A Calculus Proof of Proposition 1 in "On the Moral Hazard Problem Without the First-Order Approach"

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In Kadan and Swinkels (2012a) we establish a general expression for the effect simultaneously tightening the minimum payment and individual rationality constraints in a general moral hazard problem. The result there does not rely on the validity of the first order approach (FOA), allows for multidimensional signals and effort, and requires no order structure on signals or differentiability of the distribution function. The proof of this result is established using a variational argument, which relies on differentiability and integrability results. In standard moral hazard problems, when the FOA is valid, this result becomes quite simple. Below we provide such a simple version of this result. We also provide a calculus proof of this result that does not hinge on a variational argument. The notation we use is as in Kadan and Swinkels (2012b).

Proposition 1 *Assume the FOA is valid. For all e and m,*

$$C_m(e,m) = u'(m) \int \left[\frac{1}{u'(\pi(x,e,m))} - \lambda(e,m) \right] f(x|e)\, dx. \tag{1}$$

Proof: Fix $e > 0$ and m. We have,

$$\begin{aligned} C_m(e) &= \int_0^1 \pi_m f = \int_0^1 \frac{1}{u'(\pi)} u'(\pi)\, \pi_m f dx \qquad (2) \\ &= \int_0^{\hat{x}} \frac{1}{u'(\pi)} u'(\pi)\, \pi_m f + \int_{\hat{x}}^1 \frac{1}{u'(\pi)} u'(\pi)\, \pi_m f \\ &= \int_0^{\hat{x}} \frac{u'(m)}{u'(\pi)} f + \int_{\hat{x}}^{\bar{x}} \left[\lambda(x,e) + \mu(x,e) \frac{f_e}{f} \right] u'(\pi)\, \pi_m f \\ &= \int_0^{\hat{x}} \frac{u'(m)}{u'(\pi)} f + \lambda(e,m) \int_{\hat{x}}^1 u'(\pi)\, \pi_m + \mu(e,m) \int_{\hat{x}}^1 u'(\pi)\, \pi_m f_e \end{aligned}$$

Where the second equality follows since over $[0, \hat{x}(e,m)]$, $\pi_m = 1$ and $u'(\pi(x,e,m)) = u'(m)$.

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If IR is not binding then $\lambda(e,m)=0$. Otherwise, from IR we have that for all m,

$$\int_0^1 u(\pi) f = u_0 + c(e).$$

Differentiating by m gives

$$\int_0^1 u'(\pi)\pi_m f = 0.$$

Equivalently,

$$\begin{aligned}\lambda(e,m)\int_{\hat{x}}^1 u'(\pi)\pi_m f &= -\lambda(e,m)\int_0^{\hat{x}} u'(\pi)\pi_m f \\ &= -\lambda(e,m)\,u'(m)\,F(\hat{x}|e).\end{aligned} \tag{3}$$

From IC we have that for all m,

$$\int u(\pi) f_e = c'(e).$$

Differentiating by m gives,

$$\int u'(\pi)\pi_m f_e = 0.$$

Equivalently,

$$\begin{aligned}\mu(e,m)\int_{\hat{x}}^1 u'(\pi)\pi_m f_e &= -\mu(e,m)\int_0^{\hat{x}} u'(\pi)\pi_m f_e \\ &= -\mu(e,m)\,u'(m)\int_0^{\hat{x}} f_e = \mu(e,m)\,u'(m)\int_{\hat{x}}^1 f_e \\ &= u'(m)\int_{\hat{x}}^1 \mu(e,m)\frac{f_e}{f} f = u'(m)\int_{\hat{x}}^1\left[\frac{1}{u'(\pi)}-\lambda(e,m)\right] f.\end{aligned} \tag{4}$$

Plugging (3) and (4) back into (2) gives

$$\begin{aligned}C_m(e) &= \int_0^{\hat{x}}\frac{u'(m)}{u'(\pi)} f + \lambda(e,m)\int_{\hat{x}}^1 u'(\pi)\pi_m f + \mu(e,m)\int_{\hat{x}}^1 u'(\pi)\pi_m f_e \\ &= \int_0^{\hat{x}}\frac{u'(m)}{u'(\pi)} f - \lambda(e,m)\,u'(m)\,F(\hat{x}|e) + u'(m)\int_{\hat{x}}^1\left[\frac{1}{u'(\pi)}-\lambda(e,m)\right] f \\ &= u'(m)\int_0^{\hat{x}}\left[\frac{1}{u'(\pi)}-\lambda(e,m)\right] f + u'(m)\int_{\hat{x}}^1\left[\frac{1}{u'(\pi)}-\lambda(e,m)\right] f \\ &= u'(m)\int\left[\frac{1}{u'(\pi)}-\lambda(e,m)\right] f,\end{aligned}$$

as required. ■

References

[1] Kadan, O. and J. Swinkels, 2012*a*. On the Moral Hazard Problem without the First Order Approach. Available at http://www.kellogg.northwestern.edu/Faculty/Directory/Swinkels_Jeroen.aspx.

[2] Kadan, O. and J. Swinkels, 2012*b*. Minimum Payments and Induced Effort in Moral Hazard Problems. Available at http://www.kellogg.northwestern.edu/Faculty/Directory/Swinkels_Jeroen.aspx.