Circuit Breakers and Market Contagion: Theory and Evidence

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ABSTRACT

Circuit breakers based on market indices aim to reduce market volatility and prevent crashes during challenging market conditions. However, our continuous-time equilibrium model with multiple stocks reveals a possible contrary outcome. In times of market stress, circuit breakers can induce substantial crash and volatility contagion among otherwise independent stocks and cause heightened market volatility. Our empirical analysis, utilizing Chinese and US data, robustly supports these findings, questioning the effectiveness of circuit breakers in achieving regulatory goals. These results warrant a reevaluation of circuit breaker mechanisms and their potential unintended consequences.

JEL classification: C02, G11

Keywords: Circuit breaker, crash contagion, volatility contagion, return correlation

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1. Introduction

Circuit breakers based on market indices are widely implemented in many countries, including the United States, France, Canada, and China, as a measure to stabilize market prices during challenging times. When a predetermined percentage decline in a market index occurs, the circuit breaker is triggered, leading to a temporary halt in trading for the entire market. Recent events such as the COVID-19 pandemic triggered circuit breakers multiple times across various countries, including the United States, Japan, and South Korea. Notably, Chinese regulators removed a four-day-old circuit breaker rule after it was triggered twice during the week of January 4, 2016. Existing literature on circuit breakers, such as studies by Chen et al. (2023), Greenwald and Stein (1991), and Subrahmanyam (1994), has primarily focused on examining the impact of circuit breakers on the overall return and volatility of stock indices. However, a crucial question remains unanswered: can circuit breakers have adverse effects on return contagion and volatility contagion across individual stocks, thereby increasing systemic risk, particularly during periods of market distress, precisely when circuit breakers are intended to be helpful?

In this paper, we develop a continuous-time equilibrium model to shed light on this important question. In our model, investors can invest in a risk-free asset and two groups of risky assets (stocks) with independent dividend processes to maximize expected utility from their final wealth at time T. Investors have stochastically heterogeneous beliefs about the expected growth rates of the dividends. To cleanly identify the role of circuit breakers in causing contagion and correlations, we assume that the investors have exponential preferences: As a result, in the absence of circuit breakers, the equilibrium stock returns would be independent. The stock market is subject to a market-wide circuit breaker rule, where if the sum of two stock prices (the index) reaches a threshold, the entire stock market is closed until T.

Contrary to regulatory goals, our findings demonstrate that while circuit breakers can be beneficial in normal market conditions by reducing market volatility,[1] they can also result in crash contagion, volatility contagion, greater volatilities, and higher correlations among otherwise independent stocks during periods of market stress. Our model suggests that market-wide circuit breakers may contribute to financial contagion and serve as a channel through which idiosyncratic risks transform into systemic risks. To mitigate these issues, we propose an alternative circuit breaker approach based on individual stocks rather than the entire index. This alternative approach has no contagion or correlations effects.

The intuitions for our main results are as follows. When the circuit breaker is triggered and the market is closed, risk sharing is reduced, resulting in lower stock prices in general. Before the circuit breaker is triggered, if an idiosyncratic negative shock hits a group of stocks in an index, the index level declines, increasing the probability of reaching the circuit breaker threshold. Consequently, the prices of other group stocks may also decrease in anticipation of a likely market closure, leading to positive return correlation, even though stocks would be independent in the absence of the circuit breaker. When the idiosyncratic shock is significant and the index approaches the circuit breaker threshold, the correlation increases further due to the higher likelihood of an imminent market closure. In extreme cases where one group of stocks experiences a crash and triggers the circuit breaker, the prices of the other group of stocks (whose prices are otherwise continuous) must jump down to after-market-closure levels, causing crash contagion. In general, as stock prices decline, the index moves closer to the circuit breaker threshold, causing other stock prices to fall due to the fear of an impending market closure, thus creating a vicious cycle that increases market volatility and cross-stock contagion. Additionally, as some stocks become more volatile (e.g., due to increases in dividend volatilities), the likelihood of triggering the circuit breaker rises, leading to volatility contagion. There-

[1] Other benefits of circuit breakers may include the dampening effect on overreaction, panic, and herding on stock prices.

fore, circuit breakers can transmit crash and volatility effects across stocks, even though stocks would be independent in their absence. These contagion effects can transform idiosyncratic risks into systemic risks.

Our findings suggest that to mitigate contagion effects and systemic risks, it is more effective to impose circuit breakers on individual stocks rather than on the entire market index. In this alternative approach, the circuit breaker threshold is based on individual stock returns, triggering a halt in trading for that specific stock only. This approach avoids increasing correlations and preventing any form of contagion. We demonstrate that with individual stock-based circuit breakers, stock prices tend to be higher, and the likelihood of a market-wide large decline is reduced is lower compared to circuit breakers imposed on the index.

While our model considers only two stocks in the index, we acknowledge that real-world indices typically consist of hundreds of stocks. However, during periods of market distress, markets often focus on a small number of key factors such as Federal Reserve decisions and major economic news that can impact a large group of stocks. In our model, each stock represents a significant portion of stocks exposed to a common risk factor during bad times. Therefore, a negative shock in the risk factor can cause the prices of a large group of stocks to decline together, potentially affecting another group of stocks through the circuit breaker connection, even if they are not directly exposed to the shock.

To test our model's main prediction that circuit breakers exacerbate market contagion during bad times, we leverage a unique policy experiment that occurred in China in early 2016. The introduction of market-wide circuit breakers by the China Securities Regulation Commission (CSRC) on January 1, 2016, followed by their suspension after triggering twice in four days, provides an opportunity to compare market contagion with and without circuit breakers, free from many confounding factors.

Measuring market contagion poses challenges, and while prior studies have focused on cross-country contagion (see, e.g., King and Wadhwani, 1990; Calvo and Reinhart, 1995; Forbes and Rigobon, 2001; Bae, Karolyi, and Stulz, 2003; Bekaet, Harvey, and Ng, 2005; Diebold and Yilmaz, 2009), we adapt their approach to our setting. In our analysis, we examine whether circuit breakers induce a significant increase in simultaneous co-movements and lead-lag propagation across stocks. We particularly focus on utility stocks, known for their low comovements with stocks in other industries during normal times (Chan, Lakonishok, and Swaminathan, 2007), to capture additional co-movements exceeding normal levels as evidence of increased market contagion.

In the Chinese market, the market-wide circuit breaker was triggered on January 4, 2016, the first trading day since the introduction of the circuit breaker policy. To assess the impact of circuit breakers, we compare the events of January 4 with several similar days that did not have circuit breakers in place. Specifically, we use December 31, 2015, the last trading day before the policy was implemented, January 8, 2016, the first trading day after the suspension of the policy, and eight other days when the stock market experienced a significant drop that would have triggered the circuit breaker, had it been active, as control groups. Our analysis reveals that on January 4, both the simultaneous co-movements and the lead-lag relations between utility stocks and other stocks were significantly higher. Additionally, intra-day analysis demonstrates that cross-stock contagion became more severe as the market approaches the trading halt on January 4.

To determine whether our findings are specific to the Chinese market, we examine the more recent experiences with circuit breakers in the United States during the COVID-19 pandemic. In March 2020, the S&P 500 index triggered market-wide circuit breakers four times, resulting in 15-minute trading halts during the opening hour on March 9, 12, and 16, as well as in the early afternoon on March 18. Applying the same empirical methods to the U.S. data, we find that on the day when the circuit breaker was triggered, and especially during the period leading up to the trading halt, declines in other stocks readily spread to utility stocks. This evidence of cross-stock contagion strongly supports the predictions of our model.

Our paper draws motivation from the seminal work of Chen et al. (2023). While Chen et al. (2023) focus on a single stock (index) and demonstrated in a dynamic equilibrium setting that downside circuit breakers could lower stock prices, increase market volatility, and accelerate market decline (referred to as the "magnet effect"), our study focuses on the cross-stock contagion effect of circuit breaker rules using a dynamic equilibrium model with *multiple* stocks and potentially discontinuous stock dividend streams. We show that, in the presence of jump risk, a crash in one stock can cause a crash in an otherwise independent stock. Furthermore, unlike Chen et al. (2023), our analysis does not involve leverage constraints before or after market closure. Instead, we highlight the circuit breakers' contagion effect as the primary economic mechanism driving our results. This suggests that circuit breakers can have a significant impact on price dynamics, even without any changes in leverage constraints.

Several other theoretical studies have explored circuit breakers in different contexts. For instance, Greenwald and Stein (1991) demonstrate how circuit breakers can aid in coordinating trading for market participants in a market with limited participation. Subrahmanyam (1994) show that circuit breakers can increase price volatility due to investors shifting trades to periods with lower liquidity supply when information asymmetry exists. Hong and Wang (2000) examine the impact of periodic exogenous market closure on asset prices and highlight the rich patterns of trading and returns produced by their model, consistent with empirical findings.

Numerous empirical studies have challenged the efficacy of circuit breakers, including market-wide circuit breakers, price limits, and trading pauses. For example, using Nasdaq order book data, Hautsch and Horvath (2019) find that trading pauses increase volatility, reduce price stability and liquidity after the pause, while enhancing price discovery during the break. Kim and Rhee (1997), analyzing Tokyo Stock Exchange data, conclude that price limit mechanisms in the exchange are ineffective, leading to higher volatility levels, interference with efficient price equilibrium, and disruption of trading. Examining the behavior of the Israeli stock market during the October 1987 crash, Lauterbach and Ben-Zion (1993) find that circuit breakers reduce initial price losses and next-day opening order imbalances but have no long-run effect. Similarly, Lee, Ready, and Seguin (1994) investigate the impact of firm-specific New York Stock Exchange (NYSE) trading halts and find that they do not reduce volume or price volatility in the post-halt period. In the context of the October 1997 market crash, Goldstein and Kavajecz (2004) observe the magnet effect, indicating an acceleration of activity approaching the market-wide circuit breaker.[2]

In contrast to the existing literature, our study focuses on the dynamic interactions among stocks and examines the cross-stock contagion behavior caused by market-wide circuit breakers. To the best of our knowledge, this is the first theoretical and empirical analysis capturing and analyzing such contagion effects. The use of the sudden adoption and suspension of circuit breaker policy in China provides a unique approach to isolate the effects of circuit breakers. Although circuit breakers aim to stabilize markets during bad times, our findings indicate that market-wide circuit breakers can have significant crash and volatility contagion effects, exacerbating market volatility during downturns.

[2] A few other studies on market halts focus on other related issues. For example, Ackert, Church, and Jayaraman (2001) conduct an experimental study to analyze the effects of mandated market closures and temporary halts on market behavior. Corwin and Lipson (2000) study order submission strategies of traders around market halts, providing a detailed description of the mechanics of trading halts and identifying traders who provide liquidity. Christie, Corwin, and Harris (2002) study the impact on post-halt market prices of Nasdaq's alternative halt and reopening procedures. Their results are consistent with the hypothesis that increased information transmission during the halt reduces post-halt uncertainty.

2. The Model

We consider a continuous-time exchange economy over a finite time interval $[0, T]$. Investors can trade two risky assets, Stock 1 and Stock 2, and one risk-free asset. Each of the two stocks in our model represents a group of stocks that share the same significant risk exposure in bad times. The risk-free asset has a net supply of zero and the interest rate can be normalized to zero because there is no intertemporal consumption in our model. The total supply of each stock is one share and every stock pays only a terminal dividend at time T. The dividend processes are exogenous and publicly observed. Uncertainty about dividends is represented by a standard Brownian motion Z_t and an independent standard Poisson process N_t with jump intensity κ and jump size ν defined on a complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$. An augmented filtration $\{\mathcal{F}_t\}_{t \geq 0}$ is generated by Z_t and N_t.

There is a continuum of investors of Types A and B in the economy, with a mass of 1 for each type. For $i = A, B$ and $j = 1, 2$, Type i investors are initially endowed with θ^i_{j0} shares of Stock j but no risk-free asset, with $0 \leq \theta^i_{j0} \leq 1$ and $\theta^A_{j0} + \theta^B_{j0} = 1$. The probability measure Type A investors use is $\mathbf{P}^A$, which is the same as the true probability measure $\mathbf{P}$. Under Type A's probability measure, Stock 1's dividend process evolves as:

$$dD_{1,t} = \mu^A_1 dt + \sigma dZ_t, \tag{1}$$

and Stock 2's dividend process follows a jump process with drift:

$$dD_{2,t} = \mu^A_2 dt + \nu(dN_t - \kappa^A dt), \tag{2}$$

where Stock 1's expected dividend growth rate μ^A_1, Stock 1's dividend volatility σ, Stock 2's expected dividend growth rate μ^A_2, jump size ν, and jump intensity $\kappa^A = \kappa$ are all constants, the compensated Poisson process $N_t - \kappa^A t$ is a martingale under $\mathbf{P}^A$, and $D_{j,0} = 1$ for $j = 1, 2$.

Relative to Type A investors, Type B investors have different beliefs about the dividend process $D_{1,t}$ and employ a different probability measure $\mathbf{P}^B$, under which the dividend process $D_{1,t}$ evolves as

$$dD_{1,t} = \mu_{1,t}^B dt + \sigma dZ_t^B, \tag{3}$$

where Z_t^B is a Brownian motion under measure $\mathbf{P}^B$, and $\mu_{1,t}^B = \mu_1^A + \delta_{1,t}$ for a stochastic process $\delta_{1,t}$ (specified below) that measures the disagreement between Type A and Type B investors about the growth rate of the dividend process $D_{1,t}$. Under $\mathbf{P}^B$, the dividend process $D_{2,t}$ evolves as

$$dD_{2,t} = \mu_{2,t}^B dt + \nu(dN_t^B - \kappa_t^B dt), \tag{4}$$

where under measure $\mathbf{P}^B$, N_t^B is a non-homogeneous Poisson process with jump intensity κ_t^B and jump size ν, and $\mu_{2,t}^B$ is Stock 2's expected dividend growth rate. For simplicity of exposition, we assume $\kappa_t^B = \kappa^A \delta_{2,t}$, where $\delta_{2,t}$ (specified below) measures the disagreement between Type A and Type B investors about the jump intensity of process $D_{2,t}$. Similar to $D_{1,t}$, we assume that Type A and Type B investors also disagree on the expected growth rate of $D_{2,t}$. For simplicity, we assume that this disagreement only stems from the disagreement on the jump intensity. In other words, conditional on no jumps, Type A and B investors agree on Stock 2's expected growth rate, i.e., $\mu_{2,t}^B - \nu\kappa_t^B = \mu_2^A - \nu\kappa^A$.

The Radon-Nikodym derivative between the two probability measures can therefore be written as.

$$\eta_T = \frac{d\mathbf{P}^B}{d\mathbf{P}^A}|_{\mathcal{F}_T} = \eta_{1,T}\eta_{2,T}, \tag{5}$$

where

$$\eta_{1,T} = e^{\int_0^T \frac{\delta_{1,t}}{\sigma} dZ_t - \int_0^T \frac{\delta_{1,t}^2}{2\sigma^2} dt}, \quad \eta_{2,T} = e^{\kappa^A \int_0^T (1-\delta_{2,t}) dt} \prod_{i=1}^{N_T} \delta_{2,t_i},$$

and $t_i, i = 1, 2, ...$ are jump times before T.

For the disagreement process $\delta_{1,t}$, we assume that under the probability measure $\mathbf{P}^A$:

$$d\delta_{1,t} = -k_1(\delta_{1,t} - \bar{\delta}_1)dt + \sigma_\delta dZ_t, \tag{6}$$

where $\bar{\delta}_1$ is the constant long-time average of the disagreement (which could be zero), $k_1 > 0$ measures the speed of mean reversion in the disagreement, and σ_δ is the volatility of the disagreement.[3]

For the disagreement process $\delta_{2,t}$, we assume that under the probability measure $\mathbf{P}^A$:

$$d\delta_{2,t} = -k_2(\delta_{2,t} - \bar{\delta}_2)dt + \nu_\delta dN_t, \tag{7}$$

where $\bar{\delta}_2$ is the constant long-time average, $k_2 > 0$ is the speed of mean reversion, and $\nu_\delta > 0$ is a constant jump size of the disagreement process.[4]

In this paper we focus on the market closure effect of circuit breakers, i.e., investors cannot trade for a period of time after circuit breakers are triggered. As we show later, stochastic disagreement is necessary for the presence of the market closure effect, because in the absence of stochastic disagreement, investors would not trade after time zero even when the market is always open and thus market closure would not have any impact on asset prices. To capture the market crash risk, the fundamentals of a group of stocks must jump down with a positive probability.[5] Therefore, we assume these two features

[3]In the Appendix, we show that this $\delta_{1,t}$ process is consistent with Kalman filtering when Type B investors do not know the expected growth rate of Stock 1's dividend.

[4]With the specialized dynamics of κ_t^B, N_t^B is a Hawkes process in general. See Aït-Sahalia, Cacho-Diaz, and Laeven (2015) and Aït-Sahalia and Hurd (2016) for applications of Hawkes processes in finance.

[5]In the previous version, we show that when dividend $D_{2,t}$ is not a jump but a diffusion process with continuous paths, similar to $D_{1,t}$, our main results still hold such as increased correlations, volatility contagion and magnet effects in the presence of circuit breakers. The only exception is crash contagion,

in our model.

Hereafter, we use the notation $\mathbb{E}^i[\cdot]$ to denote the expectation under the probability measure $\mathbf{P}^i$ for $i \in \{A, B\}$.

To isolate the impact of circuit breakers on stock return correlations, we assume that, for $i \in \{A, B\}$, Type i investors have constant absolute risk averse (CARA) preferences over the terminal wealth W_T^i at time T:

$$u(W_T^i) = -\exp(-\gamma W_T^i),$$

where $\gamma > 0$ is the absolute risk aversion coefficient. With CARA preferences, there is no wealth effect and therefore in the absence of circuit breakers, it can be shown that returns of the two stocks would be independent.

Trading in the stocks is subject to a market-wide circuit breaker rule as explained next. Let $S_{j,t}$ denote the price of Stock $j = 1, 2$ at time $t \leq T$ and the index $S_t = S_{1,t} + S_{2,t}$ denote the sum of the two prices (equivalent to an equally weighted index).[6] Define the circuit breaker trigger time

$$\tau = \inf\{t : S_t \leq h, t \in [0, T)\},$$

where h is the circuit breaker threshold (hurdle). At the circuit breaker trigger time τ, the market is closed until T,[7] which results in the market closure effect. In practice, the circuit breaker threshold h is typically equal to a percentage of the previous day's closing level. In this paper, we set $h = (1 - \alpha)S_0$ for a constant α (e.g., $\alpha = 0.07$ for Level 2

since no crash (a discrete change in a short time period) occurs in continuous changes of $D_{1,t}$ or $D_{2,t}$. To save space, we do not include these results in this version, but they are available from the authors.

[6]Using a different form of the combination of the stock prices as the index would not change our main results, as long as the index is increasing in both stock prices.

[7]Assuming that markets can reopen after being halted for a period of time would not change the qualitative results on contagion. Quantitatively, the results are close in very bad times, because the fear of market closure is similar whether the closure is long or relatively short in very bad times.

market closure in the Chinese stock markets and for Level 1 market closure in the U.S. market).

3. Equilibrium without Circuit Breakers

As a benchmark case, we first solve for the equilibrium stock prices when there is no circuit breaker in place in the market. Because the market is complete in this case, it is convenient to solve the planner's problem:

$$\max_{W_T^A, W_T^B} \mathbb{E}_0^A[u(W_T^A) + \xi\eta_T u(W_T^B)], \tag{8}$$

subject to the budget constraint $W_T^A + W_T^B = D_{1,T} + D_{2,T}$, where ξ is a constant depending on the initial wealth weights of the two types of investors.

From the first order conditions, we obtain:

$$W_T^A = \frac{1}{2\gamma}\log(\frac{1}{\xi\eta_T}) + \frac{1}{2}(D_{1,T} + D_{2,T}), \tag{9}$$

$$W_T^B = -\frac{1}{2\gamma}\log(\frac{1}{\xi\eta_T}) + \frac{1}{2}(D_{1,T} + D_{2,T}). \tag{10}$$

Given the utility function $u(x) = -e^{-\gamma x}$, the state price density under Type A investors' beliefs is

$$\pi_t^A = \mathbb{E}_t^A[\zeta u'(W_T^A)] = \mathbb{E}_t^A[\gamma\zeta e^{-\gamma W_T^A}] = \gamma\zeta\xi^{\frac{1}{2}}\mathbb{E}_t^A[\eta_T^{\frac{1}{2}} \cdot e^{-\frac{\gamma}{2}(D_{1,T}+D_{2,T})}], \tag{11}$$

for some constant ζ. Therefore, the stock price in equilibrium is given by

$$\hat{S}_{j,t} = \frac{\mathbb{E}_t^A\left[\pi_T^A D_{j,T}\right]}{\mathbb{E}_t^A[\pi_T^A]} = D_{j,t} + \frac{\mathbb{E}_t^A\left[\pi_T^A(D_{j,T} - D_{j,t})\right]}{\mathbb{E}_t^A[\pi_T^A]}, \quad j = 1, 2. \tag{12}$$

Since the two dividend processes are independent, Equation (12) can be simplified into

$$\hat{S}_{1,t} = \frac{\mathbb{E}_t^A[\pi_{1,T}^A D_{1,T}]}{\mathbb{E}_t^A[\pi_{1,T}^A]}, \qquad \hat{S}_{2,t} = \frac{\mathbb{E}_t^A[\pi_{2,T}^A D_{2,T}]}{\mathbb{E}_t^A[\pi_{2,T}^A]}, \tag{13}$$

where $\pi_{1,t}^A = \mathbb{E}_t^A[\eta_{1,T}^{1/2} \cdot e^{-\frac{\gamma}{2}D_{1,T}}]$, $\pi_{2,t}^A = \mathbb{E}_t^A[\eta_{2,T}^{1/2} e^{-\frac{\gamma}{2}D_{2,T}}]$. Thus, the two prices can be computed separately when there are no circuit breakers, which implies that stock returns are independent.

Next, we derive the equilibrium prices in closed form for the two stocks and examine the impact of the jump and the stochastic disagreement on the market equilibrium.

For Stock 1, the disagreement process is governed by the mean-reverting process (6). The formula of equilibrium price $\hat{S}_{1,t}$ can be derived analytically and is presented in the following proposition.

PROPOSITION 1. *When there are no circuit breakers, the equilibrium price of Stock 1 is:*

$$\hat{S}_{1,t} = D_{1,t} + \mu_1^A (T-t) - 2\left(\frac{dA(t;\gamma)}{d\gamma} + \frac{dC(t;\gamma)}{d\gamma}\delta_{1,t}\right), \tag{14}$$

where $A(t;\gamma)$ and $C(t;\gamma)$ are given in Appendix A.

Proposition 1 shows that, in addition to the dividend payment, disagreement also affects the price of Stock 1. As a result, the instantaneous volatility of the stock price $\hat{S}_{1,t}$ is different from that of the dividend process.[8]

To show the importance of disagreement being stochastic, we next show what would happen if the disagreement were constant, that is, $\delta_{1,t} = \delta_{1,0}$ for all $t \in [0,T]$. In this

[8]It can be shown that the instantaneous volatility of the equilibrium price $\hat{S}_{1,t}$ is greater than the volatility of the dividend process $D_{1,t}$ when $T - t$ is small.

case, the equilibrium price would simplify to

$$\hat{S}_{1,t} = D_{1,t} + \frac{\mu_1^A + \mu_{1,t}^B}{2}(T-t) - \frac{\gamma}{2}\sigma^2(T-t).$$

Thus, the equilibrium price of Stock 1 would be determined by the average beliefs of Type A and B investors on the expected growth rate of the dividend and the volatility of the stock price would be the same as the volatility of its dividend. Moreover, by applying Ito's lemma to the wealth process $W_t^A = \frac{\mathbb{E}_t^A[\pi_T^A W_T^A]}{\mathbb{E}_t^A[\pi_T^A]}$, we can find that the equilibrium number of shares of Stock 1 held by Type A investors would be equal to

$$\hat{\theta}_{1,t}^A = \frac{1}{2} - \frac{1}{2\gamma}\frac{\delta_{1,0}}{\sigma^2}, \tag{15}$$

which implies that the equilibrium number of shares of Stock 1 held by Type B investors would be equal to

$$\hat{\theta}_{1,t}^B = \frac{1}{2} + \frac{1}{2\gamma}\frac{\delta_{1,0}}{\sigma^2}. \tag{16}$$

Because the number of shares held by investors in the equilibrium would be constant over time if the disagreement were constant, market closure would not have any impact on the equilibrium price in the case of constant disagreement. This result implies that stochastic disagreement is necessary for circuit breakers to have any impact through the market closure channel.

For Stock 2, an analytical expression of the equilibrium price in the case of stochastic disagreement can unlikely be obtained. However, it can be shown that if the disagreement $\delta_{2,t} = \delta_2$ is a constant, then

$$\mathbb{E}_t^A[\pi_{2,T}^A D_{2,T}] = \mathbb{E}_t^A[\eta_{2,T}^{1/2} e^{-\gamma D_{2,T}/2}] \cdot \left(D_{2,t} + (\mu_2^A - \kappa^A \nu)(T-t) + \kappa^A \sqrt{\delta_2}\nu(T-t)e^{-\frac{\gamma}{2}\nu} \right).$$

Then by Equation (13), we have the equilibrium price of Stock 2 as in the following proposition.

PROPOSITION 2. *When there are no circuit breakers and $\delta_{2,t} = \delta_2$ (i.e., constant disagreement on the jump intensity), the equilibrium price of Stock 2 is:*

$$\hat{S}_{2,t} = D_{2,t} + (\mu_2^A - \kappa^A \nu)(T-t) + \kappa^A \sqrt{\delta_2} \nu (T-t) e^{-\frac{\gamma}{2}\nu}. \tag{17}$$

Proposition 2 shows that the equilibrium price is affected by the heterogenous beliefs through the geometric average of beliefs of Type A and Type B investors on the jump intensity. In addition, the instantaneous volatility (square root of instantaneous variance) of the equilibrium price under $\mathbf{P}^A$ is the same as that of the dividend process because the rest of the terms in (17) are deterministic.

Let $\hat{\theta}_{j,t}^A$ be the optimal shares of Stock i held by Type A investors. Then $dW_t^A = \hat{\theta}_{1,t}^A d\hat{S}_{1,t} + \hat{\theta}_{2,t}^A d\hat{S}_{2,t}$. Applying Ito's formula to $W_t^A = \mathbb{E}_t^A[\pi_T^A W_T^A]/\pi_t^A$ and collecting the coefficients of stochastic terms, we obtain the optimal shares holding of Stock 2 for Type A investors as follows.

$$\hat{\theta}_{2,t}^A = \frac{1}{2} - \frac{1}{2\gamma\nu} \log \delta_2. \tag{18}$$

This shows that in the absence of circuit breakers, if the disagreement were constant, then the equilibrium trading strategy in Stock 2 for all investors would be to buy and hold and thus market closure would not have any impact on Stock 2 price. Therefore, as for Stock 1, stochastic disagreement is also important for Stock 2 to capture the market closure effect.

4. Equilibrium with Circuit Breakers

In this section, we study equilibrium prices when the circuit breaker rule is imposed in the market. We first solve for the indirect utility functions at the circuit breaker trigger time τ by maximizing investors' expected utility at $\tau \le T$:

$$\max_{\theta^i_{1,\tau},\theta^i_{2,\tau}} \mathbb{E}^i_\tau[u(W^i_\tau + \theta^i_{1,\tau}(S_{1,T} - S_{1,\tau}) + \theta^i_{2,\tau}(S_{2,T} - S_{2,\tau}))], \ i \in \{A, B\}, \tag{19}$$

with the market clearing condition $\theta^A_{j,\tau} + \theta^B_{j,\tau} = 1$ and the terminal condition $S_{j,T} = D_{j,T}$, where $\theta^i_{j,\tau}$ is the optimal number of shares of Stock j held by Type i investors at time τ, for $i \in \{A, B\}$ and $j = 1, 2$.

If the circuit breaker is triggered by a continuous decline in Stock 1's dividend, then the after-closure prices of both stocks will reflect their respective fundamental values because both dividends are continuous at the trigger time and investors can trade continuously. If there is a jump in Stock 2's dividend, then the index level corresponding to the after-jump dividend levels may fall strictly below the circuit breaker threshold h. To resolve this technical issue, as what is done in practice, we assume that investors can trade both stocks one more time to reflect the after-jump dividend levels after the circuit breaker is triggered by a jump in Stock 2's price caused by a jump in its dividend. [9] Therefore, at the market closure time, both stocks can reach their fundamental values regardless of which stock triggered the circuit breaker.

Exploiting the dynamics of $D_{j,t}$ and evaluating the expectation in the above optimization problems, we obtain a system of equations that determine $\theta^i_{j,\tau}$ for $i \in \{A, B\}, j = 1, 2$. Then the equilibrium prices are obtained through market clearing conditions. We summarize the result in the following proposition.

[9] An alternative justification is that the jump can be viewed as an approximation of a deterministic steep decline (less than but very close to a 90-degree drop) and during the fast decline, Stock 1 or Stock 2 can trade freely and reach their fundamental values.

PROPOSITION 3. *Suppose that the market is halted at a stopping time $\tau < T$.*
(1) For Stock 1, the market clearing price at τ is given by

$$S^c_{1,\tau} = D_{1,\tau} + \mu^A_1(T-\tau) - \gamma\theta^A_{1,\tau}\sigma^2(T-\tau),$$

where the optimal share holding of Type A investors is

$$\theta^A_{1,\tau} = \frac{-\frac{1}{\tilde{k}_1}(1-e^{\tilde{k}_1(\tau-T)})\delta_{1,\tau} - \frac{k_1\bar{\delta}_1}{\tilde{k}_1}(T-\tau-\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}) + I_\tau}{I_\tau + \gamma\sigma^2(T-\tau)}, \tag{20}$$

with $\tilde{k}_1 = k_1 - \frac{\sigma_\delta}{\sigma}$ and

$$I_\tau = -\gamma\sigma^2(\tau-T) + \frac{2\sigma_\delta\sigma\gamma}{\tilde{k}_1}(T-\tau-\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1}) + \frac{\sigma^2_\delta\gamma}{\tilde{k}^2_1}(T-\tau-2\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1} + \frac{1-e^{2\tilde{k}_1(\tau-T)}}{2\tilde{k}_1}).$$

If $\tilde{k}_1 = 0$, the optimal share holding is simplified into:[10]

$$\theta^A_{1,\tau} = \frac{1}{\gamma}\left(\frac{\gamma\sigma^2 - \gamma\sigma_\delta\sigma(\tau-T) + \frac{1}{2}k_1\bar{\delta}_1(\tau-T) + \frac{\sigma^2_\delta\gamma}{3}(\tau-T)^2 - \delta_{1,\tau}}{-\sigma_\delta\sigma(\tau-T) + \frac{\sigma^3_\delta}{3}(\tau-T)^2 + 2\sigma^2}\right).$$

(2) For Stock 2, the market clearing price is given by

$$S^c_{2,\tau} := D_{2,\tau} + (\mu^A_2 - \kappa^A\nu)(T-\tau) + \kappa^A\nu(T-\tau)e^{-\gamma\theta^A_{2,\tau}\nu}.$$

The optimal share holding $\theta^A_{2,\tau}$ of Type A investors at τ is specified in Appendix B.2.

As in the case of no circuit breakers, because dividend processes are independent and investors have CARA preferences, the price of a stock only depends on its own dividend process at the circuit breaker trigger time.

[10] It can be verified that as $\tau \to T^-$, $\theta^A_{1,\tau} \to \frac{1}{2} - \frac{\delta_{1,T}}{2\gamma\sigma^2}$, which coincides with the optimal share holding of Stock 1 by Type A in the case of constant disagreement.

4.1 Circuit Breaker Trigger Time τ

The circuit breaker trigger time τ can be characterized using the dividend values. Because the market is closed when the sum of prices falls below (or reaches) the threshold h, we have

$$
\begin{aligned}
h &\geq S^c_{1,\tau} + S^c_{2,\tau} \\
&= D_{1,\tau} + D_{2,\tau} + \left(\mu_1^A - \gamma\sigma^2\theta^A_{1,\tau} + (\mu_2^A - \kappa^A\nu) + \kappa^A\nu e^{-\gamma\theta^A_{2,\tau}\nu}\right)(T-\tau).
\end{aligned}
$$

It follows that we may define the stopping time τ using the dividend processes as follows.

PROPOSITION 4. *Let h be the threshold. Define a stopping time*

$$
\tau = \inf\{t \geq 0 : D_{1,t} + D_{2,t} \leq \underline{D}(t)\},
$$

where

$$
\underline{D}(t) = h - \left(\mu_1^A - \gamma\sigma^2\theta^A_{1,\tau} + (\mu_2^A - \kappa^A\nu) + \kappa^A\nu e^{-\gamma\theta^A_{2,\tau}\nu}\right)(T-\tau).
$$

Then the circuit breaker is triggered at time τ when $\tau < T$.

Note that $D_{1,t} + D_{2,t}$ is a jump diffusion process; thus, the trigger time τ is the first time the jump-diffusion process hits or goes below $\underline{D}(t)$.

4.2 Equilibrium Prices before τ

After obtaining the market clearing prices and the optimal portfolios at τ, we now study the equilibrium stock prices for $t < \tau \wedge T$. For $i \in \{A, B\}$, let

$$
G^i_\tau(\theta^{i,*}_{1,\tau}, \theta^{i,*}_{2,\tau}) = G^i_{1,\tau} + G^i_{2,\tau},
$$

where $G^i_{1,\tau}$ and $G^i_{2,\tau}$ are given by (B.5),(B.6), and (B.16) in Appendix B. It can be shown that the indirect utility function of Type $i \in \{A, B\}$ at τ can be written as follows:

$$V^i(W^i_\tau, \tau) = \max_{\theta^i_{1,\tau}, \theta^i_{2,\tau}} \mathbb{E}^i_\tau[u(W^i_\tau + \theta^i_{1,\tau}(S_{1,T} - S_{1,\tau}) + \theta^i_{2,\tau}(S_{2,T} - S_{2,\tau}))] = -e^{-\gamma(W^i_\tau + G^i_\tau(\theta^{i,*}_{1,\tau}, \theta^{i,*}_{2,\tau}))}.$$

Then we are ready to solve the planner's problem at time $t < T \wedge \tau$:

$$\max_{W^A_{T\wedge\tau}, W^B_{T\wedge\tau}} \mathbb{E}^A_t[V^A(W^A_{T\wedge\tau}, T \wedge \tau) + \xi\eta_{T\wedge\tau} V^B(W^B_{T\wedge\tau}, T \wedge \tau)], \tag{21}$$

subject to the wealth constraint $W^A_{T\wedge\tau} + W^B_{T\wedge\tau} = S_{1,T\wedge\tau} + S_{2,T\wedge\tau}$.

Similar to the case without circuit breakers, it follows from the first order conditions and the wealth constraint that

$$W^A_{T\wedge\tau} = \frac{1}{2\gamma}\log(\frac{1}{\xi\eta_{T\wedge\tau}}) + \frac{1}{2}(S_{1,T\wedge\tau} + S_{2,T\wedge\tau}) + \frac{G^B_{T\wedge\tau} - G^A_{T\wedge\tau}}{2}, \tag{22}$$

$$W^B_{T\wedge\tau} = -\frac{1}{2\gamma}\log(\frac{1}{\xi\eta_{T\wedge\tau}}) + \frac{1}{2}(S_{1,T\wedge\tau} + S_{2,T\wedge\tau}) + \frac{G^A_{T\wedge\tau} - G^B_{T\wedge\tau}}{2}. \tag{23}$$

In addition, the state price density under Type A investors' beliefs is

$$\begin{aligned}\pi^A_t &= \mathbb{E}^A_t[\zeta(V^A(W^A_{T\wedge\tau}, T\wedge\tau))'] = \mathbb{E}^A_t[\gamma\zeta e^{-\gamma(W^A_{T\wedge\tau} + G^A_{T\wedge\tau})}] \\ &= \gamma\zeta\mathbb{E}^A_t[\eta^{1/2}_{T\wedge\tau} \cdot e^{-\frac{\gamma}{2}(S_{1,T\wedge\tau} + S_{2,T\wedge\tau} + G^B_{T\wedge\tau} + G^A_{T\wedge\tau})}],\end{aligned} \tag{24}$$

for some constant ζ, where $(V^A(W^A_{T\wedge\tau}, T\wedge\tau))'$ denotes the marginal utility of wealth. Thus, the stock price at $t < T \wedge \tau$ in equilibrium is given by

$$S_{j,t} = \frac{\mathbb{E}^A_t[\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}]}{\mathbb{E}^A_t[\pi^A_{T\wedge\tau}]}, \quad j = 1, 2, \tag{25}$$

with

$$S_{j,T\wedge\tau} = \begin{cases} D_{j,T}, & \text{if } \tau \geq T, \\ S^c_{j,\tau}, & \text{if } \tau < T. \end{cases} \tag{26}$$

In Equation (25), because the stopping time τ depends on the circuit breaker threshold h, the equilibrium prices $S_{1,t}$ and $S_{2,t}$ also depend on h. On the other hand, in practice, h depends on the initial stock prices $S_{1,0}$ and $S_{2,0}$, because $h = (1-\alpha)(S_{1,0} + S_{2,0})$ (e.g., $\alpha = 0.07$ for Chinese markets). Therefore, to obtain the equilibrium prices $S_{1,t}$ and $S_{2,t}$, we need to solve the following fixed point problem in $S_{1,0}$ and $S_{2,0}$:

$$S_{j,0} = \frac{\mathbb{E}^A_0[\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}]}{\mathbb{E}^A_0[\pi^A_{T\wedge\tau}]}, j = 1, 2, \tag{27}$$

where the right hand side is implicitly a function of the initial stock prices $S_{1,0}$ and $S_{2,0}$. The following proposition guarantees the existence and uniqueness of a solution to the above fixed point problem.

PROPOSITION 5. *If the initial equilibrium index value $\hat{S}_{1,0} + \hat{S}_{2,0}$ is positive in the absence of circuit breakers, there exists a unique solution to the fixed point problem (27) in the presence of circuit breakers.*

We can then compute the trading strategies as follows. The wealth process of Type A investors is

$$W^A_t = \frac{\mathbb{E}^A_t[\pi^A_{T\wedge\tau} W^A_{T\wedge\tau}]}{\mathbb{E}^A_t[\pi^A_{T\wedge\tau}]}, t < T \wedge \tau. \tag{28}$$

From the budget constraint we have

$$dW^A_t = \bar{\theta}^A_{1,t} dS_{1,t} + \bar{\theta}^A_{2,t} dS_{2,t},$$

where $\bar{\theta}^A_{1,t}$ and $\bar{\theta}^A_{2,t}$ are share holdings of Type A for Stock 1 and Stock 2, respectively.

For $j = 1, 2$, we can recover the share holdings of Stock j at t by calculating quantities of $\mathbb{E}_t^A[dW_t^A \cdot dS_{j,t}]$, $\mathbb{E}_t^A[dS_{1,t} \cdot dS_{2,t}]$, and $\mathbb{E}_t^A[(dS_{j,t})^2]$ through simulations.

In the next section, we numerically compute the equilibrium prices and analyze the impact of circuit breakers.

5. Impact of Circuit Breakers

In this section, we examine the impact of circuit breakers on the dynamics of the market. The default parameter values for numerical analysis are set as follows, where daily growth rates and volatilities are used.[11] The algorithms used for the numerical analysis are presented in Appendix F.

$$\begin{array}{llll}
\mu_1^A = 0.10/250, & \sigma = 0.4, & \sigma_\delta = 0.5, & \\
k_1 = 0.1, & \delta_{1,0} = 0, & \bar{\delta}_1 = 0, & \\
\mu_2^A = 0.10/250, & \nu = -0.25, & \kappa^A = 1, & \\
k_2 = 0.1, & \delta_{2,0} = 1, & \bar{\delta}_2 = 1, & \nu_\delta = 0.5, \\
\gamma = 1, & \alpha = 0.07, & T = 1 \ \ (day). &
\end{array}$$

Because $\delta_{1,0} = 0$ and $\delta_{2,0} = 1$, Type B investors initially correctly estimate the expected growth rate of Dividend 1 and the jump intensity of Stock 2's dividend. Since our main goal is to examine the impact of circuit breakers in *bad* times when the market is volatile and the crash probability of some stocks is high (e.g., the U.S. market in the week of March 9, 2020 and the Chinese stock market in early January of 2016), we set the jump frequency high and the jump size large, along with a high volatility of Stock 1's dividend. Because of the CARA preferences, the initial share endowment of the investors does not affect the equilibrium. The circuit breaker is triggered when the sum of the two

[11] We have analyzed the impact using a wide range of parameter valuee and have obtained the same qualitative results.

prices (i.e., the index) first goes below the threshold $(1-\alpha)(S_{1,0}+S_{2,0})$, i.e., drops 7% from the initial value.

One alternative to the market-wide circuit breakers is to impose a circuit breaker separately on each stock (instead of on an index). With this separate circuit breaker on each stock, if a circuit breaker for a stock is triggered, only the trading in the corresponding stock is halted. For example, when the circuit breaker of Stock 1 is triggered, only the trading of Stock 1 is halted, but trading in Stock 2 is unaffected. Obviously, with separate circuit breakers, equilibrium prices remain independent, in sharp contrast to the case of market-wide circuit breakers. Let $S^{sep}_{j,t}, j = 1, 2$ denote the equilibrium prices of Stock j in this benchmark. We compare the impact of circuit breakers on the stock prices when they are on an index and when they are on individual stocks.

5.1 Equilibrium Prices

By Propositions 1 and 2, we obtain the initial equilibrium prices $\hat{S}_{1,0} = 0.8725, \hat{S}_{2,0} = 0.9703$ in the absence of circuit breakers. When there are separate circuit breakers on individual stocks, the equilibrium prices are $S^{sep}_{1,0} = 0.8719$ and $S^{sep}_{2,0} = 0.9577$, which are respectively lower than those without circuit breakers. In the presence of market-wide circuit breakers, we obtain the equilibrium prices $S_{1,0} = 0.8652$ and $S_{2,0} = 0.9418$. The prices of both stocks with separate circuit breakers are lower than those without circuit breakers, because market closure prevents risk sharing after the circuit breakers are triggered. In addition, with circuit breakers on an index, the prices are even lower. As we show later, this is because of the contagion effect of the circuit breakers on indices.

Figure 1 and Figure 2 depict sample paths of the equilibrium prices shortly before the circuit breaker are triggered. In Figure 1, the market is halted due to the continuous drop of Stock 1's price, while in Figure 2 the circuit breaker is trigged by a jump in Stock 2's price. In the first case, as Stock 1's price (the red line) approaches to the threshold and finally causes the index to drop enough to trigger the circuit breaker, Stock 2's price moves down to the corresponding equilibrium price at closure (the blue line). In the second case, when Stock 2's price crashes (jumps down), the circuit breaker is triggered, and Stock 1's price also crashes down to the equilibrium price at closure (the blue line). Figures 1 and 2 suggest circuit breakers may cause a positive correlation and a crash contagion between the two stocks, which we will investigate in details later.

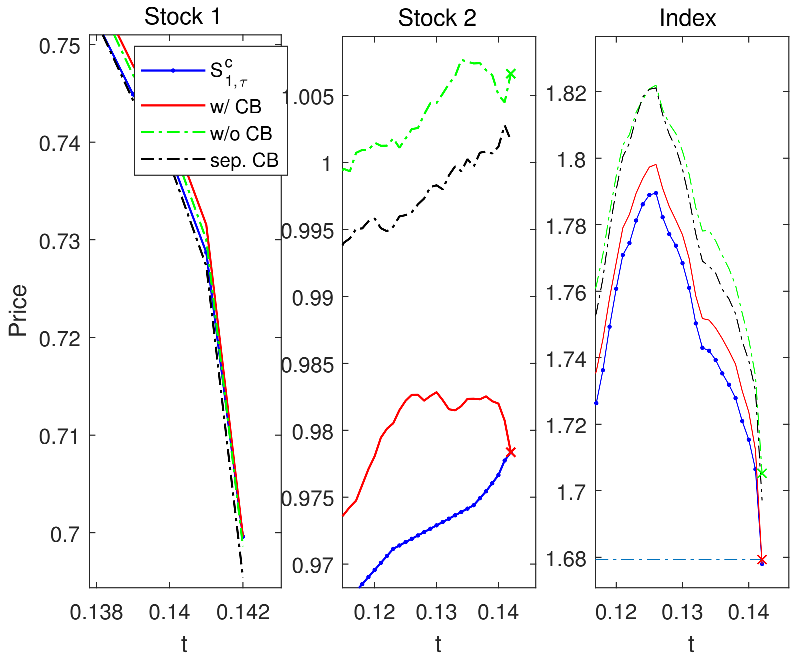

Figure 1. This figure depicts sample paths of the equilibrium prices during a period of time right before the market is halted due to continuous decreasing in Stock 1's dividend process.

5.2 Crash Contagion

Because the circuit breaker based on a stock index is triggered when the index reaches a threshold, a crash in a group of stocks (e.g., from a downward jump in their dividends) may trigger the circuit breaker and cause the entire market to be closed down. As a result, the prices of otherwise independent stocks may also jump down because of the

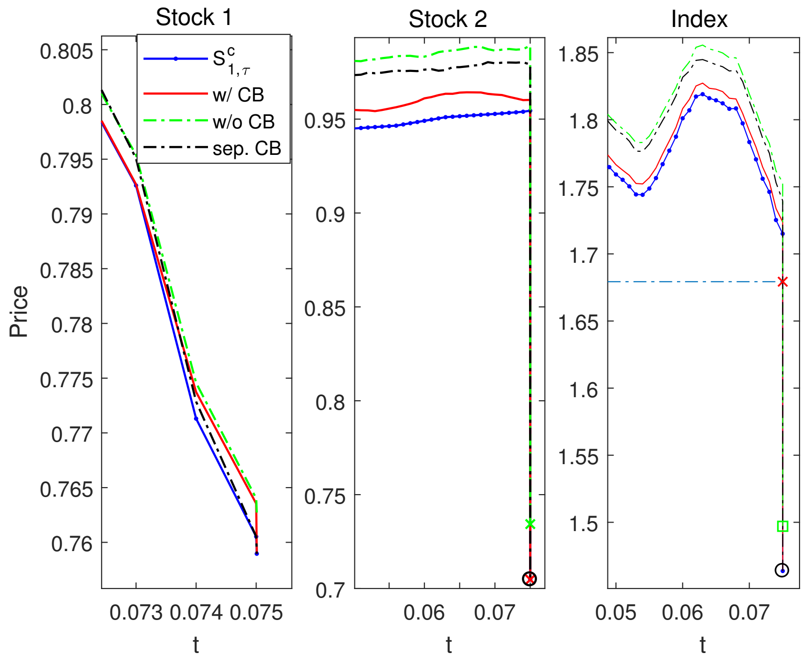

Figure 2. This figure depicts sample paths of the equilibrium prices during a period of time before the market is halted due to a jump in Stock 2's dividend process.

sudden market-wide closure. We refer this pattern of cross-stock serial crashes as *crash contagion*.

Let $\mathscr{C}$ denote the event that a crash in Stock 2's price triggers the circuit breakers at τ and Δt be a small time interval. In Figure 3, we plot the unconditional distribution of Stock 1's price change between τ and $\tau - \Delta t$ (blue line), the corresponding distribution conditional on the event $\mathscr{C}$ (red dashed line), and the conditional distribution of Stock 1's price change between τ and $\tau-$ (red line). The green line in Figure 3 shows that without circuit breakers, the price change of Stock 1 between τ and $\tau - \Delta t$, with or without a crash in Stock 2's price, is normally distributed. This implies that without circuit breakers, there is no contagion across stocks. In contrast, as the red dashed line in Figure 3 shows, in the presence of circuit breakers, after a crash of Stock 2 that triggers a circuit breaker, the distribution of Stock 1's price change between τ and $\tau - \Delta t$ shifts leftward significantly compared to the unconditional distribution. This distribution shift indicates crash contagion from Stock 2 to Stock 1 in the presence of circuit breakers. Figure 3 indicates that on average a crash of Stock 2 's price causes Stock 1 price to drop, suggesting a positive correlation between the returns of the two stocks when the market crashes. Recall that in the absence of circuit breakers, Stock 1 price is continuous and thus Stock 1's price change between τ and $\tau-$ is zero. The red line of Figure 3 implies that in the presence of circuit breakers, not only there is contagion but also a crash in Stock 2's price can cause a crash (jumping down) in Stock 1's price, i.e., circuit breakers can result in a crash contagion. This discontinuity in Stock 1's price is due to the discontinuous change in the value of the stock due to the sudden market closure. Stock 1's price jumps down because the market closure reduces risk sharing and thus increases the riskiness of the stock.

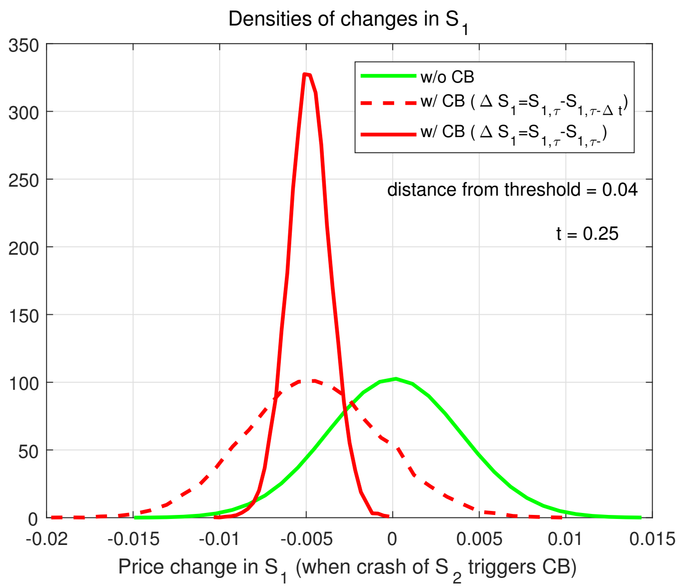

Figure 3. Distribution of changes in Stock 1's price when the circuit breaker is triggered by a jump in Stock 2's price. In the presence of a circuit breaker, the distribution is skewed negatively. Results for two methods of measuring the changes are presented. Meanwhile, in the absence of circuit breakers the price changes follows a normal distribution.

5.3 Increased Correlations and Correlation Asymmetry

With circuit breakers based on indices, a discrete jump (crash) in a stock is not necessary to adversely affect other otherwise independent stocks. Intuitively, even after a small decline in the price of a stock, the index gets closer to the circuit breaker threshold and thus the market is more likely to be closed early, which may lower the prices of other otherwise independent stocks, which in turn makes the index even closer to the circuit breaker threshold, entering into a vicious circle. This contagion magnitude is typically smaller than that caused by a crash in a stock in normal times, but can become much more significant and create strong correlations when the circuit breaker is close to being triggered because of the magnified vicious circle effect. We next show that a gradual change in the price of a stock can indeed affect the price of another stock and can also cause high correlations among otherwise independent stocks when the index gets close to the circuit breaker threshold.

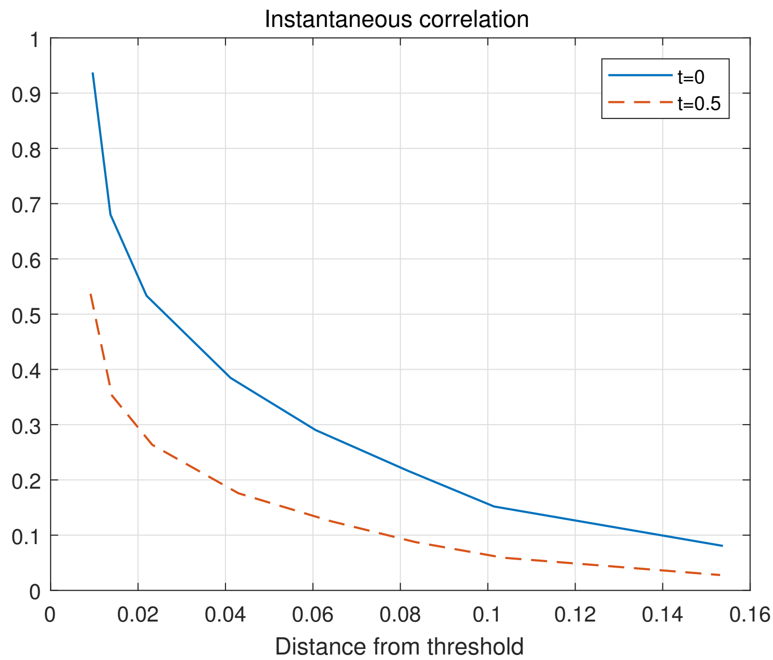

Figure 4. Instantaneous correlation.

Consistent with our intuition, Figure 4 shows that the correlation between the two prices with circuit breakers increases significantly as the index gets close to the threshold.[12] When the index is far from the threshold and thus a market closure is unlikely, the correlation becomes close to zero, because the correlation without circuit breakers is zero. In addition, when the potential market closure duration is large ($T - t$ is large), the impact of the circuit breakers on the correlation is even greater, because the fear of a market closure is stronger when the potential market closure duration is longer. For example, conditional on the same distance of 0.02 from the threshold, if it is later in the day at $t = 0.75$, the correlation is 0.2, but the correlation increases to about 0.55 if it is early in the day at $t = 0$.[13]

We further show that circuit breakers may change tail dependence of the stock returns and generate correlation asymmetry. Hong, Tu, and Zhou (2007) develop a model-free method to measure correlation asymmetry as follows. Let r_1 and r_2 be two random variables with zero mean and unit standard deviation. Define

$$\rho^+ = corr(r_1, r_2 | r_1 > c, r_2 > c), \rho^- = corr(r_1, r_2 | r_1 < -c, r_2 < -c),$$

where $c \geq 0$ can be interpreted as a multiplier of standard deviations. The correla-

[12] In the figure, "distance from threshold" is defined as the value of the index in exceed of the threshold. Because the equilibrium index level is determined jointly by the dividend levels of the two stocks, the way to vary the distance is not unique. In all the figures in this paper that plot against the distance to threshold we fix $D_{2,t}$ and vary $D_{1,t}$. We also used alternative ways such as fixing $D_{1,t}$ and varying $D_{2,t}$ and find similar results.

[13] So far the dividend processes are assumed to be uncorrelated and we show that a strong correlation of the stock prices can emerge due to circuit breakers. One concern may be that if the dividend processes are already correlated, then the additional correlation caused by the circuit breakers may be small and thus the effect of circuit breakers in increasing correlation may be small in practice. To address this concern, in an earlier version of the paper, we show that even when the dividends are correlated, the presence of circuit breakers can significantly increase the correlation of stock prices further. In addition, the presence of circuit breakers can even make negatively correlated stocks in the absence of circuit breakers become positively correlated. This reversal is because as the index gets close to the threshold, the common fear for market closure offsets the effect of the negatively correlated dividends and as a result the correlation turns positive. These results are not presented in the current version to save space, but available from the authors.

tion asymmetry is measured by $\rho^+ - \rho^-$. Given a set of observations of (r_1, r_2), correlation asymmetry can be tested for a single multiplier c or a vector of them, e.g., $c = (0, 0.5, 1, 1.5)$. We refer readers to Hong, Tu, and Zhou (2007) for more details of the statistical testing. Using our simulation data of the equilibrium stock returns, we report the testing results in Table 1. Table 1 shows that significant correlation asymmetries are present, indicating that the circuit breaker generates significantly greater tail dependence after a large drop in stock prices than after a large increase. Note that in the absence of circuit breakers, there is no correlation and thus there is no correlation asymmetry.

distance	c	0	0.5	1	1.5	J(stat.)	p-value
0.0211	$\rho^+ - \rho^-$	-0.0854	-0.1710	-0.3742	-0.5788	19.334	0.00068
0.0187	$\rho^+ - \rho^-$	-0.2145	-0.3662	-0.6147	-0.3224	82.96	$< 10^{-6}$

Table 1. This table reports correlation asymmetry testing results at different distances from the threshold. The existence of correlation asymmetry is confirmed, even more significant if the distance is smaller. The p-values are calculated by using the statistics (J) with a χ^2_3 distribution.

5.4 Volatility Contagion and Volatility Amplification

Next, we show that in addition to crash contagion, circuit breakers can also cause volatility contagion among otherwise independent stocks, i.e., an increase in the volatility of one stock can cause an increase in that of another. Figure 5 plots the instantaneous volatility of Stock 2 against that of Stock 1 for $t = 0$ and $t = 0.25$ when the index level is 0.01 above the circuit breaker threshold as we change the volatility of Stock 1's dividend. Figure 5 indicates that, indeed, a higher volatility of Stock 1's dividend can cause a higher volatility of Stock 2. Intuitively, the stock price contagion causes the volatility contagion. As explained above, after some stocks fall in prices, the index gets closer to the circuit breaker threshold, other stock prices also fall due to the fear of the more likely market closure, which in turn drives the index even closer to the threshold, and so on. This vicious cycle implies that as the price change of one stock becomes more volatile, so does the price change of the other, resulting in volatility contagion, especially when the index is close to the circuit breaker threshold. In addition, when the time to horizon is longer ($t = 0$), the effect of the circuit breakers is greater, and therefore the degree of contagion is larger as measured by the sensitivity of Stock 2's volatility change to Stock 1's volatility change (the red line slope).

One of the regulatory goals of the circuit breaker is to reduce market volatility in bad times. Because of the volatility contagion, we conjecture that contrary to regulators' intention, circuit breakers may increase the market volatility in bad times. Figure 6 plots the volatility of the index with circuit breakers against the index's distance from the circuit breaker threshold at two different time points $t = 0$ and $t = 0.5$. Figure 6 suggests that, indeed, contrary to the regulatory goal, circuit breakers can amplify the market volatility. This is because the vicious cycle effect described above can increase the sensitivity of stocks' prices to dividend shocks.

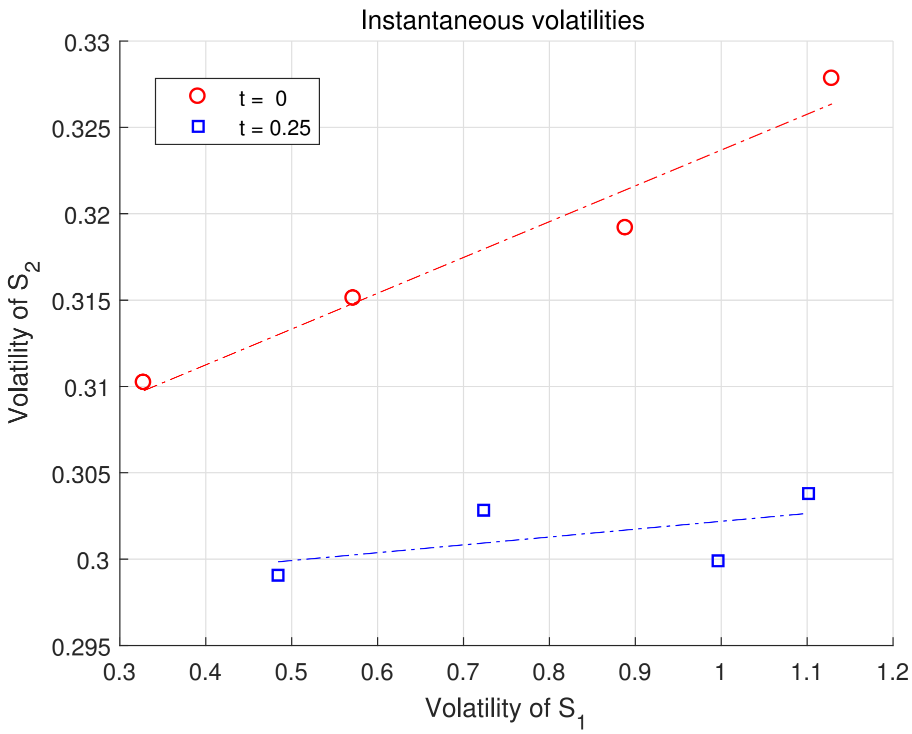

Figure 5. This figure shows that volatilities of Stocks 1 and 2 are correlated in the presence of circuit breakers.

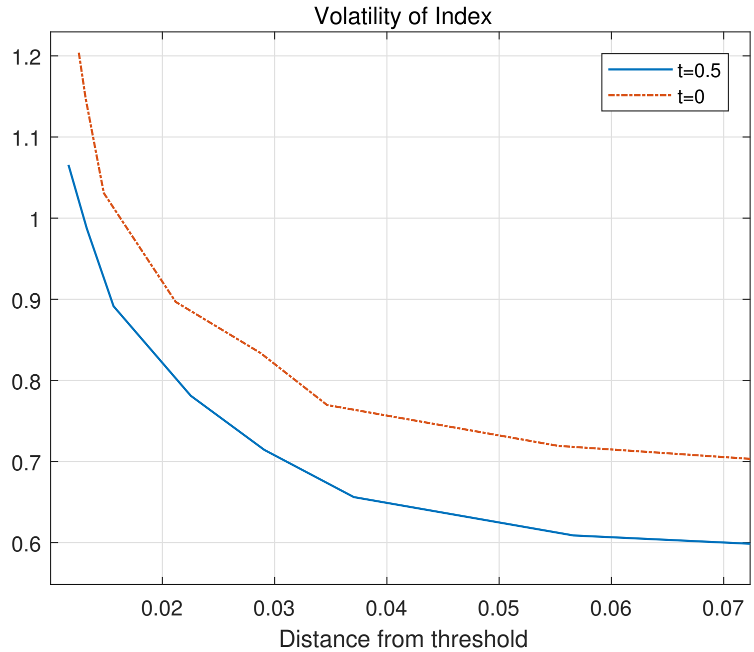

Figure 6. This figure plot the ratios of volatility with circuit breakers to that without circuit breakers against the distance from the circuit breaker threshold.

5.5 Acceleration of Market Decline: The Magnet Effect

Circuit breakers are implemented to protect the market from a fast decline. Contrary to this intention, Chen et al. (2023) show in a single-stock setting that circuit breakers can accelerate a stock price decline compared to the case without circuit breakers. This acceleration is what is called the "magnet effect" by Chen et al. (2023). However, it is not clear how the presence of multiple stocks affects this magnet effect. Our following results suggest that, in the presence of circuit breakers on stock indices, the probability of falling to the index threshold compared to the case without circuit breakers is also increased, so the magnet effect found by Chen et al. (2023) is robust to a multiple-stock setting.

Figure 7 shows the probabilities of reaching the circuit breaker index threshold in a given time interval (duration) with circuit breakers on the index (red line) and without circuit breakers (blue line). It suggests that the probability of falling to the index threshold when there is a circuit breaker on the index is higher than that without any circuit breakers. This is because with circuit breakers on indices, when one stock goes down, the distance to the circuit breaker threshold is shorter and the likelihood of an early market closure is greater. As a result, other stock prices tend to go down, which in turn drags the index further downward, resulting in a downward accelerating vicious circle, contrary to regulators' intention. Because of the contagion effect across stocks, the magnet effect in a model with multiple stocks like ours is stronger than that found in the single-stock setting of Chen et al. (2023), ceteris paribus. In addition, when the potential market closure length is longer (e.g., at $t = 0$), this magnet effect is even stronger.

Although the magnet effect is present in both Chen et al. (2023) and this paper, the main driving force of the magnet effect in our setting is different. The main driving force for the magnet effect in Chen et al. (2023) is the fear that one has to liquidate a levered position at the market closure time because after market closure, leverage is prohibited by the solvency requirement. In contrast, in this paper there is no change in the leverage level allowed before and after a market closure. Figure 8 shows that when a separate circuit breaker is imposed on Stock 1 as in Chen et al. (2023), the probability of reaching the circuit breaker threshold is almost the same as that in the absence of a circuit breaker. This implies that in our setting which does not restrict leverage, the magnet effect would be virtually zero without the contagion effect. This suggests that different from Chen et al. (2023), the driving force behind the magnet effect in our setting is the contagion effect of circuit breakers, instead of the leverage constraint effect in Chen et al. (2023).

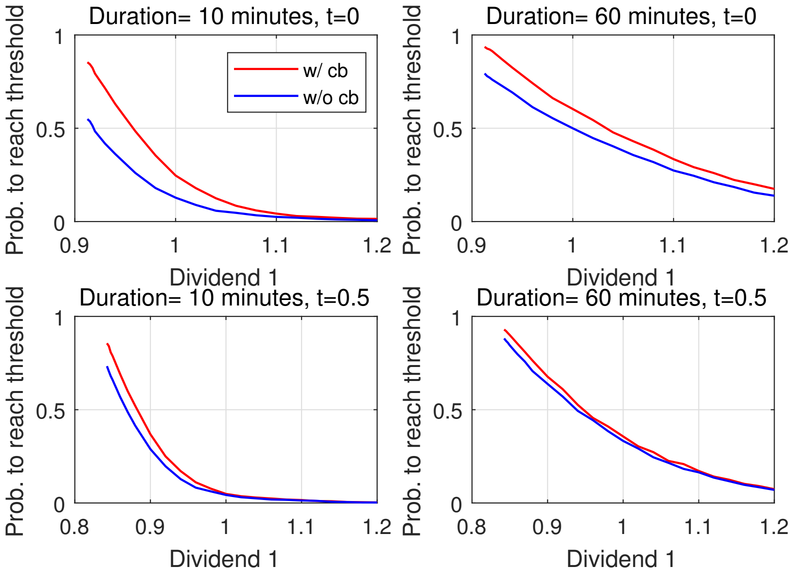

Figure 7. This figure shows the probability that prices will reach the threshold with or without a circuit breaker.

5.6 Benefits of Circuit Breakers

In this paper, we focus on the possible unintended bad effects of circuit breakers in bad states. To be clear, we are not arguing that circuit breakers are always counterproductive. On the contrary, there is no doubt that circuit breakers do have "bright sides" as recognized by a few of previous studies and our model also implies such bright sides when there is only moderate decline in the market.

As an example, given any distance of the index from the threshold, we simulate the next instant Stock 1 and Stock 2 prices and sort the index changes into quantiles with lowest quantile (≤ 0.2) representing the largest drop and the highest quantitle (≥ 0.8) representing the largest increase. In Figure 9, we plot the volatility of the index against the distance to the threshold for index changes in the lowest, the middle, and the highest quantiles. Figure 9 shows that index volatilities are reduced when the index changes moderately or increases sharply as the index approaches the circuit breaker threshold. Therefore, as a benefit of circuit breakers, they can decrease market volatility in normal

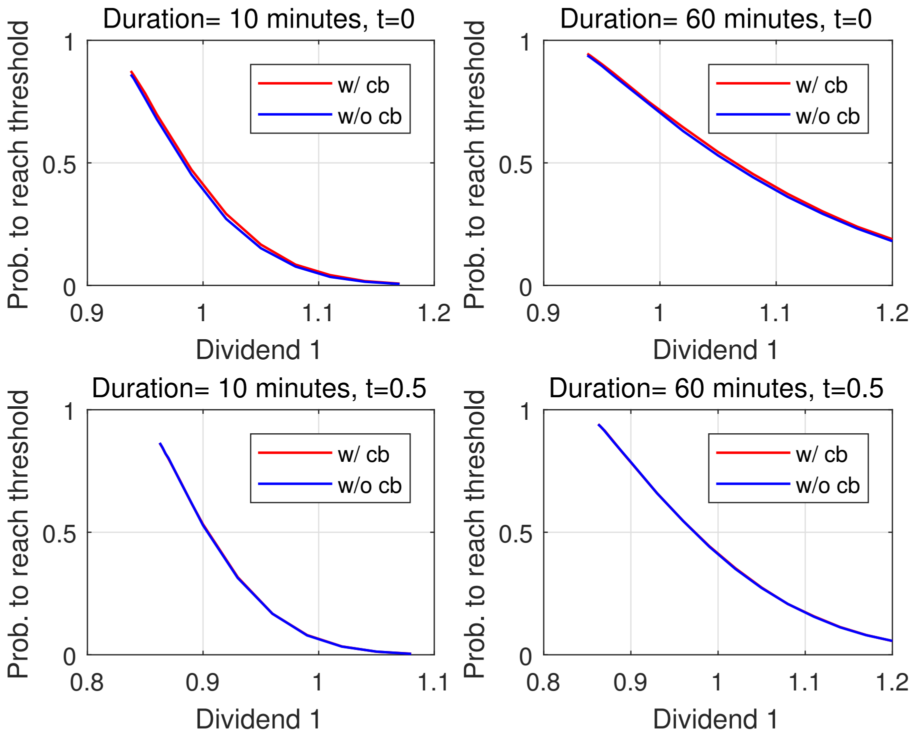

Figure 8. This figure shows the probability that prices will reach the threshold with or without a circuit breaker for Stock 1 with a separate circuit breaker.

or good times.[14]

6. Empirical Analysis

Our model has two main predictions: (1) With a market-wide circuit breaker, as market index gets closer to the circuit breaker threshold, the contagion across stocks gets stronger; and (2) The contagion with a market-wide circuit breaker is stronger than that without. In this section, we empirically test these predictions.

[14]However, large drops in the index cause increased volatilities, as found in the preceding subsections.

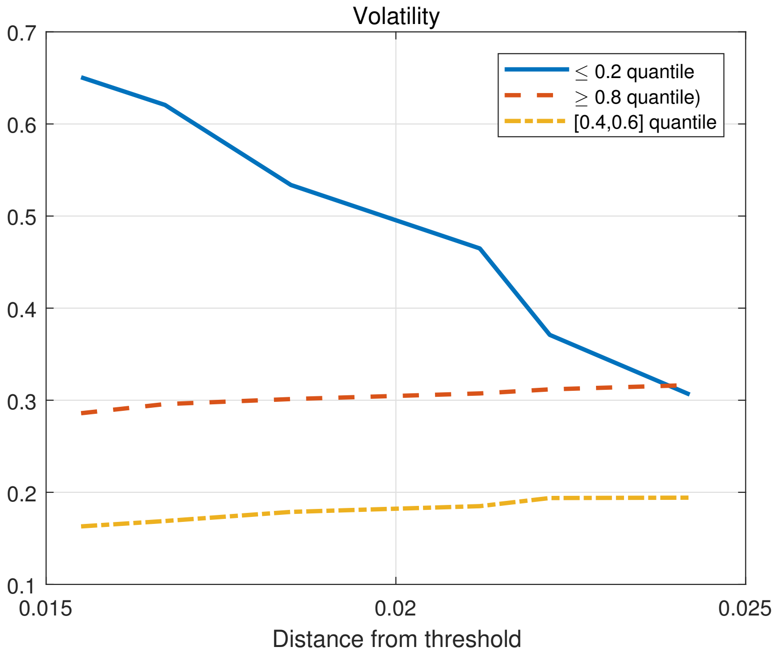

Figure 9. This figure depicts how correlation and volatility change for different change levels of the stock prices (the quantiles are sorted by changes in the index in the next instant for a given distance to the threshold).

6.1 Identification Strategy

It is inherently difficult to distinguish the circuit breaker effects from the general effects of market turbulence. To establish that it is the circuit breaker imposition that causes the change in the degree of contagion, we take advantage of a policy experiment in China in January, 2016. After the stock market tumbled in the second half of 2015, the CSRC introduced market-wide circuit breakers (CB) starting from January 1 in 2016. The Chinese CB rules mimic those in the United States, albeit with a much lower triggering threshold. The CB rule uses the CSI 300 Index (China Securities Index, a Chinese counterpart of the S&P 500 index) as reference and imposes a 15-minute trading halt when the index rises or falls by a threshold of 5% relative to the previous close level. After the initial halt, trading continues unless CSI 300's movement later reaches the 7% level, when the market will be closed for the day. The trading halt was triggered on

January 4, the very first trading day since the policy's official implementation, and later on January 7. After receiving harsh public criticisms, the CSRC suspended this policy from January 8 on. The sudden adoption and suspension of the CB policy provide a good setting to test our model implications. This approach complements (Chen et al., 2023) who construct a distance to circuit breakers threshold and examine aggregate market behavior along this distance over time.

Our empirical strategy is to compare the degree of market contagion across different periods that have different extent of impact from circuit breakers. Prior studies often consider a significant increase in the degree of co-movement between stock markets in different countries as the primary evidence of market contagion. We follow this logic but adapt such cross-country tests to suit our setting. As cautioned in Forbes and Rigobon (2001), some groups of countries (assets) are innately closely connected to each other, and therefore it would not be surprising to find a large negative shock propagating from one group to another. Therefore, we focus on groups of stocks that are least correlated with others during normal times. As in Chan, Lakonishok, and Swaminathan (2007), we find that utility stocks have substantially lower correlations with other stocks in normal times. Accordingly, we examine how utility stocks' co-movements with other industries' stocks manifest with and without circuit breakers. Specifically, we first compare stock co-movement in the period just before the trading halt with a period well before the trading halt as an intra-day test of prediction (1); we then compare stock co-movement on a day with circuit breakers with days without circuit breakers as an inter-day test of prediction (2). In addition to the simultaneous co-movement between utility stocks and other stocks, we further investigate whether other stocks' declines can lead to subsequent declines in utility stocks. We view an elevated correlation between other stocks' simultaneous/past returns and utility stocks' returns as evidence of market contagion.

To examine the contagion effect of circuit breakers across utility stocks and other stocks, we use the following regression specification:

$$
\begin{aligned}
\text{Return}_{8,j,t} =& \alpha + \beta_1 \text{Return}_{k,j,t} + \beta_2 \text{CB}_j + \beta_3 (\text{Return}_{k,j,t} * \text{CB}_j) \\
& + \beta_4 \text{Return}_{k,j,t-5} + \beta_5 (\text{Return}_{k,j,t-5} * \text{CB}_j) \\
& + \beta_6 \text{Return}_{k,j,t-10} + \beta_7 (\text{Return}_{k,j,t-10} * \text{CB}_j) + \epsilon, \quad k = 1, 2 \ldots, 7, 9 \ldots, 12
\end{aligned} \tag{R}
$$

In the above specification (R), the dependent variable $\text{Return}_{8,j,t}$ is the value-weighted utility industry return (Industry No.8 based on Fama-French 12-industry classification) at minute t on day j. We manually construct a mapping between FF12 and the CSRC's detailed industry classifications and use the end of 2015 market capitalization to compute portfolio weights in an industry. For explanatory variables, we include $\text{Return}_{k,j,t}$, the simultaneous return of the kth Fama-French industry ($k = 1, 2 \ldots, 7, 9 \ldots, 12$), to examine the co-movements between utility stocks and other stocks. We also include two lagged return variables $\text{Return}_{k,j,t-5}$ (5 minutes) and $\text{Return}_{k,j,t-10}$ (10 minutes) to test whether other industries' past returns relate to utility stocks' subsequent movements. We further interact a dummy variable CB with these three return variables to examine whether stock returns' simultaneous co-movements and lead-lag relations vary with the value of CB.[15]

Equation (R) is estimated using high-frequency trading data from Jinshuyuan.net (a Chinese stock trading data provider similar to Refinitiv). We first perform industry pairwise estimation by regressing utility stock returns on each of 11 industries' returns and obtain β coefficients for each industry. We construct a Wald-type test based on a more general test of linear restrictions on the regression parameters in linear models (Cameron and Trivedi , 2005, p.224). The null hypothesis is that the mean value of the

[15] As explained later, CB can capture the time distance from a CB halt or the presence of circuit breakers.

corresponding βs across 11 separate regressions is equal to zero. The Wald-type test statistic converges to the χ^2 distribution under the null hypothesis (Cameron and Trivedi , 2005, p.225).

6.2 Intra-day Evidence of Jan 4

The circuit breakers were triggered twice, on January 4 and 7, before the policy was suspended on January 8. On January 7, the circuit breaker was tripped only 12 minutes after market opened, which makes January 7 not suitable for testing prediction (1). Hence, we focus our analysis on January 4. On January 4, the CSI 300 dropped by 5% in the early afternoon, triggering a 15-minute trading halt at 13:13; shortly after when trading resumed at 13:28, the market tumbled again, hitting the 7% threshold at 13:34 and forced the market to close for the rest of the day. We analyze stock contagion during the period leading up to the trading halt. In particular, we divide the period before trading halt into two halves and rename the dummy variable CB in Equation (R) as a dummy variable "SH" (short for the second half) that equals one for the period immediately before the trading halt.

The results are presented in Table 2. Results in Column (1) indicate that utility stock returns are closely related with the simultaneous returns of other stocks. The average coefficient estimate of the interaction term is positive and statistically significant at 1% level. The economic magnitude is considerable: In the first 70 trading minutes on January 4, the average co-movement coefficient is 0.717; in the subsequent 73 minutes before the trading halt, the co-movement intensity increases by about 26.4% (0.189/0.717). Results in Columns (2) and (3) suggest that in the period further away from the trading halt, other stocks' past returns have very little (or even negative) association with subsequent returns of utility stocks. In contrast, in the period right before CB, both 5-minute and 10-minute lagged returns of other stocks are significantly positively associated with subsequent returns of utility stocks. In Column (4), we control for the simultaneous returns, both lagged returns, and the interaction terms. The results indicate that other stocks' past and simultaneous returns become statistically significantly associated with utility stock returns during the period right before CB. These results are consistent with model Prediction (1), namely, with CB in place, as a market index gets closer to the triggering threshold, contagion across stocks gets significantly stronger.

Table 2. Test for market contagion within the CB day of Jan 4

This table reports the results of estimating Equation R1 using data from January 4, 2016. The dependent variable is the value-weighted Fama-French utility industry return (FF8) at minute t. Explanatory variables include the returns of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous return), $t - 5$ (Lagged return 5 min), and $t - 10$ (Lagged return 10 min). We divide the trading period before CB into two halves and SH is a dummy variable that equals one for the second half right before the CB. We interact this dummy variable with the three return variables to test whether market contagion intensifies when the time draws closer to the triggering of CB. We regress utility stock returns on the simultaneous and lagged returns for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Returns of FF8			
	(1)	(2)	(3)	(4)
Simultaneous return	0.717***	0.732***	0.743***	0.747***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous return*SH	0.189***	0.177***	0.156***	0.156***
	(0.001)	(0.001)	(0.001)	(0.001)
SH (Second Half)	−0.0001**	−0.0001	0.0001	0.0001
	(0.016)	(0.379)	(0.698)	(0.103)
Lagged return (5 min)		0.005		−0.005
		(0.755)		(0.755)
Lagged return (5 min)*SH		0.080***		0.087***
		(0.005)		(0.002)
Lagged return (10 min)			−0.001	0.003
			(0.951)	(0.824)
Lagged return (10 min)*SH			0.239***	0.233***
			(0.001)	(0.001)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	143	137	132	132
Avg. R^2	0.739	0.75	0.764	0.77

6.3 Inter-day Evidence

6.3.1 CB versus Pre-CB and Post-CB Controls

Our model Prediction (2) implies that market contagion is stronger with CB than without. As the circuit breaker policy was first introduced on January 4 and later suspended from January 8 on, we use the two adjacent days: December 31, 2015, the last trading day prior to the initiation, and January 8, the first trading day since the suspension of the policy, as control groups. Accordingly, in regression (R) whose results are reported in Table 3, the dummy variable of CB equals one for January 4, and zero for December 31, 2015 and January 8, 2016.

Results in Column (1) in Table 3 show that the return co-movements between utility stocks and other stocks are significantly greater on January 4, the day with CB, than the days without CB. The point estimates indicate a 55.8% (0.327/0.585) increase from December 31 to January 4. Moreover, results in Columns (2) and (3) show a sharp contrast with and without CB in the lead-lag relations of stocks: with CB in place, other stocks' past returns propagate to utility stocks' subsequent returns, consistent with the market contagion prediction, while without CB, the propagation is much weaker or even negative. For instance, in Column (2), the coefficient of 5-minute lagged return is -0.016 but the interaction term is 0.105 with a statistical significance at 1% level. This evidence suggests that other stocks' 5-minute past returns are inversely related to utility stocks' subsequent returns on December 31 and January 8, but are positively related to utility stocks' subsequent returns on January 4. When we move our attention to Column (4), the previous conclusions continue to hold. Both the simultaneous co-movements and the lead-lag relations between utility stocks and other stocks are significantly elevated on a day when CB is implemented.

Table 3. Comparing market contagion with pre-CB and post-CB controls

This table reports the results of estimating Equation (R) combining data from January 4, 2016 (CB) with December 31, 2015 and January 8, 2016 data (Controls). The dependent variable is the value-weighted Fama-French utility industry return (FF8) at minute t. Explanatory variables include the returns of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous return), $t-5$ (Lagged return 5 min), and $t-10$ (Lagged return 10 min). CB is a dummy variable that equals one for January 4 and zero for December 31 and January 8. We interact this dummy with the three return variables to test whether market contagion intensifies when circuit breakers are triggered. We regress utility stock returns on the simultaneous and lagged returns for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Returns of FF8			
	(1)	(2)	(3)	(4)
Simultaneous return	0.585***	0.583***	0.587***	0.585***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous return*CB	0.327***	0.331***	0.316***	0.329***
	(0.001)	(0.001)	(0.001)	(0.001)
CB	−0.0001***	−0.0001***	−0.0001***	−0.0001**
	(0.001)	(0.001)	(0.001)	(0.016)
Lagged return (5 min)		−0.016**		−0.015*
		(0.046)		(0.058)
Lagged return (5 min)*CB		0.105***		0.098***
		(0.001)		(0.001)
Lagged return (10 min)			−0.001	−0.003
			(0.875)	(0.636)
Lagged return (10 min)*CB			0.240***	0.239***
			(0.001)	(0.001)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	445	442	439	439
Avg. R^2	0.689	0.694	0.704	0.709

6.3.2 CB versus Eight Days with Extraordinary Market Decline

One might worry that December 31, 2015 and January 8, 2016 are not perfect counterfactuals as the stock market only experienced moderate drops on these two days. To alleviate this concern, we choose all the days when the CSI 300 index dropped over 5% since 2016 as another group of controls. These eight days include January 11, January 26, February 25 of 2016, February 9 and October 11 of 2018, May 6 of 2019, February 3 of 2020, and April 25 of 2022. This new choice of counterfactual allows us to estimate what would have followed in the market when the CSI drops below 5% in the absence of circuit breakers. Combining these eight non-CB days with the treatment day of January 4, we re-estimate Equation (R) and report the results in Table 4.

The results in Table 4 bear a close resemblance to Table 3. Results in Column (1) indicate that utility and other stocks' simultaneous co-movements are about 25.7% (0.186/0.723) greater on January 4 than those eight days when stock markets dropped over 5%. It is worth noting that on those days with extraordinary market declines, other industries' 5 and 10 minutes past returns are both negatively associated with utility stocks' subsequent returns yet on the day with CB, this lead-lag relations are significantly positive. This evidence is consistent with the quantitative significance of market contagion caused by the circuit breakers.

6.3.3 Evidence Based on the Entire Week of CB

The above analysis compares the CB day of January 4 with various sets of controls. The short-lived policy actually lasted four days from January 4 to January 7, although circuit breakers were not triggered on January 5 and 6. In this section, we analyze whether market contagion behavior changes from the week before CB to the week after CB. To this end, we pool data for all four days (CB Week) and compare market contagion behavior with four trading days before January 4 (December 28, 29, 30 and 31 of 2015)

Table 4. Comparing market contagion with eight days of extraordinary market decline

This table reports the results of estimating Equation (R) using data on January 4 (CB) and eight days when the CSI dropped over 5% post-CB (Controls). The dependent variable is the value-weighted Fama-French utility industry return (FF8) at minute t. Explanatory variables include the returns of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous return), $t - 5$ (Lagged return 5 min), and $t - 10$ (Lagged return 10 min). CB is a dummy variable that equals one for January 4 and zero for those eight days of extraordinary market decline. We interact this dummy with the three return variables to test whether market contagion intensifies when circuit breakers are triggered. We regress utility stock returns on the simultaneous and lagged returns for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Returns of FF8			
	(1)	(2)	(3)	(4)
Simultaneous return	0.723***	0.722***	0.723***	0.721***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous return*CB	0.186***	0.189***	0.177***	0.180***
	(0.001)	(0.001)	(0.001)	(0.001)
CB	−0.0001***	−0.0001***	−0.0001***	−0.0001***
	(0.001)	(0.001)	(0.001)	(0.001)
Lagged return (5 min)		−0.013***		−0.015***
		(0.003)		(0.001)
Lagged return (5 min)*CB		0.100***		0.095***
		(0.001)		(0.001)
Lagged return (10 min)			−0.006	−0.008**
			(0.110)	(0.045)
Lagged return (10 min)*CB			0.243***	0.242***
			(0.001)	(0.001)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	1523	1521	1519	1519
Avg. R^2	0.651	0.652	0.653	0.654

and four trading days after January 7 (January 8, 11, 12, and 13 of 2016). We re-estimate Equation (R) and report the results in Table 5. Overall, the results are quite similar to those in the previous tables. These results confirm that market contagion is significantly greater when the CB policy is in place.

Table 5. Market contagion analysis based on the week of CB

This table reports the results of estimating Equation (R) using data from January 4 to January 7 (CB week) and four days before January 4 and four days after January 7. The dependent variable is the value-weighted Fama-French utility industry return (FF8) at minute t. Explanatory variables include the returns of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous return), $t-5$ (Lagged return 5 min), and $t-10$ (Lagged return 10 min). CBW (CB Week) is a dummy variable that equals one for the four days between January 4 and January 7, zero for those eight days surrounding January 4 and January 7. We interact this dummy with the three return variables to test whether market contagion intensifies when circuit breaker policy is in place. We regress utility stock returns on simultaneous and lagged returns for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Returns of FF8			
	(1)	(2)	(3)	(4)
Simultaneous return	0.783***	0.778***	0.783***	0.778***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous return*CBW	0.135***	0.156***	0.128***	0.149***
	(0.001)	(0.001)	(0.001)	(0.001)
CBW (CB Week)	−0.0001***	−0.0001***	−0.0001***	−0.0001***
	(0.001)	(0.001)	(0.001)	(0.001)
Lagged return (5 min)		−0.025***		−0.023***
		(0.001)		(0.001)
Lagged return (5 min)*CBW		0.076***		0.083***
		(0.001)		(0.001)
Lagged return (10 min)			0.012***	0.023***
			(0.001)	(0.003)
Lagged return (10 min)*CBW			0.021***	0.090***
			(0.001)	(0.001)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	1899	1894	1889	1889
Avg. R^2	0.831	0.832	0.831	0.833

6.4 Evidence from the United States

One might worry about that our results might be only valid for the Chinese market. To address this concern, we present evidence based on the occurrences of circuit breaker trips in the United States amid COVID-19. Market-wide circuit breakers were mandated by the U.S. Securities and Exchange Commission (SEC) in 1988 to prevent market crashes such as the Black Monday of 1987, when the Dow Jones Industrial Average (DJIA) plunged 22.6%. Currently, the circuit breakers can be triggered at three thresholds which measure a decrease from the previous day's closing price of the S&P 500 index – 7% (level one), 13% (level two), and 20% (level three). In March 2020, level one circuit breakers were triggered four times, each causing a 15-minute trading halt. On March 9, 12 and 16, the level one circuit breakers were triggered within the first few minutes after the market opened in the morning, while the fourth occurred at 12:56:17 PM on March 18.

In this section, we focus our attention on March 18, because the first three trading halts occurred very early in the morning. We use tick history data from Refinitiv to perform our analysis. In the intra-day test, we examine market contagion behavior in the window leading up to the trading halt. In the inter-day test, we compare market contagion between March 18 and March 19, another turbulent day with a daily swing over 6% but without triggering the CB. The intra-day results are presented in Table 6. The results are similar to the results in Table 2 with simultaneous stock co-movements experiencing a 18% to 23% increase in the period immediately before the trading halt. Also, other stocks' 5-minute lagged returns exhibit significantly greater association with utility stocks' subsequent returns. We then turn to the inter-day test results in Table 7 when we compare market contagion with March 19. The previous conclusion remains unchanged and the economic magnitude of estimate is comparable. The simultaneous stock co-movements on March 18 are about 20% greater than those on March 19. Moreover, other stocks' 10-minute lagged returns experience an elevated association with utility stocks' subsequent returns on March 18.

In summary, we find consistently supporting evidence for our model predictions of contagion in both China and the United States markets.

Table 6. Test for U.S market contagion within the CB day of March 18

This table reports the results of estimating Equation R1 using U.S. tick history data on March 18, 2020. The dependent variable is the value-weighted Fama-French utility industry return (FF8) at minute t. Explanatory variables include the returns of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous return), $t - 5$ (Lagged return 5 min), and $t - 10$ (Lagged return 10 min). We divide the trading period before CB into two halves and SH is a dummy variable that equals one for the second half right before the CB. We interact this dummy variable with the three return variables to test whether market contagion intensifies when the time draws closer to the triggering of CB. We regress utility stock returns on the simultaneous and lagged returns for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Returns of FF8			
	(1)	(2)	(3)	(4)
Simultaneous return	0.788***	0.775***	0.786***	0.775***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous return*SH	0.057	0.142***	0.162***	0.175***
	(0.178)	(0.001)	(0.001)	(0.001)
SH (Second Half)	0.0001	0.0001***	0.0001***	0.0001***
	(0.308)	(0.001)	(0.001)	(0.001)
Lagged return (5 min)		−0.114***		−0.111***
		(0.001)		(0.001)
Lagged return (5 min)*SH		0.081**		0.112***
		(0.041)		(0.005)
Lagged return (10 min)			0.031	0.014
			(0.221)	(0.547)
Lagged return (10 min)*SH			0.013	0.031
			(0.742)	(0.414)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	206	201	196	196
Avg. R^2	0.481	0.503	0.521	0.526

Table 7. Comparing U.S. market contagion between March 18 and March 19

This table reports the results of estimating Equation (R) using U.S. tick history data on March 18 (CB) and March 19 (Non-CB). The dependent variable is the value-weighted Fama-French utility industry return (FF8) at minute t. Explanatory variables include the returns of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous return), $t - 5$ (Lagged return 5 min), and $t - 10$ (Lagged return 10 min). CB is a dummy variable that equals one for March 18 and zero for March 19. We interact this dummy with the three return variables to test whether market contagion intensifies when circuit breakers are triggered. We regress utility stock returns on the simultaneous and lagged returns for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Returns of FF8			
	(1)	(2)	(3)	(4)
Simultaneous return	0.747***	0.747***	0.748***	0.749***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous return*CB	0.153***	0.150***	0.152***	0.150***
	(0.001)	(0.001)	(0.001)	(0.023)
CB	0.0001**	0.0001*	0.0001**	0.0001**
	(0.035)	(0.051)	(0.019)	(0.028)
Lagged return (5 min)		−0.017		−0.019
		(0.260)		(0.219)
Lagged return (5 min)*CB		−0.022		−0.012
		(0.386)		(0.619)
Lagged return (10 min)			0.028**	0.028**
			(0.044)	(0.042)
Lagged return (10 min)*CB			0.046*	0.044*
			(0.051)	(0.062)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	377	372	367	367
Avg. R^2	0.551	0.552	0.554	0.555

6.5 Volatility Increase and Volatility Contagion

Our model also predicts that (1) as an index gets closer to the CB threshold, the market volatility increases; and (2) a volatility changes in one group of stocks can spill over to other groups of stocks (volatility contagion). In this section, we provide some supporting evidence of such predictions using the Chinese data. First, we focus on the market volatility in the period leading up to the trading halt on January 4. In Figure 10, we plot the CSI 300's volatility (in 5-minute intervals) against the market index's distance to the triggering threshold of CB. In the figure, we add a dashed, curve line using the fitted value from a log-linear regression. The market volatility path reveals that market volatility gradually rises as the distance inches closer to the triggering threshold, consistent with model prediction (1).

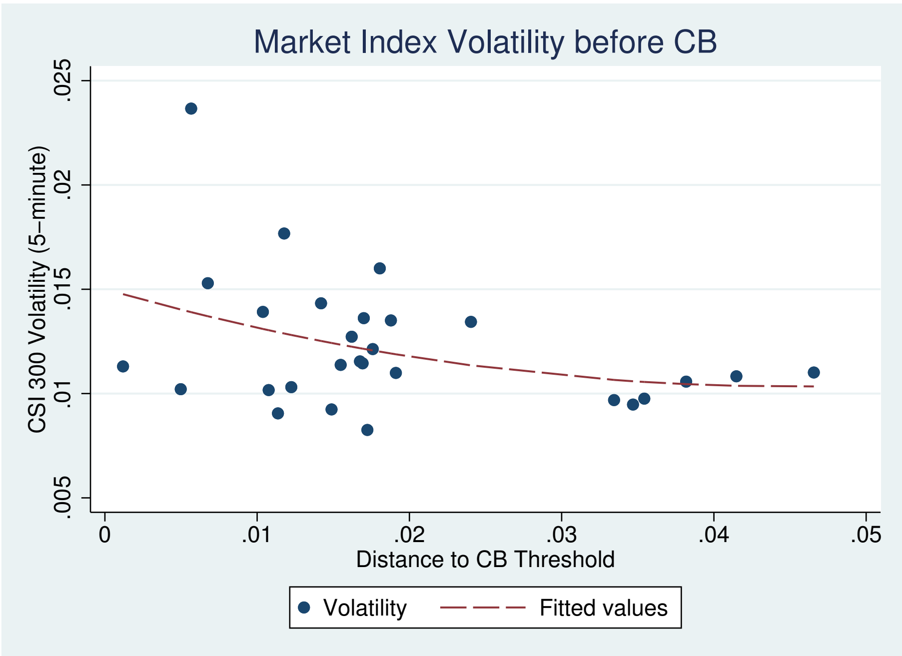

Figure 10. This figure plots the volatility of China's stock market against market index's distance to CB triggering threshold on January 4, 2016.

We next estimate the same empirical specification of Equation (R) with returns replaced by volatilities to explore whether other stocks' volatility propagate to utility stocks. Results in Table8 show that other stocks' 5-minute past volatility and simultaneous volatility are positively associated with volatility of utility stocks. This evidence is consistent with our model prediction that cross-stock volatility contagion rises when CB policy is in place.

Table 8. Comparing volatility contagion with eight days of extraordinary market decline

This table reports the results of estimating Equation (R) with returns replaced by volatilities using data on January 4 (CB) and eight days when the CSI 300 dropped over 5% post-CB. The dependent variable is the Fama-French utility industry's volatility (FF8) at minute t. Explanatory variables include the volatilities of the remaining Fama-French 11 industries $(1, 2 \ldots, 7, 9 \ldots, 12)$ at minute t (Simultaneous volatility), $t-5$ (Lagged volatility 5 min), and $t-10$ (Lagged volatility 10 min). CB is a dummy variable that equals one for January 4 and zero for those eight days of extraordinary market decline. We interact this dummy with the three volatility variables to test whether volatility contagion intensifies when circuit breakers are triggered. We regress utility stock volatility on the simultaneous and lagged volatilities for each of the remaining 11 industries separately and report the mean value of the coefficient estimates. P values from χ^2 tests for the statistical significance of this mean value are presented in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

	Volatility of FF8			
	(1)	(2)	(3)	(4)
Simultaneous volatility	0.502***	0.473***	0.488***	0.469***
	(0.001)	(0.001)	(0.001)	(0.001)
Simultaneous volatility*CB	0.274***	0.212***	0.262***	0.196***
	(0.001)	(0.001)	(0.001)	(0.001)
CB	−0.002***	−0.007***	−0.005***	−0.009***
	(0.001)	(0.001)	(0.001)	(0.001)
Lagged volatility (5 min)		0.107***		0.092***
		(0.001)		(0.001)
Lagged volatility (5 min)*CB		0.296***		0.295***
		(0.001)		(0.001)
Lagged volatility (10 min)			0.079***	0.051***
			(0.001)	(0.001)
Lagged volatility (10 min)*CB			0.143**	0.152***
			(0.012)	(0.004)
Intercept	Yes	Yes	Yes	Yes
Avg. Obs.	1523	1521	1521	1519
Avg. R^2	0.836	0.848	0.840	0.850

7. Conclusion

We present a continuous-time equilibrium model that incorporates multiple stocks to investigate the impact of circuit breakers on joint stock price dynamics and cross-stock contagion. Contrary to the intended regulatory objectives of circuit breakers, our findings reveal that while they can be effective in normal market conditions by dampening volatility, they can have unintended consequences during periods of significant market downturns.

Specifically, our research demonstrates that circuit breakers can contribute to crash contagion, volatility contagion, heightened volatilities, and increased correlations among stocks that are otherwise independent. These results are supported by our comprehensive empirical analysis, which incorporates data from both Chinese and US markets. Notably, our analysis indicates that circuit breaker rules may have exacerbated international market declines triggered by the COVID-19 pandemic due to their contagion effects. Consequently, market-wide circuit breakers have the potential to propagate financial contagion, transforming idiosyncratic risks into systemic risks, particularly during adverse market conditions.

To address these concerns, we propose an alternative circuit breaker rule based on individual stock returns rather than market indices. By implementing this revised rule, we can mitigate the aforementioned issues associated with circuit breakers and establish a more effective framework for managing market volatility while reducing contagion risks.

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Appendix

A Price of Stock 1: Without Circuit Breakers

We assume that the disagreement process $\delta_{1,t}$ is stochastic and follows Equation (6). When there are no circuit breakers, the equilibrium price of Stock 1 is independent of Stock 2 because of independent dividend processes. The price of Stock 1 can be obtained in closed-form as follows.

We first evaluate $\mathbb{E}_t^A[\pi_{1,T}^A]$. Ignoring constants, we need to calculate

$$\mathbb{E}_t^A[\eta_{1,T}^{1/2} e^{-\frac{\gamma}{2} D_{1,T}}] = \mathbb{E}_t^A[e^{Y_{1,T}}] \cdot f(t),$$

where $f(t)$ is a deterministic function and,

$$Y_{1,T} = \int_0^T (\frac{\delta_s}{2\sigma} - \frac{\gamma\sigma}{2}) dZ_s + \int_0^T (-\frac{\delta_s^2}{4\sigma^2}) ds.$$

To simplify notation, in the rest of Appendix A, we use δ_t, k, and $\bar{\delta}$ to denote $\delta_{1,t}$, k_1, and $\bar{\delta}_1$ respectively.

Conjecture $F(t, y, \delta, \delta^2) = e^{A(t)+B(t)y+C(t)\delta+\frac{H(t)}{2}\delta^2} = \mathbb{E}^A[e^{Y_T}|Y_t = y, \delta_t = \delta]$, with $A(T) = C(T) = H(T) = 0$ and $B(T) = 1$. Substituting the conjecture into the moment generating function of the process (Y_t, δ_t) and collecting the coefficients of y, δ, δ^2 and constants, we obtain four ordinary different equations:

$$\begin{aligned}
A'(t) + \frac{1}{8}\gamma^2\sigma^2 B(t)^2 + k\bar{\delta}C(t) + \frac{\sigma_\delta^2}{2}(C(t)^2 + H(t)) - \frac{\gamma\sigma\sigma_\delta}{2}B(t)C(t) &= 0, \\
B'(t) &= 0, \\
C'(t) - \frac{\gamma}{4}B(t)^2 + k\bar{\delta}H(t) - kC(t) + C(t)H(t)\sigma_\delta^2 + \frac{\sigma_\delta}{2\sigma}B(t)C(t) - \frac{\gamma\sigma\sigma_\delta}{2}B(t)H(t) &= 0, \\
\frac{H'(t)}{2} - \frac{1}{4\sigma^2}B(t) + \frac{B(t)^2}{8\sigma^2} - kH(t) + \frac{\sigma_\delta^2}{2}H(t)^2 + \frac{\sigma_\delta B(t)H(t)}{2\sigma} &= 0.
\end{aligned}$$

The solution of the ODE system is obtained as follows.

$$
\begin{aligned}
B(t) &= 1, \\
H(t) &= \frac{e^{(D^+-D^-)\sigma_\delta^2(t-T)} - 1}{e^{(D^+-D^-)v^2(t-T)}D^- - D^+} D^+D^-, \\
C(t) &= \int_t^T e^{\int_t^s f(x)ds} g(s)ds = \frac{1}{\Delta(D^- - D^+e^{2\Delta(T-t)})} \\
&\quad \cdot \Big(-\frac{\gamma}{4}((D^+ + D^-)e^{\Delta(T-t)} - D^+e^{2\Delta(T-t)} - D^-) - (k\bar{\delta} - \frac{\sigma\sigma_\delta\gamma}{2})D^+D^-(e^{\Delta(T-t)} - 1)^2 \Big), \\
A(t) &= \int_T^t (-\frac{1}{8}\gamma^2\sigma^2 - k\bar{\delta}C(s) - \frac{\sigma_\delta^2}{2}(C(s)^2 + H(s)) + \frac{\gamma}{2}\sigma_\delta\sigma C(s))ds,
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta =& \sqrt{k^2 + \frac{\sigma_\delta^2}{2\sigma^2} - \frac{\sigma_\delta k}{\sigma}}, \\
D^\pm =& \frac{k - \frac{\sigma_\delta}{2\sigma} \pm \sqrt{k^2 + \frac{\sigma_\delta^2}{2\sigma^2} - \frac{\sigma_\delta k}{\sigma}}}{\sigma_\delta^2}, \\
f(t) =& -k + \sigma_\delta^2 H(t) + \frac{\sigma_\delta}{2\sigma}, \\
g(t) =& -\frac{\gamma}{4} + k\bar{\delta}H(t) - \frac{\gamma\sigma\sigma_\delta}{2}H(t).
\end{aligned}
$$

Then

$$
\mathbb{E}_t^A[e^{Y_T}] = F(t, y, \delta, \delta^2; \gamma) = e^{A(t)+B(t)y+C(t)\delta+\frac{H(t)}{2}\delta^2}.
$$

Next, we consider the first derivative of F with respect to γ to obtain $\mathbb{E}_t^A[e^{Y_T}Z_T]$. We define

$$
A(t;\gamma) = A(t), C(t;\gamma) = C(t).
$$

Note that

$$
\begin{aligned}
\frac{dB(t)}{d\gamma} =& \frac{dH(t)}{d\gamma} = 0, \\
\frac{dC(t;\gamma)}{d\gamma} =& \int_t^T e^{\int_t^s f(x)dx}[-\frac{1}{4} - \frac{\sigma\sigma_\delta}{2}H(s)]ds, \\
\frac{dA(t;\gamma)}{d\gamma} =& \int_T^t (\frac{-\sigma^2\gamma}{4} - k\bar{\delta}\frac{dC(s;\gamma)}{d\gamma} - \sigma_\delta^2 C(s;\gamma)\frac{dC(s;\gamma)}{d\gamma} + \frac{\sigma_\delta\sigma}{2}C(s;\gamma) + \frac{\gamma\sigma_\delta\sigma}{2}\frac{dC(s;\gamma)}{d\gamma})ds.
\end{aligned}
$$

Hence

$$
\mathbb{E}_t^A[e^{Y_T}Z_T] = -\frac{2}{\sigma}\frac{d}{d\gamma}\mathbb{E}_t^A[e^{Y_T}] = -\frac{2}{\sigma}\frac{d}{d\gamma}F(t, y, \delta, \delta^2; \gamma).
$$

Finally, the stock price in the equilibrium is given by

$$\begin{aligned}
\hat{S}_{1,t} &= \frac{\mathbb{E}_t^A[\pi_{1,T}^A D_{1,T}]}{\mathbb{E}_t^A[\pi_{1,T}^A]} = \frac{\mathbb{E}_t^A[\pi_{1,T}^A D_{1,T}]}{F} = D_{1,0} + \mu_1^A T - 2\frac{\frac{dF}{d\gamma}}{F} \\
&= D_0 + \mu_1^A T - 2(\frac{dA(t;\gamma)}{d\gamma} + \frac{dy}{d\gamma} + \frac{dC(t;\gamma)}{d\gamma}\delta_t) \\
&= D_{1,0} + \mu_1^A T - 2(\frac{dA(t;\gamma)}{d\gamma} - \frac{\sigma}{2}Z_t + \frac{dC(t;\gamma)}{d\gamma}\delta_t).
\end{aligned}$$

The last equality above holds because $Y_t = \int_0^t (\frac{\delta_s}{2\sigma} - \frac{\gamma\sigma}{2})dZ_s + \int_0^t (-\frac{\delta_s^2}{4\sigma^2})ds$ and $Y_t = y$ yield $dy/d\gamma = -1/2\sigma Z_t$. By $D_{1,t} = D_{1,0} + \mu_1^A t + \sigma Z_t$ (μ_1^A is constant), we obtain

$$\hat{S}_{1,t} = D_{1,t} + \mu_1^A (T-t) - 2(\frac{dA(t;\gamma)}{d\gamma} + \frac{dC(t;\gamma)}{d\gamma}\delta_t). \tag{A.1}$$

In case δ_t is constant, i.e., $\sigma_\delta = k = 0$ and $\delta_t \equiv \delta_0$, we find that $dA(t)/d\gamma = -\sigma^2\gamma/4(t-T)$ and $dC(t;\gamma)/d\gamma = -1/4(T-t)$. Thus, $\hat{S}_{1,t} = D_{1,t} + \mu_1^A(T-t) + (\delta_0/2 - \sigma^2\gamma/2)(T-t)$. This is the equilibrium price of Stock 1 in the case of constant disagreement.

Since $H(t) \to 0$ as $t \to T$, we see that $dC(t;\gamma)/d\gamma$ is negative when $T-t$ is small. Thus, it follows (A.1) that the instantaneous volatility of the stock price $\sigma_{\hat{S}} = \sigma - 2\frac{dC(t;\gamma)}{d\gamma}\sigma_\delta$ is greater than the dividend volatility σ when $T-t$ is small, given σ_δ is positive.

B Market Clearing Prices

B.1 Stock 1: Stochastic Disagreement

In the presence of circuit breakers, we cannot obtain the equilibrium price of Stock 1 directly. In this section, we derive the market clearing price of Stock 1 when a circuit breaker is triggered and the market is closed early. Because the two dividend processes are independent and we assume no leverage constraints when the market is halted, the market clearing prices for the two stocks are independent of each other.

The disagreement $\delta_{1,t}$ is stochastic following (6), therefore $\mu_{1,t}^B = \delta_{1,t} + \mu_1^A$ is stochastic as well. In the presence of a circuit breaker, we solve for the market clearing price when the market is halted. To do so, we solve the utility maximization problem

$$\max_{\theta_{1,\tau}^A} \mathbb{E}_\tau^A[-e^{-\gamma W_T^A}],$$

subject to $W_T^A = \theta_{1,\tau}^A(D_{1,T} - S_{1,\tau}) + W_\tau^A$, where W_t^A is the wealth of Type A investors at time t.

Using the dynamics $D_{1,T} = D_{1,\tau} + \mu_1^A(T-\tau) + \sigma(Z_T - Z_\tau)$, we obtain the optimal portfolio of agent A as follows.

$$\theta_{1,\tau}^A = \frac{D_{1,\tau} - S_{2,\tau} + \mu_1^A(T-\tau)}{\gamma\sigma^2(T-\tau)}. \tag{B.1}$$

Next, we study the utility maximization problem of agent B:

$$\max_{\theta_{1,\tau}^B} \mathbb{E}_\tau^B[-e^{-\gamma(W_\tau^B + \theta_{1,\tau}^B(D_{1,T} - S_{1,\tau}))}].$$

We first prove the following lemma.

Lemma B1. *Suppose θ is a constant, then*

$$\mathbb{E}_t^B[e^{-\gamma\theta D_{1,T}}] = e^{A(t,\theta) + B(t,\theta)D_{1,t} + C(t,\theta)\delta_{1,t}},$$

where

$$\begin{aligned}
A(t,\theta) &= \gamma\theta\mu_1^A(t-T) - \frac{\sigma^2}{2}\gamma^2\theta^2(t-T) + \frac{1}{\tilde{k}_1}(-\gamma\theta k_1\bar{\delta}_1 + \sigma_\delta\sigma\gamma^2\theta^2)(T - t - \frac{1 - e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}) \\
&\quad + \frac{\sigma_\delta^2\gamma^2\theta^2}{2\tilde{k}_1^2}(T - t - 2\frac{1 - e^{\tilde{k}_1(t-T)}}{\tilde{k}_1} + \frac{1 - e^{2\tilde{k}_1(t-T)}}{2\tilde{k}_1}), \\
B(t,\theta) &= -\gamma\theta, \\
C(t,\theta) &= \frac{-\gamma\theta}{\tilde{k}_1}(1 - e^{\tilde{k}_1(t-T)}),
\end{aligned}$$

with $\tilde{k}_1 = k_1 - \sigma_\delta/\sigma$. In particular, if $\tilde{k}_1 = 0$, then

$$\begin{aligned}
A(t,\theta) &= \gamma\theta\mu_1^A(t-T) - \frac{\sigma^2}{2}\gamma^2\theta^2(t-T) + \frac{1}{2}(-\gamma\theta k_1\bar{\delta}_1 + \gamma^2\theta^2\sigma_\delta\sigma)(t-T)^2 - \frac{\sigma_\delta^2\gamma^2\theta^2}{6}(t-T)^3, \\
B(t,\theta) &= -\gamma\theta, \\
C(t,\theta) &= \gamma\theta(t-T).
\end{aligned}$$

Lemma B1 can be proved by using the moment generating function of process $D_{1,t}$ and $\delta_{1,t}$ and solving an ODE system. Detailed deviations are omitted here.

By the lemma,

$$\mathbb{E}^B_\tau[-e^{-\gamma(W^B_\tau+\theta^B_{1,\tau}(D_{1,T}-S_{1,\tau}))}] = -e^{-\gamma W^B_\tau} e^{A(t,\theta^B_{1,\tau})+C(t,\theta^B_{1,\tau})\delta_{1,\tau}} e^{-\gamma\theta^B_{1,\tau}(D_{1,\tau}-S_{1,\tau})}.$$

Then the FOC with respect to $\theta^B_{1,\tau}$ yields that

$$\gamma S_{1,\tau} - \gamma D_{1,\tau} + \frac{\partial A(\tau,\theta^B_{1,\tau})}{\partial\theta^B_{1,\tau}} + \frac{\partial C(\tau,\theta^B_{1,\tau})}{\partial\theta^B_{1,\tau}}\delta_{1,\tau} = 0$$

or

$$S_{1,\tau} - D_{1,\tau} + \mu^A_1(\tau-T) - \frac{1}{\tilde{k}_1}(1-e^{\tilde{k}_1(\tau-T)})\delta_{1,\tau} - \bar{\delta}_1\frac{k_1}{\tilde{k}_1}(T-\tau-\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1}) + \theta^B_{1,\tau}I(\tau) = 0, \tag{B.2}$$

where

$$I(t) = -\gamma\sigma^2(t-T) + \frac{2\sigma_\delta\sigma\gamma}{\tilde{k}_1}(T-t-\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}) + \frac{\sigma^2_\delta\gamma}{\tilde{k}^2_1}(T-t-2\frac{1-e^{\tilde{k}_1(t-T)}}{\tilde{k}_1}+\frac{1-e^{2\tilde{k}_1(t-T)}}{2\tilde{k}_1}).$$

It follows (B.1) that

$$S_{1,\tau} = D_{1,\tau} + \mu^A_1(T-\tau) - \theta^A_{1,\tau}\gamma\sigma^2(T-\tau). \tag{B.3}$$

Together with (B.2) and the market clearing condition $\theta^A_{1,\tau} + \theta^B_{1,\tau} = 1$, we obtain the optimal share holding of Type A for Stock 1 at the time of market closure.

$$\theta^{A,*}_{1,\tau} = \frac{-\frac{1}{\tilde{k}_1}(1-e^{\tilde{k}_1(\tau-T)})\delta_{1,\tau} - \bar{\delta}_1\frac{k_1}{\tilde{k}_1}(T-t-\frac{1-e^{\tilde{k}_1(\tau-T)}}{\tilde{k}_1}) + I(\tau)}{I(\tau)+\gamma\sigma^2(T-t)}. \tag{B.4}$$

Therefore, we find the market clearing price $S_{1,\tau}$ by (B.3) where $\theta^A_{1,\tau} = \theta^{A,*}_{1,\tau}$ given by (B.4).

In particular, in the case $\tilde{k}_1 = 0$ (or $k_1 = \sigma_\delta/\sigma$),

$$\theta^{A,*}_{1,\tau} = \frac{1}{\gamma}\left(\frac{\gamma\sigma^2 - \gamma\sigma_\delta\sigma(\tau-T) + \frac{1}{2}k_1\bar{\delta}_1(\tau-T) + \frac{\sigma^2_\delta\gamma}{3}(\tau-T)^2 - \delta_{1,\tau}}{-\sigma_\delta\sigma(\tau-T) + \frac{\sigma^2_\delta}{3}(\tau-T)^2 + 2\sigma^2}\right),$$

and substituting it into (B.1), it follows that

$$\begin{aligned} S_{1,\tau} &= D_{1,\tau} + \mu_1^A(T-\tau) \\ &+ \frac{\gamma\sigma^2 - \gamma\sigma_\delta\sigma(\tau-T) + \frac{1}{2}k\bar{\delta}(\tau-T) + \frac{\sigma_\delta^2\gamma}{3}(\tau-T)^2 - \delta_{1,\tau}}{-\sigma_\delta\sigma(\tau-T) + \frac{\sigma_\delta^2}{3}(\tau-T)^2 + 2\sigma^2}\sigma^2(\tau-T). \end{aligned}$$

Finally, it is worthy mentioning that $S_{1,\tau}$ may not be larger than $\hat{S}_{1,\tau}$ (the equilibrium price in the absence of circuit breakers at time τ). In fact, for a relative small σ_δ (say, less than half of the volatility σ), the coefficient of $\delta_{1,t}$ in (B.3) can always be less than the coefficient of $\delta_{1,t}$ in the formula of $\hat{S}_{1,\tau}$. Thus, along with a small γ, we can always have $S_{1,\tau} < \hat{S}_{1,\tau}$. Under these conditions, the market clearing price with circuit breakers can always be smaller than the price without circuit breakers at time τ.

Denote the market clearing price of Stock 1 by $S_{1,\tau}^c$. Then by (B.3),

$$S_{1,\tau}^c = D_{1,\tau} + \mu_1^A(T-\tau) - \gamma\theta_{1,\tau}^{A,*}\sigma^2(T-\tau).$$

In addition, we obtain the value function of Type B investors:

$$V_1^B(\tau, W_\tau^B) = \max_{\theta_{1,\tau}^B} \mathbb{E}_\tau^B[e^{-\gamma(W_\tau^B + \theta_{1,\tau}^B(D_{1,T} - S_{1,\tau}))}] = e^{-\gamma W_\tau^B} e^{-\gamma G_{1,\tau}^B},$$

where $-\gamma G_{1,\tau}^B = -\gamma\theta_{1,\tau}^B(D_{1,\tau} - S_{2,\tau}) + A(\tau, \theta_{1,\tau}^{B,*}) + C(\tau, \theta_{1,\tau}^{B,*})\delta_{1,\tau}$, or

$$G_{1,\tau}^B = \theta_{1,\tau}^{B,*}(D_{1,\tau} - S_{2,\tau}) - \frac{1}{\gamma}A(\tau, \theta_{1,\tau}^{B,*}) - \frac{1}{\gamma}C(\tau, \theta_{1,\tau}^{B,*})\delta_{1,\tau}, \tag{B.5}$$

and the value function of Type A investors:

$$V_1^A(\tau, W_\tau^A) = \max_{\theta_{1,\tau}^A} \mathbb{E}_\tau^A[e^{-\gamma(W_\tau^A + \theta_{1\tau}^A(D_{1,T} - S_{1,\tau}))}] = e^{-\gamma W_\tau^A} e^{-\gamma G_{1,\tau}^A},$$

where $-\gamma G_{1,\tau}^A = -\gamma\theta_{1,\tau}^{A,*}(D_{1,\tau} - S_{1,\tau}) - \gamma\theta_{1,\tau}^{A,*}\mu_1^A(T-\tau) + \frac{\gamma^2(\theta_{1,\tau}^{A,*})^2}{2}\sigma^2(T-\tau) = -\frac{\gamma^2(\theta_{1,\tau}^{A,*})^2}{2}\sigma^2(T-\tau)$, or

$$G_{1,\tau}^A = \theta_{1,\tau}^{A,*}(D_{1,\tau} - S_{1,\tau}) + \theta_{1,\tau}^{A,*}\mu_1^A(T-\tau) - \frac{\gamma(\theta_{1,\tau}^{A,*})^2}{2}\sigma^2(T-\tau) = \frac{\gamma(\theta_{1,\tau}^{A,*})^2}{2}\sigma^2(T-\tau). \tag{B.6}$$

B.2 Stock 2: Stochastic Disagreement on Jump Intensity

Note that for Stock 2 there is disagreement on the jump intensity of dividend process $D_{2,t}$, which follows

$$dD_{2,t} = (\mu_2^i - \kappa_t^i \nu)dt + \nu dN_t^i,\ i = A, B, \tag{B.7}$$

with $\kappa_t^A \equiv \kappa^A$ and $N_t^A \equiv N_t$. In this Appendix, we derive the market clearing price $S_{2,\tau}^c$ of Stock 2 when a circuit breaker is triggered.

Recall that $\delta_{2,t} = \kappa_t^B/\kappa^A$ satisfies a mean-reverting process as follows.

$$d\delta_{2,t} = -k_2(\delta_{2,t} - \bar{\delta}_2)dt + \nu_\delta dN_t. \tag{B.8}$$

Suppose that the circuit breaker is triggered at $\tau < T$. The individual optimization problem of Type $i \in \{A, B\}$ investors at τ is:

$$V_2^i(W_\tau^i, \tau) = \max_{\theta_{2,\tau}^i} \mathbb{E}_\tau^i[-\exp(-\gamma(W_\tau^i + \theta_{2,\tau}^i(D_{2,T} - S_{2,\tau})))], \tag{B.9}$$

subject to the market clearing condition $\theta_{2,\tau}^A + \theta_{2,\tau}^B = 1$, where W_τ^i is the wealth owned by Type i investors at time τ. Note that

$$\mathbb{E}_\tau^i[u(W_\tau^i + \theta_{j,\tau}^i(D_{2,T} - S_{2,\tau}))] = -e^{-\gamma W_\tau^i} e^{\gamma\theta_{2,\tau}^i S_{2,\tau}} \mathbb{E}_\tau^i[e^{-\gamma\theta_{2,\tau}^i D_{2,T}}].$$

For Type A agents we have

$$\mathbb{E}_\tau^i[e^{-\gamma\theta_{2,\tau}^i D_{2,T}}] = \exp\{-\gamma\theta_{2,\tau}^A D_{2,\tau} + (\tau - T)((\mu_2^A - \kappa^A\nu)\gamma\theta_{2,\tau}^A - \kappa^A(e^{-\gamma\theta_{2,\tau}^A\nu} - 1))\}; \tag{B.10}$$

and for Type B agents we have

$$\mathbb{E}_\tau^B[e^{-\gamma\theta_{2,\tau}^i D_{2,T}}] = \exp\{-\gamma\theta_{2,\tau}^B D_{2,\tau} + \vartheta_\tau g(\tau; -\gamma\theta_{2,\tau}^B) + \int_\tau^T (-\gamma\theta_{2,\tau}(\mu_2^A - \kappa^A\nu)) + k_2\bar{\delta}g(s; -\gamma\theta_{2,\tau}^B)ds\}$$
$$:= exp(-\gamma M_{2,\tau}^B), \tag{B.11}$$

where function $g(t; \alpha)$ satisfies

$$g'(t; \alpha) + \kappa^A(e^{\alpha\nu + \nu_\delta g(t)} - 1) - k_2 g(t; \alpha) = 0, \tag{B.12}$$

with $g(T) = 0$.

To solve the optimization problem (B.9), we find the first order conditions with respect to $\theta^i_{2,\tau}$ for $j \in \{1,2\}, i \in \{A,B\}$ as follows.

$$D_{2,\tau} - S_{2,\tau} + (T-\tau)(\mu_2^A - \kappa^A\nu + \kappa^A\nu e^{-\gamma\theta^A_{2,\tau}\nu}) = 0, \tag{B.13}$$

$$D_{2,\tau} - S_{2,\tau} + \delta_2\frac{\partial g(\tau;\alpha)}{\partial\alpha}|_{\alpha=-\gamma\theta^B_{2,\tau}} + \int_\tau^T (\mu_2^A - \kappa^A\nu) + k_2\bar{\delta}\frac{\partial g(s;\alpha)}{\partial\alpha}|_{\alpha=-\gamma\theta^B_{2,\tau}})ds = 0, \tag{B.14}$$

where $g_\alpha(t) := \frac{\partial g(t;\alpha)}{\partial\alpha}$ satisfies an ODE as follows.

$$g'_\alpha(t) + \kappa^A e^{\alpha\nu+\nu_\delta g(t)}(\nu + \nu_\delta g_\alpha(t)) - k_2 g_\alpha(t) = 0, \tag{B.15}$$

with $g_\alpha(T) = 0$.

Along with the market clearing condition: $\theta^A_{2,\tau} + \theta^B_{2,\tau} = 1$, we can solve the optimal share holdings $\theta^A_{2,\tau} = \theta^{A,*}_{2,\tau}$, $\theta^B_{2,\tau} = \theta^{B,*}_{2,\tau}$ and the market clearing price $S_{2,\tau} = S^c_{2,\tau}$ from (B.13) and (B.14). No explict solutions like for Stock 1, we rely on numerical solutions in practice.

By (B.13), the market clearing price of Stock 2 can be expressed by $\theta^{A,*}_{2,\tau}$:

$$S^c_{2,\tau} = D_{2,\tau} + (\mu_2^A - \kappa^A\nu)(T-\tau) + \kappa^A\nu(T-\tau)e^{-\gamma\theta^{A,*}_{2,\tau}\nu}.$$

Define

$$G^A_{2,\tau} = \theta^A_{2,\tau}\mu_2(T-\tau) + \theta^i_{2,\tau}(D_{2,\tau} - S^c_{2,\tau}) - \frac{\kappa^A}{\gamma}(T-\tau)(e^{-\gamma\theta^{A,*}_{2,\tau}\nu} - 1), \tag{B.16}$$

$$G^B_{2,\tau} = M^B_{2,\tau} - \theta^{B,*}_{2,\tau}S^c_{2,\tau},$$

where $M^B_{2,\tau}$ is defined in (B.11). Then by (B.10) and (B.11), the value function of Type i investors at τ can be expressed in terms of $W^i_{2,\tau}$ and $G^i_{2,\tau}$ as follows.

$$V^i_2(W^i_\tau, \tau) = -e^{-\gamma W^i_\tau}e^{-\gamma G^i_{2,\tau}}, i \in \{A,B\}.$$

By the expressions of $G^A_{2,\tau}$ and $G^B_{2,\tau}$, it is useful to notice that $S^c_{2,\tau} + G^A_{2,\tau} + G^B_{2,\tau}$ does not depend on $S^c_{2,\tau}$ directly. The quantity depends on the optimal share holdings at τ: $\theta^{A,*}_{2,\tau}$ and $\theta^{B,*}_{2,\tau}$.

C Learning and Heterogeneous Beliefs

Suppose

$$dD_t = \mu_t dt + \sigma d\bar{Z}_t.$$

The dividend D_t is observable but the growth rate μ_t is not. Agents A and B infer the value of μ_t through the information from the dividend. Assume that

$$d\mu_t = -k(\mu_t - \bar{\mu})dt + \sigma_\mu d\bar{Z}_t,$$

and $\mu_0 \sim N(a_0, b_0)$, a normal distribution with mean a_0 and standard deviation b_0. Agent $i \in \{A, B\}$ believes $k = k^i, \bar{\mu} = \bar{\mu}^i, \sigma_\mu = \sigma_\mu^i, a_0 = a_0^i, b_0 = b_0^i$. Both of them learn μ_t through $\{D_s\}_{s=0}^t$. Let $\mu_t^A = \mathbb{E}^A[\mu_t|\{D_s\}_{s=0}^t]$ and $\mu_t^B = \mathbb{E}^B[\mu_t|\{D_s\}_{s=0}^t]$. Then following the standard filtering results, we have (under the assumption: $\mu_t|\{D_s\}_{s=0}^t \sim N(\hat{\mu}, \sigma_\mu)$)

$$\begin{aligned} d\mu_t^A &= -k^A(\mu_t^A - \bar{\mu}^A)dt + \sigma_\mu^A dZ_t^A, \\ d\mu_t^B &= -k^B(\mu_t^B - \bar{\mu}^B)dt + \sigma_\mu^B dZ_t^B, \end{aligned}$$

where $dZ_t^i = \frac{1}{\sigma}(dD_t - \mu_t^i dt), i = A, B$. Then

$$dD_t = \mu_t^A dt + \sigma dZ_t^A, \qquad dD_t = \mu_t^B dt + \sigma dZ_t^B.$$

Therefore, $Z_t^B + \frac{\delta_t}{\sigma}t$ is equal to Z_t^A almost surely, where $\delta_t = \mu_t^B - \mu_t^A$. In other words, $Z_t^B + \frac{\delta_t}{\sigma}t$ is a standard Brownian motion under agent A's probability measure $\mathbf{P}^A$.

Thus,

$$d\mu_t^B = -k^B(\mu_t^B - \bar{\mu}^B)dt - \frac{\sigma_\mu^B}{\sigma}\delta_t dt + \sigma_\mu^B dZ_t^A.$$

So we can obtain the general dynamics of the stochastic disagreement δ_t under learning. To validate the setting adopted in this paper, we let $\sigma_\mu^A = 0$, $k^A = 0$, and $\mu_t^A = \mu^A$ for all t. That is, we assume that Type A investors take the long-time mean of the growth rate as the estimation and impose no learning. Then it follows that

$$\begin{aligned} d\delta_t = d(\mu_t^B - \mu^A) &= -(k^B + \frac{\sigma_\mu^B}{\sigma})\delta_t dt - k^B(\mu^A - \bar{\mu}^B)dt + \sigma_\mu^B dZ_t^A \\ &= -k^B \delta_t dt + k^B(\bar{\mu}^B - \mu^A)dt + \sigma_\mu^B dZ_t^B. \end{aligned}$$

Further, let $k_1 = k^B + \sigma_\mu^B/\sigma$, $\sigma_\delta = \sigma_\mu^B$ and $\bar{\delta}k_1 = (\bar{\mu}^B - \mu^A)k^B$ and we have

$$d\delta_t = -k_1(\delta_t - \bar{\delta})dt + \sigma_\delta dZ_t^A,$$

that is identical to the mean-reverting disagreement process (6) assumed in the paper.

D The Fixed Point Problem

We prove the existence and uniqueness of a solution to the fixed point problem. First of all, based on the explicit expressions of the prices, we restrict the model parameters and the initial conditions (e.g., $D_{1,0}, D_{2,0}$) and assume that both $\hat{S}_{j,0}$ (the price without circuit breakers) and $S_{j,0}^c$ (the market clearing price) are positive for each $j = 1, 2$.

Recall that $S_{1,0}, S_{2,0}$ impact valuation of the expectations through the sum $S_{1,0} + S_{2,0}$ only. When the initial stock prices are $S_{1,0}$ and $S_{2,0}$, the threshold h is $(S_{1,0}+S_{2,0})(1-\alpha)$. So, we define

$$f_j(S_{1,0} + S_{2,0}) = \frac{\mathbb{E}_0^A[\pi_{T\wedge\tau}^A S_{j,T\wedge\tau}]}{\pi_0^A}, j = 1, 2.$$

and define a function $f : \mathcal{R} \to \mathcal{R}^2$ such that $f(S_{1,0} + S_{2,0}) = (f_1(S_{1,0} + S_{2,0}), f_2(S_{1,0} + S_{2,0}))^\top$, where $\top$ denotes the transpose of a vector. Then the fixed point problem is expressed as follows.

$$(S_{1,0}, S_{2,0})^\top = f(S_{1,0} + S_{2,0}).$$

Define $g(x) = f_1(x) + f_2(x) - x$, where $x \in \mathcal{R}$. When the threshold is zero, the circuit breaker is hardly triggered. Thus the equilibrium prices are close to the prices $\hat{S}_{0,1}$ and $\hat{S}_{2,0}$ respectively in the absence of circuit breakers. Given positive $\hat{S}_{0,1}$ and $\hat{S}_{2,0}$, we can obtain (specifically, for a sufficiently small volatility of $D_{1,t}$ and jump intensity of $D_{2,t}$): $g(0) = f_1(0) + f_2(0) > 0$. On the other hand, if the threshold is the sum of the market clearing prices $S_{1,0}^c + S_{2,0}^c$, the market is stopped immediately and the equilibrium prices must be the market clearing prices exactly. Thus, $g(\frac{S_{1,0}^c+S_{2,0}^c}{1-\alpha}) = f_1(\frac{S_{1,0}^c+S_{2,0}^c}{1-\alpha}) + f_2(\frac{S_{1,0}^c+S_{2,0}^c}{1-\alpha}) - \frac{S_{1,0}^c+S_{2,0}^c}{1-\alpha} = S_{1,0}^c + S_{2,0}^c - \frac{S_{1,0}^c+S_{2,0}^c}{1-\alpha} < 0$. It can be shown that $g(x)$ is a continuous function. Hence, there exists $x^* \in (0, \frac{S_{1,0}^c+S_{2,0}^c}{1-\alpha})$, such that $g(x^*) = 0$. Thus $f_1(x^*) + f_2(x^*) = x^*$.

Now define $(S_{1,0}^*, S_{2,0}^*)^\top = f(x^*)$. Then $x^* = f_1(x^*) + f_2(x^*) = S_{1,0}^* + S_{2,0}^*$ and

$$(S_{1,0}^*, S_{2,0}^*)^\top = f(x^*) = f(S_{1,0}^* + S_{2,0}^*).$$

Thus $(S_{1,0}^*, S_{2,0}^*)^\top \in \mathcal{R}^2$ is a solution to the fixed problem. The existence is proved.

Next, we show that the solution is unique. To do so, it is sufficient to show that $g(x)$ is monotonic. For the sake of notional simplicity, we ignore super-script "A" of expectations and π_t^A below.

Let $D_0 = D_{1,0} + D_{2,0}$. Given an exogenous threshold h and initial dividend sum value D_0, let $S_t^{h,D_0} = S_{1,t}^c + S_{2,t}^c$, where $S_{1,t}^c$ and $S_{2,t}^c$ are the market clearing prices; let $\tau(h, D_0)$ denote the stopping time; and let π_t^{h,D_0} be the state price density, i.e.

$$\pi_t^{h,D_0} = (\eta_t)^{1/2} e^{-\frac{\gamma}{2} S_t^{h,D_0}} \cdot e^{\frac{G_t^A + G_t^B}{2}}.$$

We redefine

$$g(x) = g(x; D_0) = \frac{\mathbb{E}[\pi_{\tau(h,D_0)\wedge T} S_{\tau(h,D_0)\wedge T}^{h,D_0}]}{\mathbb{E}[\pi_{\tau(h,D_0)\wedge T}]} - x,$$

where $h = x(1 - \alpha)$. Observe that $\tau(h, D_0) = \tau(0, D_0 - h)$ because the stopping time is determined by D_t and δ_t only. Then the market clearing (sum) price $S_{\tau(h,D_0)}^{h,D_0} = S_{\tau(0,D_0-h)}^{0,D_0-h} + h$ by the expressions of $S_{j,\tau}^c$, $j = 1, 2$. In addition, by the definition of G_τ^i, we see that $G_{\tau(h,D_0)}^i = G_{\tau(0,D_0-h)}^i$, $i = A, B$. Therefore

$$\pi_{\tau(h,D_0)}^{h,D_0} = e^{-\frac{\gamma}{2} h} \cdot \pi_{\tau(0,D_0-h)}^{0,D_0-h}. \tag{D.1}$$

Thus,

$$\begin{aligned}
g(x; D_0) &= \frac{\mathbb{E}[\pi_{\tau(h,D_0)\wedge T}^{h,D_0} \cdot (S_{\tau(h,D_0)\wedge T}^{h,D_0} - x)]}{\mathbb{E}[\pi_{\tau(h,D_0)\wedge T}^{h,D_0}]} \\
&= \frac{\mathbb{E}[\pi_{\tau(0,D_0-h)\wedge T}^{0,D_0-h} \cdot (S_{\tau(0,D_0-h)\wedge T}^{0,D_0-h} - x + h)]}{\mathbb{E}[\pi_{\tau(0,D_0-h)\wedge T}^{0,D_0-h}]} \\
&= g(0; D_0 - h) - x + h = g(0; D_0 - h) - \alpha x.
\end{aligned}$$

Given $h_1 < h_2$, we have $\tau(0, D_0 - h_1) \geq \tau(0, D_0 - h_2)$. Then,

$$\begin{aligned}
&\mathbb{E}[\pi_{\tau(0,D_0-h_1)}^{0,D_0-h_1}] = \mathbb{E}[\mathbb{E}[\pi_{\tau(0,D_0-h_1)}^{0,D_0-h_1} | \tau(0, D_0 - h_2)]] = \mathbb{E}[\pi_{\tau(0,D_0-h_2)}^{0,D_0-h_1}] \\
&= \mathbb{E}[(\eta_{\tau(0,D_0-h_2)\wedge T})^{1/2} e^{-\frac{\gamma}{2} S_{\tau(0,D_0-h_2)\wedge T}^{0,D_0-h_1}} \cdot e^{\frac{G_{\tau(0,D_0-h_2)\wedge T}^A + G_{\tau(0,D_0-h_2)\wedge T}^B}{2}}] \\
&= \mathbb{E}[(\eta_{\tau(0,D_0-h_2)\wedge T})^{1/2} e^{-\frac{\gamma}{2} S_{\tau(0,D_0-h_2)\wedge T}^{0,D_0-h_2}} \cdot e^{\frac{G_{\tau(0,D_0-h_2)\wedge T}^A + G_{\tau(0,D_0-h_2)\wedge T}^B}{2}} \cdot e^{-\gamma/2(h_2-h_1)}] \\
&= \mathbb{E}[\pi_{0,D_0-h_2}^{0,D_0-h_2}] e^{-\gamma/2(h_2-h_1)}.
\end{aligned}$$

Similarly,

$$\begin{aligned}\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_1)}] &= \mathbb{E}[\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_1)} | \tau(0, D_0 - h_2)]] \\ &= \mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_2)} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_2)}] e^{-\gamma/2(h_2-h_1)} \\ &\geq \mathbb{E}[\pi^{0,D_0-h_2}_{\tau(0,D_0-h_2)} \cdot S^{0,D_0-h_2}_{\tau(0,D_0-h_2)}] e^{-\gamma/2(h_2-h_1)}.\end{aligned}$$

Finally, let $x_1 < x_2$ and $h_1 = x_1(1-\alpha), h_2 = x_2(1-\alpha)$. It follows that

$$\begin{aligned}g(x_1; D_0) = g(0; D_0 - x_1) - \alpha x_1 &= \frac{\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)\wedge T} \cdot S^{0,D_0-h_1}_{\tau(0,D_0-h_1)\wedge T}]}{\mathbb{E}[\pi^{0,D_0-h_1}_{\tau(0,D_0-h_1)\wedge T}]} - \alpha x_1 \\ &\geq \frac{\mathbb{E}[\pi^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T} \cdot S^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T}]}{\mathbb{E}[\pi^{0,D_0-h_2}_{\tau(0,D_0-h_2)\wedge T}]} - \alpha x_1 \\ &= g(0; D_0 - h_2) = g(x_2; D_0) + \alpha x_2 - \alpha x_1 > g(x_2; D_0).\end{aligned}$$

Thus, $g(\cdot, D_0)$ is monotonic. This completes the proof of uniqueness.

E The Case of Correlated Dividend Processes

To impose a correlation between dividend processes, we assume that: under $\mathbf{P}^A$,

$$dD_{1,t} = \mu_1^A dt + \sigma_1 dZ_t, \tag{E.1}$$

$$dD_{2,t} = \mu_2 dt + \sigma_2 dZ_t + \nu dN_t, \tag{E.2}$$

and under $\mathbf{P}^B$:

$$dD_{1,t} = \mu_1^B dt + \sigma_1 dZ_t^B, \tag{E.3}$$

$$dD_{2,t} = \mu_2 dt + \frac{\sigma_2}{\sigma_1}\delta_t dt + \sigma_2 dZ_t^B + \nu dN_t, \tag{E.4}$$

where $\mu_1^B = \mu_1^A + \delta_t$ and

$$d\delta_t = -k(\delta_t - \bar{\delta})dt + \sigma_\delta dZ_t,$$

or

$$d\delta_t = -k(\delta_t - \bar{\delta})dt + \frac{\sigma_\delta}{\sigma_1}\delta_t dt + \sigma_\delta dZ_t^B.$$

Then the two dividend processes are correlated with instantaneous correlation

$$\rho = \frac{\sigma_2}{\sqrt{\sigma_2^2 + \kappa\nu^2}}.$$

We assume no disagreement on jump intensity of the Poisson process N_t ($\kappa^A \equiv \kappa^B \equiv \kappa$) and study the equilibrium prices without or with circuit breakers.

E.1 The Equilibrium Prices without Circuit Breakers

The pricing formula has the same expression as that in the uncorrelated case.

$$\hat{S}_{j,t} = \mathbb{E}_t^A \left[\frac{\pi_T^A D_{j,T}}{\mathbb{E}_t^A[\pi_T^A]} \right], j = 1, 2,$$

where $\pi_T^A = \gamma\zeta\mathbb{E}_t^A[\eta_T^{1/2} \cdot e^{-\frac{\gamma}{2}(D_{1,T}+D_{2,T})}]$. However, the two prices cannot be evaluated separately anymore because the two dividend processes are correlated ($\sigma_2 \neq 0$).

E.2 The Equilibrium Prices with Circuit Breakers

We derive the market clearing prices when the market is closed early due to the circuit breaker.

Type A investors need to maximize the individual utility function

$$\max_{\theta_{1,\tau}^A, \theta_{2,\tau}^A} \mathbb{E}_t^A[-e^{-\gamma(\theta_{1,\tau}^A(D_{1,T}-S_{1,\tau})+\theta_{2,\tau}^A(D_{2,T}-S_{2,\tau}))}].$$

It results in first order conditions:

$$-\gamma(D_{1,\tau} - S_{1,\tau}) - \gamma\mu_1^A(T-\tau) + \gamma^2(\theta_{1,\tau}^A\sigma_1 + \theta_{2,\tau}^A\sigma_2)\sigma_1(T-\tau) = 0, \tag{E.5}$$
$$-\gamma(D_{2,\tau} - S_{2,\tau}) - \gamma\mu_2(T-\tau) + \gamma^2(\theta_{1,\tau}^A\sigma_1 + \theta_{2,\tau}^A\sigma_2)\sigma_2(T-\tau) - \gamma\nu\kappa e^{-\gamma\theta_2^A\nu} = 0. \tag{E.6}$$

For Type B investors, the optimization problem is

$$\max_{\theta_{1,\tau}^B, \theta_{2,\tau}^B} \mathbb{E}_t^B[-e^{-\gamma(\theta_{1,\tau}^B(D_{1,T}-S_{1,\tau})+\theta_{2,\tau}^B(D_{2,T}-S_{2,\tau}))}].$$

We first obtain an expression for the following expectation for any real numbers x and y:

$$\mathbb{E}_t^B[e^{x\int_t^T \delta_s ds + y(Z_T^B - Z_t^B)}] = e^{A(t;x,y)+C(t;x)\delta_t},$$

where

$$A(t;x,y) = \frac{y^2}{2}(T-\tau) + k\bar{\delta}\int_t^T C(s;x)ds + \frac{\sigma_{\bar{\delta}}^2}{2}\int_t^T C(s;x)^2 ds + y\sigma_\delta \int_t^T C(s;x)ds,$$
$$C(t;x) = \frac{x}{k-\frac{\sigma_\delta}{\sigma_1}}(1-e^{(k-\frac{\sigma_\delta}{\sigma_1})(\tau-T)}).$$

Then let $y = -\gamma(\theta_{1,\tau}^B\sigma_1 + \theta_{2,\tau}^B\sigma_2)$ and $x = -\gamma(\theta_{1,\tau}^B + \theta_{2,\tau}^B\frac{\sigma_2}{\sigma_1})$; we obtain the first order conditions for the maximization problem of Type B:

$$-\gamma(D_{1,\tau}-S_{1,\tau}) - \gamma\mu_1^A(T-\tau) + \frac{dA(t;x,y)}{d\theta_{1,\tau}^B} + \frac{dC(t;x)}{d\theta_{1,\tau}^B}\delta_t = 0, \tag{E.7}$$
$$-\gamma(D_{2,\tau}-S_{2,\tau}) - \gamma\mu_2(T-\tau) - \gamma\kappa\nu(T-\tau)e^{-\gamma\theta_{2,\tau}^B\nu} + \frac{dA(t;x,y)}{d\theta_{2,\tau}^B} + \frac{dC(t;x)}{d\theta_{2,\tau}^B}\delta_t = 0. \tag{E.8}$$

Along with the market clearing condition $\theta_{j,\tau}^A + \theta_{j,\tau}^B = 1, j = 1,2$, the four first order conditions determine the solution $S_{1,\tau}^*, S_{2,\tau}^*, (\theta_{1,\tau}^A)^*, (\theta_{2,\tau}^A)^*$, that is the market clearing prices and the share holdings at the market early closure time τ, respectively.

Next, as in the case of uncorrected dividend processes, we obtain the indirect utility functions for Type A and Type B investors and the state price density. The equilibrium stock prices at $t < \tau$ can be evaluated numerically by solving a fixed point problem similar to (27).

In the above, we deal with the case of no disagreement on the jump intensity. To incorporate a stochastic disagreement $\delta_{2,t}$ into the model, we can follow the procedure in Appendix B.2.

F Numerical Algorithms

P1: Solve for the Fixed Point Problem

Step 1. Initialize a solution and the threshold by letting $S_{1,0} = 1, S_{2,0} = 1$ and $h = (S_{1,0} + S_{2,0})(1-\alpha)$.

Step 2. Generate M pairs of sample paths of $D_{1,t}$ and $D_{2,t}$ according to their dynamics (1) and (2) and stochastic differential equations of (6), (7) of $\delta_{1,t}$ and $\delta_{2,t}$.

Step 3. Go through each sample ω: (1) Check whether the circuit breaker is triggered at some time τ before T. If it is triggered, let $S_{j,T\wedge\tau} = S_{j,\tau}^c$, which is the market clearing price at time τ. Otherwise, let $S_{j,T\wedge\tau} = D_{j,T}$. (2) Calculate $\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}$ and $\pi^A_{T\wedge\tau}$ for each sample, where $\pi^A_{T\wedge\tau}$ is calculate by Eq. (24). (3) Find the averages of the two quantities over all samples. (4) Then let $\tilde{S}_{j,0}$ be the ratio of the two averages.

Step 4. If $\|\tilde{S}_0 - S_0\| < tol$, we find an approximated solution to the fixed point problem with accuracy tol. Otherwise, let $S_0 = \tilde{S}_0$ and go to Step 1.

P2: Find Stock Prices at any time $t' < \tau$

Step 1. Using S_0 obtained by P1 codes, let $h = (S_{1,0} + S_{2,0})(1 - \alpha)$ be the threshold of the circuit breakers.

Step 2. Generate M pairs of sample paths of $D_{1,t}$ and $D_{2,t}$ from t' to T.

Step 3. Go through each sample ω: (1) Check whether the circuit breaker is triggered at some time τ before T. If it is triggered, let $S_{j,T\wedge\tau} = S^c_{j,\tau}$, which is the market clearing price at time τ. Otherwise, let $S_{j,T\wedge\tau} = D_{j,T}$. (2) Calculate $\pi^A_{T\wedge\tau} S_{j,T\wedge\tau}$ and $\pi^A_{T\wedge\tau}$ for each sample, where $\pi^A_{T\wedge\tau}$ is calculate by Eq. (24). (3) Find the averages of the two quantities over all samples. (4) Then let $S_{j,t'}$ be the ratio of the two averages, that is the stock price at time t'.

P3: Calculate Correlation and Volatilities of $S_{1,t}$ and $S_{2,t}$

Step 1. Find equilibrium prices $S_{1,t}$ and $S_{2,t}$ at a give time $t < T$ by using P2 codes.

Step 2. Generate M sample pairs of $(D_{1,t+\Delta t},\ D_{2,t+\Delta t})$ given $(D_{1,t}, D_{2,t})$.

Step 3. For each $(D_{1,t+\Delta t},\ D_{2,t+\Delta t})$, calculate the equilibrium price $S_{1,t+\Delta t}$ and $S_{2,t+\Delta t}$ and find the price change $\Delta S_{j,t} = S_{j,t+\Delta t} - S_{j,t}$ for $j = 1, 2$.

Step 4. Calculate the correlation of the M pairs of $(\Delta S_{1,t}, \Delta S_{2,t})$, as well as the volatilities of $\Delta S_{1,t}$ and $\Delta S_{2,t}$.