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Market making with asymmetric information and inventory risk ☆

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Abstract

Market makers in some financial markets often make offsetting trades and have significant market power. We develop a market making model that captures these market features as well as other important characteristics such as information asymmetry and inventory risk. In contrast to the existing literature, a market maker in our model can optimally shift some trades with some investors to other investors by adjusting bid or ask. As a result, we find that consistent with empirical evidence, expected bid–ask spreads may *decrease* with information asymmetry and bid–ask spreads can be positively correlated with trading volume.

JEL classification: D42; D53; D82; G12; G18

Keywords: Market making; Illiquidity; Bid–ask spread; Asymmetric information; Imperfect competition

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1. Introduction

As shown by the existing empirical literature,[1] market makers in some financial markets tend to make offsetting trades and have significant market power. In this paper, we develop a market making model that captures these market features as well as other important characteristics such as information asymmetry and inventory risk. In contrast to the existing rational expectations models and microstructure models with information asymmetry,[2] this model introduces an alternative equilibrium setting where some uninformed investor with market power (e.g., a market maker) can optimally adjust bid or ask to shift some trades with potentially informed investors to other investors. As a result, this model can help explain the puzzle that bid–ask spreads may decrease with information asymmetry, as shown by empirical studies.[3] Moreover, we show that consistent with empirical evidence,[4] bid–ask spreads can be positively correlated with trading volume. In contrast to double auction models (e.g., Kyle, 1989; Rostek and Weretka, 2012), supply and demand function competition models (e.g., Vives, 2011), and Nash Bargaining models (e.g., Atkeson et al., 2014), some agent in our model serves a dual role: a buyer in one market and simultaneously a seller in another. Our solution shows how this dual role affects the equilibrium outcome in these markets.

Specifically, we consider a one-period model with three types of risk averse investors: informed investors, uninformed investors, and an uninformed market maker. On date 0, all investors optimally choose how to trade a risk-free asset and a risky security (e.g., a less liquid stock, a corporate bond, or a derivative security) to maximize their expected constant absolute risk averse (CARA) utility from the terminal wealth on date 1. All may be endowed with some shares of the risky security whose payoff becomes public on date 1. Informed investors observe a private signal about the date 1 payoff of the security just before trading on date 0 and thus have trading demand motivated by private information. Informed investors also have non-information-based incentives to trade, which we term as a liquidity shock and model as a random endowment of a nontradable asset whose payoff is correlated with that of the risky security. It follows that informed investors also have trading demand motivated by the liquidity needs for hedging.

Both informed and uninformed investors must trade through the market maker. We assume that the market maker posts bid and ask price schedules first as in a Stackelberg game, taking into account their impact on other investors' trading demand, other investors then trade optimally taking the posted price schedules as given.[5] The equilibrium bid and ask depths are determined by the market clearing conditions at the bid and at the ask, i.e., the total amount the market maker buys (sells) at the bid (ask) is equal to the total amount other investors sell (buy). In equilibrium, the risk-free asset market also clears.

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[1] E.g., Sofianos (1993), Shachar (2012), Garman (1976), Lyons (1995), Ang et al. (2011).

[2] E.g., Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), Glosten and Milgrom (1985), Kyle (1985), Admati and Pfleiderer (1988), Boulatov et al. (2010), Goldstein et al. (2014).

[3] For example, Brooks (1996) finds a negative relationship between bid–ask spreads and information asymmetry around earnings and dividends announcements. Huang and Stoll (1997) find that the asymmetric information component of the bid–ask spread can be *negative* and statistically significant. Acharya and Johnson (2007) show that in the credit default swap (CDS) market, spreads can be lower with greater information asymmetry.

[4] See, for example, Lin et al. (1995) and Chordia et al. (2001).

[5] This is equivalent to a setting where other investors submit demand schedules to the market maker (similar to Kyle, 1989 and Cespa and Vives, 2012), who then chooses bid and ask prices. The order size dependence of price schedules is consistent with the bargaining feature in less liquid markets.

We solve the equilibrium bid and ask prices, bid and ask depths, trading volume, and inventory levels in closed-form even when investors have different risk aversion, different inventory levels, different liquidity shocks, different resale values of the risky asset and heterogeneous private information. We find that in equilibrium, both bid–ask spread and market trading volume are proportional to the absolute value of the reservation price difference between the informed and the uninformed.[6] The key intuition is that because the market maker can buy from some investors at the bid and sell to other investors at the ask, what matters for the spread and the trading volume is the reservation price difference between these investors. As in Goldstein et al. (2014), investors have different motives to trade and thus the reservation prices can change differently in response to the same information and liquidity shocks. The greater the reservation price difference, the greater the total gain from trading, the more other investors trade, and because of the market maker's market power, the higher the spread. This also implies that in contrast to the literature on portfolio selection with transaction costs (e.g., Davis and Norman, 1990; Liu, 2004), bid–ask spreads can be positively correlated with trading volume. Clearly, a market maker's market power and the feasibility of making offsetting trades, which are missing in most of the existing literature, are critical for this result. Therefore, our model predicts that in markets where market makers have significant market power and can make offsetting trades, trading volume is positively correlated with bid–ask spreads.

Empirical studies have shown that bid–ask spreads can decrease with information asymmetry. In contrast, existing asymmetric information models predict that as information asymmetry increases, bid–ask spreads also increase. We show that our model can help explain this puzzle. The main intuition is as follows. Unlike "noise traders" who have to trade the same amount at any quoted prices, in our paper the uninformed investors are discretionary in the sense that their trading amount depends on the trading price and the adverse selection problem they face. As information asymmetry increases, the adverse selection effect faced by the uninformed increases. Therefore, the market maker needs to offer a better price (lower ask or higher bid) to the uninformed investors to induce them to take some of the market maker's trades with the potentially informed investors. This results in a narrower spread on average if the uninformed do not have any initial endowment of the risky asset. On the other hand, if the uninformed have an initial endowment of the risky asset, then there is an opposing force at work: as information asymmetry increases, the uncertainty about the value of the initial endowment increases, and thus the uninformed are willing to sell at a lower bid price. This opposing force drives down the bid price when the potentially informed investors buy and thus can drive up the spread. Accordingly, our model predicts that in markets where market makers have significant market power and can make frequent offsetting trades, the average spread decreases with information asymmetry if the current risky asset holdings of the uninformed are small. We find that a market maker's risk aversion (and hence inventory risk) is not critical for this result. However, a market marker's inventory risk significantly affects prices, depths and trading volume because the required risk premium increases with inventory risk.

In most existing models on the determination of bid–ask spread in the presence of information asymmetry (e.g., Glosten and Milgrom, 1985), a market maker deals with the adverse selection problem by lowering the bid and/or increasing the ask. Our model demonstrates a second approach a market maker can use to control the adverse selection effect: shifting part of her trade

[6] The reservation price is the critical price such that an investor buys (sells) the risky security if and only if the ask (bid) is lower (higher) than this critical price.

with the potentially informed to other investors.[7] We show that it is optimal for the market maker to combine these two approaches to best manage the adverse selection effect. When a market maker only uses the first approach, bid–ask spread is higher with asymmetric information, trading volume is negatively correlated with bid–ask spread and market breaks down (i.e., no trade) when bid–ask spread is infinity. In contrast, when a market maker also uses the second approach, not only bid–ask spread can be lower with asymmetric information, trading volume can be positively correlated with bid–ask spreads, but also market can break down when the bid–ask spread is zero.

The critical driving forces behind our main results are: (1) investors trade through the market maker; (2) the market maker has market power; and (3) as the informed, the uninformed are also discretionary. Because of (1) and (2), the spread increases with the absolute value of the reservation price difference between the informed and the uninformed. Because of (3), the adverse selection effect of information asymmetry drives down the expected spread as explained above.

In the study of the impact of investor short-termism on informational price inefficiency using a two-period model, Cespa and Vives (2015) find that the information inference component of the price impact of trades can be negative in the second period. In contrast to the aforementioned mechanism underlying our model that can cause the information asymmetry component of spreads to be negative, in Cespa and Vives (2015) it is retrospective inference implied by the persistency of the liquidity trading that can make the inference component of the price impact negative.

While as cited before, there are findings where bid–ask spreads can decrease with information asymmetry, and trading volume and bid–ask spreads can be positively correlated, there are also findings where the opposite is true (e.g., Green et al., 2007; Edwards et al., 2007). The existing literature cannot reconcile these seemingly contradictory empirical evidence. Even though many factors may drive these opposite findings and it is beyond the scope of this paper to pinpoint the key drivers for these results through a thorough empirical analysis, our model provides conditions under which these opposite empirical findings can arise and can shed some light on possible sources. For example, our model might help explain the negative relationship between spread and information asymmetry found by Acharya and Johnson (2007), because they focus on more active CDS markets where dealers with significant market power can make relatively frequent offsetting trades and most (uninformed) customers have small initial holdings. In addition, consistent with the prediction of positive correlation between spreads and trading volume, Li and Schürhoff (2011) find that in municipal bond markets central dealers, who likely have greater market power and can make offsetting trades more easily than peripheral dealers, charge higher bid–ask spreads and also experience greater trading volume.

As our paper, Hall and Rust (2003) also consider a monopolist market maker who posts bid and ask prices. Different from our model, however, their model abstracts away from asymmetric information, which is the main focus of our analysis. Although our model focuses on some financial markets, minor modifications of the model can be applied to other markets such as a market where producers have market power in both input and output markets and buy from input producers and sell to output buyers.[8]

[7] In contrast, in the existing literature, while noise traders pay worse prices as information asymmetry increases, a market maker cannot transfer more of the trade with the informed to noise traders by adjusting prices because noise traders' demand is assumed to be exogenously given.

[8] For example, if the output is proportional to input due to a linear production technology, then our model only needs almost trivial changes.

The remainder of the paper proceeds as follows. In Section 2 we briefly describe some applicable markets and discuss additional related literature. We present the model in Section 3. In Section 4 we derive the equilibrium. In Section 5 we provide some comparative statics on asset prices and bid–ask spreads. We present, solve and discuss a generalized model in Section 6. We conclude in Section 7. All proofs are provided in Appendix A. In Appendix B, we present the rest of the results of Theorem 2 for the generalized model. In Appendix C, we show the robustness of our results in a similar model using an alternative measure of information asymmetry as in Easley and O'Hara (2004).

2. Applicable markets and related literature

Our model applies to some OTC markets where search and bilateral negotiation are not critical. In most of the OTC markets, investors almost always trade with designated dealers (market makers) who typically quote a pair of bid and ask prices that are explicitly or implicitly contingent on order sizes.[9] In addition, dealers in OTC markets face significant information asymmetry and inventory risk, and therefore, they frequently engage in offsetting trades within a short period of time with other customers or with other dealers when their inventory level deviates significantly from desired targets (e.g., Acharya and Johnson, 2007; Shachar, 2012). We do not explicitly model searching process for a counterparty or bilateral negotiation.[10] However, our analysis applies well to many existing OTC markets where bilateral negotiation with a market maker is not critical and searching cost for a non-market-maker counterparty is high, but searching cost for a market maker is low. For example, in bond markets after the introduction of the Transaction Reporting and Compliance Engine (TRACE) and OTCQX and OTCQB stock markets, investors can observe both bid and ask prices and do not need to conduct much search for or significant negotiation with market makers. This is because for institutional traders they know very well who are the (best) market makers and request quotes from and make trades with a few market makers with frequent trading relationship, while for retail investors most trades are done through (institutional) brokers whose search cost for a market maker is also relatively low. In addition, even if search cost for a market maker is high, our results would also likely hold. This is because after a match between non-market-makers and a market maker is found, this trading relationship would probably last a significant period of time and our results apply in this period.

One key feature of our model is that non-market-makers trade through market makers. In some centralized markets (e.g., NYSE, Toronto Stock Exchange), investors also mainly trade through market makers for some small stocks and in relatively illiquid period of trading period (e.g., Anand and Venkataraman, 2013). In a quote-driven, electronic market such as the London Stock Exchange (LSE), public investors generally cannot trade directly among themselves and need to trade through market makers (e.g., Charitou and Panayides, 2009). This suggests that our model also applies to some of these financial markets.

[9] For example, Li and Schürhoff (2011) show that dealers intermediate 94% of the trades in the municipal bond market, with most of the intermediated trades representing customer-dealer-customer transactions.

[10] The cost of searching for a counterparty can be significant in some OTC markets for some investors, which motivates some studies to use search-based or network-based models for OTC markets (e.g., Duffie et al., 2005; Vayanos and Weill, 2008). Nash bargaining has been used by the existing literature to model bilateral negotiation in OTC markets (e.g., Duffie et al., 2005; Gofman, 2011; Atkeson et al., 2014). As shown in a previous version of the paper, our model is equivalent to a Nash bargaining game between investors and the market maker where the market maker has all the bargaining power.

In contrast to this model, existing market making literature either ignores information asymmetry (e.g., Garman, 1976; Stoll, 1978; Ho and Stoll, 1981) or abstracts away a market maker's inventory risk (e.g., Kyle, 1985; Glosten and Milgrom, 1985; Admati and Pfleiderer, 1988). However, both information asymmetry and inventory risk are important determinants of market prices and market liquidity for many financial markets. Different from inventory-based models, our model takes into account the impact of information asymmetry on bid and ask prices and inventory levels. In contrast to most information-based (rational expectations) models (e.g., Grossman and Stiglitz, 1980; Kyle, 1985; Glosten and Milgrom, 1985), in our model a market maker faces discretionary uninformed investors, has significant market power, profits from bid–ask spreads, and bears significant inventory risk. Different from Kyle (1989) where the informed and the uninformed submit demand schedules to an auctioneer who sets a single market clearing price and does not trade herself, in our model investors submit demand schedules to a strategic market maker who trades separately with buyers and sellers and markets clear separately at bid and at ask. The market power of a market maker and separation of trades at bid and at ask are critical features of OTC markets that our model captures.

Different from most of the existing literature on dealership markets,[11] a market maker in our model can shift some trades with the potentially informed to other customers by adjusting bid or ask, and as a result, expected spread can decrease with information asymmetry.

3. The model

We consider a one-period setting where there are a continuum of identical informed investors with mass N_I, a continuum of identical uninformed investors with mass N_U, and $N_M = 1$ designated market maker who is also uninformed. They can trade one risk-free asset and one risky security on date 0 to maximize their expected constant absolute risk aversion (CARA) utility from their wealth on date 1. There is a zero net supply of the risk-free asset, which also serves as the numeraire and thus the risk-free interest rate is normalized to 0. The total supply of the security is $N \times \bar{\theta} \geq 0$ shares where $N = N_I + N_U + N_M$ and the date 1 payoff of each share is $\tilde{V} \sim \mathbf{N}(\bar{V}, \sigma_V^2)$, where $\bar{V}$ is a constant, $\sigma_V > 0$, and $\mathbf{N}$ denotes the normal distribution. The total risky asset endowment is $N_i \bar{\theta}$ shares for type $i \in \{I, U, M\}$ investors. No investor is endowed with any risk-free asset.

On date 0, informed investors observe a private signal

$$\hat{s} = \tilde{V} - \bar{V} + \tilde{\varepsilon} \tag{1}$$

about the payoff $\tilde{V}$, where $\tilde{\varepsilon}$ is independently normally distributed with mean zero and variance σ_ε^2.[12] To prevent the informed's private information from being fully revealed in equilibrium, following Wang (1994), O'Hara (1997), and Vayanos and Wang (2012), we assume that the informed also have non-information based trading demand. Specifically, we assume that an informed investor is also subject to a liquidity shock that is modeled as a random endowment of $\hat{X}_I \sim \mathbf{N}(0, \sigma_X^2)$ units of a non-tradable risky asset on date 0, with $\hat{X}_I$ realized on date 0 and only known to informed investors.[13] The non-tradable asset has a per-unit payoff of $\tilde{L} \sim \mathbf{N}(0, \sigma_L^2)$

[11] E.g., Leach and Madhavan (1992, 1993), Lyons (1997), Viswanathan and Wang (2004), and Duffie et al. (2005).

[12] Throughout this paper, "bar" variables are constants, "tilde" random variables are realized on date 1 and "hat" random variables are realized on date 0.

[13] The random endowment can represent any shock in the demand for the security, such as a liquidity shock or a change in the needs for rebalancing an existing portfolio or a change in a highly illiquid asset.

that has a covariance of σ_{VL} with $\tilde{V}$ and is realized and becomes public on date 1. The correlation between the non-tradable asset and the security results in a non-information based, liquidity demand for the risky asset to hedge the non-tradable asset payoff.

In addition, to provide a measure of information asymmetry, we assume that there is a public signal

$$\hat{S}_s = \hat{s} + \hat{\eta} \tag{2}$$

about the informed's private signal $\hat{s}$ that all investors can observe, where $\hat{\eta}$ is independently normally distributed with mean zero and variance σ_η^2. This public signal represents public news about the asset payoff determinants, such as macroeconomic conditions, cash flow news and regulation shocks, which is correlated with but less precise than the informed's private signal.[14] As we show later, the noisiness σ_η^2 of the public signal can serve as a measure of information asymmetry. In empirical tests, one can use the amount of relevant public news as a proxy for this information asymmetry measure, because the more relevant public news, the better the uninformed can estimate the security payoff.[15] Using an alternative measure of information asymmetry as proposed by Easley and O'Hara (2004) does not change qualitatively our results (see Appendix C).

All trades must go through the designated market maker (dealer) whose market making cost is assumed to be 0.[16] Specifically, I and U investors sell to the market maker at the bid B or buy from her at the ask A or do not trade at all. The market maker posts her price schedules first. Then informed and uninformed investors decide how much to trade. When deciding on what price schedules to post, the market maker takes into account the best response functions (i.e., the demand schedules) of the informed and the uninformed given the to-be-posted price schedules.[17]

After observing private signal $\hat{s}$ and liquidity shock $\hat{X}_I$, each informed investor chooses a demand schedule $\Theta_I(\hat{s}, \hat{X}_I;\ \cdot)$. After observing public signal $\hat{S}_s$, each uninformed trader chooses a demand schedule $\Theta_U(\hat{S}_s;\ \cdot)$. The schedules Θ_I and Θ_U are traders' strategies. Given bid price B and ask price A, the quantities demanded by informed and uninformed investors can be written $\theta_I = \Theta_I(\hat{s}, \hat{X}_I, A, B)$ and $\theta_U = \Theta_U(\hat{S}_s, A, B)$.

For $i \in \{I, U, M\}$, investors of type i are identical both before and after realizations of signals on date 0 and thus we assume they adopt the same trading strategy. Let $\mathcal{I}_i$ represent a type i investor's information set on date 0 for $i \in \{I, U, M\}$. Given A and B, for $i \in \{I, U\}$, a type i investor's problem is to choose θ_i to solve

$$\max E[-e^{-\delta \tilde{W}_i} | \mathcal{I}_i], \tag{3}$$

where

[14] More generally, one can allow the public signal to have an additional component which is conditionally independent of the informed's private signal. This generalization makes the notation and analytical expressions more complicated, but does not affect our main results.

[15] Information asymmetry proxies such as disclosure level, analyst coverage and transparency commonly used in the empirical literature are clearly some measures of the amount of relevant public news.

[16] Assuming zero market making cost is only for a better focus and expositional simplicity. Market making cost is considered in an earlier version, where a potential market maker must pay a fixed market-making utility cost on date 0 to become a market maker. We show that no results in this paper are altered by this fixed cost as long as a market maker has significant market power, which can occur when the market making cost is large and thus the entry barrier is high.

[17] This can be reinterpreted as a Stackelberg game between the market maker and other investors where the market maker moves first by posting bid and ask price schedules (that depend on order sizes), then other players move by trading the optimal amount given the price schedules.

$$\tilde{W}_i = \theta_i^- B - \theta_i^+ A + (\bar{\theta} + \theta_i)\tilde{V} + \hat{X}_i \tilde{L}, \tag{4}$$

$\hat{X}_U = 0$, $\delta > 0$ is the absolute risk-aversion parameter, $x^+ := \max(0, x)$, and $x^- := \max(0, -x)$.

Since I and U investors buy from the market maker at ask and sell to her at bid, we can view these trades as occurring in two separate markets: the "ask" market and the "bid" market. In the ask market, the market maker is the supplier, other investors are demanders and the opposite is true in the bid market. The monopolist market maker chooses bid and ask prices, taking into account other investors' demand curve in the ask market and other investors' supply curve in the bid market.

Given bid price B and ask price A, let the realized demand schedules of the informed and the uninformed be denoted as $\Theta_I(A, B)$ and $\Theta_U(A, B)$ respectively. By market clearing conditions, the equilibrium ask depth α must be equal to the total amount bought by other investors and the equilibrium bid depth β must be equal to the total amount sold by other investors, i.e.,

$$\alpha = \sum_{i=I,\, U} N_i \Theta_i(A, B)^+, \ \beta = \sum_{i=I,\, U} N_i \Theta_i(A, B)^-. \tag{5}$$

It then follows that the risk-free asset market will be automatically cleared by the Walras' law. We denote market maker's pricing strategies as $\mathbb{A}(\hat{S}_s, \cdot)$ and $\mathbb{B}(\hat{S}_s, \cdot)$. For any realized demand schedules $\Theta_I(A, B)$ and $\Theta_U(A, B)$, the designated market maker's problem is to choose ask price level $A := \mathbb{A}(\hat{S}_s, \Theta_I, \Theta_U)$ and bid price level $B := \mathbb{B}(\hat{S}_s, \Theta_I, \Theta_U)$ to solve

$$\max E\left[-e^{-\delta \tilde{W}_M} | \mathcal{I}_M\right], \tag{6}$$

subject to

$$\tilde{W}_M = \alpha(A, B)A - \beta(A, B)B + (\bar{\theta} + \beta(A, B) - \alpha(A, B))\tilde{V}. \tag{7}$$

This leads to the definition of a Bayesian Nash equilibrium as follows.

Definition 1. An equilibrium $(\Theta_I^*(A, B), \Theta_U^*(A, B), (A^*, B^*))$ given any signals $\hat{s}$, $\hat{X}_I$ and $\hat{S}_s$ is such that

1. given any A and B, $\Theta_i^*(A, B)$ solves a type i investor's Problem (3) for $i \in \{I, U\}$, where the information set of the informed is $\mathcal{I}_I = \{\hat{s}, \hat{X}_I, A, B\}$ and the information set of the uninformed is $\mathcal{I}_U = \{\hat{S}_s, A, B\}$;
2. given $\Theta_I^*(A, B)$ and $\Theta_U^*(A, B)$, A^* and B^* solve the market maker's Problem (6), where the information set of the market maker is $\mathcal{I}_M = \{\hat{S}_s, \Theta_I^*(A, B), \Theta_U^*(A, B)\}$[18]; and
3. for every realization of the signals $\hat{s}$, $\hat{X}_I$ and $\hat{S}_s$, the beliefs of all investors are consistent with the joint conditional probability distribution in equilibrium.

Our model has direct mapping into empirical studies in the literature. Take the study of Acharya and Johnson (2007) as an example. The traded assets are credit derivatives. As argued by Acharya and Johnson (2007), informed investors likely include some commercial bankers who might have private information about a company's default probability. Other holders (e.g., retail investors) of the company's debt may be the (relatively) uninformed. In addition, almost all non-market-makers trade through market makers in the credit derivative markets.

[18] The market clearing conditions (Equation (5)) are implicitly enforced in the market maker's problem.

3.1. Discussions on the assumptions of the model

In this subsection, we provide justifications for our main assumptions.

The assumption that there is only one market maker is for expositional focus. A model with multiple market makers was solved in an earlier version of this paper, the results of which are reported in Section A.3 in the Appendix. In this model with oligopolistic Cournot competition, we show that competition among market makers, while lowering spreads, does not change our main qualitative results (e.g., expected bid–ask spread can decrease with information asymmetry).[19]

In some OTC markets and for relatively illiquid stocks in some centralized markets, it can be costly for non-market-makers to find and directly trade with each other. Therefore, most trades are through market makers, as we assume in the model. One important assumption is that the market maker can buy at the bid from some investors and sell at the ask to other investors at the same time. This assumption captures the fact that in many OTC markets, when a dealer receives an inquiry from a client, she commonly contacts other clients (or dealers) to see at which price and by how much she can unload the inquired trade *before* she trades with the initial client, while in centralized markets, orders on opposite sides come in relatively frequently. In addition, even for markets where there is a delay between offsetting trades, using a dynamic model with sequential order arrival would unlikely yield qualitatively different results. For example, in such a dynamic model, spreads can still decrease with information asymmetry, because even when orders arrive sequentially and thus a market maker needs to wait a period of time for the offsetting trades, as long as she has a reasonable estimate of the next order, she will choose qualitatively the same trading strategy.

To keep information from being fully revealed in equilibrium, we assume that informed investors have liquidity shocks in addition to private information. One can interpret this assumption as there are some pure liquidity traders who trade in the same direction as the informed. Alternatively, one can view an informed investor as a broker who combines information motivated trades and liquidity motivated trades. The assumption that all informed traders have the same information and the same liquidity shock is only for simplicity so that there are only two groups of non-market-makers in the model. Our main results still hold when they have different information and different liquidity shocks. Intuitively, no matter how many heterogeneous investor groups there are, the equilibrium bid and ask prices would divide these groups into a Buy group, a Sell group, and a No trade group. Therefore, as long as the characteristics of the Buy and Sell group investors are similar to those in our model, our main results still hold. For example, if in equilibrium some informed buy, other informed sell, and the uninformed do not trade, then the reservation price difference between the two informed groups would determine the spread, which can still decrease with information asymmetry (between the two informed groups) by a similar intuition.

As in Glosten (1989) and Vayanos and Wang (2012), our model implies that the market maker knows who are the relatively more informed investors because two groups of non-market-makers trade differently. Note that, however, she does not know how much of the informed's trade is due to private information because the informed's trade can also be due to liquidity shock. The informed's trade is qualitatively the same as the observed aggregate trade of information trades

[19] In contrast to Bertrand competition, Cournot competition allows market makers to keep some market power, which is critical for our main results. As is well-known, it takes only two Bertrand competitors to reach a perfect competition equilibrium (and thus no market maker has any market power). However, market prices can be far from the perfect competition ones (e.g., Christie and Schultz, 1994; Chen and Ritter, 2000; Biais et al., 2010).

and noise trades as modeled in existing literature (e.g., Kyle, 1985). What is critical for our results is that the market maker observes orders from and trades with different groups of investors.

We also assume that the market maker posts price schedules first (after taking into account what would be the best responses of other investors), then other investors choose their optimal trading strategies taking the posted price schedules as given, and thus other investors are not strategic. As we show, this assumption is equivalent to assuming that in a Nash Bargaining game between the market maker and other investors, the market maker has all the bargaining power. This is consistent with the common practice in OTC markets that a dealer making two-sided markets typically provides a take-it-or-leave-it pair of prices, a bid and an offer, to customers (e.g., Duffie, 2012, Chapter 1). As we show in the next section, the equilibrium bid and ask prices are indeed functions of order sizes.

Different from the existing models, we assume there is a public signal that is correlated with the private signal of the informed. This additional signal is not critical for our main results (e.g., spread can be smaller with asymmetric information), but has two main benefits. In addition to providing a measure of information asymmetry, it makes our model nest models with different degrees of information asymmetry in one unified setting.[20]

4. The equilibrium

In this section, we solve the equilibrium bid and ask prices, bid and ask depths and trading volume in closed form. As in standard asymmetric information models, although the uninformed cannot observe the private signal, they can extract information about it from observing market prices.

Given A and B, the optimal demand schedule of a type i investor ($i \in \{I, U\}$) is

$$\Theta_i^*(A, B) = \begin{cases} \frac{P_i^R - A}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_i]} & A < P_i^R, \\ 0 & B \le P_i^R \le A, \\ -\frac{B - P_i^R}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_i]} & B > P_i^R, \end{cases} \tag{8}$$

where

$$P_i^R = E[\tilde{V}|\mathcal{I}_i] - \delta \mathrm{Cov}[\tilde{V}, \tilde{L}|\mathcal{I}_i]\hat{X}_i - \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_i]\bar{\theta} \tag{9}$$

is the investor's reservation price (i.e., the critical price such that a non-market-maker buys (sells, respectively) the security if and only if the ask price is lower (the bid price is higher, respectively) than this critical price).

Because the informed know exactly $\{\hat{s}, \hat{X}_I\}$ while equilibrium prices A^* and B^* and the public signal $\hat{S}_s$ are only noisy signals about $\{\hat{s}, \hat{X}_I\}$, the information set of the informed in equilibrium is equivalent to

$$\mathcal{I}_I = \{\hat{s}, \hat{X}_I\}, \tag{10}$$

which implies that

[20] For example, the case where $\sigma_\eta^2 = 0$ implies that the uninformed and the market maker can perfectly infer $\hat{s}$ from the public signal and thus represents the symmetric information case. The case where $\sigma_\eta^2 = \infty$, on the other hand, implies that the public signal is useless and thus corresponds to the asymmetric information case as modeled in the standard literature.

$$\mathrm{E}[\tilde{V}|\mathcal{I}_I] = \bar{V} + \rho_I \hat{s}, \quad \mathrm{Var}[\tilde{V}|\mathcal{I}_I] = (1-\rho_I)\sigma_V^2, \quad \mathrm{Cov}[\tilde{V}, \tilde{L}|\mathcal{I}_I] = (1-\rho_I)\sigma_{VL}, \tag{11}$$

where

$$\rho_I := \frac{\sigma_V^2}{\sigma_V^2 + \sigma_\varepsilon^2} \tag{12}$$

is the weight the informed put on the private signal. Equation (9) then implies that

$$P_I^R = \bar{V} + \rho_I \hat{S} - \delta(1-\rho_I)\sigma_V^2 \bar{\theta}, \tag{13}$$

where $\hat{S} := \hat{s} + \frac{h}{\rho_I}\hat{X}_I$ is a composite signal of $\hat{s}$ and $\hat{X}_I$, and $h = -\delta(1-\rho_I)\sigma_{VL}$ represents the hedging premium per unit of liquidity shock.

While $\hat{s}$ and $\hat{X}_I$ both affect the informed investor's demand and thus the equilibrium prices, other investors can only infer the value of the composite signal $\hat{S}$ from market prices because the joint impact of $\hat{s}$ and $\hat{X}_I$ on market prices is only through $\hat{S}$. In addition to $\hat{S}$, other investors can also observe the public signal $\hat{S}_s$ about the private signal $\hat{s}$. Thus we conjecture that the equilibrium prices A^* and B^* depend on both $\hat{S}$ and $\hat{S}_s$. To simplify exposition and highlight the essential driving forces behind our results, we restrict to equilibria where the market maker's posted prices A and B are piecewise linear in both $\hat{S}$ and $\hat{S}_s$. Accordingly, the information sets for the uninformed investors and the market maker are

$$\mathcal{I}_U = \mathcal{I}_M = \{\hat{S}, \hat{S}_s\}. \tag{14}$$

Then the conditional expectation and conditional variance of $\tilde{V}$ for the uninformed (and the market maker) are respectively

$$E[\tilde{V}|\mathcal{I}_U] = \bar{V} + \rho_U\left((1-\rho_X)\hat{S} + \rho_X \hat{S}_s\right), \tag{15}$$

$$\mathrm{Var}[\tilde{V}|\mathcal{I}_U] = (1-\rho_U)\sigma_V^2, \tag{16}$$

where

$$\rho_X := \frac{h^2\sigma_X^2}{h^2\sigma_X^2 + \rho_I^2\sigma_\eta^2}, \quad \rho_U := \frac{\sigma_V^2}{\sigma_V^2 + \rho_X\rho_I\sigma_\eta^2}\rho_I < \rho_I, \tag{17}$$

where ρ_U is the weight the uninformed put on the weighted (by ρ_X) average signal of $\hat{S}$ and $\hat{S}_s$. It follows that the reservation price for a U investor and the market maker is

$$P_U^R = P_M^R = \bar{V} + \rho_U\left((1-\rho_X)\hat{S} + \rho_X\hat{S}_s\right) - \delta(1-\rho_U)\sigma_V^2\bar{\theta}. \tag{18}$$

Let Δ denote the difference in the reservation prices of I and U investors. We then have

$$\Delta := P_I^R - P_U^R = (\rho_I - \rho_U)\left(\left(1 + \frac{\sigma_V^2}{\rho_I\sigma_\eta^2}\right)\hat{S} - \frac{\sigma_V^2}{\rho_I\sigma_\eta^2}\hat{S}_s + \delta\sigma_V^2\bar{\theta}\right). \tag{19}$$

Define

$$\nu := \frac{\mathrm{Var}[\tilde{V}|\mathcal{I}_U]}{\mathrm{Var}[\tilde{V}|\mathcal{I}_I]} = \frac{1-\rho_U}{1-\rho_I} \geq 1$$

to be the ratio of the conditional variance for the uninformed to that for the informed, and

$$\overline{N} := \nu N_I + N_U + 1 \geq N$$

to be the information weighted total population. The following theorem provides the equilibrium bid and ask prices and equilibrium quantities in closed-form.

Theorem 1. *There is a unique symmetric piecewise linear equilibrium:*

1. *The equilibrium bid and ask prices are respectively*

$$A^* = P_U^R + \frac{\nu N_I}{2\left(\overline{N}+1\right)}\Delta + \frac{\Delta^+}{2}, \tag{20}$$

$$B^* = P_U^R + \frac{\nu N_I}{2\left(\overline{N}+1\right)}\Delta - \frac{\Delta^-}{2}. \tag{21}$$

The bid–ask spread is

$$A^* - B^* = \frac{|\Delta|}{2} = \frac{1}{2}(\rho_I - \rho_U)\left|\left(1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2}\right)\hat{S} - \frac{\sigma_V^2}{\rho_I \sigma_\eta^2}\hat{S}_s + \delta\sigma_V^2\bar{\theta}\right|. \tag{22}$$

2. *The equilibrium quantities demanded are*

$$\theta_I^* = \frac{N_U + 2}{2\left(\overline{N}+1\right)}\frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}, \quad \theta_U^* = -\frac{\nu N_I}{2\left(\overline{N}+1\right)}\frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]}; \tag{23}$$

the equilibrium ask and bid depths are respectively

$$\alpha^* = N_I(\theta_I^*)^+ + N_U(\theta_U^*)^+, \tag{24}$$

$$\beta^* = N_I(\theta_I^*)^- + N_U(\theta_U^*)^-, \tag{25}$$

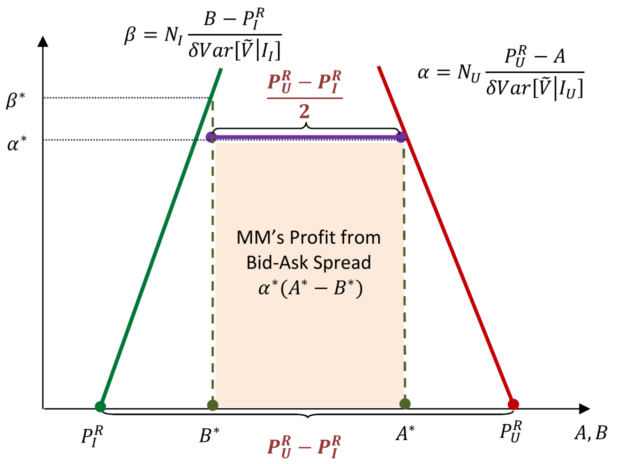
which implies that the equilibrium trading volume is

$$\alpha^* + \beta^* = \frac{N_I(N_U+1)}{\overline{N}+1}\left(\frac{|\Delta|}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}\right). \tag{26}$$

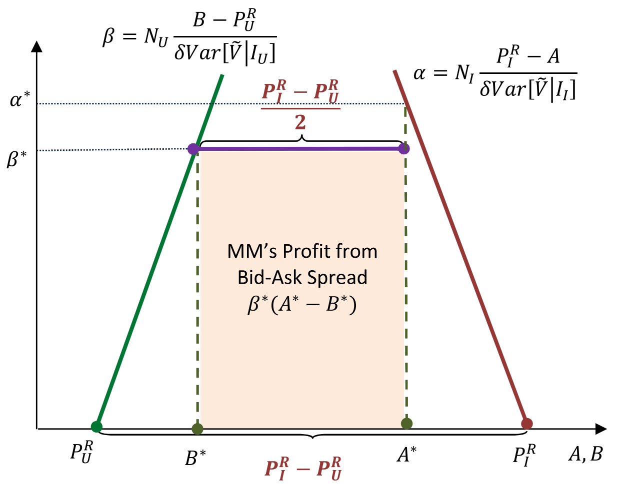
To help understand the results in Theorem 1, we first provide some graphical illustration. Suppose $P_U^R > P_I^R$ and thus U investors buy and I investors sell. The market clearing condition (5) implies that

$$\alpha = N_U \frac{P_U^R - A}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]}, \quad \beta = N_I \frac{B - P_I^R}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}.$$

We plot the above demand and supply functions and equilibrium spreads in Fig. 1(a). Similarly, Fig. 1(b) displays the demand and supply functions and equilibrium spreads for the case where the informed buy and the uninformed sell. Fig. 1 shows that the higher the bid, the more a market maker can buy from other investors, and the lower the ask, the more a market maker can sell to other investors. Facing the demand and supply functions of other investors, a monopolist market maker optimally trades off the prices and quantities, in addition to inventory risk. Similar to the results of classical models on monopolistic firms who set a market price to maximize profit, the bid and ask spread is equal to the absolute value of the reservation price difference $|\Delta|$ divided by 2 (divided by $N_M + 1$ with multiple market makers, as shown in Appendix A). Different from these monopolistic firms, however, the market maker is a seller in one market and a buyer in another, and makes profit from the spread. In addition, as implied by Theorem 1, Fig. 1(a) illustrates that the difference between P_U^R (P_I^R) and the ask (bid) price is also proportional to the absolute value of the reservation price difference $|\Delta|$. Therefore the trading amount of both I and U investors and thus the aggregate trading volume all increase with $|\Delta|$. The minimum of α



(a) The Uninformed Buy and the Informed Sell



(b) The Informed Buy and the Uninformed Sell

Fig. 1. Demand/supply functions and bid/ask spreads.

and β multiplied by the bid–ask spread (the shaded areas) captures the sure profit secured by the market maker from the intermediation role, while the difference between α and β represents her speculation on inventory investment.

We next present a benchmark case to pin down the driving forces behind the results in Theorem 1. Suppose that the market maker is risk neutral, $P_I^R < P_U^R = P_M^R$, I investors sell, and U investors buy. Then the market maker's problem is equivalent to

$$\max_{\{A,B\}} A \frac{N_U(P_U^R - A)}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]} - B \frac{N_I(B - P_I^R)}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]} + \left(\bar{\theta} - \frac{N_U(P_U^R - A)}{\delta \text{Var}[\tilde{V}|\mathcal{I}_U]} + \frac{N_I(B - P_I^R)}{\delta \text{Var}[\tilde{V}|\mathcal{I}_I]}\right) \text{E}[\tilde{V}|\mathcal{I}_M].$$

This is a problem for a monopolist who purchases in one market and sells in another completely separate market. The solution is simply

$$A = \frac{P_M^R + P_U^R}{2} = P_U^R, \quad B = \frac{P_M^R + P_I^R}{2} = P_U^R - \frac{\Delta^-}{2}, \tag{27}$$

where $P_M^R = \mathrm{E}[\tilde{V}|\mathcal{I}_M]$ because the market maker is risk neutral.

Now we are ready to explain the main driving forces at work. Comparing A and B in Equation (27) with Equations (20) and (21) shows that the first and the third terms in Equations (20) and (21) represent the choice of the trading prices for a monopolist market maker who trades in both the "ask" market and the "bid" market to maximize her expected total profit. It can be easily shown that when the market maker does not have market power (i.e., she is also a price taker), the equilibrium price P satisfies $A > P > B$. This implies that the market maker lowers the bid price below P and raises the ask price above P due to the market power.

The difference between the ask price and the bid price in Equation (27) from those in Equations (20) and (21) is the second term in Equations (20) and (21). Because the only difference in the derivation of Equation (27) from that of Equations (20) and (21) is the assumption of zero risk aversion of the market maker, the second term in Equations (20) and (21) appears because of the aversion of the market maker toward inventory risk. For a risk averse market maker, the optimal ask (bid) price is still the average of the reservation price of the uninformed (informed) and that of the market maker, as in Equation (27). The only difference is that the reservation price of the market maker is reduced by the inventory risk premium evaluated at the date 1 inventory level of θ_M (instead of at the initial endowment level of $\bar{\theta}$), i.e., it is equal to $\mathrm{E}[\tilde{V}|\mathcal{I}_M] - \delta\theta_M \mathrm{Var}[\tilde{V}|\mathcal{I}_M]$. This implies that the bid and ask spread is still half of the absolute value of the reservation price difference between the informed and the uninformed even when the market maker is risk averse and thus compared with prices in Equation (27), both the bid price and ask price are adjusted by the same amount (i.e., the second term). If the informed sell and the uninformed buy (i.e., $\Delta < 0$), the second term is negative, which implies that the market maker lowers the trading price with the informed (i.e., the bid price). Because in this case the market maker buys in the net at the decreased bid price, this lower price for the net purchase represents a discount the market maker charges for taking the inventory risk. In addition, the market maker also lowers the ask price to induce the uninformed to take part of her trade with the informed to reduce some inventory risk.

As shown in a previous version of the paper, our model can be reinterpreted as a Nash bargaining game between the market maker and other investors where the market maker has all the bargaining power. In a nutshell, in the Nash bargaining game, the market maker and an investor bargain over the trading price with the trading amount determined by the optimal demand schedule of the investor. Therefore, the Nash bargaining game where the market maker has all the bargaining power is to choose the trading price to maximize the market maker's expected utility given the demand schedule of the investor, and thus yields exactly the same outcome as our solution above.[21]

Equations (20) and (21) imply that in equilibrium both bid and ask prices can be written as piecewise linear functions of $\hat{S}$ and $\hat{S}_s$, therefore as we conjectured, the uninformed can indeed back out a unique $\hat{S}$ given the public signal $\hat{S}_s$ and the posted quotes. Even in the generalized model in Section 6 where the informed do not trade in equilibrium, the uninformed can still infer $\hat{S}$ if the equilibrium price is set such that the informed are indifferent between trading and no trading, because from market prices the uninformed can then back out $\hat{S}$ that makes the

[21] We also solve the case where other investors have bargaining power and the case where they bargain over both trade price and trade size and find that the qualitative results are the same. For example, the equilibrium bid–ask spread is still proportional to the absolute value of the reservation price difference between the informed and the uninformed if the uninformed and the market maker have the same reservation price.

informed's trade size equal to zero.[22] In addition, because the equilibrium bid and ask prices depend on $\hat{S}$ and the public signal $\hat{S}_s$, the market maker can indeed post the bid and ask price schedules *before* observing the order flow and the public signal. The bid and ask price *levels* are then determined after the realizations of the signals revealed by the orders. Because by (23) there is a one-to-one mapping between $\hat{S}$ and the informed's and the uninformed's order sizes for a given $\hat{S}_s$, the market maker can also equivalently post the price schedules as piecewise linear functions of the informed's and the uninformed's order sizes. Moreover, Equation (23) implies that I investors buy and U investors sell if and only if I investors have a higher reservation price than U investors.

Note that the ask and bid quotes in Equations (20) and (21) differ by $\Delta^+/2$ in the ask price and $-\Delta^-/2$ in the bid price. This implies when the informed buy ($\Delta > 0$), the market maker adjusts the trading price with the informed (ask) upward compared to the trading price with the uninformed (bid), and when the informed sell, she adjusts the trading price with the informed (bid) downward compared to the trading price with the uninformed (ask). In this sense, the market maker adjusts the trading prices with the informed and those with the uninformed differently.

In the standard literature on portfolio choice with transaction costs (e.g., Davis and Norman, 1990; Liu, 2004), it is well established that as the bid–ask spread increases, investors reduce trading volume to save on transaction costs and thus trading volume and bid–ask spread move in the opposite directions. In contrast, Theorem 1 implies that bid–ask spreads and trading volume can move in the same direction, because both trading volume and bid–ask spread increase with $|\Delta|$. Lin et al. (1995) find that trading volume and effective spreads are positively correlated at the beginning and the end of the day. Chordia et al. (2001) find that the effective bid–ask spread is positively correlated with trading volume. Our model suggests that these positive correlations may be caused by changes in the valuation difference of investors. There are also empirical findings that bid–ask spreads can be negatively correlated with trading volume (e.g., Green et al., 2007; Edwards et al., 2007). The negative correlation is consistent with the case where the bid–ask spread is almost exogenous, as in any partial equilibrium model (e.g., Liu, 2004). When market makers have near perfect competition, the bid–ask spread is essentially determined by the market-making cost and therefore is largely exogenous. In this case, changes in the market making costs may cause spread and trading volume to move in opposite directions, resulting in a negative correlation between spread and trading volume. Thus, one of the empirically testable implications of our model is that when market makers have significant market power and make frequent offsetting trades, bid–ask spreads and trading volume are positively correlated. This prediction seems consistent with the finding of Li and Schürhoff (2011): In municipal bond markets, central dealers, who likely have greater market power than peripheral dealers, charge higher bid–ask spreads and also enjoy greater trading volume.

5. Comparative statics

In this section, we provide some comparative statics on asset prices and market illiquidity, focusing on the impact of information asymmetry and liquidity shock volatility.

[22] As in Glosten (1989) and Vayanos and Wang (2012), the market maker in our model can infer how much informed investors are trading. However, she does not know how much is due to information on the security's payoff or how much is due to the liquidity demand.

5.1. A measure of information asymmetry

A good measure of information asymmetry should be such that a change of its value affects information asymmetry but does not affect other relevant economic variables such as the quality of aggregate information about the security payoff.[23] For example, the precision of a private signal about asset payoff would not be a good measure, because a change in the precision also changes the quality of aggregate information about the payoff and both information asymmetry and information quality can affect economic variables of interest (e.g., prices, liquidity). Even a comparison between the cases with and without asymmetric information cannot attribute the difference to the impact of information asymmetry alone, as long as the information quality is different across these two cases. We next propose a measure of information asymmetry.

One of the fundamental manifestations of asymmetric information is that the security payoff conditional variance for the uninformed is greater than that for the informed, i.e.,

$$\mathrm{Var}(\tilde{V}|\mathcal{I}_U) - \mathrm{Var}(\tilde{V}|\mathcal{I}_I) = \left(\left(\frac{\sigma_\varepsilon^2 + \sigma_V^2}{\sigma_V^2}\right)^2 \left(1 + \frac{\sigma_V^4 \sigma_\eta^2}{\delta^2 \sigma_\varepsilon^4 \sigma_{VL}^2 \sigma_X^2}\right) \frac{1}{\sigma_\eta^2} + \frac{\sigma_\varepsilon^2 + \sigma_V^2}{\sigma_V^4}\right)^{-1} \geq 0. \tag{29}$$

The greater this conditional variance difference, the greater the information asymmetry. This difference is monotonically increasing in σ_η^2, σ_{VL}^2 and σ_X^2, but nonmonotonic in σ_ε^2 and σ_V^2.[24] A change in σ_{VL}^2 would change the correlation between the nontraded asset and the risky security while a change in σ_X^2 would change the unconditional liquidity shock uncertainty. In addition to the undesirable nonmonotonicity, a change in σ_ε^2 or σ_V^2 would also change the quality of aggregate information about the security payoff. In contrast, a change in σ_η^2 only changes the information asymmetry but not the quality of aggregate information or the unconditional liquidity shock uncertainty or the correlation between the nontraded asset and the risky security. Accordingly, to isolate the impact of information asymmetry in the subsequent analysis, we use σ_η^2 as the measure of information asymmetry.[25]

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[23] The quality of aggregate information about the security payoff is measured by the inverse of the security payoff variance conditional on *all* the information in the economy, i.e.,

$$(\mathrm{Var}(\tilde{V}|\mathcal{I}_I \cup \mathcal{I}_U \cup \mathcal{I}_M))^{-1} = (\mathrm{Var}(\tilde{V}|\mathcal{I}_I))^{-1} = \frac{\sigma_V^2 + \sigma_\varepsilon^2}{\sigma_V^2 \sigma_\varepsilon^2}, \tag{28}$$

where the first equality follows from the fact that the informed have better information than the rest and the second from (11).

[24] The nonmonotonicity follows because as σ_ε^2 decreases or σ_V^2 increases, the conditional covariance magnitude $\left|\frac{\sigma_\varepsilon^2}{\sigma_V^2+\sigma_\varepsilon^2}\sigma_{VL}\right|$ decreases, thus the noise from the hedging demand decreases and hence the conditional security payoff variance of the uninformed may get closer to that of the informed.

[25] We also solved an alternative model where as in Easley and O'Hara (2004), there are a total of K signals, $K_p \leq K$ of which are private signals that only the informed can observe and the rest are public signals that everyone can observe. In this alternative model, K_p is a measure of information asymmetry, because as it increases, the total information available to the market does not change, but the information asymmetry increases. We show that our results such as expected spread may decrease with information asymmetry still hold in this alternative approach. See Appendix C for more details.

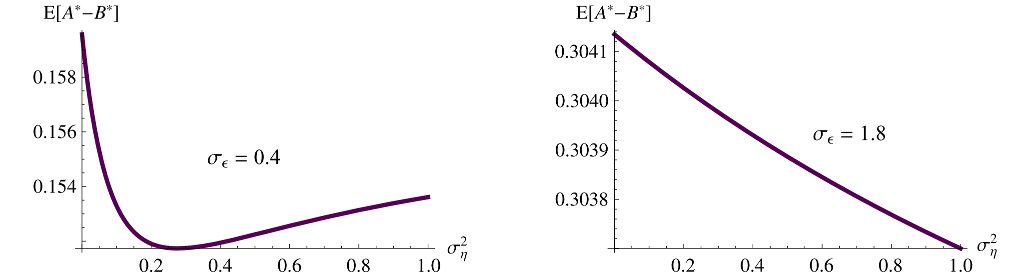

Fig. 2. Expected bid–ask spread against information asymmetry σ_η^2. The default parameter values are: $\delta = 1, \bar{\theta} = 4$, $\bar{V} = 3$, $N_I = 100$, $N_U = 1000$, $\sigma_V = 0.4$, $\sigma_X = 1$, and $\sigma_{VL} = 0.8$.

5.2. Bid–ask spread, market depths, and trading volume

The following proposition implies that in contrast to most of the existing literature (e.g., Glosten and Milgrom, 1985), not only ex post bid–ask spreads (i.e., spreads after signal realizations) but also expected bid–ask spreads can *decrease* as information asymmetry increases.

Proposition 1.

1. *The reservation price difference Δ is normally distributed with mean μ_D and variance σ_D^2, where*

$$\mu_D = \delta(\rho_I - \rho_U)\sigma_V^2\bar{\theta}, \quad \sigma_D^2 = h^2\sigma_X^2 - (\rho_I - \rho_U)\sigma_V^2, \tag{30}$$

which implies that the expected bid–ask spread is equal to:

$$E[A^* - B^*] = \frac{2\sigma_D \mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) + \mu_D\left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right)}{2}, \tag{31}$$

where **n** *and* **N** *are respectively the pdf and cdf of the standard normal distribution.*

2. *The expected bid–ask spread decreases with information asymmetry σ_η^2 if and only if*

$$\mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) - \delta\bar{\theta}\sigma_D\left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right) > 0, \tag{32}$$

which is always satisfied when $\bar{\theta} = 0$ or μ_D is small enough.

3. *The expected bid–ask spread increases with both the liquidity shock volatility σ_X and the covariance magnitude $|\sigma_{VL}|$.*

Because as σ_X^2 goes to 0 or σ_ε^2 goes to ∞, ρ_U goes to ρ_I and thus μ_D goes to zero, regardless of the magnitude of the information asymmetry σ_η^2, Part 2 of Proposition 1 implies that for small enough $\bar{\theta}$ or σ_X^2 or large enough σ_ε^2, the expected spread decreases with information asymmetry σ_η^2 even in the presence of large information asymmetry, as illustrated by Fig. 2 which suggests that this can occur when the private signal is noisy.

We next provide the main intuition for the result that expected spread may decrease with information asymmetry. In existing models with information asymmetry, an uninformed counterparty of the informed controls the adverse selection effect of information asymmetry by charging a price premium (if a purchase may be from the informed) or demanding a price discount (if a

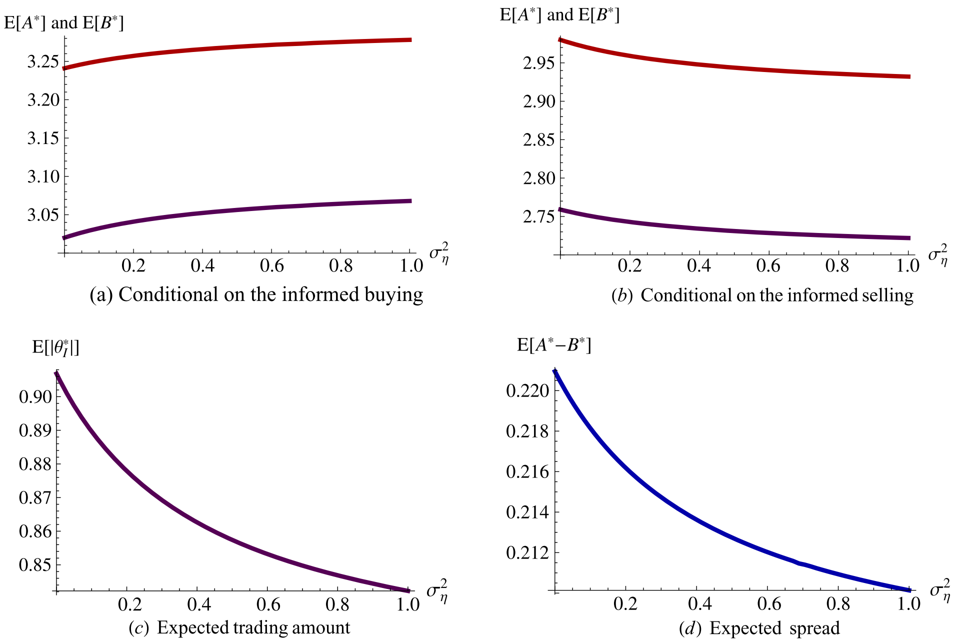

Fig. 3. Expected ask, bid, and spread conditional on the trading direction of the informed against information asymmetry σ_η^2. The default parameter values are: $\delta = 1, \bar{\theta} = 0, \bar{V} = 3, N_I = 100, N_U = 1000, \sigma_\varepsilon = 0.6, \sigma_V = 0.4, \sigma_X = 1$, and $\sigma_{VL} = 0.8$.

sale may be from the informed). In our model, to control the adverse selection effect, in addition to varying the trading price with the informed as in the existing models, the market maker who is uninformed also adjusts the trading price with other uninformed investors to induce them to take part of her trade with the informed. As information asymmetry increases, the uninformed need a lower ask price to buy at or a higher bid price to sell at for facing the increased adverse selection problem, which may cause the spread to decrease on average. To illustrate that this is indeed what occurs in our model, we plot in Fig. 3 the expected bid, ask, spread and the informed trade amount against the information asymmetry σ_η^2 conditional on the informed buying (selling) when the initial endowment $\bar{\theta} = 0$. Fig. 3 shows that conditional on the informed buying, as the information asymmetry increases, the market maker raises the ask price, as expected from the existing models with information asymmetry. As a result, the informed's purchase amount decreases on average, as shown in Fig. 3(c). To induce the uninformed to take part of her trade with the informed, the market maker also increases the bid price. Because the optimal ask is equal to bid plus a half of the reservation price difference and as explained below, the average reservation price difference conditional on the informed buying (or selling) may decrease with information asymmetry, the bid price may increase more than the ask price and thus as shown in Fig. 3(d), the expected spread may go down as the information asymmetry increases. Fig. 3(b) illustrates the same intuition and similar results for the case where the informed sell.[26]

[26] Figs. 3(c) and 3(d) also represent the corresponding quantities for the case where the informed sell because of the symmetry between the two cases when $\bar{\theta} = 0$.

Next, we explain why conditional on the informed buying (or selling), the average reservation price of the uninformed can get closer to that of the informed as information asymmetry increases. For this purpose, we can rewrite the reservation price difference (19) as

$$\Delta = \underbrace{h\hat{X}_I}_{\text{hedging effect}} + \underbrace{\left(E[\tilde{V}|\mathcal{I}_I] - E[\tilde{V}|\mathcal{I}_U]\right)}_{\text{estimation error effect}} + \underbrace{\left(\delta Var[\tilde{V}|\mathcal{I}_U]\bar{\theta} - \delta Var[\tilde{V}|\mathcal{I}_I]\bar{\theta}\right)}_{\text{estimation risk effect}}, \tag{33}$$

where the first term is from the difference in the hedging demand ("hedging effect") between the informed and the uninformed, the second term is the difference in the estimation of the expected security payoff ("estimation error effect"), and the third term is the difference in the risk premium required for the estimation risk ("estimation risk effect").

Consider first the simplest case where $\bar{\theta} = 0$, i.e., there is no estimation risk effect. On average, hedging effect and estimation error effect are equal to zero, and thus the unconditional expected reservation price difference is zero. However, conditional on the informed buying (or selling), because the expected reservation price difference is proportional to the volatility of the reservation price difference, the expected reservation price of the uninformed gets closer to that of the informed as the volatility of the reservation price difference decreases. Now, we explain why the volatility of the reservation price difference decreases with information asymmetry in this case. For given changes in $\hat{S}$ (that determines the order size of the informed) and in the public signal $\hat{S}_s$, as information asymmetry σ_η^2 increases, the uninformed attribute a greater portion of the change in $\hat{S}$ to the change in the private signal $\hat{s}$,[27] reflecting the adverse selection effect. Therefore, in the estimation of the expected payoff, as information asymmetry increases, the uninformed have closer weights on the private signal $\hat{s}$ and on the public signal to those of the informed. Thus, the estimation error effect becomes less sensitive to realizations of $\hat{S}$ and $\hat{S}_s$. Because the hedging effect does not change with information asymmetry, the volatility of the reservation price difference (which is equal to the sum of the hedging effect and the estimation error effect when $\bar{\theta} = 0$) decreases as information asymmetry increases, and so does the expected bid–ask spread.[28]

[27] In (15), as σ_η^2 increases, the weight $\rho_U(1-\rho_X)$ on $\hat{S}$ increases and the weight $\rho_U\rho_X$ on $\hat{S}_s$ decreases.

[28] Alternatively, we can rewrite the reservation price difference as

$$\Delta = \underbrace{E[h\hat{X}_I|\mathcal{I}_U]}_{\text{estimated hedging premium}} + \underbrace{\delta \mathrm{Var}[h\hat{X}_I|\mathcal{I}_U]\bar{\theta}}_{\text{estimation risk premium}}. \tag{34}$$

Intuitively, if the uninformed always had the same liquidity shock as the informed, then the uninformed would just trade the same amount as the informed (which can be inferred from the equilibrium price) and thus would have the same reservation price as the informed; thus the reservation price difference comes from the difference in the liquidity shock, which is reflected by the difference in the estimation of the liquidity shock and the associated estimation risk premium. If $\bar{\theta} = 0$, then we have

$$E|\Delta| = \sqrt{\frac{2\mathrm{Var}[E[h\hat{X}_I|\mathcal{I}_U]]}{\pi}}. \tag{35}$$

Because the variance of conditional expectation of any random variable decreases as the conditioning signal becomes noisier, by (35) the expected spread decreases with information asymmetry σ_η^2, which measures the noisiness of the conditioning signal.

If the uninformed have a positive initial endowment of the risky asset, they have a lower reservation price than the informed on average because they are more uncertain about the value of the initial endowment and thus require a higher risk premium. Therefore, on average the informed buy at the ask and the uninformed sell at the bid. As the information asymmetry increases, the average reservation price of the uninformed becomes even lower due to the increased uncertainty and thus the expected spread may get wider.

The main intuition behind the result on expected spread explained above suggests that if the uninformed's estimation risk premium is small, then expected spread decreases with information asymmetry. For most securities, on average an uninformed investor has small estimation risk premium, either because the investor has small holdings (e.g., for a retail investor $\bar{\theta}$ is small) or because the risk aversion toward the estimation risk is low (e.g., for investors who have offsetting positions elsewhere δ is small). Accordingly, one empirically testable implication is that in markets where market makers have significant market power and can offset their trades relatively frequently (e.g., relatively active derivative markets), average spreads decrease with information asymmetry.

Proposition 1 also implies that as liquidity shocks become more volatile or as the payoffs of the security and the nontraded asset covary more, the expected bid–ask spread increases. Intuitively, as σ_X^2 or $|\sigma_{VL}|$ increases, the volatilities of the hedging effect, the estimation error effect, and the estimation risk effect all increase. Therefore, the expected spread increases.

Because a market maker faces both information asymmetry and inventory risk, it would be helpful to separate the effects of information asymmetry and inventory risk on equilibrium asset prices and bid–ask spreads. However, it seems impossible to completely separate these effects in every single case for every economic variable of interest because in general these two effects interact with each other. On the other hand, we can separate them in some special cases. First, clearly, in the symmetric information case (i.e., $\sigma_\eta^2 = 0$), there is no information asymmetry effect. Second, if the market maker is risk neutral and thus has no aversion to the inventory risk, then the equilibrium results are free of inventory risk effect.[29] In general, however, both information asymmetry and inventory risk affect equilibrium bid and ask prices, expected spread, and trading volume.

Next we examine how expected market depths and trading volume change with information asymmetry and liquidity shock volatility.

Proposition 2.

1. *If N_U is large enough, then the expected trading volume increases with information asymmetry, i.e.,* $\frac{\partial E[\alpha^*+\beta^*]}{\partial \sigma_\eta^2} > 0$, *if and only if the expected spread increases with information asymmetry.*
2. *As the liquidity shock volatility σ_X or the covariance magnitude $|\sigma_{VL}|$ increases, the expected trading volume increases.*

As many studies of asymmetric information show (e.g., Akerlof, 1970), information asymmetry decreases trading volume because of the well known "lemons" problem. In contrast, as shown in Part 1 of Proposition 2 and Fig. 4, the average trading volume can increase with information asymmetry when the population of the uninformed investors is relatively large. This is

[29] See discussions on Theorem 2 for the generalized model in Section 6 where the market maker can be risk neutral.

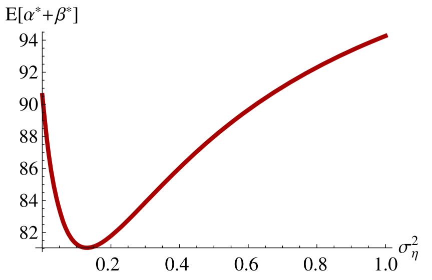

Fig. 4. The expected trading volume in equilibrium against information asymmetry σ_η^2. The default parameter values are $\delta = 1$, $\bar{\theta} = 4$, $\bar{V} = 3$, $N_M = 1$, $N_I = 100$, $N_U = 1000$, $\sigma_\varepsilon = 0.6$, $\sigma_V = 0.8$, $\sigma_{VL} = 0.8$, and $\sigma_X = 1$.

because expected trading volume increases with the expected magnitude of the reservation price difference, which can increase with information asymmetry when the marginal impact of the adverse selection effect on each uninformed investor is small that occurs when their population size is large. In addition, because as the liquidity shock volatility σ_X or the covariance magnitude $|\sigma_{VL}|$ increases, the expected magnitude of the reservation price difference increases, so does the expected trading volume.

5.3. Utility loss due to market power

In this subsection, we analyze the welfare loss due to market power. Not surprisingly, it can be shown that the market maker's market power makes herself better off but non-market-makers worse off. Also as expected, because of market friction, the social welfare loss (measured by the total certainty equivalent wealth loss defined below) due to market power is positive. This implies that there exists a Pareto improvement wealth transfer and market regulation mechanism that limit market bid–ask spreads and depths and make all investors (including the market maker) strictly better off.

Now we examine how information asymmetry affects the welfare loss from the market maker's market power. Let U_i and U_i^c denote the expected utility of type i ($i = I, U, M$) investors with a monopolistic market maker and a perfectly competitive market maker (where the market maker is also a price taker as others) respectively, given realizations of signals on date 0 and f_i and f_i^c be the corresponding certainty equivalent wealth, i.e., $U_i = -\exp(-\delta f_i)$, and $U_i^c = -\exp(-\delta f_i^c)$.

Definition 2. The certainty equivalent wealth loss of a type i investor ($i = I, U, M$) due to market power is $f_i^c - f_i$ and the total certainty equivalent wealth loss WL is $N_U(f_U^c - f_U) + N_I(f_I^c - f_I) + (f_M^c - f_M)$.

The next result shows how the expected total certainty equivalent wealth loss due to market power (before date 0 signal realizations) changes with information asymmetry.

Proposition 3. *If the difference in conditional variances between the uninformed and the informed (i.e.,* $\mathrm{Var}(\tilde{V}|\mathcal{I}_U) - \mathrm{Var}(\tilde{V}|\mathcal{I}_I)$*) is small, then the expected total certainty equivalent wealth loss due to market power decreases with information asymmetry (i.e.,* $\frac{\partial E[WL]}{\partial \sigma_\eta^2} < 0$*).*

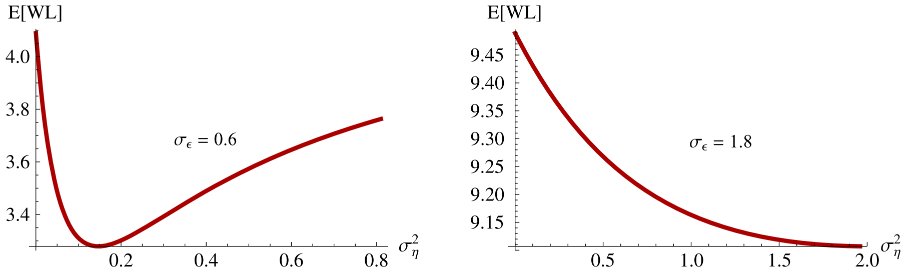

Fig. 5. The expected total certainty equivalent wealth loss due to market power against information asymmetry σ_η^2. The default parameter values are $\delta = 1$, $\bar{\theta} = 4$, $\bar{V} = 3$, $N_I = 100$, $N_U = 1000$, $\sigma_V = 0.8$, $\sigma_{VL} = 0.8$, and $\sigma_X = 1$.

Proposition 3 implies that information asymmetry may *decrease* the expected total certainty equivalent wealth loss due to market power. Consistent with this result, Fig. 5 shows that the expected total certainty equivalent wealth loss decreases with information asymmetry when the private signal is noisy. Intuitively, this is because when the private signal is noisy, the expected bid–ask spread decreases with information asymmetry (as shown in Fig. 2) and the increase in investors' utility from the smaller expected spread can offset the cost of information asymmetry.

6. A generalized model

To simplify exposition, in the main model studied above we assume that all investors have the same risk aversion, the same initial inventory, the same date 1 resale value of the security, and only the informed have private information and liquidity shocks. In this section, we relax these assumptions and still, the generalized model is tractable and solved in closed-form.

This generalized model can be used to conduct analyses such as the effect of a market maker's inventory, private information, and liquidity shocks on asset prices. Let $\bar{\theta}_i$, δ_i, $\hat{X}_i$, $\tilde{V}_i$ and $\mathcal{I}_i$ denote respectively the initial inventory, risk aversion coefficient, liquidity shock, date 1 resale value of the security and information set for a type i investor for $i \in \{I, U, M\}$. Then by the same argument as before, a type i investor's reservation price can be written as

$$P_i^R = E[\tilde{V}_i|\mathcal{I}_i] - \delta_i \mathrm{Cov}[\tilde{V}_i, \tilde{L}|I_i]\hat{X}_i - \delta_i \mathrm{Var}[\tilde{V}_i|\mathcal{I}_i]\bar{\theta}_i, \quad i \in \{I, U, M\}. \tag{36}$$

Let $\Delta_{ij} := P_i^R - P_j^R$ denote the reservation price difference between type i and type j investors for $i, j \in \{I, U, M\}$. In this generalized model, there are eight cases corresponding to eight different trading direction combinations of the informed and the uninformed, as illustrated in Fig. 6.[30] Fig. 6 shows that the trading directions are determined by the ratio of the reservation price difference between the informed and the uninformed (Δ_{IU}) to the reservation price difference between the uninformed and the market maker (Δ_{UM}). When the magnitude of this ratio is large enough (Cases (1) and (5)), the informed and the uninformed trade in opposite directions. If it is small enough (Cases (3) and (7)), on the other hand, they trade in the same direction. In between, either the informed or the uninformed do not trade.

To save space, we only present the equilibrium results for Cases (1), (2), and (5) in this section, where Cases (1) and (5) are a direct generalization of the main model in Section 3, and Case (2)

[30] The case where both informed and uninformed do not trade is a measure zero event (represented by the origin of the figure), which occurs only when the reservation prices of *all* investors are exactly the same.

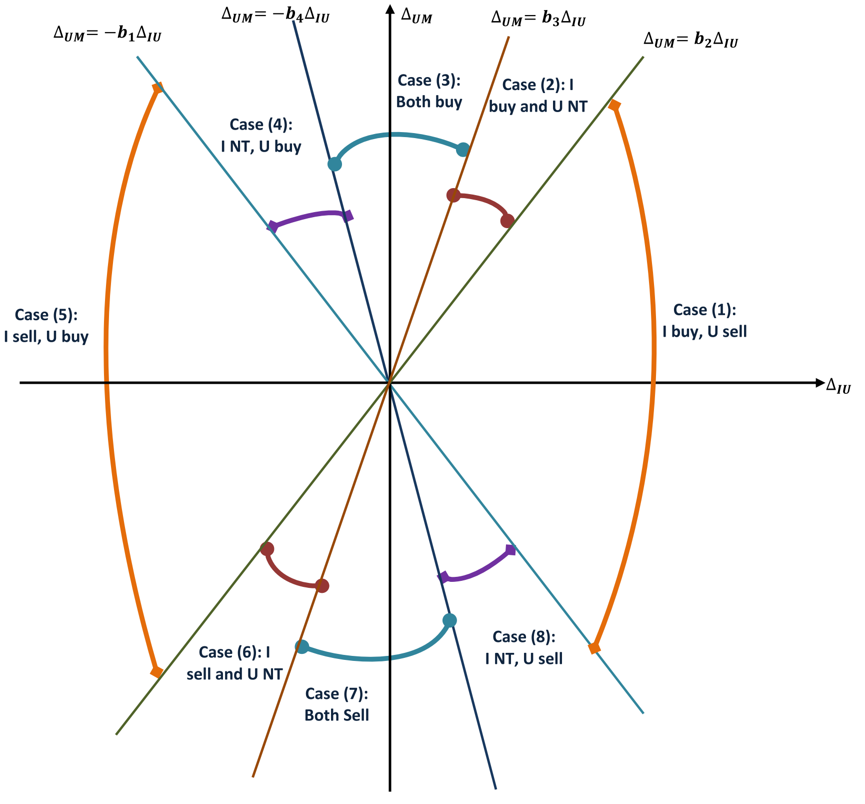

Fig. 6. Eight cases of equilibria characterized by the trading directions of the informed and the uninformed, where b_1, b_2, b_3 and b_4 are defined in (37), (38) and (B-1).

illustrates what happens if some investors do not trade. The rest are similar and are provided in Appendix B. Define

$$b_1 = \frac{\delta_M \nu_2 N_U + 2\delta_U \nu_1}{2\delta_U \nu_1}, \quad b_2 = \frac{\delta_M \nu_2 N_I}{2\delta_I}, \tag{37}$$

$$b_3 = \frac{1}{2\delta_I}\left(\delta_M \nu_2 N_I + \sqrt{\frac{\delta_I(2\delta_I + \delta_M \nu_2 N_I)(\hat{N}+1)}{\delta_I N_U/(\delta_U \nu_1 N_I) + 1}}\right) > b_2. \tag{38}$$

where

$$\nu_1 = \frac{\mathrm{Var}[\tilde{V}_U|\mathcal{I}_U]}{\mathrm{Var}[\tilde{V}_I|\mathcal{I}_I]}, \; \nu_2 = \frac{\mathrm{Var}[\tilde{V}_M|\mathcal{I}_M]}{\mathrm{Var}[\tilde{V}_I|\mathcal{I}_I]}, \; \hat{N} := \frac{\delta_M}{\delta_I}\nu_2 N_I + 1 + \frac{\delta_M \nu_2}{\delta_U \nu_1} N_U.$$

Theorem 2. *For the generalized model, we have:*

1. *The informed buy and the uninformed sell (Case (1)) if and only if*

$$-b_1\Delta_{IU} < \Delta_{UM} < b_2\Delta_{IU}. \tag{39}$$

The informed sell and the uninformed buy (Case (5)) if and only if

$$b_2\Delta_{IU} < \Delta_{UM} < -b_1\Delta_{IU}.$$

For Cases (1) and (5), the equilibrium bid and ask prices are

$$A^* = P_U^R + \frac{\nu_2 N_I \delta_M}{2\delta_I\left(\hat{N}+1\right)}\Delta_{IU} - \frac{\Delta_{UM}}{\hat{N}+1} + \frac{\Delta_{IU}^+}{2}, \tag{40}$$

$$B^* = P_U^R + \frac{\nu_2 N_I \delta_M}{2\delta_I\left(\hat{N}+1\right)}\Delta_{IU} - \frac{\Delta_{UM}}{\hat{N}+1} - \frac{\Delta_{IU}^-}{2}, \tag{41}$$

and the bid–ask spread is

$$A^* - B^* = \frac{|\Delta_{IU}|}{2}; \tag{42}$$

the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{UM} + b_1\Delta_{IU}}{(\hat{N}+1)\delta_I \operatorname{Var}[\tilde{V}_I|\mathcal{I}_I]}, \tag{43}$$

$$\theta_U^* = \frac{\Delta_{UM} - b_2\Delta_{IU}}{(\hat{N}+1)\delta_U \operatorname{Var}[\tilde{V}_U|\mathcal{I}_U]}, \tag{44}$$

$$\theta_M^* = -\left(N_I\theta_I^* + N_U\theta_U^*\right); \tag{45}$$

the equilibrium quote depths are

$$\alpha^* = N_I(\theta_I^*)^+ + N_U(\theta_U^*)^+, \tag{46}$$

$$\beta^* = N_I(\theta_I^*)^- + N_U(\theta_U^*)^-. \tag{47}$$

2. *The informed buy and the uninformed do not trade (Case (2)) if and only if*

$$b_2\Delta_{IU} \le \Delta_{UM} \le b_3\Delta_{IU}. \tag{48}$$

For Case (2), the equilibrium bid and ask prices are

$$A^* = P_I^R - \frac{\Delta_{IM}}{2 + N_I\nu_2\delta_M/\delta_I}, \quad B^* \le P_U^R; \tag{49}$$

the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{IM}}{(2\delta_I + N_I\nu_2\delta_M)\operatorname{Var}[\tilde{V}_I|\mathcal{I}_I]}, \ \theta_U^* = 0, \ \theta_M^* = -N_I\theta_I^*; \tag{50}$$

the equilibrium quote depths are

$$\alpha^* = \frac{N_I\Delta_{IM}}{(2\delta_I + N_I\nu_2\delta_M)\operatorname{Var}[\tilde{V}_I|\mathcal{I}_I]}, \quad \beta^* = 0. \tag{51}$$

As in the main model, when all investors trade in equilibrium (Cases (1) and (5)), spread and trading volume are still proportional to the absolute value of the reservation price difference between the informed and the "uninformed" and thus positively correlated. In addition, our main result that expected spread can decrease with information asymmetry still holds in the generalized model. We illustrate this robustness in Fig. 7, where we set $\delta_M = 0$, $\bar{\theta} = 2.5$ and everything else

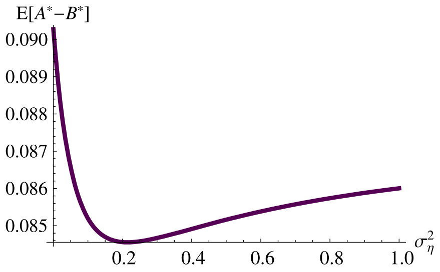

Fig. 7. Expected bid–ask spread against information asymmetry σ_η^2 when the market maker is risk neutral. The default parameter values are: $\delta = 1$, $\bar{\theta} = 2.5$, $\bar{V} = 3$, $N_I = 100$, $N_U = 1000$, $\sigma_V = 0.4$, $\sigma_\varepsilon = 0.4$, $\sigma_X = 1$, and $\sigma_{VL} = 0.8$.

the same as in the left subfigure of Fig. 2 in the paper.[31] Fig. 7 shows that the expected spread may decrease with information asymmetry σ_η^2 when market maker is risk neutral and there are multiple equilibria in some states. Setting $\delta_M = 0$ in Fig. 7 also helps to show that the market maker's risk aversion (and thus inventory risk) is not critical for our main results. The driving force behind our main results is that the market maker can shift some trades from some investors to other risk averse investors who optimally respond to prices.

As Theorem 1, Theorem 2 reveals that conditional on the uninformed and the informed trading in the opposite directions (i.e., Cases (1) and (5)), the equilibrium spread only depends on the reservation price difference between the informed and the uninformed, but not on the initial inventory, or the risk aversion, or the private valuation of a market maker. Intuitively, the initial inventory, the risk aversion, and the private valuation of a market maker only affect the optimal net trade she must make to achieve the optimal date 1 inventory level. To achieve a given level of net trade, the market maker can vary the bid price while keeping the spread constant, i.e., the ask is set to bid plus the fixed spread. This shows that spread can be chosen independently of the optimal date 1 inventory level. For cases where the market maker does not trade on both sides (e.g., Case (2)), however, the spread in general depends on the characteristics of the market maker. This is because the market maker does not make offsetting trades in these cases and thus any trades made change inventory risk exposure. This result suggests whether the initial inventory, the risk aversion, or the private valuation of a market maker is important for the spread depends on whether the market maker can relatively frequently make offsetting trades. One empirically testable implication of this result is that in relatively less active markets, the average spread is more sensitive to the inventory level and the private information of a market maker.

Although inventory risk does not affect the spread in Cases (1) and (5), it affects active depths and prices (i.e., at which trades occur) in all cases. For example, for Cases (1) and (5), Theorem 2 implies when the market maker's initial inventory is large, she reduces the inventory by lowering both the ask and the bid, which encourages purchases and discourages sales by other investors

[31] We use a different $\bar{\theta}$ value from that used in Fig. 2 so that we can still have both decreasing and increasing patterns in one figure with the changed δ_M. Because the market maker is indifferent between all possible inactive prices (i.e., at which no trade occurs), if there is no trade at bid (resp., ask), we use the highest possible bid (resp., the lowest possible ask) such that there is no trade at bid (resp., ask) to compute the expected spread, e.g., setting $B^* = P_U^R$ in Case (2). Note that even with this convention, there are still multiple equilibria in some states. For example, at the boundary between Cases (1) and (2), there are two equilibria across which the equilibrium prices and trading quantities differ but the market maker obtains the same expected utility. This multiplicity does not affect the computation because it is a measure zero event.

and thus increases equilibrium ask depth and decreases bid depth.[32] Accordingly, another empirically testable implication is that average ask depth increases, but average bid depth decreases with a market maker's initial inventory. In addition, Theorem 2 implies that as the market maker's risk aversion increases, keeping the reservation price of the market maker constant (e.g., by setting $\theta_M = 0$), her net trade amount always decreases, bid price also decreases when she buys in the net, but ask price increases when she sells in the net (due to the extra risk premium required for taking inventory risk), which implies that expected spread increases with the market maker's risk aversion.

Theorem 2 shows when the market maker and the uninformed have different reservation prices, there may exist equilibria where some investors do not trade and the market maker only trades on one side. For example, in Case (2), the reservation price of the uninformed is lower than that of the informed but higher than that of the market maker, the market maker chooses not to trade with the uninformed. This is because in this case the profit from the spread and the benefit from shifting the trade with the potentially informed are dominated by the cost from the price required for trading with the uninformed. Other examples include Cases (3), (4), (6), (7), and (8) presented in Appendix B. This shows that while the market maker can trade both at the bid and at the ask on date 0, she may choose to trade only on one side. These equilibria where the market maker trades only on one side at a time imply similar trading behaviors to those implied by a sequential trading model. Cases (1) and (5) are more applicable to more active markets such as OTCQX and OTCQB stock markets where the frequency of making offsetting trades is relatively high, while the rest is more representative of less active markets where time between offsetting trades is relatively long (e.g., pink sheet stock markets).[33]

7. Summary and conclusions

Market makers in some financial markets often make offsetting trades and have significant market power. In this paper, we develop a market making model that captures these market features as well as other important characteristics such as information asymmetry and inventory risk. We solve the equilibrium bid and ask prices, bid and ask depths, trading volume, and inventory levels in closed-form. Our model can accommodate substantial heterogeneity across investors in preferences, endowment, informativeness, and liquidity demand (as in Section 6). The trading behavior in these equilibria is largely consistent with those observed in a wide range of financial markets.

In contrast to the existing literature, a market maker in our model can optimally shift some trade with potentially informed investors to other discretionary investors by adjusting bid or ask. As a result, we find that consistent with empirical evidence, expected bid–ask spreads may *decrease* with information asymmetry and bid–ask spreads can be positively correlated with trading volume. Our analysis shows that when market makers can make offsetting trades and have significant market power, their pricing, liquidity provision, and inventory decisions as well as the impact of information asymmetry on these decisions can be qualitatively different from those predicted by the existing literature. In addition, information asymmetry may decrease the welfare loss due to market power.

[32] This is because the reservation price of a market maker decreases with the initial inventory, and thus Δ_{UM} increases with it.

[33] OTCQX and OTCQB are top tier OTC markets for equity securities (more than 3700 stocks) with a combined market capitalization of more than \$1 trillion and more than 2 billion daily share trading volume.

Important empirical implications of our analysis include:

1. In markets where market makers have significant market power and frequently offset their trades, average spreads decrease with information asymmetry and spreads are positively correlated with trading volume.
2. Average spread is more sensitive to a market maker's inventory level in markets where market makers cannot frequently make offsetting trades.
3. As a market maker's initial inventory increases, average ask depth increases, but average bid depth decreases.

We hope future empirical investigations will study the importance of a market maker's offsetting trades and market power in affecting asset pricing and market liquidity.

Appendix A

In this appendix, we provide proofs of analytical results and an extension of our model to allow multiple market makers.

A.1. Proof of Theorem 1

We prove the case when $\Delta < 0$. In this case, we conjecture that I investors sell and U investors buy. Given bid price B and ask price A, the optimal demand of I and U are:

$$\theta_I^* = \frac{P_I^R - B}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]} \quad \text{and} \quad \theta_U^* = \frac{P_U^R - A}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]}. \tag{A-52}$$

Substituting (A-52) into the market clearing condition (5), we get that the market clearing bid and ask depths are:

$$\alpha = N_U \theta_U^* = N_U \frac{P_U^R - A}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]}, \quad \beta = -N_I \theta_I^* = N_I \frac{B - P_I^R}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}. \tag{A-53}$$

The market maker's problem is equivalent to:

$$\max_{A,B} \alpha A - \beta B + (\bar{\theta} + \beta - \alpha) E[\tilde{V}|\mathcal{I}_M] - \frac{1}{2}\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_M](\bar{\theta} + \beta - \alpha)^2, \tag{A-54}$$

subject to (A-53). The FOC with respect to B (noting that β is a function of B) gives us:

$$-\beta - B\frac{N_I}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]} + E[\tilde{V}|\mathcal{I}_M]\frac{N_I}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]} - \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_M](\bar{\theta} + \beta - \alpha)\frac{N_I}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]} = 0,$$

which is reduced to

$$(\nu N_I + 2)\beta - \nu N_I \alpha = -\frac{N_I \Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}, \tag{A-55}$$

by (9), (18), and expressing B in terms of β using (A-53).

Similarly using the FOC with respect to A, we get:

$$\alpha - \frac{N_U A}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]} + \frac{N_U E[\tilde{V}|\mathcal{I}_M]}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]} + \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_M](\bar{\theta} + \beta - \alpha)\left(-\frac{N_U}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]}\right) = 0,$$

which can be reduced to

$$(N_U+2)\alpha - N_U\beta = 0, \tag{A-56}$$

by using (9), expressing A in terms of α using (A-53), and noting that $\mathcal{I}_M = \mathcal{I}_U$.

Solving (A-56) and (A-55), we can get the equilibrium ask depth and bid depth α^* and β^* as in (24) and (25). Substituting α^* and β^* into (A-53), we can get the equilibrium ask and bid prices A^* and B^* as in (20) and (21). In addition, by the market clearing condition, we have $\theta_U^* = \alpha^*/N_U$ and $\theta_I^* = -\beta^*/N_I$ as in (23). Also, $A^* < P_U^R$ and $B^* > P_I^R$ are equivalent to $\Delta < 0$, which is exactly the condition we conjecture for I investors to sell and U investors to buy. Similarly, we can prove Theorem 1 for the other case where I investors buy and U investors sell.

Because all utility functions are strictly concave and all budget constraints are linear in the amount invested in the security, there is a unique solution to the problem of each informed and each uninformed given the bid and ask prices. Because the market clearing bid and ask depths are linear in bid and ask prices, there is a unique solution to her utility maximization problem (which already takes into account the market clearing conditions). This implies that there is a unique equilibrium. □

A.2. Proofs of Propositions 1–3

Proof of Proposition 1. Part 1:

$$\Delta = (\rho_I - \rho_U)\left(\hat{s} + \frac{h}{\rho_I}\left(1 + \frac{\sigma_V^2}{\rho_I\sigma_\eta^2}\right)\hat{X}_I - \frac{\sigma_V^2}{\rho_I\sigma_\eta^2}\hat{\eta} + \delta\sigma_V^2\bar{\theta}\right),$$

which implies that Δ is normally distributed with mean $\mu_D = \delta(\rho_I - \rho_U)\sigma_V^2\bar{\theta}$ and variance

$$\sigma_D^2 = (\rho_I - \rho_U)^2\left(\sigma_V^2 + \sigma_\varepsilon^2 + \left(\frac{h}{\rho_I}\left(1 + \frac{\sigma_V^2}{\rho_I\sigma_\eta^2}\right)\right)^2\sigma_X^2 + \frac{\sigma_V^4}{\rho_I^2\sigma_\eta^2}\right)$$
$$= h^2\sigma_X^2 - (\rho_I - \rho_U)\sigma_V^2, \tag{A-57}$$

where the last equality follows from simplification using the law of total variance. Direct integration then yields (31).

Part 2: Taking the derivative of the right hand side of (31) with respect to ρ_U, we have

$$\frac{\partial E[A^* - B^*]}{\partial \rho_U} = \frac{\sigma_V^2}{2\sigma_D}\left[\mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) - \delta\bar{\theta}\sigma_D\left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right)\right].$$

Because ρ_U is decreasing in σ_η^2, we have (32). When $\bar{\theta} = 0$ or μ_D is small enough, the above expression is always positive.

Part 3: It can be shown that μ_D increases but μ_D/σ_D decreases in σ_X^2. $\frac{\partial E[A^*-B^*]}{\partial \sigma_X^2} > 0$ then follows from taking derivatives with respect to μ_D and μ_D/σ_D after factoring out μ_D in (31). Similarly, it follows from straightforward (but tedious) computation that $\frac{\partial E[A^*-B^*]}{\partial |\sigma_{VL}|} > 0$. □

Proof of Proposition 2. Part 1: From the expression of trading volume in Theorem 1, we have

$$Sign\left(\frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_\eta^2}\right) = Sign\left(\frac{\partial E|\Delta|}{\partial \sigma_\eta^2} - \frac{E|\Delta|}{\overline{N}+1}\frac{\partial \overline{N}}{\partial \sigma_\eta^2}\right).$$

Because $\frac{\partial \overline{N}}{\partial \sigma_\eta^2} = A_1 N_I$, where $A_1 := \frac{\rho_X^2 \rho_U^2}{\rho_I \sigma_\varepsilon^2}$. Therefore, if N_U is large enough, we have $\frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_\eta^2} > 0$ if and only if $\frac{\partial E|\Delta|}{\partial \sigma_\eta^2} > 0$.

Part 2: It can be shown that $\frac{\partial (\mu_D/(\overline{N}+1))}{\partial \sigma_X^2} > 0$. From (26), (31), and taking derivatives of $\mu_D/(\overline{N}+1)$ and μ_D/σ_D after factoring out μ_D with respect to σ_X^2, we have $\frac{\partial E[\alpha^* + \beta^*]}{\partial \sigma_X^2} > 0$. Similarly, straightforward computation yields $\frac{\partial E[\alpha^* + \beta^*]}{\partial |\sigma_{VL}|} > 0$. □

Proof of Proposition 3. The total equivalent wealth loss due to market power with asymmetric information is: $WL = \frac{\Delta^2}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]} D$, where D is defined as follows.

$$D = \frac{N_I \left(N_U(\overline{N}+1)^2 + N_U(\overline{N}+1) + 2(N_U+2)\right)}{8\overline{N}(\overline{N}+1)^2}. \tag{A-58}$$

Since $\overline{N}$ increases in σ_η^2, D decreases with information asymmetry σ_η^2. In addition, because

$$\frac{\partial E[\Delta^2]}{\partial \sigma_\eta^2} = -\frac{\rho_I^2 \sigma_X^4 \sigma_V^8 \left(\sigma_X^2 \rho_I^2 \sigma_\eta^2 (\sigma_\varepsilon^2 + \sigma_V^2 - 2\delta^2 \sigma_V^4 \bar{\theta}^2) + (\sigma_X^2 + \rho_I^2 \sigma_\eta^2 / h^2)\sigma_V^4\right)}{\left(\sigma_X^2 \rho_I^2 \sigma_\eta^2 (\sigma_\varepsilon^2 + \sigma_V^2) + \sigma_V^4 (\rho_I^2 \sigma_\eta^2 / h^2 + \sigma_X^2)\right)^3}, \tag{A-59}$$

we have, $\frac{\partial E[\Delta^2]}{\partial \sigma_\eta^2} < 0$, if and only if

$$\begin{aligned} \mathrm{Var}(\tilde{V}|\mathcal{I}_U) - \mathrm{Var}(\tilde{V}|\mathcal{I}_I) &= \left(\left(\frac{\sigma_\varepsilon^2 + \sigma_V^2}{\sigma_V^2}\right)^2 \left(1 + \frac{\sigma_V^4 \sigma_\eta^2}{\delta^2 \sigma_\varepsilon^4 \sigma_{VN}^2 \sigma_X^2}\right) \frac{1}{\sigma_\eta^2} + \frac{\sigma_\varepsilon^2 + \sigma_V^2}{\sigma_V^4}\right)^{-1} \\ &< (2\delta^2 \bar{\theta}^2)^{-1}. \end{aligned} \tag{A-60}$$

Therefore, $E[WL]$ decreases with information asymmetry σ_η^2 if $\mathrm{Var}(\tilde{V}|\mathcal{I}_U) - \mathrm{Var}(\tilde{V}|\mathcal{I}_I)$ is small. □

A.3. An Extension with multiple market makers

In contrast to the standard microstructure literature where market makers directly choose market prices (Bertrand competition), for this multiple market maker case, we model the oligopolistic competition among the market makers as a Cournot competition that is well studied and understood.[34] Specifically, we assume that market makers simultaneously choose how much to buy at bid given the inverse supply function (a function of the market makers' purchasing quantity) of all other participants and how much to sell at ask given the inverse demand function (a function of the market makers' selling quantity) of all other participants. The posted bid and ask prices are the required prices to achieve the optimal amount market makers choose to trade. The equilibrium definition for the main model can be directly extended to this multiple market maker case. *Note that in the monopoly case, the Bertrand competition formulation and the demand/supply function competition formulation (e.g., Vives, 2011) yield the same results as the Cournot competition formulation.*

[34] Similar extension and closed form results can be obtained for the generalized model.

Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{N_M})^\top$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_{N_M})^\top$ be the vector of the number of shares market makers sell at ask (i.e., ask depth) and buy at bid (i.e., bid depth) respectively. Given the demand schedules of the informed and the uninformed ($\Theta_I^*(A, B)$ and $\Theta_U^*(A, B)$), the bid price $B(\beta)$ (i.e., the inverse supply function) and the ask price $A(\alpha)$ (i.e., the inverse demand function) can be determined by the following stock market clearing conditions at the bid and ask prices.[35]

$$\sum_{j=1}^{N_M} \alpha_j = \sum_{i=I,\,U} N_i \Theta_i^*(A, B)^+, \quad \sum_{j=1}^{N_M} \beta_j = \sum_{i=I,\,U} N_i \Theta_i^*(A, B)^-, \tag{A-61}$$

where the left-hand sides represent the total sales and purchases by market makers respectively and the right-hand sides represent the total purchases and sales by other investors respectively.

Then for $j = 1, 2, \ldots, N_M$, the designated market maker M_j's problem is

$$\max_{\alpha_j \geq 0, \beta_j \geq 0} E\left[-e^{-\delta \tilde{W}_{M_j}} | \mathcal{I}_M\right], \tag{A-62}$$

where

$$\tilde{W}_{M_j} = \alpha_j A(\alpha) - \beta_j B(\beta) + (\bar{\theta} + \beta_j - \alpha_j)\tilde{V}. \tag{A-63}$$

We now state the multiple-market-maker version of Theorem 1 for the $N_M \geq 1$ case (setting $N_M = 1$ yields Theorem 1). The proof is very similar to that of Theorem 1 and thus omitted.

Theorem 1 *(With multiple market makers).*

1. *The equilibrium bid and ask prices are*

$$A^* := A(\alpha^*) = P_U^R + \frac{\nu N_M N_I}{(N_M + 1)\left(\overline{N} + 1\right)}\Delta + \frac{\Delta^+}{N_M + 1},$$

$$B^* := B(\beta^*) = P_U^R + \frac{\nu N_M N_I}{(N_M + 1)\left(\overline{N} + 1\right)}\Delta - \frac{\Delta^-}{N_M + 1},$$

and we have $A^* > P^* > B^*$, where

$$P^* = \frac{\nu N_I}{\overline{N}} P_I^R + \frac{N_U}{\overline{N}} P_U^R + \frac{N_M}{\overline{N}} P_M^R \tag{A-64}$$

is the equilibrium price of a perfect competition equilibrium where market makers are also price takers. The bid–ask spread is

$$A^* - B^* = \frac{|\Delta|}{N_M + 1} = \frac{(\rho_I - \rho_U)\left|\left(1 + \frac{\sigma_V^2}{\rho_I \sigma_\eta^2}\right)\hat{S} - \frac{\sigma_V^2}{\rho_I \sigma_\eta^2}\hat{S}_s + \delta\sigma_V^2\bar{\theta}\right|}{N_M + 1}. \tag{A-65}$$

2. *The equilibrium quantities demanded are*

[35] The risk-free asset market will be automatically cleared by the Walras' law. A buyer's (seller's) trade only depends on ask A (bid B). So A only depends on α and B only depends on β.

$$\theta_I^* = \frac{N_M(N_U + N_M + 1)}{(N_M + 1)\left(\overline{N} + 1\right)} \frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}, \quad \theta_U^* = -\frac{\nu N_M N_I}{(N_M + 1)\left(\overline{N} + 1\right)} \frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]}; \tag{A-66}$$

the equilibrium quote depths are

$$\alpha^* = \frac{N_I}{N_M}(\theta_I^*)^+ + \frac{N_U}{N_M}(\theta_U^*)^+, \tag{A-67}$$

$$\beta^* = \frac{N_I}{N_M}(\theta_I^*)^- + \frac{N_U}{N_M}(\theta_U^*)^-, \tag{A-68}$$

which implies that the equilibrium trading volume is

$$N_M(\alpha^* + \beta^*) = \frac{N_M N_I(N_M + 2N_U + 1)}{(N_M + 1)(\overline{N} + 1)} \left(\frac{|\Delta|}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}\right). \tag{A-69}$$

Appendix B

In this appendix, we report the remaining results on the generalized model. Define

$$b_4 = \frac{N_I \delta_U \nu_1}{\sqrt{N_U\delta_I + N_I\delta_U\nu_1}\left(\sqrt{N_U\delta_I + N_I\delta_U\nu_1} - \sqrt{\frac{N_U\delta_I\delta_U\nu_1(\hat{N}+1)}{2\delta_U\nu_1 + N_U\delta_M\nu_2}}\right)} (> b_1). \tag{B-1}$$

The rest of Theorem 2 is as follows.

1. *Both the informed and uninformed buy (Case (3)) if and only if*

   $$\Delta_{UM} \geq \max\{-b_4\Delta_{IU},\ b_3\Delta_{IU}\}.$$

   For Case (3), the equilibrium prices are

   $$A^* = \frac{N_I\nu_1\delta_U P_I^R + N_U\delta_I P_U^R}{N_I\nu_1\delta_U + N_U\delta_I} - \frac{N_I\nu_1\delta_U\Delta_{IM} + N_U\delta_I\Delta_{UM}}{(\hat{N}+1)(N_I\nu_1\delta_U + N_U\delta_I)}, \quad B^* \leq A^*;$$

   the equilibrium security quantities demanded are

   $$\theta_I^* = \frac{\Delta_{UM} + \left(1 + \frac{N_U\delta_I}{N_U\delta_I + N_I\delta_U\nu_1} + \frac{N_U\delta_M\nu_2}{\delta_U\nu_1}\right)\Delta_{IU}}{(\hat{N}+1)\delta_I \mathrm{Var}[\tilde{V}_I|\mathcal{I}_I]},$$

   $$\theta_U^* = \frac{\Delta_{UM} - \left(\frac{N_I\delta_U\nu_1}{N_U\delta_I + N_I\delta_U\nu_1} + \frac{N_I\delta_M\nu_2}{\delta_I}\right)\Delta_{IU}}{(\hat{N}+1)\delta_U \mathrm{Var}[\tilde{V}_U|\mathcal{I}_U]},$$

   $$\theta_M^* = -N_I\theta_I^* - N_U\theta_U^*;$$

   and the equilibrium depths are

   $$\alpha^* = N_I\theta_I^* + N_U\theta_U^*, \quad \beta^* = 0.$$

2. *The informed do not trade and uninformed buy (Case (4)) if and only if*

   $$-b_1\Delta_{IU} \leq \Delta_{UM} \leq -b_4\Delta_{IU}.$$

   For Case (4), the equilibrium prices are

$$A^* = P_U^R - \frac{\Delta_{UM}}{2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)}, \quad B^* \leq P_I^R;$$

the equilibrium security quantities demanded are

$$\theta_I^* = 0, \; \theta_U^* = \frac{\Delta_{UM}}{(2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)) \delta_U \operatorname{Var}[\tilde{V}_U | \mathcal{I}_U]}, \; \theta_M^* = -N_U \theta_U^*;$$

and the equilibrium depths are

$$\alpha^* = N_U \theta_U^*, \quad \beta^* = 0.$$

3. *The informed sell and the uninformed do not trade (Case (6)) if and only if*

$$b_3 \Delta_{IU} \leq \Delta_{UM} \leq b_2 \Delta_{IU}.$$

For Case (6), the equilibrium prices are

$$B^* = P_I^R - \frac{\Delta_{IM}}{2 + N_I \nu_2 \delta_M / \delta_I}, \quad A^* \geq P_U^R;$$

the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{IM}}{(2 + N_I \nu_2 \delta_M / \delta_I) \delta_I \operatorname{Var}[\tilde{V}_I | \mathcal{I}_I]}, \; \theta_U^* = 0, \; \theta_M^* = -N_I \theta_I^*;$$

and the equilibrium depths are

$$\alpha^* = 0, \quad \beta^* = -N_I \theta_I^*.$$

4. *Both the informed and uninformed sell (Case (7)) if and only if*

$$\Delta_{UM} \leq \min\{-b_4 \Delta_{IU}, \; b_3 \Delta_{IU}\}.$$

For Case (7), the equilibrium prices are

$$B^* = \frac{N_I \nu_1 \delta_U P_I^R + N_U \delta_I P_U^R}{N_I \nu_1 \delta_U + N_U \delta_I} - \frac{N_I \nu_1 \delta_U \Delta_{IM} + N_U \delta_I \Delta_{UM}}{(\hat{N} + 1)(N_I \nu_1 \delta_U + N_U \delta_I)},$$

and $A^* \geq B^*$; the equilibrium security quantities demanded are

$$\theta_I^* = \frac{\Delta_{UM} + \left(1 + \frac{N_U \delta_I}{N_U \delta_I + N_I \delta_U \nu_1} + \frac{N_U \delta_M \nu_2}{\delta_U \nu_1}\right) \Delta_{IU}}{(\hat{N} + 1) \delta_I \operatorname{Var}[\tilde{V}_I | \mathcal{I}_I]},$$

$$\theta_U^* = \frac{\Delta_{UM} - \left(\frac{N_I \delta_U \nu_1}{N_U \delta_I + N_I \delta_U \nu_1} + \frac{N_I \delta_M \nu_2}{\delta_I}\right) \Delta_{IU}}{(\hat{N} + 1) \delta_U \operatorname{Var}[\tilde{V}_U | \mathcal{I}_U]},$$

and the equilibrium depths are

$$\alpha^* = 0, \quad \beta^* = -N_I \theta_I^* - N_U \theta_U^*.$$

5. *The informed do not trade and the uninformed sell (Case (8)) if and only if*

$$-b_4 \Delta_{IU} \leq \Delta_{UM} \leq -b_1 \Delta_{IU}.$$

For Case (8), the equilibrium prices are

$$B^* = P_U^R - \frac{\Delta_{UM}}{2 + N_U \nu_2 \delta_M / (\nu_1 \delta_U)}, \quad A^* \geq P_I^R.$$

the equilibrium security quantities demanded are

$$\theta_I^* = 0,\ \theta_U^* = \frac{\Delta_{UM}}{(2 + N_U \nu_2 \delta_M/(\nu_1 \delta_U))\delta_U \mathrm{Var}[\tilde{V}_U|\mathcal{I}_U]},\ \theta_M^* = -N_U \theta_U^*;$$

and the equilibrium depths are

$$\alpha^* = 0,\ \ \beta^* = -N_U \theta_U^*.$$

Proof of Theorem 2. This is similar to the proof of Theorem 1. We only sketch the main steps. First, for each case, conditional on the trading directions (or no trade), we derive the equilibrium depths, prices, and trading quantities, similar to the proof of Theorem 1. Then we verify that under the specified conditions the assumed trading directions are indeed optimal. □

Appendix C

In this appendix, we provide an alternative model where an alternative measure of information asymmetry is used and show that our main results still hold.

Similar to Easley and O'Hara (2004), assume that there are K signals, $\hat{s}_\iota(\iota = 1, 2, \cdots, K)$, where $\hat{s}_\iota = \tilde{V} - \bar{V} + \tilde{\varepsilon}_\iota$ and $\tilde{\varepsilon}_\iota \sim \mathbf{N}(0, \sigma_\varepsilon^2)$ are independently distributed. Among these signals, K_p signals are private and $K - K_p$ signals are public. Informed traders observe all K signals. Uninformed traders and market maker only observe the $K - K_p$ public signals. Let

$$P_i^R = E[\tilde{V}|\mathcal{I}_i] - \delta \mathrm{Cov}[\tilde{V}, \tilde{L}|\mathcal{I}_i]\hat{X}_i - \delta \mathrm{Var}[\tilde{V}|\mathcal{I}_i]\bar{\theta} \tag{C-1}$$

be investor i's ($i \in \{I, U\}$) reservation price.

Because the informed know exactly $\{\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_K, \hat{X}_I\}$, the information set of the informed in equilibrium is

$$\mathcal{I}_I = \{\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_K, \hat{X}_I\}, \tag{C-2}$$

which implies that

$$\mathrm{E}[\tilde{V}|\mathcal{I}_I] = \bar{V} + \rho_I \sum_{\iota=1}^{K} \hat{s}_\iota,\quad \mathrm{Var}[\tilde{V}|\mathcal{I}_I] = (1 - K\rho_I)\sigma_V^2,\quad \mathrm{Cov}[\tilde{V}, \tilde{L}|\mathcal{I}_I] = (1 - K\rho_I)\sigma_{VL}, \tag{C-3}$$

where

$$\rho_I := \frac{\sigma_V^2}{K\sigma_V^2 + \sigma_\varepsilon^2}, \tag{C-4}$$

is the weight the informed put on the sum of all signals. Equation (C-1) then implies that

$$P_I^R = \bar{V} + \rho_I \hat{S} + \rho_I \sum_{\iota=K_p+1}^{K} \hat{s}_\iota - \delta(1 - K\rho_I)\sigma_V^2 \bar{\theta}, \tag{C-5}$$

where $\hat{S} := \sum_{\iota=1}^{K_p} \hat{s}_\iota + \frac{h}{\rho_I}\hat{X}_I$ and $h = -\delta(1 - K\rho_I)\sigma_{VL}$ represents the hedging premium per unit of liquidity shock.

While $\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_K$, and $\hat{X}_I$ both affect the informed investor's demand and thus the equilibrium prices, other investors observe public signals $\hat{s}_{K_p+1}, \hat{s}_{K_p+2}, \cdots, \hat{s}_K$ and can infer the value of $\hat{S}$ from market prices because the joint impact of $\hat{s}_1, \hat{s}_2, \cdots, \hat{s}_{K_p}$, and $\hat{X}_I$ on market prices is only through $\hat{S}$. Thus we conjecture that the equilibrium prices A^* and B^* depend on both $\hat{S}$ and $\sum_{\iota=K_p+1}^{K} \hat{s}_\iota$. Accordingly, the information sets for the uninformed investors and the market maker are

$$\mathcal{I}_U = \mathcal{I}_M = \{\hat{S}, \hat{s}_{K_p+1}, \hat{s}_{K_p+2}, \cdots, \hat{s}_K\}. \tag{C-6}$$

Then the conditional expectation and conditional variance of $\tilde{V}$ for the uninformed and the market maker are respectively

$$E[\tilde{V}|\mathcal{I}_U] = \bar{V} + \rho_U \left(\left(1 - (K - K_p)\rho_X\right) \hat{S} + (1 + K_p \rho_X) \sum_{\iota=K_p+1}^{K} \hat{S}_\iota \right), \tag{C-7}$$

$$\mathrm{Var}[\tilde{V}|\mathcal{I}_U] = (1 - K\rho_U)\sigma_V^2, \tag{C-8}$$

where

$$\rho_X := \frac{h^2\sigma_X^2}{(K - K_p)h^2\sigma_X^2 + K\rho_I^2 K_p \sigma_\varepsilon^2}, \quad \rho_U := \frac{\sigma_V^2}{\sigma_V^2 + \rho_X \rho_I K_p \sigma_\varepsilon^2}\rho_I < \rho_I, \tag{C-9}$$

where ρ_U is the weight the uninformed put on the average signals of $\hat{S}$ and $\sum_{\iota=K_p+1}^{K} \hat{S}_\iota$.

It follows that for $i \in \{U, M\}$,

$$P_i^R = \bar{V} + \rho_U \left(\left(1 - (K - K_p)\rho_X\right) \hat{S} + (1 + K_p \rho_X) \sum_{\iota=K_p+1}^{K} \hat{S}_\iota \right) - \delta(1 - K\rho_U)\sigma_V^2 \bar{\theta}. \tag{C-10}$$

Let $\Delta := P_I^R - P_U^R$ denote the difference in the reservation prices of I and U investors. We then have

$$\Delta = (\rho_I - \rho_U) \left(\left(1 + \left(\frac{K}{K_p} - 1 \right) \frac{\sigma_V^2}{\rho_I \sigma_\varepsilon^2} \right) \hat{S} + \left(1 - \frac{\sigma_V^2}{\rho_I \sigma_\varepsilon^2} \right) \sum_{\iota=K_p+1}^{K} \hat{S}_\iota + \delta K \sigma_V^2 \bar{\theta} \right). \tag{C-11}$$

Let

$$\nu := \frac{\mathrm{Var}[\tilde{V}|\mathcal{I}_U]}{\mathrm{Var}[\tilde{V}|\mathcal{I}_I]} = \frac{1 - K\rho_U}{1 - K\rho_I} \geq 1$$

be the ratio of the security payoff conditional variance of the uninformed to that of the informed, and

$$\overline{N} := \nu N_I + N_U + 1 \geq N$$

be the information weighted total population. The following theorem provides the equilibrium bid and ask prices and equilibrium security demand in closed-form.

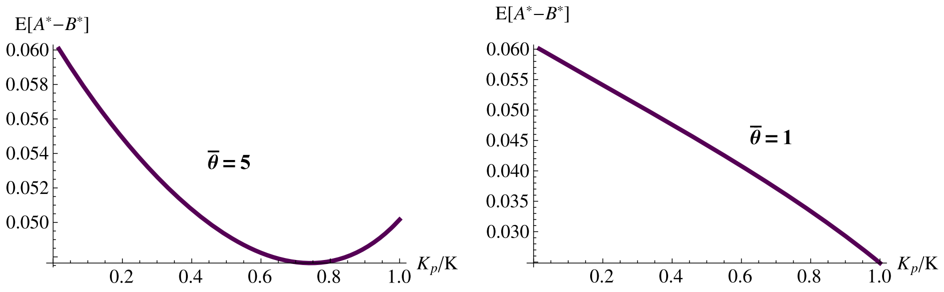

Fig. 8. Expected bid–ask spread against information asymmetry $\frac{K_p}{K}$. The default parameter values are: $\delta = 1, \bar{\theta} = 1$, $\bar{V} = 3$, $N_I = 100$, $N_U = 1000$, $\sigma_\varepsilon = 1.5$, $\sigma_V = 0.4$, $\sigma_X = 1$, and $\sigma_{VL} = 0.8$, $K = 40$.

Theorem 3. *There is a unique symmetric equilibrium (i.e., the same type investors adopt the same strategies):*

1. *The equilibrium bid and ask prices are respectively*

$$A^* = P_U^R + \frac{\nu N_I}{2\left(\overline{N}+1\right)}\Delta + \frac{\Delta^+}{2},$$

$$B^* = P_U^R + \frac{\nu N_I}{2\left(\overline{N}+1\right)}\Delta - \frac{\Delta^-}{2}.$$

The bid–ask spread $A^* - B^*$ is equal to

$$\frac{|\Delta|}{2} = \frac{\rho_I - \rho_U}{2}\left|\left(1 + \left(\frac{K}{K_p} - 1\right)\frac{\sigma_V^2}{\rho_I \sigma_\varepsilon^2}\right)\hat{S} + \left(1 - \frac{\sigma_V^2}{\rho_I \sigma_\varepsilon^2}\right)\sum_{\iota=K_p+1}^{K}\hat{S}_\iota + \delta K \sigma_V^2 \bar{\theta}\right|.$$

2. *The equilibrium quantities demanded are*

$$\theta_I^* = \frac{N_U + 2}{2\left(\overline{N}+1\right)}\frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}, \quad \theta_U^* = -\frac{\nu N_I}{2\left(\overline{N}+1\right)}\frac{\Delta}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_U]};$$

the equilibrium ask and bid depths are respectively

$$\alpha^* = N_I(\theta_I^*)^+ + N_U(\theta_U^*)^+,$$

$$\beta^* = N_I(\theta_I^*)^- + N_U(\theta_U^*)^-,$$

which implies that the equilibrium trading volume is

$$\alpha^* + \beta^* = \frac{N_I(N_U+1)}{\overline{N}+1}\left(\frac{|\Delta|}{\delta \mathrm{Var}[\tilde{V}|\mathcal{I}_I]}\right).$$

As in our main model, Theorem 3 shows that both the spread and the trading volume are proportional to the reservation price difference between the informed and the uninformed. In addition, using the expressions in Proposition 4, we can examine how expected spread changes with the information asymmetry K_p/K. As shown in Fig. 8, expected spread can still decrease with information asymmetry.

Proposition 4.

1. *The reservation price difference* Δ *is normally distributed with mean* μ_D *and variance* σ_D^2*, where*

$$\mu_D = \delta(\rho_I - \rho_U) K \sigma_V^2 \bar{\theta}, \quad \sigma_D^2 = h^2 \sigma_X^2 - K(\rho_I - \rho_U)\sigma_V^2, \tag{C-12}$$

which implies that the expected bid–ask spread is equal to:

$$E[A^* - B^*] = \frac{2\sigma_D \mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) + \mu_D \left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right)}{2}, \tag{C-13}$$

where $\mathbf{n}$ *and* $\mathbf{N}$ *are respectively the pdf and cdf of the standard normal distribution.*

2. *The expected bid–ask spread decreases with information asymmetry* $\frac{K_p}{K}$ *if and only if*

$$\mathbf{n}\left(\frac{\mu_D}{\sigma_D}\right) - \delta\bar{\theta}\sigma_D \left(2\mathbf{N}\left(\frac{\mu_D}{\sigma_D}\right) - 1\right) > 0, \tag{C-14}$$

which is always satisfied when $\bar{\theta} = 0$ *or* μ_D *is small enough.*

The proofs of Theorem 3 and Proposition 4 are very similar to those for our main model and thus omitted.

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