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Asset Pricing Implications of Short-Sale Constraints in Imperfectly Competitive Markets

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Abstract. We study the impact of short-sale constraints on market prices and liquidity in imperfectly competitive markets in which market makers have market power. In contrast to the existing literature, we show that because competition is imperfect, short-sale constraints decrease bid prices, increase ask prices, and drive up bid-ask spread volatility, with or without information asymmetry. If market makers are risk neutral, then short-sale constraints do not affect ask prices or ask depths. In addition, the impact of short-sale constraints can increase with market transparency. Our main results are unaffected by endogenous information acquisition or reduced information revelation because of short-sale constraints.

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1. Introduction

Implicit and explicit short-sale constraints are prevalent in many financial markets, and competition in most of these markets is far from perfect (e.g., Christie and Schultz 1994, Biais et al. 2010).[1] However, to the best of our knowledge, extant theories regarding how short-sale constraints affect asset prices and market liquidity exclusively focus on perfectly competitive markets.[2] In addition, they cannot explain a robust empirical finding that impositions of regulatory short-sale bans cause significant increases in bid-ask spreads in many financial markets.[3] Regulatory short-sale bans are more likely imposed and more likely to bind when market conditions have deteriorated significantly and a large number of investors can only trade with a small number of designated market makers who have significant market power (e.g., Anand and Venkataraman 2016).[4] This motivates us to study the impact of short-sale constraints in an imperfectly competitive market in which investors trade through a small number of designated market makers with market power. We find that short-sale constraints have qualitatively different impacts in the presence of market power. In particular, our analysis predicts that short-sale constraints decrease bid prices and increase bid-ask spreads. In addition, if market makers are risk averse, then short-sale constraints also increase ask prices. Furthermore, our model suggests that the impact of short-sale constraints tends to be greater in markets with more transparency.

More specifically, we consider an equilibrium model with three types of risk-averse investors: hedgers, nonhedgers, and a designated market maker. Investors can trade one risk-free asset and one risky security. Hedgers have trading demand motivated by hedging. In addition, hedgers may observe a private signal about the risky security's future payoff, and thus may also have information-motivated trading demand. Both hedgers and nonhedgers are subject to short-sale constraints and trade through the designated market maker. As in Goldstein et al. (2014), because investors have different motives to trade, their reservation prices may differ,[5] which causes trading in equilibrium.

Because short-sale constraints restrict sales, one might expect that bid prices increase in equilibrium, as predicted by most of the extant theories (e.g., Harrison and Kreps 1978, Yuan 2006, Wang 2016). However, this is exactly the opposite to what we find. One key difference of our model from the extant literature is that competition among short-sellers' counterparty (i.e., the market maker) is imperfect in our model. The intuition for our opposite result can be illustrated with a simple example. Suppose that, without short-sale constraints, a short seller short sells ten shares (at the bid) in equilibrium, but with short-sale constraints, the short seller can only short sell five shares. Because the optimal number of shares that the short seller chooses to short decreases as the bid price decreases, a market maker with market power can lower the bid price to the level at which the constraints just start to bind (i.e., at this lower bid price, the short seller shorts five shares even when unconstrained). By doing this, the market maker pays a lower price for the shares without any adverse impact on the number of shares she can buy (i.e., still five shares).[6] Therefore, because of the market power of the market maker, the equilibrium bid price is lower with short-sale constraints.[7] More generally, when some investors are restricted from selling more, if buyers do not have market power, they will then compete for the reduced supply, and thus drive up the equilibrium trading price, as found in the extant literature that considers competitive markets. On the other hand, if buyers have market power, then the equilibrium price goes down, as we show in this paper. This is because a lower price is better for buyers, and if it is set at the level at which short-sale constraints just start to bind, it does not affect the number of shares buyers can buy. Our paper is the first to demonstrate how short-sale constraints affect the price at which short-sales occur (i.e., the bid) critically depends on whether buyers have market power.

Because the market maker buys less from short sellers when short-sale constraints bind, she also sells less at the ask by charging a higher ask price to achieve the optimal inventory risk exposure. This results in a higher ask price and a smaller ask depth. The simplest example to explain the intuition is when the market maker is infinitely risk averse. In this case, the market maker does not carry any inventory (and makes profit only from the spread). Therefore, when her purchase at the bid is reduced by short-sale constraints imposed on other investors, she reduces her sale by the same amount by charging a higher ask price to avoid any net inventory position. Our analysis also implies that as the midquote price volatility increases, the probability that short-sale constraints bind also increases.

On the other hand, if the market maker were risk neutral, then the change in the inventory risk because of the reduction of purchases at the bid caused by short-sale constraints would be irrelevant for her, and thus short-sale constraints would not affect the ask price or the ask depth. This demonstrates how short-sale constraints affect the ask price, and the ask depth critically depends on a market maker's risk aversion. However, unless the reservation price of the market maker is so high that she wants to buy more with a significantly higher price from other unconstrained investors, the same intuition as previously stated would still apply for the determination of the bid price and the bid depth, and thus short-sale constraints would still lower the bid and increase the spread. Therefore, while the market maker's market power is the key driving force behind the result that short-sale constraints decrease the bid price, the market maker's aversion to inventory risk is the channel through which short-sale constraints increase the ask price and decrease the ask depth.[8]

We show that, even in the presence of information asymmetry, our main qualitative results still hold. In addition, because more public disclosure about asset payoff reduces overall uncertainty and increases investors' trading demand, short-sale constraints bind more often and thus have a greater effect with more public disclosure. Thus, our model predicts that, ceteris paribus, the adverse impact of short-sale constraints on prices and market liquidity is greater in more transparent markets.[9]

To the best of our knowledge, Diamond and Verrecchia (1987) (hereafter DV) is the only theoretical paper in the existing literature that examines the effect of short-sale constraints on bid-ask spreads.[10] Because the uninformed are unlikely to short even without a short-sale ban (e.g., Boehmer et al. 2008), as Boehmer et al. (2013) point out, DV predict that if short sales are banned, then bid-ask spreads will narrow. This is because the ban prevents the informed from shorting, and thus other traders will face less adverse selection after the ban. On the other hand, if there is no information asymmetry, then DV predict that short-sale constraints have no impact on the bid or the ask or the spread.[11] In contrast, the extant empirical literature finds that bid-ask spreads significantly increase as a result of the 2008 short-sale bans (e.g., Beber and Pagano 2013, Boehmer et al. 2013, Ang et al. 2013), which is exactly what our model predicts. As most of the rational expectations models in market microstructure literature (e.g., Glosten and Milgrom 1985, Admati and Pfleiderer 1988), DV consider a perfect competition market with risk-neutral market makers. However, the presence of a market maker's market power is an important characteristic in the markets studied by the above empirical work around the 2008 short-sale bans. One of the reasons for this market power is that other liquidity providers in normal times tend to exit markets during bad times, and only a small number of market makers remain active (e.g., Anand and Venkataraman 2016). The difference in the prediction of DV and that of ours highlights the importance of a market-maker's market power in affecting the impact of short-sale bans.

2. Applicable Markets and Additional Related Literature

Even relatively more liquid markets, such as the New York Stock Exchange (NYSE), NASDAQ, and Paris Bourse, employ designated market makers to facilitate trading, especially during financial market stress.[12] These market makers are required to maintain two-sided markets during exchange hours and are obligated to buy and sell at their displayed bids and offers.

Designated market makers are core liquidity providers in many of these markets, even under normal market conditions. For example, in 2015, designated market makers accounted for about approximately 12% of liquidity adding volume in NYSE-listed securities, on average. Anand and Venkataraman (2016) find that endogenous liquidity providers scale back their participation in unison when market conditions are unfavorable. Around the imposition of the short-sale bans during the financial crisis in 2008, designated market makers tend to play an even more important role in making the market because many endogenous liquidity providers become liquidity demanders at that time. Accordingly, to capture this feature, we focus on the trades that investors made with the designated market makers to study the impact of short-sale constraints, although there are limit-order-book driven transactions in these markets.

Competition among market makers is imperfect in many financial markets. For example, Christie and Schultz (1994) suggest that NASDAQ dealers may implicitly collude to maintain wide spreads. (Biais et al. 2010) analyze trades and order placement on NASDAQ and a competing electronic order book, Island. They conclude that competition among market makers in these markets is still imperfect even after the introduction of electronic markets. In addition, the opaqueness and illiquidity of many dealers' markets make these markets even less competitive (e.g., Ang et al. 2013).

Because there tend to be less liquidity and less trading volume in imperfectly competitive markets, implicit and explicit short-sale constraints are more prevalent in these markets. For example, short selling of small stocks is difficult and rare, possibly because of low ownership by market makers and institutions (the main lenders of shares), which leads to high short-sale costs. Even though we model short-sale constraints in the form of an explicit limit on short positions instead of in the form of short-sale costs, the qualitative results from these two alternative approaches are the same if the short-sale costs are sufficiently high to reduce short sales, on average.[13] In addition, explicit short-sale constraints are also often imposed by market-making firms in many imperfectly competitive markets. For example, Ang et al. (2013) collect short-selling data for a sample of 50 over-the-counter (OTC) stocks and 50 similarly sized (small) listed stocks in June 2012. They find that short sales are prohibited for a large number of the listed stocks and even more for the OTC stocks. For instance, they state that "a retail customer of Fidelity could buy all 100 of these stocks, but the broker would allow short selling in only one of the OTC stocks and eight of the listed stocks" (p. 2990).

A vast literature exists on the impact of short-sale constraints on asset prices in competitive markets. Most of these models, except Hong and Stein (2003) and Bai et al. (2006), find that short-sale constraints drive up trading prices (e.g., Scheinkman and Xiong 2003, Wang 2016). Hong and Stein (2003) and Bai et al. (2006) show that short-sale constraints can cause trading prices to go down, as in our model. However, the driving force in Hong and Stein (2003) and Bai et al. (2006) is the assumption that short-sale constraints prevent some investors from revealing negative information. For example, when the negative information initially prevented from being revealed is disclosed later, prices decrease, as shown in Hong and Stein (2003). In contrast, the driving force behind our result that short-sale constraints can lower trading prices is buyers' market power, and therefore our result holds even when there is no information asymmetry.[14] In addition, different from these two papers, our paper predicts that bid price decreases by a greater amount in more transparent markets. Furthermore, neither Hong and Stein (2003) nor Bai et al. (2006) examine the impact of short-sale constraints on bid-ask spreads. Liu and Wang (2016) study market making in the presence of asymmetric information and inventory risk and demonstrate that bid-ask spreads may decrease with information asymmetry. Short-sale constraints are absent in Liu and Wang (2016), and thus they are silent on the impact of short-sale constraints on market prices and liquidity. Nezafat et al. (2014) consider an equilibrium model with endogenous information acquisition and short-sale constraints. In contrast to our model, they do not study the impact of short-sale constraints on *equilibrium* bid-ask spreads, and there are no strategic traders in their model.

3. The Model

We consider a one-period setting with dates 0 and 1.[15] There are a continuum of identical hedgers with mass N_h, a continuum of identical nonhedgers with mass N_n, and $N_m = 1$ designated market maker. They can trade one risk-free asset and one risky security on date 0 to maximize their expected constant absolute risk aversion (CARA) utility from the terminal wealth on date 1. No investor is endowed with any amount of the risk-free asset. The risk-free asset serves as the numeraire, and thus the risk-free interest rate is normalized to 0. For type $i \in \{h, n, m\}$ investors, the total risky security endowment is $N_i \bar{\theta}$ shares.[16] The aggregate supply of the risky security is $N \times \bar{\theta} > 0$ shares where $N = N_h + N_n + N_m$ and the date 1 payoff of each share is $\tilde{V}$, where $\tilde{V} \sim \mathbf{N}(\bar{V}, \sigma_V^2)$, $\bar{V}$ is a constant, $\sigma_V > 0$, and $\mathbf{N}(\cdot)$ denotes the normal distribution.

Hedgers are subject to a liquidity shock that is modeled as a random endowment of $\hat{X}_h \sim \mathbf{N}(0, \sigma_X^2)$ units of a nontradable risky asset on date 0, with $\hat{X}_h$ realized on date 0.[17] The nontradable asset has a per-unit payoff of $\tilde{L} \sim \mathbf{N}(0, \sigma_L^2)$ that has a covariance of σ_{VL} with the risky security's payoff $\tilde{V}$. The payoff of the nontradable asset is realized and becomes public on date 1. The correlation between the nontradable asset and the risky security results in a liquidity demand for the risky security to hedge the nontradable asset payoff. The nonhedgers do not have any liquidity shocks, that is, $\hat{X}_n = 0$.

All trades go through the designated market maker.[18] As required by regulators, the designated market maker must provide quotes on both sides of the market. Accordingly, we assume that the market maker posts her price schedules first. Then hedgers and nonhedgers decide how much to sell to the designated market maker at the bid B or buy from her at the ask A or do not trade at all. When deciding on what price schedules to post, the market maker takes into account the best response functions (i.e., the demand schedules) of other investors given the to-be-posted price schedules.[19] In equilibrium, the risk-free asset market also clears.

We assume that both hedgers and nonhedgers are subject to short-sale constraints, that is, the after-trade position $\theta_i + \bar{\theta} \geq -\kappa_i$, $i = h, n$, where θ_i is the quantities demanded by trader i and $\kappa_i \geq 0$ can be different for the hedgers and the nonhedgers.[20] A smaller κ_i means a more stringent short-sale constraint: If $\kappa_i = 0$, then type i investors are prohibited from short selling; and if $\kappa_i = \infty$, then it is equivalent to the absence of short-sale constraints. Heterogeneous short-sale constraint stringencies for hedgers and nonhedgers capture the essence of possibly different short-sale costs across them and allow us to examine the impact of a short-sale ban when some investors cannot short sell even without the ban (e.g., Kolasinski et al. 2013). In most markets, a designated market maker is exempted from short-sale constraints by regulators to facilitate her liquidity provision. Accordingly, we assume that the market maker is not subject to short-sale constraints.[21]

Because there is a continuum of hedgers and nonhedgers, we assume that they are price takers.[22] After observing liquidity shock $\hat{X}_h$, each hedger chooses a demand schedule $\Theta_h(\hat{X}_h, \cdot)$. Because from equilibrium prices, nonhedgers can infer out the liquidity shock realized $\hat{X}_h$, there is no information asymmetry in equilibrium. Thus, each nonhedger chooses a demand schedule $\Theta_n(\hat{X}_h, \cdot)$ that can also directly depend on $\hat{X}_h$. The schedules Θ_h and Θ_n are traders' strategies. Given bid price B and ask price A, the quantities demanded by hedgers and nonhedgers can be written as $\theta_h = \Theta_h(\hat{X}_h, A, B)$ and $\theta_n = \Theta_n(\hat{X}_h, A, B)$.

Given A and B, for $i \in \{h, n\}$, a type i investor's problem is to choose θ_i to solve

$$\max E[-e^{-\delta \tilde{W}_i}], \tag{1}$$

subject to the budget constraint

$$\tilde{W}_i = \theta_i^- B - \theta_i^+ A + (\bar{\theta} + \theta_i)\tilde{V} + \hat{X}_i \tilde{L}, \tag{2}$$

and the short-sale constraint

$$\theta_i + \bar{\theta} \geq -\kappa_i, \tag{3}$$

where $\delta > 0$ is the absolute risk-aversion parameter, $\hat{X}_n = 0$, $x^+ := \max(0, x)$, and $x^- := \max(0, -x)$.[23]

Since h and n investors buy from the designated market maker at ask and sell to her at bid, we can view these trades as occurring in two separate markets: the "ask" market and the "bid" market. In the ask market, the market maker is the supplier, and other investors are demanders; and the opposite is true in the bid market. The monopolistic market maker chooses bid and ask prices, taking into account other investors' demand curves in the ask market and supply curves in the bid market.

Given liquidity shock $\hat{X}_h$, let the realized demand schedules of hedgers and nonhedgers be denoted as $\Theta_h(A, B)$ and $\Theta_n(A, B)$, respectively, where A is the ask price and B is the bid price. By market-clearing conditions, the equilibrium ask depth α must be equal to the total amount bought by other investors, and the equilibrium bid depth β must be equal to the total amount sold by other investors, that is,[24]

$$\alpha = \sum_{i=h,n} N_i \Theta_i(A, B)^+, \quad \beta = \sum_{i=h,n} N_i \Theta_i(A, B)^-. \tag{4}$$

The risk-free asset market will be automatically cleared by the Walras' law. Note that, if an investor decides to buy (sell), then only the ask (bid) price affects how much he buys (sells), that is, $\Theta_i(A, B)^+$ only depends on A and $\Theta_i(A, B)^-$ only depends on B. Therefore, the bid depth β only depends on B, henceforth referred to as $\beta(B)$, and the ask depth α only depends on A, henceforth referred to as $\alpha(A)$.

We denote the market maker's pricing strategies as $\mathbb{A}(\cdot)$ and $\mathbb{B}(\cdot)$. For any realized demand schedules $\Theta_h(A, B)$ and $\Theta_n(A, B)$, the designated market maker's problem is to choose an ask price level $A := \mathbb{A}(\Theta_h, \Theta_n)$ and a bid price level $B := \mathbb{B}(\Theta_h, \Theta_n)$ to solve[25]

$$\max E[-e^{-\delta \tilde{W}_m}], \tag{5}$$

subject to

$$\tilde{W}_m = \alpha(A)A - \beta(B)B + (\bar{\theta} + \beta(B) - \alpha(A))\tilde{V}. \tag{6}$$

This leads to the definition of an equilibrium.

Definition 1. Given any liquidity shock $\hat{X}_h$, an equilibrium $(\Theta_h^*(A, B), \Theta_n^*(A, B), (A^*, B^*))$ is such that[26]

1. given any A and B, $\Theta_i^*(A, B)$ solves a type i investor's problem (1)–(3) for $i \in \{h, n\}$;
2. given $\Theta_h^*(A, B)$ and $\Theta_n^*(A, B)$, A^* and B^* solve the market maker's problem (5)–(6).

3.1. Discussions on the Assumptions of the Model

In this subsection, we discuss our main assumptions and whether these assumptions are important for our main results.

The assumption that there is only one market maker is for expositional simplicity. In Appendix B, we present the extension to the case with multiple market makers. In this more general model with Cournot competition, we show that our main qualitative results still hold (e.g., short-sale constraints increase the expected bid-ask spread).[27]

The existing empirical analyses of how short-sale constraints affect spreads focus on the spread difference shortly after the constraint imposition dates. Accordingly, we use a one-period setting to examine the immediate impact of short-sale constraints. This one-period setting also helps highlight the main driving forces behind our results and simplifies exposition. As we show in the online appendix, extending to a dynamic model does not change the immediate impact of short-sale constraints. The assumption that the market maker can make offsetting trades at bid and ask simultaneously is not critical for our main results. Even when the market maker *cannot* make an offsetting trade, short-sale constraints still decrease bid and bid depth. This is because as we show later, the market-maker's market power in the bid market is the key driving force for the result.

To keep the exposition as simple as possible to show the key driving force behind our main results, we assume that there is no information asymmetry in the main model. We relax this assumption in Section 6 and the online appendix to demonstrate the robustness of our results to the presence of information asymmetry even with reduced information revelation due to short-sale constraints. We assume that the market maker posts price schedules first and then other investors submit orders to the market maker. This is consistent with the common practice in less competitive markets in which a designated market maker making two-sided markets typically provides a take-it-or-leave-it pair of prices, a bid and an offer, to customers (e.g., Duffie 2012, chapter 1).

In accordance with the existing literature on the impact of short-sale constraints, we do not explicitly take into account the possibility that the imposition of short-sale constraints itself may convey negative information about the stock payoff. However, the effect of this negative signal is clear from our model, that is, it decreases both bid and ask prices. Therefore, while the result that short-sale constraints increase the ask price might be reversed if this negative information effect dominates, the main result that short-sale constraints decrease bid price would be strengthened. In addition, if the negative information effect lowers bid and ask by a similar amount, the result that short-sale constraints increase the spread would also likely hold. Moreover, empirical studies show that the increases in bid-ask spreads following short-sale bans are not driven by any negative information possibly conveyed by the impositions themselves (e.g., Beber and Pagano 2013, Boehmer et al. 2013, Ang et al. 2013).

4. The Equilibrium

In this section, we solve for the equilibrium bid and ask prices, bid and ask depths, and trading volume in closed form.

Given A and B, the optimal demand schedule for a type i investor for $i \in \{h, n\}$ is

$$\theta_i^*(A,B) = \begin{cases} (P_i^R - A)/(\delta\sigma_V^2) & A < P_i^R, \\ 0 & B \le P_i^R \le A, \\ \max\{-(\kappa_i + \bar{\theta}), -(B - P_i^R)/(\delta\sigma_V^2)\} & B > P_i^R, \end{cases} \quad (7)$$

where

$$P_i^R = \bar{V} + \omega \hat{X}_i - \delta\sigma_V^2 \bar{\theta} \quad (8)$$

is the reservation price of a type i investor (i.e., the critical price such that a nonmarket maker buys (sells, respectively) the security if and only if the ask price is lower (the bid price is higher, respectively) than this critical price) and

$$\omega = -\delta\sigma_{VL} \quad (9)$$

represents the hedging premium per unit of the liquidity shock.

Let Δ denote the difference between the reservation prices of h and n investors, that is,

$$\Delta := P_h^R - P_n^R = \omega \hat{X}_h. \quad (10)$$

The following theorem provides the equilibrium bid and ask prices and equilibrium security demand in closed form.

Theorem 1. 1. *If* $-2(N+1)\delta\sigma_V^2(\kappa_h + \bar{\theta})/(N_n + 2) < \Delta < 2(N+1)\delta\sigma_V^2(\kappa_n + \bar{\theta})/N_h$, *then short-sale constraints do not bind for any investors,*

(a) *the equilibrium bid and ask prices are*

$$A^* = P_n^R + \frac{N_h}{2(N+1)}\Delta + \frac{\Delta^+}{2}, \quad (11)$$

$$B^* = P_n^R + \frac{N_h}{2(N+1)}\Delta - \frac{\Delta^-}{2}, \quad (12)$$

the bid-ask spread is

$$A^* - B^* = \frac{|\Delta|}{2} = \frac{|\omega \hat{X}_h|}{2}, \quad (13)$$

(b) *the equilibrium quantities demanded are*

$$\theta_h^* = \frac{N_n+2}{2(N+1)\delta\sigma_V^2}\Delta, \quad \theta_n^* = -\frac{N_h}{2(N+1)\delta\sigma_V^2}\Delta, \quad \theta_m^* = 2\theta_n^*, \tag{14}$$

and the equilibrium quote depths are

$$\alpha^* = N_h(\theta_h^*)^+ + N_n(\theta_n^*)^+, \tag{15}$$
$$\beta^* = N_h(\theta_h^*)^- + N_n(\theta_n^*)^-. \tag{16}$$

2. *If* $\Delta \leq -2(N+1)\delta\sigma_V^2(\kappa_h+\bar{\theta})/(N_n+2)$, *then short-sale constraints bind for hedgers,*

(a) *the equilibrium bid and ask prices are*

$$A_{c1}^* = P_n^R - \frac{\delta N_h \sigma_V^2(\kappa_h+\bar{\theta})}{N_n+2}, \tag{17}$$
$$B_{c1}^* = P_h^R + \delta\sigma_V^2(\kappa_h+\bar{\theta}), \tag{18}$$

the bid-ask spread is

$$A_{c1}^* - B_{c1}^* = -\Delta - \frac{N+1}{N_n+2}\delta\sigma_V^2(\kappa_h+\bar{\theta}), \tag{19}$$

(b) *the equilibrium quantities demanded are*

$$\theta_{hc1}^* = -(\kappa_h+\bar{\theta}), \quad \theta_{nc1}^* = \frac{N_h(\kappa_h+\bar{\theta})}{N_n+2}, \quad \theta_{mc1}^* = \frac{2N_h(\kappa_h+\bar{\theta})}{N_n+2}, \tag{20}$$

and the equilibrium quote depths are

$$\alpha_{c1}^* = N_n\theta_{nc1}^*, \quad \beta_{c1}^* = N_h(\kappa_h+\bar{\theta}). \tag{21}$$

3. *If* $\Delta \geq 2(N+1)\delta\sigma_V^2(\kappa_n+\bar{\theta})/N_h$, *then short-sale constraints bind for nonhedgers,*

(a) *the equilibrium bid and ask prices are*

$$A_{c2}^* = P_h^R - \frac{\Delta + \delta N_n \sigma_V^2(\kappa_n+\bar{\theta})}{N_h+2}, \tag{22}$$
$$B_{c2}^* = P_n^R + \delta\sigma_V^2(\kappa_n+\bar{\theta}), \tag{23}$$

the bid-ask spread is

$$A_{c2}^* - B_{c2}^* = \frac{N_h+1}{N_h+2}\Delta - \frac{N+1}{N_h+2}\delta\sigma_V^2(\kappa_n+\bar{\theta}), \tag{24}$$

(b) *the equilibrium quantities demanded are*

$$\theta_{hc2}^* = \frac{\Delta + \delta N_n\sigma_V^2(\kappa_n+\bar{\theta})}{(N_h+2)\delta\sigma_V^2}, \quad \theta_{nc2}^* = -(\kappa_n+\bar{\theta}), \tag{25}$$
$$\theta_{mc2}^* = \frac{-N_h\Delta + 2\delta N_n\sigma_V^2(\kappa_n+\bar{\theta})}{(N_h+2)\delta\sigma_V^2}, \tag{26}$$

and the equilibrium quote depths are

$$\alpha_{c2}^* = N_h\theta_{hc2}^*, \quad \beta_{c2}^* = N_n(\kappa_n+\bar{\theta}). \tag{27}$$

Theorem 1 shows that whether short-sale constraints bind for some investors depends on whether the magnitude of the reservation price difference is sufficiently large. If hedgers' reservation price is close to that of nonhedgers, then no one trades a large amount in equilibrium, and thus short-sale constraints do not bind for any of the investors (Case 1). If hedgers' reservation price is much lower than that of nonhedgers, then the equilibrium bid price in the no-constraint case is much higher than the reservation price of hedgers, hedgers would like to sell a large amount, and thus short-sale constraints bind for hedgers (Case 2). The opposite is true if nonhedgers' reservation price is much lower than that of hedgers (Case 3). The thresholds for the reservation price difference such that short-sale constraints bind for some investors are determined by equalizing the unconstrained equilibrium short-sale quantities (θ_h^* or θ_n^*) to the short-sale bounds ($-(\kappa_h+\bar{\theta})$ or $-(\kappa_n+\bar{\theta})$, respectively).

Part 1 of Theorem 1 implies that when short-sale constraints do not bind, in equilibrium both bid and ask prices are nonlinear functions of the reservation prices of hedgers and nonhedgers. In addition, nonhedgers can indeed infer $\hat{X}_h$ from observing the equilibrium trading price as we conjectured, because of the one-to-one mapping between the two. Furthermore, part 1 shows that, when short-sale constraints do not bind, the equilibrium bid-ask spread is equal to half of the absolute value of the reservation price difference between hedgers and nonhedgers.

When short-sale constraints bind for hedgers or nonhedgers, the maximum amount of purchase that the market maker can make with the constrained investors is fixed, and thus the market maker's utility always decreases in the bid price in the region in which the constraints bind. As explained in the next section, the market power of the market maker then indicates that the optimal bid price when short-sale constraints bind in equilibrium must be such that short-sale constraints just start to bind, which gives rise to the constrained equilibrium bid prices as in (18) and (23), and bid depths as in (21) and (27). Given these bid prices and depths, ask prices and depths are then determined optimally by the market maker to trade off profit from the spread and the inventory risk, taking into account the demand schedules of the buyers.

5. The Effect of Short-Sale Constraints

In this section, we analyze the effect of short-sale constraints on bid prices, ask prices, bid-ask spreads, and bid-ask spread volatility.

By Theorem 1, we have the following:

Proposition 1. 1. *As short-sale constraints become more stringent for hedgers or nonhedgers, the equilibrium bid price decreases, the equilibrium ask price increases, and so does the equilibrium bid-ask spread.*

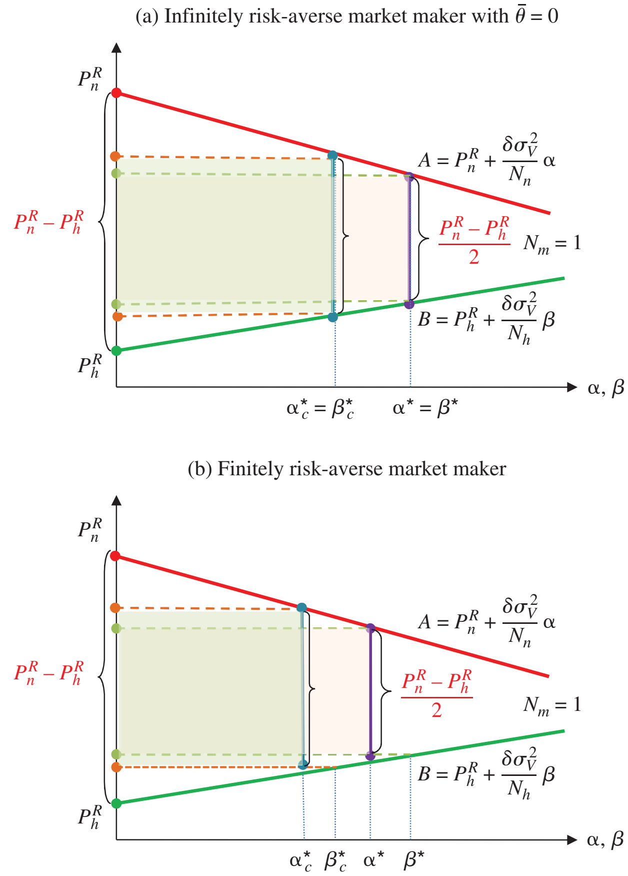
2. *As short-sale constraints become more stringent for hedgers or nonhedgers, the equilibrium bid depth, the equilibrium ask depth, and the equilibrium trading volume decrease.*

Because short-sale constraints restrict sales at the bid, one might expect that short-sale constraints increase the equilibrium bid price. In contrast, Proposition 1 implies that prohibition of short sales decreases the bid. We next provide the essential intuition for this seemingly counterintuitive result and other implications of Proposition 1 through graphical illustrations. Suppose $P_h^R < P_n^R$, and thus hedgers sell and nonhedgers buy in equilibrium. The market clearing condition (4) implies that the inverse demand and supply functions faced by the market maker are, respectively,

$$A = P_n^R - \frac{\delta\sigma_V^2}{N_n}\alpha, \quad B = P_h^R + \frac{\delta\sigma_V^2}{N_h}\beta. \tag{28}$$

To make the intuition as simple as possible, we first plot the above inverse demand and supply functions and equilibrium spreads in Figure 1(a) for the extreme case in which the market maker has infinite risk aversion and no initial endowment of the risky security.[28] Then we illustrate in Figure 1(b) the case in which the market maker has the same risk aversion and initial endowment as other investors. Figure 1 shows that, as the market maker decreases bid (increases ask) other investors sell (buy) less. Facing the inverse supply and demand functions, a monopolistic market maker optimally trades off profit from the spread and inventory risk. Similar to the results of monopolistic competition models, the bid and ask spread is equal to the absolute value of the reservation price difference $|\Delta|$, divided by 2 (by $N_m + 1$ with multiple market makers engaging in Cournot competition). In Figure 1(a) because the market maker has infinite risk aversion and no initial endowment, the market maker buys the same amount at the bid as the amount she sells at the ask, so that there is zero inventory carried to date 1. With short-sale constraints binding for hedgers, a market maker can only buy from hedgers up to $N_h(\kappa_h + \bar{\theta})$, no matter how high the bid price is. Because the market maker has market power and obtains a greater utility with a lower bid price when the amount of purchase at the bid is fixed, the market maker chooses a lower bid price such that the short-sale constraint never strictly binds. Therefore, if the unconstrained equilibrium sale amount from hedgers is larger than the upper bound $N_h(\kappa_h + \bar{\theta})$ permitted by the short-sale constraints, the market maker lowers the bid price such that in the constrained equilibrium, hedgers sell less and the short-sale constraints just start to bind. Because the market maker buys less from hedgers in equilibrium, the market maker must sell less to nonhedgers at the ask than in the unconstrained case to avoid inventory risk. Therefore, the market maker optimally increases the ask price to achieve the desired reduced amount of sale. When the market maker has positive but finite risk aversion, the same motive of reducing inventory risk also drives up the ask price and drives down the ask depth, although the market maker may choose to carry some inventory.

Figure 1. Inverse Demand/Supply Functions and Bid/Ask Prices With and Without Short-Sale Constraints



Notes. P_i^R (defined in Equation (8)) is the reservation price of trader i, for $i \in \{h, n\}$. In both figures, α_c^* (β_c^*, respectively) is the ask depth (bid depth, respectively) when short-sale constraints bind for some traders, and α^* (β^*, respectively) is the ask depth (bid depth, respectively) when short-sale constraints do not bind for any investors. The shaded areas denote the market maker's profits from bid-ask spreads.

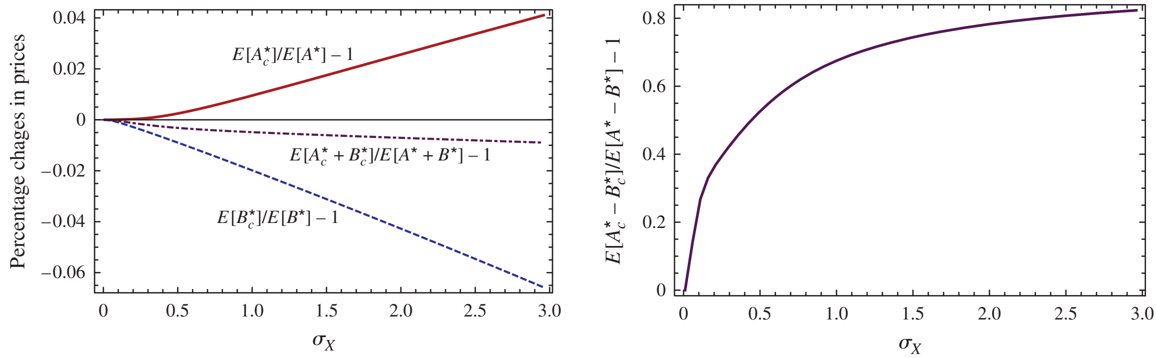
On the other hand, as shown in the following proposition, if the market maker is risk neutral, then short-sale constraints do not affect ask prices or ask depth, because inventory risk is irrelevant for her.

Proposition 2. *For a risk-neutral market maker, short-sale constraints have no impact on ask prices or ask depth.*

In addition, as long as the reservation price of the market maker is not so high that she wants to buy more from all other investors, short-sale constraints still reduce bid price, in which case short-sale constraints still increase bid-ask spreads.

The above intuition suggests that position limits on long positions would have the same qualitative

Figure 2. Percentage Changes in the Expected Bid Price (Dashed), the Expected Ask Price (Solid), the Expected Midprice (Dot-Dashed), and the Expected Bid-Ask Spread with Short-Sale Constraints Against σ_X



Note. The parameter values are $\delta = 1$, $\sigma_V = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\bar{V} = 3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, $\bar{\theta} = 1/(N_h + N_n + N_m)$, and $\kappa_h = \kappa_n = 0$.

impact: increasing ask prices, decreasing bid prices, and thus increasing bid-ask spread; and decreasing bid and ask depths, thus also reducing trading volume.[29]

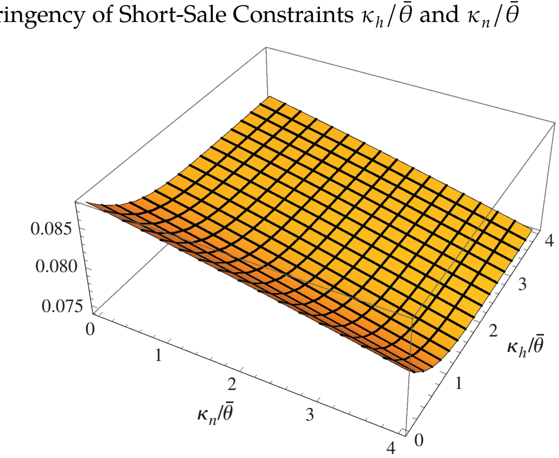
To further identify the driving force behind the reduction of bid price due to short-sale constraints, in Theorem 4 in Appendix B.2 we report the equilibrium results for an alternative model in which the market maker is a price taker in the bid market as in most of the extant literature, but a monopolist in the ask market as in the main model. Theorem 4 shows the same qualitative results for the impact of short-sale constraints on the ask price, bid and ask depths, and trading volume. However, in contrast to the main model, Theorem 4 implies that short-sale constraints *increase* equilibrium bid price. Because this alternative model differs from our main model only in that the market maker is a price taker in the bid market, this shows that the driving force behind our result that short-sale constraints decrease bid price is indeed the market maker's market power. If buyers do not have market power (i.e., are price takers), then they compete for the reduced supply and thus the constrained equilibrium price becomes higher.

Our model predicts that in markets in which market makers have market power and are risk averse, imposing short-sale constraints will cause bid prices to go down and ask prices to go up. There is a caveat for this result: as in the existing literature, we do not model explicitly the information content of the imposition itself. The imposition of short-sale constraints by regulators may signal some negative information about the stocks being regulated. If this negative information content was taken into account, then the joint impact of this negative signal and short-sale constraints would lower the bid price further, but might also lower the ask price in the net. This is because negative information drives both bid price and ask price down, as implied by Theorem 1. On the other hand, because negative information drives both bid price and ask price down, the information content of the imposition of the constraints does not significantly affect the result that short-sale constraints increase bid-ask spread, as long as the magnitude of the impact on bid is similar to that on ask.[30]

To illustrate the average magnitude of the impact across all possible realizations of the liquidity shock $\hat{X}_h$, we plot the percentage changes in expected bid, expected ask, expected midquote price, and expected spread in Figure 2 against the liquidity shock volatility σ_X. Figure 2 shows that short-sale prohibition can have significant impact on expected bid and ask prices and even greater impact on expected spread.[31] In addition, the average midquote price can also go down with short-sale constraints, but the magnitude is smaller. In general, whether midquote price increases or decreases depends on the relative magnitudes of the elasticities of demand and supply.

Figure 2 shows that the impact of short-sale constraints increases with the liquidity shock volatility. Intuitively, as the liquidity shock volatility increases, not only the probability that short-sale constraints

Figure 3. Volatility of the Bid-Ask Spread Against the Stringency of Short-Sale Constraints $\kappa_h/\bar{\theta}$ and $\kappa_n/\bar{\theta}$



Note. The default parameters are $\delta = 1$, $\sigma_V = 0.9$, $\sigma_L = 0.9$, $\sigma_{VL} = 0.3$, $\sigma_X = 0.5$, $\bar{V} = 3$, $N_h = 10$, $N_m = 1$, $N_n = 100$, and $\bar{\theta} = 1/(N_h + N_n + N_m)$.

bind increases, but also conditional on constraints binding, the average impact on bid and ask prices increases. Consequently, the unconditional average impact becomes greater with a higher liquidity shock volatility.

We next study how short-sale constraints affect the time 0 volatility of bid-ask spread.[32] To this end, we have the following:

Proposition 3. *Short-sale constraints increase the time* 0 *bid-ask spread volatility, that is,* $\mathrm{Vol}(A_c^* - B_c^*) \geq \mathrm{Vol}(A^* - B^*)$.

Thus one empirically testable prediction of our model is that after the imposition of short-sale constraints, the volatility of spread increases. The main intuition for Proposition 3 is as follows. When short-sale constraints bind, there is less risk sharing among investors and thus bid and ask prices change more in response to a random shock. For example, for the same change in the reservation price difference, spread changes more, which in turn implies that the volatility of spread goes up.

To examine the impact of short-sale constraints on the spread volatility, we plot bid-ask spread volatilities against κ_h and κ_n. Consistent with Proposition 3, Figure 3 shows that the spread volatility increases with the stringency of short-sale constraints.

6. Equilibrium with Asymmetric Information

In this subsection, we extend our model to incorporate asymmetric information to facilitate comparisons with DV and to derive additional empirical predictions. To ensure that the private information about the risky security's payoff does not affect hedging demand, we decompose the date 1 payoff $\tilde{V}$ of each share into $\tilde{v} + \tilde{u}$, where $\tilde{v} \sim \mathbf{N}(\bar{V}, \sigma_v^2)$ and $\tilde{u} \sim \mathbf{N}(0, \sigma_u^2)$ are independent with $\sigma_v > 0$, $\sigma_u > 0$, $\mathrm{Cov}(\tilde{u}, \tilde{L}) = \sigma_{VL}$, and $\mathrm{Cov}(\tilde{v}, \tilde{L}) = 0$.

We assume that on date 0, hedgers observe a private signal

$$\hat{s} = \tilde{v} - \bar{V} + \tilde{\epsilon} \tag{29}$$

about the payoff $\tilde{v}$, where $\tilde{\epsilon}$ is independently normally distributed with mean zero and variance σ_ϵ^2.[33] Because the payoff of the nontraded asset $\tilde{L}$ is independent of the first component $\tilde{v}$ (i.e., $\mathrm{Cov}(\tilde{v}, \tilde{L}) = 0$), private information about the security payoff does not affect the hedging demand.[34] Thus, for hedgers, information motivated trades are separated from hedging motivated trades.[35] Assuming it is hedgers who observe the private signal is to preserve information asymmetry in equilibrium.[36] Because hedgers have private information and nonhedgers do not, we will also refer to hedgers as the informed, and nonhedgers as the uninformed in this and subsequent extensions with asymmetric information.

To examine how information asymmetry affects the impact of short-sale constraints, we need a measure of information asymmetry. To this end, we assume that there is a public signal

$$\hat{S}_s = \hat{s} + \hat{\eta} \tag{30}$$

about hedgers' private signal $\hat{s}$ that all investors (i.e., nonhedgers, the designated market maker, and hedgers) can observe, where $\hat{\eta}$ is independently normally distributed with mean zero and volatility $\sigma_\eta > 0$. This public signal represents public disclosure about the asset payoff determinants, such as macroeconomic conditions, cash flow news, and regulation shocks, which is correlated with but less precise than hedgers' private signal. As demonstrated in Liu and Wang (2016), the volatility σ_η can serve as a clean measure of information asymmetry that does not affect aggregate information quality in the economy (measured by the precision of security payoff distribution conditional on *all* information in the economy).[37]

We restrict our analysis to symmetric equilibria in which all investors of the same type adopt the same trading strategy. Investors' problems are exactly the same as those in the main model, except that their information sets are different. Let $\mathcal{I}_i$ represent a type i investor's information set on date 0 for $i \in \{h, n, m\}$. Because hedgers know exactly $\{\hat{s}, \hat{X}_h\}$, we have $\mathcal{I}_h = \{\hat{s}, \hat{X}_h\}$,

$$\mathrm{E}[\tilde{V} \mid \mathcal{I}_h] = \bar{V} + \rho_h \hat{s}, \quad \mathrm{Var}[\tilde{V} \mid \mathcal{I}_h] = (1 - \rho_h)\sigma_v^2 + \sigma_u^2, \tag{31}$$

where

$$\rho_h := \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}. \tag{32}$$

The hedgers' reservation price becomes

$$P_h^R = \bar{V} + \hat{S} - \delta((1 - \rho_h)\sigma_v^2 + \sigma_u^2)\bar{\theta}, \tag{33}$$

where $\hat{S} := \rho_h \hat{s} + \omega \hat{X}_h$.

Given that the joint impact of $\hat{s}$ and $\hat{X}_h$ on hedgers' demand is through the composite signal $\hat{S}$, we restrict our analysis to equilibrium prices A^* and B^* that are piecewise linear in the composite signal $\hat{S}$ and the public signal $\hat{S}_s$ and conjecture that other investors can infer the value of $\hat{S}$ (but not $\hat{s}$) from the realized market prices.[38] Accordingly, the information sets for the nonhedgers and the market maker are

$$\mathcal{I}_n = \mathcal{I}_m = \{\hat{S}, \hat{S}_s\}. \tag{34}$$

Then the conditional expectation and conditional variance of $\tilde{V}$ for nonhedgers and the market maker are, respectively,

$$E[\tilde{V} \mid \mathcal{I}_n] = \bar{V} + \rho_n(1 - \rho_X)\hat{S} + \rho_n \rho_X \rho_h \hat{S}_s, \tag{35}$$

$$\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n] = (1 - \rho_n \rho_h)\sigma_v^2 + \sigma_u^2, \tag{36}$$

where

$$\rho_X := \frac{\omega^2\sigma_X^2}{\omega^2\sigma_X^2 + \rho_h^2\sigma_\eta^2}, \quad \rho_n := \frac{\sigma_v^2}{\sigma_v^2 + \rho_X\rho_h\sigma_\eta^2}. \tag{37}$$

It follows that the reservation prices for the nonhedgers and the market maker are equal to

$$P_n^R = P_m^R = \bar{V} + \rho_n(1-\rho_X)\hat{S} + \rho_n\rho_X\rho_h\hat{S}_s - \delta((1-\rho_n\rho_h)\sigma_v^2 + \sigma_u^2)\bar{\theta}. \tag{38}$$

We solve this model with information asymmetry and report the equilibrium results in Theorem 2 in Appendix A, of which Theorem 1 is a special case with $\sigma_\epsilon = \infty$. Theorem 2 shows that information asymmetry quantitatively changes the prices and quantities, but qualitative results remain the same. For example, when short-sale constraints do not bind, the spread is still equal to half of the absolute value of the reservation price difference between hedgers and nonhedgers. When short-sale constraints bind, the bid price is still such that the constraints just start to bind. Theorem 2 also shows that in equilibrium either the short-sale constraints do not bind or just start to bind and thus the trading quantity reveals the composite signal $\hat{S}$, consistent with our conjecture. In addition, in contrast to the perfect competition case, there is no equilibrium where the composite signal $\hat{S}$ is not fully revealed. This is because (1) as argued previously, it is suboptimal for the market maker to set a bid price such that the short-sale constraints strictly bind; and (2) if the equilibrium price without short-sale constraints would make the short-sale constraints strictly bind, then a bid price that is lower than the threshold price at which the short-sale constraints start to bind would make the short-sale constraints not binding, and the market maker can be better off by increasing the bid price so that she can buy more from the sellers. This result demonstrates that the market power that can separate the bid market from the ask market may help improve the informativeness of market prices in the presence of short-sale constraints.

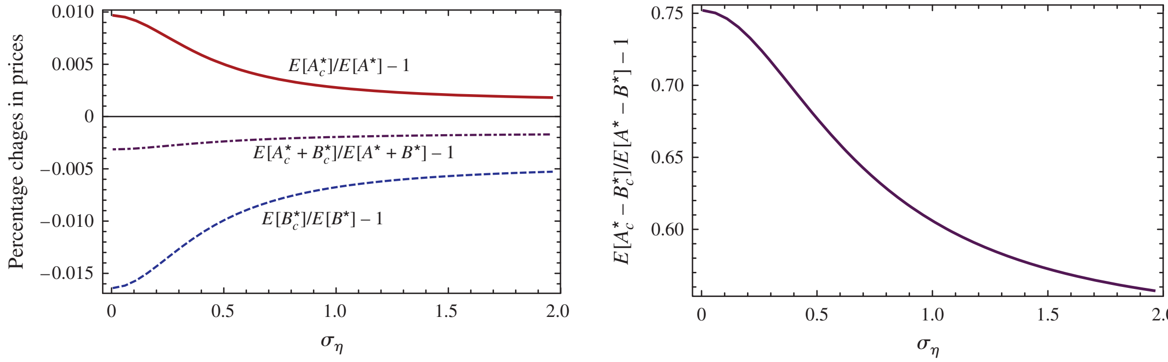
More importantly, we show in Appendix A that Proposition 1 holds with information asymmetry. Proposition 1 suggests that as long as short-sale constraints become more stringent for some investors, the bid price and depths decrease, but the ask price and spread increase. In particular, if some investors (e.g., the uninformed) cannot short sell (possibly because of high short-sale costs) before a short-sale ban, then the imposition of the short-sale ban that prevents other investors (e.g., the informed) from shorting will make the bid price and depths decrease, but the ask price and spread increase. This is in sharp contrast with the conclusions of DV. To facilitate future empirical analysis, we next compare the predictions of our model with information asymmetry to those of DV using three main cases—Case 1: no short-sale constraints for any investors (i.e., $\kappa_n = \kappa_h = \infty$); Case 2: only the informed can short and without constraints (i.e., $\kappa_n = 0$, $\kappa_h = \infty$); and Case 3: short-sale prohibition for both the informed and the uninformed (i.e., $\kappa_n = \kappa_h = 0$). Case 2 is motivated by empirical evidence that short-sale costs can be smaller for relatively informed investors and thus short sellers tend to be more informed (e.g., Boehmer et al. 2008). Proposition 1 implies that, in our model, whenever short-sale constraints are imposed on additional investors (Case 1 to Case 2 or Case 2 to Case 3), whether informed or uninformed, the expected bid price goes down, while the expected ask price and spread go up. In contrast, in DV, whether the informed or the uninformed become constrained is critical for their prediction. First, in contrast to our model, DV predict that immediately after a change from Case 1 to Case 3, neither the bid nor the ask changes and thus the spread also stays the same (see corollary 2 in DV). The intuition in DV is that since short-sale prohibition restricts both the informed and the uninformed symmetrically, conditional on a sell order, the percentage of the informed trading does not change and thus the conditional expected payoff remains the same. Because for a risk-neutral, competitive market maker, the bid price is equal to the conditional expected payoff, the equilibrium bid price also remains the same. In addition, since the ask price is equal to the expected payoff conditional on a *buy* order and short-sale prohibition does not affect an investor's purchasing decision in their model, the ask price also remains the same. Second, consider a change from Case 2 to Case 3. Because the ban prohibits the informed from shorting, and thus a sell order becomes less likely from the informed, the DV model implies that in these markets the ban increases the expected bid price. As explained above, in the DV model, short-sale constraints do not have any impact on the ask price. This indicates that, as Boehmer et al. (2013) pointed out, the DV model predicts that the expected spread will go down after the additional short-sale ban on the

Table 1. Comparison of Predictions on Average Bid, Ask, and Spread

	This paper			Diamond and Verrecchia (1987)		
Changes	Bid	Ask	Spread	Bid	Ask	Spread
Case 1 to Case 3	↓	↑	↑	—	—	—
Case 2 to Case 3	↓	↑	↑	↑	—	↓

Notes. Case 1: both unconstrained ($\kappa_n = \kappa_h = \infty$); Case 2: only the informed can short and without constraints ($\kappa_n = 0$, $\kappa_h = \infty$); Case 3: short-sale prohibition for both hedgers and nonhedgers ($\kappa_n = \kappa_h = 0$). "↑" means that the value is increased, "↓" means that the value is decreased, and "—" means that the value is unchanged.

Figure 4. Percentage Changes in the Expected Bid Price (Dashed), the Expected Ask Price (Solid), the Expected Midprice (Dot-Dashed), and the Expected Bid-Ask Spread with Short-Sale Constraints Against σ_η



Note. The parameter values are $\delta=1$, $\sigma_u=0.4$, $\sigma_v=0.9$, $\sigma_L=0.9$, $\sigma_{VL}=0.3$, $\bar{V}=3$, $\sigma_X=0.8$, $N_h=10$, $N_m=1$, $N_n=100$, $\bar{\theta}=1/(N_h+N_n+N_m)$, and $\kappa_h=\kappa_n=0$.

informed. The main driving forces for the stark difference between the conclusions of these two models are the market power and the risk aversion of the market maker in our model. We summarize the main differences in predictions in Table 1. One can use these differences in predictions to test which theory applies better in which markets.

To illustrate the magnitudes of the impact of information asymmetry, we plot the percentage changes in expected bid, expected ask, and expected spread against the information asymmetry measure σ_η in Figure 4. The figure shows that indeed short-sale constraints always decrease expected bid and increase expected ask even in the presence of asymmetric information. In addition, as information asymmetry increases, the magnitudes of the percentage changes in the expected ask, the expected bid, and the expected spread can all decrease. This is because as information asymmetry increases, both the adverse selection effect and the uncertainty faced by the uninformed increase. Consequently, investors may trade less on average, which results in the constraints binding less. Thus, our model predicts that, ceteris paribus, the impact of short-sale constraints is greater for stocks with less information asymmetry.[39]

7. Conclusions

Regulatory short-sale constraints are often imposed when market conditions deteriorate and markets become much less competitive. In contrast, extant theories on how short-sale constraints affect asset prices and market liquidity exclusively focus on perfectly competitive markets, and cannot explain the robust empirical finding that impositions of regulatory short-sale bans cause significant increases in bid-ask spreads in many financial markets. In this paper, we demonstrate that the impact of short-sale constraints in an imperfectly competitive market in which market makers have market power is qualitatively different from that in a perfectly competitive market. Our model predicts that short-sale constraints drive bid prices down and bid-ask spreads up. If, in addition, market makers are risk averse, then short-sale constraints also drive the ask price up. Furthermore, short-sale constraints increase the volatility of bid-ask spreads. The main results are largely unaffected by the presence of information asymmetry, endogenization of information acquisition, reduced information revelation, or dynamic trading. Moreover, more public disclosure can further magnify the adverse impact of short-sale constraints on asset prices and market liquidity.

Our model provides some novel empirically testable implications. For example, in markets in which market makers have significant market power, short-sale constraints decrease average bid, but increase average spread and spread volatility; and the impact of short-sale constraints is greater in more transparent markets.

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Appendix A

We first state the main results for the extended model with asymmetric information. Let Δ denote the difference in the reservation prices of h and n investors, that is,

$$\Delta := P_h^R - P_n^R = (1-\rho_n)\left(\left(1+\frac{\sigma_v^2}{\rho_h\sigma_\eta^2}\right)\hat{S} - \frac{\sigma_v^2}{\sigma_\eta^2}\hat{S}_s + \delta\rho_h\sigma_v^2\bar{\theta}\right). \quad \text{(A.1)}$$

Let

$$\nu := \frac{\mathrm{Var}[\tilde{V}\mid \mathcal{I}_n]}{\mathrm{Var}[\tilde{V}\mid \mathcal{I}_h]} \geq 1$$

be the ratio of the security payoff conditional variance of nonhedgers to that of hedgers, and

$$\bar{N} := \nu N_h + N_n + 1 \geq N$$

be the information weighted total population. The following theorem provides the equilibrium bid and ask prices and equilibrium security demand in closed form.

Theorem 2. 1. *If* $-(2(\bar{N}+1)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}))/(N_n+2) < \Delta < (2(\bar{N}+1)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta}))/(\nu N_h)$, *then short-sale constraints do not bind for any investors,*

(a) *the equilibrium bid and ask prices are, respectively,*

$$A^* = P_n^R + \frac{\nu N_h}{2(\bar{N}+1)}\Delta + \frac{\Delta^+}{2}, \tag{A.2}$$

$$B^* = P_n^R + \frac{\nu N_h}{2(\bar{N}+1)}\Delta - \frac{\Delta^-}{2}, \tag{A.3}$$

and the bid-ask spread is

$$A^* - B^* = \frac{|\Delta|}{2}; \tag{A.4}$$

(b) *the equilibrium quantities demanded are*

$$\theta_h^* = \frac{N_n+2}{2(\bar{N}+1)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]}\Delta,$$
$$\theta_n^* = -\frac{\nu N_h}{2(\bar{N}+1)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}\Delta, \quad \theta_m^* = 2\theta_n^*, \tag{A.5}$$

and the equilibrium quote depths are

$$\alpha^* = N_h(\theta_h^*)^+ + N_n(\theta_n^*)^+, \quad \beta^* = N_h(\theta_h^*)^- + N_n(\theta_n^*)^-. \tag{A.6}$$

2. *If* $\Delta \leq -(2(\bar{N}+1)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}))/(N_n+2)$, *then short-sale constraints bind for hedgers,*

(a) *the equilibrium bid and ask prices are, respectively,*

$$A_{c1}^* = P_n^R - \frac{\delta \nu N_h\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta})}{N_n+2}, \tag{A.7}$$

$$B_{c1}^* = P_h^R + \delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}), \tag{A.8}$$

and the bid-ask spread is

$$A_{c1}^* - B_{c1}^* = -\Delta - \frac{\bar{N}+1}{N_n+2}\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}); \tag{A.9}$$

(b) *the equilibrium quantities demanded are*

$$\theta_{hc1}^* = -(\kappa_h+\bar{\theta}), \quad \theta_{nc1}^* = \frac{N_h(\kappa_h+\bar{\theta})}{N_n+2}, \quad \theta_{mc1}^* = \frac{2N_h(\kappa_h+\bar{\theta})}{N_n+2}, \tag{A.10}$$

and the equilibrium quote depths are

$$\alpha_{c1}^* = N_n\theta_{nc1}^*, \quad \beta_{c1}^* = N_h(\kappa_h+\bar{\theta}). \tag{A.11}$$

3. *If* $\Delta \geq (2(\bar{N}+1)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta}))/(\nu N_h)$, *then short-sale constraints bind for nonhedgers,*

(a) *the equilibrium bid and ask prices are, respectively,*

$$A_{c2}^* = P_h^R - \frac{\Delta + \delta N_n\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta})}{\nu N_h+2}, \tag{A.12}$$

$$B_{c2}^* = P_n^R + \delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta}), \tag{A.13}$$

and the bid-ask spread is

$$A_{c2}^* - B_{c2}^* = \frac{\nu N_h+1}{\nu N_h+2}\Delta - \frac{\bar{N}+1}{\nu N_h+2}\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta}); \tag{A.14}$$

(b) *the equilibrium quantities demanded are*

$$\theta_{hc2}^* = \frac{\Delta + \delta N_n\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta})}{(\nu N_h+2)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]}, \quad \theta_{nc2}^* = -(\kappa_n+\bar{\theta}), \tag{A.15}$$

$$\theta_{mc2}^* = \frac{-\nu N_h\Delta + 2\delta N_n\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta})}{(\nu N_h+2)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}, \tag{A.16}$$

and the equilibrium quote depths are

$$\alpha_{c2}^* = N_h\theta_{hc2}^*, \quad \beta_{c2}^* = N_n(\kappa_n+\bar{\theta}). \tag{A.17}$$

Proof of Theorems 1 and 2. We only prove the generalized model with information asymmetry, because it nests the main model with symmetric information by setting $\sigma_\epsilon = \infty$. We consider the case when $\Delta < 0$, and the other case is similar. In this case, we conjecture that hedgers sell and nonhedgers buy. First, suppose no investors are constrained. Given bid price B and ask price A, the optimal demand of h and n are, respectively,

$$\theta_h^* = \frac{P_h^R - B}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]} \quad \text{and} \quad \theta_n^* = \frac{P_n^R - A}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}. \tag{A.18}$$

Substituting (A.18) into the market-clearing condition (4), we obtain that the market-clearing ask and bid depths are, respectively,

$$\beta = -N_h\theta_h^* = N_h\frac{B - P_h^R}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]}, \quad \alpha = N_n\theta_n^* = N_n\frac{P_n^R - A}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}. \tag{A.19}$$

Because of the CARA utility and the normal distribution of the date 1 wealth, the market maker's problem is equivalent to

$$\max_{A,B}\{\alpha A - \beta B + (\bar{\theta}+\beta-\alpha)E[\tilde{V} \mid \mathcal{I}_m] - \tfrac{1}{2}\delta\mathrm{Var}[\tilde{V} \mid \mathcal{I}_m](\bar{\theta}+\beta-\alpha)^2\}, \tag{A.20}$$

subject to (A.19). The first-order condition with respect to B (noting that β is a function of B) gives us

$$-\beta - B\frac{N_h}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]} + E[\tilde{V} \mid \mathcal{I}_m]\frac{N_h}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]} - \delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_m](\bar{\theta}+\beta-\alpha)\frac{N_h}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]} = 0,$$

which can be reduced to

$$(\nu N_h+2)\beta - \nu N_h\alpha = -\frac{N_h\Delta}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h]}, \tag{A.21}$$

by using (38) and expressing B in terms of β using (A.19).

Similarly using the first-order condition with respect to A, we obtain

$$\alpha + A\left(-\frac{N_n}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}\right) - E[\tilde{V} \mid \mathcal{I}_m]\left(-\frac{N_n}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}\right) + \delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_m](\bar{\theta}+\beta-\alpha)\left(-\frac{N_n}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}\right) = 0, \tag{A.22}$$

which can be reduced to

$$(N_n+2)\alpha - N_n\beta = 0, \tag{A.23}$$

by using (38), expressing A in terms of α using (A.19), and noting that $\mathcal{I}_m = \mathcal{I}_n$.

Solving (A.23) and (A.21), we can obtain the equilibrium ask depth and bid depth α^* and β^* as in (A.6). Substituting α^* and β^* into (A.19), we can obtain the equilibrium ask and bid prices A^* and B^* as in (A.2) and (A.3). In addition, by the market-clearing condition, we have $\theta_n^* = \alpha^*/N_n$, $\theta_h^* = -\beta^*/N_h$, $\theta_m^* = \beta^* - \alpha^*$, which can be simplified into Equation (A.5).

The short-sale constraints bind for hedgers if and only if $\theta_h^* \le -(\kappa_h + \bar{\theta})$, equivalently, if and only if $\Delta \le -(2(\bar{N}+1)\cdot \delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}))/(N_n+2)$. When short-sale constraints bind for hedgers, we have $\theta_{hc1}^* = -(\kappa_h+\bar{\theta})$ and $\beta_{c1}^* = N_h(\kappa_h+\bar{\theta})$. Because the first-order condition (A.23) with respect to α remains the same, we have

$$\alpha_{c1}^* = \frac{N_h N_n}{N_n+2}(\kappa_h+\bar{\theta}).$$

Then from (A.19), we get the equilibrium bid price B_{c1}^* and ask price A_{c1}^* when short-sale constraints bind for hedgers. Other quantities can then be derived. Similarly, we can prove Theorem 2 for the other case in which hedgers buy and nonhedgers sell. In addition, there is no equilibrium where the composite signal $\hat{S}$ is not fully revealed. This is because if the composite signal $\hat{S}$ is low enough such that the equilibrium price without short-sale constraints would make the short-sale constraints strictly bind, then a bid price that is lower than the threshold price at which the short-sale constraints start to bind would make the short-sale constraints nonbinding, and the market maker would be better off by increasing the bid price so that she can buy more from the sellers. Q.E.D.

Proof of Proposition 1. We prove this proposition for the case in which hedgers sell; the proof of the other case is very similar and we thus skip it here. Conditional on the constraint binding for hedgers, it is clear from Theorem 2 that A_{c1}^* decreases in κ_h, B_{c1}^* increases in κ_h, and $(A_{c1}^* - B_{c1}^*)$ decreases in κ_h. We next show that compared to the case without short-sale constraints, the bid price is lower and the ask price is higher with the constraints. By Theorem 2, we have

$$B_{c1}^* - B^* = \frac{N_n+2}{2(\bar{N}+1)}\Delta + \delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}) \tag{A.24}$$

and

$$A_{c1}^* - A^* = -\frac{\nu N_h}{2(\bar{N}+1)}\Delta - \frac{N_h}{N_n+2}\delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_h+\bar{\theta}). \tag{A.25}$$

The condition $\Delta < -(2(\bar{N}+1))/(N_n+2)\delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta})$ implies that $B_{c1}^* \le B^*$ and $A_{c1}^* \ge A^*$, which leads to $A_{c1}^* - B_{c1}^* \ge A^* - B^*$. Similarly, the results on depths and trading volume can be demonstrated. Q.E.D.

The following lemma is used to prove Proposition 3.

Lemma 1. *Let* $f(x) = h(x)1_{\{x>0\}} + h(-x)1_{\{x\le 0\}}$ *and* $g(x) = g_1(x)1_{\{x>0\}} + g_2(x)1_{\{x\le 0\}}$. *If* $h(x)$ *and* $g_1(x)$ *change in the same direction as* $x>0$ *changes and* $h(-x)$ *and* $g_2(x)$ *change in the same direction as* $x \le 0$ *changes, then we have* $\operatorname{Cov}(f(x), g(x)) > 0$.

Proof.

$$\begin{aligned}
\operatorname{Cov}(f(x), g(x)) &= E(f(x)g(x)) - E(f(x))E(g(x)) \\
&= \int_{-\infty}^{+\infty} f(x)g(x)p(x)\,dx - \int_{-\infty}^{+\infty} f(x)p(x)\,dx \int_{-\infty}^{+\infty} g(x)p(x)\,dx \\
&= \int_{-\infty}^{+\infty} p(y)\,dy \int_{-\infty}^{+\infty} f(x)g(x)p(x)\,dx \\
&\quad - \int_{-\infty}^{+\infty} f(y)p(y)\,dy \int_{-\infty}^{+\infty} g(x)p(x)\,dx \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (f(x)g(x) - f(y)g(x))p(x)p(y)\,dx\,dy \\
&= \tfrac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (f(x)-f(y))(g(x)-g(y))p(x)p(y)\,dx\,dy.
\end{aligned} \tag{A.26}$$

Since $p(-x) = p(x)$ and $p(-y) = p(y)$, we have

$$\begin{aligned}
&\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (f(-x)-f(-y))(g(-x)-g(-y))p(x)p(y)\,dx\,dy \\
&\quad = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (f(x)-f(y))(g(x)-g(y))p(x)p(y)\,dx\,dy.
\end{aligned} \tag{A.27}$$

From (A.26) and (A.27), we have $\operatorname{Cov}(f(x), g(x)) =$

$$\begin{aligned}
&\tfrac{1}{4}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (f(x)-f(y))(g(x)+g(-x)-g(y)-g(-y)) \\
&\quad \cdot p(x)p(y)\,dx\,dy.
\end{aligned} \tag{A.28}$$

(1) If x and y have the same sign, the term inside of the integral can be written as

$$(f(x)-f(y))(g(x)-g(y)) + (f(-x)-f(-y))(g(-x)-g(-y)),$$

which is ≥ 0.

(2) If $x < 0$ and $y > 0$, the term inside of the integral can be written as

$$(f(-x)-f(y))(g(-x)-g(y)) + (f(x)-f(-y))(g(x)-g(-y)),$$

which is ≥ 0.

(3) If $x > 0$ and $y < 0$, the term inside of the integral can be written as

$$(f(x)-f(-y))(g(x)-g(-y)) + (f(-x)-f(y))(g(-x)-g(y)),$$

which is ≥ 0. In addition, at least for some x and y, the term inside of the integral is nonzero. Therefore, $\operatorname{Cov}(f(x), g(x)) > 0$. Q.E.D.

Proof of Proposition 3. The spread with short-sale constraints $A_c^* - B_c^*$ can be written as $f(\Delta) + g(\Delta)$, where

$$f(\Delta) = A^* - B^* = \frac{|\Delta|}{2}$$

and

$$g(\Delta) = \begin{cases}
\dfrac{\nu N_h}{2(\nu N_h+2)}\Delta - \dfrac{\bar{N}+1}{\nu N_h+2}\delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta}) \\
\qquad \Delta \ge \dfrac{2(\bar{N}+1)}{\nu N_h}\delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n+\bar{\theta}), \\
-\dfrac{1}{2}\Delta - \dfrac{\bar{N}+1}{N_n+2}\delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}) \\
\qquad \Delta \le -\dfrac{2(\bar{N}+1)}{N_n+2}\delta\operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h+\bar{\theta}), \\
0 \quad \text{otherwise.}
\end{cases}$$

It can be easily verified that $f(\Delta)$ and $g(\Delta)$ satisfy the conditions of Lemma 1. Therefore, $f(\Delta)$ and $g(\Delta)$ are positively correlated. Then it follows that $\mathrm{Var}(A_c^* - B_c^*) > \mathrm{Var}(A^* - B^*)$. Q.E.D.

Proof of Proposition 2. Because the market maker is risk neutral, she chooses A and B to maximize $A\,\alpha(A) - B\,\beta(B) + (\bar{\theta} + \beta(B) - \alpha(A))\bar{V}$ and thus the choices of B and A are independent. Therefore, short-sale constraints have no impact on ask or ask depth. Q.E.D.

Appendix B

B.1. A Simple Model with Only Hedgers and a Market Maker

To illustrate the impact of short-sale constraints on asset prices, we now present a simple model with only hedgers and a market maker who has market power. Hedgers are subject to a liquidity shock that is modeled as a random endowment of $\hat{X}_h \sim \mathbf{N}(0, \sigma_X^2)$ units of a nontradable risky asset on date 0, with $\hat{X}_h$ realized on date 0. The nontradable asset has a per-unit payoff of $\tilde{L} \sim \mathbf{N}(0, \sigma_L^2)$ that has a covariance of σ_{VL} with the risky security's payoff $\tilde{V}$. Given P, hedgers' problem is to choose θ_h to solve

$$\max E[-e^{-\delta(-\theta_h P + (\bar{\theta}+\theta_h)\tilde{V} + \hat{X}_h \tilde{L})}], \tag{B.1}$$

subject to the short-sale constraint

$$\theta_h + \bar{\theta} \geq -\kappa_h. \tag{B.2}$$

The designated market maker's problem is to choose price level P to solve

$$\max E[-e^{-\delta(-\theta_m(P)P + (\bar{\theta}+\theta_m(P))\tilde{V})}], \tag{B.3}$$

where

$$\theta_m(P) = -N_h \theta_h(P). \tag{B.4}$$

Define $\Delta_{mh} := P_m^R - P_h^R$ as the difference between the reservation prices of the market maker and hedgers. We obtain the following results.[40]

Theorem 3. 1. *If $\Delta_{mh} < (N_h + 2)\delta\sigma_V^2(\kappa_h + \bar{\theta})$, then short-sale constraints do not bind for hedgers,*

(a) *the equilibrium price is*

$$P = P_h^R + \frac{\Delta_{mh}}{N_h + 2}; \tag{B.5}$$

(b) *the equilibrium quantities demanded are*

$$\theta_h^* = -\frac{\Delta_{mh}}{(N_h+2)\delta\sigma_V^2}, \quad \theta_m^* = \frac{N_h \Delta_{mh}}{(N_h+2)\delta\sigma_V^2}. \tag{B.6}$$

2. *If $\Delta_{mh} \geq (N_h + 2)\delta\sigma_V^2(\kappa_h + \bar{\theta})$, then short-sale constraints bind for hedgers,*

(a) *the equilibrium price is*

$$P_c^* = P_h^R + \delta\sigma_V^2(\kappa_h + \bar{\theta}); \tag{B.7}$$

(b) *the equilibrium quantities demanded are*

$$\theta_{hc}^* = -(\kappa_h + \bar{\theta}), \quad \theta_{mc}^* = N_h(\kappa_h + \bar{\theta}). \tag{B.8}$$

Since $P_c^* \leq P^*$ when short-sale constrains bind for hedgers, Theorem 3 implies that short-sale constraints increase asset prices when buyers have market power.

B.2. Equilibrium with a Price-Taking Market Maker in the Bid Market

To isolate the impact of the market power on how short-sale constraints affect bid price, we assume that the market maker is a monopolist in the ask market, as in our main model, but is a price taker in the bid market. Under this assumption, we have the following:

Theorem 4. 1. *If $0 < \Delta < ((\nu N_h + 2(N_n+1))/(\nu N_h))\delta \cdot \mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})$, then no one is constrained, and the equilibrium prices are*

$$A_1^* = P_n^R + \frac{\bar{N}}{2(N_n+1) + \nu N_h}\Delta, \quad B_1^* = P_n^R + \frac{\nu N_h}{2(N_n+1) + \nu N_h}\Delta, \tag{B.9}$$

the equilibrium depths are

$$\alpha_1^* = \frac{\nu N_h (N_n+1)}{2(N_n+1)+\nu N_h}\frac{\Delta}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}, \quad \beta_1^* = \frac{N_n}{N_n+1}\alpha_1^*,$$

and the investors' optimal stock demand is given by

$$\theta_{h1}^* = \frac{\alpha_1^*}{N_h}, \quad \theta_{n1}^* = -\frac{\beta_1^*}{N_n}, \quad \theta_{m1}^* = -\frac{\alpha_1^*}{N_n+1}. \tag{B.10}$$

2. *If $-((2(\nu N_h+1)+N_n)/(N_n+2))\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h + \bar{\theta}) < \Delta < 0$, then no one is constrained, and the equilibrium prices are*

$$A_2^* = P_n^R + \frac{\nu N_h}{2\nu N_h + N_n + 2}\Delta, \quad B_2^* = P_n^R + \frac{2\nu N_h}{2\nu N_h + N_n + 2}\Delta, \tag{B.11}$$

the equilibrium depths are

$$\alpha_2^* = -\frac{\nu N_h N_n}{N_n + 2 + 2\nu N_h}\frac{\Delta}{\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}, \quad \beta_2^* = \frac{N_n+2}{N_n}\alpha_2^*,$$

and the investors' optimal stock demand is given by

$$\theta_{h2}^* = -\frac{\beta_2^*}{N_h}, \quad \theta_{n2}^* = \frac{\alpha_2^*}{N_n}, \quad \theta_{m2}^* = \frac{2}{N_n+2}\beta_2^*. \tag{B.12}$$

3. *If $\Delta \geq ((\nu N_h + 2(N_n+1))/(\nu N_h))\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})$, then short-sale constraints bind for nonhedgers, and the equilibrium prices are*

$$A_{c1}^* = P_n^R + \frac{(\nu N_h + 1)\Delta - N_n\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + 2}, \tag{B.13}$$

$$B_{c1}^* = P_n^R + \frac{\nu N_h \Delta - 2N_n\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + 2}, \tag{B.14}$$

the equilibrium depths are

$$\alpha_{c1}^* = \frac{\nu N_h \Delta + \nu N_h N_n \delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + 2)\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n]}, \quad \beta_{c1}^* = N_n(\kappa_n + \bar{\theta}),$$

and the investors' optimal stock demand is given by

$$\theta_{hc1}^* = \frac{\alpha_{c1}^*}{N_h}, \quad \theta_{nc1}^* = -(\kappa_n + \bar{\theta}), \quad \theta_{mc1}^* = N_n(\kappa_n + \bar{\theta}) - \alpha_{c1}^*. \tag{B.15}$$

4. *If $\Delta < -((2(\nu N_h+1)+N_n)/(N_n+2))\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_h + \bar{\theta})$, then short-sale constraints bind for hedgers, and the equilibrium prices are*

$$\begin{aligned} A_{c2}^* &= P_n^R - \frac{\nu N_h}{N_n+2}\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h + \bar{\theta}), \\ B_{c2}^* &= P_n^R - \frac{2\nu N_h}{N_n+2}\delta\,\mathrm{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h + \bar{\theta}), \end{aligned} \tag{B.16}$$

the equilibrium depths are

$$\alpha^*_{c2} = \frac{N_n}{N_n+2} N_h(\kappa_h + \bar{\theta}), \quad \beta^*_{c2} = N_h(\kappa_h + \bar{\theta}),$$

and the investors' optimal stock demand is given by

$$\theta^*_{hc2} = -(\kappa_h + \bar{\theta}), \quad \theta^*_{nc2} = \frac{1}{N_n+2} N_h(\kappa_h + \bar{\theta}), \quad \theta^*_{mc2} = \frac{2}{N_n+2} N_h(\kappa_h + \bar{\theta}). \tag{B.17}$$

Given the results stated in Theorem 4, it is easy to show that as in our main model, short-sale constraints increase the equilibrium ask price and decrease bid/ask depths. In contrast to our main model, however, Theorem 4 suggests that short-sale constraints *increase* the equilibrium bid price (i.e., $B^*_{c1} > B^*_1$ and $B^*_{c2} > B^*_2$). Because the absence of market power of the market maker in the bid market is the only difference from the main model, this shows that the key driving force behind the result that short-sale constraints decrease equilibrium bid prices is the market power of the market maker.

B.3. An Extension with Multiple Market Makers

As we have shown, when a market maker is a monopolist, the bid price goes down with short-sale constraints, while the bid price goes up when market makers are perfectly competitive. One natural question is then what happens to the bid price with oligopolistic competition among multiple market makers. As is well known, with Bertrand competition, it only takes two competitors to reach perfect competition, which is counterfactual in most financial markets. Therefore, we model the oligopolistic competition among market makers as a Cournot competition. Specifically, we assume that given other participants' trades, a market maker simultaneously chooses how much to buy at bid and how much to sell at ask taking into account the price impact of her trades. The equilibrium definition for the main model can be directly extended to this multiple market maker case.

Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{N_m})^\top$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_{N_m})^\top$ be the vector of the number of shares market makers sell at ask (i.e., ask depth) and buy at bid (i.e., bid depth), respectively. Given the unconstrained demand schedules of the hedgers and the nonhedgers ($\Theta^*_h(A,B)$ and $\Theta^*_n(A,B)$), the bid price $B(\beta)$ (i.e., the inverse supply function) and the ask price $A(\alpha)$ (i.e., the inverse demand function) can be determined by the following stock market clearing conditions at the bid and ask prices:

$$\sum_{j=1}^{N_m} \alpha_j = \sum_{i=h,n} N_i \Theta^*_i(A,B)^+, \quad \sum_{j=1}^{N_m} \beta_j = \sum_{i=h,n} N_i \Theta^*_i(A,B)^-, \tag{B.18}$$

where the left-hand sides represent the total sales and purchases by market makers, respectively, and the right-hand sides represent the total purchases and sales by other investors, respectively,

Then for $j = 1, 2, \ldots, N_m$, the designated market maker m_j's problem is

$$\max_{\alpha_j \ge 0, \beta_j \ge 0} E[-e^{-\delta \tilde{W}_{m_j}} \mid \mathcal{I}_m], \tag{B.19}$$

subject to (B.18), where

$$\tilde{W}_{m_j} = \alpha_j A(\alpha) - \beta_j B(\beta) + (\bar{\theta} + \beta_j - \alpha_j)\tilde{V}, \tag{B.20}$$

subject to the constraint that $\beta_j \le (N_i(\kappa_i + \bar{\theta}))/N_m$ when short-sale constraints bind for $i \in \{h, n\}$.[41]

This leads to our definition of the Nash equilibrium of the Cournot competition.[42]

Definition 2. An equilibrium $(\theta^*_h(A,B), \theta^*_n(A,B), A^*, B^*, \alpha^*, \beta^*)$ is such that

1. given any A and B, $\theta^*_i(A,B)$ solves a type i investor's problem (1)–(3) for $i \in \{h, n\}$;
2. given $\theta^*_h(A,B)$ and $\theta^*_n(A,B)$, α^*_j and β^*_j solve market maker m_j's problem (B.19), for $j = 1, 2, \ldots, N_m$.

Define

$$C_h := \frac{N_m(N_n + N_m + 1)}{(N_m+1)(\bar{N}+1)}, \quad C_n := \frac{\nu N_m N_h}{(N_m+1)(\bar{N}+1)}. \tag{B.21}$$

We now state the multiple-market-maker version of Theorem 2 for the $N_m \ge 1$ case (setting $N_m = 1$ yields Theorem 2). The proof is very similar to that of Theorem 2 and thus omitted.

Theorem 5. 1. *If* $-\delta \operatorname{Var}[\tilde{V}|\mathcal{I}_h](\kappa_h + \bar{\theta})/C_h < \Delta < \delta \operatorname{Var}[\tilde{V}|\mathcal{I}_n] \cdot (\kappa_n + \bar{\theta})/C_n$, *then no investor bind in short-sale constraints.*

(a) *The equilibrium bid and ask prices are*

$$A^* := A(\alpha^*) = P^R_n + C_n \Delta + \frac{\Delta^+}{N_m + 1},$$
$$B^* := B(\beta^*) = P^R_n + C_n \Delta - \frac{\Delta^-}{N_m + 1},$$

and we have $A^* > P^* > B^*$, *where*

$$P^* = \frac{\nu N_h}{\bar{N}} P^R_h + \frac{N_n}{\bar{N}} P^R_n + \frac{N_m}{\bar{N}} P^R_m \tag{B.22}$$

is the equilibrium price of a perfect competition equilibrium where market makers are also price takers.

(b) *The equilibrium quantities demanded are*

$$\theta^*_h = C_h \frac{\Delta}{\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h]}, \quad \theta^*_n = -C_n \frac{\Delta}{\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n]}, \quad \theta^*_m = \frac{N_m + 1}{N_m} \theta^*_n; \tag{B.23}$$

the equilibrium quote depths are

$$\alpha^* = \frac{N_h}{N_m}(\theta^*_h)^+ + \frac{N_n}{N_m}(\theta^*_n)^+, \quad \beta^* = \frac{N_h}{N_m}(\theta^*_h)^- + \frac{N_n}{N_m}(\theta^*_n)^-, \tag{B.24}$$

which implies that the equilibrium trading volume is

$$N_m(\alpha^* + \beta^*) = \frac{N_m N_h (N_m + 2N_n + 1)}{(N_m + 1)(\bar{N} + 1)} \left(\frac{|\Delta|}{\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h]} \right). \tag{B.25}$$

2. *If* $\Delta \le -(\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h + \bar{\theta}))/C_h$, *then short-sale constraints bind for hedgers and*

(a) *the equilibrium bid and ask prices are*

$$A^*_{c1} = P^R_n - \frac{\delta \nu N_h \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h + \bar{\theta})}{N_n + N_m + 1}, \tag{B.26}$$

$$B^*_{c1} = P^R_h + \delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h](\kappa_h + \bar{\theta}); \tag{B.27}$$

(b) *the equilibrium quantities demanded are*

$$\theta^*_{hc1} = -(\kappa_h + \bar{\theta}), \quad \theta^*_{nc1} = \frac{N_h(\kappa_h + \bar{\theta})}{N_n + N_m + 1}, \quad \theta^*_{mc1} = -\frac{N_h \theta^*_{hc1} + N_n \theta^*_{nc1}}{N_m}, \tag{B.28}$$

the equilibrium quote depths are

$$\alpha^*_{c1} = \frac{N_n \theta^*_{nc1}}{N_m}, \quad \beta^*_{c1} = \frac{N_h(\kappa_h + \bar{\theta})}{N_m}. \tag{B.29}$$

3. *If* $\Delta \geq (\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta}))/C_n$, *then short-sale constraints bind for nonhedgers and*

(a) *the equilibrium bid and ask prices are*

$$A^*_{c2} = P^R_h - \frac{N_m \Delta + \delta N_n \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})}{\nu N_h + N_m + 1}, \tag{B.30}$$

$$B^*_{c2} = P^R_n + \delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta}); \tag{B.31}$$

(b) *the equilibrium quantities demanded are*

$$\begin{aligned} \theta^*_{hc2} &= \frac{N_m \Delta + \delta N_n \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n](\kappa_n + \bar{\theta})}{(\nu N_h + N_m + 1)\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h]} \\ \theta^*_{nc2} &= -(\kappa_n + \bar{\theta}), \quad \theta^*_{mc2} = -\frac{N_h \theta^*_{hc2} + N_n \theta^*_{nc2}}{N_m}, \end{aligned} \tag{B.32}$$

the equilibrium quote depths are

$$\alpha^*_{c2} = \frac{N_h \theta^*_{hc2}}{N_m}, \quad \beta^*_{c2} = \frac{N_n(\kappa_n + \bar{\theta})}{N_m}. \tag{B.33}$$

Theorem 5 implies that as in the main model with a monopolist market maker, for any $N_m > 1$, the bid price with short-sale constraints is always lower than that without. This is because when short-sale constraints bind, market makers equally split the maximum amount allowed by the constraints, the perfect competition among the short-sellers then drives the bid price to the level where the short-sale constraints just start to bind, as in the monopolist case.

B.4. The Effect of Short-Sale Constraints on Welfare

We now examine the effect of short-sale constraints on investors' welfare. Let U_{ic} and U_i denote the expected utility of type i ($i = h, n, m$) investors with and without short-sale constraints, respectively, given realizations of signals on date 0 and f_{ic} and f_i be the corresponding certainty equivalent wealth, that is, $U_{ic} = -\exp(-\delta f_{ic})$, and $U_i = -\exp(-\delta f_i)$. The certainty equivalent wealth loss of a type i investor ($i = h, n, m$) because of short-sale constraints is $f_i - f_{ic}$. We have the following:

Proposition 4. *When short-sale constraints bind for either hedgers or nonhedgers, both hedgers and nonhedgers are worse off, but market makers can be better off if there are two or more market makers.*

From Proposition 4, when short-sale constraints bind for some investors, all nonmarket makers trade at worse prices and with lower quantities and thus both of them are worse off. However, the presence of short-sale constraints may make market makers better off because they can trade at better prices. Market makers can buy at lower bid prices and sell at higher ask prices though both bid depth and ask depth are lower than the case without short-sale constraints. This positive price effect of short-sale constraints from reducing competition may dominate the negative quantity effect from reducing trading if there is more than one market maker. This dominance happens when short-sale constraints do not reduce trading by a large amount (i.e., when $|\Delta|$ is not too large). In the monopolistic case, the market maker is always worse off with short-sale constraints because of the absence of the benefit of limiting competition among market makers.

Proof of Proposition 4. It can be shown that

$$\begin{aligned} f_h - f_{hc} &= \tfrac{1}{2}\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_h][(\theta^*_h)^2 - (\theta^*_{hc})^2], \\ f_n - f_{nc} &= \tfrac{1}{2}\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n][(\theta^*_n)^2 - (\theta^*_{nc})^2], \end{aligned}$$

where $(\theta^*_h)^2 \geq (\theta^*_{hc})^2$ and $(\theta^*_n)^2 \geq (\theta^*_{nc})^2$. Therefore, both hedgers and nonhedgers are worse off when short-sale constraints bind for some investors. In addition,

$$\begin{aligned} f_m - f_{mc} &= \alpha^*_j(A^* - P^R_m) - \alpha^*_{jc}(A^*_c - P^R_m) - \beta^*_j(B^* - P^R_m) + \beta^*_{jc}(B^*_c - P^R_m) \\ &\quad - \tfrac{1}{2}\delta \operatorname{Var}[\tilde{V} \mid \mathcal{I}_n]((\beta^*_j - \alpha^*_j)^2 - (\beta^*_{jc} - \alpha^*_{jc})^2). \end{aligned} \tag{B.34}$$

Equation (B.34) implies that $f_m - f_{mc}$ is a convex quadratic function of Δ and the equation $f_m - f_{mc} = 0$ has two real roots. This implies that short-sale constraints make market makers better off if and only if Δ is between the two roots. In addition, if $N_m = 1$, then the two roots are the same. Therefore, a monopolistic market maker is always worse off with short-sale constraints. Q.E.D.

Endnotes

[1] Explicit short-sale constraints mean that it is explicitly stated that there is a maximum amount an investor can short sell. Implicit short-sale constraints mean that while it is not explicitly stated that there is a maximum amount an investor can short sell, but the short-sale cost is so high that investors do not short sell above the maximum amount.

[2] See, for example, Harrison and Kreps (1978), Diamond and Verrecchia (1987), Bai et al. (2006), Yuan (2006), and Wang (2016).

[3] See, for example, Beber and Pagano (2013), Boehmer et al. (2013), and Ang et al. (2013).

[4] This market power is necessary for compensating them for the increased risk during this period.

[5] The reservation price is the critical price, such that an investor buys (sells) the security if and only if the ask (bid) is lower (higher) than this critical price.

[6] Put differently, when short-sale constraints bind for short sellers, a monopolist market maker can commit to just buy the maximum amount allowed by the short-sale constraints, and as a result, the perfect competition among short sellers drives the bid price to the level at which the constraints just start to bind.

[7] To pinpoint market makers' market power as the cause of the opposite result, we show in Theorem 4 in Appendix B.2 that keeping everything else the same as in our model, if market makers did not have market power, then short-sale constraints would indeed increase equilibrium bid prices.

[8] Although our model focuses on short-sale constraints, our main results also apply to any constraints that restrict the amount of sales or purchases by nonmarket makers through the same mechanism. For example, our model indicates that limits on long positions drive up ask prices, drive down bid prices, reduce bid and ask depths, and increase bid-ask spread volatility. This is because with reduced demand at the ask price because of the limits, the market maker increases the ask price, decreases the ask depth, and if she is risk averse, she also buys less to control the inventory risk by lowering the bid price.

[9] In the online appendix, we show our results are robust to endogenous information acquisition, reduced information revelation as a result of short-sale constraints, and an extension to a dynamic setting. This is because the key driving force behind the main results is the market power, which can still exist even with these changes.

[10] Goldstein et al. (2017) show that in rational expectation equilibrium models, bid-ask spreads can be related to Kyle's lambda.

However, empirical studies on the impact of short-sale constraints on bid-ask spreads directly use the observed spreads in data and not an estimated price impact. Accordingly, to explain the empirical findings it is more direct to explicitly model the determination of spreads as in our model.

[11] If both the uninformed and the informed short sell before the ban, DV predict that, immediately after the imposition of the ban, there is no change in the bid or the ask, and thus the spread also remains the same. Over time, DV predict that the spread narrows more slowly, and thus becomes greater relative to that without the ban.

[12] See, for example, "Designated Market Makers" (2016, http://www.nyse.com) and Venkataraman and Waisburd (2007).

[13] As an extreme example, if short-sale costs are infinity, then it is equivalent to imposing short-sale bans.

[14] In Appendix B.1, we present a simple symmetric-information model where sellers are subject to short-sale constraints, buyers have market power, and all trade at one price (instead of separately at bid and/or ask). The key difference of the simple model from Hong and Stein (2003) and Bai et al. (2006) is the market power of the buyers and the absence of information asymmetry. We show that the equilibrium price is indeed lower when the constraints bind because of the market power channel.

[15] We show in the online appendix that our main results still hold in a two-period dynamic setting.

[16] Given the CARA preferences, having different cash endowment would not change any of the results. Throughout this paper, "bar" variables are constants, "tilde" random variables are realized on date 1, and "hat" random variables are realized on date 0.

[17] The random endowment can represent any shock in the demand for the security, such as a liquidity shock or a change in the needs for rebalancing an existing portfolio or a change in a highly illiquid asset.

[18] As demonstrated by Anand and Venkataraman (2016), many liquidity providers exit markets in bad times, and a large number of investors can only trade with a small number of designated market makers. We assume zero market-making cost because a positive cost complicates analysis and does not change our main results as will become clear later.

[19] This is equivalent to a setting in which other investors submit demand schedules to the market maker, similar to Kyle (1989), Glosten (1989), and Biais et al. (2000).

[20] An alternative way of modeling short-sale constraints is to impose short-sale costs. This alternative approach would yield the same qualitative results, because as the costs increase, the amount and frequency of short sales decrease, which is qualitatively the same as the effect of decreasing the stringency parameter κ_i in our model.

[21] If the designated market maker was also subject to short-sale constraints, then the qualitative results would stay the same. This is because the short-sale constraints for the market maker restrict her sale at the ask. When the constraints bind for her, she cannot sell more at the ask price and therefore the ask price becomes higher, while the impact of the short-sale constraints on other investors remains qualitatively the same at the bid.

[22] Even if they had market power, the qualitative results would be the same, because short-sale constraints would still restrict their sales even when they have market power and the market power of the market maker would still imply that the bid price goes down.

[23] We have solved the more general case in which investors have different liquidity shocks, different information, different initial endowment, and different risk aversions, the results of which are presented in the online appendix.

[24] To help remember, α (alpha) denotes ask depth and β (beta) denotes bid depth.

[25] One of the roles of a designated market maker is to provide liquidity. As shown later, in our model the market maker always trades when others have needs to trade and thus in this sense always provides liquidity.

[26] The market clearing conditions (Equation (4)) are implicitly enforced in the market maker's problem.

[27] Market makers in Cournot competition still retain some market power even when there are more than two market makers. In contrast, as is well known, it takes only two Bertrand competitors to reach a perfect competition equilibrium. However, market prices can be far from those of perfect competition (e.g., Christie and Schultz 1994, Chen and Ritter 2000, Biais et al. 2010).

[28] Even though in the main model, we assume that the market maker has the same risk aversion as other investors, this extreme case can be easily solved to yield the results shown in Figure 1(a). In this special case, the ask depth is always equal to the bid depth and the market maker maximizes only the profit from the spread and carries no inventory.

[29] The results on the effect of position limits (on both long and short positions) on prices and depths are available from the authors.

[30] If one models the impact of the information content of short-sale constraints imposition as having a lower unconditional expected payoff $\bar{V}$ in the case with short-sale constraints than without, then in our model, equilibrium bid and ask prices decrease by the same amount and thus the spread would be unaffected.

[31] All of the figures in the paper are for illustrations of qualitative results only and we do not attempt to calibrate to imperfectly competitive markets. $\bar{\theta}$ is chosen to normalize the total supply of the security to 1 share.

[32] Note that the time 1 bid-ask spread is zero, because the payoff becomes publicly known at time 1 and thus both bid and ask prices are equal to the payoff. This indicates that just prior to time 0, there is only uncertainty about the time 0 bid-ask spread, but no uncertainty about time 1 spread. Thus the time 0 spread volatility can also be interpreted as the volatility of the change in the spread between time 0 and time 1, that is, time series volatility.

[33] Observing the private signal may also be reinterpreted as extracting more precise information from public news than the uninformed (e.g., Engelberg et al. 2012).

[34] Ganguli and Yang (2009) show that if endowment shocks are also correlated with a forecastable term, then there can be multiple equilibria or no equilibrium.

[35] This way, hedgers' trades can also be viewed as pooled trades from pure information traders and pure liquidity traders.

[36] If it were nonhedgers who observe the private signal, then because hedgers know their own liquidity shock, they would be able to infer the private signal precisely from equilibrium prices and thus there would be no information asymmetry in equilibrium.

[37] For example, the precision of a private signal about the risky security payoff would not be a good measure of information asymmetry, because a change in the precision also changes the quality of aggregate information about the payoff and both information asymmetry and information quality can affect economic variables of interest (e.g., prices, liquidity).

[38] Given that the market maker takes into account the best response of other investors in the posted price schedules, the equilibrium trading quantities (equivalently the realized prices implied by these quantities) reveal $\hat{S}$.

[39] We also find spread volatility always increases as a result of short-sale constraints, even in the presence of asymmetric information. In addition, as information asymmetry increases, the volatility increase caused by the short-sale constraints decreases.

[40] The proof of Theorem 3 is similar to those of Theorems 1 and 2, and is thus omitted.

[41] This constraint reflects the market power of market makers and ensures that the short-sale constraints are satisfied. One justification for this constraint is that market makers know that trying to buy more would only drive up price and would not affect how much they can buy in equilibrium and market makers are identical.

[42] Deviations by undercutting prices can be prevented by matching prices by other market makers in subsequent periods in a repeated-game setting. As in standard Cournot competition models, varying prices is not in the strategy space.

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