Asymmetric Information, Endogenous Illiquidity, and Asset Pricing With Imperfect Competition*

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Abstract

We study how asymmetric information, imperfect competition among market makers, and risk aversion affect equilibrium illiquidity and asset pricing. All the main results are obtained in closed-form. In our model, market power, asymmetric information, and market-making cost drive market illiquidity. This model can potentially explain some of the puzzling empirical findings such as (1) the bid-ask spread can be lower with asymmetric information; (2) the bid-ask spread can be positively correlated with trading volume; (3) stock volatility tends to decrease trading volume; (4) the bid-ask spread is positively correlated with market makers' inventory. In addition, we find that information asymmetry may *reduce* the welfare loss due to market power and market depth always decreases with information asymmetry. Furthermore, the equilibrium number of market makers decreases with the market-making cost and increases with trading volume, and may increase with information asymmetry.

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I. Introduction

The recent financial crisis highlights the importance of understanding the effect of asymmetric information and stock volatility on market illiquidity and trading volume. Some puzzling findings of the vast literature on asymmetric information and market illiquidity include (1) the bid-ask spread can be lower with asymmetric information (e.g., Kini and Mian (1995), Huang and Stoll (1997), Chordia, Roll, and Subrahmanyam (2001)); (2) the bid-ask spread can be positively correlated with trading volume (e.g., Easley and O'Hara (1987, 1992), Chordia, Roll, and Subrahmanyam (2001)); (3) stock volatility tends to decrease trading volume (e.g., Chordia, Roll, and Subrahmanyam (2001)); (4) the bid-ask spread is positively correlated with market makers' inventory (e.g., Hendershott, Moulton, Seasholes (2007)); (5) trading volume may increase with information asymmetry (e.g., Chordia, Roll, and Subrahmanyam (2001)). For example, Huang and Stoll (1997) find that for NYSE stocks the asymmetric information component of the bid-ask spread is *negative* and statistically significant. Chordia, Roll, and Subrahmanyam (2001) find that the effective bid and ask spread is positively correlated with trading volume unconditionally (Table III). They also find that just before GDP announcement, the spread is lower and trading volume is higher (Table V). In addition, high level of market volatility is associated with a reduction in the trading activity (Table V). Hendershott, Moulton, Seasholes (2007) conclude that when market makers' inventory is larger, the bid-ask spread is significantly greater. As far as we know, none of the extant models can explain all of these findings. For example, standard asymmetric information models in the market microstructure literature predict that bid-ask spread increases with asymmetric information, stock volatility does not affect trading volume, higher bid-ask spread reduces trading volume, and inventory is irrelevant for liquidity, in sharp contrast to (1)–(4)(e.g., Copeland and Galai (1983), Glosten and Milgrom (1985), and Easley and O'Hara (1987)).

In this paper, we develop a flexible but tractable equilibrium model to study the impact of asymmetric information, imperfect competition among market makers, and risk aversion on market illiquidity, trading volume, bid and ask prices, bid and ask depths, and social welfare. This model integrates the microstructure framework with the standard asset pricing framework in a unifying setting. Specifically, we consider an economy that consists of three types of risk averse investors: informed traders, uninformed traders, and (uninformed) market makers. All investors optimally choose how to trade a riskfree asset and a risky stock to maximize their expected utility. All investors are endowed with some finite shares of the stock but no riskfree asset. Some investors are also endowed with some nontraded asset (e.g., labor income) whose payoff is correlated with the stock payoff, and thus these investors have also trading demand for hedging. Informed investors have some private information about the expected payoff of the stock before the terminal date and thus they have additional trading demand from the private information. Neither informed traders nor uninformed traders trade strategically. All trades go through the market makers who need to pay a fixed utility cost to make the market. Different from the standard literature which assumes market makers directly post bid and ask prices, we assume that market makers take the demand and supply curves of the informed and uninformed traders as given and choose how much to buy at the bid (bid depth) and how much to sell at the ask (ask depth). We model the competition among market makers for purchasing and selling as the standard Cournot competition, i.e., they take into account the impact of their trades on the market prices when they choose simultaneously how much to buy and sell. As they buy more at the bid, the bid price is driven up and as they sell more at the ask, the ask price is driven down. The equilibrium bid and ask prices are determined by the market clearing conditions at the bid and at the ask. In equilibrium, both the riskfree asset and the stock market clear. We solve the equilibrium bid and ask prices, bid and ask depths in closed forms and conduct an extensive analysis on the the effect of market power and information asymmetry on equilibrium illiquidity and social welfare.

In contrast to the existing models, our model can help explain all of the five puzzling findings listed above. Specifically, we find that the equilibrium bid-ask spread may *decrease* with information asymmetry and thus the bid-ask spread can be lower and trading volume can be higher with asymmetric information. In addition, before the informed observe their private information, the (ex ante) expected bid-ask spread in the asymmetric information case may also be lower than that in the symmetric information case. As investors' trading demand increases or the competition among market makers decreases, the bid-ask spread, the bid and ask depths, and the inventory of the market makers all increase, which implies that the bid-ask spread may be positively correlated with trading volume and inventory level. As stock volatility increases, the hedging demand and the risk bearing capacity of the market makers decrease and so does the trading volume.

To help explain the main intuitions behind our findings, consider the case where only the informed have some endowment of the nontraded asset and their hedging demand for this endowment is positive. In addition, suppose they buy the stock while the uninformed sell it in equilibrium. Unlike "noise traders" as modeled in most of the microstructure models, uninformed investors in our model optimally react to market prices in determining their trades. Define the reservation price as the critical price such that an investor buys (sells) the stock if and only if stock price is lower (higher) than this critical price. Since the informed and the uninformed trade in opposite directions, the bid and ask prices must be between their reservation prices. Let ΔRP denote the difference between the informed's and the uninformed's reservation prices. Since the informed buy and the uninformed sell, we have $\Delta RP > 0$. Similar to the standard result in the classical Cournot competition models, the spread is equal to ΔRP divided by one plus the number of market makers. The difference ΔRP is the sum of three differences across the informed and the informed: (1) the difference in the hedging demand ("hedging demand effect"); (2) the difference in the estimation of the expected stock payoff ("estimation error effect"); and (3) the difference in the risk premium required for estimation risk ("estimation risk effect"). Since only the uninformed are subject to estimation risk and they are risk averse, they require a higher risk premium and thus the estimation risk effect always drives up the reservation price difference ΔRP. In contrast, since the uninformed can overestimate or underestimate the expected stock payoff, the estimation error effect can drive ΔRP downward or upward. When the uninformed overestimate and thus the estimation error effect is negative, the net of the estimation error effect and the estimation risk effect can cancel out some of the hedging demand effect. In these cases, the reservation price difference with asymmetric information can be lower than that with symmetric information and accordingly the bid-ask spread with asymmetric information can be lower than that with symmetric information. Since asymmetric information can result in a smaller bid-ask spread which implies lower trading costs, the trading volume can also be higher with asymmetric information.

As the number of market makers decreases, the competition among market makers goes down, the total number of shares market makers sell or buy decrease. Therefore the ask price goes up and the bid price goes down, which implies that the spread goes up. In addition, if market makers are net buyers, then as the number of market makers decreases, they need to buy more and thus their inventory also increases. Therefore the bid-ask spread can be positively correlated with inventory level. As investors' trading demand increases, the difference in the reservation prices increases, and therefore the spread increases. In addition, the difference between the lower (higher) reservation price and the bid (ask) price also increases. This greater difference drives up the trading volume and bid-ask depths. Therefore, the bid-ask spread, market depth, and trading volume can all be positively correlated. As stock volatility increases, both the hedging benefit from the stock and the risk bearing capacity of market makers decrease, which in turn reduces trading volume.

In addition, we show that as the number of market makers (a proxy for competition) increases, the equilibrium bid-ask spread decreases and trading volume increases. Therefore, our model can also allow for negative correlation between the bid-ask spread and trading volume. Since as the number of market makers increases, the net benefit from being a market maker decreases, the maximum number of market markers that can exist in equilibrium is finite if market making cost is positive. We find that the maximum number of market makers in equilibrium increases in the trading volume and decreases in the market making cost. When the bid-ask spread increases in information asymmetry, so does the maximum number of market makers in equilibrium.

We also find that even though market makers gain from their market power, both the informed and uninformed investors suffer significant welfare loss from the market-power driven illiquidity. More importantly, the market makers' welfare gain is *smaller* than the welfare losses of other investors and thus social welfare is reduced by the presence of market power. This finding suggests the importance of increasing competition among market makers through some systematic mechanism (e.g., expanding electronic markets). It also suggests that some restrictions on bid-ask spreads and bid-ask depths with appropriate compensation for market making may also increase social welfare. In addition, consistent with the finding that asymmetric information may reduce the bid-ask spread, we find that more information asymmetry can *reduce* the social welfare loss due to market power.

Furthermore, we decompose the bid-ask spread into three components: (1) due to market-making cost; (2) due to market power; (3) due to asymmetric information. We show that the percentage of bid-ask spread due to market power decreases with the number of market makers and market-making cost, while the opposite is true for the percentage of bid-ask spread due to market-making cost. The bid-ask spread due to asymmetric information can be negative, consistent with the findings in Huang and Stoll (1997). Finally, we find market depth (the reciprocal of λ in Kyle (1985)) decreases with information asymmetry, the stock volatility, and the investors' risk aversion.

Our paper provides a new framework that integrates standard asset pricing and microstructure models in a unified setting. This framework provides a tractable setting for studying a variety of interesting, but complicated problems such as the relationships among market liquidity, asset pricing, trading volume, and market competition. Standard microstructure models assume both the supply of and the demand for a stock can be infinite, and the market clearing price is set to be the conditional expected payoff. In contrast, as in standard asset pricing models, we assume that the supply of the stock is finite and market makers compete for order flow under the Cournot competition. All investors choose their optimal investment strategies and the equilibrium stock price and interest rate clear both the risk free asset market and the stock market.

In contrast to our model, most of the existing microstructure literature assume risk neutral market makers and perfect competition among them (e.g., Copeland and Galai (1983), Kyle (1985), Glosten and Milgrom (1985)). However, market makers can be very risk-averse in practice (e.g., Lyons (1995)),[1] and the competition among market makers can be far from perfect (e.g., Christie and Schultz (1994), Chen and Ritter (2000), and Biais, Bisière and Spatt (2003)). There also exists a large literature on the effect of illiquidity on portfolio choice and asset pricing (e.g., Constantinides (1986), Vayanos (1998), Lo, Mamaysky and Wang (2004), Liu (2004), Liu and Loewenstein (2002)). In this literature, illiquidity is generally modeled as exogenous transaction costs and therefore the fundamental question of what affects illiquidity (which in turn affects asset pricing) is largely unanswered.[2]

Our model is also related to Kyle (1989), Subrahmanyam (1991), Diamond and Verrechia (1991), and Naik, Neuberger, and Viswanathan (1999). Kyle (1989) con-

[1]The popularity of various hedging trades (e.g., delta hedging) by market makers also suggests they are typically risk averse.

[2]In addition, in most of this literature, it is not clear where transaction costs paid by the investors go and the impact of the agents who receive these transaction costs is thus not examined.

sider the imperfect competition among risk averse informed traders. He shows that informed traders reveal less information when competition is imperfect. Subrahmanyam (1991) studies a noncompetitive market where informed traders and market makers are risk averse and informed traders trade strategically. He finds that increasing the precision of private information intensifies competition between risk averse informed traders and thus can increase market liquidity. Diamond and Verrechia (1991) shows that reducing information asymmetry can increase liquidity and thus can reduce the cost of capital. In addition, security prices may be nonmonotonic in information asymmetry because of the potential exit of market makers. In all these three papers, market makers post a single price, the trading needs of some of the uninformed traders (i.e., "noise traders") are exogenous and thus do not respond to price changes. Therefore if market makers were allowed to post bid and ask prices, then in contrast to our predictions, the bid-ask spread would always be increasing in information asymmetry. Naik, Neuberger, and Viswanathan (1999) examine whether full and prompt disclosure of public-trade details improves the welfare of a risk-averse investor in a two-stage dealership market where one market maker first executes an order and then offset his position by trading with other market makers. Similar to the other three papers, market makers post a single price which is the conditional expected payoff of the stock.

The rest of the paper proceeds as follows. In Section II we present the model. In Section III we solve the case with symmetric information, and in Section IV we derive the equilibrium under asymmetric information. In Section V we examine the impact of asymmetric information on asset prices, illiquidity, and welfare. We conclude in Section VI. All proofs are in the Appendix.

II. The Model

In a one period setting, there are N investors who maximize their expected constant absolute risk aversion (CARA) utility from the terminal wealth on date 1. They can trade one risk-free asset and one risky asset ("stock") on date 0. There is a zero net supply for the risk-free asset, which also serves as the numeraire and thus the risk-free interest rate is normalized to 0. The total supply of the stock is $N\bar{\theta}$ shares and the date 1 payoff of each share is $\tilde{V} = \bar{V} + \hat{F} + \tilde{u}$, where $\bar{V}$ is a constant representing the publicly known expected payoff, $\hat{F}$ is a zero-mean random variable that is realized on date 0 and may be observed only by some investors on date 0 and $\tilde{u}$ is an independent zero-mean random variable that no one can observe until it becomes public on date 1.[3]

Every investor is endowed with $\bar{\theta}$ shares of the stock but no risk-free asset. There are three types of investors: N_I informed investors (I), N_U uninformed investors (U), and $N_M = N - (N_I + N_U)$ potential market makers (M). To become a market maker, an investor must be a potential market maker and must pay a fixed market-making utility cost c on date 0 before making the market. We assume that both N_U and N_I are large such that all I and U investors are price takers and there are no strategic interactions among them or with market makers. In addition to the stock, a type i ($i \in \{I, U, M\}$) investor is also endowed with $\hat{X}_i$ units of a non-traded risky asset on date 0. The nontraded asset has a per-unit payoff of $\tilde{N}$ that is realized and becomes public on date 1 and a covariance of σ_{uN} with $\tilde{u}$. $\hat{X}_i$ ($i \in \{I, U, M\}$) is a random variable that is realized and only directly known to type-i investor on date 0.

[3]Throughout this paper, "bar" variables are constant, "tilde" variables are realized on date 1 and "hat" variables are realized on date 0.

For tractability, we assume that $\hat{F}$, $\hat{X}_i$, $\tilde{u}$, and $\tilde{N}$ are all zero-mean normally distributed random variables with variances σ_F^2, σ_i^2, σ_u^2, and σ_N^2 respectively, for $i \in \{I, U, M\}$.

All trades must go through market makers. Specifically, given market bid price B and ask price A, I and U investors sell to market makers at the bid or buy from them at the ask.

Given that investors of type i ($i \in \{I, U, M\}$) are ex ante identical, we restrict our analysis to symmetric equilibria where all type i investors adopt the same trading strategy. Let I_i represent investor i's information set on date 0 for $i \in \{I, U, M\}$. Given B and A, for $i \in \{I, U\}$, investor i's problem is

$$\max_{\theta_i} E[-e^{-\delta_i \tilde{W}_i} | I_i], \tag{1}$$

subject to the budget constraint

$$\tilde{W}_i = (\bar{\theta} - \theta_i) \left(B \mathbf{1}_{\{\theta_i < \bar{\theta}\}} + A \mathbf{1}_{\{\theta_i > \bar{\theta}\}} \right) + \theta_i \tilde{V} + \hat{X}_i \tilde{N}, \tag{2}$$

where $\delta_i > 0$ is the absolute risk-aversion parameter, θ_i is the number of shares held until date 1 by investor i, and $\mathbf{1}$ is the indicator function.

Since all trades must go through market makers, market makers can have market powers even though the number of market makers is relatively small in the economy. To model the oligopolistic game among the market makers, we use the notion of the Cournot competition that is well studied and understood in economics, especially in the industrial organization literature.[4] Specifically, we assume that market makers

[4] An obvious alternative form of competition is Bertrand competition where market makers compete directly by choosing the bid and ask prices. However, the standard Bertrand competition model

simultaneously choose the optimal number of shares to sell at ask and to buy at bid, taking into account the equilibrium price impact of their trades.

Let $\alpha = (\alpha_1, \alpha_2, ..., \alpha_{N_M})^\top$ and $\beta = (\beta_1, \beta_2, ..., \beta_{N_M})^\top$ be the vector of the number of shares market makers want to buy (bid depth) and sell (ask depth) respectively. Then the potential market maker M_j's $(j = 1, 2, ..., N_M)$ problem is

$$\max_{\alpha_j \geq 0, \beta_j \geq 0, R_j \in \{0,1\}} E\left[\left(-e^{-\delta_M \tilde{W}_{M_j}} - c\right) R_j + \left(-e^{-\delta_M \tilde{W}_{M_j}}\right)(1 - R_j) | I_M\right], \quad (3)$$

subject to the budget constraint

$$\begin{aligned} \tilde{W}_{M_j} &= \left(\beta_j A(\alpha_j, \beta_j, \alpha_{-j}, \beta_{-j}) - \alpha_j B(\alpha_j, \beta_j, \alpha_{-j}, \beta_{-j}) + (\bar{\theta} + \alpha_j - \beta_j)\tilde{V}\right) R_j \\ &\quad + \left(\beta_j B - \alpha_j A + (\bar{\theta} + \alpha_j - \beta_j)\tilde{V}\right)(1 - R_j) + \hat{X}_{M_j}\tilde{N}, \end{aligned} \quad (4)$$

where $\delta_M > 0$ is the absolute risk-aversion parameter for market makers and $R_j \in \{1, 0\}$ indicates the choice of being a market maker or not. Note that if the potential market maker M_j chooses to be a market maker (i.e., $R_j = 1$), then she takes into account the price impact of her own trades. On the other hand, if M_j chooses not to be a market maker (i.e., $R_j = 0$), then she takes prices B and A as given, as I and U investors do.

In equilibrium, the bid price $B^*(\alpha^*, \beta^*)$ and the ask price $A^*(\alpha^*, \beta^*)$ must solve the following stock market clearing conditions.[5]

$$\sum_{j=1}^{N_M} \alpha_j^* = \sum_{i=I,U} N_i(\bar{\theta} - \theta_i^*)\mathbf{1}_{\{\theta_i^* < \bar{\theta}\}}, \quad \sum_{j=1}^{N_M} \beta_j^* = \sum_{i=I,U} N_i(\theta_i^* - \bar{\theta})\mathbf{1}_{\{\theta_i^* > \bar{\theta}\}}, \quad (5)$$

yields the counterfactual result that it only takes two competitors to reach the perfect competition equilibrium price. We view the posted bid and ask prices as the required prices to achieve the optimal amounts market makers choose to buy or sell.

[5]The risk-free asset market will be automatically cleared by the Walras' law.

where the left-hand sides represent the total purchases and sales by market makers respectively and the right-hand sides represent the total sales and purchases by other investors respectively.

This leads to our definition of the Nash equilibrium of the Cournot competition where all potential market makers choose to be market makers.[6]

Definition 1 *An equilibrium* $(\theta_I^*, \theta_U^*, \alpha^*, \beta^*, A^*, B^*)$ *is such that*

1. θ_i^* *(*$i \in \{I, U\}$*) solves investor i's Problem* (1) *for given* A^* *and* B^**;*

2. α_j^**,* β_j^* *and* $R_j^* = 1$ *solve potential market maker* M_j*'s Problem (3), for* $j = 1, 2, ..., N_M$*; and*

3. A^* *and* B^* *clear both the stock and the risk-free asset markets.*

III. The Equilibrium under Symmetric Information

As a benchmark, in this section we study the case with symmetric information where investors' non-traded asset endowments and $\hat{F}$ are publicly known at date 0. In this case, the equilibrium illiquidity arises from the market making cost and the market power of market makers.

[6] This is without loss of generality, because the case where some potential market makers choose not to be market makers is equivalent to the case with less potential market makers.

A. Perfect Competition with Symmetric Information

We first examine the simplest subcase where all investors are price takers and there is no market-making cost.[7] With perfect competition and zero market-making cost, equilibrium bid and ask prices must be the same and thus all investors trade at the same price. Let P_s^* denote the equilibrium stock price in this subcase. With symmetric information, investors' information sets are such that $I_I = I_U = I_M = \{\hat{F}, \hat{X}_I, \hat{X}_U, \hat{X}_M, P_s^*\}$. Therefore, type-$i$ $(i = I, U, M)$ investor's problem is equivalent to

$$\max_{\theta_i} \quad -e^{-\delta_i(\bar{\theta}-\theta_i)P_s^* - \delta_i\theta_i\bar{V} - \delta_i\theta_i\hat{F}} E[e^{-\delta_i\theta_i\tilde{u} - \delta_i\hat{X}_i\tilde{N}}], \tag{6}$$

which can be simplified to

$$\min_{\theta_i} \quad \delta_i\theta_i(P_s^* - \bar{V} - \hat{F}) + \frac{1}{2}\delta_i^2(\theta_i^2\sigma_u^2 + \hat{X}_i^2\sigma_N^2 + 2\theta_i\sigma_{uN}\hat{X}_i). \tag{7}$$

From the first order condition, we get:

$$P_s^* - \bar{V} - \hat{F} - \hat{H}_i - \delta_i\theta_i\sigma_u^2 = 0, \tag{8}$$

which leads to the optimal demand

$$\theta_i^* = \frac{\bar{V} + \hat{F} + \hat{H}_i - P_s^*}{\delta_i\sigma_u^2}, \quad i = I, U, M, \tag{9}$$

[7] With positive market making cost, no competitive equilibrium exists. This is because on one hand market makers need compensation in terms of a positive bid-ask spread for the market making cost, on the other hand, a positive bid-ask spread implies infinite demand and supply by market makers since they no longer internalize their trades' price impact.

where

$$\hat{H}_i = -\delta_i \sigma_{uN} \hat{X}_i \tag{10}$$

represents investor i's hedging demand. Equation (9) implies that type-i investors buy (sell) if and only if the stock price is lower (greater) than the reservation price

$$P_i^R \equiv \bar{V} + \hat{F} + \hat{H}_i - \delta_i \sigma_u^2 \bar{\theta}, \quad i = I, U, M. \tag{11}$$

(11) implies that type-i investors' reservation price increases with expected stock payoff and decreases with stock payoff volatility. In addition, investors' hedging demand also impacts their reservation prices. For example, if the endowed non-traded asset is positively correlated with the stock (i.e., $\sigma_{uN} \hat{X}_i > 0$), then type-$i$ investors want to hold less shares of the stock and thus their reservation price of the stock is lower.

Let

$$\bar{\delta} = \left(\frac{N_I}{N} \frac{1}{\delta_I} + \frac{N_U}{N} \frac{1}{\delta_U} + \frac{N_M}{N} \frac{1}{\delta_M} \right)^{-1}$$

be the harmonic mean of the risk aversion coefficients of the investors. The following theorem provides the equilibrium price and equilibrium stock holdings.

Theorem 1 *With symmetric information, zero market-making cost, and perfect competition,*

1. *the equilibrium price of the stock is*

$$P_s^* = \bar{V} + \hat{F} - \bar{\delta} \sigma_u^2 \bar{\theta} + \bar{\delta} \sum_{i=I,U,M} \frac{N_i}{N} \frac{\hat{H}_i}{\delta_i}; \tag{12}$$

2. the equilibrium stock holdings are

$$\theta_i^* = \frac{\bar{\delta}}{\delta_i}\bar{\theta} + \left(1 - \frac{N_i}{N}\frac{\bar{\delta}}{\delta_i}\right)\frac{\hat{H}_i}{\delta_i \sigma_u^2} - \frac{\bar{\delta}}{\delta_i}\sum_{j \neq i}\frac{N_j}{N}\frac{\hat{H}_j}{\delta_j \sigma_u^2}, i = I, U, M. \tag{13}$$

Theorem 1 implies that the equilibrium price increases with the expected payoff $(\bar{V} + \hat{F})$, decreases with the volatility of the payoff and the supply of the stock. In addition, investors' hedging demand also impacts the equilibrium price. In particular, if the endowed nontraded asset is negatively correlated with the stock (i.e., $\sigma_{uN}\hat{X}_i < 0$), then type-i investors have positive hedging demand for the stock and thus the stock price is driven higher. Interestingly, the equilibrium price can increase with the average risk aversion in our model. This is because the risk from the non-traded asset may dominate the risk from the stock and thus investors may be willing to buy more shares of the stock to hedge the non-traded asset risk as they become more risk averse. With risk neutral potential market makers (i.e., $\delta_M = 0$, which implies $\bar{\delta} = 0$), the equilibrium price is equal to the expected payoff. Due to perfect competition, hedging demand of other risk averse investors no longer affects the equilibrium stock price.

B. Symmetric Information with Imperfect Competition

When the market-making cost is positive, a bid-ask spread is required to compensate a market maker for making the market. As the number of market makers increases, the benefit from market making decreases due to increased competition. The following proposition shows that if the market making cost is below the utility gain from being the monopolistic market maker, then there always exists a unique (symmetric) equilibrium.

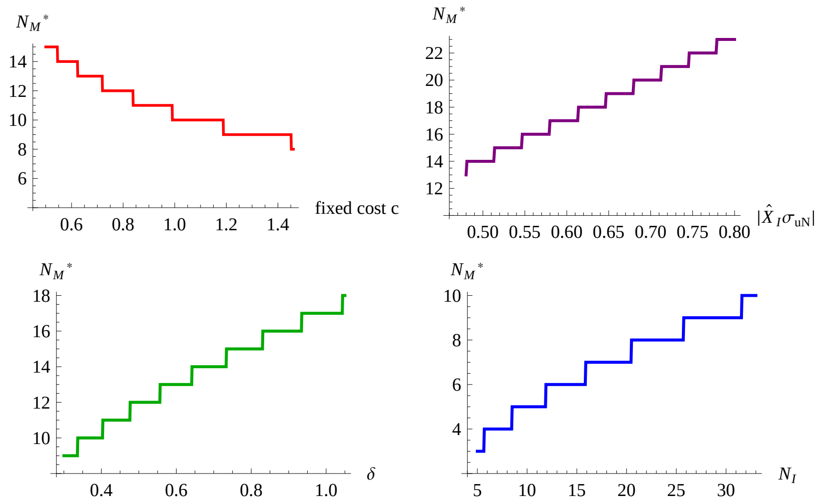

Figure 1: The maximum number of market makers in equilibrium against c, $|\hat{H}_I|$, δ and N_I. The default parameter values are $\bar{\theta} = 1, \delta = 0.8, \hat{F} = 0.5, \hat{H}_I = -0.48, N_I = 100, N_U = 1000, \bar{V} = 3, \sigma_u = 0.4, \hat{X}_U = \hat{X}_M = 0$.

Proposition 1 *For any given $c \in [0, \bar{c}]$, where $\bar{c}$ is a monopolistic market maker's utility gain from making the market in equilibrium, there exists a unique N_M^* such that for any $N_M \leq N_M^*$, there is a unique equilibrium.*

N_M^* represents the maximum number of market makers that an equilibrium can have for a given market-making cost. For expositional simplicity, from now on we will focus on the case where only type-I investors have non-zero endowment of the non-traded asset (i.e., $\hat{X}_U = \hat{X}_M = 0$) and all investors have the same risk aversion (i.e., $\delta_M = \delta_I = \delta_U = \delta$ and thus $\bar{\delta} = \delta$).[8] As illustrated in Figure 1, the maximum number of market makers in equilibrium increases in investors' hedging demand

[8]For the more general cases where all investors have non-traded asset endowment and different risk aversions, although we can also obtain closed-form solutions, it would take too much space and yield no qualitative difference. For example, even assuming $\hat{X}_M = 0$, there are still eight possible subcases when $\hat{X}_U \neq 0$ or $\delta_M \neq \delta$.

magnitude measured by $|\hat{H}_I|$, the risk-aversion coefficient and the number of investors with hedging demand. On the other hand, as the fixed cost c increases, the maximum number of market makers in equilibrium decreases because market makers require a higher benefit from making the market.

Now we explicitly solve for the equilibrium assuming $N_M \leq N_M^*$. Theorem 1 implies that with our assumptions ($\hat{X}_U = \hat{X}_M = 0$ and $\delta_M = \delta_I = \delta_U = \delta$), I investors sell and U investors buy if and only if $\hat{H}_I < 0$. It is natural to conjecture that both investors trade in the same directions in the presence of market-making cost and market power as those in the perfect competition case. We find that this conjecture is indeed correct, as implied by the following theorem.

Theorem 2 *Suppose $N_M \leq N_M^*$, in the presence of market-making cost and market power,*

1. *the equilibrium ask and bid prices are such that*

$$A_s^* = \bar{V} + \hat{F} - \delta\sigma_u^2\bar{\theta} + \frac{N_M N_I}{(N+1)(N_M+1)}\hat{H}_I + \frac{1}{N_M+1}(\hat{H}_I)^+, \tag{14}$$

$$B_s^* = \bar{V} + \hat{F} - \delta\sigma_u^2\bar{\theta} + \frac{N_M N_I}{(N+1)(N_M+1)}\hat{H}_I - \frac{1}{N_M+1}(\hat{H}_I)^-, \tag{15}$$

which implies that $A_s^* > P_s^* > B_s^*$, where P_s^* is the perfect competition equilibrium price as defined in (12) (with $\bar{\delta} = \delta$ and $\hat{X}_U = \hat{X}_M = 0$), and the bid-ask spread

$$A_s^* - B_s^* = \frac{|P_I^R - P_U^R|}{N_M+1} = \frac{|\hat{H}_I|}{N_M+1};$$

2. *the equilibrium stock holdings are such that*

$$\theta_I^* = \bar{\theta} + \frac{N_M(N_U + N_M + 1)}{(N+1)(N_M+1)} \frac{\hat{H}_I}{\delta\sigma_u^2}, \quad \theta_U^* = \bar{\theta} - \frac{N_M N_I}{(N+1)(N_M+1)} \frac{\hat{H}_I}{\delta\sigma_u^2}, \tag{16}$$

$$\alpha_j^* = \frac{(N_M + N_U + 1)N_I(\hat{H}_I)^- + N_I N_U (\hat{H}_I)^+}{(N+1)(N_M+1)\delta\sigma_u^2},$$

$$\beta_j^* = \frac{(N_M + N_U + 1)N_I(\hat{H}_I)^+ + N_I N_U (\hat{H}_I)^-}{(N+1)(N_M+1)\delta\sigma_u^2},$$

and

$$\theta_{M_j}^* = \bar{\theta} - \frac{N_I}{N+1} \frac{\hat{H}_I}{\delta\sigma_u^2}, j = 1, 2, ..., N_M, \tag{17}$$

which implies that the presence of market-making cost and market power does not alter investors' trading directions.

Corollary 1 1. *As hedging demand $|\hat{H}_I|$ increases, the bid-ask spread $A_s^* - B_s^*$, bid depth α_j^*, ask depth β_j^*, the net order size $|\alpha_j^* - \beta_j^*|$, and the trading volume $\sum_{j=1}^{N_M}(\alpha_j^* + \beta_j^*)$ all increase.*

2. *As N_M increases, the bid-ask spread, bid depth, and ask depth all decrease, but trading volume increases when the number of uninformed investors is large.*[9]

3. *As the stock payoff volatility σ_u increases, the bid depth α_j^*, the ask depth β_j^*, the net order size $|\alpha_j^* - \beta_j^*|$, and the trading volume $\sum_{j=1}^{N_M}(\alpha_j^* + \beta_j^*)$ all decrease.*

4. *The bid-ask spread is increasing in the risk aversion of non-market-makers, but independent of the market makers' risk aversion.*

Theorem 2 implies that the bid price converges from below and ask price converges from above to the competitive market equilibrium price as N_M increases if the market

[9] $N_U + N_M$ is fixed.

making cost is zero, as illustrated in Figure 2. Moreover, the equilibrium bid price always increases in N_M while the equilibrium ask price always decreases in N_M due to the more intensive competition among market makers.[10] Because the equilibrium bid (ask) price is lower (higher) than the equilibrium price with competitive market, the equilibrium trading volume (as measured by the sum of sales and purchases $(N_M(\alpha_j^* + \beta_j^*))$ is lower than that in the perfect competition case. Therefore, market power and market making cost increase the spread and decrease the equilibrium trading volume. Thus market power and market making cost tend to make the bid-ask spread negatively correlated with trading volume, as expected. However, as shown in Corollary 1, an increase in the hedging demand can increase both spread and trading volume and thus induces a positive correlation between the two. Intuitively, as the hedging demand increases, the difference in the reservation prices increases and thus market makers charge a higher spread and also trade more with other investors. Therefore in contrast to standard models, our model predicts that bid-ask spread can be negatively or positively correlated with trading volume depending on which of the two forces dominates. Consistent with our prediction, Chordia, Roll, and Subrahmanyam (2001) find that the effective bid-ask spread is positively correlated with trading volume.

As the stock payoff volatility increases, the volatility of stock return also increases. Therefore Corollary 1 also implies that as stock volatility increases, market liquidity as measured by depth and trading volume decrease, consistent with empirical evidence (e.g., Chordia, Roll, and Subrahmanyam (2001)). Intuitively, as volatility increases,

[10]If we measure the stock return of a non-market-maker by $\frac{\tilde{V}}{A^*}$, these results suggest that market maker competition increases expected return and return volatility, but does not affect the Sharpe-ratio.

the risk bearing capacity of the risk-averse market makers decreases and thus the market depth and trading volume decrease. Note that the expression for the bid-ask spread in Theorem 2 shows that the stock volatility does not affect the spread and therefore the decrease in the trading volume and market depth is indeed from the reduction in the risk bearing capacity of the market makers, and not from an increase in the bid-ask spread.[11]

Consistent with Easley and O'Hara (1987, 1992), Corollary 1 implies that the net order size is positively correlated with the bid-ask spread. However, a typical justification of this finding (e.g., Easley and O'Hara (1987, 1992)) is that as the net order size increases, the adverse effect of information asymmetry increases and thus the bid-ask spread increases. In contrast, we view the net order size as the net trade that the market makers are willing to make, because $|\alpha_j^* - \beta_j^*| = |\theta_{M_j}^* - \bar{\theta}|$. In our model, the positive correlation results are driven by the change in the hedging demand. As hedging demand increases, the spread increases and thus the market makers are willing to sell or buy more in the net at the better price.

Interestingly, Corollary 1 shows that market makers' risk aversion does not affect the bid and ask spread. This is because the difference between the reservation prices of the I and U investors is independent of market makers' risk aversion. However, it can be shown that market makers' risk aversion does affect the bid and ask prices and their inventory level.

[11]The result that bid-ask spread does not depend on stock volatility depends crucially on the assumption that I and U investors have the same risk aversion. For example, if I investors buy, U investors sell, and I investors are less risk averse than U investors, then we show that the spread increases with stock volatility.

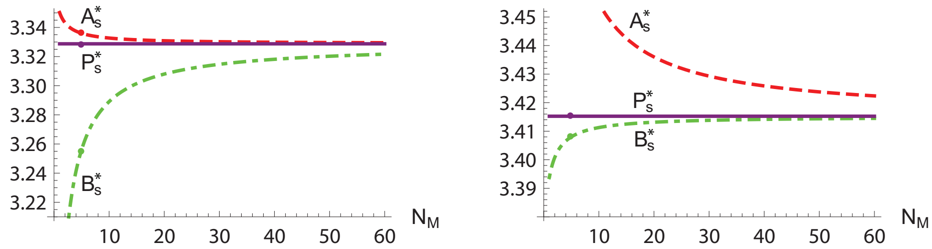

Figure 2: The bid , ask and competitive market equilibrium price as N_M increases (with fixed $N_M + N_U$). The parameter values are: $\bar{\theta} = 1, \delta = 0.8, N_I = 100, N_U = 1000, \bar{V} = 3, c = 0, \sigma_u = 0.4$, and $\hat{H}_I = -0.48$ in the left figure, $\hat{H}_I = 0.48$ in the right figure.

Since the bid-ask spread is a result of market power and market-making cost, we can decompose the bid-ask spread into two components: one due to market making cost ($\frac{|\hat{H}_I|}{N_M^*+1}$) and the other due to the market power $((\frac{1}{N_M+1} - \frac{1}{N_M^*+1})|\hat{H}_I|)$.

Corollary 2

$$\frac{bid - ask\ spread\ due\ to\ market\ making\ cost}{bid - ask\ spread} = \frac{N_M + 1}{N_M^* + 1},$$

and

$$\frac{bid - ask\ spread\ due\ to\ market\ power}{bid - ask\ spread} = 1 - \frac{N_M + 1}{N_M^* + 1}.$$

Since N_M^* decreases with the market making cost c, Corollary 2 implies that the percentage of bid-ask spread due to market power (due to market making cost) decreases (increases) with competition and market making cost c.

Another measure of illiquidity is the magnitude of the price impact of a trade. To examine the price impact, we change the non-market-makers' total purchases and sales by ε_b and ε_s respectively, *i.e.*, we rewrite (5) by

$$\sum_{j=1}^{N_M} \alpha_j^* = \sum_{i=I,U} N_i(\bar{\theta} - \theta_i^*) \mathbf{1}_{\{\theta_i^* < \bar{\theta}\}} + \varepsilon_s, \quad \sum_{j=1}^{N_M} \beta_j^* = \sum_{i=I,U} N_i(\theta_i^* - \bar{\theta}) \mathbf{1}_{\{\theta_i^* > \bar{\theta}\}} + \varepsilon_b. \tag{18}$$

Then we have:

Proposition 2 *The new equilibrium bid and ask prices are as follows:*

$$A(\varepsilon_s, \varepsilon_b) = \bar{V} + \hat{F} - \delta\sigma_u^2\bar{\theta} + \frac{1 + N + N_M N_U}{(1 + N_M)N_U(1 + N)}\delta\sigma_u^2\varepsilon_b - \frac{N_M}{(1 + N_M)(1 + N)}\delta\sigma_u^2\varepsilon_s$$
$$+ \frac{N_I N_M \hat{H}_I}{(1 + N_M)(1 + N)} + \frac{1}{1 + N_M}\left(\hat{H}_I - \frac{N_I - N_U}{N_I N_U}\delta\sigma_u^2\varepsilon_b\right)\mathbf{1}_{\{\hat{H}_I > 0\}}, \tag{19}$$

and

$$B(\varepsilon_s, \varepsilon_b) = \bar{V} + \hat{F} - \delta\sigma_u^2\bar{\theta} + \frac{N_M}{(1 + N_M)(1 + N)}\delta\sigma_u^2\varepsilon_b - \frac{1 + N + N_M N_I}{(1 + N_M)N_I(1 + N)}\delta\sigma_u^2\varepsilon_s$$
$$+ \frac{(N_I N_M + 1 + N)\hat{H}_I}{(1 + N_M)(1 + N)} - \frac{1}{1 + N_M}\left(\hat{H}_I - \frac{N_U - N_I}{N_I N_U}\delta\sigma_u^2\varepsilon_s\right)\mathbf{1}_{\{\hat{H}_I > 0\}}. \tag{20}$$

Corollary 3 *If* $\hat{H}_I < 0$,[12] *then* $\frac{\partial A^*}{\partial \varepsilon_s} = -\frac{N_M \delta\sigma_u^2}{(1+N_M)(1+N)}$, $\frac{\partial B^*}{\partial \varepsilon_s} = -\frac{(N+1+N_M N_I)\delta\sigma_u^2}{(1+N_M)N_I(1+N)}$, $\frac{\partial (A^*-B^*)}{\partial \varepsilon_s} = \frac{\delta\sigma_u^2}{(1+N_M)N_I}$, $\frac{\partial A^*}{\partial \varepsilon_b} = \frac{(1+N+N_M N_U)\delta\sigma_u^2}{(1+N_M)N_U(1+N)}$, $\frac{\partial B^*}{\partial \varepsilon_b} = \frac{N_M \delta\sigma_u^2}{(1+N_M)(1+N)}$, *and* $\frac{\partial (A^*-B^*)}{\partial \varepsilon_b} = \frac{\delta\sigma_u^2}{(1+N_M)N_U}$.

Proposition 2 and Corollary 3 imply that both the bid and the ask prices are linear in the extra purchases and sales and they decrease (increase) when there are more sales (purchases). Figure 3 shows the percentage changes of bid and ask prices against the percentage increases of total purchases or sales. As we can see from Figure 3, larger trades will result in a larger price impact (less market depth). In addition, Corollary 3 shows that both extra sales and extra purchases drive up the

[12]The results for the other case are similar.

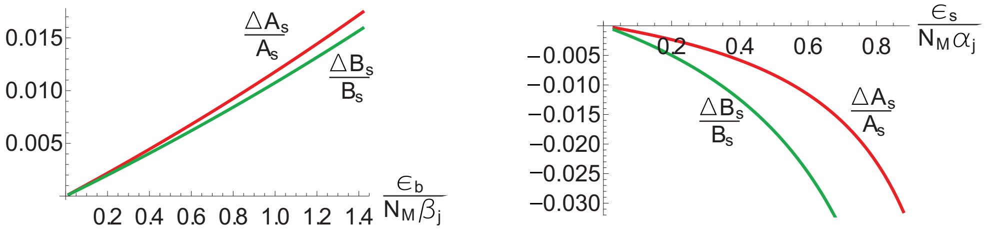

Figure 3: Percentage Price Impact against Additional Trading Size. The parameter values are $\bar{\theta} = 1, \delta = 0.8, N_I = 100, N_U = 1000, N_M = 10, \hat{H}_I = -0.48, \bar{V} = 3, \sigma_u = 0.4$.

bid-ask spread, because market makers have limited risk bearing capacity and thus want to discourage additional sales or purchases and pass some of the extra sales or purchases to other investors by lowering (raising) the bid (ask) more than the ask (bid). Moreover, the price impact and the impact on the bid-ask spread increase with risk aversion and the payoff volatility.[13]

C. Investors' Utility Loss Due to Market Illiquidity

To isolate the effect of market power on welfare, in this subsection, we assume that the market-making cost $c = 0$. Let U_i and $\bar{U}_i$ denote the utility of i ($i = I, U, M$) investors with imperfect and perfect competition respectively and f_i and $\bar{f}_i$ be the corresponding certainty equivalent wealth, i.e., $U_i = -\exp(-\delta f_i)$, and $\bar{U}_i = -\exp(-\delta \bar{f}_i)$.

Definition 2 *The certainty equivalent wealth loss of investor i ($i = I, U, M$) due to market illiquidity is $\bar{f}_i - f_i$.*

The following proposition shows how market power affects welfare.[14]

[13]Since the additional purchases or sales are not driven by hedging demand, the hedging demand does not affect the price impact.

[14]Closed-form expressions for the equivalent wealth losses are available from the authors.

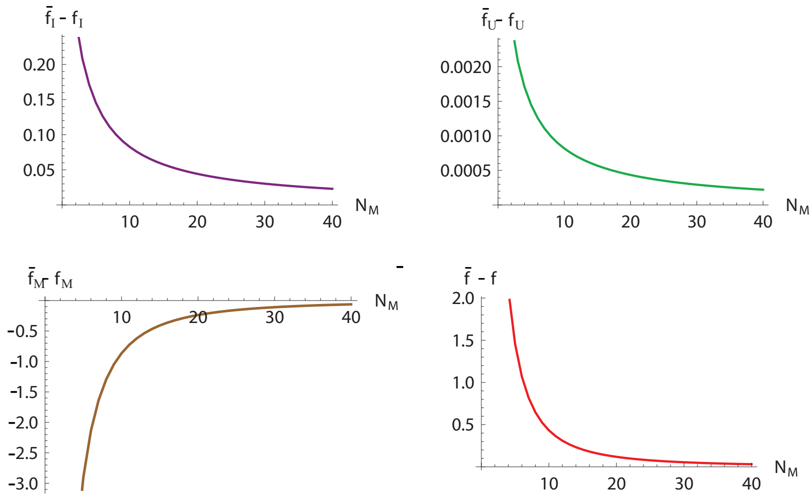

Figure 4: Certainty equivalent wealth loss due to market power against N_M (with fixed $N_U + N_M$). The default parameter values are: $\bar{\theta} = 1, \delta = 0.8, N_I = 100, N_U + N_M = 1010, \bar{V} = 3, \hat{F} = 0.5, \hat{X}_I \sigma_{uN} = 0.6, c = 0, \sigma_u = 0.4$.

Proposition 3 1. *Market makers' market power makes themselves better off and non-market-makers worse off.*

2. *More importantly, market makers' market power reduces the total welfare.*

3. *Both the certainty equivalent wealth losses for I and U investors and the certainty equivalent wealth gain for M investors decrease with N_M and σ_u^2, and increase with hedging demand and risk aversion δ.*

Not surprisingly, market makers benefit from their market power by earning a higher bid-ask spread. Other investors are worse off because they have to trade at a worse price. More importantly, Proposition 3 shows that market makers' welfare gain is less than the welfare loss of the other investors. This is because when determining their trades, market makers do not internalize other investors' losses. As N_M increases, market power decreases and thus both market makers' utility gain and other investors' utility loss decrease. This implies that there exists a Pareto improvement wealth transfer and market regulation mechanism that restricts market bid-ask spread and make all investors (including market makers) strictly better off. It also suggests the importance of promoting competition among market makers on improving market liquidity and social welfare.

As the stock payoff volatility increases, the hedging benefit goes down, investors trade less with market makers, and thus the utility loss decreases. As hedging demand and risk aversion δ increases, however, investors trade more with market makers, get worse prices, and therefore suffer greater loss compared to the perfect competition case.

IV. The Equilibrium Under Asymmetric Information

We now assume that $\hat{F}$ and $\hat{X}_I$ are only observable to the informed investors. Therefore, informed investors' trades can be motivated by both risk-sharing and private information. As before, we first consider the perfect competition case.

A. Perfect Competition with Asymmetric Information

Let P_a^* denote the competitive equilibrium price with asymmetric information. The optimal demand of an informed investor is then

$$\theta_I^* = \frac{\bar{V} + \hat{S} - P_a^*}{\delta \sigma_u^2}, \tag{21}$$

where $\hat{S} = \hat{F} - \delta\sigma_{uN}\hat{X}_I = \hat{F} + \hat{H}_I$ measures the combined demand from hedging needs and private information about the expected payoff. (21) implies that the reservation price for I investors is now

$$P_{aI}^R = \bar{V} + \hat{S} - \delta\bar{\theta}\sigma_u^2. \tag{22}$$

Since the informed investor's demand is a monotonically increasing function of $\hat{S}$, his order reveals the value of $\hat{S}$ to market makers and the equilibrium price should also be monotonic in $\hat{S}$. Thus we conjecture that the equilibrium price depends on $\hat{S}$. Since the uninformed investors can then infer the value of $\hat{S}$ from market prices, the information sets for the informed and uninformed investors are $I_I = \{\hat{F}, \hat{X}_I, P_a^*\}$ and $I_U = I_M = \{P_a^*\} = \{\hat{S}\}$ respectively. Therefore, the uninformed investor's problem is

$$\max_{\theta_U} -e^{-\delta\bar{\theta}P_a^* + \delta\theta_U(P_a^* - \bar{V}) + \frac{1}{2}\delta^2\theta_U^2\sigma_u^2} \times E[e^{-\delta\theta_U\hat{F}}|\hat{S}]. \tag{23}$$

Letting $\sigma_H = \delta|\sigma_{uN}|\sigma_I$ be the volatility of the hedging demand, then the conditional expectation of $\hat{F}$ is

$$E[\hat{F}|\hat{S}] = \frac{\sigma_F^2\hat{S}}{\sigma_F^2 + \sigma_H^2}, \tag{24}$$

and the conditional variance of $\hat{F}$ is

$$\text{Var}[\hat{F}|\hat{S}] = \frac{\sigma_F^2\sigma_H^2}{\sigma_F^2 + \sigma_H^2}. \tag{25}$$

Therefore, the optimal demand of $U-$investors is:

$$\theta_U^* = \frac{\bar{V} + \frac{\sigma_F^2\hat{S}}{\sigma_F^2 + \sigma_H^2} - P_a^*}{\delta\left(\sigma_u^2 + \frac{\sigma_H^2\sigma_F^2}{\sigma_F^2 + \sigma_H^2}\right)}. \tag{26}$$

Equation (26) implies that the reservation prices for U investors is now

$$P_{aU}^R = \frac{\bar{V} + \sigma_F^2 \hat{S}}{\sigma_F^2 + \sigma_H^2} - \delta\bar{\theta}\left(\sigma_u^2 + \frac{\sigma_H^2 \sigma_F^2}{\sigma_F^2 + \sigma_H^2}\right). \tag{27}$$

Letting $d_1 = \frac{\sigma_F^2}{(\sigma_F^2+\sigma_H^2)\sigma_u^2+\sigma_H^2\sigma_F^2}$, $d_2 = (1+\frac{\sigma_H^2}{\sigma_F^2})d_1$, the following theorem provides the equilibrium price and equilibrium stock holdings.

Theorem 3 *In the presence of asymmetric information, there exists a unique competitive equilibrium with stock price being linear in $\hat{S}$. More specifically,*

$$P_a^* = \bar{V} + \frac{N_I + d_1(N_M+N_U)\sigma_u^2}{N_I + d_2(N_M+N_U)\sigma_u^2}(\hat{S} + \delta\sigma_F^2\bar{\theta}) - \frac{N + \left(\frac{N_I}{\sigma_u^2} + d_1(N_M+N_U)\right)\sigma_F^2}{N_I + d_2(N_M+N_U)\sigma_u^2}\delta\sigma_u^2\bar{\theta},$$

and the investors' optimal stock demands are given by

$$\theta_I^* = \bar{\theta} + \frac{(d_2-d_1)(N_M+N_U)}{\delta\left(N_I + d_2(N_M+N_U)\sigma_u^2\right)}(\hat{S} + \delta\sigma_F^2\bar{\theta}), \tag{28}$$

$$\theta_U^* = \theta_M^* = \bar{\theta} + \frac{(d_1-d_2)N_I}{\delta\left(N_I + d_2(N_M+N_U)\sigma_u^2\right)}(\hat{S} + \delta\sigma_F^2\bar{\theta}). \tag{29}$$

Clearly, the equilibrium price is strictly increasing in $\hat{S}$, which implies that in equilibrium all investors can indeed infer the value of $\hat{S}$ from observing the market price. Equations (28) and (29) imply that $I-$investors buy and both $U-$investors and $M-$investors sell if and only if $\hat{S} > -\delta\sigma_F^2\bar{\theta}$.[15] This result can be understood

[15] In our model, market makers observe order flow and can infer how much informed investors are trading. However, they do not know how much of the informed investor's order is due to information on the stock's payoff or how much is due to the hedging demand. This is similar to the set-up of Glosten (1989). The informed's trading demand can be viewed as the pooling of the demand of an informed investor who only trades for information purposes and the demand of an uninformed who only trades for noninformational purposes such as hedging and liquidity. Indeed, we analyzed an alternative model where we have four types of investors: (1) informed without non-traded asset, (2) uninformed without non-traded asset, (3) uninformed with privately observed $\hat{X}_U$ unit of non-traded

by computing the difference in the reservation prices in the presence of asymmetric information:

$$P_{aI}^R - P_{aU}^R = \frac{\sigma_H^2}{\sigma_F^2 + \sigma_H^2}(\hat{S} + \delta\sigma_F^2\bar{\theta}), \tag{30}$$

which implies that if $\hat{S} > -\delta\sigma_F^2\bar{\theta}$ then the reservation price of the informed investor is higher than that of the uninformed investor. Therefore the equilibrium price P_a^* is in between of P_{aI}^R and P_{aU}^R, I investors sell, and U and M investors buy. Since *ex ante* $E[\hat{S}] = 0$, on average informed investors buy in equilibrium.

B. Asymmetric Information with Imperfect Competition

As in the symmetric information case, there exists a maximum number of market makers N_M^* when the market-making cost is not very large. More specifically,

Proposition 4 *For any given $c \in [0, \bar{c}_a]$, where $\bar{c}_a$ is a monopolistic market maker's utility gain from making the market in equilibrium in the presence of asymmetric information, there exists a unique N_M^* such that for any $N_M \leq N_M^*$, there is a unique equilibrium.*

Now we derive the equilibrium bid and ask price in the presence of asymmetric information and market power when $N_M \leq N_M^*$. Let B_a^* and A_a^* be the equilibrium bid price and ask price respectively. If $P_{aI}^R > A_a^* > B_a^* > P_{aU}^R$, then I investors buy and U investors sell. If $P_{aU}^R > A_a^* > B_a^* > P_{aI}^R$, however, then I investors sell and U investors buy. Since $P_{aI}^R > P_{aU}^R$ if and only if $\hat{S} > -\delta\sigma_F^2\bar{\theta}$, we conjecture that as in the

asset, and (4) uninformed market makers without non-traded asset. In this case, the equilibrium price is a linear function of $\hat{F}$ and $\hat{X}_U$ and market makers do not know whether a trader is informed or uninformed. We show that our qualitative results in this alternative model stay the same as in our current model. However, the alternative model involves much more notations and becomes much harder to explain the main results.

perfect competition case, in equilibrium I investors buy if and only if $\hat{S} > -\delta\sigma_F^2\bar{\theta}$. After solving for the equilibrium bid and ask prices B_a^* and A_a^*, we then verify that our conjecture is correct.

The following theorem provides the equilibrium bid and ask prices and equilibrium stock holdings in the presence of market power.

Theorem 4 *Suppose $N_M \leq N_M^*$, in the presence of asymmetric information and market power, in equilibrium, we have*

1. *the equilibrium bid and ask prices are*

$$A_a^* = \bar{V} + a_1(\hat{S} + \delta\sigma_F^2\bar{\theta}) + (b_1 - a_1)(\hat{S} + \delta\sigma_F^2\bar{\theta})^+ - \kappa\delta\sigma_u^2\bar{\theta}$$

$$B_a^* = \bar{V} + b_1(\hat{S} + \delta\sigma_F^2\bar{\theta}) + (a_1 - b_1)(\hat{S} + \delta\sigma_F^2\bar{\theta})^+ - \kappa\delta\sigma_u^2\bar{\theta},$$

where $b_1 > a_1 > 0$, and $\kappa > 0$ are constants as stated in (66), (67) and (68) in the Appendix. The bid and ask spread is

$$A_a^* - B_a^* = \frac{|P_{aI}^R - P_{aU}^R|}{N_M + 1} = \frac{\sigma_H^2}{(N_M + 1)(\sigma_F^2 + \sigma_H^2)}|\hat{S} + \delta\sigma_F^2\bar{\theta}|, \tag{31}$$

and we have

$$A_a^* > P_a^* > B_a^*; \tag{32}$$

2. *the equilibrium stock holdings are*

$$\theta_I^* = \bar{\theta} + C_I(\hat{S} + \delta\sigma_F^2\bar{\theta}),\ \theta_U^* = \bar{\theta} - C_U(\hat{S} + \delta\sigma_F^2\bar{\theta}),\ \theta_{M_j}^* = \bar{\theta} - C_M(\hat{S} + \delta\sigma_F^2\bar{\theta}), \tag{33}$$

$$\alpha_j^* = C_\alpha(\hat{S} + \delta\sigma_F^2\bar{\theta})^- + \frac{N_U}{N_U + N_M + 1}C_\alpha(\hat{S} + \delta\sigma_F^2\bar{\theta})^+, \tag{34}$$

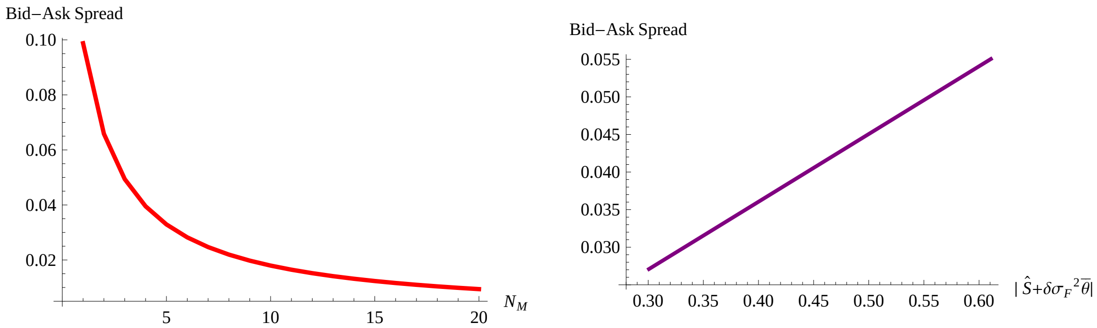

Figure 5: Bid and Ask Spread against N_M and $|\hat{S} + \delta\sigma_F^2\bar{\theta}|$. The parameter values are: $\delta = 0.8, \bar{\theta} = 1, N_M = 10, \sigma_F = 0.03, \hat{S} = -0.2$.

and

$$\beta_j^* = \frac{N_U}{N_U + N_M + 1} C_\alpha(\hat{S} + \delta\sigma_F^2\bar{\theta})^- + C_\alpha(\hat{S} + \delta\sigma_F^2\bar{\theta})^+, \tag{35}$$

where $C_i > 0$ ($i = \{I, U, M, \alpha\}$) are constants stated in (69), (70), (71) and (72) in the Appendix.

Theorem 4 implies that as conjectured, $I-$investors buy and $U-$investors sell if and only if $\hat{S} > -\delta\sigma_F^2\bar{\theta}$. Under asymmetric information, both the bid and ask prices increase in $\hat{S}$. As in the symmetric information case, the bid and ask spread is equal to the absolute value of the difference between the reservation prices, divided by $N_M + 1$.[16] This implies that market makers equally split the market making profit which increases in the difference between the reservation prices. Thus the bid-ask spread decreases in competition among market makers and increase in $|\hat{S} + \delta\sigma_F^2\bar{\theta}|$, as illustrated in Figure 5.

[16]This is similar to the results of classical Cournot competition models of multiple firms who compete for the amount of output of a homogeneous product.

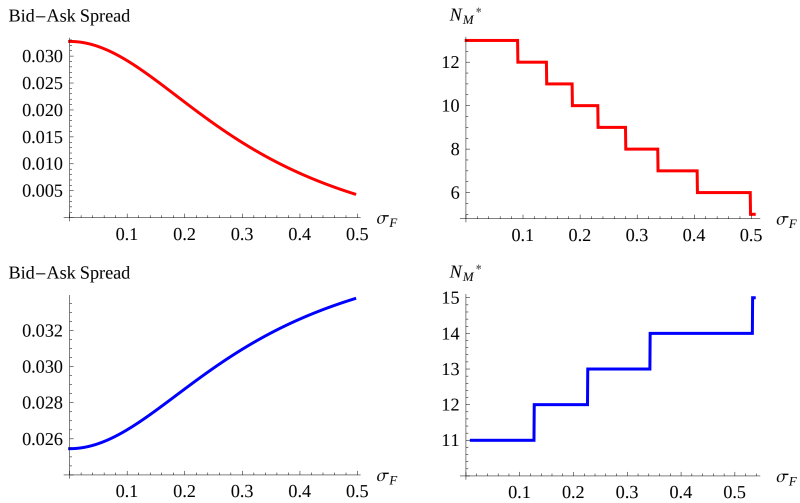

Figure 6: The bid-ask spread and the maximum number of market makers in equilibrium against σ_F. The parameter values are $\delta = 0.8, N_M = 10, N_I = 100, N_U = 1000, \bar{V} = 3, \sigma_F = 0.03, c = 0.4, \sigma_u = 0.4$. And $\bar{\theta} = 1, \hat{S} = -0.36$ in the upper panel graphes, and $\bar{\theta} = 5, \hat{S} = 0.28$ in the lower panel graphes.

The extent of the information asymmetry about $\hat{F}$ and $\hat{H}_I$ can be measured by the uncertainty parameters σ_F and σ_H. Since information about the hedging demand is unrelated to the expected payoff of the stock, we will focus our analysis on information asymmetry measure σ_F (if $\sigma_F = 0$, then there is no information asymmetry about $\hat{F}$ or $\hat{H}_I$ (by inference)).[17] Next we examine how information asymmetry affects the bid-ask spread.

Proposition 5 $\frac{\partial(A_a^* - B_a^*)}{\partial \sigma_F} < 0$ *iff* $\hat{S} < -\delta\sigma_F^2\bar{\theta}$ *or* $\hat{S} > \delta\sigma_H^2\bar{\theta}$.

The difference ΔRP is the sum of three differences across the informed and the informed: (1) the difference in the hedging demand ("hedging demand effect"); (2) the difference in the estimation of the expected stock payoff ("estimation error effect");

[17]It can be easily shown that $\frac{\partial(A_a^* - B_a^*)}{\partial \sigma_H} > 0$ and thus the spread always increases in the information asymmetry about the hedging demand. This is because uncertainty about the hedging demand does not affect the conditional distribution of the stock payoff.

and (3) the difference in the risk premium required for estimation risk ("estimation risk effect"). Since only the uninformed are subject to estimation risk and they are risk averse, they require a higher risk premium and thus the estimation risk effect always drives up the reservation price difference ΔRP. In contrast, since the uninformed can overestimate or underestimate the expected stock payoff, the estimation error effect can drive ΔRP downward or upward. When the uninformed overestimate and thus the estimation error effect is negative, the net of the estimation error effect and the estimation risk effect can cancel out some of the hedging demand effect. In these cases, the reservation price difference with asymmetric information can be lower than that with symmetric information and accordingly the bid-ask spread with asymmetric information can be lower than that with symmetric information. Since asymmetric information can result in a smaller bid-ask spread which implies lower trading costs, the trading volume can also be higher with asymmetric information.

In contrast to most of the existing literature (e.g., Glosten and Milgrom (1990)), Proposition 5 shows that the bid-ask spread may *decrease* as information asymmetry increases. More specifically, the bid-ask spread decreases with the information asymmetry if and only if informed traders have strong incentive to trade, i.e., if and only if $|\hat{S}|$ is large. As shown by (30), the difference between investors' reservation prices is the sum of three differences across the informed and the informed: (1) the difference in the hedging demand ("hedging demand effect"); (2) the difference in the estimation of the expected stock payoff ("estimation error effect"); and (3) the difference in the risk premium required for estimation risk ("estimation risk effect"). By (30), the hedging demand effect is equal to $\hat{H}$, the estimation error effect is equal to

$$\hat{F} - \frac{\sigma_F^2 \hat{S}}{\sigma_F^2 + \sigma_H^2},$$

and the estimation risk effect is equal to

$$\delta\bar{\theta}\frac{\sigma_H^2 \sigma_F^2}{\sigma_F^2 + \sigma_H^2}.$$

The information asymmetry σ_F has opposite effects on the estimation error effect and the estimation risk effect: it decreases the magnitude of the estimation error effect and increases that of the estimation risk effect. For the essential intuition, consider the case where both $\hat{H}$ and $\hat{F}$ are positive and the uninformed underestimates the expected payoff $\hat{F}$. As information asymmetry σ_F increases, the uninformed attributes a larger fraction of the sum $\hat{F} + \hat{H}$ ($\hat{S}$) to $\hat{F}$ and thus the underestimation becomes smaller. Therefore the estimation error effect decreases with information asymmetry. On the other hand, as information asymmetry increases the estimation risk effect increases because the uninformed becomes more uncertain about the expected payoff. When $|\hat{S}|$ is large, the decrease in the estimation error effect dominates and therefore the difference between investors' reservation prices and thus the bid-ask spread decreases in the information asymmetry. When $|\hat{S}|$ is small, then the increase in the estimation risk effect dominates and thus the bid-ask spread increases with information asymmetry.

As we can see from Figure 6, when the bid-ask spread increases (decreases) in σ_F, so does the maximum number of market makers in equilibrium. Similar to the symmetric information case, the maximum number of market makers in equilibrium decreases in the fixed cost c. As the bid-ask spread, the maximum number of market

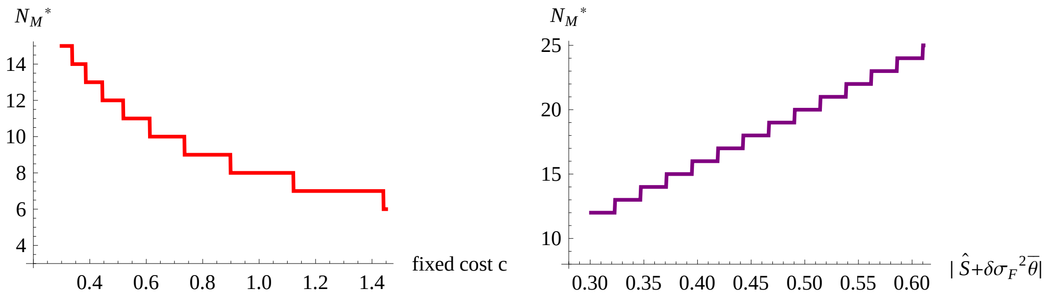

Figure 7: The maximum number of market makers in equilibrium against c, $|\hat{S}+\delta\sigma_F^2\bar{\theta}|$. The parameter values are $\bar{\theta} = 1, \delta = 0.8, \hat{S} = 0.28, N_I = 100, N_U = 1000, \bar{V} = 3, \sigma_F = 0.03, c = 0.4, \sigma_u = 0.4$.

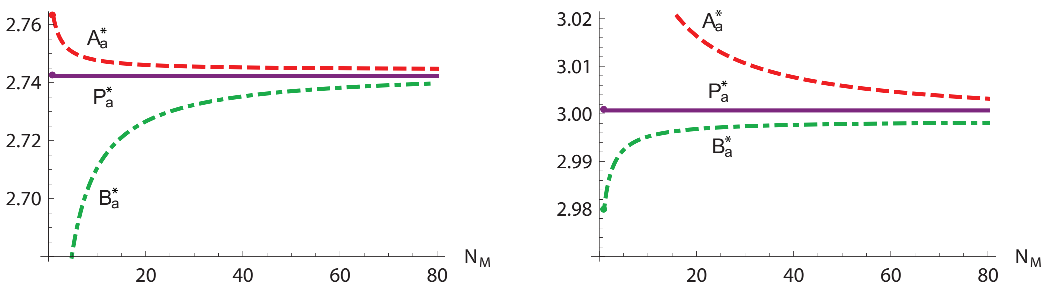

Figure 8: The bid (lower curve), ask (upper curve) and competitive market equilibrium price (middle line) when N_M increases (with fixed $N_U + N_M$). The parameter values are: $\bar{\theta} = 1, \delta = 0.8, N_I = 100, N_U = 1000, \bar{V} = 3, \sigma_F = 0.03, \sigma_I = 0.4, \sigma_u = 0.4$, $\hat{S} = -0.5$ in the left figure, and $\hat{S} = 0.5$ in the right figure.

makers increases in $|\hat{S} + \delta\sigma_F^2\bar{\theta}|$, as illustrated in Figure 7.

As in the symmetric information case, the bid and ask prices converge to the perfect competition equilibrium price when N_M is large (see Figure 8). In addition, the equilibrium ask price is monotonically decreasing in N_M and the equilibrium bid price is monotonically increasing in N_M due to intensified competition among market makers.

In the presence of asymmetric information, we can decompose the bid-ask spread into three components: market power, market-making cost, and asymmetric information.

Corollary 4

$$bid-ask\ spread = bid-ask\ due\ to\ market\ power + bid-ask\ due\ to\ market-making\ cost$$

$$+ bid - ask\ due\ to\ asymmetric\ information,$$

where the first two components are as defined in Corollary 2 and the spread due to asymmetric information is:

$$\frac{\sigma_H^2}{(N_M+1)(\sigma_F^2+\sigma_H^2)}|\hat{S}+\delta\sigma_F^2\bar{\theta}| - \frac{1}{N_M+1}|\hat{H}_I|.$$

As we will see in Section V, the spread due to asymmetric information can be negative under certain conditions. It can be shown that the presence of information asymmetry does not change the results in Corollary 1 for the symmetric information case, with $|\hat{H}_I|$ replaced by $|\hat{S}+\delta\sigma_F^2\bar{\theta}|$. The same intuitions for these results apply too. The following corollary shows how trading volume and net order size change with information asymmetry.

Corollary 5 $\frac{\partial(N_M(\alpha_j^*+\beta_j^*))}{\partial\sigma_F} < 0$ *and* $\frac{\partial|\alpha_j^*-\beta_j^*|}{\partial\sigma_F} < 0$ *iff* $\hat{S} < -\delta\sigma_F^2\bar{\theta}$ *or* $\hat{S} > \frac{(1+N)\delta\sigma_H^2\sigma_u^2\bar{\theta}}{N_I\sigma_H^2+(1+N)\sigma_u^2}$.

Corollary 5 shows that as information asymmetry increases, both the trading volume and the net order size decrease if the informed sell or buy a lot. Intuitively, as we show in Proposition 5, the difference in the reservation prices becomes smaller and the difference between the trading price and the reservation price also shrinks.

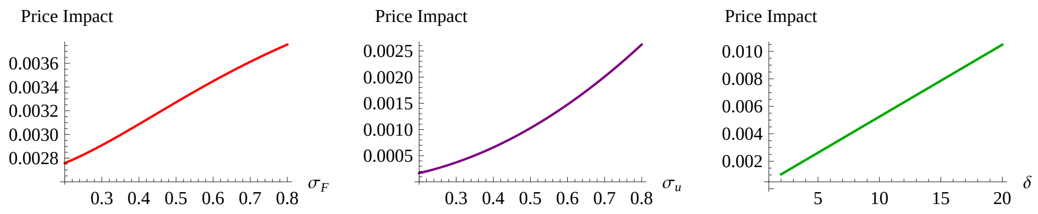

Figure 9: Price Impact Against σ_F, σ_u and δ. The parameter values are: $\bar{\theta} = 1, \delta = 5, N_M = 10, N_I = 100, N_U = 1000, \sigma_F = 0.03, \sigma_I = 0.4, \sigma_u = 0.4$.

Therefore trading volume decreases. The reason for the decrease in the net order size is that the market makers are less willing to take a large net opposite trade to that of the informed trader, because in this case the informed traders' trades are more likely to be information based.

Similar to the symmetric information case, we also study the price impact of a purchase ε_b or a sale ε_s and obtain:[18]

Corollary 6 $\frac{\partial A_a^*}{\partial \varepsilon_s} = -\frac{\delta N_M \sigma_u^2}{(1+N_M)(N_I+d_2(1+N_M+N_U)\sigma_u^2)}$, $\frac{\partial B_a^*}{\partial \varepsilon_s} = -\frac{(N_I(N_M+1)+d_2(1+N_M+N_U)\sigma_u^2)\delta\sigma_u^2}{N_I(1+N_M)(N_I+d_2(1+N_M+N_U)\sigma_u^2)}$, $\frac{\partial A_a^*}{\partial \varepsilon_b} = \frac{\delta(N_I+d_2(1+N_M)(1+N_U)\sigma_u^2)}{d_2 N_U(1+N_M)(N_I+d_2(1+N_M+N_U)\sigma_u^2)}$, *and* $\frac{\partial B_a^*}{\partial \varepsilon_b} = \frac{\delta N_M \sigma_u^2}{(1+N_M)(N_I+d_2(1+N_M+N_U)\sigma_u^2)}$.

Corollary 6 shows that all qualitative results remain the same as in the symmetric information case. However, the magnitude of the effect is smaller with asymmetric information.

C. Investors' Utility Loss

As in the symmetric information case, investors have some utility loss due to market power.

[18]To save space, we only list the results for $\hat{S} < -\delta\sigma_F^2\bar{\theta}$, the price impacts for $\hat{S} > -\delta\sigma_F^2\bar{\theta}$ are very similar and the closed form results are available from the authors.

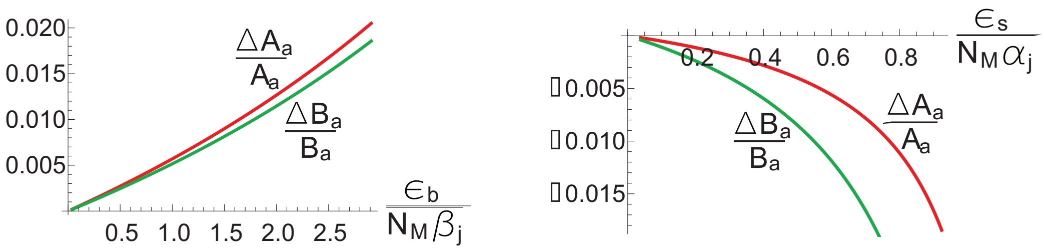

Figure 10: Percentage Price Impact Against Additional Trading Size. The parameter values are: $\bar{\theta} = 1, \delta = 0.8, N_M = 10, N_I = 100, N_U = 1000, \sigma_F = 0.03, \hat{S} = -0.2, \sigma_I = 0.4, \sigma_u = 0.4$.

Proposition 6 *1. Market makers' market power makes themselves better off and non-market-makers worse off. More importantly, the total welfare is reduced.*

2. Both the certainty equivalent wealth losses for I and U investors and the certainty equivalent wealth gain for M investors decrease with N_M, and increase with $|\hat{S} + \delta\sigma_F^2\bar{\theta}|$.

3. The total certainty equivalent wealth loss due to market power can increase or decrease with information asymmetry σ_F.

As in the symmetric information case, I and U have some utility loss and M has some utility gain due to market makers' market power, however, the total utility is always reduced by the presence of market power. If the combined trading demand due to private information and hedging needs increases, then investors trade more with market makers and therefore investors' certainty equivalent wealth loss increases as we can see in Figure 11. In addition, Proposition 6 shows that the total certainty equivalent wealth loss due to the market power can increase or decrease in σ_F, as illustrated in Figure 12. Intuitively, since the bid-ask spread can increase or decrease

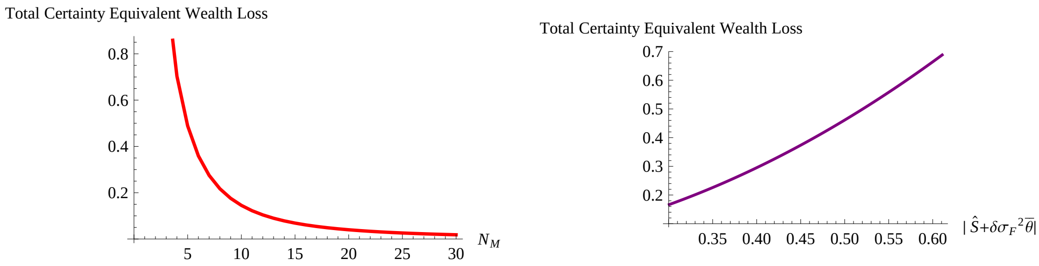

Figure 11: The total certainty equivalent wealth loss due to market power against N_M and $|\hat{S} + \delta\sigma_F^2\bar{\theta}|$. The parameter values are: $\bar{\theta} = 1, \delta = 0.8, N_M = 10, N_I = 100, N_U = 1000, \bar{V} = 3, \sigma_F = 0.03, \sigma_I = 0.4, c = 0, \sigma_u = 0.4$.

with information asymmetry, so can the total welfare loss.

V. Spread and Utility Loss with and without Asymmetric Information

In this section, we compare the competitive equilibrium price, the bid-ask spread, and the total welfare loss due to market power with and without asymmetric information.

A. Competitive Equilibrium Prices, Bid and Ask Spreads with and without Asymmetric Information

We first compare the competitive equilibrium prices with and without asymmetric information, we have

Proposition 7

$$P_a^* \leq P_s^* \quad if\ and\ only\ if\ \hat{F} \geq -\delta\sigma_F^2\bar{\theta} + \frac{(N\sigma_u^2 + N_I\sigma_H^2)\sigma_F^2}{N\sigma_H^2\sigma_u^2}\hat{H}_I. \tag{36}$$

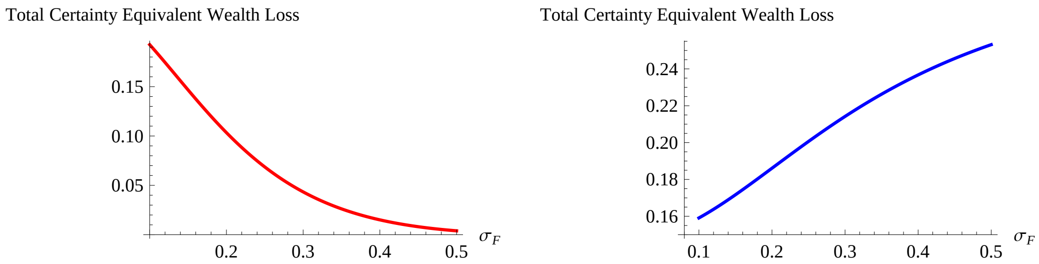

Figure 12: The total certainty equivalent wealth loss due to market power against σ_F. The parameter values are: $N_M = 10, N_I = 100, N_U = 1000, \bar{V} = 3, \sigma_F = 0.03, \sigma_I = 0.4, c = 0, \sigma_u = 0.4, \bar{\theta} = 1$. And $\delta = 0.8, \hat{S} = -0.36$ in the left graph, and $\delta = 5, \hat{S} = 0.28$ in the right graph.

and

$$E[P_a^*] < E[P_s^*]. \tag{37}$$

Proposition 7 shows that the expected equilibrium price with asymmetric information is lower than that with symmetric information. This is because non-I investors require a higher risk premium in the presence of asymmetric information. However, ex-post, when the realization of the private information about the expected payoff of the stock $\hat{F}$ is low, the equilibrium price in the asymmetric information case is *higher* than that in the symmetric information case.

We now compare bid and ask spread and depths with and without asymmetric information. We have the following results:

Proposition 8 *1.* $A_a^* - B_a^* < A_s^* - B_s^*$ *iff* $|\hat{S} + \delta\sigma_F^2\bar{\theta}| < \frac{\sigma_F^2+\sigma_H^2}{\sigma_H^2}|\hat{H}_I|$*;*

2. $(\alpha_j^* + \beta_j^*)_a \leq (\alpha_j^* + \beta_j^*)_s$ *iff* $|\hat{S} + \delta\sigma_F^2\bar{\theta}| < \frac{(N_I+d_2(1+N_M+N_U)\sigma_u^2)\sigma_F^2}{\sigma_H^2 d_1(1+N)\sigma_u^2}|\hat{H}_I|$.

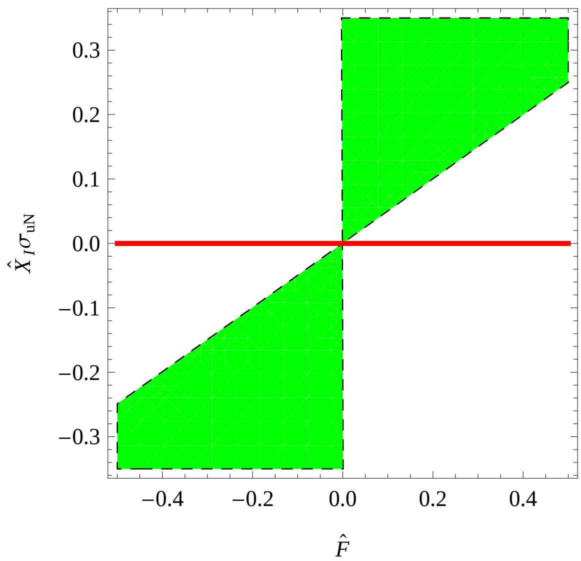

Figure 13: The bid and ask spread with and without asymmetric information. The colored area denotes those states where the bid-ask spread is wider with symmetric information. Other parameter values are: $\bar{\theta} = 1, \delta = 0.8, \sigma_I = 0.4, \sigma_F = 0.03, \bar{V} = 3, \sigma_u = 0.4, N_M = 10$.

Proposition 8 implies that the bid-ask spread with asymmetric information can be smaller than that in the symmetric information case. This occurs if and only if $\hat{F}$ is relatively small comparing to the hedging demand as illustrated in Figure 13. Under symmetric information, the difference in the reservation prices for the I and U investors is $|P_I^R - P_U^R| = |\hat{H}_I|$. Accordingly, the bid-ask spread in the symmetric information case only depends on $\hat{H}_I$ but not $\hat{F}$. In the presence of asymmetric information, the difference in the reservation prices for the I and U investors is $|P_{aI}^R - P_{aU}^R| = \frac{\sigma_H^2}{\sigma_F^2+\sigma_H^2}|\hat{F} + \hat{H}_I + \delta\sigma_F^2\bar{\theta}|$. In equilibrium, we must have that both the bid and ask be between P_{aI}^R and P_{aU}^R so that I and U trade in opposite directions and therefore the bid-ask spread is always smaller than $|P_{aI}^R - P_{aU}^R|$. To help explain the intuition behind the results in Proposition 8, we fix $\hat{H}_I$. Then the bid-ask spread under symmetric information is a constant when we change $\hat{F}$, and in the presence of asymmetric information, if $\hat{F} = F^* \equiv -\hat{H}_I - \delta\sigma_F^2\bar{\theta}$, the reservation prices are the same for I and U, therefore, the equilibrium bid-ask spread must be zero (and no trade). When $\hat{F}$ is near F^*, then investors' reservation prices are close, and thus the bid-ask spread with asymmetric information is small and can be smaller than that with symmetric information. On the other hand, when $\hat{F}$ is far from F^*, then investors' reservation prices are significantly different from each other, thus market makers can take advantage of this difference by increasing the bid-ask spread. Therefore, when $\hat{F}$ is very far from F^*, the bid-ask spread with asymmetric information is wider than that with symmetric information.

Proposition 8 also shows that the equilibrium market depth and trading volume with asymmetric information can be lower or higher than that with symmetric information, depending on the relative magnitude of $\hat{F}$ and $\hat{H}_I$. Interestingly, if the bid-ask spread is lower than the symmetric information case then the trading volume and market depth are also lower. This is one of the testable empirical predictions of this model.

Proposition 8 is an ex-post result which is dependent on the realized values of $\hat{F}$ and $\hat{H}_I$. We next provide an an-ante result on the expected bid-ask spread before the realization of $\hat{F}$ and $\hat{H}_I$ with and without asymmetric information.

Proposition 9 *1. The expected bid-ask spreads under symmetric and asymmetric information are:*

$$E[A_s^* - B_s^*] = \frac{2}{N_M + 1} \frac{\sigma_H}{\sqrt{2\pi}}, \tag{38}$$

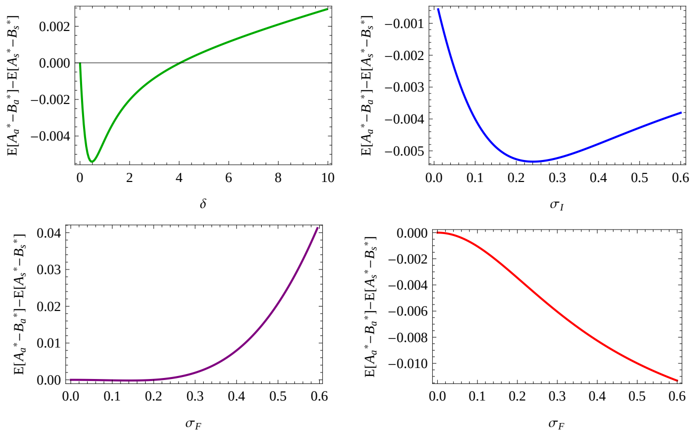

Figure 14: The difference of the expected bid-ask spreads with and without asymmetric information when changing the parameters, δ, σ_F and σ_I. The parameter values are: $\sigma_I = 0.4, N_M = 10, \sigma_F = 0.25, \bar{V} = 3, \bar{\theta} = 1$, and $\sigma_I = 0.4$. And $\delta = 5$ in the lower left graph and $\delta = 0.8$ in other three graphs.

$$E[A_a^* - B_a^*] = \frac{\sigma_H^2}{(N_M+1)(\sigma_H^2+\sigma_F^2)} \left(\frac{2b\sigma_F}{\sqrt{2\pi}} e^{-\frac{\delta^2\bar{\theta}^2\sigma_F^2}{2b^2}} + \delta\sigma_F^2\bar{\theta}\left(2\boldsymbol{N}\left(\frac{\delta\sigma_F\bar{\theta}}{b}\right) - 1\right)\right), \tag{39}$$

where $b = \sqrt{1 + \sigma_H^2/\sigma_F^2}$ and $\boldsymbol{N}$ is the cdf of the standard normal distribution.

2. *If $\sigma_H < \frac{\sigma_F}{1+\delta\bar{\theta}\sigma_F\sqrt{\frac{\pi}{2}}}$, then $E[A_a^* - B_a^*] < E[A_s^* - B_s^*]$, and if $\sigma_F^2 > \frac{\frac{2}{\sqrt{2\pi}}\sigma_H^2}{\delta\sigma_H\bar{\theta} - \frac{2}{\sqrt{2\pi}}}$, then $E[A_a^* - B_a^*] > E[A_s^* - B_s^*]$.*

3. *As $\sigma_F \downarrow 0$, $E[A_a^* - B_a^*]$ converges to $E[A_s^* - B_s^*]$.*

Proposition 9 shows that the expected bid-ask spread with asymmetric information converges to the expected bid-ask spread with symmetric information as σ_F tends to zero, as expected. The second result in Proposition 9 implies that in the presence of asymmetric information, if the uncertainty of the hedging demand is very small, then the average bid-ask spread with asymmetric information is smaller than that with symmetric information. If the uncertainty of the private information is large, then the average bid-ask spread with asymmetric information is larger than that with symmetric information. Figure 14 shows how the difference between the expected spread changes with risk aversion, the uncertainty with respect to the private endowment and the stock payoff.

B. Investors' Utility Loss with and without Asymmetric Information

We can compare investors' total certainty equivalent wealth loss due to market power and the implied illiquidity with and without asymmetric information. Since the bid-ask spread can be lower with asymmetric information and investors welfare increases when the spread is smaller, one expects that the presence of asymmetric information may *decrease* the welfare loss from market power. The following proposition confirms this expectation.

Proposition 10 *Let ULA and UL be the certainty equivalent wealth loss due to market power and the implied illiquidity with and without asymmetric information respectively, then $ULA < UL$ if and only if $|\hat{S} + \delta\sigma_F^2\bar{\theta}| < C_1|\hat{H}_I|$, where $C_1 > 0$ is as defined in (78) in the Appendix.*

Proposition 10 implies that the presence of asymmetric information indeed may *decrease* the investors' total certainty equivalent wealth loss due to market power and the implied illiquidity. This decrease typically occurs when $\hat{F}$ is relatively small comparing to investors' hedging needs measured by $|\hat{H}_I|$, as illustrated in Figure 15. In most of these cases, the bid-ask spread is smaller in the presence of asymmetric

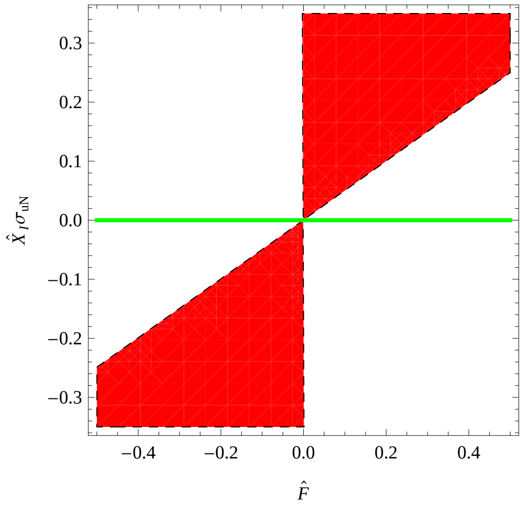

Figure 15: The total certainty equivalent wealth loss with and without asymmetric information. The colored area denotes those states where the total certainty equivalent wealth loss is bigger with symmetric information. The parameter values are: $\bar{\theta} = 1, \delta = 0.8, \sigma_I = 0.4, \sigma_F = 0.03, \bar{V} = 3, \sigma_u = 0.4, N_M = 10, N_I = 100, N_U = 1000.$

information. Figure 16 shows how the difference between the expected utility loss due to market power with and without asymmetric information with risk aversion, the uncertainty with respect to the private endowment and the stock payoff.

VI. Concluding Remarks

In this paper we develop a general equilibrium model with endogenous illiquidity in an integrated framework with asymmetric information and oligopolistic competition. All the main results are obtained in closed-form. This model can potentially help many empirical findings such as why equilibrium bid-ask spread may decrease with asymmetric information and why the bid-ask spread can be positively correlated with trading volume and market depth. In addition, we find that information asymmetry may *reduce* the social welfare loss suffered from market makers' market power. Our

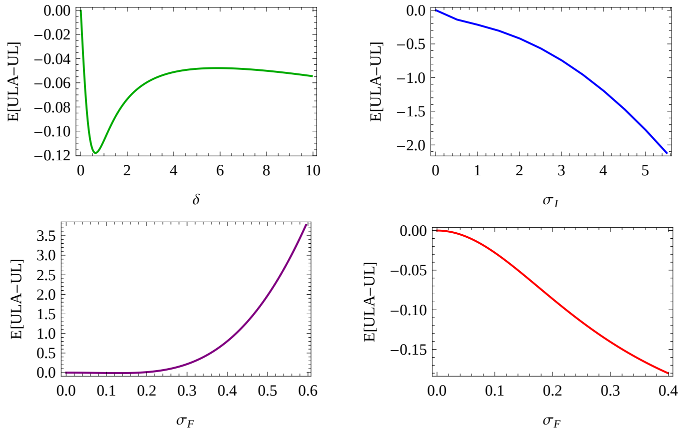

Figure 16: The difference of the total expected certainty equivalent wealth loss with and without asymmetric information when changing the parameters $\bar{\theta}$, δ, σ_F and σ_I. The parameter values are: $\sigma_I = 0.4, N_M = 10, N_I = 100, N_U = 1000, \sigma_F = 0.25, \bar{V} = 3, \bar{\theta} = 1$ and $\sigma_I = 0.4$. And $\delta = 5$ in the lower left graph and $\delta = 0.8$ in other three graphs.

findings suggest the great importance of enhancing market liquidity through systematic mechanisms such as restrictions on the bid-ask spread and depths.

Appendix

Proof of Theorem 1: It is not difficult to get that:

$$\theta_M = \frac{\bar{V} + \hat{F} - P_s^*}{\delta_M \sigma_u^2}, \tag{40}$$

by the market clearing condition

$$N_I \theta_I + N_U \theta_U + N_M \theta_M = N\bar{\theta}. \tag{41}$$

We can get the perfect competition equilibrium. Substituting (9) into the market clearing condition (41), we get the equilibrium stock price P_s^*. Substituting P_s^* into (9), we get investors' optimal stock holdings as in the Theorem. *Q.E.D.*

Proof of Theorem 2: We only prove Case 1, *i.e.*, I investors sell and U investors buy, the proof of Case 2 is very similar and thus we skip it here. Given bid price B and ask price A, the optimal demand of I and U are:

$$\theta_I = \frac{\tilde{F} + \bar{V} - B - \delta \sigma_{uN} \hat{X}_I}{\delta \sigma_u^2} \quad and \quad \theta_U = \frac{\tilde{F} + \bar{V} - A}{\delta \sigma_u^2}. \tag{42}$$

The market clearing conditions (5) become:

$$\sum_{j=1}^{N_M} \alpha_j = N_I(\bar{\theta} - \theta_I) \quad and \quad \sum_{j=1}^{N_M} \beta_j = N_U(\theta_U - \bar{\theta}). \tag{43}$$

Substituting (42) into the market clearing conditions (43), we get that the market clearing bid and ask prices are:

$$A = \tilde{F} + \bar{V} - \delta\sigma_u^2\bar{\theta} - \frac{\delta\sigma_u^2}{N_U}\sum_{j=1}^{N_M}\beta_j, \quad and \quad B = \tilde{F} + \bar{V} - \delta\sigma_u^2\bar{\theta} - \delta\sigma_{uN}\hat{X}_I + \frac{\delta\sigma_u^2}{N_I}\sum_{j=1}^{N_M}\alpha_j. \quad (44)$$

Market maker M_j's problem becomes:

$$\min_{\alpha_j,\beta_j} -\delta(\beta_j A - \alpha_j B) - \delta(\bar{\theta} + \alpha_j - \beta_j)(\hat{F} + \bar{V}) + \frac{1}{2}\delta^2(\bar{\theta} + \alpha_j - \beta_j)^2\sigma_u^2, \quad (45)$$

substituting (44) into (45) and the first order condition w.r.t to α_j gives us:

$$\frac{1}{N_I}\delta\sigma_u^2\sum_{j=1}^{N_M}\alpha_j - \delta\sigma_{uN}\hat{X}_I + \delta(\alpha_j - \beta_j)\sigma_u^2 + \alpha_j\frac{1}{N_I}\delta\sigma_u^2 = 0, \quad (46)$$

sum all, we get:

$$\sum_{j=1}^{N_M}\alpha_j = \frac{N_I}{N_I + N_M + 1}\sum_{j=1}^{N_M}\beta_j + \frac{N_M N_I \sigma_{uN}\hat{X}_I}{(N_I + N_M + 1)\sigma_u^2}. \quad (47)$$

The first order condition w.r.t to β_j gives us:

$$\frac{1}{N_U}\delta\sigma_u^2\sum_{j=1}^{N_M}\beta_j - \delta(\alpha_j - \beta_j)\sigma_u^2 + \beta_j\frac{1}{N_U}\delta\sigma_u^2 = 0, \quad (48)$$

sum all, we get:

$$\sum_{j=1}^{N_M}\beta_j = \frac{N_U}{N_U + N_M + 1}\sum_{j=1}^{N_M}\alpha_j. \quad (49)$$

From (47) and (49), we get:

$$\sum_{j=1}^{N_M}\alpha_j = \frac{N_I N_M(N_U + N_M + 1)}{(N+1)(N_M+1)}\frac{\sigma_{uN}\hat{X}_I}{\sigma_u^2}, \quad \sum_{j=1}^{N_M}\beta_j = \frac{N_M N_I N_U \sigma_{uN}\hat{X}_I}{(N+1)(N_M+1)\sigma_u^2}. \quad (50)$$

Substituting (50) into (46) and (48), we can get α_j^* and β_j^* as given in Theorem 2. Substituting (50) into (44) and (42), we get the equilibrium bid and ask prices, I and U investors' optimal demand as given in Theorem 2. *Q.E.D.*

Proof of Corollary 1: The net order size and trading volume are

$$|\alpha_j^* - \beta_j^*| = \frac{N_I}{N+1}\frac{|\sigma_{uN}\tilde{X}_I|}{\sigma_u^2}, \ \sum_{j=1}^{N_M}(\alpha_j^* + \beta_j^*) = \frac{N_M(N_M + 2N_U + 1)N_I|\sigma_{uN}\hat{X}_I|}{(N+1)(N_M+1)\sigma_u^2}.$$

We have: $A_s^* - B_s^* = \frac{(N+1)\sigma_u^2}{(N_M+1)N_I}|\alpha_j^* - \beta_j^*|$, $A_s^* - B_s^* = \frac{(N+1)\delta\sigma_u^2}{N_M N_I(N_M+2N_U+1)}\sum_{j=1}^{N_M}(\alpha_j + \beta_j)$, *i.e.*, the spread is a linear increasing function of the trading volume.

$$\frac{\partial\alpha_j^*}{\partial N_M} = \frac{\partial\beta_j^*}{\partial N_M} = -\frac{N_I\,(1 + N - N_I)\,|\sigma_{uN}\hat{X}_I|}{(1+N_M)^2(1+N)\sigma_u^2} < 0.$$

Proof of Proposition 3: The utility of I investors in a competitive market is:

$$U_I = -\exp(-\delta f_I) = -\exp\left(-\delta(\bar{\theta} - \theta_I^*)P_s^* - \delta\theta_I^*(\bar{V} + \tilde{F}) + \frac{1}{2}\delta^2(\theta_I^{*2}\sigma_u^2 + \tilde{X}_I^2\sigma_N^2 + 2\theta_I^*\sigma_{uN}\hat{X}_I)\right), \tag{51}$$

where P_s^*, θ_I^* are defined in Proposition 1. I's utility with illiquidity is:

$$\hat{U}_I = -\exp(-\delta\hat{f}_I) = -\exp\left(-\delta(\bar{\theta} - \theta_I^*)B_1^* - \delta\theta_I^*(\bar{V} + \tilde{F}) + \frac{1}{2}\delta^2(\theta_I^{*2}\sigma_u^2 + \tilde{X}_I^2\sigma_N^2 + 2\theta_I^*\sigma_{uN}\hat{X}_I)\right), \tag{52}$$

where B^*, θ_I^* are defined as in Case 1 of Proposition 2. It is not difficult to see that I investors have some utility loss due to the presence of illiquidity. And I investors' utility loss measured in certainty equivalent wealth is given as in Proposition 3. Similarly, we can get that U investors also have some utility loss and market makers have some utility gain due to the presence of illiquidity. It is not difficult to see that the expressions for certainty equivalent wealth losses for both I, U and the welfare gain for M in Case 2 are the same as those in Case 1. *Q.E.D.*

Sketch Proof of Proposition 1: We prove this Proposition by comparing the certainty equivalent wealth ($\hat{f}_M$) of the N_M^{th} potential market maker(with N_U uninformed investor) and the certainty equivalent wealth ($\hat{f}_U$) of the $(N_U+1)^{th}$ uninformed investor (with N_M market makers), where

$$\hat{f}_M = (\beta_j A - \alpha_j B) + (\bar{\theta} + \alpha_j - \beta_j)(\bar{V} + \hat{F}) - \frac{1}{2}\delta(\bar{\theta} + \alpha_j - \beta_j)^2 \sigma_u^2,$$

and for the case when U investors are buyers,

$$\hat{f}_U = (\bar{\theta} - \theta_U^*)A + \theta_U^*(\bar{V} + \hat{F}) - \frac{1}{2}\delta {\theta_U^*}^2 \sigma_u^2, \tag{53}$$

where α_j, β_j, A, B are given in Theorem 2. It is not difficult to show that if $c = 0$, then $\hat{f}_M(N_M, N_U) > \hat{f}_U(N_M - 1, N_U + 1)$ for $N_M < \infty$, and $\hat{f}_M(N_M, N_U) = \hat{f}_U(N_M - 1, N_U + 1)$ for $N_M = \infty$. Also, $\hat{f}_M(N_M, N_U) - \hat{f}_U(N_M - 1, N_U + 1)$ strictly decreases in N_M. Therefore, if $c < \bar{c}$, then there is a unique $N_M^* < \infty$, where

$$\bar{c} = \hat{f}_M(1, N_U) - f_U(0, N_U + 1), \tag{54}$$

i.e., $\bar{c}$ is the monopolistic market maker's certainty equivalent wealth gain for making the market. Therefore, there exists a unique N_M^* (If $c = 0$, then $N_M^* = \infty$), such that market makers are better off (resp. worse off) for being market makers when $N_M < N_M^* (N_M > N_M^*)$. In other words, there are no more entries to the market makers' market when $N_M \geq N_M^*$. *Q.E.D.*

Proof of Theorem 3: From our assumption that $\hat{F}$ and $\sigma_{uN}\hat{X}_I$ are *i.i.d* normally distributed, we know that $\hat{S}$ is normally distributed with mean zero and variance $\sigma_F^2 + \sigma_H^2$. The covariance between $\hat{F}$ and $\hat{S}$ is $Cov(\hat{F}, \hat{S}) = Cov(\hat{F}, \hat{F} - \delta\sigma_{uN}\hat{X}_I) = Var(\hat{F}) = \sigma_F^2$, therefore, the correlation coefficient of $\hat{F}$ and $\hat{S}$ is $\rho_{\hat{F},\hat{S}} = \frac{\sigma_F}{\sqrt{\sigma_F^2+\sigma_H^2}}$.

$$E[\hat{F}|\hat{S}] = \rho_{\hat{F},\hat{S}} \times \sigma_F \times \frac{\hat{S}}{\sqrt{\sigma_F^2 + \sigma_H^2}} = \frac{\hat{S}\sigma_F^2}{\sigma_F^2 + \sigma_H^2},$$

$$Var[\hat{F}|\hat{S}] = (1 - \rho_{\hat{F},\hat{S}}^2)\sigma_F^2 = \frac{\sigma_H^2\sigma_F^2}{\sigma_F^2 + \sigma_H^2}.$$

Also, we know that

$$E[e^{-\delta\theta_U\hat{F}}|\hat{S}] = e^{-\delta\theta_U E[\hat{F}|\hat{S}] + \frac{1}{2}\delta^2\theta_U^2 Var[\hat{F}|\hat{S}]}. \tag{55}$$

Plugging (55) into the uninformed investor's objective function (23) and taking first order condition, we get type-U investor's optimal stock demand given P_a^* is

$$\theta_U^* = \frac{\frac{\hat{S}\sigma_F^2}{\sigma_F^2+\sigma_H^2} - P_a^* + \bar{V} - \delta\hat{X}_U\sigma_{uN}}{\delta\left(\sigma_u^2 + \frac{\sigma_H^2\sigma_F^2}{\sigma_F^2+\sigma_H^2}\right)}. \tag{56}$$

Similarly, we get:

$$\theta_M^* = \frac{\frac{\hat{S}\sigma_F^2}{\sigma_F^2+\sigma_H^2} - P_a^* + \bar{V}}{\delta\left(\sigma_u^2 + \frac{\sigma_H^2\sigma_F^2}{\sigma_F^2+\sigma_H^2}\right)}. \tag{57}$$

Substituting (21), (56) and (57) into market clearing condition

$$N_I\theta_I^* + N_U\theta_U^* + N_M\theta_M^* = N\bar{\theta},$$

we get the equilibrium stock price P_a^*. Substituting P_a^* into (21), (26) and (57), we can get I, U and M investors' optimal stock holdings. *Q.E.D.*

Proof of Theorem 4: We prove Case 1 when $\hat{S} < -\delta\sigma_F^2\bar{\theta}$. Given bid price B and ask price A, the optimal demand of I and U are:

$$\theta_I^* = \frac{\hat{S} + \bar{V} - B}{\delta\sigma_u^2} \quad and \quad \theta_U^* = \frac{\frac{\hat{S}\sigma_F^2}{\sigma_F^2+\sigma_H^2} - A + \bar{V}}{\delta\left(\sigma_u^2 + \frac{\sigma_H^2\sigma_F^2}{\sigma_F^2+\sigma_H^2}\right)}. \tag{58}$$

Substituting (58) into the market clearing conditions (43), we get that the market clearing bid and ask prices are:

$$A = \frac{1}{1+\frac{\sigma_H^2}{\sigma_F^2}}\hat{S} + \bar{V} - \frac{\delta\bar{\theta}}{d_2} - \frac{\delta}{N_U d_2}\sum_{j=1}^{N_M}\beta_j, \quad and \quad B = \hat{S} + \bar{V} - \delta\sigma_u^2\bar{\theta} + \frac{\delta\sigma_u^2}{N_I}\sum_{j=1}^{N_M}\alpha_j, \tag{59}$$

where α_j and β_j are the optimal shares of stock M_j choose to buy from I investors and sell to U investors respectively. Market maker M_j's problem is:

$$\min_{\alpha_j,\beta_j} -\delta(\beta_j A - \alpha_j B) - \delta(\bar{\theta} + \alpha_j - \beta_j)\left(\frac{\hat{S}}{1+\frac{\sigma_H^2}{\sigma_F^2}} + \bar{V}\right) + \frac{1}{2}\delta^2(\bar{\theta} + \alpha_j - \beta_j)^2\frac{1}{d_2}, \tag{60}$$

where A and B are the market clearing prices given in (59). F.O.C with respect to α_j gives us:

$$\frac{\sigma_H^2}{\sigma_F^2+\sigma_H^2}\hat{S} + \delta\left(\frac{1}{d_2} - \sigma_u^2\right)\bar{\theta} + \frac{\delta\sigma_u^2}{N_I}\sum_{j=1}^{N_M}\alpha_j + \left(\frac{\sigma_u^2}{N_I} + \frac{1}{d_2}\right)\delta\alpha_j - \frac{\delta}{d_2}\beta_j = 0. \tag{61}$$

Sum all, we get:

$$\frac{N_M\sigma_H^2}{\sigma_F^2+\sigma_H^2}\hat{S} + N_M\delta\left(\frac{1}{d_2} - \sigma_u^2\right)\bar{\theta} + \left(\frac{(N_M+1)\sigma_u^2}{N_I} + \frac{1}{d_2}\right)\delta\sum_{j=1}^{N_M}\alpha_j - \frac{\delta}{d_2}\sum_{j=1}^{N_M}\beta_j = 0. \tag{62}$$

F.O.C with respect to β_j, we get:

$$\frac{\delta}{N_U d_2}\sum_{j=1}^{N_M}\beta_j - \delta(\alpha_j - \beta_j)\frac{1}{d_2} + \frac{\delta}{N_U d_2}\beta_j = 0. \tag{63}$$

Sum all, we get:

$$\sum_{j=1}^{N_M}\alpha_j = \frac{N_U + N_M + 1}{N_U}\sum_{j=1}^{N_M}\beta_j. \tag{64}$$

Substituting (64) into (62), we get

$$\sum_{j=1}^{N_M}\beta_j = -\frac{N_M N_I N_U d_1 \frac{\sigma_H^2}{\sigma_F^2}}{\delta(N_M+1)\left(N_I + (N_U + N_M + 1)d_2\sigma_u^2\right)}\left(\hat{S} + \delta\sigma_F^2\bar{\theta}\right). \tag{65}$$

Substituting (65) into (59), we can get the equilibrium bid and ask prices A_{a1}^* and B_{a1}^*. And then substituting A_{a1}^* and B_{a1}^* into (58), we can get the optimal stock holdings of I and U investors.

$$a_1 = \frac{\left(N_M + \frac{1}{1+\frac{\sigma_H^2}{\sigma_F^2}}\right)N_I + (N_M+1)(N_U+N_M+1)d_1\sigma_u^2}{(N_M+1)\left(N_I + (N_U+N_M+1)d_2\sigma_u^2\right)}, \tag{66}$$

$$\kappa = a_1\frac{\sigma_F^2}{\sigma_u^2} + \frac{\frac{N_I}{d_2} + (N_M N_I + (N_M+1)N_U + (N_M+1)^2)\sigma_u^2}{(N_M+1)\left(N_I + (N_U+N_M+1)d_2\sigma_u^2\right)\sigma_u^2}, \tag{67}$$

$$b_1 = \frac{(N_M+1)N_I + (N_M + 1 + \frac{\sigma_H^2}{\sigma_F^2})(N_U+N_M+1)d_1\sigma_u^2}{(N_M+1)\left(N_I + (N_U+N_M+1)d_2\sigma_u^2\right)}, \tag{68}$$

$$C_I = \frac{\frac{\sigma_H^2}{\sigma_F^2}N_M\left(N_U+N_M+1\right)}{\delta(1+N_M)\left(N_I\sigma_H^2 + (N+1)(1+\frac{\sigma_H^2}{\sigma_F^2})\sigma_u^2\right)}, \tag{69}$$

$$C_U = \frac{\frac{\sigma_H^2}{\sigma_F^2}N_M N_I}{\delta(1+N_M)\left(N_I\sigma_H^2 + (N+1)(1+\frac{\sigma_H^2}{\sigma_F^2})\sigma_u^2\right)}, \tag{70}$$

$$C_M = \frac{\frac{\sigma_H^2}{\sigma_F^2} N_I}{\delta \left(N_I \sigma_H^2 + (N+1)(1+\frac{\sigma_H^2}{\sigma_F^2})\sigma_u^2 \right)}, \tag{71}$$

and

$$C_\alpha = \frac{N_I d_1 \frac{\sigma_H^2}{\sigma_F^2}(N_U + N_M + 1)}{\delta(N_M+1)\left(N_I + (N_U + N_M + 1) d_2 \sigma_u^2\right)}. \tag{72}$$

It is not difficult to derive that $A_{a1}^* < P_{aU}^*$ and $B_{a1}^* > P_{aI}^*$ are equivalent to $\hat{S} < -\delta\sigma_F^2\bar{\theta}$ which is just the condition we conjecture for I investors to sell and U investors to buy. Similarly, we can prove Case 2 of this Theorem. *Q.E.D.*

Proof of Proposition 7: We have

$$P_a^* - P_s^* = \frac{(d_1-d_2)(N_M+N_U)\sigma_u^2}{N_I + d_2(N_M+N_U)\sigma_u^2}\hat{F} + \left(\frac{N_I}{N} - \frac{N_I + d_1(N_M+N_U)\sigma_u^2}{N_I + d_2(N_M+N_U)\sigma_u^2}\right)\delta\sigma_{uN}\hat{X}_I$$
$$+\delta\sigma_u^2\bar{\theta}\left(1 - \frac{N}{N_I + d_2(N_M+N_U)\sigma_u^2}\right).$$

It is not difficult to get

$$If\ \hat{F} \le (>) -\delta\sigma_F^2\bar{\theta} - \frac{N\sigma_u^2 + N_I\sigma_H^2}{N\frac{\sigma_H^2}{\sigma_F^2}\sigma_u^2}\delta\sigma_{uN}\hat{X}_I,\ then\ P_a^* \ge (<) P_s^*. \tag{73}$$

Since

$$E[P_s^*] = \bar{V} - \delta\sigma_u^2\bar{\theta},\ and\ E[P_a^*] = \bar{V} - \frac{\delta N\sigma_u^2\bar{\theta}}{N_I + d_2(N_M+N_U)\sigma_u^2},$$

and $N_I + d_2(N_M + N_U)\sigma_u^2 < N$, we have $E[P_a^*] < E[P_s^*]$. *Q.E.D.*

Proof of Proposition 8: There are four cases: (1)informed investors sell in both symmetric and asymmetric information case; (2) informed investors buy in both symmetric and asymmetric information case; (3) informed investors sell in symmetric information case but buy in asymmetric information case; (4) informed investors buy in symmetric information case but sell in asymmetric information case. For case (1),

$$(A_a^* - B_a^*) - (A_s^* - B_s^*) = -\frac{1}{(N_M+1)(1+\frac{\sigma_H^2}{\sigma_F^2})}\delta \hat{X}_I \sigma_{uN} - \frac{\sigma_H^2}{(N_M+1)(\sigma_F^2+\sigma_H^2)}(\hat{F} + \delta\sigma_F^2\bar{\theta}),$$

so if $\sigma_{uN}\hat{X}_I \geq 0$ & $-\delta\sigma_F^2\bar{\theta} - \frac{\delta\sigma_{uN}\hat{X}_I\sigma_F^2}{\sigma_H^2} < \hat{F} \leq -\delta\sigma_F^2\bar{\theta} + \delta\sigma_{uN}\hat{X}_I$, then $A_a^* - B_a^* < A_s^* - B_s^*$. For case (3),

$$(A_a^* - B_a^*) - (A_s^* - B_s^*) = -\frac{\sigma_F^2 + 2\sigma_H^2}{(N_M+1)(\sigma_F^2+\sigma_H^2)}\delta \hat{X}_I \sigma_{uN} + \frac{\sigma_H^2}{(N_M+1)(\sigma_F^2+\sigma_H^2)}(\hat{F} + \delta\sigma_F^2\bar{\theta}),$$

so if $\sigma_{uN}\hat{X}_I \geq 0$ & $-\delta\sigma_F^2\bar{\theta} + \delta\sigma_{uN}\hat{X}_I < \hat{F} < -\delta\sigma_F^2\bar{\theta} + \frac{\sigma_F^2+2\sigma_H^2}{\sigma_H^2}\delta\sigma_{uN}\hat{X}_I$, then $A_a^* - B_a^* < A_s^* - B_s^*$. Combine (1) and (3), we get: if $\sigma_{uN}\hat{X}_I \geq 0$ & $-\delta\sigma_F^2\bar{\theta} - \frac{\delta\sigma_{uN}\hat{X}_I\sigma_F^2}{\sigma_H^2} < \tilde{F} < -\delta\sigma_F^2\bar{\theta} + \frac{\sigma_F^2+2\sigma_H^2}{\sigma_H^2}\delta\sigma_{uN}\hat{X}_I$. This is the first case in 1 of Proposition 8. Similarly, we can prove other cases. *Q.E.D.*

Proof of Proposition 9:

$$E[A_s^* - B_s^*] = \frac{1}{N_M+1}\delta|\sigma_{uN}|\left(\int_0^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_I}e^{-\frac{x^2}{2\sigma_I^2}}xdx - \int_{-\infty}^0\frac{1}{\sqrt{2\pi}\sigma_I}e^{-\frac{x^2}{2\sigma_I^2}}xdx\right) \tag{74}$$

$$= \frac{2}{N_M+1}\delta|\sigma_{uN}|\int_0^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_I}e^{-\frac{x^2}{2\sigma_I^2}}xdx = \frac{2}{N_M+1}\frac{\sigma_H}{\sqrt{2\pi}}. \tag{75}$$

We know from previous section,

$$f(\hat{S}) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_F^2+\sigma_H^2}}e^{-\frac{\hat{S}^2}{2(\sigma_H^2+\sigma_F^2)}}, \ \hat{S} = \tilde{F} - a\hat{X}_I.$$

Therefore,

$$E[A_a^* - B_a^*] = -\frac{\frac{\sigma_H^2}{\sigma_F^2}}{(N_M+1)(1+\frac{\sigma_H^2}{\sigma_F^2})}\left(\int_{-\infty}^{-\delta\sigma_F^2\bar{\theta}}(\hat{S}+\delta\sigma_F^2\bar{\theta})f(\hat{S})d\hat{S} - \int_{-\delta\sigma_F^2\bar{\theta}}^{+\infty}(\hat{S}+\delta\sigma_F^2\bar{\theta})f(\hat{S})d\hat{S}\right)$$

$$= \frac{\frac{\sigma_H^2}{\sigma_F^2}}{(N_M+1)(1+\frac{\sigma_H^2}{\sigma_F^2})}\left(\frac{2\sqrt{1+\frac{\sigma_H^2}{\sigma_F^2}}\sigma_F}{\sqrt{2\pi}}e^{-\frac{\delta^2\bar{\theta}^2\sigma_F^2}{2(1+\frac{\sigma_H^2}{\sigma_F^2})}} + \delta\sigma_F^2\bar{\theta}\left(2N\left(\frac{\delta\sigma_F\bar{\theta}}{\sqrt{1+\frac{\sigma_H^2}{\sigma_F^2}}}\right)-1\right)\right).$$

We use the fact that $\frac{x}{1+x^2}n(x) < 1 - N(x) < \frac{n(x)}{x},\ x > 0$, where $n(x)$ is the *pdf* for standard normal distribution and $N(x)$ is the *cdf* for standard normal distribution. We have:

$$E[A_a^* - B_a^*] > \frac{\delta\sigma_H^2\sigma_F^2\bar{\theta}}{(N_M+1)(\sigma_H^2+\sigma_F^2)}.$$

And we have

$$E[A_a^* - B_a^*] < \frac{\sigma_H^2}{(N_M+1)(\sigma_F^2+\sigma_H^2)}(|\hat{F}| + |-\delta\sigma_{uN}\hat{X}_I| + \delta\sigma_F^2\bar{\theta})$$

$$< \frac{\sigma_H^2}{(N_M+1)(\sigma_F^2+\sigma_H^2)}\left(\frac{2\sigma_F}{\sqrt{2\pi}} + \frac{2\sigma_H}{\sqrt{2\pi}} + \delta\sigma_F^2\bar{\theta}\right)$$

Therefore, if $\frac{\delta\sigma_H^2\sigma_F^2\bar{\theta}}{\sigma_H^2+\sigma_F^2} > \frac{2\sigma_H}{\sqrt{2\pi}}$, then $E[A_a^* - B_a^*] > E[A_s^* - B_s^*]$, and if $\sigma_H < \frac{\sigma_F}{1+\delta\bar{\theta}\sigma_F\sqrt{\frac{\pi}{2}}}$, then $E[A_a^* - B_a^*] < E[A_s^* - B_s^*]$.

Proof of Proposition 10: The total utility loss with symmetric information is: $UL = \delta\sigma_{uN}^2\hat{X}_I^2D^2$, and the total utility loss with asymmetric information is: $ULA =$

$(\hat{S} + \delta\sigma_F^2\bar{\theta})^2\frac{E^2}{\delta}$, where D^2 and E^2 are as follows.

$$D^2 = \frac{N_I\left(N_M^3 + N_U(1+N_I+N_U)^2 + N_M^2(2+3N_U) + N_M(1+N_U(4+3N_I+3N_U))\right)}{2(1+N_M)^2N^2(1+N)^2\sigma_u^2}, \tag{76}$$

$$E^2 = \frac{\frac{\sigma_H^4}{\sigma_F^4}N_I}{2(1+\frac{\sigma_H^2}{\sigma_F^2})(N_M+1)^2\left(\frac{\sigma_H^2}{\sigma_F^2}N_I\sigma_F^2 + (1+\frac{\sigma_H^2}{\sigma_F^2})N\sigma_u^2\right)\left(\frac{\sigma_H^2}{\sigma_F^2}N_I\sigma_F^2 + (1+\frac{\sigma_H^2}{\sigma_F^2})(1+N)\sigma_u^2\right)^2}$$

$$\Big(\left(N_M^3 + N_U(1+N_I+N_U)^2 + N_M^2(2+3N_U) + N_M\left(1+N_U(4+3N_I+3N_U)\right)\right)\sigma_u^4$$

$$+\frac{\sigma_H^2}{\sigma_F^2}\sigma_u^2\Big(\sigma_F^2N_IN_U(2+2N+N_M) + 2\Big(N_M^3 + N_U(1+N_I+N_U)^2 + N_M^2(2+3N_U)$$

$$+N_M(1+N_U(4+3N_I+3N_U))\Big)\sigma_u^2\Big) + \frac{\sigma_H^4}{\sigma_F^4}\Big(\sigma_F^4N_I^2N_U + \sigma_F^2N_IN_U(2+2N+N_M)\sigma_u^2$$

$$+\left(N_M^3+N_U(1+N_I+N_U)^2+N_M^2(2+3N_U)+N_M(1+N_U(4+3N_I+3N_U))\right)\sigma_u^4\Big)\Big). \tag{77}$$

Letting

$$C_1 = \frac{D}{E}, \tag{78}$$

then we have $ULA < UL$ if and only if $|\hat{S}+\delta\sigma_F^2\bar{\theta}| < C_1|\delta\sigma_{uN}\hat{X}_I|$. *Q.E.D.*

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