Bitcoin: Predictability and Profitability via Technical Analysis*

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Abstract

We document that Bitcoin returns, while largely unpredictable by macroeconomic variables, are predictable by 5- to 100-day moving averages (MAs) of its prices, both in- and out-of-sample. Trading strategies based on MAs generate substantial alpha, utility and Sharpe ratios gains, and significantly reduce the severity of drawdowns relative to a buy-and-hold position in Bitcoin, which already has a Sharpe ratio of 1.9. We explain these facts with a novel equilibrium model that demonstrates, in the absence of cashflows, rational learning leads to joint predictability of returns by different horizon's MAs.

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I. Introduction

Bitcoin is one of the most speculative assets in the history of finance. Its rapid price increase surprised even the most optimistic of market observers and early investors. One dollar invested in Bitcoin on October 27, 2010 grew to $103,453 by January 31, 2018, while the same investment in the S&P500 stock index grew to only $2.65 over the same period. Moreover, Bitcoin is only the first digital coin in the rapidly growing cryptocurrency market that has a capitalization exceeding $800 billion at the end of 2017. Bitcoin's dramatic price growth accompanies similarly dramatic volatility. For example, Bitcoin experienced three large drawdowns averaging 30% in 2017, in spite of ultimately realizing a more than 1,300% return over the year. Few, if any, measurable fundamentals explain Bitcoin's explosive price growth and high volatility. In contrast to common financial assets such as stocks and bonds, Bitcoin does not pay dividends or interest. Since Bitcoin has no widely accepted fundamentals, existing asset pricing theories are difficult to apply. In this paper, we address two natural questions of investors: what explains the dynamics of Bitcoin prices and are Bitcoin returns predictable?

Given the absence of observable fundamentals such as dividends, interest payments, or book values, Bitcoin traders must rely heavily on the path of prices, which signal information of others. Theoretically, under various imperfect market conditions, **?**, **?**, **?**, **?**, **?**, **?**, **?**, among others, show that past stock prices can predict future prices. These results imply that technical indicators, which are functions of past prices, can represent useful trading signals. Empirically, **?** and **?**, among others, show that trading based on technical indicators, especially the moving averages of prices, can be profitable in the stock market. **?** and **?** further provide insightful comments about the effectiveness of technical strategies from top practitioners. A crucial assumption in existing equilibrium models featuring technical analysis is that assets generate observable cashflows, and thus their assumptions are not applicable to Bitcoin.

To motivate the applicability of technical analysis to Bitcoin and other assets that lack fundamentals in general, we provide the first equilibrium model featuring technical traders and assets without cash flows. Our model differs from that of **?** in two key respects. First, our model features heterogeneous technical traders who are rational investors that optimally learn from past prices, though each over a different horizon. Second, our model's risky asset does not pay dividends. Rather, the asset produces a random flow of utility-providing benefits, which are a latent state variable investors learn about. The central novel prediction of our model is that the moving averages of these assets' prices over different horizons jointly forecast returns.

Consistent with our model, we find that Bitcoin returns are jointly predictable in-sample by ratios of 5- to 100-day moving averages (MAs) of price relative to current price. In-sample predictive regressions can overstate the significance of predictability to investors in real-time. (e.g., **?**). Thus, we assess the out-of-sample predictability of Bitcoin returns by MA-price ratios. To incorporate information across horizons, we apply out-of-sample mean-combination or three-pass-regression-filter methods and find statistically significant out-of-sample R^2s (e.g., **??**). Moreover, encompassing tests reject the null that the out-of-sample forecasts from any one horizon's MA subsumes the predictive information for returns from forecasts based on each other MA. These results indicate that the MA-price ratios of different horizons jointly forecast Bitcoin returns out-of-sample. We also test whether Bitcoin returns are predictable by the VIX, Treasury bill rate, term spread, and the default spread, which are common predictors of stock returns. While these variables have some degree of in-sample return predictability for Bitcoin returns, this predictability fails out of sample.

To assess the economic significance of Bitcoin-return predictability to investors, we form a trading strategy that goes long Bitcoin when the price is above the MA, and long cash otherwise. We find that these trading strategies using MA lag lengths ranging from 5 to 100 days, significantly outperform the buy-and-hold benchmark, increasing Sharpe ratios by 0.2 to 0.6 per year from 1.9. The alphas and mean-variance-utility gains are also large for most MA strategies. Moreover, average returns on Bitcoin on days when the different horizon MA signals indicate a long Bitcoin position are 11 to 58 times as large as when the signals indicate investment in cash. These results are robust across both halves of the sample. The MA strategies also outperform the buy-and-hold benchmark when applied to other cryptocurrencies, such as Ripple and Ethereum, Bitcoins two largest competitors.

To further test the prediction of our theory, we also consider the NASDAQ portfolio during a ten-year window (1996–2005) that includes the dot.com boom-and-bust of the early 2000's. In this period, many emerging technologies associated with the internet and other communication advances introduced fundamentals that at the time were difficult to assess, much like Bitcoin. Many NASDAQ companies possessed no earnings (or negative earnings) for years before they became viable, and other company's innovations never proved valuable and eventually failed. We show that our technical trading strategies applied to NASDAQ outperform the buy-and-hold benchmark in this ten-year window. This outperformance largely derives from avoiding the length and severity of the major NASDAQ drawdowns during this period. We also show that the profitability of the moving average-based strategies declines after 2005 as internet-based technology matured and investors became better able to evaluate price using fundamentals. These results demonstrate wider applicability of our model to other emerging assets characterized by fundamentals that are difficult to value besides just Bitcoin and other cryptocurrencies.

Our model provides a refutable economic implication in addition to the joint predictability of Bitcoin returns by MAs of prices. Specifically, in our model, trading results from variation across MA horizon indicators. Consistent with this implication, controlling for the absolute value of price shocks, the chief empirical determinant of volume, we show that proxies for disagreement across horizons and total turnover implied by the MA signals are significantly and positively associated with trading volume. Hence, overall, results demonstrate that Bitcoin returns are jointly predictable by MAs of different horizons, investors can profit from this predictability, and Bitcoin's trading volume is at least partially explained by differing MA trading signals across horizons.

A. Related Literature

Our paper contributes to the growing literature on the economics of cryptocurrencies and the associated blockchain technology. Relatively few papers in this vein study the asset pricing properties of Bitcoin. Using the Cagan model of hyperinflation, **?** empirically examines the relative contribution of shocks to volume and velocity on variation in Bitcoin's price. Jermann finds that most of the variation in Bitcoin's price is attributable to volume shocks, consistent with stochastic adoption dominating technology innovations. **?** explains how cryptocurrencies can have positive value given limited supply. **?**, **?**, and **?** all provide models in which the value of cryptocurrencies depends on some combination of (i) usage and the degree of adoption, (ii) the scarcity of Bitcoin, and (iii) the value of anonymity.

Our model differs from those used by these prior studies in at least two important respects. First, we do not model Bitcoin as a currency per se. Accordingly, we do not specify currency-related determinants of its value (e.g. (i)–(iii) above). Rather, we model the flow of utility-providing benefits as a random state variable. This generality is important because some market participants argue that Bitcoin is better thought of as a speculative asset than a currency (e.g., **?**). For example Bitcoin's high volatility eliminates its use a store of value, a defining feature of money. Second, the papers cited above all assume full-information, however, our model features learning. This feature is critical given the lack of agreement on what determines the value of Bitcoin.[1] The learning aspect of our model also helps us to answer novel questions relative to the prior studies such as: what predicts Bitcoin returns?

Relative to asset pricing inquiries, most of the literature on the economics of Bitcoin seeks to identify problems, implementation issues, and uses of cryptocurrencies. **?** discuss the virtual currency's potential to disrupt existing payment systems and perhaps even monetary systems. **?** describe immense possibilities for the future for Bitcoin and its underlying blockchain technology. **?** describe conditions and practical steps necessary for using blockchain technology as a global currency. **?** provide a model of Bitcoin trading fees. **?** discusses use of Blockchain for trading equities and the corresponding governance implications. **?** document Bitcoin price manipulation. **?** model the reliability of the Blockchain mechanism. **?** discusses how blockchain technology will shape the rate and direction of innovation. **?** study the optimal design of cryptocurrencies and assess quantitatively how well such currencies can support bilateral trade. **?** model the impact of blockchain technology on information environments. **?** model competition among privately issued currencies. **?** document that a large portion of Bitcoin transactions represent illegal activity. **?** model fees and self-propagation mechanism of the Bitcoin payment system. **?** model the use of blockchain in trading financial assets. **?** examines economic viability of blockchain price-formation mechanism. **?** show theoretically and empirically that Bitcoin prices forecast Bitcoin production.

The rest of the paper is organized as follows. Section II introduces the model and discusses its implications. Section III provides the data and summary statistics. Section IV reports the main empirical results, and Section V concludes.

[1]In a Bloomberg interview on December 4, 2013, Alan Greenspan stated: "You have to really stretch your imagination to infer what the intrinsic value of Bitcoin is. I havent been able to do it. Maybe somebody else can."

II. The Model

Moving averages of prices have been widely used in practice for forecasting and trading risky assets such as stocks. However, to the best of our knowledge, no rational model justifies such practice.[2] In this section, we develop the first rational equilibrium model that provides a theoretical foundation for using the moving averages of prices for forecasting and trading risky assets including cryptocurrencies such as Bitcoin.

In the model, there is one risky asset ("Bitcoin") whose service flow provides utility to investors. There are two types of investors with different prior beliefs about the future benefit of Bitcoin. Trading occurs between the two types of investors as their beliefs evolve over time. When type 1 investors are more optimistic (pessimistic), they buy from (sell to) type 2 investors, and vice versa. Due to lack of full information about the risky asset, we show next that in equilibrium the optimal investment strategies of the two types of investors can be approximated by functions of MAs with different lags which is endogenously determined. The main insight of the model is that, with incomplete information, MAs of past prices can be useful trading signals. Moreover, trading volume can be predicted by the MAs which indicates how disagreement changes over time between the two types of investors.

We start out the model with a set of assumptions.

Assumption 1. There is one unit of Bitcoin in the economy and each unit provides a stream of service/enjoyment δ_t, where

$$\frac{d\delta_t}{\delta_t} = X_t dt + \sigma_\delta dZ_{1t}, \tag{1}$$

$$dX_t = \mu(\bar{X} - X_t)dt + \rho\sigma_X dZ_{1t} + \sqrt{1-\rho^2}\sigma_X dZ_{2t}, \tag{2}$$

where σ_δ, μ, $\bar{X}$, σ_X, and $\rho \in [-1, 1]$ are all known constants and (Z_{1t}, Z_{2t}) is a two-dimensional standard Brownian motion, and the expected growth rate X_t is an unobservable state variable that affects the service flow of Bitcoin.

While Bitcoin does not provide any cash flows, we assume that it offers service flow to investors who derive utility from it. As a result, there is a market for its trading. For risky assets like stocks,

[2] In contrast to our model, Han et. al. (2016) exogenously assume there are some investors who use moving average rules in trading.

the service flow can represent dividend payment. Different from most stocks, the uncertainty about the expected growth rate X_t of Bitcoin service flow is much greater. On the investors, we make the assumptions below.

Assumption 2. There are two types of investors who differ by their priors about the state variable X_t and possibly initial endowment of Bitcoin.[3] Type i investor is endowed with $\eta_i \in (0,1)$ units of Bitcoin with $\eta_1 + \eta_2 = 1$ and has a prior that X_0 is normally distributed with mean $M^i(0^-)$ and variance $V^i(0^-)$, $i = 1, 2$.

Denote by $\mathcal{F}_t$ the filtration at time t generated by the Bitcoin price process $\{B_s\}$ and the prior $(M^i(0^-), V^i(0^-))$ for all $s \leq t$ and $i = 1, 2$. Further let $M_t^i \equiv E[X_t|\mathcal{F}_t^i]$ be the conditional expectation of X_t. Below we will solve the equilibrium price explicitly in terms of M_t^i. For this, we need two additional assumptions.

Assumption 3. All investors have log preference over the service flow provided by Bitcoin with discount rate β until time T. Specifically, the investor's expected utility is

$$E \int_0^T e^{-\beta t} \log C_t^i dt,$$

where C_t^i denotes the service flow received by a Type i investor from owning Bitcoin.

Assumption 4. Investors can trade one risk free asset and the Bitcoin with the risk free rate r_t and the Bitcoin price B_t to be determined in equilibrium. We conjecture and later verify that B_t satisfies

$$\frac{dB_t}{B_t} = (\mu_t^i B_t - \delta_t)dt + \sigma_\delta B_t d\hat{Z}_{1t}^i, \tag{3}$$

where μ_t^i is an adapted stochastic process and $\hat{Z}_{1t}^i$ is an innovation process to be determined in equilibrium.

With the above assumptions, we have

Proposition 1: In an economy defined by *Assumption 1-4*, there exists an equilibrium, in which

$$dB_t = ((\beta + M_t^i)B_t - \delta_t)dt + \sigma_\delta B_t d\hat{Z}_{1t}^i,$$

[3] Since investors can continuously observe δ_t, they can directly calculate the volatility σ_δ and therefore there is no disagreement about the volatility.

the fraction of wealth invested in the Bitcoin by Investor 1 is

$$1 + \frac{\alpha_t}{1+\alpha_t}\frac{M_t^1 - M_t^2}{\sigma^2}, \tag{4}$$

and by Investor 2 is

$$1 - \frac{1}{1+\alpha_t}\frac{M_t^1 - M_t^2}{\sigma^2}, \tag{5}$$

where

$$M_t^i = h^i(t) + f^i(0,t)\log\frac{B_t}{B_0} + (f^i(t,t) - f^i(0,t))\left(\log B_t - \frac{\int_0^t g^i(u,t)\log B_u du}{\int_0^t g^i(u,t)du}\right) \tag{6}$$

is the ith investor's conditional expectation of X_t, $h^i(.)$, $f^i(.,.)$, and $g^i(.,.)$ are as defined in the Appendix for $i = 1, 2$, and α_t is as defined in (**??**), denoting the ratio of the marginal utility of type 1 investor to that of type 2 investor.

Proof. See the Appendix.

There are two important implications of Proposition 1. First, it implies that the Bitcoin price is predictable by the moving averages, because

$$\frac{\int_0^t g^i(u,t)\log B_u du}{\int_0^t g^i(u,t)du}$$

is simply a weighted moving average of the log prices of Bitcoin. With linearization, we can approximate this moving average with the arithmetic MAs of prices, as is more commonly used in practice.

The second implication is that the optimal trading strategy is a linear function of the MAs. In our model setting, investors trade with each other due to their differences in prior beliefs. Because of the lack of complete information, investors use MA signals to extract information from the market. As a result, their optimal trading strategies are functions of the MAs. As far as we know, this is the first rational equilibrium model that justifies the usage of MAs for forecasting and trading risky assets.

In practice, due to difficulties in estimating the exact functionals, we use the MAs only as timing signals. This is for simplicity, similar to technical stock trading in practice where most technical traders apply the MAs to time a stock or the market. The simple trading strategy is then: When the MAs are high enough, we either buy or hold the position if bought already; Otherwise, we sell or do not trade if we do not own any Bitcoin.

III. Data

Bitcoin trades continuously and electronically on multiple exchanges around the world. We obtain daily Bitcoin prices from the news and research site Coindesk.com, which is frequently cited in professional publications such as the *Wall Street Journal*, over the sample period July 18, 2010 (first day available) through January 31, 2018. Starting July 1, 2013, Coindesk reports a Bitcoin price equal to the average of those listed on large high-volume exchanges. Prior to July 2013, Coindesk reported the price from Mt. Gox, an exchange that handled most of the trading volume in Bitcoin at the time.[4] We also obtain data on a second cryptocurrency, Ripple, from coinmarketcap.com. Ripple is one of the two largest competitors to Bitcoin, and of these two, has a much longer time series of data available.[5]

We obtain the daily risk-free rate and market excess return (MKT) through December 29, 2017 from the website of Kenneth French. To measure the risk-free rate on weekends and through January 31, 2017 (during which time the FOMC has not changed the short-term rates, we use the most recently available one-day risk-free rate. The average risk free rate over this time (see below) was multiple orders of magnitude smaller than the average Bitcoin return over this time so our risk-free rate assumptions can not have an economically meaningful impact on our results.

We obtain daily prices and total returns on the NASDAQ composite index, Gold, and the Barclay's aggregate bond market index from Bloomberg. We obtain daily levels of the S&P500 index, VIX, 3-month and 10-year Treasury yields ($BILL$ and LTY, respectively), and Moody's BAA- and AAA-bond index yields (BAA and AAA, respectively) from the St. Louis Federal Reserve Bank website over the sample period July 18, 2010–January 31, 2018. We define $TERM = LTY - BILL$ and $DEF = BAA - AAA$. VIX, $BILL$, $TERM$, and DEF are commonly used returns predictors and among the few available at the daily frequency (e.g., **???**).

Figure **??** depicts the time-series of $1 invested in Bitcoin or the S&P500 at the beginning of our

[4] For details on the history of the Bitcoin market, see **?**.

[5] Ethereum is the remaining cryptocurrency from the three largest by market cap.

sample period. Over the roughly seven year sample, \$1 investment in the S&P500 increased to about \$2.65. Over the same period, \$1 invested in Bitcoin grew to \$103,453! Table 1 presents summary statistics for our main variables of interest. Panel A shows that Bitcoin earns an annualized daily excess return of 213.6% and a Sharpe ratio of 1.9 with an annualized volatility of 111.0%. In contrast, MKT has a much lower average return and volatility over the period of 14.4% and 14.6%, respectively. Although far less than the Sharpe ratio of Bitcoin, the resulting MKT Sharpe ratio of 0.99 is relatively high by historical standards. The aggregate bond market, proxied by AGG, also earns a high Sharpe ratio during our sample of 0.82. In contrast to other markets, Gold earned slightly less per year than the risk-free rate. All of the returns have small and insignificant daily autocorrelations.

Panel B presents summary statistics for several benchmark return predictor variables used in the next section. All four are highly persistent, with an autoregressive coefficient of 0.96–1.0. Moreover, Augmented Dickey-Fuller tests fail to reject the null that any of the return predictors except VIX contain a unit root.

IV. Empirical results

Based on the theory of Section **??**, we predict that returns on Bitcoin are jointly predictable by moving averages of price and technical-analysis strategies based on Bitcoin should produce even greater performance than the buy-and-hold strategy. We test these predictions in this section.

A. Random walk test

Following **?**, Table 2 reports variance ratio tests of the hypothesis that log Bitcoin prices follow a random walk. Under this hypothesis, the variance of price changes should scale linearly with horizon and the variance ratios (VR(q)) should be one for each horizon. Rejection of the null hypothesis implies predictability of price changes via the history of prices. Ratios below one are consistent with mean reversion of log prices. If the variance ratio is above one, log prices are mean averting (**?**)), and returns are positively autocorrelated. Consistent with the latter, results for k=2, 4, 8 and 16 weeks all reveal ratios above one, and VR is significantly and substantially above one for horizons of four or more weeks.

The fact that Bitcoin's variance ratio is above one encourages the use of technical analysis, because it indicates positive autocorrelation of returns. Thus, when the current price is rising and above its 5-, 10-, 20-, 50- or 100-day moving average, investors can likely profit from purchasing Bitcoin as the positive trend is likely to continue. In contrast, if a negative shock causes the price to fall below this horizons moving average trends, investors should sell as the negative trend is likely to continue. In short, the random walk hypothesis of Bitcoin price is rejected, and we show below that its price is predictable by the MAs, consistent with our model and suggested by the variance ratio tests.

B. In-sample predictability

Following **?**, **?**, and **?**, we form technical strategies based on the prior day's price relative to a lagged moving average (MA). Specifically, we define a moving average ($MA(L)$) of daily Bitcoin prices (P_t) as:

$$MA_t(L) = (1/L)\sum_{l=0}^{L-1} P_{t-l} \tag{7}$$

We choose moving average horizons of L = 5, 10, 20, 50, or 100 days. These lags choices are common, see, e.g., **?** and **?**. Our main return predictors of interest are the ratios of prices to moving averages: $(MA_{t-1}(L)/P_{t-1})$.

Table 3 evaluates in-sample predictability and presents estimates of predictive regressions of the form:

$$r_{t+1,t+5} = a + b'X_t + \varepsilon_{t+1,t+h}, \tag{8}$$

where $r_{t+1,t+5}$ denotes the log excess return on Bitcoin over business days $t+1$ through $t+5$. We use **?** standard errors with a bandwidth of 5 to account for heteroskedasticity and 4 days of overlap in retuarn observations.

In Panel A, columns (1)–(5) present results with $X_t = \log(MA_{t-1}(L)/P_{t-1})$ for $L = 5,\ 10,\ 20,\ 50,$ or 100. The $\log(MA_{t-1}(L)/P_{t-1})$ significantly predict $r_{t+1,t+5}$ for L = 20, 50, and 100. The moving averages of different horizons will mechanically be highly correlated with each other. Hence, to account for multicollinearity, while also testing whether different MA horizons jointly predict $r_{t+1,t+5}$, we construct three principal components of the $\log(MA_{t-1}(L)/P_{t-1})$ and use them as predictors in Column (6). The first two principal components significantly predict $r_{t+1,t+5}$, supporting joint predictability of information contained in multiple horizon forecasts.

Panel B presents predictive regressions of the form Eq. (**??**) using the "common" return predictors VIX, $BILL$, $TERM$, and DEF. Columns (1)–(5) show that VIX and $BILL$ individually and significantly predict $r_{t+1,t+5}$, and all four predictors jointly predict $r_{t+1,t+5}$. Moreover, column (6) shows that controlling for the four common predictors renders the three PCs of $\log(MA_{t-1}(L)/P_{t-1})$ insignificant. Thus, it appears that the predictive signals contained in the MA are related to the equity- and bond-predictor variables. Taken together, the evidence in Panel E initially appears to suggest that the equity- and bond-return predictors at least partially subsume the technical indicators. However, it is well-established that highly persistent regressors such as VIX, $BILL$, $TERM$, and DEF can generate spuriously high in-sample return predictability (e.g., **???**); these biases and parameter instabilty often imply that in-sample estimates poorly capture real-time predictability, which actually impacts investors (e.g., **?**). For this reason among others, we next assess the out-of-sample predictability of Bitcoin returns.

C. Out-of-sample predictability

Table 4 presents out-of-sample R^2 (R^2_{OS}) for recursively estimated predictive regressions of Eq. (**??**) at the horizon of $h = 7$ days. For robustness, we report R^2_{OS} using several split dates between the in-sample and out-of-sample periods (e.g., **?**). The first five columns of Panels A and B present results using univariate predictive regressions based on $\log(MA_{t-1}(L)/P_{t-1})$. The sixth column (denoted PLS_MA) uses the three-pass-regression filter of **?** to combine the predictive information in the $\log(MA_{t-1}(L)/P_{t-1})$. The last column (denoted MEAN), follows **?** and presents R^2_{OS} for the MEAN combination forecast, which is the simple average of the univariate predictive regression-based forecasts (Eq. (**??**) with $X_{it} = \log(MA_{t-1}(L)/P_{t-1})$). Recent studies show strong evidence of out-of-sample return predictability using three-pass-filter methodology (e.g., **??**). Prior evidence however reveals that simple-average forecasts are robust, and frequently outperform more sophisticated combination methods in forecasting returns and other macroeconomic time-series out-of-sample (e.g., **??**).

Panel A shows that several of the $\log(MA_{t-1}(L)/P_{t-1})$ individually frequently predict returns out-of-sample with $R^2_{OS} > 0$. Moreover, at each horizon, the MEAN forecasts forecasts predict returns with an R^2_{OS} of 1.17%–3.66%, which is high at the weekly horizon. For comparison, **?** report comparable R^2_{OS} but at the quarterly returns on the S&P500, and predictability should increase with horizon. The R^2_{OS} for the MEAN forecast are significant at the 10% or 5% level for each choice of in-sample period. The R^2_{OS} for the three-pass-filter forecast range from 0.91%–2.46% and are at least marginally significant when the in-sample period is at least two years.

Panel B presents similar results as Panel A, but uses 5-days per week Bitcoin returns so they that can be matched with the equity- and bond-predictor variables. The R^2_{OS} drop relative to those in Panel A, but remain positive and non-trivial for several of the $\log(MA_{t-1}(L)/P_{t-1})$ and the associated three-pass-filter and MEAN forecasts. Moreover, the R^2_{OS} remain at least marginally significant for the three-pass-filter and MEAN forecasts using the three-year in-sample period.

In contrast, Panel C highlights that VIX, $BILL$, $TERM$, and DEF all have $R^2_{OS} < 0$ individually, regardless of in-sample length. Moreover the PLS forecasts based on these predictors also have $R^2_{OS} < 0$ as does the MEAN forecast for all but one in-sample length; both combination procedures are also insignificant for all sample periods. Thus, the technical indicators predict Bitcoin returns in real time while other common return predictors do not.

Figure 2 depicts the time-series of differences in cumulative-squared prediction errors for each forecast in Panel A. An upward slope indicates that the stated forecast is more accurate than the historical average. Each forecast exhibits sharp swings in predictability during the beginning of the sample and weak predictability for the 3-year window spanning mid-2013 through mid-2017 before mostly increasing in the second half of 2017.

It is not entirely clear from the results in Table 4, however, that multiple MA horizons jointly predict Bitcoin returns out-of-sample. For example, the combination forecasts' R^2_{OS} are not greater than the greatest R^2_{OS} of the individual $\log(MA_{t-1}(L)/P_{t-1})$-based forecasts. Hence, we examine whether the different out-of-sample forecasts "encompass" each other, that is subsume each others' predictive information or conversely the different strategies contain useful different marginal information. To demonstrate that the different MA strategies possess different information we use Harvey, Leybourne and Newbold out-of-sample encompassing tests in Table 5. The null hypothesis is that the out-of-sample forecasts in the column heading encompasses the forecast in the row implying that the column heading forecasts contain all the relevant information; whereas, the alternative hypothesis (a probability less than .05) is that the column's forecast does not encompass the forecast in the row and hence the row forecasts contain additional useful information.

Results in the first column of Table 5 shows that the historical average forecast does not encompass any other forecast; this implies the moving average forecasts and their combinations (PLS or MEAN) contain useful incremental out-of-sample predictive information. More importantly, not one of the $\log(MA_{t-1}(L)/P_{t-1})$-based forecasts that have a corresponding $R^2_{OS} > 0$ in Table 4 encompass all of the others. For instance, inspection of the MA20 column indicates that we can reject that the MA5, MA10, and M100 have similar information; however, we can not reject the null of encompassing for the MA50, suggesting that only the MA20 and MA50 forecasts contain similar information. Overall, most different MA horizons add significantly distinct out-of-sample forecasts information for out-of-sample forecasting of Bitcoin returns.

D. Performance of trading strategies

We define the buy-sell indicator (buy=1, sell=0) associated with each MA strategy as:

$$S_{L,t} = \begin{cases} 1, \text{ if } P_{t-1}/MA_{t-1}(L) > 1 \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

The return on the Bitcoin MA strategies on day t are given by:

$$r_t^{MA(L)} = S_{n,t} \cdot R_t + (1 - S_{n,t}) \cdot R_{ft}, \; n = 5, \; 10, \; 20, \; 50, \; 100, \tag{10}$$

where r_t and r_{ft} denote, respectively, the return on Bitcoin and the risk-free rate on day t. Intuitively, the trading strategy defined by Eq. (**??**) captures the predictability of Bitcoin by short-term trends discussed theoretically in Section **??**. We denote the excess return of the buy-and-hold position in Bitcoin as rx_t and the excess return on the MA strategies by $rx_t^{MA(L)}$.

To assess the economic significance of Bitcoin-return predictability to investors, we examine the performance of the MA strategy's performance relative to a buy-and-hold position (e.g., **???**). Table 6 presents summary statistics for the buy-and-hold and MA strategies. Panel A, which uses the full sample (10/27/2010–1/31/2018), shows that all strategies are right-skewed and have fat tails. The Sharpe ratio of Bitcoin is 1.9, which is about five times the historical Sharpe ratio of the stock market (e.g., **?**). All of the MA strategies further increase this ratio to 2.1 to 2.5. The maximum drawdown of Bitcoin is 89.5%, while those of the MA strategies are all lower, ranging from 64.3% to 70.3%. Panel B indicates that the performance of Bitcoin was higher during the first half of the sample, although the performance gains of the MA strategies are similar in magnitude as those exhibited during the full sample. Similarly, Panel C shows that the baseline performance of Bitcoin remains strong in the second half of the sample with a Sharpe ratio of 1.5, however the MA strategies still improve performance (with Sharpe ratios up to 2.1 and drawdowns reduced from 73.4 to as low as 33.5%).

Panel A of Figure 3 plots the cumulative value of $1 invested in Bitcoin and the MA10 strategy at the beginning of the sample. At the end of our sample, the $1 in Bitcoin grew to $103,453 while the $1 in the buy-and-hold strategy grew to approximately $305,306, a difference of about $200,000 over 7 years! Panel B plots the drawdowns of Bitcoin and the MA10 strategy. As Panel B shows, the out-performance of the MA strategies relative to the buy and hold largely stems from the MA strategy having less severe and much shorter drawdowns than the buy-and-hold.

Table 7 presents average returns of Bitcoin on days when the five moving-average strategies indicate investment in Bitcoin (IN) and when these strategies indicate investment in Treasury bills (OUT). The results clearly show that vast majority of positive returns accrue to Bitcoin obtain on IN days. Moreover, this out-sized performance occurs over both halves of the sample.

Table 8 formally tests the performance of MA strategies relative to the buy-and-hold. Specifically, we regress the excess returns of the MA strategies on the buy-and-hold benchmark:

$$rx_t^{MA(L)} = \alpha + \beta \cdot rx_t + \varepsilon_t. \tag{11}$$

A positive alpha indicates that access to rx_t^{MAn} increases the maximum possible Sharpe ratio relative to that of a buy-and-hold Bitcoin position. The ultimate benefit of such increases to an investor is increased utility from a higher maximum Sharpe ratio for their whole portfolio. Thus, alpha only matters to the extent that it expands the mean-variance frontier. Intuitively, this expansion depends on the alpha relative to the residual risk investors must bear to capture it. The maximum Sharpe ratio (SR_{New}) attainable from access to rx_t and $rx_t^{MA(L)}$ is given by:

$$SR_{New} = \sqrt{\left(\frac{\alpha}{\sigma(\varepsilon_t)}\right)^2 + SR_{Old}^2}, \tag{12}$$

where SR_{Old} is the Sharpe ratio of rx_t (e.g., **?**). Hence, we use the appraisal ratio $\left(\frac{\alpha}{\sigma(\varepsilon_t)}\right)$ as one measure of the benefits of technical analysis to investors.

A disadvantage of the appraisal ratio is that its effect on Sharpe ratios is nonlinear. The same appraisal ratio has a greater impact on a lesser SR_{Old} than vice versa. Thus, to further facilitate comparison across assets, we measure the percentage increase in mean-variance utility, which—for any level of risk aversion—is equal to:

$$\text{Utility gain} = \frac{SR_{New}^2 - SR_{Old}^2}{SR_{Old}^2}. \tag{13}$$

? find that timing expected returns on the stock market increases mean-variance utility by approximately 35%, providing a useful benchmark utility gain.

Panel A shows that over the entire sample period, the MA strategies earn significant α with respect to rx_t of 0.11% to 0.24% per day. These alphas lead to economically large utility gains of 18.5% to 70.3%. Panel B shows these results remain strong in the second half of the sample.

Panel C presents results with sampling over the five U.S. business days per week. The αs and utility gains remain close to those in Panel A. Panel D adds the excess return on the stock market (MKT), the Barclay's Aggregate Bond Index (AGG), and gold over the same sample as Panel C. These variables proxy for exposure to other major asset classes as well as innovations in important macroeconomic news. Panel D shows that the $rx_t^{MA(L)}$ has little exposure to the other markets and in particular, this exposure has no impact on α. Thus, the incremental performance of the MA strategies relative to the buy-and-hold position is completely unexplained by correlation with important news that drives other major markets.

A naive alternative to our discrete buy-or-sell strategies defined by Eq. (**??**) would be estimating mean-variance weights using our Bitcoin-return forecasts, and then testing whether the resulting strategy out-performs the buy-and-hold benchmark (e.g., **???**). However, this approach has several theoretical and empirical shortcomings relative to our simple MAn strategies. First, the mean-variance weights assume the investor is choosing between the market return and the risk-free asset. However, Bitcoin is a poor theoretical proxy to the market portfolio of risky assets. Second, prior to 2017, investors could not sell Bitcoin short or buy it on margin or via futures contracts. Hence, the weights on Bitcoin should be constrained between zero and one. Thus, the mean-variance-weights approach could only outperform the MA strategies by choosing optimal variation between zero and one. This in turn exacerbates the following two problems: (i) that the mean-variance weights require at least two estimated forecasts, and therefore come with substantial estimation error, and (ii) the mean-variance weights assume for tractability the mean-variance functional form of investor utility. While a common assumption, mean-variance utility is unlikely to precisely capture the behavior of a representative investor.

In contrast, our discrete MA strategies satisfy are based on a directly observable out-of-sample signals and require no estimation error. They also make no assumption about the utility of underlying investors. Thus, the strong performance of our MA strategies relative to the buy-and-hold position already provide evidence of great economic significance of out-of-sample predictability of Bitcoin returns by technical indicators based on multiple MA horizons. Moreover, this strong performance relative to the buy-and-hold precludes the need for more sophisticated methods to demonstrate the economic significance of out-of-sample predictability by technical indicators.

E. Subperiods and major episodes

Despite Bitcoin's rapid increases over the past few year, its performance has been plagued by several bear markets. A Fortune magazine article (**?**) examines the explanations behind five major Bitcoin crashes, and a more recent article (**?**) details explanations behind the three bear market crashes of 2017. We briefly summarizes the cause of these downturns from these articles, and document the performance of our technical trading strategies during these identified episodes. The evidence clearly shows that the moving average indicators relatively quickly indicated exit and thus minimized large draw-downs.

An outage at Mt Gox, the prominent Bitcoin trading platform at the time in April of 2013, lead to uncertainty about the platform and a collapse in Bitcoin from \$230 to \$93, a decline of nearly 60%. The MA5, MA10 and MA20 declined however to \$165, \$125 and 117, or declines of 28%, 46% and 49%, respectively. Fortune further details the second decline to overzealous US regulators in the late fall of 2013; Bitcoin fell from $1125 to $522, or nearly 54%. The MA5, MA10 and MA20 declined however to $900, $603 and $820, or declines of 20%, 46% and 27%. Third, the collapse of Mt Gox then in February 2014 lead to an additional fall from $855 to $598, nearly 40%. The MA5, MA10 and MA20 declined however to $900, $603 and $820, or declines of 20%, 46% and 27%. In all three cases, the drawdown's figure clearly illustrates that the MA predictors quickly indicated exit and hence saved investors money.

Fortune and Cointelegraph both discuss the Summer of 2017 due to uncertainty about the hard-fork of Bitcoin splitting into two coins: Bitcoin and Bitcoin cash. Bitcoin fell from $3019 to $1939, or approximately 56%. The MA5, MA10 and MA20 declined however to $2605, $2417 and $2183, or declines of 16%, 25% and 38%. Then the Chinese in September of 2017 threatened to crack down on Bitcoin, contributing to fall from $4950 to $3003 or 27%. The MA5, MA10 and MA20 declined however to $4663, $3581 and $4217, or declines of 14%, 28% and 15%. Cointelegraph lastly documents a recent crash attributable to Bitcoin limiting its blocksize, and prompted an exit to Bitcoin cash which increased 40%, while Bitcoin fell from $7559 to $5857, more than 21%. The MA5, MA10 and MA20 declined to $7147, $6570, $6337, or falls of 4%, 12% and 15%, respectively.

During the writing of this article in early 2018, Bitcoin experienced its first downturn of 2018 due in part to crackdown in China and Korea and fears of regulation of the U.S. that began in mid December. Increased scrutiny by regulators lead to a fall of 59.9%; however, the MA5, MA10 and MA20 experienced losses of 26.4%, 23.7% and 38.8%. Similarly to the above patterns, the MA strategy relative quickly indicated to sell Bitcoin and hence minimized large losses.[6] Overall, in all seven episodes, the three sharp declines in 2013-2014, the three bear markets of 2017, and the recent bear market in early 2018, the MA5, MA10 and MA20 decisively outperformed a buy-and-hold strategy since they relatively quickly indicated exit and hence minimized large losses.

F. Performance of trading strategies applied to Ripple

To examine the robustness of our trading strategy performance, Table 9 presents performance results similar to those above for Ripple, which is the third largest digital currency by market

[6] These percentages reflect declines to early February, 2018; even if Bitcoin losses continue, the MA indicators have all indicated exit.

capitalization and has existed since 2013.[7] Panel A shows that all the MA strategies except MA100 increase Sharpe ratios relative to the buy-and-hold strategy by up to 0.55. Each strategy also reduces the maximum drawdown of the buy-and-hold by about 10%-24%.

Figure **??** further shows that the representative MA10 strategy not only substantially limits the Ripple's large maximum drawdowns over multiple episodes, but also the duration of drawdowns. Panel B shows that the MA5, MA10, and MA20 strategies earn significant alpha with respect to the buy-and-hold Ripple strategy and lead to large utility gains. Overall, the performance of our MA strategy does not appear unique to Bitcoin.

G. Comparison with NASDAQ

Over our sample, Bitcoin exhibits a substantial run-up in prices, with many market observers anticipating a crash, citing the eventual bursting of many so-called "bubbles".[8] It is therefore interesting to assess how MA strategies would help investors improve performance in prior extreme run-ups that were followed by extreme market crashes. The equilibrium theory underlying technical analysis is not restricted to Bitcoin, so this exercise also assesses the robustness of our MA strategy results using Bitcoin. We apply each of the MA strategies defined by Eq. (**??**) to the total return on the NASDAQ composite index using daily data over the sample 1996–2005, a ten year window approximately centered around the peak of the NASDAQ "bubble".

Bitcoin shares several common features with the NASDAQ in the late 1990s. These include difficulty valuing the underlying asset with fundamentals and rapid run-ups in price. Thus, the current intense interest in blockchain technology including its value as a medium of exchange and its ultimate long-run commercial monetary worth is similar to the frenzy around internet stock in the late 1990s. Many dot-com companies for years had no earnings and lacked measurable fundamentals. Several of these companies went bankrupt such as Pets.com, while others proved commercially viable. If our technical strategies are applied to the NASDAQ during this period, could they have out-performed a buy-and-hold during the run-up, and could they have avoided large losses on the way-down? We demonstrate that the answer to both questions is—yes. Hence the NASDAQ stock price episode provides robust evidence of technical trading for firms in an

[7] Ethereum is the second largest by market capitalization but its data began several years after Ripple. Hence, though the results are similar, we report only those on Ripple for brevity.

[8] E.g., the "Dutch Tulip mania", the "dot.com crash", the "CDO/MBS" crash, etc.

industry that is newly emerging without clear fundamentals.

Figure **??** depicts the performance of the buy-and-hold position in NASDAQ relative to the MA10 strategy. Panel A shows the MA10 increases more steadily than NASDAQ, avoiding much of the latter's peaks and valleys. The MA10 returns \$3.66 at the end of 2005 to an investor with a \$1 investment at the beginning of 1996. Conversely, a buy-and-hold investor in NASDAQ would have about half (\$1.85) of the accumulated value. Panel A shows that much of the performance gains from the MA strategy come from avoiding most of NASDAQ's large crash in the early 2000's. Panel B further shows that the MA10 did not just avoid the largest crash, however. The strategy avoids the majority of each of the non-trivial NASDAQ drawdown before the crash as well.

Table 10 documents the performance of the MA strategies applied to the NASDAQ relative to the buy-and-hold strategy. Results in Panel A show that the MA10, MA20 and MA50 methods particularly possess mean returns more than four percent greater than the 7.3% buy-and-hold of the NASDAQ. Further, all five methods substantially boost the NASDAQ Sharpe ratio of 0.25; e.g., MA10, MA20 and MA50 possess Sharpe ratios of .65 to .71. The last column documents that the MA strategies also greatly reduce the maximum drawdown of NASDAQ (77.9%) to 25.7%–44.7%.[9]

Panel B of Table 10 presents the alphas, appraisal and utility gain for the NASDAQ. Results document significant alpha for MA10 to MA100 strategies. It also reveals very high utility gains for all five strategies; for investors that use the MA10, MA20 and MA50 methods, the utility gains exceed 600% relative to a buy-and-hold. Overall, the same technical strategies that produce significant performance improvements in Bitcoin and Ripple also produce significant performance improvements in NASDAQ, largely by reducing the length and severity of drawdowns.

H. Volume implications of our model

In our model, trading is based on moving-average indicators across different horizons. Testing this refutable implication provides an opportunity to validate our model's mechanism in explaining the predictability of Bitcoin by moving averages of multiple horizons.

We test for volume generated by technical trading in two ways. First, we evaluate whether increases in total turnover implied by different MA signals also leads to higher Bitcoin volume.

[9]We also applied our strategies to the Nasdaq over the past ten years using our framework, which follows the maturation of internet-based technologies and an increased understanding of fundamentals. These untabulated results show that the MA strategies no longer earn significant alpha or produce Sharpe ratio gains.

We measure this total turnover by the sum of the turnover generated by each moving-average buy-sell indicator $S_{L,t}$: $\sum_L |\Delta S_{L,t}|$. Second, we evaluate whether disagreement among MA buy-sell indicators ($S_{L,t}$) is associated with higher trading volume. Intuitively, if technical traders disagree, they will trade with each other. As a measure of disagreement, we use the cross-sectional standard deviation of the signed turnover implied by each MA strategy, denoted $\sigma_L(\Delta S_{L,T})$. For each measure, Table 11 presents estimations of regressions of the form:

$$\Delta \log(volume_t) = a + b \cdot X_t + c|r_t| + \varepsilon_t, \tag{14}$$

where the X_t denotes one or both of our two volume-inducing variables. Because large price shocks are the main empirical determinant of volume and are likely correlated with our price-based indicators, we also control for the absolute value of returns (see, e.g., **?**). We use change in log volume as the dependent variable because the level of volume is not stationary.

Results in column (1) of Table 11 demonstrate that increases in turnover across multiple horizons lead to increases in volume, controlling for price shocks. Then, column (2) shows that increases in disagreement among MA traders also leads to significant increases in volume. Finally, column (3) indicates that turnover and disagreement across MA traders jointly predicts volume, with both measures positively related to volume. Overall, the results in Table 11 are consistent with traders using MA strategies significantly impacting trading volume in Bitcoin.

V. Conclusion

Bitcoin and cryptocurrencies are increasingly attracting attention from investors and financial institutions. This has lead for instance to recent Bitcoin ETF trading by large financial institutions such as Fidelity and futures trading in the US. Yet, there is a lack of both empirical and theoretical evidence on the investment properties of Bitcoin.

Since Bitcoin has no obvious fundamentals to analyze, it is therefore a natural laboratory to use technical analysis. In this paper, we propose a new theory that predicts that moving averages of prices over different horizons should jointly predict Bitcoin returns. Predictive regressions confirm the joint predictability of Bitcoin returns by moving averages over multiple horizons. Moreover, combining these signals yields significant out-of-sample predictability. Simple real-time trading strategies that exploit this predictability significantly outperform the buy-and-hold position in Bitcoin, with increases in Sharpe ratio of 0.2–0.6. We show technical strategies relative to a buy-and-hold stems from its ability to exit a down market, thereby decreasing the length and severity of drawdowns. We further test an implication from our model and find that increases in turnover from differences in signals across multiple horizons boosts trading volume.

Bitcoin skeptics argue that the behavior of cryptocurrency prices resembles that of the NASDAQ "bubble". Applying the same trading strategies to the NASDAQ index around the rise and fall in the early 2000's would have spared investors from much of the crash. Our trading-strategy results suggest that if Bitcoin exhibits a massive devaluation similar to that of "bursting of the NASDAQ bubble", technical analysis could protect investors from much of resulting losses.

Appendix: Proof of Proposition 1

In this appendix, we present the proof of Proposition 1.

First we provide the evolution equations for conditional expectation and the conditional variance. Following the standard filtering theory, $\forall i = 1, 2$, M_t^i satisfies

$$dM_t^i = \mu(\bar{X} - M_t^i)dt + \sigma_M^i(t)d\hat{Z}_{1t}^i, \quad M_0^i = M^i(0^-), \tag{A.1}$$

where $\hat{Z}_{1t}^i$ is the (observable) innovation processes satisfying

$$\hat{Z}_{1t}^i = \int_0^t \frac{X_s - M_s^i}{\sigma_\delta} ds + Z_{1t},$$

$\sigma_M^i(t) = \frac{V^i(t)}{\sigma_\delta} + \rho\sigma_X$, $V^i(t) \equiv E[(X_t - M_t^i)^2 | \mathcal{F}_t^i]$ is the conditional variance of X_t satisfying

$$\frac{dV^i(t)}{dt} = -2\mu V^i(t) + \sigma_X^2 - \left(\frac{1}{\sigma_\delta}V^i(t) + \rho\sigma_X\right)^2, \quad V^i(0) = V^i(0^-). \tag{A.2}$$

This implies that

$$\frac{d\delta_t}{\delta_t} = M_t^i dt + \sigma_\delta d\hat{Z}_{1t}^i, \quad i = 1, 2. \tag{A.3}$$

Next we derive the optimal trading strategy and equilibrium Bitcoin price. As in **?**, **?**, and **?**, the investor's problem is separable in inference and optimization.[10] In particular, given the initial endowment $\eta_i > 0$ and the prior $(M_i(0^-), V_i(0^-))$, Investor i's portfolio selection problem is equivalent to

$$\max_{\theta^i, C^i} E \int_0^T e^{-\beta t} \log C_t^i dt,$$

subject to

$$dW_t = r_t W_t dt + \theta_t^i(\mu_t^i - r_t)dt + \theta_t^i \sigma_\delta d\hat{Z}_{1t}^i - C_t^i dt. \tag{A.4}$$

Define π_t^i as the state price density for investor i. Then

$$d\pi_t^i = -r_t \pi_t^i dt - \kappa_t^i \pi_t^i d\hat{Z}_{1t}^i, \tag{A.5}$$

[10] The separation principle trivially applies because the objective function is independent of the unobservable state variable (see, e.g., Fleming and Rishel (1975, Chap. 4, Sec. 11) .

where κ_t^i is the price of risk perceived by investor i, i.e.,

$$\kappa_t^i = \frac{\mu_t^i - r_t}{\sigma}. \tag{A.6}$$

Using the standard dual approach (e.g., **?**) to solve Investor i's problem, we have

$$e^{-\beta t}(C_t^i)^{-1} = \lambda_i \pi_t^i. \tag{A.7}$$

Define

$$\alpha_t = \frac{\lambda_1 \pi_t^1}{\lambda_2 \pi_t^2} \tag{A.8}$$

to be the ratio of the marginal utilities. Then α_t evolves as

$$d\alpha_t = -\alpha_t \mu_t^d d\hat{Z}_{1t}^1, \quad \mu_t^d = \frac{\mu_t^1 - \mu_t^2}{\sigma}, \quad \alpha_0 = \frac{\eta_2}{\eta_1}, \tag{A.9}$$

where the first equality is from Ito's lemma and the consistency condition (i.e., the Bitcoin price is the same across all investors):

$$\frac{\mu_t^1 - \mu_t^2}{\sigma} = \frac{M_t^1 - M_t^2}{\sigma_\delta},$$

and the last equality follows from the budget constraints.

By market clearing condition $C_t^1 + C_t^2 = \delta_t$, we have

$$C_t^1 = \frac{\delta_t}{1+\alpha_t}, \quad C_t^2 = \frac{\alpha_t \delta_t}{1+\alpha_t},$$

$$\kappa_t^1 = \sigma_\delta + \frac{\alpha_t}{1+\alpha_t}\mu_t^d, \quad \kappa_t^2 = \sigma_\delta - \frac{1}{1+\alpha_t}\mu_t^d,$$

$$r_t = \beta + \frac{1}{1+\alpha_t}M_t^1 + \frac{\alpha_t}{1+\alpha_t}M_t^2 - \sigma_\delta^2.$$

Therefore, the fraction of wealth invested in the Bitcoin by Investor 1 is

$$\kappa_t^1/\sigma,$$

i.e.,

$$1+\frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma}, \tag{A.10}$$

and by Investor 2 is

$$1-\frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma}. \tag{A.11}$$

So if $\mu_t^d > 0$, i.e., Investor 1 is more optimistic than Investor 2, then Investor 1 borrows to buy the Bitcoin, and Investor 2 sells the Bitcoin and lends.

The Bitcoin price

$$B_t = E_t^1 \int_t^T \frac{\pi_s^1}{\pi_t^1}\delta_s ds = \frac{1-e^{-\beta(T-t)}}{\beta}\delta_t,$$

which implies that

$$dB_t = ((\beta + M_t^i)B_t - \delta_t)dt + \sigma_\delta B_t d\hat{Z}_{1t}^i,$$

$$\mu_t^i = \beta + M_t^i, \mu_t^d = \frac{M_t^1 - M_t^2}{\sigma_\delta}, \sigma = \sigma_\delta.$$

This implies that

$$d\hat{Z}_{1t}^i = \frac{1}{\sigma_\delta}\left(d\log B_t - \left(M_t^i - \frac{\beta}{1-e^{-\beta(T-t)}} - \frac{1}{2}\sigma_\delta^2\right)dt\right).$$

Investor 1's wealth is

$$W_{1t} = E_t^1 \int_t^T \frac{\pi_s^1}{\pi_t^1} C_s^1 ds = \frac{1-e^{-\beta(T-t)}}{\beta} C_t^1 = \frac{1}{1+\alpha_t} B_t$$

and Investor 2's wealth is

$$W_{2t} = E_t^1 \int_t^T \frac{\pi_s^2}{\pi_t^2} C_s^2 ds = \frac{1-e^{-\beta(T-t)}}{\beta} C_t^2 = \frac{\alpha_t}{1+\alpha_t} B_t.$$

The number of Bitcoin Investor 1 holds is equal to

$$N_{1t} = W_{1t}(1+\frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma})/B_t = \frac{1}{1+\alpha_t}(1+\frac{\alpha_t}{1+\alpha_t}\frac{\mu_t^d}{\sigma}).$$

The number of Bitcoin Investor 2 holds is equal to

$$N_{2t} = W_{2t}(1 - \frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma})/B_t = \frac{\alpha_t}{1+\alpha_t}(1 - \frac{1}{1+\alpha_t}\frac{\mu_t^d}{\sigma}).$$

We have

$$\frac{\partial N_{1t}}{\partial \alpha_t} = \frac{-(1+\alpha_t) + (1-\alpha_t)\mu_t^d/\sigma_\delta}{(1+\alpha_t)^3},$$

which is < 0 if and only if

$$\alpha_t > \frac{\mu_t^d/\sigma_\delta - 1}{\mu_t^d/\sigma_\delta + 1}.$$

Next we derive the expression of the conditional expectation M_t^i in the form of moving averages. We have

$$dM_t^i = (a^i(t) - b^i(t)M_t^i)dt + c^i(t)d\log B_t, \tag{A.12}$$

where

$$a^i(t) = \mu\bar{X} + \left(\frac{\beta}{1-e^{-\beta(T-t)}} + \frac{1}{2}\sigma_\delta^2\right)c^i(t),$$

$$b^i(t) = \mu + c^i(t), \quad c^i(t) = \frac{\sigma_M^i(t)}{\sigma_\delta}.$$

Equation (**??**) implies that

$$M_t^i = h^i(t) + \int_0^t f^i(u,t)d\log B_u,$$

where

$$h^i(t) = e^{-\int_0^t b^i(s)ds}\int_0^t a^i(u)e^{\int_0^u b^i(s)ds}du, f^i(u,t) = c^i(u)e^{\int_t^u b^i(s)ds}.$$

Then by integration by parts, we have

$$\begin{aligned} M_t^i &= h^i(t) - f^i(0,t)\log B_0 + c^i(t)\log B_t - \int_0^t \log B_u df^i(u,t) \\ &= h^i(t) + f^i(0,t)\log\frac{B_t}{B_0} + (f^i(t,t) - f^i(0,t))\left(\log B_t - \frac{\int_0^t \log B_u g^i(u,t)du}{\int_0^t g^i(u,t)du}\right), \end{aligned} \tag{A.13}$$

where

$$g^i(u,t) = \frac{\partial f^i(u,t)}{\partial u}.$$

It can be shown that $g^i(u,t) > 0$ for any u and t, and thus $\frac{\int_0^t g^i(u,t) \log B_u du}{\int_0^t g^i(u,t) du}$ is a weighted average of $\log(B_u)$ over the interval $[0,t]$. In addition, this implies that $f^i(t,t) - f^i(0,t) > 0$ for any t. This complete the proof of Proposition 1.

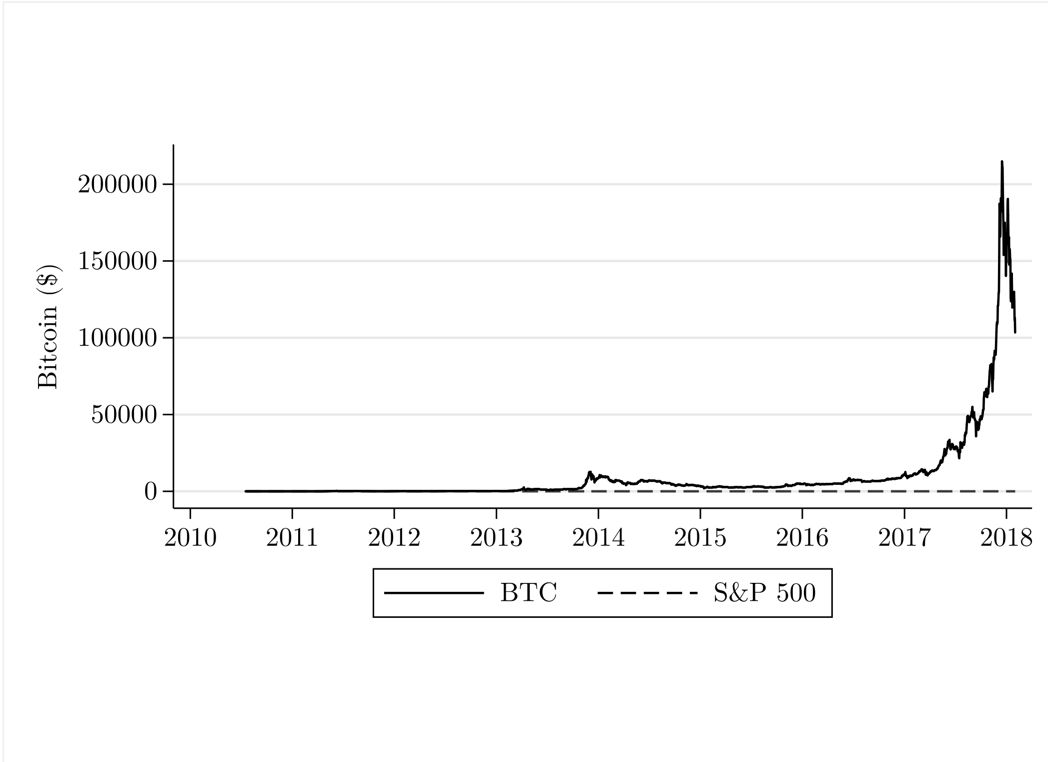


Figure 1: Value of $1 invested in Bitcoin or S&P500 on 7/18/2010

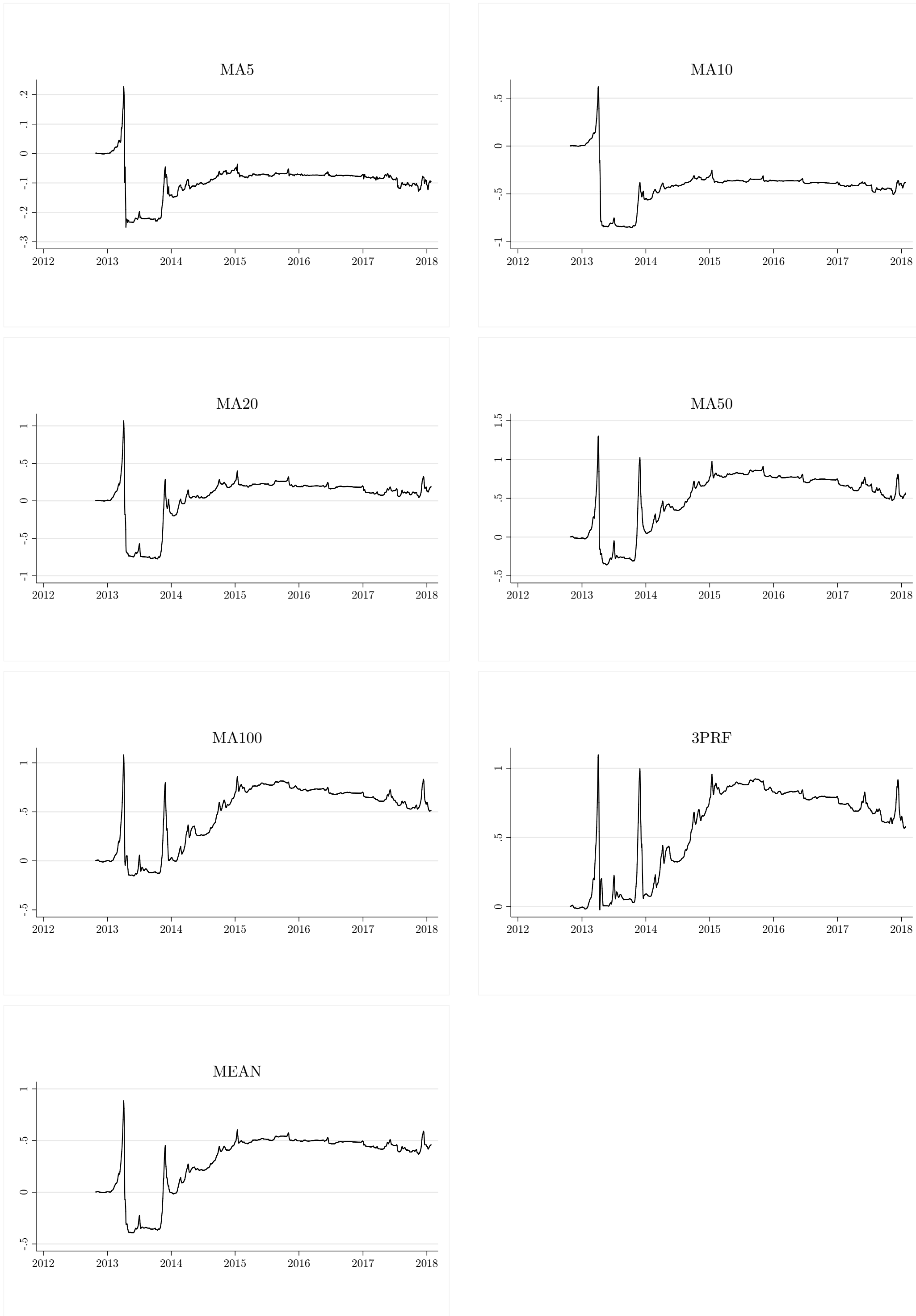
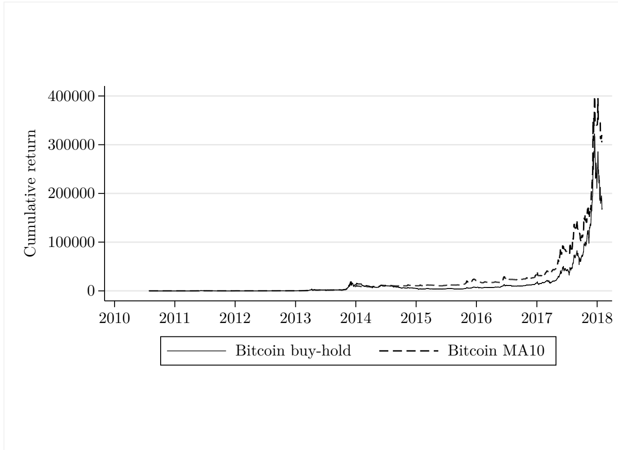
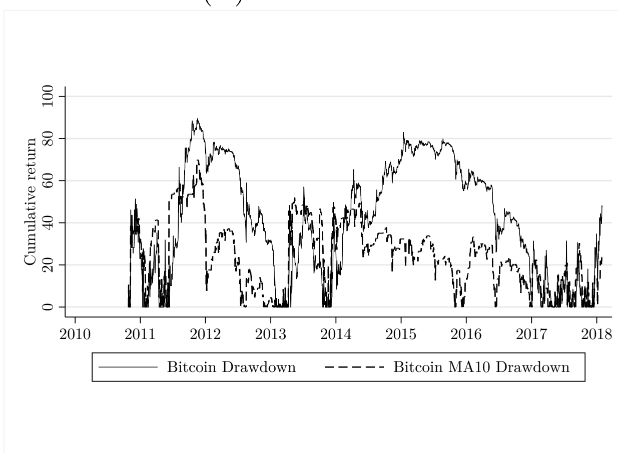


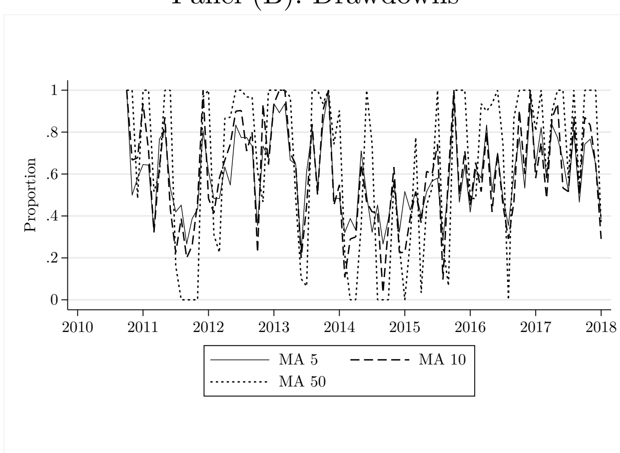
Figure 2: Cumulative squared forecast errors of Bitcoin returns based on historical-average forecasts minus those of MA strategies



Panel (A): Cumulative returns



Panel (B): Drawdowns



Panel (C): Proportion of days long in Bitcoin

Figure 3: Performance of investment in Bitcoin buy-and-hold and MA strategies.
Panel A presents cumulative returns to $1 invested in the buy-and-hold and MA10 Bitcoin strategies on 7/18/2010. Panel B presents drawdowns of each strategy in Panel A. Panel C presents the proportion of days each month that the MA strategies listed invest in Bitcoin as opposed to the risk-free rate.

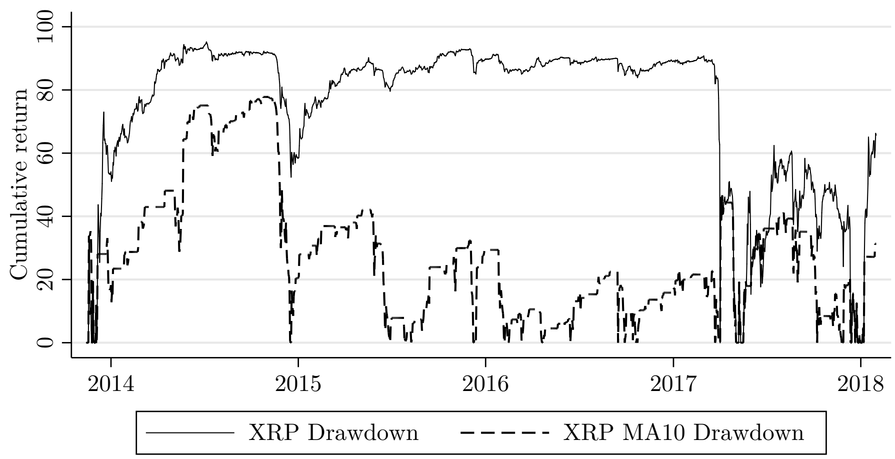
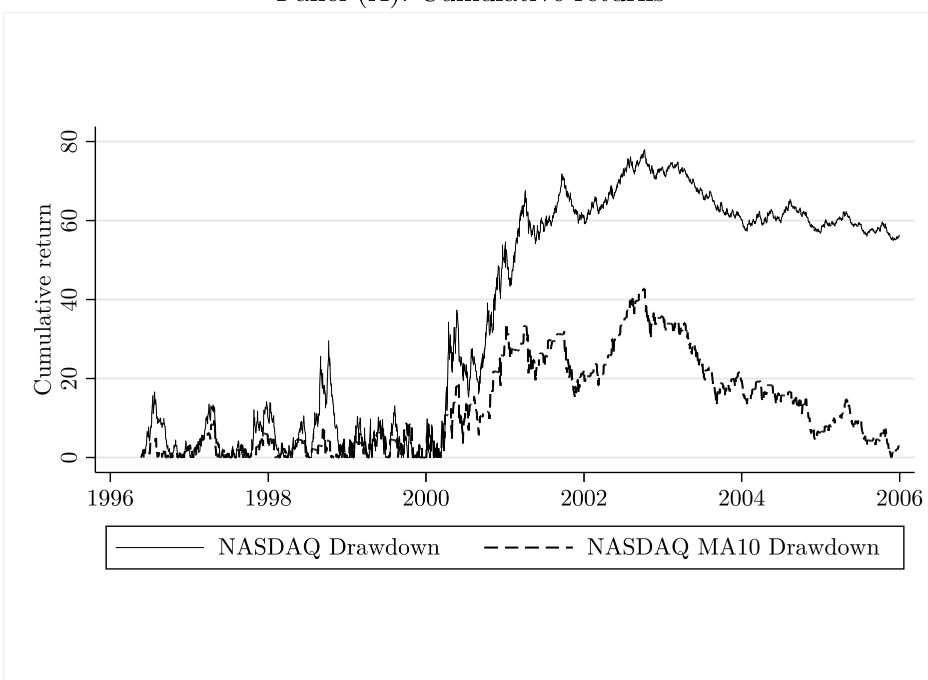


Figure 4: Drawdowns of buy-and-hold and MA10 strategies applied to Ripple.
This figure presents drawdowns of the buy-and-hold and MA10 strategies applied to Ripple from 11/12/2013 through 1/31/2018.

Panel (A): Cumulative returns



Panel (B): Drawdowns

Figure 5: Performance of investment in NASDAQ buy-and-hold and MA10 strategies.
Panel A presents cumulative returns to $1 invested in the buy-and-hold and MA10 NASDAQ strategies on 1/2/1996 through 12/30/2005. Panel B presents drawdowns of each strategy.

Table 1: Summary statistics

Panel A of this Table presents summary statistics of the returns in excess of the 1-day risk-free rate on Bitcoin (BTC), the U.S. stock market (MKT), the Barclays aggregate bond market index (AGG), and gold. Means, standard deviations, and Sharpe ratios are annualized. Panel B presents summary statistics of other relevant variables. AR1 denotes the first-order autoregressive coefficient and p_{df} denotes the p-value from an augmented Dickey-Fuller test for the null of a unit root. The sample period is daily from 10/27/2010−1/31/2018. Bitcoin returns trade 7 days a week and have 2654 observations during the sample period. Other variables are available only 5 days a week and have 1,896 observations during this period.

Panel A: Returns								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	AR1
BTC	213.58	111.04	1.92	-38.83	52.89	0.92	15.65	0.06
MKT	14.43	14.6	0.99	-6.97	4.97	-0.45	8.23	-0.08
AGG	2.68	3.27	0.82	-1.01	0.74	-0.3	4.13	-0.05
GOLD	-0.69	16.2	-0.04	-9.07	4.69	-0.52	8.71	-0.03

Panel B: Predictor variables								
	Mean(%)	SD(%)	Min(%)	Max(%)	Skewness	Kurtosis	AR1	p_{df}
VIX	16.29	5.55	9.14	48	2.05	8.42	0.96	0
BILL	0.23	0.34	-0.02	1.45	2	5.94	1	1
TERM	2.05	0.59	1	3.6	0.42	2.5	1	0.45
DEF	0.96	0.25	0.53	1.54	0.53	2.22	1	0.62

Table 2: Variance ratio tests of the random walk hypothesis

The variance ratios are denoted $VR(q)$ with the corresponding heteroscedasticity-robust test statistics and p-values denoted z^* and p_{z^*}, respectively. Under the random walk null hypothesis, the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). The sample period is 7/18/2010−1/31/2018.

q	$VR(q)$	z_s^*	p_{z^*}
14	1.237	1.623	0.104
28	1.575	2.967	0.003
56	2	3.919	0
112	2.211	3.649	0

Table 3: In-sample predictability of Bitcoin returns

This table presents estimates of predictive regressions of the form: $r_{t+1,t+5} = a + b'X_t + \epsilon_{t+1,t+5}$, where $r_{t+1,t+5}$ denotes the log return on Bitcoin over business days $t+1$ through $t+5$ (one week). We use present Newey and West (1987) standard errors below the point estimates to correct for heteroscedasticity and serial correlation in return observations. In Panel A, the predictors are the log moving average/price ratios $(\log(MA(L)/P))$ or their first three principal components $(PC1, PC2, or\ PC3)$. In Panel B, the predictors include these principal components along with the other return predictors (*VIX*, *BILL*, *TERM*, and *DEF*). The sample period is 7/18/2010—1/31/2018 ($n = 1896$).

Panel A: Predictability of Bitcoin returns by log moving average/price ratios							
	(1)	(2)	(3)	(4)	(5)		(6)
$\log(MA_t(5)/P_t)$	-17.90					$PC1_t$	-1.36
	(-14.43)						(0.64)
$\log(MA_t(10)/P_t)$		-18.10				$PC2_t$	1.67
		(-12.75)					(-0.79)
$\log(MA_t(20)/P_t)$			-17.95			$PC3_t$	1.36
			(-8.47)				(1.86)
$\log(MA_t(50)/P_t)$				-12.00			
				(-4.60)			
$\log(MA_t(100)/P_t)$					-6.61		
					(-2.95)		
R^2	0.00	0.01	0.03	0.04	0.03		0.04

Panel B: Predictability of Bitcoin returns by common return predictors						
	(1)	(2)	(3)	(4)	(5)	(6)
VIX_t	-1.84				-2.54	-1.74
	(-0.65)				(-0.80)	(-0.94)
$BILL_t$		0.73			2.14	1.55
		(0.65)			(1.06)	(1.27)
$TERM_t$			0.99		2.94	2.19
			(0.86)		(1.18)	(1.30)
DEF_t				-0.49	2.05	1.54
				(-.58)	(0.77)	(0.92)
$PC1_t$						-1.01
						(-0.69)
$PC2_t$						1.09
						(0.92)
$PC3_t$						1.86
						(2.02)
R^2	0.02	0.01	0.00	0.00	0.04	0.05

Table 4: Out-of-sample predictability of Bitcoin-returns

This table presents out-of-sample R^2 for predictive regressions of the form: $r_{t+1,t+7} = a + b'X_t + \epsilon_{t+1,t+7}$, where $r_{t+1,t+7}$ denotes the log return on Bitcoin over (calendar) days $t+1$ through $t+7$. T_0 denotes the in-sample period in days. In columns denoted MAL, the only predictor is $\log(MA(L)/P)$. In columns denoted 3PRF, the forecast is based on the 3-pass regression filter of Kelly and Pruitt (2013) using all five $\log(MA(L)/P)$. In columns denoted MEAN, the forecast is the simple average of the five forecasts based on each $\log(MA(L)/P)$. Panel A presents results using 7-day frequency. Panel B presents similar results as Panel A for 5-day-per week data to facilitate comparison with Panel C, which presents results using *VIX*, *BILL*, *DEF*, and *TERM* as predictors. The in-sample plus out-of-sample period is 10/27/2010−1/31/2018 (n=2,654 in Panel A, and n=1,823 in Panels B and C). Brackets present p-values for the null that the adjacent out-of-sample R^2 is zero.

Panel A: 7-day-per-week observations							
T_0	MA5	MA10	MA20	MA50	MA100	3PRF	MEAN
365	-0.41	-1.28	0.15	1.46	0.65	0.91 [.124]	1.17 [.050]
730	-0.28	-1.12	0.57	1.68	1.53	1.71 [.067]	1.37 [.098]
1095	0.56	2.04	4.23	3.91	2.87	2.46 [.033]	3.66 [.016]
Panel B: 5-day-per-week observations							
T_0	MA5	MA10	MA20	MA50	MA100	3PRF	MEAN
252	-0.82	-2.7	-1.02	0.63	0.17	0.54 [.098]	0.22 [.249]
504	-0.56	-2.49	-0.68	0.7	0.83	1.07 [.088]	0.35 [.277]
756	0.71	2.38	4.33	3.69	2.64	2.16 [.055]	3.6 [.022]
Panel C: 5-day-per-week observations							
T_0	VIX	BILL	TERM	DEF		3PRFcro	MEANcro
252	-5.61	-2.19	-7.56	-4.46		-5.45 [.541]	-1.31 [.343]
504	-1.89	-1.65	-3.32	-2.36		-1.9 [.333]	-0.07 [.282]
756	-3.56	-3.73	-2.94	-3.64		-3.61 [.503]	-0.85 [.516]

Table 5: Encompassing tests

This table reports p-values for the Harvey, Leybourne, and Newbold (1998) statistic. The statistic corresponds to a one-sided (upper-tail) test of the null hypothesis that the 7-day Bitcoin-return forecast defined in the column heading encompasses the forecast defined by the row heading, against the alternative hypothesis that the forecast given in the column does not encompass the forecast given in the row. The forecasts are the out-of-sample forecasts defined in Table 4 Panel A along with the historical-average forecast ($\bar{r}$). The sample period is 10/27/2010−1/31/2018 with an in-sample period of $T_0 = 730$.

	$\bar{r}$	MA5	MA10	MA20	MA50	MA100	3PRF	MEAN
$\bar{r}$		0.349	0.274	0.313	0.302	0.352	0.389	0.402
MA5	0.000		0.000	0.000	0.000	0.000	0.000	0.000
MA10	0.000	0.999		0.000	0.000	0.000	0.000	0.000
MA20	0.000	0.997	0.998		0.000	0.000	0.000	0.989
MA50	0.000	0.720	0.703	0.672		0.960	0.344	1.000
MA100	0.000	0.270	0.160	0.000	0.000		0.001	0.995
3PRF	0.000	0.188	0.112	0.001	0.000	0.852		0.811
MEAN	0.000	0.764	0.234	0.000	0.000	0.000	0.000	

Table 6: Performance of Bitcoin trading strategies

This table presents summary statistics of the returns in excess of the 1-day risk-free rate on Bitcoin (BTC) and each of the MA(L) Bitcoin strategies. Means, standard deviations, and Sharpe ratios are annualized. The sample period is daily from 10/27/2010−1/31/2018. Panel A presents full sample results (n=2,654). Panels B and C, respectively, present results for the first and second halves (n=1,327) of the sample. MDD denotes maximum drawdown.

	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
Panel A: Full-sample								
BTC	213.58	111.04	1.92	-38.83	52.89	0.92	15.65	89.48
MA5	209.78	83.59	2.51	-29.41	52.89	2.62	31.84	66.44
MA10	208.49	85.1	2.45	-38.83	52.89	2.21	32.38	69.75
MA20	196.91	86.13	2.29	-38.83	52.89	2.08	31.71	64.25
MA50	195.98	91.81	2.13	-38.83	52.89	1.6	26.03	70.28
MA100	203.37	97.54	2.09	-38.83	52.89	1.61	22.76	70.28
Panel B: First-half (10/27/2010-6/25/2014, n=1,327)								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
BTC	349.71	142.6	2.45	-38.83	52.89	0.81	11.53	89.48
MA5	339.7	108.66	3.13	-29.41	52.89	2.25	22.44	66.44
MA10	331.54	111.98	2.96	-38.83	52.89	1.77	22.07	69.75
MA20	316.66	114.11	2.77	-38.83	52.89	1.7	21.3	64.25
MA50	309.67	121.24	2.55	-38.83	52.89	1.33	17.68	70.28
MA100	341.12	127.76	2.67	-38.83	52.89	1.37	15.68	70.28
Panel C: Second-half (6/26/2014-1/31/2018, n=1,327)								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
BTC	104.64	71.95	1.45	-21.9	25.41	0.18	9.39	73.37
MA5	108.74	51.97	2.09	-14.24	25.41	1.63	17.84	34.28
MA10	103.43	51.12	2.02	-11.13	25.41	1.68	18.53	33.5
MA20	95.15	50.04	1.9	-11.13	22.97	0.93	14.17	38.76
MA50	100.65	55.77	1.8	-16.51	22.97	0.52	11.91	34.71
MA100	88.96	61.37	1.45	-16.73	25.41	0.39	12.7	46.77

Table 7: Returns to Bitcoin when MA strategies are invested vs not invested

This table presents average returns to Bitcoin on days when the $\mathtt{MA}(L)$ strategy defined in the column heading is long Bitcoin (IN) and days when the strategy is not (OUT). The sample period is daily from 10/27/2010–1/32/2018. Panel A presents full sample results (n=2,654). Panels B and C, respectively, present results for the first and second halves (n=1,327) of the sample.

	All Days	MA5	MA10	MA20	MA50	MA100
Panel A: Full sample						
OUT	0.59%	0.01%	0.01%	0.05%	0.05%	0.03%
IN		0.58%	0.57%	0.54%	0.54%	0.56%
Panel B: First half						
OUT	0.90%	0.06%	0.07%	0.17%	0.25%	0.04%
IN		1.41%	1.37%	1.31%	1.20%	1.23%
Panel C: Second Half						
OUT	0.29%	-0.01%	0.00%	0.03%	0.01%	0.04%
IN		0.30%	0.28%	0.26%	0.28%	0.24%

Table 8: Alphas of MA Bitcoin strategies relative to buy-and-hold benchmark and other asset classes

Panels A, B and C presents regressions of the form: $rx_t^{\mathsf{MA}(L)} = \alpha + \beta_{BTC} \cdot rx_t^{BTC} + \epsilon_t$, where rx_t denotes the day-t buy-and-hold excess return on Bitcoin and $rx_t^{\mathsf{MA}(L)}$ denotes the excess return on the $\mathsf{MA}(L)$ Bitcoin strategy. Beneath each regression is the Sharpe ratio and appraisal ratio of the MA strategy as well as the utility gain from access to $rx_t^{\mathsf{MA}(L)}$. Panel A also reports the average daily turnover (TO) of the MA strategies, the one-way transaction cost (FEE) that would be required to eliminate the alpha of the MA strategy, and the percentage of days when the return on the strategy is at least that of Bitcoin (WIN(%)). The sample period is 10/27/2010−1/31/2018. Panel A presents results for 7-day-per-week observations (n=2,654). Panel B presents the same results as Panel A, but for the second half of the sample (n=1,302). Panel C presents similar results as Panel A but using 5-day-per-week observations. Panel D uses the same sample as Panel C but also includes the excess returns on the stock market (MKT), the Barclay's Aggregate Bond Index (AGG), and gold as factors. Newey-West standard errors are below point estimates in parentheses.

Panel A: 7-day-per-week observations (N=2654)					
	MA5	MA10	MA20	MA50	MA100
β_{BTC}	0.57	0.59	0.60	0.68	0.77
	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)
$\alpha(\%)$	0.24	0.23	0.19	0.14	0.11
	(0.05)	(0.06)	(0.06)	(0.06)	(0.05)
R^2	0.57	0.59	0.60	0.68	0.77
MA Sharpe	2.51	2.45	2.29	2.13	2.09
Appraisal	1.61	1.52	1.26	0.97	0.83
Utility gain(%)	70.29	62.35	42.83	25.33	18.52
TO(%)	21.75	12.93	7.95	5.01	2.68
FEE(%)	1.12	1.76	2.36	2.74	3.96
WIN(%)	79.70	80.32	80.86	84.02	84.57

Panel B: 7-day-per-week observations, second-half subsample (N=1327)					
	MA5	MA10	MA20	MA50	MA100
β_{BTC}	0.52	0.50	0.48	0.60	0.73
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
$\alpha(\%)$	0.15	0.14	0.12	0.10	0.04
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
R^2	0.52	0.50	0.48	0.60	0.73
MA Sharpe	2.09	2.02	1.90	1.80	1.45
Appraisal	1.51	1.41	1.24	1.07	0.40
Utility gain(%)	107.25	93.42	72.52	54.28	7.63

Table 8: (continued)

Panel C: 5-day-per-week data (N=1807)					
	MA5	MA10	MA20	MA50	MA100
β_{BTC}	0.58	0.60	0.60	0.67	0.77
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)
$\alpha(\%)$	0.28	0.24	0.22	0.17	0.13
	(0.07)	(0.07)	(0.07)	(0.06)	(0.06)
R^2	0.58	0.60	0.59	0.67	0.77
MA Sharpe	2.42	2.30	2.23	2.13	2.08
Appraisal	1.48	1.29	1.18	0.97	0.84
Utility gain(%)	59.70	45.57	38.08	25.85	19.15

Panel D: Controlling for other factors at 5-day frequency (N=1793)					
	MA5	MA10	MA20	MA50	MA100
β_{BTC}	0.58	0.60	0.59	0.67	0.77
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)
β_{MKT}	8.32	10.51	8.26	-8.31	-19.86
	(12.06)	(12.22)	(12.14)	(-12.59)	(-13.15)
β_{AGG}	0.03	0.43	0.25	-0.31	-0.58
	(0.43)	(0.45)	(0.43)	(-0.43)	(-0.42)
β_{GOLD}	-0.02	-0.05	-0.07	-0.03	-0.07
	(-0.10)	(-0.08)	(-0.08)	(-0.072)	(-0.07)
$\alpha(\%)$	0.27	0.23	0.22	0.19	0.16
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
R^2	0.58	0.60	0.59	0.67	0.77

Table 9: Performance of trading strategies for Ripple

Panel A presents summary statistics of the returns in excess of the 1-day risk-free rate on Ripple and each of the $\mathsf{MA}(L)$ strategies for Ripple. The data are from 11/12/2013-1/31/2018 (n=1540). Means, standard deviations, and Sharpe ratios are annualized. MDD denotes maximum drawdown. Panel B presents regressions of the form: $rx_t^{\mathsf{MA}(L)} = \alpha + \beta_{RIPPLE} \cdot rx_t + \epsilon_t$, where rx_t denotes the day-t buy-and-hold excess return on Ripple and $rx_t^{\mathsf{MA}(L)}$ denotes the excess return on the $\mathsf{MA}(L)$ Ripple strategy. Beneath each regression is the appraisal ratio $\left(\frac{\alpha}{\sigma(\epsilon)}\right)$ of the MA strategy and the utility gain from access to $rx_t^{\mathsf{MA}(L)}$. Newey-West standard errors are below point estimates in parentheses.

Panel A: Summary Statistics for Ripple strategies

	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
Ripple	230.78	173.15	1.33	-46.01	179.37	6.69	112.33	95.21
MA5	286.22	156.52	1.83	-46.01	179.37	9.02	166.58	70.8
MA10	290.91	154.47	1.88	-46.01	179.37	9.31	175.24	77.81
MA20	255.08	155.15	1.64	-46.01	179.37	9.18	172.73	77.26
MA50	234.46	157.3	1.49	-46.01	179.37	8.74	163.85	85.52
MA100	203.94	156.98	1.3	-46.01	179.37	8.9	165.22	79.95

Panel B: Strategy alphas

	(1)	(2)	(3)	(4)	(5)
	MA5	MA10	MA20	MA50	MA100
β_{RIP}	0.82	0.80	0.80	0.83	0.82
	(0.05)	(0.05)	(0.05)	(0.05)	(0.05))
α(%)	0.27	0.29	0.19	0.12	0.04
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
R^2	0.82	0.80	0.80	0.83	0.82
Appraisal	1.47	1.54	1.01	0.67	0.22
Utility gain(%)	120.82	134.01	57.88	25.21	2.64

Table 10: Performance of trading strategies for NASDAQ: 1996-2005

Panel A presents summary statistics of the returns in excess of the 1-day risk-free rate on NASDAQ and each of the $\mathsf{MA}(L)$ NASDAQ strategies. Means, standard deviations, and Sharpe ratios are annualized. MDD denotes maximum drawdown. Panel B presents regressions of the form: $rx_t^{\mathsf{MA}(L)} = \alpha + \beta_{NASDAQ} \cdot rx_t + \epsilon_t$, where rx_t denotes the day-t buy-and-hold excess return on NASDAQ and $rx_t^{\mathsf{MA}(L)}$ denotes the excess return on the $\mathsf{MA}(L)$ NASDAQ strategy. Beneath each regression is the appraisal ratio $\left(\frac{\alpha}{\sigma(\epsilon)}\right)$ of the MA strategy and the utility gain from access to $rx_t^{\mathsf{MA}(L)}$. The sample period is daily from 1/2/1996−12/30/2005 yielding n=2,419 strategy-return observations. Newey-West standard errors are below point estimates in parentheses.

Panel A: Summary Statistics of NASDAQ strategies								
	Mean(%)	SD(%)	Sharpe	Min(%)	Max(%)	Skewness	Kurtosis	MDD(%)
NASDAQ	7.26	29.45	0.25	-9.69	14.15	0.20	6.99	77.93
MA5	7.81	18.54	0.42	-6.23	8.10	0.07	9.46	44.72
MA10	11.61	17.8	0.65	-6.23	8.10	0.05	9.12	42.72
MA20	12.45	17.6	0.71	-5.59	8.10	-0.06	8.30	25.66
MA50	12.05	17.54	0.69	-7.66	4.84	-0.43	7.30	37.24
MA100	7.16	17.22	0.42	-7.66	4.28	-0.50	7.77	41.26

Panel B: Strategy alphas					
	(1)	(2)	(3)	(4)	(5)
	MA5	MA10	MA20	MA50	MA100
β_{NASDAQ}	0.40	0.37	0.36	0.35	0.34
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
$\alpha(\%)$	0.02	0.04	0.04	0.04	0.02
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
R^2	0.40	0.37	0.36	0.35	0.34
appraisal	0.34	0.63	0.70	0.67	0.34
Utility gain(%)	193.1	656.1	803.3	743.9	184.7

Table 11: Volume and technical trading indicators

This table presents regressions of the form:

$$\Delta \log(volume)_t = a + b \cdot X_t + c \cdot |r_t| + \epsilon_t,$$

where $volume_t$ denotes the trading volume in Bitcoin on day t, $|r_t|$ denotes the absolute return on Bitcoin on day t, and X_t denotes one of two predictors. In column (1), X_t is the sum $\left(\sum_L |\Delta S_{L,t}|\right)$ of the absolute turnover's $|\Delta S_{L,t}|$ from each of the MA strategies. In column (2), X_t is the cross-sectional standard deviation $\left(\sigma_L(\Delta S_{L,t})\right)$ of $\Delta S_{L,t}$, a measure of the "disagreement" among technical traders using the different MA strategies ($L = 5, 10, 20, 50,$ or 100). In column (3), X_t includes $\sum_L |\Delta S_{L,t}|$ and $\sigma_L(\Delta S_{L,t})$. The sample is 12/27/2013–1/31/2018 (n=1496). Newey-West standard errors are below point estimates in parentheses.

	(1)	(2)	(3)
$\sum_L(\lvert\Delta S_{L,t}\rvert)$	0.31		0.32
	(0.06)		(0.06)
$\sigma_L(\Delta S_{L,t})$		0.03	0.03
		(0.01)	(0.01)
$\lvert r_t\rvert$	5.33	5.75	5.33
	(0.55)	(0.57)	(0.55)
R^2	0.18	0.17	0.18