{2(\bar{N} + 1)} \Delta + \frac{\Delta^+}{2}, \quad (20)$$

$$B^* = P_U^R + \frac{\nu N_I}{2(\bar{N} + 1)} \Delta - \frac{\Delta^-}{2}. \quad (21)$$

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<sup>25</sup>Because all utility functions are strictly concave and all budget constraints are linear in the amount invested in the security, there is a unique solution to the problem of each informed and each uninformed given the bid and ask prices. Because the market clearing bid and ask depths are linear in bid and ask prices, there is a unique solution to her utility maximization problem (which already takes into account the market clearing conditions). This implies that there is a unique equilibrium when all investors trade in equilibrium. When some investors do not trade in equilibrium as illustrated in Section 5, there are multiple equilibria because either bid or ask would not be unique (see Theorem 2).



The bid-ask spread is

$$A^* - B^* = \frac{|\Delta|}{2} = \frac{1}{2}(1 - \rho_U) \left| \left( 1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2} \right) \hat{S} - \frac{\sigma_V^2}{\sigma_\eta^2} \hat{S}_s + \delta \rho_I \sigma_V^2 \bar{\theta} \right|. \quad (22)$$

2. The equilibrium quantities demanded are

$$\theta_I^* = \frac{N_U + 2}{2(\bar{N} + 1)} \frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}, \quad \theta_U^* = -\frac{\nu N_I}{2(\bar{N} + 1)} \frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}; \quad (23)$$

the equilibrium ask and bid depths are respectively

$$\alpha^* = N_I(\theta_I^*)^+ + N_U(\theta_U^*)^+, \quad (24)$$

$$\beta^* = N_I(\theta_I^*)^- + N_U(\theta_U^*)^-, \quad (25)$$

which implies that the equilibrium trading volume is

$$\alpha^* + \beta^* = \frac{N_I(N_U + 1)}{\bar{N} + 1} \left( \frac{|\Delta|}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} \right). \quad (26)$$

As shown in Section A.2 in Appendix A, the above equilibrium can be reinterpreted as the solution to a Nash bargaining game between the market maker and other investors where the market maker has all the bargaining power. In a nutshell, in the Nash bargaining game, the market maker and an investor bargain over the trading price with the trading amount determined by the optimal demand schedule of the investor. Therefore, the Nash bargaining game where the market maker has all the bargaining power is to choose the trading price to maximize the market maker's expected utility given the demand schedule of the investor, and thus yields exactly the same outcome as our solution above.<sup>26</sup>

Equations (20) and (21) imply that in equilibrium both bid and ask prices are determined by the reservation price of the uninformed and the reservation price difference between the informed and the uninformed. In existing models with information asymmetry, an uninformed

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<sup>26</sup>We also solve the case where other investors have bargaining power and the case where they bargain over both trade price and trade size, the qualitative results are the same. For example, the equilibrium bid-ask spread is still proportional to the absolute value of the reservation price difference between the informed and the uninformed.

counterparty of the informed controls the adverse selection effect of information asymmetry by charging a price premium (if the informed buy) or demanding a price discount (if the informed sell). In our model, to control the adverse selection effect, the market maker also adjusts the trading price with the uninformed to induce them to take part of her trade with the informed, in addition to varying the trading price with the informed. Consider the case where the reservation price of the informed is higher than that of the uninformed (i.e.,  $\Delta > 0$ ) and thus the informed buy and the uninformed sell in equilibrium. Because the information weighted total population  $\bar{N}$  increases with the uninformed population  $N_U$ , Equation (20) implies that as  $N_U$  decreases, the ask price paid by the informed increases. This is because as the uninformed population  $N_U$  decreases, the market maker can shift less of her trade with the informed to the uninformed. In the extreme case where there is no (discretionary) uninformed investor (as in many existing models), the market maker charges the highest ask price because she can no longer shift any of her trade with the informed to other investors.

In addition, given the public signal  $\hat{S}_s$ , all investors can indeed infer  $\hat{S}$  from observing their trading prices as conjectured, because of the one-to-one mapping between the two. Even in the generalized model in Section 5 where the informed do not trade in equilibrium, the uninformed can still infer  $\hat{S}$  if the equilibrium price is set such that the informed are indifferent between trading and no trading, because the uninformed can then back out  $\hat{S}$  that makes the informed's trade size equal to zero from market prices.<sup>27</sup>

Note that because the equilibrium bid and ask price *schedules* depend on  $\hat{S}$  and the public signal  $\hat{S}_s$ , the market maker can indeed post the bid and ask price schedules *before* observing the order flow and the public signal. The bid and ask price *levels* are then determined after the realizations of the signals revealed by the orders. Because by (23) there is a one-to-one mapping between  $\hat{S}$  and the informed's and the uninformed's order sizes for a given  $\hat{S}_s$ , the market maker can also equivalently post the price schedules as (nonlinear) functions of the informed's and the uninformed's order sizes. The order size dependence is similar to the "quantity discounts" allowed by Biais, Foucault and Salanié (1998) for dealership markets, although there is no information asymmetry considered in their model.

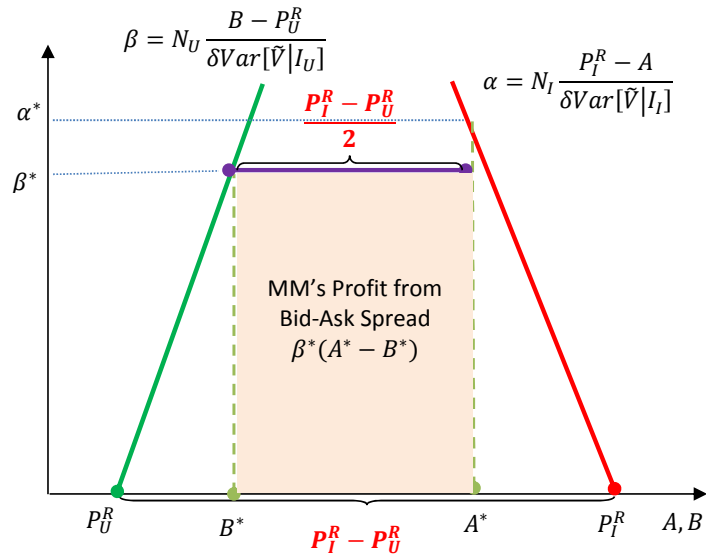
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<sup>27</sup>As in Glosten (1989) and Vayanos and Wang (2012), the market maker in our model can infer how much informed investors are trading. However, she does not know how much is due to information on the security's payoff or how much is due to the liquidity demand.

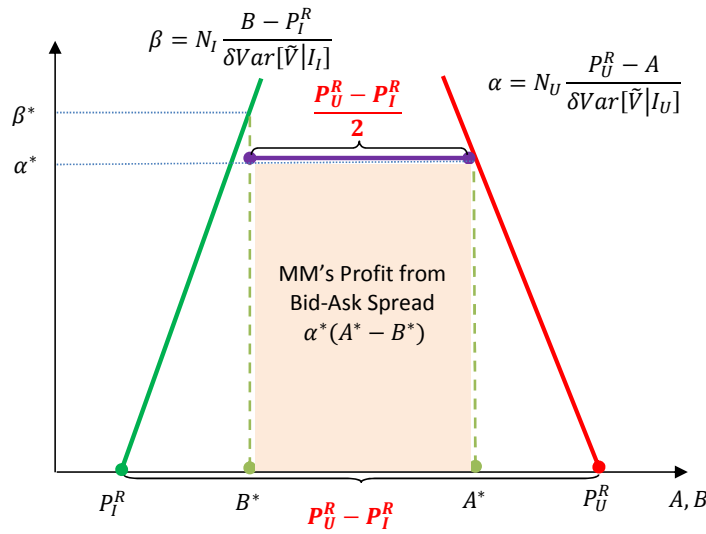
Theorem 1 also shows that the equilibrium bid-ask spread is equal to the absolute value of the reservation price difference between the informed and the uninformed, divided by 2 (more generally, as shown in an earlier version of the paper, divided by  $N_M + 1$ ). This is similar to the results of classical models on monopolistic firms who set the market price to maximize profit. Different from these monopolistic firms, however, the market maker both buys and sells and makes profit from the spread. As conjectured, Equation (23) implies that  $I$  investors buy and  $U$  investors sell if and only if  $I$  investors have a higher reservation price than  $U$  investors. Because the market maker has the same reservation price as the  $U$  investors, in the net she trades in the same direction as  $U$  investors.

In the standard literature on portfolio choice with transaction costs (e.g., Davis and Norman (1990), Liu (2004)), it is well established that as the bid-ask spread increases, investors reduce trading volume to save on transaction costs and thus trading volume and bid-ask spread move in the opposite directions. In contrast, Theorem 1 implies that bid-ask spreads and trading volume can move in the same direction, because both trading volume and bid-ask spread increase with  $|\Delta|$ . Lin, Sanger and Booth (1995) find that trading volume and effective spreads are positively correlated at the beginning and the end of the day. Chordia, Roll, and Subrahmanyam (2001) find that the effective bid-ask spread is positively correlated with trading volume. Our model suggests that these positive correlations may be caused by changes in the valuation difference of investors. There are also empirical findings that bid-ask spreads can be negatively correlated with trading volume (e.g., Green, Hollifield and Schürhoff (2007), Edwards, Harris and Piwowar (2007)). The negative correlation is consistent with the case where the bid-ask spread is almost exogenous, as in any partial equilibrium model (e.g., Liu (2004)). When market makers have near perfect competition, the bid-ask spread is essentially determined by the market-making cost and therefore is largely exogenous. Thus, one of the empirically testable implications of our model is that when market makers have significant market power, bid-ask spreads and trading volume are positively correlated. This prediction seems consistent with the finding of Li and Schürhoff (2011): In municipal bond markets, central dealers, who likely have greater market power than peripheral dealers, charge higher bid-ask spreads and also enjoy greater trading volume.

Next we provide the essential intuition for the results in Theorem 1 through graphical



(a) The Informed Buy and the Uninformed Sell



(b) The Uninformed Buy and the Informed Sell

Figure 1: Demand/Supply Functions and Bid/Ask Spreads.

illustrations. Suppose  $P_I^R > P_U^R$  and thus  $I$  investors buy and  $U$  investors sell. The market clearing condition (5) implies that

$$\alpha = N_I \frac{P_I^R - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}, \quad \beta = N_U \frac{B - P_U^R}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}.$$

We plot the above demand and supply functions and equilibrium spreads in Figure 1 (a). Similarly, we present Figure 1 (b) for the case where the informed sell and the uninformed buy. Figure 1 shows that the higher the bid, the more a market maker can buy from other investors, and the lower the ask, the more a market maker can sell to other investors. Facing the demand and supply functions of other investors, a monopolist market maker optimally trades off the prices and quantities. Similar to the results of classical models on monopolistic firms who set a market price to maximize profit, the bid and ask spread is equal to the absolute value of the reservation price difference  $|\Delta|$  divided 2. In addition, as implied by Theorem 1, Figure 1 (a) illustrates that the difference between  $P_I^R$  ( $P_U^R$ ) and the ask (bid) price is also proportional to the reservation price difference magnitude  $|\Delta|$ . Therefore the trading amount of both  $I$  and  $U$  investors and thus the aggregate trading volume all increase with  $|\Delta|$ . The shaded areas represent the profits  $(\min(\alpha^*, \beta^*)(A^* - B^*))$  the market maker makes from the bid-ask spread at time 0.

In contrast to the existing literature that assumes zero expected profit for each trade (e.g., Glosten and Milgrom (1985)), Theorem 1 implies that a market maker may lose money in expectation on a particular trade. For example, suppose  $\Delta > 0$  (which implies that the informed buy at the ask and the uninformed sell at the bid), the per share expected profit of the market maker from the trade at the bid (not including the profit from the spread) is equal to

$$E[\tilde{V}|\mathcal{I}_M] - B^* = \delta \text{Var}[\tilde{V}|\mathcal{I}_M] \bar{\theta} - \frac{N_I \nu}{2(\bar{N} + 1)} \Delta, \quad (27)$$

which can be negative if  $\Delta$  is large, in which case the market maker on average loses to the uninformed and makes money from the informed. The market maker is willing to buy from the uninformed in anticipation of a loss from this trade because she can sell the purchased shares at a higher price (i.e., ask). Because of the hedging benefit, the informed may be willing to buy from the market maker in anticipation of a loss from this purchase. This same

intuition applies to a dynamic setting where orders arrive sequentially. For example, seeing an order to sell at the bid, if the market maker expects that she will be able to unwind part of her purchase later at a higher price, she would be willing to accommodate the sell order even in anticipation of a loss for this purchase. This suggests that using a dynamic model does not change these qualitative results, while making the analysis less tractable.

Theorem 1 and Equation (27) imply when  $\Delta < 0$ , a market maker buys in the net and she makes positive expected profit from inventory carried over if she does not have any initial inventory (i.e.,  $\bar{\theta} = 0$ ), because of the required inventory risk premium. This is consistent with the findings of Hendershott, Moulton, and Seasholes (2007).

## 4. Comparative statics

In this section, we provide some comparative statics on asset prices and market illiquidity, focusing on the impact of information asymmetry and liquidity shock volatility.

### 4.1. A measure of information asymmetry

While there is a vast literature on the impact of information asymmetry on asset pricing and market liquidity, to our knowledge, if the informed do not know exactly the future payoff (as in our model), then there is still not a good measure of information asymmetry, i.e., a change of which does not affect other relevant economic variables such as the quality of aggregate information about the security payoff.<sup>28</sup> For example, the precision of a private signal about asset payoff would not be a good measure, because a change in the precision also changes the quality of aggregate information about the payoff and both information asymmetry and information quality can affect economic variables of interest (e.g., prices, liquidity). Even a comparison between the cases with and without asymmetric information

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<sup>28</sup>The quality of aggregate information about the security payoff is measured by the inverse of the security payoff variance conditional on *all* the information in the economy, i.e.,

$$(\text{Var}(\tilde{V}|\mathcal{I}_I \cup \mathcal{I}_U \cup \mathcal{I}_M))^{-1} = (\text{Var}(\tilde{V}|\mathcal{I}_I))^{-1} = \frac{\sigma_V^2 + \sigma_\varepsilon^2}{\sigma_V^2 \sigma_\varepsilon^2}, \quad (28)$$

where the first equality follows from the fact that the informed have better information than the rest and the second from (11).

cannot attribute the difference to the impact of information asymmetry alone, as long as the information quality is different across these two cases. We next propose a measure of information asymmetry.

One of the fundamental manifestations of asymmetric information is that the security payoff conditional variance for the uninformed is greater than that for the informed, i.e.,

$$\text{Var}(\tilde{V}|\mathcal{I}_U) - \text{Var}(\tilde{V}|\mathcal{I}_I) = \left( \left( \frac{\sigma_\varepsilon^2 + \sigma_V^2}{\sigma_V^2} \right)^2 \left( 1 + \frac{\sigma_V^4 \sigma_\eta^2}{\delta^2 \sigma_\varepsilon^4 \sigma_{VN}^2 \sigma_X^2} \right) \frac{1}{\sigma_\eta^2} + \frac{\sigma_\varepsilon^2 + \sigma_V^2}{\sigma_V^4} \right)^{-1} \geq 0. \quad (29)$$

The greater this conditional variance difference, the greater the information asymmetry. This difference is monotonically increasing in  $\sigma_\eta^2$ ,  $\sigma_{VN}^2$  and  $\sigma_X^2$ , but nonmonotonic in  $\sigma_\varepsilon^2$  and  $\sigma_V^2$ .<sup>29</sup> A change in  $\sigma_{VN}^2$  would change the correlation between the nontraded asset and the risky security while a change in  $\sigma_X^2$  would change the unconditional liquidity shock uncertainty. In addition to the undesirable nonmonotonicity, a change in  $\sigma_\varepsilon^2$  or  $\sigma_V^2$  would also change the quality of aggregate information about the security payoff. In contrast, a change in  $\sigma_\eta^2$  only changes the information asymmetry but not the quality of aggregate information or the unconditional liquidity shock uncertainty or the correlation between the nontraded asset and the risky security. Accordingly, to isolate the impact of information asymmetry in the subsequent analysis, we use  $\sigma_\eta^2$  as the measure of information asymmetry. Similar idea behind the noisiness of the public signal ( $\sigma_\eta^2$ ) about the private signal serving as a measure of information asymmetry extends to other models with information asymmetry. For example, in a model where informed investors have heterogeneous private information, one can still use the noisiness of a public signal that has already been reflected in every private signal to measure the information asymmetry.

## 4.2. Bid-ask spread, market depths, and trading volume

The following proposition implies that in contrast to most of the existing literature (e.g., Glosten and Milgrom (1985)), not only ex post bid-ask spreads (i.e., spreads after signal real-

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<sup>29</sup>The nonmonotonicity follows because as  $\sigma_\varepsilon^2$  decreases or  $\sigma_V^2$  increases, the conditional covariance magnitude  $\left| \frac{\sigma_\varepsilon^2}{\sigma_V^2 + \sigma_\varepsilon^2} \sigma_{VN} \right|$  decreases, thus the noise from the hedging demand decreases and hence the conditional security payoff variance of the uninformed may get closer to that of the informed.

izations) but also expected bid-ask spreads across all realizations can *decrease* as information asymmetry increases.

**Proposition 1** 1. *The reservation price difference  $\Delta$  is normally distributed with mean  $\mu_D$  and variance  $\sigma_D^2$ , where*

$$\mu_D = \delta\rho_I(1 - \rho_U)\sigma_V^2\bar{\theta}, \quad \sigma_D^2 = h^2\sigma_X^2 - \rho_I(1 - \rho_U)\sigma_V^2, \quad (30)$$

*which implies that the expected bid-ask spread is equal to:*

$$E[A^* - B^*] = \frac{2\sigma_D\mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) + \mu_D\left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right)}{2}, \quad (31)$$

*where  $\mathbf{n}$  and  $\mathbf{N}$  are respectively the pdf and cdf of the standard normal distribution.*

2. *The expected bid-ask spread decreases with information asymmetry  $\sigma_\eta^2$  if and only if*

$$\mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) - \delta\bar{\theta}\sigma_D\left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right) > 0, \quad (32)$$

*which is always satisfied when  $\bar{\theta} = 0$  or  $\mu_D$  is small enough.*

3. *The expected bid-ask spread increases with both the liquidity shock volatility  $\sigma_X$  and the covariance magnitude  $|\sigma_{VN}|$ .*

Because  $\rho_U$  goes to 1 as  $\sigma_X^2$  goes to 0 and  $\rho_I$  goes to 0 as  $\sigma_\varepsilon^2$  goes to  $\infty$ , Part 2 of Proposition 1 implies that for small enough  $\bar{\theta}$  or  $\sigma_X^2$  or large enough  $\sigma_\varepsilon^2$ , which leads to small enough  $\mu_D$ , the expected spread decreases with information asymmetry  $\sigma_\eta^2$ . Therefore, the expected spread with even large information asymmetry (e.g.,  $\sigma_\eta^2 = \infty$ ) can be smaller than that with symmetric information. Consistent with these results, Figure 2 shows that when  $\sigma_\varepsilon = 1.2$ , for example, the expected spread decreases with information asymmetry even when information asymmetry is large.

The fundamental driving force of the results on expected spreads is that information asymmetry can reduce the reservation price difference between the buyer and the seller because of the well known adverse selection effect. A sell order by the informed on average



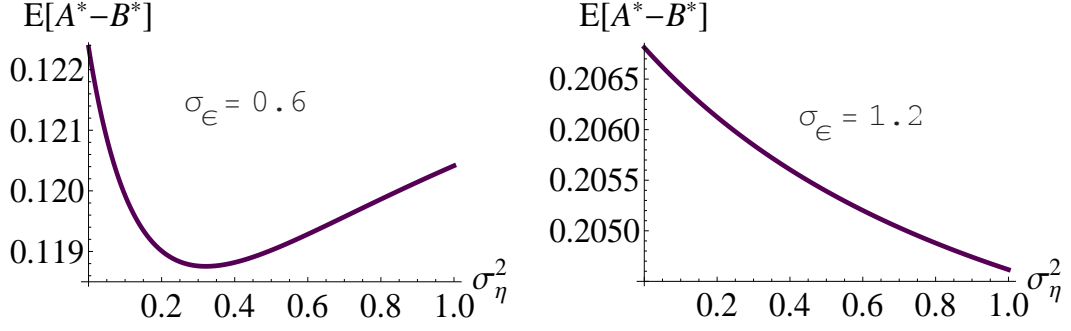


Figure 2: Expected bid-ask spread against information asymmetry  $\sigma_\eta^2$ . The default parameter values are:  $\delta = 1$ ,  $\bar{\theta} = 4$ ,  $\bar{V} = 3$ ,  $N_I = 100$ ,  $N_U = 1000$ ,  $\sigma_V = 0.4$ ,  $\sigma_X = 1$ , and  $\sigma_{VN} = 0.8$ .

conveys negative information about the asset payoff, and therefore the uninformed's reservation price becomes lower and they are thus only willing to purchase it at a lower ask price. Similarly, a buy order by the informed on average implies positive information about the payoff, and therefore the uninformed's reservation price becomes higher and they thus demand a higher bid price to sell it for. As information asymmetry increases, the informed's reservation price on average gets closer to that of the informed, and as a result the average spread goes down. If the uninformed have an initial endowment of the asset ( $\bar{\theta} > 0$ ), then there is an opposing force: as information asymmetry increases, the uncertainty about the value of the initial endowment increases, and thus the uninformed are willing to accept a lower bid price to sell it for. This opposing force drives down the bid price when the informed buy and thus can drive up the spread. Accordingly, our model predicts that in markets where market makers have significant market power and the current holdings of the uninformed are small, the average spread decreases with information asymmetry. Next, we provide more detailed explanations of this result.

We can rewrite the reservation price difference (19) as

$$\Delta = \underbrace{h\hat{X}_I}_{\text{hedging effect}} + \underbrace{\left(E[\tilde{V}|\mathcal{I}_I] - E[\tilde{V}|\mathcal{I}_U]\right)}_{\text{estimation error effect}} + \underbrace{\left(\delta\text{Var}[\tilde{V}|\mathcal{I}_U]\bar{\theta} - \delta\text{Var}[\tilde{V}|\mathcal{I}_I]\bar{\theta}\right)}_{\text{estimation risk effect}}, \quad (33)$$

where the first term is from the difference in the hedging demand (“hedging effect”) between the informed and the uninformed, the second term is the difference in the estimation of the expected security payoff (“estimation error effect”), and the third term is the difference in the risk premium required for the estimation risk (“estimation risk effect”). Consider first the simplest case where  $\bar{\theta} = 0$ , i.e., there is no estimation risk effect. On average, hedging effect and estimation error effect are equal to zero, and thus the expected reservation price difference is zero. However, because the spread is proportional to the *absolute value* of the reservation price difference, the expected spread becomes greater both when the reservation price difference is more positive and when it is more negative. Therefore, the expected spread increases as the volatility of the reservation price difference increases. As information asymmetry increases, the volatility of the reservation price difference becomes smaller because of the adverse selection effect of the information asymmetry. More specifically, for given changes in  $\hat{S}$  (that determines the order size of the informed) and in the public signal  $\hat{S}_s$ , as information asymmetry  $\sigma_\eta^2$  increases, the uninformed attribute a greater portion of the change in  $\hat{S}$  to the change in the private signal  $\hat{s}$ ,<sup>30</sup> reflecting the adverse selection effect, and also put less weight on the public signal. Therefore, in the estimation of the expected payoff, as information asymmetry increases, the uninformed have closer weights on the private signal  $\hat{s}$  and the public signal  $\hat{S}_s$  to those of the informed. Thus, the estimation error effect becomes less sensitive to realizations of  $\hat{S}$  and  $\hat{S}_s$ . Because the hedging effect does not change with information asymmetry, the volatility of the reservation price difference (which is equal to the sum of the hedging effect and the estimation error effect when  $\bar{\theta} = 0$ ) decreases as information asymmetry increases, and so does the expected bid-ask spread. If the uninformed have some initial endowment of the asset, then the uninformed have a higher risk premium and thus on average a lower reservation price than the informed. Therefore, on average the informed buy at the ask and the uninformed sell at the bid. As the information asymmetry increases, the reservation price of the uninformed becomes lower and the expected spread gets greater, because the uninformed’s uncertainty about the value of the initial holdings increases.

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<sup>30</sup>I.e.,  $\rho_U(1 - \rho_X)$  in (15) increases with  $\sigma_\eta^2$ .

With the understanding of the main intuition behind the result on expected spread and of the fact that the reservation price of the informed does not depend on the information asymmetry  $\sigma_\eta^2$ , it is clear that, as also confirmed by the generalized model presented later, as long as the uninformed's estimation risk premium is small, then expected spread decreases with information asymmetry. For most securities, on average an uninformed investor has small estimation risk premium, either because the investor has small holdings (e.g., for a retail investor  $\bar{\theta}$  is small) or because the risk aversion toward the estimation risk is low (e.g., for investors who have offsetting positions elsewhere  $\delta$  is small). Accordingly, one empirically testable implication is that in markets where market makers have significant market power and can offset their trades relatively frequently (e.g., relatively active derivative markets), average spreads decrease with information asymmetry.

Proposition 1 also implies that as liquidity shocks become more volatile or the payoffs of the security and the nontraded asset covary more, the expected bid-ask spread increases. Intuitively, as  $\sigma_X^2$  or  $|\sigma_{VN}|$  increases, the volatilities of the hedging effect, the estimation error effect, and the estimation risk effect all increase. Therefore, the expected spread increases.

Because market makers face both information asymmetry and inventory risk, it would be helpful to separate the effects of information asymmetry and inventory risk on equilibrium asset prices and bid-ask spreads. However, it seems impossible to completely separate these effects in every single case for every economic variable of interest because in general these two effects interact with each other. On the other hand, we can separate them for some important economic variables in some important cases. First, clearly, in the symmetric information case, there is no information asymmetry effect. Second, the effect of inventory risk is through the market maker's risk aversion. For example, if the market maker were risk neutral, then the market maker's inventory risk would have no effect on asset prices. Because the spread is determined by the reservation price difference between the informed and the uninformed and this difference is independent of the market maker's risk aversion, the spread is not affected by the market maker's inventory risk. Therefore our results in Proposition 1 and Figure 2 on how information asymmetry affects expected spread are free of the inventory risk effect.

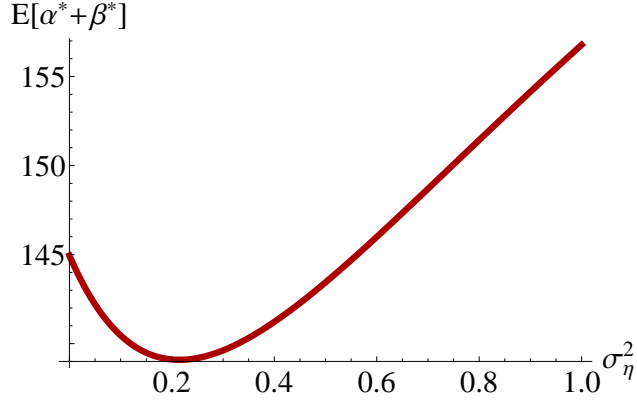


Figure 3: The expected trading volume in equilibrium against information asymmetry  $\sigma_\eta^2$ . The default parameter values are  $\delta = 1$ ,  $\bar{\theta} = 4$ ,  $\bar{V} = 3$ ,  $N_M = 1$ ,  $N_I = 100$ ,  $N_U = 1000$ ,  $\sigma_\varepsilon = 0.8$ ,  $\sigma_V = 0.8$ ,  $\sigma_{VN} = 0.8$ , and  $\sigma_X = 2$ .

Next we examine how expected market depths and trading volume change with information asymmetry and liquidity shock volatility.

**Proposition 2** 1. *If  $N_U$  is large enough, then the expected trading volume increases with information asymmetry, i.e.,  $\frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_\eta^2} > 0$ , if and only if the expected spread increases with information asymmetry.*

2. *As the liquidity shock volatility  $\sigma_X$  or the covariance magnitude  $|\sigma_{VN}|$  increases, the expected trading volume increases.*

As many studies of asymmetric information show (e.g., Akerlof (1970)), information asymmetry decreases trading volume because of the well known “lemons” problem. In contrast, as shown in Part 1 of Proposition 2 and Figure 3, the average trading volume can increase with information asymmetry when the population of the uninformed investors is relatively large. This is because expected trading volume increases with the expected magnitude of the reservation price difference, which can increase with information asymmetry when the marginal impact of the adverse selection effect on each uninformed investor is small that occurs when their population size is large. In addition, because as the liquidity shock volatility or the covariance magnitude  $|\sigma_{VN}|$  increases, the expected magnitude of the reservation price difference increases as implied by Part 2 of Proposition 1, so does the expected trading volume.

## 5. A generalized model

To simplify exposition, in the main model studied above we assume that all investors have the same risk aversion, the same initial inventory, the same date 1 resale value of the security, and only the informed have private information and liquidity shocks. In this section, we relax these assumptions and still, the generalized model is tractable and solved in closed-form.

This generalized model can be used to conduct many interesting analyses such as the effect of a market maker's inventory (e.g., Garman (1976)), private information (Van der Wel et. al. (2009)), and liquidity shocks (e.g., Acharya and Pedersen (2005)) on asset prices. Let  $\bar{\theta}_i$ ,  $\delta_i$ ,  $\hat{X}_i$ ,  $\tilde{V}_i$  and  $\mathcal{I}_i$  denote respectively the initial inventory, risk aversion coefficient, liquidity shock, date 1 resale value of the security and information set for a type  $i$  investor for  $i \in \{I, U, M\}$ . Then by the same argument as before, a type  $i$  investor's reservation price can be written as

$$P_i^R = E[\tilde{V}_i|\mathcal{I}_i] - \delta_i \text{Cov}[\tilde{V}_i, \tilde{N}|I_i]\hat{X}_i - \delta_i \text{Var}[\tilde{V}_i|\mathcal{I}_i]\bar{\theta}_i, \quad i \in \{I, U, M\}. \quad (34)$$

Let  $\Delta_{ij} := P_i^R - P_j^R$  denote the reservation price difference between type  $i$  and type  $j$  investors for  $i, j \in \{I, U, M\}$ . In this generalized model, there are eight cases corresponding to eight different trading direction combinations of the informed and the uninformed, as illustrated in Figure 4.<sup>31</sup> Figure 4 shows that the trading directions are determined by the ratio of the reservation price difference between the informed and the uninformed ( $\Delta_{IU}$ ) to the reservation price difference between the uninformed and the market maker ( $\Delta_{UM}$ ). When the magnitude of this ratio is large enough (Cases (1) and (5)), the informed and the uninformed trade in opposite directions. If it is small enough (Cases (3) and (7)), on the other hand, they trade in the same direction. In between, either the informed or the uninformed do not trade.

To save space, we only present the equilibrium results for Cases (1), (2), and (5) in this section, where Cases (1) and (5) are a direct generalization of the main model in Section 2. and Case (2) illustrates what happens if some investors do not trade. The rest are similar

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<sup>31</sup>The case where both informed and uninformed do not trade is a measure zero event that occurs only when the reservation prices of *all* investors are exactly the same, i.e., at the origin of the figure.

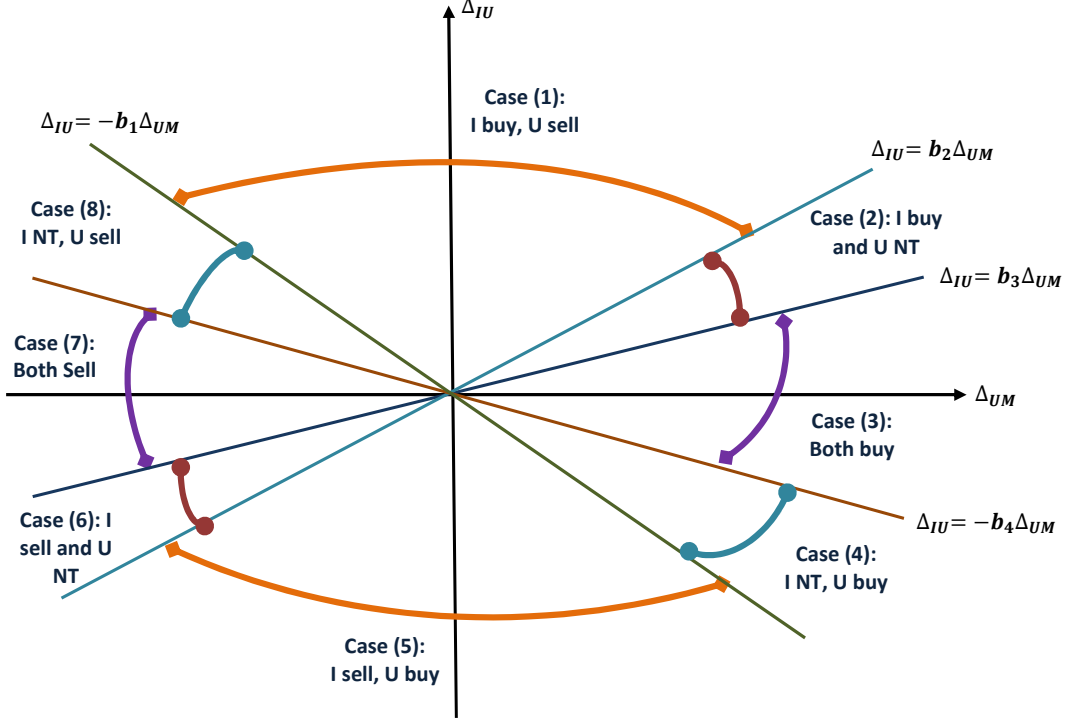


Figure 4: Eight cases of equilibria characterized by the trading directions of the informed and the uninformed, where  $b_1, b_2, b_3$  and  $b_4$  are defined in (35), (36) and (B-1).

and are provided in Appendix B. Define

$$b_1 = \frac{2\delta_U\nu_1}{\delta_M\nu_2N_U + 2\delta_U\nu_1}, \quad b_2 = \frac{2\delta_I}{\delta_M\nu_2N_I}, \quad (35)$$

$$b_3 = \frac{\delta_I}{\delta_I + \delta_M\nu_2N_I} \leq b_2, \quad (36)$$

and

$$C_U := \frac{\nu_2N_I\delta_M}{2\delta_I(\hat{N} + 1)}, \quad (37)$$

where

$$\nu_1 = \frac{\text{Var}[\tilde{V}_U|\mathcal{I}_U]}{\text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad \nu_2 = \frac{\text{Var}[\tilde{V}_M|\mathcal{I}_M]}{\text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad \hat{N} := \frac{\delta_M}{\delta_I}\nu_2N_I + 1 + \frac{\delta_M\nu_2}{\delta_U\nu_1}N_U.$$

**Theorem 2** *For the generalized model, we have:*

1. The informed buy and the uninformed sell (Case (1)) if and only if

$$\Delta_{IU} > \max\{-b_1\Delta_{UM}, b_2\Delta_{UM}\}. \quad (38)$$

The informed sell and the uninformed buy (Case (5)) if and only if

$$\Delta_{IU} < \min\{-b_1\Delta_{UM}, b_2\Delta_{UM}\}.$$

For Cases (1) and (5), the equilibrium bid and ask prices are

$$A^* = P_U^R + C_U\Delta_{IU} - \frac{\Delta_{UM}}{\hat{N} + 1} + \frac{\Delta_{IU}^+}{2},$$

$$B^* = P_U^R + C_U\Delta_{IU} - \frac{\Delta_{UM}}{\hat{N} + 1} - \frac{\Delta_{IU}^-}{2},$$

and the bid-ask spread is

$$A^* - B^* = \frac{|\Delta_{IU}|}{2}; \quad (39)$$

the equilibrium security quantities demanded are

$$\theta_I^* = \frac{(\delta_M\nu_2N_U + 2\delta_U\nu_1)\Delta_{IU} + 2\delta_U\nu_1\Delta_{UM}}{2(\hat{N} + 1)\delta_U\delta_I\nu_1 \text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad (40)$$

$$\theta_U^* = \frac{-\delta_M\nu_2N_I\Delta_{IU} + 2\delta_I\Delta_{UM}}{2(\hat{N} + 1)\delta_U\delta_I\nu_1 \text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad (41)$$

$$\theta_M^* = -(N_I\theta_I^* + N_U\theta_U^*); \quad (42)$$

the equilibrium quote depths are

$$\alpha^* = N_I(\theta_I^*)^+ + N_U(\theta_U^*)^+, \quad (43)$$

$$\beta^* = N_I(\theta_I^*)^- + N_U(\theta_U^*)^-. \quad (44)$$

2. *The informed buy and the uninformed do not trade (Case (2)) if and only if*

$$b_3\Delta_{UM} \leq \Delta_{IU} \leq b_2\Delta_{UM}. \quad (45)$$

*For Case (2), the equilibrium bid and ask prices are*

$$A^* = P_I^R - \frac{\Delta_{IM}}{2 + N_I\nu_2\delta_M/\delta_I}, \quad B^* \leq P_U^R; \quad (46)$$

*the equilibrium security quantities demanded are*

$$\theta_I^* = \frac{\Delta_{IM}}{(2\delta_I + N_I\nu_2\delta_M) \text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad \theta_U^* = 0, \quad \theta_M^* = -N_I\theta_I^*; \quad (47)$$

*the equilibrium quote depths are*

$$\alpha^* = \frac{N_I\Delta_{IM}}{(2\delta_I + N_I\nu_2\delta_M) \text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad \beta^* = 0. \quad (48)$$

In the generalized model, all investors can receive private signals about the security payoff, and thus the market maker and the “uninformed” can both be viewed as informed investors who might have different information. Our main results that expected spread can decrease with information asymmetry and that trading volume can be positively correlated with bid-ask spreads still hold under some conditions in the generalized model. For example, Theorem 2 implies when the market maker and the “uninformed” have the same reservation price, only Cases (1) and (5) are possible, i.e., all investors trade in equilibrium and the market maker trades at both the bid and the ask as long as the reservation price of the informed is different. The spread and trading volume are still both proportional to the absolute value of the reservation price difference between the informed and the “uninformed.” Thus our main results follow by the same intuitions as in the main model, even when the informed have different risk aversion, different initial endowment, and the uninformed and the market maker also have liquidity shocks.

In addition, Part 2 of Theorem 2 shows when the market maker and the uninformed have different reservation prices, there may exist equilibria where some investors do not trade and



the market maker only trades on one side. For example, in Case (2), the reservation price of the uninformed is lower than that of the informed but higher than that of the market maker, the market maker chooses not to trade with the uninformed to avoid buying from the uninformed at a price that is significantly higher than her reservation price. This is because in this case the profit from the spread and the benefit from shifting the trade with the informed are relatively small. Other examples include Cases (3), (4), (6), (7), and (8) presented in Appendix B. This shows that while the market maker can trade both at the bid and at the ask on date 0, she may choose to trade only on one side, as in all the cases except (1) and (5). These equilibria where the market maker trades only on one side at a time imply similar trading behaviors to those implied by a sequential trading model. Cases (1) and (5) are more applicable to more active markets such as OTCQX and OTCQB stock markets where search cost is low, trading frequency is relatively high and thus a market maker has a better estimate of the order flow on the other side, while the rest is more representative of less active markets where search cost is high and time between trades is relatively long (e.g., bond markets and pink sheets markets).<sup>32</sup>

As Theorem 1, Theorem 2 reveals that conditional on the uninformed and the informed trading in the opposite directions (i.e., Cases (1) and (5)), the equilibrium spread only depends on the reservation price difference between the informed and the uninformed, but not on the initial inventory, or the risk aversion, or the private valuation of a market maker. Intuitively, the initial inventory, the risk aversion, and the private valuation of a market maker only affect the certainty equivalent wealth corresponding to the net inventory and a market maker can change the spread without changing inventory by varying the bid and the ask such that equilibrium bid and ask depths change by the same amount. For Case (2), however, the spread in general depends on the characteristics of the market maker as implied by (46) with  $B^*$  set to  $P_U^R$ , this is because the market maker is not making offsetting trades at the bid and thus any trade at the ask changes inventory. This result suggests whether the initial inventory, the risk aversion, or the private valuation of a market maker is important for the spread depends on whether the market maker can relatively frequently

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<sup>32</sup>OTCQX and OTCQB are top tier OTC markets for equity securities (more than 3,700 stocks) with a combined market capitalization of more than \$1 trillion and more than 2 billion daily share trading volume.

make offsetting trades. One empirically testable implication of this result is that in relatively less active markets, the average spread is more sensitive to the inventory level and the private information of a market maker.

Although inventory risk does not affect the spread in Cases (1) and (5), it always affects active depths and prices (i.e., at which trades occur). For example, for Cases (1) and (5), Theorem 2 implies when the initial inventory is large and the market maker's risk aversion is high, she reduces the inventory by lowering both the ask and the bid, which encourages purchases and discourages sales by other investors and thus increases equilibrium ask depth and decreases bid depth.<sup>33</sup> Accordingly, another empirically testable implication is that average ask depth increases, but average bid depth decreases with a market maker's initial inventory.

The generalized version can also serve as a reduced form model that captures some additional costs for liquidation of inventory on date 1. The date 1 resale value  $\tilde{V}_M$  of the security represents what price a market maker can sell the security for on date 1. In the model in Section 2, for expositional simplicity, we assume that the true value of the security is publicly announced on date 1 and thus the resale value on date 1 is the same across all investors and does not vary with market features like search costs. In the generalized model, the date 1 utility function can represent the continuation value function in a multi-period setting and one can adapt the distribution of  $\tilde{V}_M$  to model indirectly market conditions such as searching costs and opacity. For example, when search cost is high, search takes a long time, and the resale value of the security is with large uncertainty, one can approximate this situation by using a low mean and high volatility distribution for  $\tilde{V}_M$ . This is clearly just a reduced form, but likely indirectly captures the first order effect of these features. For example, when search cost is high and the uncertainty about the resale value of the security is high, a market maker charges a higher premium for the security on date 0 and the bid-ask spread increases in a search model (e.g., Duffie, Gârleanu, and Pedersen (2005, 2007)). With a lower mean and higher volatility for  $\tilde{V}_M$ , it can be shown that our model can generate the same result. Intuitively, an increase in the volatility or a decrease in the mean of the resale

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<sup>33</sup>This is because the reservation price of a market maker decreases with the initial inventory and risk aversion, and thus  $\Delta_{UM}$  increases with it.

value on date 1 reduces the value of the security on date 0. In addition, when the market maker buys from one type of investors and the other type do not trade in equilibrium (Cases (6) and (8) in Appendix B), the bid price goes down and ask price does not change, and thus the spread goes up.

## 6. Summary and conclusions

Market makers in over-the-counter markets often make offsetting trades and have significant market power. In this paper, we develop a market making model that captures this market feature as well as other important characteristics such as information asymmetry and inventory risk. We solve the equilibrium bid and ask prices, bid and ask depths, trading volume, and inventory levels in closed-form. Our model can accommodate substantial heterogeneity across investors in preferences, endowment, informativeness, and liquidity demand (as in Section 5). The trading behavior in these equilibria is largely consistent with those observed in a wide range of financial markets.

In contrast to the existing literature, a market maker in our model can optimally shift some trade with the informed to other discretionary investors by adjusting bid or ask. As a result, we find that consistent with empirical evidence, expected bid-ask spreads may *decrease* with information asymmetry and bid-ask spreads can be positively correlated with trading volume. Our analysis shows that when market makers can make offsetting trades and have significant market power, their pricing, liquidity provision, and inventory decisions as well as the impact of information asymmetry on these decisions can be qualitatively different from those predicted by the existing literature. The important empirical implications of our analysis include:

1. In markets where market makers have significant market power and can frequently offset their trades, average spreads decrease with information asymmetry and are positively correlated with trading volume.
2. Average spread is more sensitive to a market maker's inventory level in relatively inactive OTC markets.

3. As a market maker's initial inventory increases, average ask depth increases, but average bid depth decreases.

We hope future empirical investigations will study the importance of a market maker's offsetting trades and market power in affecting asset pricing and market liquidity.

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## Appendix A

In this Appendix, we provide the proofs of Theorem 1 and Propositions 1-2. We also show that our equilibrium is equivalent to the solution to a Nash Bargaining game between other investors and the market maker when the market maker has all the bargaining power.

### A.1 Proof of Theorem 1

We prove the case when  $\Delta < 0$ . In this case, we conjecture that  $I$  investors sell and  $U$  investors buy. Given bid price  $B$  and ask price  $A$ , the optimal demand of  $I$  and  $U$  are:

$$\theta_I^* = \frac{P_I^R - B}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} \quad \text{and} \quad \theta_U^* = \frac{P_U^R - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}. \quad (\text{A-49})$$

Substituting (A-49) into the market clearing condition (5), we get that the market clearing bid and ask depths are:

$$\alpha = N_U \theta_U^* = N_U \frac{P_U^R - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}, \quad \beta = -N_I \theta_I^* = N_I \frac{B - P_I^R}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}. \quad (\text{A-50})$$

The market maker's problem is equivalent to:

$$\max_{A,B} \alpha A - \beta B + (\bar{\theta} + \beta - \alpha) E[\tilde{V}|\mathcal{I}_M] - \frac{1}{2} \delta \text{Var}[\tilde{V}|\mathcal{I}_M] (\bar{\theta} + \beta - \alpha)^2, \quad (\text{A-51})$$

subject to (A-50). The F.O.C with respect to  $B$  (noting that  $\beta$  is a function of  $B$ ) gives us:

$$-\beta - B \frac{N_I}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} + E[\tilde{V}|\mathcal{I}_M] \frac{N_I}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} - \delta \text{Var}[\tilde{V}|\mathcal{I}_M] (\bar{\theta} + \beta - \alpha) \frac{N_I}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} = 0,$$

which is reduced to

$$(\nu N_I + 2)\beta - \nu N_I \alpha = -\frac{N_I \Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}, \quad (\text{A-52})$$

by (9), (18), and expressing  $B$  in terms of  $\beta$  using (A-50).

Similarly using the F.O.C with respect to  $A$ , we get:

$$\alpha + A \left( -\frac{N_U}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]} \right) - E[\tilde{V}|\mathcal{I}_M] \left( -\frac{N_U}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]} \right) + \delta \text{Var}[\tilde{V}|\mathcal{I}_M] (\bar{\theta} + \beta - \alpha) \left( -\frac{N_U}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]} \right) = 0,$$

which can be reduced to

$$(N_U + 2)\alpha - N_U\beta = 0, \quad (\text{A-53})$$

by using (9), expressing  $A$  in terms of  $\alpha$  using (A-50), and noting that  $\mathcal{I}_M = \mathcal{I}_U$ .

Solving (A-53) and (A-52), we can get the equilibrium ask depth and bid depth  $\alpha^*$  and  $\beta^*$  as in (24) and (25). Substituting  $\alpha^*$  and  $\beta^*$  into (A-50), we can get the equilibrium ask and bid prices  $A^*$  and  $B^*$  as in (20) and (21). In addition, by the market clearing condition, we have  $\theta_U^* = \alpha^*/N_U$  and  $\theta_I^* = -\beta^*/N_I$  as in (23). Also,  $A^* < P_U^R$  and  $B^* > P_I^R$  are equivalent to  $\Delta < 0$ , which is exactly the condition we conjecture for  $I$  investors to sell and  $U$  investors to buy. Similarly, we can prove Theorem 1 for the other case where  $I$  investors buy and  $U$  investors sell. *Q.E.D.*

## A.2 Equivalence to the Nash bargaining solution

Now we show that our equilibrium result is equivalent to the solution to a Nash Bargaining game between investors and the market maker when the market maker has all the bargaining power. We show this for the case where  $\Delta < 0$ , the other case follows from the same argument. In this case,  $I$  investors sell at the bid and  $U$  investors buy at the ask. Given  $A$  and  $B$ , let  $\varphi_U$  and  $\varphi_I$  be the vectors of the number of shares the market maker sells to the uninformed and buys from the informed respectively. In general, suppose a market maker has bargaining power of  $\lambda$  and other investors have bargaining power of  $1 - \lambda$ , then the Nash Bargaining solution is to

$$\max_A (U_M(\varphi_U, \varphi_I) - U_M(\varphi_U^{-i}, \varphi_I))^\lambda (U_U(\varphi_U^i) - U_U(0))^{1-\lambda}, \quad i = 1, 2, \dots, N_U, \quad (\text{A-54})$$

$$\max_B (U_M(\varphi_U, \varphi_I) - U_M(\varphi_U, \varphi_I^{-j}))^\lambda (U_I(\varphi_I^j) - U_I(0))^{1-\lambda}, \quad j = 1, 2, \dots, N_I, \quad (\text{A-55})$$

where  $U_M(\varphi_U, \varphi_I)$  is the utility of the market maker when she buys  $\varphi_I^j$  from the  $j$ th  $I$  investor and sells  $\varphi_U^i$  to the  $i$ th  $U$  investor given the trades with other investors,  $U_M(\varphi_U^{-i}, \varphi_I)$  (resp.  $U_M(\varphi_U, \varphi_I^{-j})$ ) is the utility when she does not trade with the  $i$ th uninformed investor (resp.,  $j$ th informed investor),  $U_I(\varphi_I^j)$  (resp.  $U_U(\varphi_U^i)$ ) is the utility of the  $j$ th  $I$  investor (resp.,  $i$ th  $U$  investor) when he trades, and  $U_I(0)$  (resp.  $U_U(0)$ ) is the utility of an  $I$  (resp.,

a  $U$ ) investor when he does not trade. If  $\lambda = 1$ , then (A-54) and (A-55) are respectively equivalent to

$$\max_A \left[ A\varphi_U^i(A) + \left( \bar{\theta} + \sum_{j=1}^{N_I} \varphi_I^j - \varphi_U^i(A) - \sum_{k \neq i}^{N_U} \varphi_U^k \right) E[\tilde{V}|\mathcal{I}_M] - 1/2\delta \text{Var}[\tilde{V}|\mathcal{I}_M] \left( \bar{\theta} + \sum_{j=1}^{N_I} \varphi_I^j - \varphi_U^i(A) - \sum_{k \neq i}^{N_U} \varphi_U^k \right)^2 \right], \quad (\text{A-56})$$

and

$$\max_B \left[ -B\varphi_I^j(B) + \left( \bar{\theta} + \varphi_I^j(B) + \sum_{k \neq j}^{N_I} \varphi_I^k - \sum_{i=1}^{N_U} \varphi_U^i \right) E[\tilde{V}|\mathcal{I}_M] - 1/2\delta \text{Var}[\tilde{V}|\mathcal{I}_M] \left( \bar{\theta} + \varphi_I^j(B) + \sum_{k \neq j}^{N_I} \varphi_I^k - \sum_{i=1}^{N_U} \varphi_U^i \right)^2 \right], \quad (\text{A-57})$$

where

$$\varphi_U^i(A) = \frac{P_U^R - A}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}, \quad \varphi_I^j(B) = \frac{B - P_I^R}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}$$

are the optimal demand schedules.

Using first order conditions and symmetry among I and U investors (i.e.,  $\varphi_U^i = \varphi_U^k$ ,  $\varphi_I^j = \varphi_I^k$ , for all  $i, j, k$ ) gives us

$$N_I \varphi_I^j = (N_U + 2) \varphi_U^i, \quad (\nu N_I + 2) \varphi_I^j - \nu N_U \varphi_U^i = -\frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}, \quad (\text{A-58})$$

which is equivalent to (A-52) and (A-53) because  $\alpha = N_U \varphi_U^i$  and  $\beta = N_I \varphi_I^j$ . It follows that

$$\varphi_U^i = -\frac{\nu N_I}{2(\bar{N} + 1)} \frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}, \quad i = 1, 2, \dots, N_U, \quad \varphi_I^j = -\frac{N_U + 2}{2(\bar{N} + 1)} \frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}, \quad j = 1, 2, \dots, N_I, \quad (\text{A-59})$$

$$A = P_U^R - \delta \text{Var}[\tilde{V}|\mathcal{I}_U] \varphi_U^i = P_U^R + \frac{\nu N_I}{2(\bar{N} + 1)} \Delta, \quad B = P_I^R + \delta \text{Var}[\tilde{V}|\mathcal{I}_I] \varphi_I^j = P_U^R + \frac{\nu N_I}{2(\bar{N} + 1)} \Delta - \frac{\Delta^-}{2}. \quad (\text{A-60})$$

Comparing to the results in Theorem 1 and noting that  $\Delta < 0$ ,  $\theta_U^* = \varphi_U^i$  and  $\theta_I^* = -\varphi_I^j$  (because  $I$  sells in this case), we have that our equilibrium is equivalent to the solution to

a Nash Bargaining game between investors and the market maker when the market maker has all the bargaining power.

### A.3 Proofs of Propositions 1-2

**Proof of Proposition 1:** Part 1:

$$\Delta = \rho_I(1 - \rho_U) \left( \hat{s} + \frac{h}{\rho_I} \left( 1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2} \right) \hat{X}_I - \frac{\sigma_V^2}{\rho_I \sigma_\eta^2} \hat{\eta} + \delta \sigma_V^2 \bar{\theta} \right),$$

which implies that  $\Delta$  is normally distributed with mean  $\mu_D = \delta \rho_I(1 - \rho_U) \sigma_V^2 \bar{\theta}$  and variance

$$\sigma_D^2 = \rho_I^2(1 - \rho_U)^2 \left( \sigma_V^2 + \sigma_\varepsilon^2 + \left( \frac{h}{\rho_I} \left( 1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2} \right) \right)^2 \sigma_X^2 + \frac{\sigma_V^4}{\rho_I^2 \sigma_\eta^2} \right) = h^2 \sigma_X^2 - \rho_I(1 - \rho_U) \sigma_V^2, \quad (\text{A-61})$$

where the last equality follows from simplification using the law of total variance. Direct integration then yields (31).

Part 2: Taking the derivative of the right hand side of (31) with respect to  $\rho_U$ , we have

$$\frac{\partial E[A^* - B^*]}{\partial \rho_U} = \frac{\rho_I \sigma_V^2}{2\sigma_D} \left[ \mathbf{n} \left( \frac{\mu_D}{\sigma_D} \right) - \delta \bar{\theta} \sigma_D \left( 2\mathbf{N} \left( \frac{\mu_D}{\sigma_D} \right) - 1 \right) \right].$$

Because  $\rho_U$  is decreasing in  $\sigma_\eta^2$ , we have (32). When  $\bar{\theta} = 0$  or  $\mu_D$  is small enough, the above expression is always positive.

Part 3: It can be shown that  $\mu_D$  increases but  $\mu_D/\sigma_D$  decreases in  $\sigma_X^2$ .  $\frac{\partial E[A^* - B^*]}{\partial \sigma_X^2} > 0$  then follows from taking derivatives with respect to  $\mu_D$  and  $\mu_D/\sigma_D$  after factoring out  $\mu_D$  in (31). Similarly, it follows from straightforward (but tedious) computation that  $\frac{\partial E[A^* - B^*]}{\partial |\sigma_{VN}|} > 0$ . *Q.E.D.*

**Proof of Proposition 2:** Part 1: From the expression of trading volume in Theorem 1, we have

$$\text{Sign} \left( \frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_\eta^2} \right) = \text{Sign} \left( \frac{\partial E|\Delta|}{\partial \sigma_\eta^2} - \frac{E|\Delta|}{\bar{N} + 1} \frac{\partial \bar{N}}{\partial \sigma_\eta^2} \right). \quad (\text{A-62})$$

Because  $\frac{\partial \bar{N}}{\partial \sigma_\eta^2} = A_1 N_I$ , where  $A_1 := \frac{\rho_I \rho_X^2 \rho_U^2}{\sigma_\varepsilon^2}$ . Therefore, if  $N_U$  is large enough, we have  $\frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_\eta^2} > 0$  if and only if  $\frac{\partial E[\Delta]}{\partial \sigma_\eta^2} > 0$ .

Part 2: It can be shown that  $\frac{\partial(\mu_D/(\bar{N}+1))}{\partial \sigma_X^2} > 0$ . From (26), (31), and taking derivatives of  $\mu_D/(\bar{N}+1)$  and  $\mu_D/\sigma_D$  after factoring out  $\mu_D$  with respect to  $\sigma_X^2$ , we have  $\frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_X^2} > 0$ . Similarly, straightforward computation yields  $\frac{\partial E[\alpha^* + \beta^*]}{\partial |\sigma_{VN}|} > 0$ . *Q.E.D.*

## A.4 An Extension with multiple market makers

In contrast to the standard microstructure literature where market makers directly choose market prices (Bertrand competition), for this multiple market maker case, we model the oligopolistic competition among the market makers as a Cournot competition that is well studied and understood.<sup>34</sup> Specifically, we assume that market makers simultaneously choose how much to buy at bid given the inverse supply function (a function of the market makers' purchasing quantity) of all other participants and how much to sell at ask given the inverse demand function (a function of the market makers' selling quantity) of all other participants. The posted bid and ask prices are the required prices to achieve the optimal amount market makers choose to trade. The equilibrium definition for the main model can be directly extended to this multiple market maker case. *Note that in the monopoly case, the Bertrand competition formulation and the demand/supply function competition formulation (e.g., Vives (2011)) yield the same results as the Cournot competition formulation.*

Let  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_{N_M})^\top$  and  $\beta = (\beta_1, \beta_2, \dots, \beta_{N_M})^\top$  be the vector of the number of shares market makers sell at ask (i.e., ask depth) and buy at bid (i.e., bid depth) respectively. Given the demand schedules of the informed and the uninformed ( $\theta_I^*(A, B)$  and  $\theta_U^*(A, B)$ ), the bid price  $B(\beta)$  (i.e., the inverse supply function) and the ask price  $A(\alpha)$  (i.e., the inverse demand function) can be determined by the following stock market clearing conditions at the bid and ask prices.

$$\sum_{j=1}^{N_M} \alpha_j = \sum_{i=I, U} N_i \theta_i^*(A, B)^+, \quad \sum_{j=1}^{N_M} \beta_j = \sum_{i=I, U} N_i \theta_i^*(A, B)^-, \quad (\text{A-63})$$

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<sup>34</sup>Similar extension and closed form results can be obtained for the generalized model.

where the left-hand sides represent the total sales and purchases by market makers respectively and the right-hand sides represent the total purchases and sales by other investors respectively.

Then for  $j = 1, 2, \dots, N_M$ , the designated market maker  $M_j$ 's problem is

$$\max_{\alpha_j \geq 0, \beta_j \geq 0} E \left[ -e^{-\delta \tilde{W}_{M_j}} | \mathcal{I}_M \right], \quad (\text{A-64})$$

where

$$\tilde{W}_{M_j} = \alpha_j A(\alpha) - \beta_j B(\beta) + (\bar{\theta} + \beta_j - \alpha_j) \tilde{V}. \quad (\text{A-65})$$

We now state the multiple-market-maker version of Theorem 1 for the  $N_M \geq 1$  case (setting  $N_M = 1$  yields Theorem 1).

**Theorem 1 (with multiple market makers):**

1. *The equilibrium bid and ask prices are*

$$A^* := A(\alpha^*) = P_U^R + \frac{\nu N_M N_I}{(N_M + 1)(\bar{N} + 1)} \Delta + \frac{\Delta^+}{N_M + 1},$$

$$B^* := B(\beta^*) = P_U^R + \frac{\nu N_M N_I}{(N_M + 1)(\bar{N} + 1)} \Delta - \frac{\Delta^-}{N_M + 1},$$

and we have  $A^* > P^* > B^*$ , where

$$P^* = \frac{\nu N_I}{\bar{N}} P_I^R + \frac{N_U}{\bar{N}} P_U^R + \frac{N_M}{\bar{N}} P_M^R \quad (\text{A-66})$$

is the equilibrium price of a perfect competition equilibrium where market makers are also price takers. The bid-ask spread is

$$A^* - B^* = \frac{|\Delta|}{N_M + 1} = \frac{(1 - \rho_U) \left| \left( 1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2} \right) \hat{S} - \frac{\sigma_V^2}{\sigma_\eta^2} \hat{S}_s + \delta \rho_I \sigma_V^2 \bar{\theta} \right|}{N_M + 1}. \quad (\text{A-67})$$

2. The equilibrium quantities demanded are

$$\theta_I^* = \frac{N_M(N_U + N_M + 1)}{(N_M + 1)(\bar{N} + 1)} \frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}, \quad \theta_U^* = -\frac{\nu N_M N_I}{(N_M + 1)(\bar{N} + 1)} \frac{\Delta}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]}; \quad (\text{A-68})$$

the equilibrium quote depths are

$$\alpha^* = \frac{N_I}{N_M}(\theta_I^*)^+ + \frac{N_U}{N_M}(\theta_U^*)^+, \quad (\text{A-69})$$

$$\beta^* = \frac{N_I}{N_M}(\theta_I^*)^- + \frac{N_U}{N_M}(\theta_U^*)^-, \quad (\text{A-70})$$

which implies that the equilibrium trading volume is

$$N_M(\alpha^* + \beta^*) = \frac{N_M N_I (N_M + 2N_U + 1)}{(N_M + 1)(\bar{N} + 1)} \left( \frac{|\Delta|}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} \right). \quad (\text{A-71})$$

## Appendix B

In this Appendix, we report the remaining results on the generalized model. Define

$$b_4 = \frac{N_U \delta_I + \nu_1 N_I \delta_U}{\delta_I N_U (\hat{N} + 1) + \nu_1 N_I \delta_U} (< b_1). \quad (\text{B-1})$$

The rest of Theorem 2 is as follows.

1. Both the informed and uninformed buy (Case (3)) if and only if

$$-b_4 \Delta_{UM} < \Delta_{IU} < b_3 \Delta_{UM}. \quad (\text{B-2})$$

For Case (3), the equilibrium prices are

$$A^* = \frac{N_I \nu_1 \delta_U P_I^R + N_U \delta_I P_U^R}{N_I \nu_1 \delta_U + N_U \delta_I} - \frac{N_I \nu_1 \delta_U \Delta_{IM} + N_U \delta_I \Delta_{UM}}{(\hat{N} + 1)(N_I \nu_1 \delta_U + N_U \delta_I)}, \quad B^* \leq A^*; \quad (\text{B-3})$$

the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{IM}}{(\hat{N} + 1) \delta_I \text{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad \theta_U^* = \frac{\Delta_{UM}}{(\hat{N} + 1) \delta_U \text{Var}[\tilde{V}_U|\mathcal{I}_U]}, \quad \theta_M^* = -N_I \theta_I^* - N_U \theta_U^*; \quad (\text{B-4})$$

and the equilibrium depths are

$$\alpha^* = N_I \theta_I^* + N_U \theta_U^*, \quad \beta^* = 0. \quad (\text{B-5})$$

2. The informed do not trade and uninformed buy (Case (4)) if and only if

$$-b_1 \Delta_{UM} \leq \Delta_{IU} \leq -b_4 \Delta_{UM}. \quad (\text{B-6})$$

For Case (4), the equilibrium prices are

$$A^* = P_U^R - \frac{\Delta_{UM}}{2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)}, \quad B^* \leq P_I^R; \quad (\text{B-7})$$

the equilibrium security quantities demanded are

$$\theta_I^* = 0, \quad \theta_U^* = \frac{\Delta_{UM}}{(2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)) \delta_U \text{Var}[\tilde{V}_U | \mathcal{I}_U]}, \quad \theta_M^* = -N_U \theta_U^*; \quad (\text{B-8})$$

and the equilibrium depths are

$$\alpha^* = N_U \theta_U^*, \quad \beta^* = 0. \quad (\text{B-9})$$

3. The informed sell and the uninformed do not trade (Case (6)) if and only if

$$b_2 \Delta_{UM} \leq \Delta_{IU} \leq b_3 \Delta_{UM}. \quad (\text{B-10})$$

For Case (6), the equilibrium prices are

$$B^* = P_I^R - \frac{\Delta_{IM}}{2 + N_I \nu_2 \delta_M / \delta_I}, \quad A^* \geq P_U^R; \quad (\text{B-11})$$

the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{IM}}{(2 + N_I \nu_2 \delta_M / \delta_I) \delta_I \text{Var}[\tilde{V}_I | \mathcal{I}_I]}, \quad \theta_U^* = 0, \quad \theta_M^* = -N_I \theta_I^*; \quad (\text{B-12})$$



and the equilibrium depths are

$$\alpha^* = 0, \quad \beta^* = -N_I \theta_I^*. \quad (\text{B-13})$$

4. Both the informed and uninformed sell (Case (7)) if and only if

$$b_3 \Delta_{UM} < \Delta_{IU} < -b_4 \Delta_{UM}. \quad (\text{B-14})$$

For Case (7), the equilibrium prices are

$$B^* = \frac{N_I \nu_1 \delta_U P_I^R + N_U \delta_I P_U^R}{N_I \nu_1 \delta_U + N_U \delta_I} - \frac{N_I \nu_1 \delta_U \Delta_{IM} + N_U \delta_I \Delta_{UM}}{(\hat{N} + 1)(N_I \nu_1 \delta_U + N_U \delta_I)}, \quad (\text{B-15})$$

and  $A^* \geq B^*$ ; the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{IM}}{(\hat{N} + 1) \delta_I \text{Var}[\tilde{V}_I | \mathcal{I}_I]}, \quad \theta_U^* = \frac{\Delta_{UM}}{(\hat{N} + 1) \delta_U \text{Var}[\tilde{V}_U | \mathcal{I}_U]}, \quad (\text{B-16})$$

and the equilibrium depths are

$$\alpha^* = 0, \quad \beta^* = -N_I \theta_I^* - N_U \theta_U^*. \quad (\text{B-17})$$

5. The informed do not trade and the uninformed sell (Case (8)) if and only if

$$-b_4 \Delta_{UM} \leq \Delta_{IU} \leq -b_1 \Delta_{UM}. \quad (\text{B-18})$$

For Case (8), the equilibrium prices are

$$B^* = P_U^R - \frac{\Delta_{UM}}{2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)}, \quad A^* \geq P_I^R. \quad (\text{B-19})$$

the equilibrium security quantities demanded are

$$\theta_I^* = 0, \quad \theta_U^* = \frac{\Delta_{UM}}{(2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)) \delta_U \text{Var}[\tilde{V}_U | \mathcal{I}_U]}, \quad \theta_M^* = -N_U \theta_U^*; \quad (\text{B-20})$$

and the equilibrium depths are

$$\alpha^* = 0, \quad \beta^* = -N_U \theta_U^*. \quad (\text{B-21})$$

**Proof of Theorem 2:** This is similar to the proof of Theorem 1. We only sketch the main steps. First, for each case, conditional on the trading directions (or no trade), we derive the equilibrium depths, prices, and trading quantities, similar to the proof of Theorem 1. Then we verify that under the specified conditions the assumed trading directions are indeed optimal. *Q.E.D.*