A Portfolio Rebalancing Theory of Disposition Effect

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Abstract

The disposition effect (i.e., the tendency of investors to sell winners while holding on to losers) has been widely documented, and behavioral explanations have dominated the extant literature. In this paper, we develop a portfolio rebalancing model with transaction costs to explain the disposition effect. We show that almost all of the disposition effect patterns found in the existing literature are consistent with the optimal trading strategies implied by our model, with or without capital gains tax. In addition, selling winners that tend to outperform held losers subsequently can be optimal.

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1 Introduction

The disposition effect, i.e., the tendency of investors to sell winners while holding on to losers, has been widely documented in the empirical literature. For example, using data containing 10,000 stock investment accounts in a U.S. discount brokerage from 1987 through 1993, Odean (1998) conducts a careful set of tests of the disposition effect hypothesis and concludes that the disposition effect exists across years and investors.[1] Behavioral types of explanations, such as loss aversion, mental accounting, regret aversion, and utility from gain/loss realizations have dominated the extant literature.[2] In this paper, we develop an optimal portfolio rebalancing model with transaction costs and time-varying expected returns to show that almost all of the disposition effect patterns found in the existing literature are consistent with the optimal trading strategies implied by our model. In addition, our model can also help to explain why investors may sell winners that subsequently outperform losers that they hold, as found by some previous studies.

More specifically, we consider a model in which a small investor (i.e., who has no price impact) can trade a risk free asset and multiple risky assets ("stocks") to maximize the expected utility from the final wealth at a finite horizon. Trading in any of the stocks is subject to fixed transaction costs and short-sale constraints. The expected returns may depend on stochastic predictive variables. We solve for the optimal trading strategies and compute various disposition measures using numerical and Monte Carlo simulation methods. We show that not only can our portfolio rebalancing model generate the disposition effect qualitatively, but also the magnitude of the implied disposition effect can closely match those found in the empirical literature. For example, for some reasonable parameter values with 10 stocks in a portfolio, the probability that a sale is a gain is much greater than that it is a loss. In addition, the ratio (PGR) of realized gains to the sum of realized gains and paper gains is about 0.140, while the ratio (PLR) of realized losses to the sum of realized losses and paper losses is about 0.026. For

[1] See also, Shefrin and Statman (1985), Grinblatt and Keloharju (2001), Kumar (2009), and Ivkovi and Weisbenner (2009).

[2] See e.g., Shefrin and Statman (1985), Odean (1998), and Barberis and Xiong (2009). In contrast to Shefrin and Statman (1985) and Odean (1998), Barberis and Xiong (2009) conclude that loss aversion may lead to selling losses sooner than selling gains, the opposite of the observed disposition effect, and suggest that assuming utility on realization of gains and disutility from realization of losses can better explain the disposition effect.

comparison, Odean (1998) reports these ratios of 0.148 and 0.098, respectively (Table I in Odean (1998)). In addition, among all of the sales, gains account for more than 90%, which also suggests that an investor is much more likely to realize a gain than a loss. The main intuition for why our model can generate the disposition effect is as follows. To trade off risk and returns, it is optimal for an investor to keep the stock risk exposure within a certain range (e.g., between 10,000 and 12,500 U.S. dollars of market value in a stock for CARA preferences). If the risk exposure increases beyond the upper limit after rises in the stock price, the investor sells, and the sale is more likely a gain. If the risk exposure decreases beyond the lower limit after decreases in the stock price, the investor *buys* additional shares instead of selling.[3] In addition, because stocks that are bought have positive expected returns, overall there are gains more often than losses. As a result, the investor realizes gains more often than losses, consistent with the disposition effect. In our model, it is this portfolio rebalancing need to keep risk exposure within a certain range that drives the disposition effect.[4]

The key role of transaction costs is to make it possible to match closely the magnitudes of the disposition effect reported in the existing literature. Without transaction costs, the investor trades continuously, and thus there are no paper gains or losses, given any time interval. Therefore, both PLR and PGR ratios would be equal to 1 almost surely, and thus a portfolio model without transaction costs would not be able to explain the empirically found disposition effect magnitudes as measured by $PGR - PLR$. With transaction costs, however, when the investor sells a stock, it might be optimal not to trade some other stocks. As a result, there can be paper gains and paper losses, which implies that both PGR and PLR are less than 1 almost surely. In addition, as the number of stocks in a portfolio increases, the number of stocks with paper gains and paper losses also increases. In the extreme, when the number of stocks tends to infinity, the numbers of stocks with paper gains and paper losses also tend to infinity, and thus both PGR

[3] Selling a stock with a loss requires the upper limit of the risk exposure to fall faster than the decline in the stock price. However, *Ceteris paribus*, after a decrease in the stock price, the risk exposure decreases and thus it is more likely that the investor needs to buy.

[4] Although we consider a small investor whose trades have no price impact and thus adopt a partial equilibrium model, this contrarian type of strategy can arise in equilibrium. For example, Dorn and Strobl (2009) show that in the presence of information asymmetry, the less informed can be contrarians while the more informed can be momentum traders in equilibrium. An equilibrium model with heterogeneous agents can also justify the contrarian type of trading strategies for some investors (e.g., Basak (2005)).

and PLR tend to zero. Therefore, the existence of transaction costs in practice can be important for explaining the empirically found magnitudes of the disposition effect.

Our model can also help to explain why investors may sell winners that subsequently outperform the losers that they hold. The intuition is that if the expected return of a stock increases with a predictive variable that is positively correlated with the stock return, then conditional on a positive shock to the stock return, the average level of the predictive variable is greater than that conditional on a negative shock. Because a rise in the stock's price can increase the investor's risk exposure beyond the optimal range, it can still be optimal to sell it even though its expected return becomes higher. In addition, because of the presence of transaction costs, it can be optimal to hold on to losers even though the expected return may have decreased after a negative shock. Therefore, the average return of a winner sold can subsequently outperform that of a loser held.

For a portfolio rebalancing model with multiple stocks but without transaction costs or other frictions (e.g., Merton (1971)), it is optimal to buy some amount of another stock after a sale of a stock to rebalance risk exposure. Odean (1998) finds that even among the sales after which there are no new purchases in three weeks, the disposition effect still exists, which suggests that portfolio rebalancing is unlikely to explain the disposition effect in this subsample. In the presence of transaction costs, however, when it is optimal to sell a stock, it is possible (and likely) that, for other stocks, the risk exposures are still within the respective optimal ranges, and thus it can be optimal not to buy any of the other stocks after a sale for a period of time. To examine if our model can also generate the disposition effect conditional on there not being an immediate purchase of another stock after a sale, we conduct a similar analysis using only the sample paths along which there is no additional purchase of another stock in three weeks after a sale. We find that our model can indeed produce the disposition effect in this subsample (with a PGR of 0.139 and a PLR of 0.026).

Odean (1998) also considers a subsample in which investors sell the entire position of a stock and shows that even in this subsample, the disposition effect still appears. He suggests that portfolio rebalancing motives are unlikely to explain the disposition effect in this subsample, because for portfolio rebalancing purposes, an investor is unlikely to sell the entire position since keeping a positive exposure to the stock risk seems optimal. We show that it can be optimal to liquidate the entire position of a stock for portfolio

rebalancing purposes. For example, if at the time of a sale, the expected excess return is negative at least for a short period of time and investors cannot short sell, then it is optimal to sell the entire position. In addition, the disposition effect can arise even when the investor sells all of the holdings in a stock, consistent with the findings of Odean (1998). Intuitively, if the stock price and the expected return are negatively correlated, then after an increase in the stock price (and thus likely a gain), the expected return of the stock may turn significantly negative, and thus it may be optimal to completely liquidate the entire position in the presence of short-sale constraints.

In addition, Odean (1998) finds a "reverse disposition effect" when he computes similar ratios to PGR and PLR, but at the time of a purchase. More specifically, at each purchase time, he computes the ratio $PGPA$ of the number of stocks with a gain purchased again to the total number of stocks with a gain in a portfolio at the purchase time and the corresponding ratio $PLPA$ for losses. He finds that $PLPA$ is significantly greater than $PGPA$, i.e., an investor tends to buy again those stocks that experienced losses rather than gains. He argues that this result is consistent with the prospect theory. However, as Barberis and Xiong (2009) show, the prospect theory can predict the opposite. On the other hand, the result that $PLPA > PGPA$ is clearly in support of portfolio rebalancing, because as discussed above, when there is a loss, the exposure becomes smaller, and thus the investor tends to purchase again. To verify this intuition, we compute the $PGPA$ and $PLPA$ ratios using our model. Indeed, we find that $PLPA$ is significantly greater than $PGPA$, consistent with the finding of Odean (1998).

The extant literature finds that institutional investors tend to have a weaker disposition effect than retail investors (e.g., Locke and Mann (2005)). We show that this is consistent with our model if institutional investors hold more stocks or have smaller transaction costs. Intuitively, the more stocks an investor has, the more stocks with paper gains and paper losses. As a result, both PGR and PLR decrease, but PGR decreases more than PLR because paper gains occur more frequently than paper losses for stocks with positive expected returns. Accordingly, $PGR - PLR$ decreases as the number of stocks held increases. As transaction costs decrease, the investor trades more often, and both the number of paper gains and the number of paper losses decrease. However, the

number of paper gains decreases more slowly because gains occur more often, and thus PGR increases more slowly than PLR.[5]

Kumar (2009) investigates stock level determinants of the disposition effect and finds that the disposition effect is stronger for stocks with higher volatility. Kumar argues that this is consistent with behavioral biases being stronger for stocks that are more difficult to value. We show that our model of portfolio rebalancing can also generate such a disposition effect pattern. The main intuition for this result implied by our model is that as volatility increases, the trading boundaries are reached more frequently, even with the widened no-transaction-region. In addition, when the sell boundary is reached, the investor more likely has a gain, and thus gains are realized more often with a higher volatility.

It is well known that with capital gains tax, realizing losses sooner and deferring capital gains can provide significant benefits (e.g., Constantinides (1983)). This force acts against the disposition effect. We show that consistent with the empirical findings of Lakonishok and Smidt (1986), the disposition effect can still arise in an optimal portfolio rebalancing model with capital gains tax and transaction costs. Intuitively, when a stock price appreciates sufficiently, the investor's risk exposure can become too high, and the benefit from lowering the exposure by a sale can dominate the benefit from deferring the realization of gains. In addition, with transaction costs, it is no longer optimal to realize any losses immediately, and it may be optimal to defer even large capital losses. This is because the extra time value obtained by realizing losses sooner can be outweighed by the necessary transaction cost payment.

Overall, we find that our model can generate almost all of the disposition effect patterns and can closely match the magnitude found in the empirical literature. Obviously, this does not imply that portfolio rebalancing is the only driving force. However, our analysis suggests that in empirical investigations, one needs to separate portfolio rebalancing motivation before attributing to other potential justifications. How important is portfolio rebalancing in driving the disposition effect constitutes an important empirical question.

[5] As mentioned above, in the limit in which the transaction cost decreases to zero, the investor trades continuously. Thus, for any given time interval with positive length, there are no paper gains or paper losses almost surely, which implies that both PGR and PLR are equal to 1, and hence $PGR - PLR$ decreases to 0.

This paper also contributes to the literature on portfolio choice with transaction costs and the literature on portfolio choice with capital gains tax.[6] Our model differs from these literatures in four important aspects: (1) multiple risky assets subject to both fixed transaction costs and capital gains tax; (2) stochastically changing expected returns; (3) short-sale constraints; and (4) different capital gains tax rates for different risky assets (e.g., municipal bonds vs. stocks) .[7] As a result of these differences, our model can generate almost all of the patterns of the disposition effect in one unified setting. It is also a workhorse model that can be used to study other interesting questions. For example, how do differential tax rates for assets and return predictability affect the optimal tax timing strategy? How do return predictability and capital gains tax affect liquidity premium? How do short-sale constraints impact portfolio rebalancing and tax revenue in the presence of time-varying returns?

In an empirical study of the relationship between changes in past prices and trading volume, Lakonishok and Smidt (1986) provide some empirical evidence that rebalancing of incompletely diversified portfolio may contribute to the disposition effect. Dorn and Strobl (2009) show that in the presence of information asymmetry, the less informed can be contrarians while the more informed can be momentum traders in equilibrium and thus the less informed may display the disposition effect. In contrast to our paper, neither Lakonishok and Smidt (1986) nor Dorn and Strobl (2009) develop a portfolio rebalancing model to show that not only qualitatively but also quantitatively, portfolio rebalancing can help to explain almost all of the empirically found patterns of the disposition effect, including those in the subsamples in which there are no new purchases of another stock immediately after a sale and in which investors liquidate the entire stock positions.

The remainder of the paper proceeds as follows. We first present the main model and theoretical analysis in the next section. In Section 3, we numerically solve the model and conduct simulations to illustrate that our model can generate the disposition effect that also closely matches the empirically found magnitudes. We also show that the disposition effect can be robust with capital gains tax. We conclude in Section 4. All proofs are in

[6] For example, Davis and Norman (1990), Liu and Loewenstein (2002), Liu (2004), Constantinides (1983), (1984), Dammon and Spatt (1996), Dammon, Spatt, and Zhang (2001), Gallmeyer, Kaniel, and Tompaidis (2006), and Dai, Liu, Yang and Zhong (2015).

[7] Regarding the importance of these differentiating features, see Constantinides (1983), Fama and French (1988, 1989), Campbell, Lo, and MacKinlay (1997), and Boehmer, Jones and Zhang (2013).

the Appendix. In Section A.5 in the Appendix, we show that CRRA preferences with correlated returns do not change our main qualitative results.

2 The Model

2.1 Economic Setting

We consider the optimal investment problem of an investor who maximizes the expected constant absolute risk averse (CARA) utility from the final wealth at time $T > 0$. We assume that the investor can invest in one risk free money market account and $N \geq 1$ risky assets (stocks). A large literature has found that there is predictable variation in equity premium (e.g., Fama and French (1988, 1989), Campbell, Lo, and MacKinlay (1997)). Accordingly, similar to Campbell and Viceira (1999), we assume that for $i = 1, 2, ..., N$, the ith risky asset (Stock i hereinafter) price S_{it} follows

$$\frac{dS_{it}}{S_{it}} = (\mu_{0i} + \mu_{1i} Z_{it})dt + \sigma_{Si} dB^S_{it}, \tag{1}$$

$$dZ_{it} = (g_{0i} + g_{1i} Z_{it})dt + \sigma_{Zi} dB^Z_{it}, \tag{2}$$

where μ_{0i}, μ_{1i}, σ_{Si}, g_{0i}, g_{1i}, and σ_{Zi} are all constants; Z_{it} is the predictive variable for Stock i's return; and $\mathbf{B}^S_t = (B^S_{1t}, ..., B^S_{Nt})'$, $\mathbf{B}^Z_t = (B^Z_{1t}, ..., B^Z_{Nt})'$ are two standard N dimensional Brownian motions with pairwise correlations $E[dB^S_{it} dB^Z_{ht}] = \rho_i dt$ if $h = i$ and 0 otherwise, for $h = 1, 2, ..., N$, where ρ_i represents the correlation coefficient between the stock return and the predictive variable for Stock i. $\mathbf{B}^S$ and $\mathbf{B}^Z$ generate the filtration $\{\mathcal{F} : 0 \leq t \leq T\}$. Trading Stock i incurs a fixed (i.e., independent of the number of shares traded) transaction cost of $F_i > 0$.[8]

[8] We assume CARA preferences and uncorrelated stock returns as our main model because, as shown by Liu (2004), under this assumption, the portfolio rebalancing problem with multiple stocks subject to transaction costs can be decomposed into separate rebalancing problems, each for a stock. This decomposition greatly simplifies computation and simulation of the optimal trading strategies. If the investor has a non-CARA preference or stock returns are correlated, then this decomposition would be impossible and one needs to solve for high-dimensional free boundaries because all boundaries have to be solved for jointly. We show in Section A.5 in the Appendix that our qualitative results remain the same when we use constant relative risk averse (CRRA) preferences with correlated stock returns. This is because the critical driving force of maintaining a certain risk exposure for our main results is still present, and thus qualitatively our results remain valid. Adding proportional transaction costs does not change our main results, but makes the analysis more complicated.

As shown in the existing literature, when investors are subject to capital gains tax, it is optimal to realize losses immediately and defer realizations of gains (e.g., Constantinides (1983), Dammon, Spatt, and Zhang (2001)), which acts against displaying the disposition effect. To determine if portfolio rebalancing can still produce the disposition effect in the presence of capital gains tax, we also allow the investor to be subject to capital gains tax. According to the current tax code, capital gains tax depends on the final sale price and the exact initial purchase price ("exact basis"). Therefore, the optimal investment strategy becomes path dependent (e.g., Dybvig and Koo (1996)), and the optimization problem is of infinite dimension.[9] As in most of the extant literature on portfolio choice with capital gains tax (e.g., Dammon, Spatt, and Zhang (2001); Gallmeyer, Kaniel, and Tompaidis (2006)), we approximate the exact cost basis using the average cost basis of a position to simplify analysis. In addition, we assume full tax rebate for capital losses and symmetric tax rates for short-term and long-term investments. [10] The risk free asset pays a constant after-tax interest rate of $r > 0$.

Due to the presence of fixed transaction costs, the investor only executes a finite number of transactions in any finite time interval. Therefore, we define the investor's trading policy as follows.

Definition 1. *The investor's trading policy is a set of controls* $\mathcal{S} = \{(\tau^j, \delta^j) : j = 1, ..., J\}$*, where* J *is a random variable taking values in* $[0, 1, ...) \cup \{\infty\}$*,* $\delta^j = \{\delta_i^j : 1 \leq i \leq N\}$*, satisfying*

i. $0 \leq \tau^1 < \tau^2 < ...$ *is a sequence of* $\{\mathcal{F}_t\}$ *stopping times;*

ii. δ_i^j *is* $\{\mathcal{F}_{\tau^j}\}$*-measurable,* $1 \leq i \leq N$*.*

[9] As an example of the exact-basis system, suppose that an investor bought 10 shares at \$50/share one year ago and purchased 20 more shares at \$60/share three months ago. The first 10 shares have a cost basis of \$50/share, and the remaining 20 shares have a cost basis of \$60/share. If the investor sells the entire position at \$65/share, the early purchased 10 shares have a capital gain of $65 \times 10 - 50 \times 10 = \$150$, and the remaining 20 shares have a capital gain of $65 \times 20 - 60 \times 20 = \$100$.

[10] The limited tax rebate for up to \$3,000 in losses per year as stipulated in the U.S. tax code would reduce the benefit of realizing losses and thus strengthen our results. As in most models on optimal investment with capital gains tax, we further assume: (i) capital gains tax is realized immediately after the sale; (ii) there is no wash sale restriction; and (iii) shorting against the box is prohibited. For the justification of these additional assumptions, see e.g., Constantinides (1983) and Gallmeyer, Kaniel, and Tompaidis (2006). These additional assumptions are only for expositional simplicity because they do not change the main driving force for the disposition effect in our model.

In Definition 1, J is the total number of trading times; τ^j is the jth trading time of the investor; and δ_i^j is the dollar amount of the purchase (sale, if negative) of Stock i at τ^j.

Let Y_{it}, $i = 1, ..., N$, be the dollar amount invested in Stock i; X_t be the dollar amount invested in the money market account; and K_{it} be the total cost basis of Stock i, all at time t. Then, between trading times (i.e., for $t \in (\tau^j, \tau^{j+1})$), we have:

$$\begin{aligned} dX_t &= rX_t dt, && (3) \\ \frac{dY_{it}}{Y_{it}} &= (\mu_{0i} + \mu_{1i} Z_{it})dt + \sigma_{Si} dB_{it}^S, && (4) \\ dK_{it} &= 0. && (5) \end{aligned}$$

Between trading times, i. Equation (3) follows because the risk free asset grows at the constant rate of r; ii. Equation (4) holds because the stock value in Stock i grows at an expected rate of $\mu_{0i} + \mu_{1i} Z_{it}$ with volatility of σ_{Si}; and iii. Equation (5) reflects that the total cost basis does not change.

At trading time τ^j, we have

$$\begin{aligned} X_{\tau^j} &= X_{\tau^j -} - \sum_{i=1}^{N} \left(\delta_i^j + F_i 1_{\{\delta_i^j \neq 0\}} + \alpha_i \left((Y_{i\tau^j -} - K_{i\tau^j -}) \frac{-\delta_i^j}{Y_{i\tau^j -}} - F_i \right) 1_{\{\delta_i^j < 0\}} \right), && (6) \\ Y_{i\tau^j} &= Y_{i\tau^j -} + \delta_i^j, && (7) \\ K_{i\tau^j} &= K_{i\tau^j -} + (\delta_i^j + F_i) 1_{\{\delta_i^j > 0\}} - K_{i\tau^j -} \frac{-\delta_i^j}{Y_{i\tau^j -}} 1_{\{\delta_i^j < 0\}}, && (8) \end{aligned}$$

where $\alpha_i \in [0, 1)$ is the constant tax rate for Stock i.[11] The first term inside of the summation sign in Equation (6) is the amount of purchase, the second term the amount of fixed trading costs, and the third term the capital gains tax for Stock i when the trade is a sale. To understand the capital gains tax term, note that $Y_{i\tau^j -} - K_{i\tau^j -}$ represents the capital gains for the entire position in Stock i and $\frac{-\delta_i^j}{Y_{i\tau^j -}}$ is the proportion of the position sold, and thus the product of the two is equal to capital gains corresponding to the amount of sale $-\delta_i^j$ under the assumption of average basis. The $-F_i$ term appears because capital gains tax is levied on capital gains net of trading costs. Note that the summation term in (6) implies that our model allows offsetting tax liabilities across

[11] In Equations (6) and (8), we use the convention that $\frac{0}{0} = 1$ to deal with the case in which both $K_{i\tau^j -}$ and $Y_{i\tau^j -}$ are equal to zero.

stocks. Equation (7) states that the amount in Stock i is increased by the amount of the purchase. Equation (8) determines how the total tax basis changes at the trading time: i. if it is a purchase, then the basis is increased by the total cost of the purchase, i.e., the amount of the purchase plus the trading cost and ii. if it is a sale, then the basis is reduced proportionally.[12]

In addition, since individual investors and many institutional investors rarely short-sell stocks,[13] we require that:

$$Y_{it} \geq 0, \forall i = 1, 2, ...N, \ t \leq T. \tag{9}$$

Now, we are ready to define the admissible policies.

Definition 2. *A trading policy* $\{(\tau^j, \delta^j) : j = 1, ..., J\}$ *is admissible if*

i. Equations (3)-(8) admit a unique solution satisfying (9);

ii. With probability one, either $J(\omega) < \infty$ *or* $J(\omega) = \infty$ *implies* $\lim_{j\to\infty} \tau^j(\omega) = \infty$*;*

iii. $E\left[\int_0^T Y_{it}^2 dt\right] < \infty, i = 1, 2, ..., N.$[14]

The investor chooses her optimal policy $\{(\tau^j, \delta^j) : j = 1, ..., J\}$ among all of the admissible policies to maximize

$$E\left[u(W_T)\right],$$

subject to Equations (3)-(9), where

$$u(W) = -e^{-\beta W}, \tag{10}$$

$\beta > 0$ is the constant absolute risk-aversion coefficient, and

$$W_t = X_t + \sum_{i=1}^{N} \left(Y_{it} - \alpha_i(Y_{it} - F_i - K_{it}) - F_i\right)^+$$

[12] To understand the average-basis approximation in Equation (6), let n be the number of Stock i shares sold at time t and N be the total number of shares that the investor holds just before the sale. Then $\frac{-\delta_{it}}{Y_{it}} = \frac{n}{N}$ and the realized capital gain is equal to $n \times (S_{it} - B_{it}) = n \times (\frac{Y_{it}}{N} - \frac{K_{it}}{N}) = (Y_{it} - K_{it})\frac{-\delta_{it}}{Y_{it}}$, where B_{it} is the average basis.

[13] For example, the results of Anderson (1999) and Boehmer, Jones, and Zhang (2008) imply that only about 1.5% of short sales come from individual investors.

[14] This integrability condition prevents arbitrage strategies, such as a doubling strategy.

is the time t after-liquidation net wealth for which we allow forfeiture of the remaining position if its after-tax value is less than the fixed cost.

Our model differs from the vast literature on portfolio choice with transaction costs[15] by simultaneously allowing three important features: (1) multiple risky assets subject to fixed transaction costs; (2) stochastically changing expected returns as supported by empirical evidence (e.g., Fama and French (1988, 1989), Campbell, Lo, and MacKinlay (1997)); and (3) short-sale constraints.

Our model also differs from a large literature on portfolio choice with capital gains tax (e.g., Constantinides (1983, 1984), Dammon and Spatt (1996), Dammon, Spatt, and Zhang (2001), and Gallmeyer, Kaniel, and Tompaidis (2006)) in three major aspects: (1) there are multiple risky assets subject to fixed transaction costs in addition to capital gains tax;[16] (2) we allow stochastically changing expected returns; and (3) we allow investment in different assets to be subject to different capital gains tax rates (e.g., municipal bonds vs. stocks).

2.2 Solution

In this subsection, we characterize the solution to our model. For notational convenience, we denote by $\mathbf{Y}_t = (Y_{1t}, ..., Y_{Nt})$, $\mathbf{K}_t = (K_{1t}, ..., K_{Nt})$, $\mathbf{Z}_t = (Z_{1t}, ..., Z_{Nt})$, $\mathbf{F} = (F_1, ..., F_N)$, $\mathbf{y} = (y_1, ..., y_N)$, $\mathbf{k} = (k_1, ..., k_N)$, $\mathbf{z} = (z_1, ..., z_N)$, $\delta = (\delta_1, ..., \delta_N)$, and let $V(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z})$ be the running value function

$$V(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) = \sup_{\{(\tau^j, \delta^j): j=1,...,J\}} E\left[u(W_T) | X_t = x, \mathbf{Y}_t = \mathbf{y}, \mathbf{K}_t = \mathbf{k}, \mathbf{Z}_t = \mathbf{z}\right], \tag{11}$$

subject to Equations (3)-(9), and $\Omega \equiv [0, T] \times R \times R_+^N \times R_+^N \times R^N$ denote the entire solution region. Define

$$\begin{aligned} \mathcal{L}V &= rx\frac{\partial V}{\partial x} + \sum_{i=1}^{N}(\mu_{0i} + \mu_{1i}z_i)y_i\frac{\partial V}{\partial y_i} + \frac{1}{2}\sum_{i=1}^{N}\sigma_{Si}^2 y_i^2\frac{\partial^2 V}{\partial y_i^2} \\ &+ \sum_{i=1}^{N}(g_{0i} + g_{1i}z_i)\frac{\partial V}{\partial z_i} + \frac{1}{2}\sum_{i=1}^{N}\sigma_{Zi}^2\frac{\partial^2 V}{\partial z_i^2} + \sum_{i=1}^{N}\rho_i\sigma_{Si}\sigma_{Zi}y_i\frac{\partial^2 V}{\partial y_i \partial z_i}, \end{aligned} \tag{12}$$

[15] For example, Davis and Norman (1990), Liu and Loewenstein (2002), and Liu (2004).

[16] Gallmeyer, Kaniel, and Tompaidis (2006) also consider multiple stocks. However, given that the investor's problem that they consider cannot be decomposed into individual one-asset optimization problems, the number of risky assets that they can compute optimal strategies for is significantly limited.

and

$$\mathcal{M}V = \sup_{\{\delta \geq -\mathbf{y}, \delta \neq \mathbf{0}\}} \mathcal{M}^{\delta} V,$$

where

$$\mathcal{M}^{\delta} V \quad = \quad V(t, \xi(x, \mathbf{y}, \mathbf{k}; \delta), \mathbf{y} + \delta, \zeta(\mathbf{y}, \mathbf{k}; \delta), \mathbf{z}), \tag{13}$$

with

$$\xi(x, \mathbf{y}, \mathbf{k}; \delta) \quad = \quad x - \sum_{i=1}^{N} \left(\delta_i + F_i 1_{\{\delta_i \neq 0\}} + \alpha_i \left((y_i - k_i) \frac{-\delta_i}{y_i} - F_i \right) 1_{\{\delta_i < 0\}} \right), \tag{14}$$

$$\zeta(\mathbf{y}, \mathbf{k}; \delta) \quad = \quad \mathbf{k} + (\delta + \mathbf{F}) 1_{\{\delta > 0\}} - \mathbf{k} \frac{-\delta}{\mathbf{y}} 1_{\{\delta < 0\}}, \tag{15}$$

and the vector operations in (15) being component-wise.[17]

Then, the Hamilton-Jacobi-Bellman equation for V can be formally written as

$$\max \left\{ \frac{\partial V}{\partial t} + \mathcal{L}V, \;\; \mathcal{M}V - V \right\} = 0, \tag{16}$$

$\forall (t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) \in \Omega$, with terminal condition

$$V(T, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) = -\exp \left\{ -\beta \left[x + \sum_{i=1}^{N} (y_i - \alpha_i (y_i - F_i - k_i) - F_i)^+ \right] \right\}. \tag{17}$$

Because of the independence of returns and the separability of tax liabilities across all of the stocks, the solution can be constructed stock-by-stock as follows.[18]

Proposition 2.1. *For $i = 1, 2, ..., N$, assume that function $\varphi_i(t, y_i, k_i, z_i)$ satisfies the following variational inequality equation*

$$\begin{cases} \max\{\frac{\partial \varphi_i}{\partial t} + \mathcal{L}_i \varphi_i, \; \mathcal{M}_i \varphi_i - \varphi_i\} = 0, \\ \varphi_i(T, y_i, k_i, z_i) = \beta \left[y_i - \alpha_i (y_i - F_i - k_i) - F_i \right]^+ \end{cases} \tag{18}$$

[17] $(\xi(x, \mathbf{y}, \mathbf{k}; \delta), \mathbf{y} + \delta, \zeta(\mathbf{y}, \mathbf{k}; \delta))$ is the investor's new position value and cost basis after purchasing δ_i amount of Stock i.

[18] Because investors can get a full tax rebate for all losses, paying tax on the net gain/loss is equivalent to treating tax liability separately. For example, if one stock has \$1 gain and another stock has \$1 loss, then the net tax liability is zero if tax is paid on the net gain/loss. If tax is paid for each individual stock separately, one pays tax on the \$1 gain, and gets the same amount of tax rebate on the \$1 loss, so in the net, the tax payment is also zero. This separability of tax liabilities across stocks makes the stock by stock decomposition possible.

$\forall (t, y_i, k_i, z_i) \in \Omega_i \equiv [0,T] \times R_+ \times R_+ \times R$, *where*

$$\begin{aligned}
\mathcal{L}_i \varphi_i &= (\mu_{0i} + \mu_{1i} z_i) y_i \frac{\partial \varphi_i}{\partial y_i} + \frac{1}{2}\sigma_{Si}^2 y_i^2 \left[\frac{\partial^2 \varphi_i}{\partial y_i^2} - \left(\frac{\partial \varphi_i}{\partial y_i} \right)^2 \right] + (g_{0i} + g_{1i} z_i) \frac{\partial \varphi_i}{\partial z_i} \\
&+ \frac{1}{2}\sigma_{Zi}^2 \left[\frac{\partial^2 \varphi_i}{\partial z_i^2} - \left(\frac{\partial \varphi_i}{\partial z_i} \right)^2 \right] + \rho_i \sigma_{Si} \sigma_{Zi} y_i \left[\frac{\partial^2 \varphi_i}{\partial y_i \partial z_i} - \frac{\partial \varphi_i}{\partial y_i} \frac{\partial \varphi_i}{\partial z_i} \right]
\end{aligned}$$

and

$$\mathcal{M}_i \varphi_i = \sup_{\delta_i \in [-y_i, \infty) \setminus \{0\}} \mathcal{M}_i^{\delta_i} \varphi_i,$$

with

$$\begin{aligned}
\mathcal{M}_i^{\delta_i} \varphi_i &= \varphi_i \left(t, y_i + \delta_i, k_i + (\delta_i + F_i) 1_{\{\delta_i > 0\}} - k_i \frac{-\delta_i}{y_i} 1_{\{\delta_i < 0\}}, z_i \right) \\
&- \beta \left(\delta_i + F_i + \alpha_i \left((y_i - k_i) \frac{-\delta_i}{y_i} - F_i \right) 1_{\{\delta_i < 0\}} \right) e^{r(T-t)}.
\end{aligned}$$

Then

$$v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) = -\exp\{-\beta x e^{r(T-t)} - \sum_{i=1}^{N} \varphi_i(t, y_i, k_i, z_i)\} \tag{19}$$

satisfies the HJB equation (16) with terminal condition (17).

Let

$$NTR_i = \{(t, y_i, k_i, z_i) \in \Omega_i : \mathcal{M}_i \varphi_i < \varphi_i\}$$

denote the no-transaction-region for Stock i and

$$TR_i = \{(t, y_i, k_i, z_i) \in \Omega_i : \mathcal{M}_i \varphi_i = \varphi_i\}$$

be the transaction-region.

The following verification theorem indicates that under certain regularity conditions, the function v constructed in (19) is indeed the value function, and in addition, it characterizes the optimal trading strategies.

Proposition 2.2. *(Verification theorem)*

For $i = 1, ..., N$, let $\varphi_i(t, y_i, k_i, z_i)$ be a solution to Equation (18). For $i = 1, ..., N$ and any $0 \leq t \leq T$, let $\tau_i^0 = t$ and define a sequence of stopping times $t \leq \tau_i^1 < ... < \tau_i^n < ...$ recursively as

$$\tau_i^j = \inf\{s \in [\tau_i^{j-1}, T) : (s, Y_{is}, K_{is}, Z_{is}) \in \overline{TR_i}\}$$

with the convention that $\inf \emptyset = \infty$. *In addition, if $\tau_i^j < T$, define δ_i^j as*

$$\delta_i^j \quad = \quad \arg \max_{\delta_i \in [-Y_{i\tau_i^j}, \infty) \setminus \{0\}} \mathcal{M}_i^{\delta_i} \varphi_i(\tau_i^j, Y_{i\tau_i^j}, K_{i\tau_i^j}, Z_{i\tau_i^j}).$$

Then, under some regularity conditions, the trading policy of purchasing δ_i^j dollars of Stock i if and only if at times τ_i^j, $i = 1, ..., N$, $j = 1, 2, ...$ is optimal, and $v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z})$ defined in (19) *is the value function.*

Proposition 2.2 suggests that the optimal trading strategy for each stock is to trade if and only if the stock value is outside of the no-transaction-region. In addition, the optimal amount to trade in the buy (sell) region is such that the after trade position is on the buy (sell) target surface. Without capital gains tax, buy and sell target surfaces coincide and become one.

3 Numerical Results

In this section, we numerically solve the investor's portfolio rebalancing problem to obtain the optimal trading strategies and conduct Monte Carlo simulations to show that the optimal rebalancing strategies in our model can imply the widely documented disposition effect.

We use the penalty method (cf. Forsyth and Vetzal (2002), and Dai and Zhong (2010)) to numerically solve (18). For $i = 1, 2, ..., N$, the penalty approximation of Equation (18) is given by

$$\begin{cases} \frac{\partial \varphi_i}{\partial t} + \mathcal{L}_i \varphi_i + \lambda \left(\mathcal{M}_i \varphi_i - \varphi_i\right)^+ = 0, \\ \varphi_i(T, y_i, k_i, z_i) = \beta \left[y_i - \alpha_i(y_i - F_i - k_i) - F_i\right]^+, \end{cases} \tag{20}$$

where $\lambda \gg 0$ is a large penalty parameter. As $\lambda \to \infty$, the solution to Equation (20) converges to the solution to Equation (18).

3.1 The Case without Capital Gains Tax

Even with the stock-by-stock decomposition, the optimization problem is computationally intensive. This is because in addition to time t, there are three state variables that affect the optimal free boundaries: (1) the Stock i position Y_{it}; (2) the predictive variable Z_{it}; and (3) the cost basis K_{it}. To simplify the analysis, we first show that our model can generate almost all of the empirical patterns of the disposition effect in the case without capital gains tax, i.e., $\alpha_i = 0$ for $i = 1, ..., N$.[19] Then in Section 3.2, we show that the disposition effect can still arise even when investors are subject to capital gains tax.

The main point that we try to present in this paper is that the disposition effect can be a result of portfolio rebalancing, and therefore one cannot attribute all of the disposition effect to non-portfolio-rebalancing reasons, such as regret aversion, without estimating how much the observed disposition effect can be attributed to portfolio rebalancing. Accordingly, we do not attempt to calibrate our model to a particular data set or choose a set of parameter values to exactly match various numbers reported in the empirical literature.[20] Instead, we use one set of baseline parameter values, as reported in Table 1, to demonstrate that our model can generate not only qualitatively the same as, but also quantitatively close to, the disposition effect found in the literature. For simplicity, we assume that the investor can invest in $N = 10$ stocks, indexed by 1,2,...,10. All stocks are assumed to have the same volatility of $\sigma_{Si} = 25\%$, their predictors have a long-term mean of $g_{0i} = 0$, a mean reverting speed of $-g_{1i} = 0.5$, and a volatility of $\sigma_{Zi} = 1.5\%$. We set the interest rate r to 1%, the fixed trading cost F_i to \$0.5, the absolute risk-aversion coefficient β to 0.001, and the investment horizon T to five years. Stocks 1 to 9 have the same long-term expected return of $\mu_{0i} = 10\%$, the same loading on the predictive variable of $\mu_{1i} = 1.2$, and the same correlation of $\rho_i = 0.6$ between the shocks in stock returns and the shocks in the predictors. For Stock 10, we set the long-term expected return at 3%, the loading on the predictive variable at 0.8, and the correlation between the shocks in the stock return and the shocks in its predictor at -0.6. The different parameter values for Stock 10 are used to show that the disposition effect can exist in the subsample of

[19] The simplified HJB equations for this case are provided in Appendix A.3.1.

[20] Calibration using a particular data set would require the estimation of parameter values such as investors' risk-aversion coefficients, expected returns and correlations between stock return and predictive variables at each time of trading, which is demonstrably unreliable. In addition, the limited availability of retail trading data makes this task even more difficult.

Table 1: Baseline parameter values.

This table summarizes the baseline parameter values that we use to illustrate our results. Annualization applies whenever applicable.

Parameter	Symbol	Baseline value
General		
Investment horizon (years)	T	5
Absolute risk-aversion coefficient	β	0.001
Risk free rate	r	0.01
Stock specific		
Number of stocks	N	10
Long-term average return of Stock i	μ_{0i}	0.1, $i = 1, ..., 9$ 0.03, $i = 10$
Loading on the predictor of Stock i	μ_{1i}	1.2, $i = 1, ..., 9$ 0.8, $i = 10$
Volatility of Stock i	σ_{Si}	25%, $i = 1, ..., 10$
Average predictor value of Stock i	g_{0i}	0, $i = 1, ..., 10$
Mean reverting speed of the predictor for Stock i	$-g_{1i}$	0.5, $i = 1, ..., 10$
Volatility of the predictor of Stock i	σ_{Zi}	1.5%, $i = 1, ..., 10$
Correlation between shocks in stock return and shocks in the predictor of Stock i	ρ_i	0.6, $i = 1, ..., 9$ -0.6, $i = 10$
Fixed transaction cost of trading Stock i	F_i	\$0.5, $i = 1, ..., 10$
Capital gains tax rate of Stock i	α_i	0%, $i = 1, ..., 10$

total liquidation, as documented in Odean (1998). Because of the well-known absence of the wealth effect, the level of initial wealth is not important for the numerical results.[21]

3.1.1 Optimal Trading Policies

We plot the optimal trading strategy at the half horizon time point $t = 2.5$ for Stocks 1 to 9 in the left subfigure of Figure 1 and for Stock 10 in the right subfigure to illustrate how the investor trades in a stock as the predictive variable Z_t changes.[22] Let $y = u(t, z)$ denote the sell boundary, $y = l(t, z)$ the buy boundary, and $y = y^*(t, z)$ the target level. Figure 1 illustrates that when a stock price rises enough to reach the sell boundary, the investor sells to the target level (e.g., A to B). In contrast, when stock prices decrease enough to reach the buy boundaries, the investor *buys* additional shares to reach the target level (e.g., C to D). In contrast, when the stock value is between the buy and the sell boundaries, i.e., inside the no-transaction region, it is not optimal to trade. This trading policy implies that it is optimal to keep risk exposure in each stock within a certain range. In other words, if the exposure becomes too great, the investor sells, and if it becomes too small, the investor buys. The only times that an investor sells with a loss are when a decrease in the predictive variable significantly reduces the expected return and lowers the sell boundary after a drop in the stock price (e.g., from E to F in the left subfigure of Figure 1). As we explain later, it is this optimality of keeping risk exposure in a certain range that drives the disposition effect in our model.

In addition, as the predictive state variable increases, the expected return of the stock increases, the investor on average invests more in the stock, and therefore the no-transaction region shifts upward. The right subfigure shows that complete liquidation of a stock is optimal for portfolio rebalancing purposes when the predictive variable is negative enough to make the risk premium negative in the presence of the short-sale constraint (e.g., from G to H).

[21] We find that the qualitative results for a large set of other parameter values are the same.

[22] We restrict our attention to the six standard deviation range of Z_t, so that the probability of Z_t being outside of this range is very small.

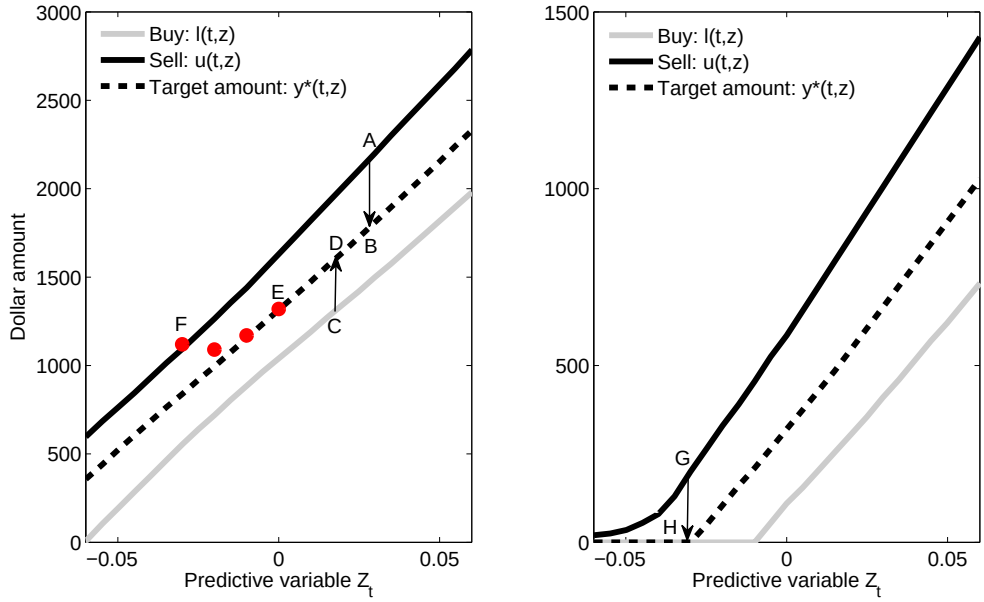

Figure 1: Optimal trading strategy for a single stock.
This figure shows the optimal trading boundaries at $t = 2.5$ years. Default parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $\mu_0 = 0.10$, $\mu_1 = 1.20$, $\sigma_S = 0.25$, $g_0 = 0$, $g_1 = -0.50$, $\sigma_Z = 0.015$, $\rho = 0.60$, $F = \$0.5$, and $\alpha = 0$; for the right subfigure: $\mu_0 = 0.03$, $\mu_1 = 0.80$, and $\rho = -0.60$.

3.1.2 Disposition Effect

To determine whether the widely documented disposition effect is consistent with the optimal trading strategies implied by our model, we conduct simulations of these optimal trading strategies, keeping track of quantities such as purchase prices, sale prices, and transaction times. Following Odean (1998), each day that a sale takes place, we compare the selling price for each stock sold to its average purchase price to determine whether that stock is sold for a gain or a loss. Each stock that is in that portfolio at the beginning of that day, but is not sold, is considered to be a paper (unrealized) gain or loss or neither. Whether it is a paper gain or a paper loss is determined by comparing its highest and lowest price for that day to its average purchase price. If its daily low is above its average purchase price, it is counted as a paper gain; if the daily high is below its average purchase price, it is counted as a paper loss; if its average purchase price lies between the high and the low, neither a gain or loss is counted. On days when no sales take place, no gains or losses (realized or paper) are counted.[23]

[23] As in Odean (1998), when a sale occurs, we assume that the average purchasing price of the remaining shares does not change. For robustness check, we also use alternative counting methods, such

For each simulated path of the 10 stocks and on each day, using the above definitions, we compute the number of realized gains/losses (# Realized Gains/Losses) and the number of paper gains/losses (# Paper Gains/Losses) for the optimal trading strategy. Then, we sum the numbers across each path to calculate the following ratios as used by Odean (1998):

$$\begin{aligned} PGR &= \frac{\#\text{Realized Gains}}{\#\text{Realized Gains} + \#\text{Paper Gains}}, \\ PLR &= \frac{\#\text{Realized Losses}}{\#\text{Realized Losses} + \#\text{Paper Losses}}. \end{aligned}$$

We also compute the fraction of sales that are gains for each path, i.e.,[24]

$$PGL = \frac{\#\text{Realized Gains}}{\#\text{Realized Gains} + \#\text{Realized Losses}}.$$

These ratios are then averaged across all of the simulated paths. We report their means in Panel A of Table 2.[25] Panel A of Table 2 shows that the disposition effect documented in the existing literature is indeed consistent with the optimal portfolio rebalancing strategy implied by our model. For example, in Table I of Odean (1998), he reports a PLR of 0.098 and a PGR of 0.148. In comparison, our model implies a PLR of 0.026 and a PGR of 0.140, with very small standard errors (not reported in the table to save space). The disposition effect measure $DE \equiv PGR - PLR$ is equal to 0.114 and statistically significant at 1%. In addition, among all of the sales, gains realizations account for more than 91%. The main intuition for our results is as follows. To maximize the expected utility from investment, it is optimal for the investor to keep the stock risk exposure within a certain range (e.g., between 10,000 and 12,500 dollars of market value in a stock for CARA preferences). If the risk exposure increases beyond

as first-in-first-out, last-in-first-out, and highest-purchase-price-first-out for the purpose of computing the average purchasing price for the current position. We find that the results are similar.

[24] In Section A.4, we also numerically solve PDEs, instead of conducting Monte Carlo simulations, to compute the probability that an initial purchase of a stock will be sold as a gain versus as a loss and the average time that it takes from the initial purchase to such a sale. We show that consistent with the disposition effect, it is much more likely that a share is sold with a gain and this probability can be greater than 0.9. In addition, the expected time to a sale with a loss can be significantly longer than that with a gain.

[25] We can view each sample path as tracking the realization of an investment account. Then, averaging across sample paths is equivalent to averaging across investment accounts. We also pooled all accounts to calculate the total number of realized gains/losses and paper gains/losses as defined previously, and then compute the disposition effect ratios. The results are very close to what we report here.

Table 2: Disposition effect

This table shows the average disposition effect measures. The results are obtained from 10,000 simulated paths for each stock from Monte Carlo simulations of the model with 10 independent stocks and 10 independent return predictors. Panel A shows the average disposition effect measures in the entire trading history; Panel B shows the average disposition effect measures when restricted to the subsample of sales in which there is no new purchase in the following three weeks; and Panel C shows the average disposition effect measures when restricted to the subsample of sales in which the entire position in at least one stock is sold. Parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $N = 10$, $\mu_{0i} = 0.10$, $\mu_{1i} = 1.20$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = 0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 1, ..., 9$, $\mu_{0i} = 0.03$, $\mu_{1i} = 0.80$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = -0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 10$. The symbol *** indicates a statistical significance level of 1%.

	Panel A: Entire history	Panel B: No new purchase	Panel C: Complete liquidations
PGR	0.140	0.139	0.114
PLR	0.026	0.026	0.074
DE	0.114***	0.113***	0.040***
PGL	0.912	0.911	0.796

the range after rises in the stock price, the investor sells and the sale is more likely a gain. If the risk exposure decreases beyond the range after decreases in the stock price, the investor buys additional shares instead of selling. Selling a stock with a loss requires that the sell boundary be reached after a drop in the stock price.[26] However, *Ceteris paribus*, after a decrease in the stock price, the (higher) sell boundary is less likely to be reached than the (lower) buy boundary. In addition, because stocks that are bought have positive expected returns, overall there are gains more often than losses. As a result, the investor realizes gains more often than losses, consistent with the disposition effect. In our model, it is this portfolio rebalancing need to keep risk exposure within a certain range that drives the disposition effect.

The key role of transaction costs is to make it possible to match closely the magnitudes of the disposition effect reported in the extant literature. Without transaction costs, the investor trades continuously, and thus there are no paper gains or losses, given any time interval. Thus, both PLR and PGR would be equal to 1 almost surely, and thus a portfolio model without transaction costs would not be able to explain the empirically found disposition effect magnitudes as measured by $PGR-PLR$. With transaction costs, however, when the investor sells a stock, it might be optimal not to trade some other stocks. As a result, there can be paper gains and paper losses, which implies that both PGR and PLR are less than 1 almost surely on each day with a sale. In addition, as the number of stocks in a portfolio increases, the number of stocks with paper gains and paper losses also increases. Therefore, the existence of transaction costs in practice can be important for explaining the empirically found magnitudes of the disposition effect.

Odean (1998) shows that among the sales after which there were no purchases in three weeks and among the sales in which the investor sells the entire position of at least one stock, the disposition effect still appears. Because in most of the existing portfolio rebalancing models (e.g., Merton (1971)), it is not optimal for an investor to sell a stock without immediately purchasing some other stocks or to sell the entire position of a stock, Odean (1998) concludes that portfolio rebalancing is unlikely to explain the disposition effect in these subsamples. In the presence of transaction costs, however, it can be optimal for an investor to sell a stock without purchasing another for an extended period

[26] For example, it can occur when both the sell and the buy boundaries move downward faster than the stock value declines, as shown in Figure 1.

of time. This is because as long as other stock positions are inside of their no-transaction regions, it is not optimal for the investor to buy any additional amount of these stocks. In addition, as shown in the right subfigure of Figure 1, it is optimal to liquidate the entire position when the expected excess return turns sufficiently negative, due to the short-sale constraints.

To determine if our model can generate the disposition effect in the subsample with no immediate purchases of any other stocks after a sale of a stock and in the subsample of complete liquidations, we compute the PLR and PGR ratios when restricted to these subsamples. We report the results in Panel B and Panel C of Table 2. The results are very similar to the results obtained for the entire sample. For example, Panel C of Table 2 shows that across all of the sample paths with complete liquidations, PGR is equal to 0.114, PLR is equal to 0.074, and DE is equal to 0.040 with high statistical significance. Intuitively, if the stock price and the expected return are negatively correlated (e.g., for Stock 10), then after an increase in the stock price (and thus there is likely a gain), the expected return of the stock may turn significantly negative, and thus it may be optimal to completely liquidate the entire position in the stock. This implies that conditional on a complete liquidation, the sale is more likely a realization of a gain for the stock than of a loss, which is consistent with the findings of Odean (1998). These results show that the disposition effect found in the no-new-purchase and complete-liquidation subsamples that Odean (1998) considered can also be consistent with portfolio rebalancing strategies implied by a model such as ours.

Existing literature finds that institutional investors tend to have weaker disposition effects than retail investors (e.g., Locke and Mann (2005)). We next show that this could be because institutional investors hold more stocks, or have smaller transaction costs, or are more sophisticated in terms of estimating the correct return dynamics. Figure 2 shows how the magnitudes of the disposition effect measure DE monotonically decreases as we increase the number of stocks in a portfolio. In addition, the disposition effect tends to be weaker when transaction costs are smaller. Intuitively, the more stocks an investor has, the more stocks with paper gains and paper losses. As a result, both PGR and PLR decrease, but PGR decreases more than PLR because paper gains occur more often than paper losses for stocks with positive expected returns. Accordingly, DE decreases as the number of stocks held increases. As transaction cost decreases, the investor trades

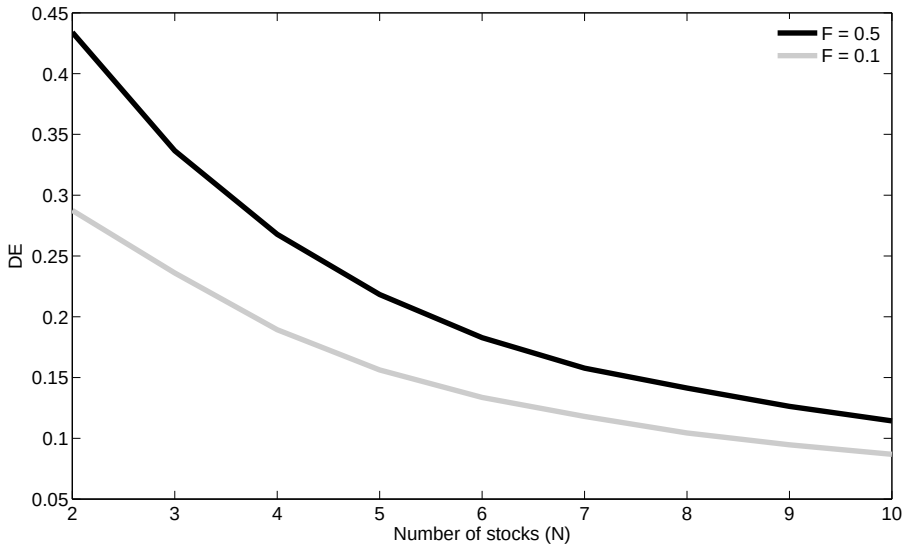

Figure 2: Disposition effect measure (DE) and the number of stocks in the portfolio. This figure shows how the disposition effect measure ($DE = PGR - PLR$) changes with respect to the number of stocks in a portfolio. Results are exhibited for two transaction cost levels: $F = \$0.5$ and $F = \$0.1$. Other parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $\mu_{0i} = 0.10$, $\mu_{1i} = 1.20$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = 0.60$, $\alpha_i = 0$, for $i = 1, ..., N-1$, $\mu_{0i} = 0.03$, $\mu_{1i} = 0.80$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = -0.60$, $\alpha_i = 0$, for $i = N$.

more often and both #Paper Gains and #Paper Losses decrease. However, #Paper Gains decreases more slowly because gains occur more often and thus PGR increases slower than PLR. In the limit in which the transaction cost decreases to zero, the investor trades continuously. Thus, for any given time interval with positive length, there are no paper gains or paper losses almost surely, which implies that both PGR and PLR are equal to 1 and hence DE decreases to 0. This illustrates that the DE measure may decrease as transaction costs decrease. Therefore, if institutional investors hold more stocks and/or have smaller transaction costs, they may display a weaker disposition effect, as measured by $PGR - PLR$.

There are many types of investor sophistications. Next, we show that one type of sophistication can result in a weaker disposition effect, as found in the existing literature (e.g., Feng and Seasholes (2005), Dhar and Zhu (2006)). Consider an extreme case in which an unsophisticated investor correctly estimates the expected return of the stock,

Table 3: Disposition effect: Sophisticated vs. Unsophisticated

This table shows the average disposition effect measures for sophisticated and unsophisticated investors. The results are obtained from 10,000 simulated paths. Parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $N = 10$, $\mu_{0i} = 0.10$, $\mu_{1i} = 1.20$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = 0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 1, ..., 9$, $\mu_{0i} = 0.03$, $\mu_{1i} = 0.80$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = -0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 10$. The symbol *** indicates a significance level of 1%.

Investor type	Sophisticated	Unsophisticated
PGR	0.140	0.149
PLR	0.026	0.019
DE	0.114***	0.130***
$\Delta(DE)$	0.016***	

but incorrectly concludes that the expected return is constant over time, i.e., the investor assumes:

$$\frac{dS_{it}}{S_{it}} = \left(\mu_{0i} - \frac{g_{0i}}{g_{1i}}\right) dt + \sigma_{Si} dB^S_{it},$$

instead of the correct processes of (1) and (2). This (incorrect) estimation could be due to a lack of experience or the high cost of gathering information (e.g., Huang and Liu (2007)).

Table 3 shows a significantly stronger disposition effect for the unsophisticated investor when shocks on the stock price and on the predictor are positively correlated for most stocks. Intuitively, after a positive shock in the stock price, the predictor is more likely to increase because of the positive correlation, which drives up the expected return, and thus makes a sophisticated investor less willing to sell. However, for an unsophisticated investor, the expected return does not go up after the stock price rises, and thus the investor is more willing to realize the gain. Therefore, an unsophisticated investor realizes gains more frequently.

Kumar (2009) investigates stock level determinants of the disposition effect and finds that the disposition effect is stronger for stocks with higher volatility. Kumar argues that this is consistent with behavioral biases being stronger for stocks that are more difficult to value. We next show that this disposition effect pattern can also be a result of portfolio rebalancing. For example, the second column of Table 4 shows when the volatility increases to 30%, PGR increases by 0.010, while PLR decreases by 0.004, which leads to a significant increase of 0.014 in the disposition effect measure. The main

Table 4: Disposition effect and volatility

This table shows the average disposition effect measures with two different volatility levels. The results are obtained from 10,000 paths of Monte Carlo simulation in the model with CARA utility, 10 independent stocks and 10 independent return predictors. Other parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $N = 10$, $\mu_{0i} = 0.10$, $\mu_{1i} = 1.20$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = 0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 1, ..., 9$, $\mu_{0i} = 0.03$, $\mu_{1i} = 0.80$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = -0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 10$. The symbol *** indicates a significance level of 1%.

Volatility level σ_S	0.25	0.30
PGR	0.140	0.150
PLR	0.026	0.022
DE	0.114***	0.128***
$\Delta(DE)$	0.014***	

Table 5: Ex post returns

This table shows the average ex post returns of the stocks sold as winners and of the stocks held as losers. The results are obtained from 10,000 simulated paths. Parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $N = 10$, $\mu_{0i} = 0.10$, $\mu_{1i} = 1.20$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = 0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 1, ..., 9$, $\mu_{0i} = 0.03$, $\mu_{1i} = 0.80$, $\sigma_{Si} = 0.25$, $g_{0i} = 0.00$, $g_{1i} = -0.50$, $\sigma_{Zi} = 0.015$, $\rho_i = -0.60$, $F_i = \$0.5$, $\alpha_i = 0$, for $i = 10$. The symbol *** indicates a significance level of 1%.

	Over the next 84 trading days	Over the next 252 trading days
Stocks sold as winners	3.43%	10.59%
Stocks held as losers	2.77%	8.85%
Difference	0.66%***	1.74%***

intuition is that as volatility increases, both the sell boundary and the buy boundary are reached more frequently, even with the widened no-transaction region. Consequently, gains are realized more often. Because losses are more likely followed by purchases, this implies a stronger disposition effect for stocks with a higher volatility.

It has been found that investors tend to sell winners that subsequently outperform losers held (unrealized) (e.g., Odean (1998)). As a result, investors tend to sell winners too soon and hold losers too long. We report the average ex post returns of stocks sold as winners and of stocks held as losers from simulations of our model in Table 5. Table 5 shows that it can be optimal to sell stocks whose future expected returns are greater than those of the stocks held. For example, over the next 84 days after a sale, the average return of the winners sold is 0.66% higher than the losers held. Over the next 252 days, it grows to 1.74%. Thus, optimal portfolio rebalancing can produce the pattern in which sold winners subsequently outperform held losers. The intuition is that if the expected

return of a stock increases with a predictive state variable that is positively correlated with the stock return, then conditional on a positive shock to the stock return, the average level of the predictive state variable is greater than that conditional on a negative shock. Therefore, the expected return of a winner can subsequently be higher than that of a loser. However, after an increase in the stock price, the investor's risk exposure increases, and thus it can still be optimal to sell winners to reduce risk exposure. On the other hand, even though after a decrease in the stock price, the expected return of the stock decreases, it can still be optimal to hold on to the losing stock because the risk exposure is reduced after the price drop and in addition the investor is subject to transaction costs.

Odean (1998) also documents a "reverse disposition effect" in which, relative to winning stocks, investors have a higher tendency to purchase additional shares of losing stocks. This is clearly consistent with portfolio rebalancing, which predicts that after a drop in price, an investor is more likely to buy the stock to increase risk exposure. To confirm this intuition, we calculate the two measures $PLPA$ and $PGPA$ used by Odean (1998):

$$\begin{aligned} PGPA &= \frac{\#\text{Gains Purchased Again}}{\#\text{Gains Purchased Again} + \#\text{Gains Potentially Purchased Again}}, \\ PLPA &= \frac{\#\text{Losses Purchased Again}}{\#\text{Losses Purchased Again} + \#\text{Losses Potentially Purchased Again}}. \end{aligned}$$

These measures are similar to PGR and PLR, except that they are computed at the time when a purchase, instead of a sale, is made. For example, #Gains Purchased Again is the number of times when a purchase is made on a stock that has a gain as of the purchasing time, and #Gains Potentially Purchased Again is the number of other stocks that have a paper gain but not purchased again at the aforementioned purchasing time.

Using the baseline parameter values in Table 1, we obtain $PLPA = 0.189$, and $PGPA = 0.053$. For comparison, Odean (1998) reports $PLPA = 0.135$ and $PGPA = 0.094$. This suggests that the "reverse disposition effect" may well be a result of optimal portfolio rebalancing.

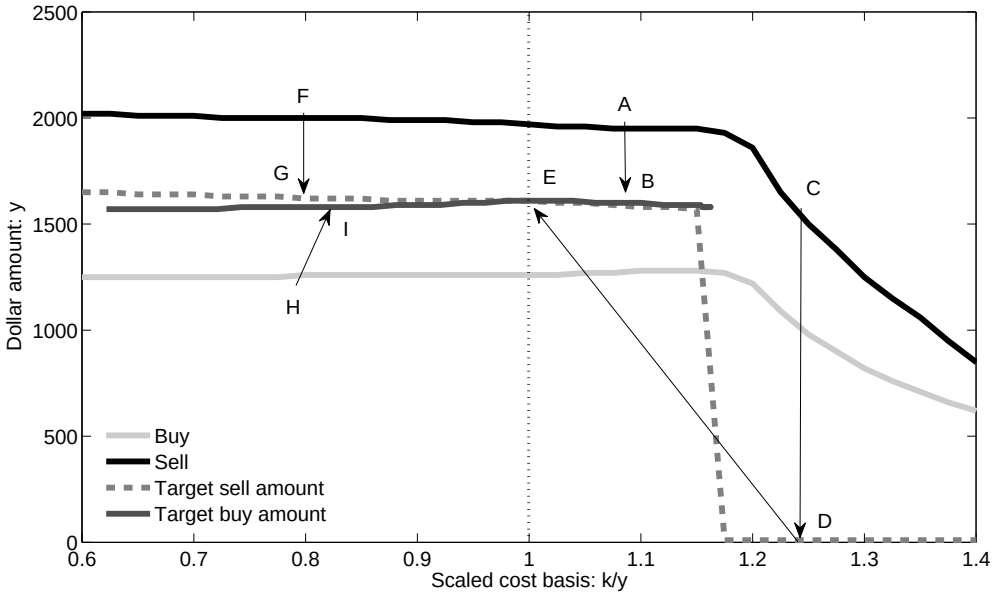

Figure 3: Optimal trading strategy for a single stock in the presence of capital gains tax. This figure shows the snapshot of optimal trading boundaries at time $t = 2.5$ years. Parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $\mu_0 = 0.10$, $\sigma_S = 0.25$, $F = \$0.5$, and $\alpha = 0.15$.

3.2 The Case with Capital Gains Tax

We now turn to the case with capital gains tax. It is well known that with capital gains tax and full capital loss tax rebate, it is optimal to realize losses immediately and delay capital gains (e.g., Constantinides (1983)). This force acts against the disposition effect. In this section, we show that even in the presence of capital gains tax and fully rebatable capital losses, the disposition effect can still arise for portfolio rebalancing purposes.

To keep tractability and to show that return predictability is not critical for the overall disposition effect, we assume the expected returns are constant in this section, i.e., $\mu_{1i} = 0$ for all i.[27] In addition, we assume Stock 10 also has the same parameter values as the first nine stocks to show that our main results are not driven by the different parameter values of Stock 10. Similar to Section 3.1, we solve the model numerically and conduct Monte Carlo simulations to calculate the disposition effect measures.

3.2.1 Optimal Trading Policy

We plot the optimal trading boundaries for a stock against the ratio $\frac{k}{y}$ in Figure 3. Note that there is a gain if and only if $\frac{k}{y} < 1$. As in the case without capital gains tax, it is also optimal to keep stock risk exposure within a certain range, as suggested by the sell region above the sell boundary and the buy region below the buy boundary. In contrast to the case with capital gains tax but without transaction costs (as considered by the existing literature, e.g., Constantinides (1983)), Figure 3 shows that it can be optimal to defer the realization of capital losses, as indicated by the no-transaction region to the right of $\frac{k}{y} = 1$. In addition, even when it is optimal to realize capital losses, the realization can be only a fraction of the losses, e.g., point A to point B. This is because the time value of the tax rebate can be smaller than the transaction costs required, and realizing only part of the losses can avoid the transaction costs needed for buying back some shares. Only when the capital losses are large enough and the risk exposure is high enough does the investor immediately realize all of the losses and then buy back some shares to achieve the optimal risk exposure (e.g., point C to point D, and then to point E). Different from the case without capital gains tax, the target amount for buying after the buy boundary is reached is different from the target amount for selling after the sell boundary is reached (e.g., point F to point G, point H to point I). This difference is a result of the additional "transaction costs" in the form of tax payment/rebate upon selling. As shown in the extant literature (e.g., Liu (2004)), the buy target lines and sell target lines are different in the presence of both proportional and fixed transaction costs. The case with capital gains tax and fixed transaction costs is similar to the case with proportional and fixed transactions in this aspect, except that the buy target line may be above the sell target line because of the tax rebate for losses (similar to a negative transaction cost).

3.2.2 Disposition Effect

As expected, the presence of capital gains tax increases the investor's propensity to realize losses and decrease her propensity to realize gains. However, there is still a significant disposition effect even with capital gains tax. For example, Table 6 shows that all else

[27] With predictability as in the model in Section 2, we would have one more dimension for each stock, which makes the optimization problem and simulation much more difficult. See Section A.3.2 for the HJB equations in this case.

Table 6: Disposition effect: the effect of capital gains tax

This table shows the average disposition effect measures with/without capital gains tax. The results are obtained from 10,000 paths of Monte Carlo simulation in the model with CARA utility, and 10 independent stocks. Parameter values: $r = 0.01$, $\beta = 0.001$, $T = 5$, $\mu_{0i} = 0.10$, $\sigma_{Si} = 0.25$, $F_i = \$0.5$, $i = 1, ..., 10$.

	No capital gains tax	15% capital gains tax
PGR	0.145	0.115
PLR	0.019	0.080
DE	0.126***	0.035***
PGL	0.941	0.841

being equal, absent capital gains tax, PGR is equal to 0.145 and PLR is equal to 0.019, which implies that the disposition measure $DE = 0.126$. With a capital gains tax rate of 15%, PGR reduces to 0.115 and PLR increases to 0.080, which implies $DE = 0.035$. Therefore, although capital gains tax tends to reduce the disposition effect, it does not seem to eliminate it. The intuition is that with transaction costs, it is optimal to defer even some large capital losses even when capital losses are fully rebatable. With this deferral of losses, the main intuition provided previously still applies, although the magnitude of the disposition effect is reduced due to the additional benefit of realizing losses and deferring gains.

4 Conclusion

The disposition effect, i.e., the tendency of investors to sell winners while holding on to losers, has been widely documented. Behavioral explanations, such as loss aversion and regret aversion, have dominated the extant literature. In this paper, we develop an optimal portfolio rebalancing model in the presence of transaction costs. We show that almost all of the disposition effect patterns found in the existing literature are consistent with the optimal trading strategies implied by our model. In addition, our model can also imply a disposition effect that closely matches the magnitudes found in empirical studies. Therefore, how much of the disposition effect cannot be explained by optimal portfolio rebalancing and must be attributed to other explanations constitutes an empirical question for future studies.

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Appendix

In this appendix, we collect the proofs for our analytical results, provide some details on HJB equations for various cases, present results on the probability of a sale and expected time to sale for a gain versus a loss, and show that using CRRA preferences with correlated stock returns does not change our main qualitative results.

A.1 Proof of Proposition 2.1:

Proof. Equation (16) can be rewritten as a linear complementarity problem (LCP):

$$\frac{\partial V}{\partial t} + \mathcal{L}V \leq 0, \tag{A-1}$$

$$V(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) \geq \mathcal{M}V(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}), \tag{A-2}$$

and in addition, if (A-1) holds with a strict inequality, then (A-2) must hold with equality.

We first verify that $v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z})$ in (19) satisfies (A-1). It follows by

$$\frac{\partial v}{\partial t} + \mathcal{L}v = -v \sum_{i=1}^{N} \left(\frac{\partial \varphi_i}{\partial t} + \mathcal{L}_i \varphi_i \right) \leq 0 \tag{A-3}$$

since $-v \geq 0$ by construction. To verify (A-2), we show for any $\delta \geq -\mathbf{y}, \delta \neq \mathbf{0}$

$$v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) \geq \mathcal{M}^{\delta} v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}). \tag{A-4}$$

For any fixed $\delta = \{\delta_1, ..., \delta_N\}$, denote by $I^{\delta} = \{i : \delta_i \neq 0\}$, then we have

$$\begin{aligned} \varphi_i(t, y_i, k_i, z_i) \quad \geq \quad & \varphi_i \left(t, y_i + \delta_i, k_i + (\delta_i + F_i) 1_{\{\delta_i > 0\}} + \frac{k_i}{y_i} \delta_i 1_{\{\delta_i < 0\}}, z_i \right) \\ & -\beta \left(\delta_i + F_i - \alpha_i \left(\left(1 - \frac{k_i}{y_i} \right) \delta_i + F_i \right) 1_{\{\delta_i < 0\}} \right) e^{r(T-t)} \end{aligned}$$

for $i \in I^\delta$. Therefore,

$$
\begin{aligned}
& v(t,x,\mathbf{y},\mathbf{k},\mathbf{z}) \\
= \; & -\exp\Bigg\{ -\beta x e^{r(T-t)} - \sum_{i\in I^\delta} \varphi_i(t,y_i,k_i,z_i) - \sum_{i\notin I^\delta} \varphi_i(t,y_i,k_i,z_i) \Bigg\} \\
\geq \; & -\exp\Bigg\{ -\beta x e^{r(T-t)} - \sum_{i\in I^\delta} \varphi_i\left(t, y_i+\delta_i, k_i + (\delta_i+F_i)1_{\{\delta_i>0\}} + \frac{k_i}{y_i}\delta_i 1_{\{\delta_i<0\}}, z_i\right) \\
& + \sum_{i\in I^\delta} \beta\left(\delta_i + F_i - \alpha_i\left(\left(1-\frac{k_i}{y_i}\right)\delta_i + F_i\right)1_{\{\delta_i<0\}}\right)e^{r(T-t)} - \sum_{i\notin I^\delta}\varphi_i(t,y_i,k_i,z_i)\Bigg\} \\
= \; & -\exp\Bigg\{ -\beta\xi(x,\mathbf{y},\mathbf{k};\delta)e^{r(T-t)} - \sum_{i\notin I^\delta}\varphi_i(t,y_i,k_i,z_i) \\
& - \sum_{i\in I^\delta}\varphi_i\left(t, y_i+\delta_i, k_i + (\delta_i+F_i)1_{\{\delta_i>0\}} + \frac{k_i}{y_i}\delta_i 1_{\{\delta_i<0\}}, z_i\right)\Bigg\} \\
= \; & \mathcal{M}^\delta v(t,x,\mathbf{y},\mathbf{k},\mathbf{z}).
\end{aligned}
$$

Because of the arbitrariness of δ we obtain (A-2).

Now, we verify that the complementarity condition is satisfied. Suppose

$$
v(t,x,\mathbf{y},\mathbf{k},\mathbf{z}) > \mathcal{M}v(t,x,\mathbf{y},\mathbf{k},\mathbf{z}),
$$

then it can be easily shown that

$$
\varphi_i(t,y_i,k_i,z_i) \;\; > \;\; \mathcal{M}_i\varphi_i(t,y_i,k_i,z_i),
$$

for all i. Therefore

$$
\frac{\partial \varphi_i}{\partial t} + \mathcal{L}_i\varphi_i = 0
$$

for all i. Hence, we have

$$
\frac{\partial v}{\partial t} + \mathcal{L}v = -v\sum_{i=1}^{N}\left(\frac{\partial\varphi_i}{\partial t} + \mathcal{L}_i\varphi_i\right) = 0.
$$

The terminal condition is clearly matched. □

A.2 Proof of Proposition 2.2:

Proof. We divide the proof into two steps. (a) It follows from Proposition 2.1 that $v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z})$ in (19) satisfies the HJB equation (16) with boundary condition (17). To proceed, let (s, η) be an arbitrary set of admissible trading policy. For notational simplicity, we denote by

$$v(t) = v(t, X_t, \mathbf{Y}_t, \mathbf{K}_t, \mathbf{Z}_t).$$

Under regularity conditions, when $s_j < s_{j+1} \leq T$, we can apply the generalized version of Itô's lemma to yield

$$\begin{aligned}
& v(s_j) - v(s_{j+1}-) \\
= & \int_{s_j}^{s_{j+1}-} \left(-\frac{\partial v}{\partial t} - \mathcal{L}v \right) ds - \sum_{i=1}^{N} \frac{\partial v}{\partial y_i} \sigma_{Si} Y_{is} dB_{is}^S - \sum_{i=1}^{N} \frac{\partial v}{\partial z_i} \sigma_{Zi} dB_{is}^Z \\
\geq & \int_{s_j}^{s_{j+1}-} - \sum_{i=1}^{N} \frac{\partial v}{\partial y_i} \sigma_{Si} Y_{is} dB_{is}^S - \sum_{i=1}^{N} \frac{\partial v}{\partial z_i} \sigma_{Zi} dB_{is}^Z,
\end{aligned}$$

where all of the partial derivatives are evaluated at $(s, X_s, \mathbf{Y}_s, \mathbf{K}_s, \mathbf{Z}_s)$. Under certain integrability conditions, the optional stopping theorem implies that

$$E_t \left[\int_t^{t^*} \frac{\partial v}{\partial y_i} \sigma_{Si} Y_{is} dB_{is}^S \right] = 0$$

and

$$E_t \left[\int_t^{t^*} \frac{\partial v}{\partial z_i} \sigma_{Zi} dB_{is}^Z \right] = 0$$

for any bounded stopping time $t^* \in [t, T]$ and $i = 1, ..., N$. Therefore,

$$E_t \left[v(s_j) - v(s_{j+1}-) \right] \geq 0,$$

and consequently,

$$E_t \left[v(s_j) - v(s_{j+1}) \right] \geq E_t \left[v(s_{j+1}-) - v(s_{j+1}) \right].$$

At time s_{j+1}, we have

$$v(s_{j+1}) = \mathcal{M}^\eta v(s_{j+1}-) \leq \mathcal{M} v(s_{j+1}-) \leq v(s_{j+1}-).$$

Hence

$$E_t[v(s_j)] \geq E_t\left[v(s_{j+1})\right].$$

Starting from $j = 0$ and using iterative expectation, we have

$$E_t[v(s_0)] \geq E_t\left[v(s_n)\right]$$

for any n such that $s_n \leq T$. Since $t \leq s_0$, similar analysis yields

$$v(t) = E_t\left[v(t)\right] \geq E_t\left[v(s_0)\right] \geq E_t\left[v(s_n)\right].$$

Due to Definition 2, for each sample path ω, we can find $n^*(\omega) < \infty$ such that $n^*(\omega) = \sup\{n : s_n \leq T\}$. Redefine $s_{n^*+1} = T$, then we have

$$v(t) \geq E_t\left[v(s_{n^*+1})\right],$$

which is equivalent to

$$\begin{aligned} v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) &\geq E_t\left[v(s_{n^*+1}, X_{s_{n^*+1}}, \mathbf{Y}_{s_{n^*+1}}, \mathbf{K}_{s_{n^*+1}}, \mathbf{Z}_{s_{n^*+1}})\right] \\ &= E_t\left[v(T, X_T, \mathbf{Y}_T, \mathbf{K}_T, \mathbf{Z}_T)\right] \\ &= E_t\left[u\left(X_T + \sum_{i=1}^{N}\left(Y_{iT} - \alpha_i(Y_{iT} - F_i - K_{iT}) - F_i\right)^+\right)\right]. \end{aligned}$$

Since the strategy (s, η) is arbitrary, we have

$$v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) \geq V(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}).$$

(b) Under some regularity conditions, the stated policy (denoted by (τ, δ)) is admissible. In addition, when the stated policy is chosen as the trading policy, all of the inequalities in the proof of part (a) become equalities, which implies that

$$v(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) = V^{\tau,\delta}(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}) \leq V(t, x, \mathbf{y}, \mathbf{k}, \mathbf{z}),$$

where $V^{\tau,\delta}$ is the value function when (τ,δ) is chosen as the trading policy. Combined with (a), we have

$$v(t,x,\mathbf{y},\mathbf{k},\mathbf{z}) = V(t,x,\mathbf{y},\mathbf{k},\mathbf{z})$$

and (τ,δ) is the optimal trading policy. □

A.3 HJB equations in special cases

A.3.1 The case without capital gains tax

In this case, the value function V does not depend on $\mathbf{k}$, and φ_i does not depend on k_i. With a slight abuse of notations, we still denote by $\varphi_i(t,y_i,z_i) = \varphi_i(t,y_i,k_i,z_i)$. Equation (18) then reduces to

$$\begin{cases} \max\{\frac{\partial \varphi_i}{\partial t} + \mathcal{L}_i\varphi_i, \sup\limits_{\delta_i\in[-y_i,\infty)\setminus\{0\}} \{\varphi_i(t,y_i+\delta_i,z_i) - \beta(\delta_i+F_i)e^{r(T-t)}\} - \varphi_i(t,y_i,z_i)\} = 0, \\ \varphi_i(T,y_i,z_i) = \beta(y_i-F_i)^+. \end{cases} \tag{A-5}$$

To further simplify, we define a new function

$$\phi_i(t,y_i,z_i) = \varphi_i(t,y_i,z_i) - \beta y_i e^{r(T-t)}.$$

It follows that the constraint

$$\varphi_i(t,y_i,z_i) \geq \sup_{\delta_i\in[-y_i,\infty)\setminus\{0\}} \{\varphi_i(t,y_i+\delta_i,z_i) - \beta(\delta_i+F_i)e^{r(T-t)}\}$$

implied by Equation (A-5) reduces to

$$\begin{aligned} \phi_i(t,y_i,z_i) &\geq \sup_{\delta_i\in[-y_i,\infty)\setminus\{0\}} \{\phi_i(t,y_i+\delta_i,z_i) - \beta F_i e^{r(T-t)}\} \\ &= \sup_{y_i^*\in[0,\infty)\setminus\{y_i\}} \phi_i(t,y_i^*,z_i) - \beta F_i e^{r(T-t)}. \end{aligned}$$

Straightforward calculation yields that $\phi_i(t, y_i, z_i)$ satisfies

$$\begin{cases} \max\{\frac{\partial \phi_i}{\partial t} + \mathcal{L}_i' \phi_i, \sup\limits_{y_i^* \in [0,\infty)\setminus\{y_i\}} \phi_i(t, y_i^*, z_i) - \beta F_i e^{r(T-t)} - \phi_i(t, y_i, z_i)\} = 0, \\ \phi_i(T, y_i, z_i) = \beta_i((y_i - F_i)^+ - y_i), \end{cases} \tag{A-6}$$

where

$$\begin{aligned} \mathcal{L}_i' \phi_i &= \mathcal{L}_i \phi_i - \sigma_{Si}^2 y_i^2 \beta e^{r(T-t)} \frac{\partial \phi_i}{\partial y_i} - \rho_i \sigma_{Si} \sigma_{Zi} y_i \beta e^{r(T-t)} \frac{\partial \phi_i}{\partial z_i} + \\ &\quad (\mu_{0i} + \mu_{1i} z_i - r) y_i \beta e^{r(T-t)} - \frac{1}{2} \sigma_{Si}^2 y_i^2 \beta^2 e^{2r(T-t)}. \end{aligned}$$

Note that portfolio rebalancing takes place whenever

$$\phi_i(t, y_i, z_i) = \sup_{y_i^* \in [0,\infty)\setminus\{y_i\}} \phi_i(t, y_i^*, z_i) - \beta F_i e^{r(T-t)}.$$

Clearly this is equivalent to

$$\phi_i(t, y_i, z_i) = \sup_{y_i^* \in [0,\infty)} \phi_i(t, y_i^*, z_i) - \beta F_i e^{r(T-t)},$$

otherwise, we have $y_i^* = y_i$, and hence $\beta F_i e^{r(T-t)} = 0$, which is a contradiction. Therefore

$$y_i^*(t, z_i) = \arg \max_{y_i^* \in [0,\infty)} \phi_i(t, y_i^*, z_i)$$

is the target amount towards which the investor rebalances her position in Stock i, given $Z_{it} = z_i$. The continuity of solution $\phi_i(t, y_i, z_i)$ implies that, near the curve $y_i = y_i^*(t, z_i)$, one has:

$$\phi_i(t, y_i, z_i) > \phi_i(t, y_i^*(t, z_i), z_i) - \beta F_i e^{r(T-t)}$$

since $\beta F_i e^{r(T-t)} > 0$. Therefore, the investor should not trade Stock i if the value of her position in Stock i is sufficiently close to its target level. We define the sell boundary as:

$$u(t, z_i) = \inf\{y_i > y_i^*(t, z_i) : \phi_i(t, y_i, z_i) = \phi_i(t, y_i^*(t, z_i), z_i) - \beta F_i e^{r(T-t)}\}, \tag{A-7}$$

and the buy boundary as:

$$l(t, z_i) = \sup\{y_i < y_i^*(t, z_i) : \phi_i(t, y_i, z_i) = \phi_i(t, y_i^*(t, z_i), z_i) - \beta F_i e^{r(T-t)}\}. \tag{A-8}$$

Then, the investor's optimal trading policy is: purchasing stock whenever $Y_{it} \leq l(t, Z_{it})$ if $l(t, Z_{it}) > 0$, and selling stock whenever $Y_{it} \geq u(t, Z_{it})$, so that the after-trade position is exactly on the target amount line.[28]

A.3.2 The case with capital gains tax but no predictability

In this case, the value function V does not depend on $\mathbf{z}$, and φ_i does not depend on z_i. Equation (18) reduces to

$$\begin{cases} \max\{\frac{\partial \varphi_i}{\partial t} + \hat{\mathcal{L}}_i \varphi_i,\ \hat{\mathcal{M}}_i \varphi_i - \varphi_i\} = 0, \\ \varphi_i(T, y_i, k_i) = \beta \left[y_i - \alpha_i(y_i - F_i - k_i) - F_i\right]^+, \end{cases} \tag{A-9}$$

for $(t, y_i, k_i) \in [0, T] \times R_+ \times R_+$, where

$$\hat{\mathcal{L}}_i \varphi_i = \mu_{0i} y_i \frac{\partial \varphi_i}{\partial y_i} + \frac{1}{2}\sigma_{Si}^2 y_i^2 \left[\frac{\partial^2 \varphi_i}{\partial y_i^2} - \left(\frac{\partial \varphi_i}{\partial y_i}\right)^2\right]$$

and

$$\begin{aligned} \hat{\mathcal{M}}_i \varphi_i = & \sup_{\delta_i \in [-y_i, \infty) \setminus \{0\}} \left\{ \varphi_i \left(t, y_i + \delta_i, k_i + (\delta_i + F_i) 1_{\{\delta_i > 0\}} + \frac{k_i}{y_i} \delta_i 1_{\{\delta_i < 0\}}\right) \right. \\ & \left. -\beta \left(\delta_i + F_i - \alpha_i \left(\left(1 - \frac{k_i}{y_i}\right) \delta_i + F_i\right) 1_{\{\delta_i < 0\}}\right) e^{r(T-t)} \right\}. \end{aligned}$$

A.4 Probability of a sale as a gain and expected time to sale

We can also compute the probability that an initial purchase of a stock will be sold as a gain versus a loss and the average time it takes to such a sale through solving partial differential equations instead of by simulations. If there are additional purchases after the initial purchase but before reaching the sell boundary, then whether the sale is a gain or a loss depends on the additional purchase prices. Accordingly, to simplify the

[28] In definitions (A-7) and (A-8), we use the conventions that $\inf \emptyset = \infty$ and $\sup \emptyset = -\infty$.

analysis, we restrict the computation to the paths in which after the initial purchase, the sell boundary is reached before the buy boundary. We illustrate this approach for a single stock, say Stock 1. Let $\tau_l^t = \inf\{s \geq t : Y_s = l(s, Z_s)\}$ and $\tau_u^t = \inf\{s \geq t : Y_s = u(s, Z_s)\}$ be the first times to reach the buy boundary $y = l(t, z)$ and the sell boundary $y = u(t, z)$, respectively. Let y_0 and z_0 denote the initial purchase amount and the initial value of the predictive variable respectively. Then we can use the Laplace transform of the stopping time τ_u^t to compute the probability and the expected time as follows. Define

$$g(t, y, z; y_0, z_0, \alpha) = E_t \left[e^{-\alpha \tau_u^t} 1_{\{Y_{\tau_u^t} > y_0\}} | \tau_u^t < \min\{\tau_l^t, T\}, Y_t = y, Z_t = z \right],$$

which is equal to:

$$f(t, y, z; y_0, z_0)/h(t, y, z; y_0, z_0),$$

where

$$f(t, y, z; y_0, z_0) = E_t \left[e^{-\alpha \tau_u^t} 1_{\{y_{\tau_u} > y_0, \tau_u < \min\{\tau_l, T\}\}} | y_t = y, z_t = z \right]$$

and

$$h(t, y, z; y_0, z_0) = E_t \left[1_{\{\tau_u < \min\{\tau_l, T\}\}} | y_t = y, z_t = z \right].$$

Applying Ito's lemma, we have

$$f_t + \mathcal{L} f - \alpha f = 0, \quad h_t + \mathcal{L} h - \alpha h = 0,$$

where $\mathcal{L}$ is as defined in (12) with $N = 1$, and the boundary and terminal conditions are

$$f(t, l(t, z), z; y_0, z_0) = 0, \quad f(t, u(t, z), z; y_0, z_0) = 1_{\{u(t,z) > y0\}}, \quad f(T, y, z; y_0, z_0) = 0$$

and

$$h(t, l(t, z), z; y_0, z_0) = 0, \quad h(t, u(t, z), z; y_0, z_0) = 1, \quad h(T, y, z; y_0, z_0) = 0.$$

The conditional probability that the initial position at (y_0, z_0) is sold with a gain, conditional on the time t position of (y, z) and reaching the sell boundary before the buy boundary, is then equal to $g(t, y, z; y_0, z_0, 0)$, and the corresponding expected time is equal to $-\frac{\partial g(t,y,z;y_0,z_0,0)}{\partial \alpha}$.

We plot the conditional probability and the ratio of corresponding expected time for a loss to that for a gain against the stock return volatility σ and the long-term average return μ_0 for Stock 1 in Figures 4 and 5, respectively. Figure 4 shows that it is much more likely that a share bought is sold with a gain. Consistent with the disposition effect, this probability is far above 0.5 and can be more than 0.9. Figure 5 shows that the expected time to a sale for a loss can be significantly longer than that for a gain, which is also consistent with the disposition effect. In addition, both the probability and the ratio increase with volatility σ_S and the long-term average return μ_0. Greater volatility and greater average return increase the chance and reduce the time that it takes for the sell boundary to be reached with a gain.

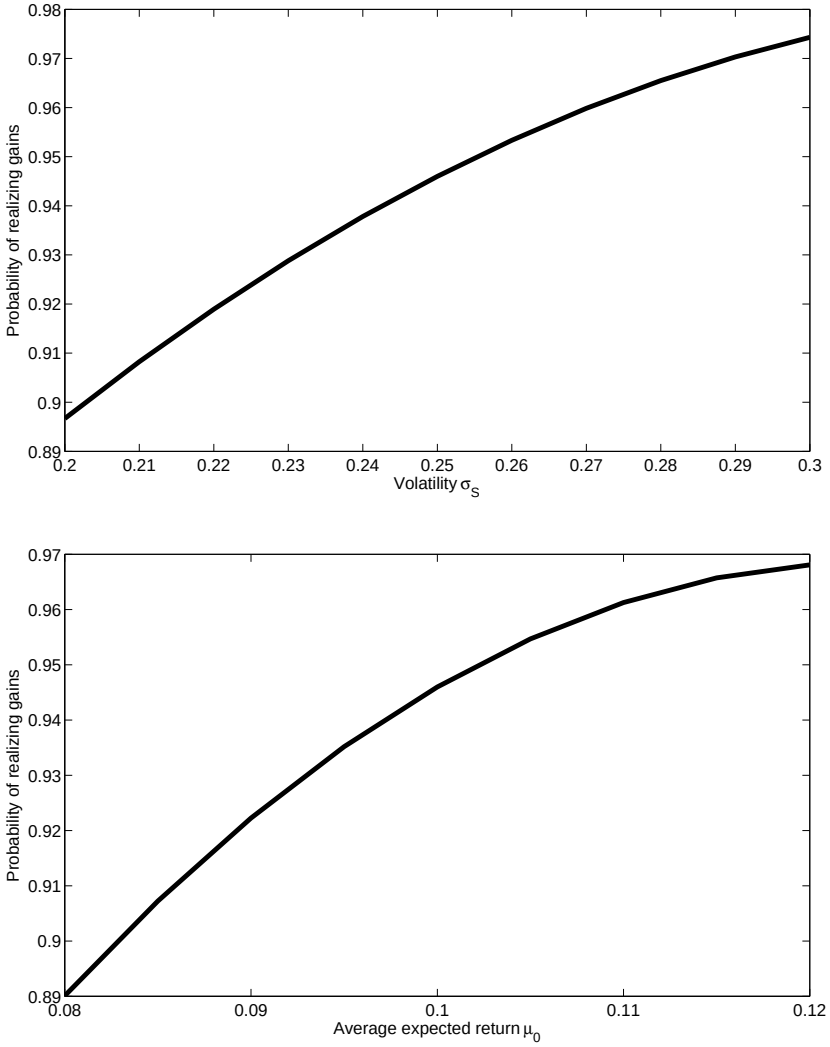

Figure 4: The probability that a sale is a gain.
Parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $\mu_0 = 0.10$, $\mu_1 = 1.20$, $\sigma_S = 0.25$, $g_0 = 0.00$, $g_1 = -0.50$, $\sigma_Z = 0.015$, $\rho = 0.60$. $z_0 = 0$, $F = \$0.5$, $\alpha = 0$, y_0 is the target amount associated with z_0.

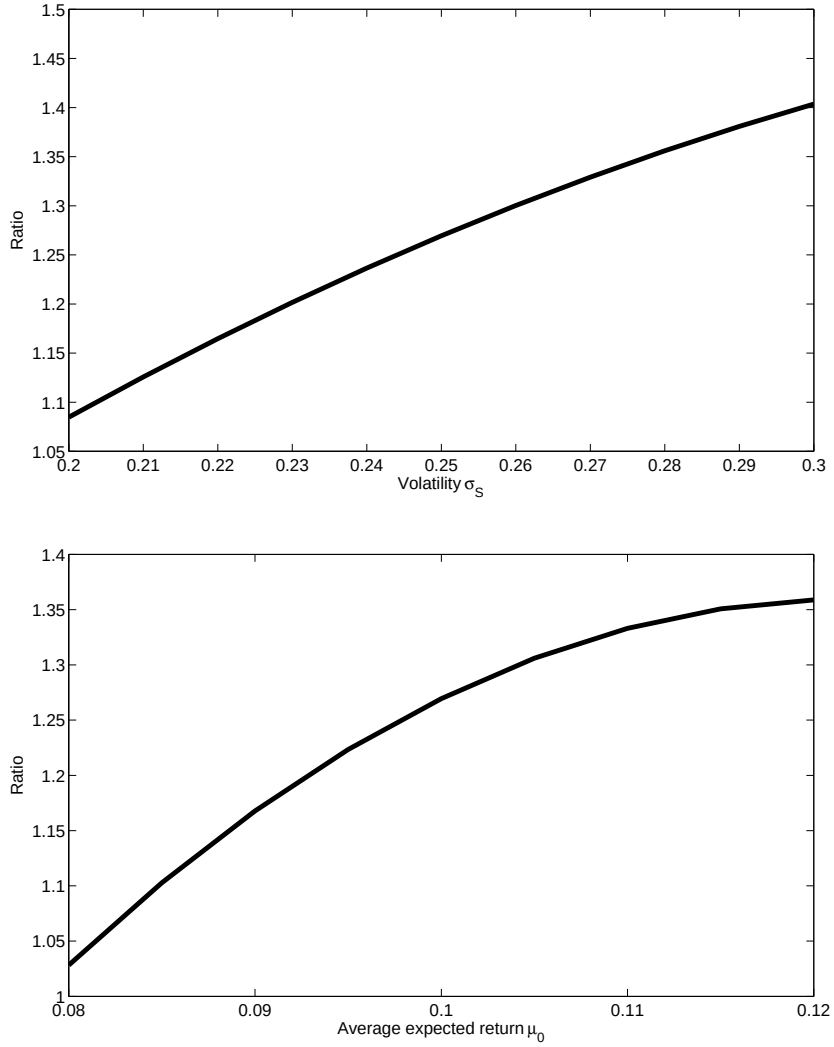

Figure 5: The ratio of the expected time to a sale for a loss to that for a gain. Parameter values: $T = 5$, $\beta = 0.001$, $r = 0.01$, $\mu_0 = 0.10$, $\mu_1 = 1.20$, $\sigma_S = 0.25$, $g_0 = 0.00$, $g_1 = -0.50$, $\sigma_Z = 0.015$, $\rho = 0.60$. $z_0 = 0$, $F = \$0.5$, $\alpha = 0$, y_0 is the target amount associated with z_0.

A.5 Results for CRRA utility function

In this section, we show that the assumptions of CARA preferences, return predictability, and return independence are not critical for the presence of the disposition effect in a portfolio rebalancing model. We also provide an alternative portfolio rebalancing justification for complete liquidation of a stock position: committed consumption.

It is widely documented that a large portion of the average household's budget is committed to ensure a certain critical level of consumption (e.g., Fratantoni (2001), Chetty and Szeidl (2007)). Committed consumption can be caused by sources such as housing and other durable goods consumption that is costly to adjust, habit formation, meeting fixed financial obligations (e.g., mortgage and tuition payments), and precautionary savings against unemployment or health shocks. In addition, investors rarely short stocks or borrow to buy stocks. Accordingly, we assume that the investor must invest at least $\underline{C}e^{-r(T-t)}$ in the risk free asset at time t to ensure that the terminal wealth at time T is above a minimum level of $\underline{C}$.[29]

To understand why it can be optimal to liquidate the entire position in a stock and to display the disposition effect in a portfolio rebalancing model, in this section we focus on the simplest case in which there are only two stocks: Stock 1 and Stock 2.[30] In addition, in order to maintain tractability, we assume that Stock 1 is perfectly liquid (i.e., $F_1 = 0$) and stock returns are constant (i.e., $\mu_{1i} = 0$, $i = 1, 2$). The two Brownian motions B^S_{1t} and B^S_{2t} are allowed to have a correlation coefficient of $\rho \in [-1, 1]$. In addition, we assume that there is no capital gains tax.

[29] We view a continuous-time model as an approximation of discrete-time trading in practice. As Liu (2014) shows, with discrete-time trading and a solvency constraint, an investor must invest at least the present value of the committed consumption level in the risk-free asset.

[30] To match the magnitude of the disposition effect, one needs to extend to the case with multiple illiquid stocks as we did in the main text. With CRRA preferences, it is no longer feasible to separate the investor's multi-stock rebalancing problem into individual stock rebalancing problems. Therefore, one needs to solve numerically a multi-dimensional optimal impulse control problem, which is much more time-consuming than solving the model in the main text.

We denote by X_t and Y_t the time t dollar amount invested in the liquid assets (Stock 1 plus the risk free asset) and in Stock 2 , respectively. Let $0 \leq \tau_1 < \tau_2 < \cdots < \tau_n < ... \leq T$ be the time points when Stock 2 is traded. Then we have

$$dX_t = rX_t dt + \xi_t(\mu_{01} - r)dt + \xi_t \sigma_{S1} dB_{1t}^S, \tag{A-10}$$

$$dY_t = \mu_{02} Y_t dt + \sigma_{S2} Y_t dB_{2t}^S, \tag{A-11}$$

for $t \in [\tau_i, \tau_{i+1})$, where ξ_t represents the dollar amount invested in Stock 1 at time t. At the transaction time τ_i, we have

$$X_{\tau_i} = X_{\tau_i^-} - \delta_{\tau_i} - F_2, \tag{A-12}$$

$$Y_{\tau_i} = Y_{\tau_i^-} + \delta_{\tau_i}, \tag{A-13}$$

where δ_{τ_i} represents the trading amount of Stock 2 .

We assume no short-selling or borrowing, and the investor is required to be solvent at all time. Thus we have the following constraints:[31]

$$X_t - \xi_t \geq \underline{C} e^{-r(T-t)}, \quad \xi_t \geq 0, \quad Y_t \geq 0. \tag{A-14}$$

Given an initial endowment (X_0, Y_0) that satisfies the solvency constraints, the investor's objective is to choose $\{\tau, \delta, \xi\}$ to maximize the expected utility $E[u(W_T)]$ derived from her wealth at time T, subject to Equations (A-10)-(A-13) and the solvency constraints (A-14), where

$$u(W) = \frac{W^{1-\gamma}}{1-\gamma},$$

and $\gamma > 0$ but $\neq 1$ is the relative risk-aversion coefficient.

A.5.1 Optimal trading policy

In the numerical calculations, we assume that Stock 1 has an expected return of 10% and a volatility of 25%, Stock 2 has an expected return of 8% and a volatility of 25%, the correlation coefficient between the two stocks is -0.3, the interest rate is 1%, the fixed trading cost is \$0.5, the committed consumption level is \$10,000, the relative risk-

[31] Even when $Y_t < F_2$, the investor can hold her stock position without violating the solvency.

Table 7:
Baseline parameter values: CRRA preference with committed consumption

This table summarizes the baseline parameter values that we use to illustrate our results in the model with CRRA preference and committed consumption. Annualization applies whenever applicable.

Parameter	Symbol	Baseline value
Risk-free rate	r	0.01
Expected return of Stock 1 S_1	μ_1	0.10
Volatility of Stock 1 S_1	σ_1	0.25
Expected return of Stock 2 S_2	μ_2	0.08
Volatility of Stock 2 S_2	σ_2	0.25
Correlation between the returns	ρ	-0.3
Committed consumption level	$\underline{C}$	\$10,000
Fixed transaction cost	F	\$0.5
Investment horizon (years)	T	5
Relative risk-aversion coefficient	γ	3

aversion coefficient is 3, and the investment horizon is five years. We report these baseline parameter values in Table 7.

We first plot the optimal trading policy at time $t = 2.5$ in Figure 6. When the portfolio weight (rather than dollar amount, as in the CARA case) in Stock 2 exceeds the sell boundary, the investor sells a lump sum amount of the stock so that the portfolio weight in the stock drops to the middle target ratio line. When the portfolio weight drops below the buy boundary, the investor buys a lump sum amount so that the portfolio weight increases to the target ratio line. Trading direction is marked on the figure, and note that the total liquidation of Stock 2 is possible when the disposable wealth is small. Intuitively, this is because when the disposable wealth is low, the investment in risky assets is small, and thus the expected return dominates risk and the investor optimally chooses to hold only the stock with the highest expected return, i.e., Stock 1 . Different from the model in the main text, the total liquidation is driven by a decline in wealth instead of the expected excess return turning sufficiently negative.

A.5.2 Disposition effects

We now conduct the same analysis as previously by computing the ratios PLR, PGR, and PGL using Monte Carlo simulations. We report the results in Table 8. Table 8 shows that even with a CRRA preference, the disposition effect exists across a large range of

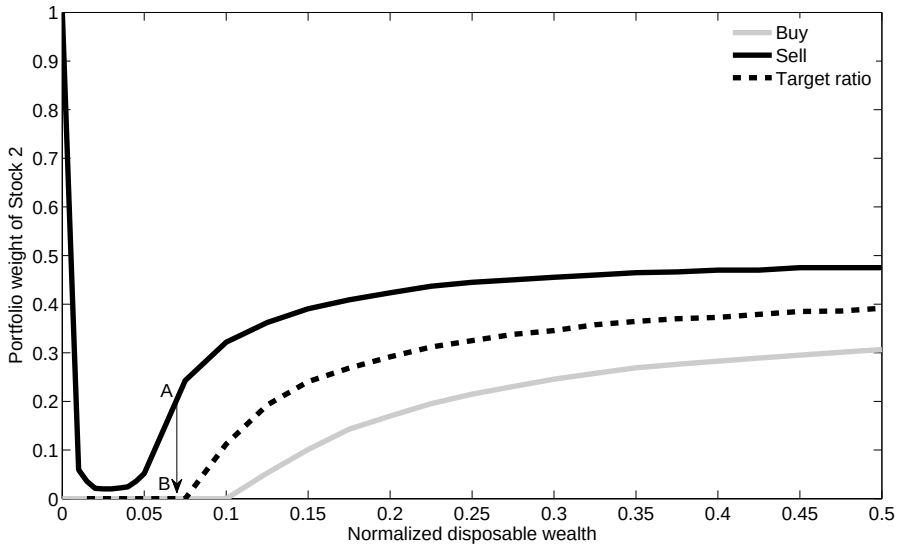

Figure 6: Optimal buy and sell boundary with CRRA utility.
This figure shows the optimal trading boundaries in the model with CRRA preference and committed consumption, at $t = 2.5$ years. Parameter values: $\mu_1 = 0.10$, $\sigma_1 = 0.25$, $\mu_2 = 0.08$, $\sigma_2 = 0.25$, $\rho = -0.3$, $r = 0.01$, $\underline{C} = \$10,000$, $F = \$0.5$, $T = 5$, and $\gamma = 3$.

Table 8: Overall disposition effect

This table shows the results obtained from 100,000 simulated paths, with various initial disposable wealth w_0 (normalized by the committed consumption $\underline{C}$). $DE = PGR - PLR$. Parameter values: $\mu_1 = 0.10$, $\sigma_1 = 0.25$, $\mu_2 = 0.08$, $\sigma_2 = 0.25$, $\rho = -0.3$, $r = 0.01$, $\underline{C} = \$10,000$, $F = \$0.5$, $T = 5$, and $\gamma = 3$. The symbol *** indicates a statistical significance level of 1%.

$w_0/\underline{C}$	0.1	0.15	0.2	0.25	0.3	0.4
PGR	0.763	0.673	0.739	0.729	0.718	0.696
PLR	0.195	0.271	0.103	0.107	0.121	0.140
DE	0.568***	0.402***	0.636***	0.622***	0.597***	0.556***

Table 9: Disposition effect among sales with a complete liquidation of Stock 2

This table shows the results obtained from 100,000 simulated paths, with various initial disposable wealth w_0 (normalized by the committed consumption $\underline{C}$). $DE = PGR - PLR$. Parameter values: $\mu_1 = 0.10$, $\sigma_1 = 0.25$, $\mu_2 = 0.08$, $\sigma_2 = 0.25$, $\rho = -0.3$, $r = 0.01$, $\underline{C} = \$10,000$, $F = \$0.5$, $T = 5$, and $\gamma = 3$. The symbol *** indicates a statistical significance level of 1%.

$w_0/\underline{C}$	0.1	0.15	0.2	0.25	0.3	0.4
PGR	0.907	0.997	1.000	1.000	1.000	1.000
PLR	0.171	0.154	0.196	0.233	0.286	0.325
DE	0.736***	0.843***	0.804***	0.767***	0.714***	0.675***

wealth levels. For example, at $w_0/\underline{C} = 0.1$, the PGR is equal to 0.763, while PLR is only 0.195. In addition, in all of the sales, more than 81% are with a gain. These results suggest that the assumption of a CARA preference in the main text is not critical for our results.

To determine if the disposition effect can still appear in the subsample in which an investor sells the entire position in a stock, we next restrict our analysis to the subsample paths along which the investor liquidates the entire position of Stock 2 and report the corresponding results in Table 9. Table 9 shows that indeed our portfolio rebalancing model can generate the disposition effect. For example, when the disposable wealth to the committed consumption ratio is 0.2, we find that the PGR is equal to 1.000, while PLR is only 0.196, implying a statistically significant disposition effect of 0.804.