Dynamic Pricing and Price Commitment of New Experience Goods

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Abstract

This article develops a dynamic model to examine how a firm selling non-durable experience goods can signal its high quality with dynamic spot-pricing or price commitment. Since consumers who buy and use the product will learn the quality of the product, the firm’s early-period price will endogenously determine the fraction of informed consumers in the later period. Without credible price commitment, the high-quality firm will prefer the pooling outcome in the first period, and in the second period, both types of firms will separate and target only the first-period buyers. By contrast, with credible price commitment, the high-quality firm will be able to profitably signal its quality by lowering its first-period price or increasing its second-period price from its first-best price. Credible price commitment will benefit the high-quality firm by lowering its signaling cost, but can either increase or decrease consumer surplus and social welfare. Furthermore, we show that a longer time horizon can allow the high-quality firm to signal its quality costlessly, by maintaining its high first-best price for all periods.

Key words: dynamic signaling, pricing, price commitment, experience goods, learning, inference, game theory
1. Introduction

An experience good is a product whose quality consumers cannot readily determine until they have used the product after purchase (Nelson 1970). An important problem for the firm selling a new, high-quality experience good is how to credibly signal its quality. Although consumers do not \textit{ex ante} know the true quality, those who buy and use the product can learn its quality, and their future purchase decisions will be based on the true quality whereas non-buyers may remain uninformed. We develop a dynamic pricing framework to examine how a firm with a new, non-durable, experience good can signal its quality in two different situations. First, when the firm cannot credibly commit its future price, it can only adopt dynamic spot-pricing to signal its quality. Second, when the firm can credibly commit its future price, its signal consists of either dynamic spot-pricing or initial pricing plus price commitment. This article analyzes the interaction of price signaling and learning through consumption experience. Our analysis also examines how price commitment can allow the high-quality firm to signal its quality more efficiently. We show that with price commitment the high-quality firm may be able to costlessly signal its quality when the time horizon is long enough.

Strategic price commitment is an efficient way to convey the quality of experience goods, because over time more consumers will learn the true quality through purchase or use of the product and they will buy the product again only if its price is in line with the true quality. If the firm commits too high a future price relative to its true quality, the firm will not attract many informed customers. In other words, it will be more costly for the low-quality firm to mimic the high-quality firm’s high future price.
So, this price commitment can be an efficient tool for the high-quality firm to signal its true quality. Our analysis reveals that the firm may commit a future price that is even higher than its first-best price. We will examine the effects of price commitment on the firm’s profit as well as on consumer surplus and social welfare.

Our research provides an alternative explanation why high-end firms (e.g., luxury-goods firms, upscale restaurants, and many professional services such as legal services) maintain high prices for their products and offer few discounts or promotions. By maintaining high prices over time, these firms can signal their high quality to new customers; having a history of not offering promotions or price reductions can to some extent be considered as price commitment. For example, Louis Vuitton (LV) does not offer price discounts and adopts a strict no-discount policy worldwide. Even though consumers *ex ante* do not directly observe the new product’s quality (especially when consumers order the product from the firm online), they may infer the firm’s high quality from its high-price commitment. Note that the level of credibility of a firm’s price commitment depends on its past behavior or reputation. Breaking the high-price commitment will lose credibility for future new product introductions. For LV, the high-price (no-discount) credibility may be close to 100% whereas for another firm it might be much less so. We will discuss in the Conclusion section how the level of credibility might influence the market outcomes. Our research suggests that when a firm has a long time horizon for its product or business, it

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1 It is worth recalling that LV is a luxury brand with an ego-expressive purpose: quality consists of the brand value. However, to sustain LV’s brand value, its product quality should also be high (e.g., the make or materials), which is not publicly known but can be learned by usage experience. So, even when a well-known brand launches new products, it still needs to convince consumers of the product quality through some marketing mix.
will tend to have “truthful” and stable pricing (at first-best price levels) according its true quality. We acknowledge that real-world examples can often have alternative explanations (or confounding factors) for the consumer’s inference of the firm’s high quality, e.g., the firm’s long-term brand positioning or free-return policies. Our price-commitment explanation offers an economic mechanism that reinforces such explanations as branding.

This article contributes to the literature on dynamic pricing of experience goods. When a firm launches a new product, it may adopt different pricing strategies (e.g., penetration or skimming pricing strategies). Shapiro (1983) argues that if consumers can learn the firm’s product quality after purchase, the monopolist’s optimal pricing strategy depends on whether its reputation is below or above its true quality. Bils (1989) studies the monopolist’s tradeoff between exploiting past customers (informed) and attracting new ones (uninformed). Bergemann and Välimäki (2006) find that the firm’s strategy depends on whether it is in a niche or a mass market, where the firm’s current price will affect the fraction of consumers being informed in the future. Our paper contributes to the aforementioned literature in two aspects. First, these articles do not consider the possibility that the firm’s strategy can signal its product quality to consumers; that is, they assume that consumers do not make rational inferences on quality from the firm’s actions. By contrast, we explicitly study how a firm’s multiple-period pricing strategy can signal its quality information to uninformed consumers, because firms of different quality levels have different optimal pricing strategies. By considering the firm’s profit incentives, the consumer will make a rational inference about the firm’s quality based on its pricing decision; such rational inferences
about quality directly affect the consumer’s expected valuation and purchase decision, therefore influencing the firm’s optimal pricing strategies. Second, we also examine the role price commitment plays in the experience goods market. We find that the firm can credibly signal its high quality by committing high future prices, especially as the number of periods increases, because such commitments would impose higher opportunity costs on the low-quality firm than on the high-quality firm. We show that a longer time horizon can allow the high-quality firm to costlessly signal its quality by committing prices to its first-best prices.

This article also contributes to the literature on firms’ strategic commitment. Commitment to a future output or price level can mitigate the time-inconsistency and incentive problem. The monopolist can attain its maximum profit when it can credibly commit to producing only the monopoly quantity (Suslow 1986) or to a static monopoly price in every period (Sobel 1991). Moreover, when the monopolist sells a durable good, price commitment (Dudine et al. 2006) and a best-price provision (Butz 1990) can reduce the consumers’ incentives to delay purchases and increase social welfare. In contrast to this stream of literature that focuses on durable goods, our article examines the effect of price commitment on non-durable experience goods in an explicitly dynamic setting. We show that since repeat customers will be informed of the true product quality, the high-quality firm will find it less costly to commit a high future price than will a low-quality firm. As a result, price commitment or the lack thereof can convey information about product quality.

Finally, our research contributes to the extensive literature on signaling games. When consumers
do not directly observe product quality before purchase, the firm can potentially convey to consumers its quality using signals such as prices (Wolinsky 1983, Gerstner 1985, Riordan 1986, Bagwell and Riordan 1991), advertised prices (Simester 1995 and Shin 2005), nonlinear price contracts (Desai and Srinivasan 1995), dissipative advertising (Nelson 1974, Schmalensee 1978, Kihlstrom and Riordan 1984, Milgrom and Roberts 1986), warranties (Lutz 1989), money-back guarantees (Moorthy and Srinivasan 1995), umbrella branding (Wernerfelt 1988), and slotting allowance (Desai 2000). To credibly signal its high quality, the firm must carry out some strategy that low-quality firms will not have any profitable incentive to mimic. When different types of firms have different marginal costs, the high-quality firm may be able to use prices to signal its quality. For example, Bagwell and Riordan (1991) provide a static model for durable goods and interpret the comparative statics to show that if the number of informed consumers exogenously increases over time, the price distortion needed for the high-quality firm to signal its quality will decrease. Note that in the interpretation of their static model, consumers make inferences about the firm’s quality only from its current price and not from past or future prices; in addition, they do not model any consumer learning since the dynamic interpretation of their static game implies that in each period new consumers arrive and exit the market. Judd and Riordan (1994) study learning of product quality but assume that quality is unknown to both consumers and the firm, who will observe independent private signals of the quality after the first period. In contrast to these works, we study dynamic pricing of non-durable goods and explicitly model the consumer’s

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2 There is also some literature on the firm’s signaling of costs or margins (e.g., Guo and Jiang 2016, Kuksov and Lin 2016, Jiang et al. 2016).
learning of the firm’s quality (through purchase and use of the product), and in our dynamic-signaling model the uninformed consumers in later periods can make inferences based on both the current and the past prices of the product. In addition, we study the case where consumers can make inferences about the firm’s quality from not only its current price but also any price commitment. Conceptually speaking, credible price commitment converts the dynamic-signaling problem into a two-dimensional signaling problem. To the best of our knowledge, our article is among the first to study a dynamic and non-durable experience-goods setting where the firm’s pricing strategy in the early period will endogenously determine how many consumers will be informed of the firm’s quality in the later period, giving rise to a dynamic signaling issue.\(^3\) This is, this paper analyzes the interaction of price signaling and consumer learning through usage experience in addition to price commitment.

We highlight some main results from our analysis. Without credible price commitment, the high-quality firm will prefer the pooling outcome in the first period, and in the second period, both types of firms will separate and target only their first-period buyers. With credible price commitment, the high-quality firm may be able to more efficiently signal its quality by either lowering its first-period price or raising its second-period price from its first-best price. Price commitment will benefit the high-quality firm by reducing its signaling cost and will make the low-quality firm worse off, because it increases the low-quality firm’s cost of mimicking the high-quality firm. The effects of the firm’s price

\(^3\) Chen and Jiang (2015) also study dynamic pricing of non-durable experience goods but focus on how the demand uncertainty and the firm’s lack of information on demand will influence its pricing strategies. Jiang and Yang (2015) examine experience goods in a dynamic setting where quality is the firm’s endogenous choice and where consumers do not directly observe the firm’s quality choice or its cost efficiency.
commitment on consumer surplus and social welfare depend on whether the firm chooses higher or lower prices than its first best. If the high-quality firm chooses a lower-than-first-best first-period price to signal its quality, both consumer surplus and social welfare will be higher. By contrast, if the high-quality firm commits to higher-than-first-best future prices to signal its quality, both consumer surplus and social welfare will be lower. We also show that a longer time horizon can allow the high-quality firm to signal its quality costlessly, by maintaining its high first-best price for all periods. A longer time horizon will enlarge the parameter region in which the high-quality firm can costlessly signal its quality.

The rest of the article is organized as follows. Section 2 presents our base model. Section 3 analyzes the dynamic-signaling case, where the firm cannot credibly commit future prices. In Section 4, we examine the multi-dimensional signaling case, where the firm can commit future prices; we extend our model to a multi-period setting and discuss consumers’ social learning. Section 5 concludes the article with discussions of forward-looking consumers, partial credibility of price commitment, and alternative distributions for consumer valuation. All proofs are provided in the Online Appendix.

2. Model

We develop a dynamic model where a monopolist sells a new, non-durable experience good in two periods \((t = 1, 2)\). With probability \(\alpha \in (0,1)\), the firm is \(H\)-type with product quality \(q^H > 0\); with probability \(1 - \alpha\), the firm is \(L\)-type with product quality \(q^L < q^H\). The firm’s product quality is constant across the two periods. The firm knows the true quality, but consumers \textit{ex ante} know only its prior distribution. The firm is risk neutral and chooses its price \((p_t)\) in each period to maximize its
total expected profit. We assume that the firm cannot price discriminate consumers within any given period. The marginal cost of production is the same for both types of firms. Note that even when consumers observe the firm’s cost, they may still be uncertain about their exact valuations of the product because \textit{ex ante} they do not know the exact quality of the experience good. By assuming that both types of firms have the same cost, we remove the cost difference as a factor for the firm’s credible signal of its quality, and focus purely on the effects of dynamic pricing and price commitment. Without loss of generality, we normalize that cost to zero. The firm’s profit in each period is thus its price times the number of consumers who purchase the product.

In each period, each consumer can decide either to buy one unit of the product or not to buy any. If a consumer buys the product of known quality $q^i$ at price $p_t$, her economic surplus or utility is $u_t(q^i, p_t) = q^i \theta - p_t$ for $i \in \{H, L\}$ and $t \in \{1, 2\}$, where $\theta$ represents the consumer’s willingness to pay for quality. Consumers are heterogeneous in $\theta$, which is assumed to be uniformly distributed: $\theta \sim \text{uniform}[0,1]$. Without loss of generality, we normalize the total number of consumers to one (i.e., there is a unit mass of consumers). Each consumer has a unit demand in each period, and can potentially buy the non-durable product (which is good for the consumer’s consumption of one period) in both periods, depending on the consumer’s expected valuation and the prices. The consumer gets zero utility from the outside option when she does not buy the product in the corresponding period. We assume that if consumers expect the quality of the product to be $q$, a consumer of type $\theta$ will purchase the product in period $t$ if and only if $\theta \geq \frac{p_t}{q}$, that is, the consumer’s purchase decision in each period is based only
on her expected utility in that period.

Before analyzing the incomplete-information model, we examine the complete-information benchmark, in which the firm’s quality is common knowledge. In equilibrium, in each period both the high-quality and low-quality firm sell to consumers with $\theta \geq \frac{1}{2}$. Lemma 1 provides the firm’s equilibrium price and profit (with the overbar indicating the case of complete information).

**LEMMA 1. (SYMMETRIC-INFORMATION BENCHMARK)** If product quality is common knowledge, the $i$-type firm’s optimal price in period $t$ is $p_t^i = \frac{q_i}{2}$ and its total profit is $\Pi_t^i = \frac{q_i}{2}$.

Although consumers do not *ex ante* know the true quality of the firm’s product, they can learn it after their post-purchase usage of the product. So, a consumer who purchases the product in the first period will become informed in the second period. Those who do not buy the product remain uninformed about the true quality, but the uninformed consumers will update their belief about quality after observing the firm’s price(s). We examine two scenarios based on whether the firm can credibly commit its future (second-period) price in the first period. If the firm cannot commit its future price, the first-period consumers will make inferences about the firm’s quality from the firm’s first-period price only, whereas the uninformed consumers in the second period—those who did not make purchases in the first period—will update their belief based on both the first-period and the second-period prices. However, if the firm can and does commit its future price, the signal to the first-period consumers will be the prices for both periods.$^4$ Since in that case the firm can no longer adjust its price in the second

$^4$ Note that whether the firm commits a future price together with the current-period price serves as a signal for
period, the uninformed consumers’ posterior belief in the second period is unchanged from the first period. We will analyze the two cases in Sections 3 and 4, respectively.

3. Pricing without Credible Price Commitment

In this section, we focus on the case where consumers do not believe any price commitment to be credible, i.e., any first-period announcement of the firm’s second-period price is cheap talk. In the first period, the firm decides its first-period price $p_1^1$. Consumers decide whether to buy based on $p_1^1$ and their updated belief about product quality. In the second period, the firm decides its second-period price $p_1^2$. The first-period buyers learned the true quality whereas the non-buyers update their belief about quality having observed both $p_1^1$ and $p_1^2$. This is thus a case of dynamic signaling.

In signaling games, there are multiple perfect Bayesian equilibria (PBE) since the PBE concept imposes no restrictions on out-of-equilibrium beliefs. The problem is more severe in a multi-period dynamic signaling setting, where in some parameter regions even the intuitive criterion refinement (Cho and Kreps 1987) leaves infinitely many possible equilibria. To further refine equilibrium outcomes, we apply the lexicographically maximum sequential equilibrium (LMSE) concept introduced by Mailath et al. (1993), which is often used as an alternative equilibrium-selection criterion (e.g., Taylor 1999, Gomes 2000, Jiang et al. 2014 and 2016). Below we adapt the definition of LMSE to our setting.

**DEFINITION. (LEXICOGRAPHICALLY MAXIMUM SEQUENTIAL EQUILIBRIUM)** In a signaling game the firm’s type (quality). For expositional convenience and without loss of generality, we assume that the firm will commit if it is indifferent between committing and not committing the future price. One can show that, when future price can be credibly committed, both types of firm will commit future prices in either separating or pooling equilibrium.
\( G \), we denote the set of types by \( \{H, L\} \), the \( i \)-type player’s payoff by \( \pi_i(\cdot) \), and the set of pure-strategy

perfect Bayesian equilibria by \( \text{PBE}(G) \). The strategy profile \( \sigma' \in \text{PBE}(G) \) lexicographically

dominates (l-dominates) \( \sigma \in \text{PBE}(G) \) if \( \pi_H(\sigma') > \pi_H(\sigma) \), or \( \pi_H(\sigma') = \pi_H(\sigma) \) and \( \pi_L(\sigma') > \pi_L(\sigma) \). The strategy profile \( \sigma \in \text{PBE}(G) \) is a LMSE if there does not exist \( \sigma' \in \text{PBE}(G) \) that l-
dominates \( \sigma \).

This equilibrium refinement concept has a desirable property that a unique, Pareto-dominant

LMSE outcome always exists among all PBE when the payoff function is concave. In our model, a PBE

is an LMSE if it is the \( H \)-type firm’s most profitable outcome among all PBE and if, conditional on

being the most profitable for the \( H \)-type firm, it is also the \( L \)-type firm’s most profitable outcome. Note

that we also apply the LMSE refinement to all continuation games in our dynamic model. That is, in

the absence of price commitment, we require that the second-period price be an LMSE in the

continuation games conditional on the first-period outcome.

When firms cannot credibly commit their future prices, three types of equilibrium outcomes are

possible. First, the first-period price \( (p_1^1) \) is separating so that all consumers can \textit{ex ante} correctly infer

the firm’s type (quality) from \( p_1^1 \). Second, the first-period price is pooling and so is the second-period

price. In this case, consumers cannot correctly infer the firm’s type in the first period. In the second

period, the first-period buyers are informed, but non-buyers, having observed \( p_1^1 \) and \( p_2^1 \), remain

uninformed and unable to infer the firm’s true type. Third, the first-period price is pooling but the

second-period price is separating—consumers cannot infer the firm’s type in the first period but in the
second period the first-period buyers know the firm’s type from use experience and non-buyers can correctly infer the firm’s type by observed both \( p^H_1 \) and \( p^L_2 \). Let \( \bar{q} \) denote the consumers’ *ex ante* expected quality of the product, i.e., \( \bar{q} \equiv \alpha q^H + (1 - \alpha) q^L \). Proposition 1 shows that the LMSE outcome of the entire game entails a first-period pooling outcome and a second-period separating outcome. In addition, in each period both the \( H \)-type and \( L \)-type firms serve consumers with \( \theta \geq \frac{1}{2} \).

**Proposition 1.** When price commitment is not credible, the LMSE is \( p^H_1 = p^L_1 = \frac{\bar{q}}{2}, \) \( p^H_2 = \frac{q^H}{2}, \) \( p^L_2 = \frac{q^L}{2}, \) and the corresponding profits are \( \pi^H = \frac{q^H + q^L}{4} \) and \( \pi^L = \frac{q^L}{4} \), respectively.\(^5\)

Generally speaking, the \( H \)-type firm wants to reveal its true type so as to charge consumers a price in line with its high quality. If the \( H \)-type firm reveals its type by charging a separating price in the first period, its true quality will be known by consumers in both periods. However, to credibly signal its quality, the \( H \)-type firm has to distort its first-period price—charge a very high or very low first-period price—to remove the \( L \)-type firm’s mimicking incentive. It turns out that the price distortion needed to separate leads to such a low first-period profit for the \( H \)-type firm that the pooling outcome in the first period will be more profitable.

Under the first-period pooling price, consumers with high enough willingness to pay will purchase the product in the first period and learn the firm’s true quality. These informed consumers are unwilling to pay too high a price for the low-quality product in the second period and hence it discourages the \( L \)-

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\(^5\) Many belief systems with different off-the-equilibrium-path beliefs can support the equilibrium. In this article, we will use as an example the strict belief system that all deviations come from the \( L \)-type firm. Furthermore, one can show that this LMSE survives the intuitive criterion.
type firm from pretending as the $H$-type firm. The more first-period buyers (who will become informed in the second period), the lower the $H$-type firm’s signaling cost in the second period—the closer the $H$-type firm can get to its first-best outcome in the second period. By charging a relatively low first-period pooling price $(p_1^H^* = p_1^L^* = \frac{q}{2})$, the $H$-type firm can attract enough first-period buyers $(\theta \geq \frac{1}{2})$ to achieve its first-best outcome in the second period, targeting only the informed customers.\(^6\)

Note that when the firm cannot credibly commit future prices, asymmetric information about its type results in higher profits for the $L$-type firm and lower profits for the $H$-type firm. In particular, the $H$-type firm cannot achieve its first-best profit for both periods. We will show later that with credible price commitment, the $H$-type firm may be able to achieve first-best profits for both periods.

### 4. Pricing with Credible Price Commitment

We now analyze the case where the firm can, in the first period, commit its second-period price, e.g., by advertising its future price, by creating price menus or catalogues that are expensive to update, or by offering price-matching guarantees. At the beginning of the first period, the firm decides whether to commit a price scheme $(\hat{p}_1^1, \hat{p}_2^1)$, which removes the firm’s ability to adjust its price in the second period. Note that even though price commitment is the firm’s choice, one can easily show that in equilibrium, both types of firms at least weakly prefer committing its second-period price. Thus, with credible price commitment, we have in essence a multi-dimensional static signaling setting in contrast to the dynamic

\(^6\) In the LMSE, the consumers with $\theta \geq \frac{1}{2}$ buy in the first period. Note that under perfect information, the firm also targets those consumers with $\theta \geq \frac{1}{2}$. Hence, the high-quality firm can earn its first-best profit by targeting to only those informed in the second period.
signaling setting in Section 3. In the first period, consumers will update their beliefs about the firm’s quality based on prices \( \left( \hat{p}_1^l, \hat{p}_2^l \right) \), and decide whether to make a purchase. Those consumers who buy the product in the first period will learn the true quality. In the second period, both informed and uninformed consumers can decide again whether to buy the product at the firm’s previously committed second-period price.

4.1. LMSE Outcome

With credible price commitment, \( \hat{p}_1^l \) and \( \hat{p}_2^l \) as a pair form a signal for the firm’s quality. Two types of equilibrium outcomes are possible: pooling and separating. In pooling equilibria, in the first period consumers cannot infer from \( \hat{p}_1^l \) and \( \hat{p}_2^l \) the firm’s true type though the first-period buyers will know the true quality in the second period (through their use of the product in the first period). In separating equilibria, consumers can in the first period correctly infer the firm’s true type from its price scheme \( \left( \hat{p}_1^l, \hat{p}_2^l \right) \). After characterizing both the pooling and separating outcomes, we can determine the LMSE outcome, which is given in Proposition 2.

**Proposition 2.** When price commitment is credible, the LMSE is

(i) If \( \frac{q^l}{q^H} \geq \frac{a^2}{2(1-\sqrt{1-a^2})} \) or \( \frac{q^l}{q^H} \leq \frac{a}{a+1} \), the price scheme is pooling:

\[
\left( \hat{p}_1^{l*}, \hat{p}_2^{l*} \right) = \left( \bar{p}_1^l, \bar{p}_2^l \right) \text{ and }
\]

(ii) If \( \frac{a}{a+1} \leq \frac{q^l}{q^H} < \frac{a^2}{2(1-\sqrt{1-a^2})} \), the price scheme is separating: \( \left( \hat{p}_1^{k*}, \hat{p}_2^{k*} \right) = \left( \frac{q^l}{\sqrt{2}}, \frac{q^l}{\sqrt{2}} \right) \) and
\[(p_1^{H*}, p_2^{H*}) = \begin{cases} 
\frac{q^H}{2}, \frac{q^L + \sqrt{q^L(q^H - q^L)}}{2} & \text{if } \frac{1}{2} \leq \frac{q^L}{q^H} < \frac{\alpha^2}{2(1 - \sqrt{1 - \alpha^2})} \\
\frac{q^H}{2}, \frac{q^H}{2} & \text{if } \frac{q^L}{q^H} = \frac{1}{2} \\
\frac{q^H - \sqrt{q^L(q^H - 2q^L)} + q^L}{2}, \frac{q^H}{2} & \text{if } \frac{\alpha}{\alpha + 1} \leq \frac{q^L}{q^H} \leq \frac{1}{2}.
\end{cases}\]

Under the pooling equilibrium, the \(H\)-type firm targets consumers with \(\theta \geq \frac{1}{2}\) in both periods, whereas the \(L\)-type firm targets consumers with \(\theta \geq \frac{1}{2}\) in the first period but those with \(\theta \geq \frac{q^H}{2q^L}\) in the second period. Given the pooling price scheme, all consumers in the first period will evaluate the product as of average quality. The \(H\)-type firm will earn a lower-than-first-best profit in the first period since the consumers’ expected product quality is lower than its true quality. By contrast, under the separating equilibrium, the \(H\)-type firm serves consumers with \(\theta \geq \frac{p_1^{H*}}{q^H}\) in period 1, whereas the \(L\)-type firm serves consumers with \(\theta \geq \frac{1}{2}\) in both periods. Although the \(H\)-type firm can reveal its type under a separating equilibrium, it may have to bear some signaling cost by committing a non-first-best price scheme to remove the \(L\)-type firm’s mimicking incentives. Whether the LMSE outcome is pooling or separating depends on the \(H\)-type firm’s signaling cost. Figure 1 illustrates the LMSE outcomes.

**Figure 1** LSME Outcome under Credible Price Commitment.
With credible price commitment, the $H$-type firm can signal its type with both its first- and second-period prices. Note that if $q^L_H = \frac{1}{2}$, a costless separating outcome is achieved—the $H$-type firm will be able to separate from the $L$-type firm by simply charging its first-best price in both periods; the $L$-type firm cannot improve its profit by mimicking the $H$-type firm. The two types of firms “naturally” separate, each achieving its own first-best profit without any incentive for deviation.

When $q^L_H \neq \frac{1}{2}$, the LMSE will be pooling if the prior probability (\(\alpha\)) of the firm being $H$-type is high enough (i.e., in the upper parameter regions in Figure 1). This makes intuitive sense. When \(\alpha\) is large, the difference between the $H$-type firm’s quality and the average expected quality will be small, making the $H$-type firm’s benefit from credibly signaling its quality small relative to the required signaling cost; hence the equilibrium outcome will be pooling as shown in Proposition 2.

If the prior probability (\(\alpha\)) of the firm being $H$-type is not very high (i.e., in the lower parameter regions in Figure 1), then the LMSE will be a separating outcome as characterized in Proposition 2. If
\( \frac{q^L}{q^H} > \frac{1}{2} \), the H-type firm will charge the first-best first-period price and simply choose a higher-than-first-best second-period price to discourage the L-type firm from mimicking. In contrast, if \( \frac{q^L}{q^H} < \frac{1}{2} \), the H-type firm will charge a lower-than-first-best first-period price to separate from the L-type firm.

Because of the large difference in quality between two types of firms, the H-type firm must sufficiently reduce its first-period price to remove the L-type firm’s mimicking benefit in the first period. Note that as \( \frac{q^L}{q^H} \) moves away from the costless-separating ratio \( \left( \frac{1}{2} \right) \), the H-type firm’s price distortion needed to separate will increase.

Intuitively, the H-type firm should find it more efficient to separate from the L-type firm by adjusting the second-period price rather than the first-period price. This is because some informed consumers in the second period will not buy the low-quality product at a high price, thus a higher second-period price imposes a higher opportunity cost to the L-type firm than to the H-type firm. To separate, the H-type firm does not have to distort its second-period price to as high an extent as the first-period price distortion required. As it turns out, if \( \frac{q^L}{q^H} > \frac{1}{2} \) the H-type firm can charge a first-best first-period price and be able to simply choose a higher-than-first-best second-period price to separate itself from the L-type firm. But when \( \frac{q^L}{q^H} < \frac{1}{2} \), the H-type firm will not be able to separate by charging its first-best first-period price, because at that price the L-type firm will prefer mimicking even if it receives zero profit in the second period. Thus, when the H-type firm has a very high quality, it has to sufficiently reduce its first-period price from its first-best to prevent the L-type firm from being able to profitably mimic the H-type firm. Proposition 3 shows the profits of the two types of firms.
PROPOSITION 3. With credible price commitment, the firm’s LMSE profit is as follows.

(i) When the price scheme is pooling,

\[ \hat{p}^{H*} = \frac{q + q^H}{4} \text{ and } \hat{p}^{L*} = \begin{cases} \frac{q}{4} & \text{if } \frac{q^L}{q^H} \leq \frac{\alpha}{\alpha + 1} \\ \frac{q + (2q^L - q^H)q^H}{4q^L} & \text{if } \frac{q^L}{q^H} \geq \frac{\alpha^2}{2(1 - \sqrt{1 - \alpha^2})} \end{cases} \]

(ii) When the price scheme is separating,

\[ \hat{p}^{H*} = \begin{cases} \frac{q^H(q^H + q^L) + 2(q^H - q^L)\sqrt{(q^H - q^L)q^L}}{4q^H} & \text{if } \frac{1}{2} \leq \frac{q^L}{q^H} < \frac{\alpha^2}{2(1 - \sqrt{1 - \alpha^2})} \\ \frac{q^L}{2} & \text{if } \frac{q^L}{q^H} = \frac{1}{2} \\ \frac{q^H + 2q^L}{4} & \text{if } \frac{\alpha}{\alpha + 1} \leq \frac{q^L}{q^H} \leq \frac{1}{2} \end{cases} \text{ and } \hat{p}^{L*} = \frac{q^L}{2} \]

4.2. Effects of Price Commitment

We now examine the impacts of price commitment on the firm’s profit, the total consumer surplus, and social welfare by comparing the LMSE outcomes in section 3 and section 4.1.

PROPOSITION 4. The H-type firm’s profit is weakly higher with credible price commitment than without; the H-type firm’s profit is strictly higher if and only if \( \frac{\alpha}{\alpha + 1} < \frac{q^L}{q^H} < \frac{\alpha^2}{2(1 - \sqrt{1 - \alpha^2})} \). By contrast, the L-type firm’s profit is strictly lower when price commitment is credible.\(^7\)

If in the pooling parameter regions in Figure 1, credible commitment will have no effect on the H-type firm’s profit since its price scheme will be the same whether price commitment is credible or not. By contrast, the L-type firm’s profit is lower when price commitment is credible than when it is not. Note that when no credible price commitment is possible, the pooling L-type firm needs to mimic

\(^7\) When future price can be credibly committed, the H-type firm will commit to some high future prices, while the L-type firm will charge its (relatively low) first-best price in both periods.
only the $H$-type firm’s first-period price and can freely choose its first-best price in the second period.

However, when credible price commitment is possible, the pooling $L$-type firm will have to mimic the $H$-type firm’s committed prices for both periods. That is, credible price commitment increases the $L$-type firm’s cost to pool with the $H$-type firm, hence making the $L$-type firm worse off in the pooling parameter regions.

If $\frac{a}{a+1} < \frac{q^H}{q^H - q^L} < \frac{a^2}{2(1-\sqrt{1-a^2})}$, i.e., in the separating parameter regions in Figure 1, the $H$-type firm is better off when it can commit its future price—a new way to signal its quality in the first period. Intuitively, credible price commitment gives the $H$-type firm an efficient way to punish the $L$-type firm for mimicking, which lowers the $H$-type firm’s signaling cost and improves its profit. For example, when $\frac{q^H}{q^H - q^L} > \frac{1}{2}$, the $H$-type firm can commit a high second-period price to credibly signal its quality, even without any need to distort its first-period price. The $L$-type firm can, under no credible price commitment, benefit from pooling with the $H$-type firm in the first period. But when credible price commitment is possible, the $L$-type firm will now find it too costly to pool in the separating parameter regions, where it resorts back to its first-best outcome. So, credible price commitment also makes the $L$-type firm strictly worse off in the separating parameter regions.

**PROPOSITION 5.** Credible price commitment improves consumer surplus if $\frac{a}{a+1} < \frac{q^H}{q^H - q^L} < \frac{1}{2}$; it does not affect consumer surplus if $\frac{q^H}{q^H - q^L} = \frac{1}{2}$; it reduces consumer surplus, otherwise.

One might intuit that consumers will be worse off when the firm has the capability to make a credible price commitment at its own discretion. Proposition 5 shows that the effect of credible price commitment
commitment on consumer surplus critically depends on the quality ratio of two types of firms. In the pooling regions in Figure 1, the consumers are worse off under credible price commitment, because in the second period the \( L \)-type firm will serve a smaller fraction of consumers when price commitment is credible than when it is not. In the right-hand-side costly-separating region in Figure 1 (i.e., when \( \frac{1}{2} \leq \frac{q^L}{q^H} < \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})} \)), credible commitment also makes consumers worse off; this is because the \( H \)-type firm will signal its quality by committing to a higher-than-first-best second-period price, serving fewer second-period customers at a higher price than in the case of no credible commitment. By contrast, in the left-hand-side costly-separating region in Figure 1 (i.e., when \( \frac{\alpha}{\alpha+1} \leq \frac{q^L}{q^H} < \frac{1}{2} \)), credible price commitment will increase consumer surplus, because the \( H \)-type firm will charge a lower-than-first-best price (and commit to its first-best price for the second period) to signal its quality. Lastly, if \( \frac{q^L}{q^H} = \frac{1}{2} \), the market outcome is costless separating under credible price commitment, leading to the same consumer surplus as in the case of no credible commitment.

**Proposition 6.** Credible price commitment improves social welfare if \( \frac{\alpha}{\alpha+1} < \frac{q^L}{q^H} < \frac{1}{2} \); it does not affect social welfare if \( \frac{q^L}{q^H} = \frac{1}{2} \); it reduces social welfare, otherwise.

Proposition 6 shows that the effect of credible price commitment on social welfare also depends on the quality ratio of two types of firms. Note that, given the firm’s market coverage, social welfare does not depend on the firm’s price (which is simply an internal transfer price within society). Credible price commitment will lower social welfare in both the pooling parameter regions (because of the \( L \)-type firm’s lowered second-period coverage) and the right-hand-side separating region (because of the
H-type firm’s lowered second-period coverage). By contrast, in the left-hand-side costly-separating region in Figure 1, credible price commitment will increase social welfare because the H-type firm’s first-period market coverage is larger with credible commitment (due to the lowered first-period price needed to signal its type). Lastly, if \( q^2_H = \frac{1}{2} \), social welfare remains the same regardless of whether price commitment is credible since the market coverage is the same for both cases.

4.3. Extension to Multi-Period Model

In this section, we extend the model to consider the situation where the firm can sell its product in \( T > 2 \) periods. In this scenario, consumers who buy the product will become informed of the true quality for all subsequent periods. When price commitment is not credible, the first-period price distortion needed to separate from the L-type firm is still too large so that the H-type firm would rather pool with the L-type firm in the first period. So, in equilibrium, both types of firms will pool at price \( \frac{q}{2} \) in the first period and the H-type firm will attract enough first-period customers (with \( \theta \geq \frac{1}{2} \)) such that the first-best separating outcome will be achieved (at price \( \frac{q^*}{2} \)) in every later period.

When the firm can in the first period credibly commit its prices for all \( T \) periods, the H-type firm will be able to signal its quality more easily, since intuitively there are more periods for the informed consumers to punish (by not buying the low-quality product at too high prices) an L-type firm that pretends to be an H-type firm. As the number of periods \( (T) \) increases, the L-type firm’s cost for mimicking the H-type firm will increase and price commitment will make it more likely that the true quality of the product will be revealed early on.
Proposition 7. With credible price commitment, the $H$-type firm can costlessly separate from the $L$-type firm if $T \geq \max \left( \frac{q^H}{q^L}, \frac{q^H}{q^H - q^L} \right)$.

Proposition 7 shows that, regardless of the quality ratio of two types of firms, the $H$-type firm can achieve costly separating as long as $T$ is sufficiently large. The $H$-type firm can signal its type by simply committing to keeping its price at the first-best price for all periods, which the $L$-type firm will find unprofitable to mimic, because such high prices will reduce its demand from informed consumers for all future periods.

4.4. Extension to Model with Social Learning

In the current age of online social media, the early customers often share the quality information that they have learned through usage experience, e.g., via online product reviews or information-sharing forums. We have extended our core model to analyze such a social-learning scenario. We assume that social learning from the early-period customers is perfect, i.e., all consumers will become informed in the second period. Within our current model framework, the LMSE outcomes actually remain the same in all parameter regions.

Let us first consider the case of no credible price commitment. In the second period, since all consumers have learned the product quality via word of mouth, the $H$-type firm will no longer need to worry about the $L$-type firm’s mimicking, so both types of firms charge their first-best prices. In this scenario, the firm’s first-period price does not affect the fraction of consumers who will know the true quality in the second period (as long as some consumers buy in the first period). In other words, the
first-period price can still signal product quality but is irrelevant for the second-period outcome. Thus, in this degenerate model, the first-period outcome is pooling, because without cost differences the \( L \)-type firm will always mimic the \( H \)-type firm.

When price commitment is credible, it is a powerful signaling tool because a high (first-best) price commitment comes at zero cost to the \( H \)-type firm but is very costly for the \( L \)-type firm when the fraction of consumers who have become informed is sufficiently high. However, to separate from the \( L \)-type firm, the \( H \)-type firm requires only its targeted consumers being informed, i.e., those with high enough willingness to pay \( \theta \geq \frac{1}{2} \). In our core model, the first-period pooling price happens to attract the consumers with \( \theta \geq \frac{1}{2} \) to buy in the first period, so it effectively enables price commitment to signal the firm’s high quality. Thus, even if those consumers with lower willingness to pay \( \theta < \frac{1}{2} \) will also learn the true quality via word of mouth, the firm’s optimal price scheme is unchanged since the firm will not want to target those consumers anyway. In other words, interestingly, even without any social learning, the firm will price in a way to generate enough early-period buyers, who will learn the firm’s quality, such that the firm’s optimal strategy for the later period is the same as when all consumers have socially learned the true quality in the later period. The formal analysis for this extension is provided in the Online Appendix.

5. Conclusion

This article examines a firm’s dynamic pricing and price commitment strategies for its new, non-durable, experience goods. It is difficult for the high-quality firm to separate from the low-quality firm when
both types of firms have the same marginal cost and price commitment is not credible. If firms can make credible price commitments (e.g., through offering price-matching guarantees or having reputations for keeping high-price commitment in the past), the high-quality firm may be able to efficiently signal its quality by either lowering its first-period price from its first-best price or increasing its second-period price. The possibility of credible price commitment benefits the high-quality firm by reducing its signaling cost, but will make the low-quality firm worse off. The effects of credible price commitment on consumer surplus and social welfare depend on the high-quality firm’s optimal price signal for its quality (higher or lower than first-best prices), which depends on the quality difference between the two types of firms. We also show that a longer time horizon will enlarge the parameter region in which the high-quality firm can signal its quality without incurring any signaling cost, i.e., by committing its first-best prices. Finally, our analysis shows that, in the current model framework, the possibility of social learning does not change the equilibrium outcomes.

We conclude by discussing the robustness of our results and pointing out some caveats about our model. First, we have assumed that the consumer’s purchase decision in each period is based only on her expected utility for that period. More specifically, consumers do not consider the “option value” of overpaying in the first period (i.e., paying a higher price than the expected consumption value) to learn about quality for their future purchase decisions. In this sense, consumers are implicitly assumed to be myopic. Note that the firm’s equilibrium pricing strategy in the original core model will actually not provide the consumer with any option/information value if that consumer has a negative consumption
surplus. Put differently, in our core model, if a consumer gets a negative first-period surplus from buying the product in the first period (e.g., any consumer with $\theta < \frac{1}{2}$), then the consumer will not buy the product even if she is forward-looking, because in equilibrium neither type of firms will target consumers with $\theta < \frac{1}{2}$ (i.e., the consumer will also get negative surplus buying the product in the second period). Thus, in our core model, whether consumers are myopic or forward-looking will not affect the equilibrium outcome.

Second, when price commitment power is available, the firm’s commitment decision itself is part of its strategy space. However, the commitment decision itself cannot alone signal quality; it must include what prices the firm is committing to. For example, if the firm commits the future prices to be the $L$-type firm’s first-best price, the fact that the firm committed its future prices does not signal it has a high quality, because the committed prices would have been most profitable to the low-quality ($L$-type) firm. If the firm is truly a high-quality firm, it would definitely not prefer committing to the $L$-type’s first-best price. To most profitably convince consumers of its high quality, the $H$-type firm can commit to the optimal price scheme $(\hat{p}^H_1, \hat{p}^H_2)$ based on the parameter regions as we have shown. Furthermore, the high-quality firm will always decide to commit since its equilibrium profit is weakly higher with credible price commitment than without.

Third, we have analyzed two extreme cases of price commitment—the firm’s price commitment is either totally credible or completely cheap talk. In practice, consumers’ perception of the credibility of the firm’s price commitment may not be so extreme. One may extend our model by introducing a
parameter $\rho$ indicating the consumer’s perception of how credible the firm’s price commitment is (i.e., the probability that the firm’s price commitment will be upheld). This probability $\rho$ will depend on the firm’s past behavior or reputation. The $H$-type firm’s price commitment will punish the mimicking $L$-type firm only with probability $\rho$ rather than with probability 1 as assumed in our core analysis. We expect the parameter region for the separating LMSE outcome to become smaller than what we have identified. But our main results will likely stay qualitatively the same.

Fourth, our model assumes that consumers’ willingness to pay for quality is uniformly distributed. If the distribution is not uniform, the firm’s equilibrium prices may change and the parameter regions for the different types of equilibrium outcomes may shift accordingly. Intuitively, for example, if the distribution has a disproportionately high fraction of high-valuation consumers, the first-period pooling price (under no credible price commitment) should be higher than in the case of uniform distribution. Without formal analysis, it is unclear whether that will result in more consumers or fewer consumers buying the product in the first period. But we would expect that the size of the firm’s optimal first-period target segment is probably highly correlated with that of the second-period for the case of no price commitment. Similarly, for the case with credible price commitment, the committed prices may differ based on the nature of the distribution, but our qualitative result should still hold, albeit under correspondingly different parameter regions.

References


Online Appendix for “Dynamic Pricing and Price Commitment of New Experience Goods”

Online Appendix A

PROOF OF LEMMA 1. Under complete information, the firm’s optimization problem is

$$\max_{p_t^i} p_t^i(1 - \theta) \text{ subject to } \theta \geq \min \{\frac{p_t^i}{q_t}, 1\}.$$ 

It is clear that the firm’s optimal price is $p_t^i = \frac{a_t}{2}$ and those consumers with $\theta \geq \frac{1}{2}$ will purchase the product.

PROOF OF PROPOSITION 1. When price commitment is not credible, three types of equilibrium outcomes are possible. First, the first-period price ($p_1^i$) is separating so that all consumers can ex ante correctly infer the firm’s type (quality) from $p_1^i$. Second, the first-period price is pooling and so is the second-period price. In this case, consumers cannot correctly infer the firm’s type in the first period. In the second period, the first-period buyers are informed, but non-buyers, having observed $p_1^i$ and $p_2^i$, remain uninformed and unable to infer the firm’s true type. Third, the first-period price is pooling but the second-period price is separating—consumers cannot infer the firm’s type in the first period but in the second period the first-period buyers know the firm’s type from use experience and non-buyers can correctly infer the firm’s type by observed both $p_1^i$ and $p_2^i$.

We first determine the second-period outcomes based on the whether the first-period price is separating or pooling: If the firm’s first-period price is separating ($p_{1,sep}^H \neq p_{1,sep}^L$), consumers will in the first period correctly infer the firm’s type. Thus, the second-period outcome is the same as that under the
symmetric-information case—each type of firm will optimally charge its first-best price in the second period: $p_{2,sep}^i = \frac{q^i}{2}$.

If the firm’s first-period price is pooling ($p_{1,pool}^H = p_{1,pool}^L \equiv p_{1,pool}$), the second-period price may be pooling or separating. We now analyze the case in which the firm’s prices in both periods are pooling prices ($p_{1,pool}$ and $p_{2,pool}$). In the second period, the first-period buyers have learned the true product quality, but the non-buyers are not directly informed of the true quality and can only make inferences based on the first- and second-period prices. The $L$-type firm’s incentive-compatible constraint in the second period (for it not to deviate from $p_{2,pool}$) is that it must earn a (weakly) higher second-period profit than its first-best profit ($\frac{q^L}{4}$), which it can earn even admitting its low quality.

Under the first-period pooling price, only those consumers with high enough willingness to pay ($\theta \geq \frac{p_{1,pool}}{q}$) purchase the product. If the first-period price is too high ($p_{1,pool} \geq \bar{q}$), no consumers will purchase the product in the first period and all consumers will remain uninformed of the true product quality in the second period.\(^1\) Since the second-period pooling price does not reveal the firm’s type or quality, only consumers with $\theta \geq \frac{p_{2,pool}}{\bar{q}}$ will purchase the product in the second period. Thus, the $L$-type firm’s second-period profit is given by $p_{2,pool}(1 - \min\left\{\frac{p_{2,pool}}{\bar{q}}, 1\right\})$ and its incentive-compatible constraint to prefer pooling in the second period is $p_{2,pool}(1 - \min\left\{\frac{p_{2,pool}}{\bar{q}}, 1\right\}) \geq \frac{q^L}{4}$.

\(^1\) However, such a high first-period price is clearly a dominated strategy for the $H$-type firm. The $H$-type firm will find it more profitable to charge a low enough first-period price such that at least some consumers will buy its product in the first period (and become informed consumers in the later period). Therefore, we will not discuss such clearly dominated pricing strategies in the following sections.
If $p_{1,\text{pool}} < q$, some consumers with high enough $\theta$ will buy the product in the first period and learn the firm’s true quality. In the second period, these informed consumers will buy the product again if they have $\theta \geq \frac{p_{2,\text{pool}}}{q^l}$ whereas the uninformed consumers will buy it only if $\theta \geq \frac{p_{2,\text{pool}}}{q}$. The market coverage in the second period may consist of both informed and uninformed consumers; three possible scenarios for the $L$-type firm’s market coverage are illustrated in Figure A.1.

**Figure A.1 H-Type Firm’s Second-Period Market Coverage Given First-Period Pooling Price.**

In the first scenario, $p_{2,\text{pool}} \geq p_{1,\text{pool}}$, the $L$-type firm targets only the informed consumers in the second period. Consumers (with low $\theta$) who do not purchase the product at $p_{1,\text{pool}}$ in the first period remain uninformed of product quality in the second period; if the second-period pooling price ($p_{2,\text{pool}}$) is even higher than before, those uninformed consumers will still not buy the product. In the second scenario,

---

2 The market coverage is marked with the bold line.
\[
\frac{q}{q} p_{1,pool} \leq p_{2,pool} \leq p_{1,pool};
\]
\( p_{2,pool} \) is low enough that some uninformed consumers will buy the product in the second period. In addition, though the first-period buyers have learned of the \( L \)-type firm’s low quality, some of them with \( \theta \geq \frac{p_{2,pool}}{q^L} \) will buy the product again. As a result, at price \( p_{2,pool} \), the \( L \)-type firm can sell its product to some informed and some uninformed consumers. In the third scenario, \( p_{2,pool} \leq \frac{q}{q} p_{1,pool} \), i.e., the second-period price is low enough that all informed consumers will buy the low-quality product. The \( L \)-type firm’s second-period market coverage includes all informed consumers and some uninformed consumers. From the \( L \)-type firm’s second-period market coverage, we can write its incentive-compatible constraint for not deviating from \( p_{2,pool} \):

\[
\begin{cases}
\frac{q}{q} p_{2,pool} (1 - \min\{\frac{p_{2,pool}}{q^L}, 1\}) \geq \frac{q^L}{4} & \text{if } p_{2,pool} \geq p_{1,pool} \\
p_{2,pool} (1 - \min\{\frac{p_{2,pool}}{q^L}, 1\}) + \frac{p_{1,pool}}{q} - \frac{p_{2,pool}}{q} \geq \frac{q^L}{4} & \text{if } \frac{q}{q} p_{1,pool} \leq p_{2,pool} \leq p_{1,pool} \\
p_{2,pool} (1 - \frac{p_{2}}{q}) \geq \frac{q^L}{4} & \text{if } p_{2,pool} \leq \frac{q}{q} p_{1,pool}
\end{cases}
\]

Now let us examine the \( H \)-type firm’s second-period market coverage. We will focus on the non-trivial pooling case of \( p_{1,pool} < q \), i.e., some consumers make a purchase in the first period. In the second period, the first-period buyers (with \( \theta \geq \frac{p_{1,pool}}{q} \)) have learned the true quality through product use and will buy the high-quality product in the second period as long as \( p_{2,pool} \leq \frac{q}{q} p_{1,pool} \). If \( p_{2,pool} \geq \frac{q}{q} p_{1,pool} \), then only a fraction \( 1 - \min\{\frac{p_{2,pool}}{q^H}, 1\} \) of the first-period customers will purchase the product again in the second period. The uninformed consumers with \( \theta \geq \frac{p_{2,pool}}{q} \) will also buy the product. Thus, the \( H \)-type firm’s second-period profit under the pooling price \( p_{2,pool} \) is given by
\[ π_H^{\text{pool}} = \begin{cases} 
\frac{p_{2,\text{pool}}}{q_H} (1 - \min\{\frac{p_{2,\text{pool}}}{q_H}, 1\}) & \text{if } p_{2,\text{pool}} \geq \frac{q_H}{\bar{q}} p_{1,\text{pool}} \\
p_{2,\text{pool}} (1 - \frac{p_{1,\text{pool}}}{\bar{q}}) & \text{if } p_{1,\text{pool}} \leq p_{2,\text{pool}} \leq \frac{q_H}{\bar{q}} p_{1,\text{pool}}, \\
p_{2,\text{pool}} (1 - \frac{p_{2,\text{pool}}}{\bar{q}}) & \text{if } p_{2,\text{pool}} \leq p_{1,\text{pool}}. 
\end{cases} \]

We then characterize the second-period pooling price for the \( H \)-type firm’s most profitable second-period pooling outcome conditional on the first-period pooling price: When price commitment is not credible and the first-period price is pooling, the \( H \)-type firm’s most profitable second-period pooling outcome entails \( p_{2,\text{pool}} \leq \frac{\bar{q}}{2} \) where “\( = \)” holds if and only if \( p_{1,\text{pool}} \geq \min\{\frac{\bar{q}^2 - \bar{q}q_L + (q_L^2/2)}{2q_L}, \bar{q} + q_L\} \).

The concept is that we need to find the second-period pooling price \( (p_{2,\text{pool}}) \) that maximizes the \( H \)-type firm’s second-period profit, given the first-period pooling price \( (p_{1,\text{pool}}) \), subject to the \( L \)-type firm’s incentive-compatible constraints (ICs). First, if \( p_{2,\text{pool}} \geq p_{1,\text{pool}} \), only \( p_{2,\text{pool}} = \frac{q_L}{2} \) satisfies the ICs. Second, if \( p_{2,\text{pool}} < p_{1,\text{pool}} \), the unconstrained optimization price is \( p_{2,\text{pool}} = \frac{\bar{q}}{2} \). For this price to satisfy the ICs, we need

\[
\left\{ \begin{array}{l}
p_{1,\text{pool}} \geq \frac{\bar{q} - \bar{q}q_L + (q_L^2/2)}{2q_L} & \text{if } q_L \geq \frac{\bar{q}}{2} \\
p_{1,\text{pool}} \geq \frac{\bar{q} + q_L}{2} & \text{if } q_L \leq \frac{\bar{q}}{2}
\end{array} \right.
\]

Otherwise, the ICs must be binding: \( p_{2,\text{pool}}(1 - \min\{\frac{p_{2,\text{pool}}}{q_L}, 1\} + \frac{p_{1,\text{pool}}}{\bar{q}} - \frac{p_{2,\text{pool}}}{\bar{q}}) = \frac{q_L}{4} \). We can derive the second-period pooling price in two cases.

(i) If \( \frac{\bar{q}}{2} \leq q_L \),

\[
p_{2,\text{pool}} = \begin{cases} 
\frac{q_L}{2(\bar{q} + q_L)} (p_{1,\text{pool}} + \bar{q} + \sqrt{p_{1,\text{pool}}^2 + 2p_{1,\text{pool}}\bar{q} - \bar{q}q_L}) & \text{if } p_{1,\text{pool}} \leq \frac{q_L}{2} \\
\bar{q} & \text{if } \frac{q_L}{2} \leq p_{1,\text{pool}} \leq \frac{\bar{q}^2 - \bar{q}q_L + (q_L^2/2)}{2q_L} \\
& \text{if } p_{1,\text{pool}} \geq \frac{\bar{q}^2 - \bar{q}q_L + (q_L^2/2)}{2q_L}.
\end{cases}
\]

(ii) If \( \frac{\bar{q}}{2} \geq q_L \),
\[ p_{2, pool} = \begin{cases} \frac{q^L}{2 (\bar{q} + q^L)} (p_{1, pool} + \bar{q} + \sqrt{p_{1, pool}^2 + 2 p_{1, pool} \bar{q} - \bar{q} q^L}) & \text{if } p_{1, pool} \leq \frac{q^L}{2} \\ \frac{q^L}{2} (p_{1, pool} + \sqrt{p_{1, pool}^2 - \bar{q} q^L}) & \text{if } \frac{q^L}{2} \leq p_{1, pool} \leq \frac{\bar{q} + 4q^L}{4} \\ \frac{q^L}{4} (p_{1, pool} + \bar{q} + \sqrt{2 p_{1, pool}^2 + 2 \bar{q} q^L}) & \text{if } \frac{\bar{q} + 4q^L}{4} \leq p_{1, pool} \leq \frac{\bar{q} + q^L}{2} \\ \frac{q^L}{2} & \text{if } p_{1, pool} \geq \frac{\bar{q} + q^L}{2} \end{cases} \]

It is straightforward to show \( p_{2, pool} \leq \frac{\bar{q}}{2} \) for any \( p_{1, pool} \) and \( p_{2, pool} = \frac{\bar{q}}{2} \) holds if and only if \( p_{1, pool} \geq \min\{\frac{q^L - \bar{q} q^L - (q^L)^2}{2 q^L}, \frac{\bar{q} + q^L}{2}\} \).

We then examine the case where after non-trivial pooling in the first period (i.e., \( p_{1, pool} < \bar{q} \)) the firm chooses a separating second-period price \( (p_{2, sep}^H \neq p_{2, sep}^L) \), i.e., in the second period the first-period non-buyers can correctly infer the quality based on the prices in both periods. Conditional on the first-period pooling price, the \( H \)-type firm’s most profitable separating second-period price is \( p_{2, sep}^H \in \arg \max_p \{p (1 - \min\{\frac{p,q^H}{q^L} , 1\})\} \) subject to the \( L \)-type firm’s incentive-compatible constraints.

Note that, to achieve a separating outcome in the second period, the \( H \)-type firm must choose a price at which the \( L \)-type firm will earn a profit lower than its first-best level. Under a second-period separating outcome, when the \( L \)-type firm mimics the \( H \)-type firm’s price, its product will be taken as high quality by the uninformed consumers. To specify the incentive-compatible constraints for the second-period separating outcome, three scenarios need to be considered for the \( L \)-type firm’s second-period market coverage when it deviates to the \( H \)-type firm’s price \( p_{2, sep}^H \). First, if \( p_{2, sep}^H \geq \frac{q^H}{q} p_{1, pool} \), the \( L \)-type firm can serve only the informed consumers with higher willingness-to-pay \( (\theta \geq \frac{p_{2, sep}^H}{q^L}) \) when it deviates to \( p_{2, sep}^H \). Second, if \( \frac{q^L}{q} p_{1, pool} \leq p_{2, sep}^H \leq \frac{q^H}{q} p_{1, pool} \), the \( L \)-type firm can serve some informed consumers
(with $\theta \geq \frac{p_{2, sep}^H}{q^L}$) and some uninformed consumers (with $\theta \geq \frac{p_{2, sep}^H}{q^H}$). Third, if $p_{2, sep}^H \leq \frac{q^L}{q} p_{1, pool}$, the $L$-type firm will serve all informed and some uninformed consumers at price $p_{2, sep}^H$. Therefore, the $L$-type firm’s incentive-compatible constraints can be written as

$$\begin{align*}
p_{2, sep}^H (1 - \min\{\frac{p_{2, sep}^H}{q^L}, 1\}) &\leq \frac{q^L}{4} \quad \text{if } p_{2, sep}^H \geq \frac{q^H}{q} p_{1, pool} \\
p_{2, sep}^H (1 - \min\{\frac{p_{2, sep}^H}{q^L}, 1\} + \frac{p_{1, pool}}{q} - \frac{p_{2, sep}^H}{q^H}) &\leq \frac{q^L}{4} \quad \text{if } \frac{q^L}{q} p_{1, pool} \leq p_{2, sep}^H \leq \frac{q^H}{q} p_{1, pool} \\
p_{2, sep}^H (1 - \frac{p_{2, sep}^H}{q^H}) &\leq \frac{q^L}{4} \quad \text{if } p_{2, sep}^H \leq \frac{q^L}{q} p_{1, pool}
\end{align*}$$

In the second-period outcome (of the continuation game), since uninformed consumers can infer the firm’s true type, the $L$-type firm will charge its first-best price ($\frac{q^L}{2}$). In contrast, to credibly convince uninformed consumers of its high quality, the $H$-type firm may have to distort its price from its first-best level ($\frac{q^H}{2}$) enough to ensure that the $L$-type firm’s mimicry is unprofitable. As the number of first-period customers increases, the $H$-type firm can charge a price closer to its first-best price because the $L$-type firm can cheat only the uninformed consumers—some informed consumers may not buy the low-quality product at the $H$-type firm’s price, thereby reducing the $L$-type firm’s incentives to mimic the $H$-type firm’s price. Thus, if the first-period pooling price is low enough, there will be enough informed consumers in the second period to discourage the $L$-type firm from mimicking even the $H$-type firm’s first-best price. That is, when price commitment is not credible and the first-period price is pooling ($p_{1, pool}^H = p_{1, pool}^L \equiv p_{1, pool}$), the $H$-type firm’s most profitable second-period separating outcome ($p_{2, sep}^H \neq p_{2, sep}^L$) entails $p_{2, sep}^L = \frac{q^L}{2}$, while $p_{2, sep}^H = \frac{q^H}{2}$ if and only if $p_{1, pool} \leq \min\{\frac{q^L}{2} (\frac{q^H}{q^L} + \frac{q^L}{q^H} - 1), (\frac{q^H + q^L}{2q^H})\}$. 

7
If the second-period price is separating \((p_{2,sep}^H \neq p_{2,sep}^L)\), the L-type firm will charge its first-best price: \(p_{2,sep}^L = \frac{q^L}{2}\). For the H-type firm, this separating second-period price must maximize its second-period profit subject to the L-type firm’s ICs. The unconstrained optimization price is \(p_{2,sep}^H = \frac{q^H}{2}\). For this price to satisfy the ICs, we need:

\[
\begin{align*}
    p_{1,pool} &\leq \frac{\bar{q}}{4} \left(\frac{q^H + q^L}{q^H} - 1\right) \text{ if } q^L \geq \frac{q^H}{2} \\
    p_{1,pool} &\leq \frac{\sqrt{q^H q^L} (q^H - q^L)}{2q^H} \text{ if } q^L \leq \frac{q^H}{2}
\end{align*}
\]

Otherwise, the ICs should be binding:

\[
\begin{align*}
    p_{2,sep}^H \left(1 - \min\left\{\frac{p_{2,sep}^L}{q^L}, 1\right\} + \frac{p_{1,pool}}{q} - \frac{p_{2,sep}^H}{q^H}\right) &= \frac{q^L}{4} \text{ if } \frac{q^L}{q} p_{1,pool} \leq p_{2,sep}^H \leq \frac{q^H}{q} p_{1,pool} \\
    p_{2,sep}^H \left(1 - \frac{p_{2,sep}^L}{q^H}\right) &= \frac{q^L}{4} \text{ if } p_{2,sep}^H \leq \frac{q^L}{q} p_{1,pool} \text{ or } p_{1,pool} \geq \bar{q}
\end{align*}
\]

We can derive the second-period separating price in three cases.

(i) If \(q^L \geq \frac{3q^H}{4}\)

\[
p_{2,sep}^H = \begin{cases} 
    \frac{q^H}{2} & \text{if } p_{1,pool} \leq \frac{\bar{q}}{4} \left(\frac{q^H + q^L}{q^H} - 1\right) \\
    \frac{q^H}{2} \sqrt{\frac{q^H (q^H - q^L)}{q^L}} \left(p_{1,pool} + \bar{q} + \sqrt{p_{1,pool}^2 + 2p_{1,pool}\bar{q} - \frac{q^L q^2}{q^H}}\right) & \text{otherwise}
\end{cases}
\]

(ii) If \(\frac{q^H}{4} \leq q^L \leq \frac{3q^H}{4}\)

\[
p_{2,sep}^H = \begin{cases} 
    \frac{q^H}{2} & \text{if } p_{1,pool} \leq \frac{\bar{q}}{4} \left(\frac{q^H + q^L}{q^H} - 1\right) \\
    \frac{q^H}{2} \sqrt{\frac{q^H (q^H - q^L)}{q^L}} \left(p_{1,pool} + \bar{q} + \sqrt{p_{1,pool}^2 + 2p_{1,pool}\bar{q} - \frac{q^L q^2}{q^H}}\right) & \text{if } \frac{(q^H + 4q^L)\bar{q}}{4q^H} \leq p_{1,pool} \leq \bar{q} \\
    \frac{q^H}{2} \sqrt{\frac{q^H (q^H - q^L)}{q^L}} \left(p_{1,pool} + \bar{q} + \sqrt{p_{1,pool}^2 + 2p_{1,pool}\bar{q} - \frac{q^L q^2}{q^H}}\right) & \text{if } p_{1,pool} \geq \bar{q}
\end{cases}
\]

(iii) If \(q^L \leq \frac{q^H}{2}\),
\[ p_{2,\text{sep}}^H = \begin{cases} \frac{q^H}{2} & \text{if } p_{1,\text{pool}} \leq \frac{(q^H+q^L)\bar{q}}{2q^H} \\ \frac{q^H}{2q} \left( p_{1,\text{pool}} + \sqrt{p_{1,\text{pool}}^2 - \frac{q^Lq^2}{q^H}} \right) & \text{if } \frac{(q^H+q^L)\bar{q}}{2q^H} \leq p_{1,\text{pool}} \leq \bar{q} \\ \frac{q^H}{2} \pm \sqrt{q^H(q^H-q^L)} & \text{if } p_{1,\text{pool}} \geq \bar{q} \end{cases} \]

One can easily show that \( p_{2,\text{sep}}^H = \frac{q^H}{2} \) if and only if \( p_{1,\text{pool}} \leq \min \left\{ \frac{\bar{q}}{2} \left( \frac{q^H}{q^L} + \frac{q^L}{q^H} - 1 \right), \frac{(q^H+q^L)\bar{q}}{2q^H} \right\} \).

Finally, we claim the LMSE that the first period-price is pooling \( (p_{1,\text{pool}}^i = \frac{\bar{q}}{2}) \) and the second-period price is separating \( (p_{2,\text{sep}}^i = \frac{q^i}{2}) \). Clearly, it is the \( H \)-type firm’s most profitable outcome when the first-period price is pooling and the second-period price is separating. Since in the first period the product is believed to come from either type of firms, the most profitable pooling price is \( p_{1,\text{pool}} = \frac{\bar{q}}{2} \). In addition, this first-period pooling price is small enough for the \( H \)-type firm to charge its first-best price without the \( L \)-type firm’s mimicking in the second period, so \( p_{2,\text{sep}}^i = \frac{q^i}{2} \).

It dominates all the possible outcomes when the first- and second-period prices are pooling. Note that by charging any pooling second-period price the \( H \)-type firm cannot earn a higher second-period profit than its first-best profit. In addition, the most profitable pooling first-period price is \( p_{1,\text{pool}} = \frac{\bar{q}}{2} \), which is the same as the price scheme above. Thus, the \( H \)-type firm cannot earn a higher profit than the case above in both periods. It also dominates all the possible outcomes when the first-period price is separating. It is because, to separate from the \( L \)-type firm, the \( H \)-type firm needs to charge a very high (or very low) first-period price: \( p_{1,\text{sep}}^H = \frac{q^H}{2} \pm \sqrt{q^H(q^H-q^L)} \), which leads to a very low profit.

From the discussion above, the LMSE is that the first period-price is pooling \( (p_{1,\text{pool}}^i = \frac{\bar{q}}{2}) \) and the
second-period price is separating \( p_{2, \text{sep}}^* = \frac{q^i}{2} \). Under these prices, those consumers with \( \theta \geq \frac{1}{2} \) will buy in the both periods; hence, the corresponding profits are: \( (\pi^H, \pi^L) = \left( \frac{\bar{q} + q^H}{4}, \frac{\bar{q} + q^L}{4} \right) \).

**Proof of Proposition 2.** With credible price commitment, two types of equilibrium outcomes are possible: pooling and separating. We first examine the pooling outcome: Given the pooling price scheme denoted by \( (\hat{p}_{1, \text{pool}}, \hat{p}_{2, \text{pool}}) \), all consumers in the first period will evaluate the product as of average quality. For a positive number of consumers to buy the product, the firm must have \( \hat{p}_{1, \text{pool}} < \bar{q} \). Consumers with \( \theta \geq \frac{\hat{p}_{1, \text{pool}}}{\bar{q}} \) will purchase the product in the first period and become informed of the true product quality. In the second period, both informed and uninformed consumers will then make a purchase decision based on the second-period price \( \hat{p}_{2, \text{pool}} \).

Note that for the pooling outcome to hold, the \( L \)-type firm’s overall pooling profit must be higher than its first-best profit \( (\frac{q^L}{2}) \), otherwise it will deviate from the pooling prices to its first-best price scheme \( (\frac{q^L}{2}, \frac{q^L}{2}) \), which essentially reveals its low type. For the nontrivial case of \( \hat{p}_{1, \text{pool}} < \bar{q} \), the \( L \)-type firm’s incentive-compatible constraints are

\[
\begin{align*}
\hat{p}_{1, \text{pool}} (1 - \frac{\hat{p}_{1, \text{pool}}}{\bar{q}}) + \hat{p}_{2, \text{pool}} (1 - \min\{\frac{\hat{p}_{2, \text{pool}}}{q^L}, 1\}) &\geq \frac{q^L}{2} \quad \text{if } \hat{p}_{2, \text{pool}} \geq \hat{p}_{1, \text{pool}} \\
\hat{p}_{1, \text{pool}} (1 - \frac{\hat{p}_{1, \text{pool}}}{\bar{q}}) + \hat{p}_{2, \text{pool}} (1 - \min\{\frac{\hat{p}_{2, \text{pool}}}{q^L}, 1\} + \frac{\hat{p}_{1, \text{pool}} - \hat{p}_{2, \text{pool}}}{\bar{q}}) &\geq \frac{q^L}{2} \quad \text{if } \frac{\hat{p}_{1, \text{pool}}}{\bar{q}} \leq \frac{\hat{p}_{2, \text{pool}}}{\bar{q}} \leq \hat{p}_{1, \text{pool}} \\
\hat{p}_{1, \text{pool}} (1 - \frac{\hat{p}_{1, \text{pool}}}{\bar{q}}) + \hat{p}_{2, \text{pool}} (1 - \frac{\hat{p}_{2, \text{pool}}}{\bar{q}}) &\geq \frac{q^L}{2} \quad \text{if } \hat{p}_{2, \text{pool}} \leq \frac{q^L}{\bar{q}} \hat{p}_{1, \text{pool}}
\end{align*}
\]

In the second period, depending on how high \( \hat{p}_{2, \text{pool}} \) is, the \( H \)-type firm will serve a fraction of the informed consumers, all informed consumers, or all informed consumers together with some uninformed consumers. The \( H \)-type firm’s total profit for both periods is computed as follows:
We show that under two scenarios the most profitable pooling outcome entails the price scheme 

\((\frac{q^H}{2}, \frac{q^H}{2})\), which implies that the H-type firm charges its first-best price in the second period: When price commitment is credible, the most profitable pooling price scheme is \((\hat{p}_{1,pool},\hat{p}_{2,pool}) = (\frac{q^H}{2}, \frac{q^H}{2})\) iff \(q^L \leq q^H \leq (1 + \alpha)q^L \) or \(\frac{\alpha + 1}{\alpha}q^L \leq q^H\).

For this pooling outcome to be sustained, the L-type firm should have no incentive to deviate from this pooling price scheme. The first scenario is when \(q^L\) is high enough \((q^L \geq \frac{q^H}{1 + \alpha})\) so that even at the H-type firm’s first-best price, a large enough fraction of the informed consumers in the second period will still buy the low-quality product. The second scenario for the pooling outcome is when \(q^H\) is high enough \((q^H \geq \frac{\alpha + 1}{\alpha}q^L)\) so that the L-type firm will make a large enough pooling first-period profit that it will not deviate from the pooling price scheme even though no consumers will buy its product at the committed high second-period price.

Note that, among all the pooling price schemes \((\hat{p}_{1,pool},\hat{p}_{2,pool})\), \((\frac{q^H}{2}, \frac{q^H}{2})\) is the unconstrained optimization where the H-type firm earns the highest first-period profit when its product is evaluated as the average quality and its first-best profit in the second period. For this price scheme to satisfy the incentive-compatible constraints for the pooling outcome, we need \(\frac{q^L}{2}(1 - \frac{1}{2}) + \frac{q^H}{2}(1 - \min\{\frac{q^H}{2q^L}, 1\}) \geq \frac{q^L}{2}\). From the ICs, we showed \((\hat{p}_{1,pool},\hat{p}_{2,pool}) = (\frac{q^H}{2}, \frac{q^H}{2})\) if and only if \(q^L \leq q^H \leq (1 + \alpha)q^L\) or \(\frac{\alpha + 1}{\alpha}q^L \leq q^H\).
Second, we examine the separating outcome: All consumers can correctly infer the firm’s true product quality from its committed price scheme. If the $L$-type firm commits to the $H$-type firm’s price scheme $(\hat{p}_{1,\text{sep}}^H, \hat{p}_{2,\text{sep}}^H)$, in the first period consumers will believe that its product is of high quality, but in the second period, the first-period buyers will know the true quality and may not buy the product again if the second-period price is high. For a separating outcome to hold, the $L$-type firm must make a lower profit than its first-best profit if it mimics the $H$-type firm’s price scheme. This incentive-compatible constraint (for the non-trivial case of $\hat{p}_{1,\text{sep}}^H < q^H$) is derived:

$$
\begin{align*}
\hat{p}_{1,\text{sep}}^H (1 - \frac{\hat{p}_{1,\text{sep}}^H}{q^H}) + \hat{p}_{2,\text{sep}}^H (1 - \min\{\frac{\hat{p}_{2,\text{sep}}^H}{q^L}, 1\}) &\leq \frac{q^L}{2} & \text{if } \hat{p}_{2,\text{sep}}^H \geq \hat{p}_{1,\text{sep}}^H \\
\hat{p}_{1,\text{sep}}^H (1 - \frac{\hat{p}_{1,\text{sep}}^H}{q^H}) + \hat{p}_{2,\text{sep}}^H (1 - \min\{\frac{\hat{p}_{2,\text{sep}}^H}{q^L}, 1\} + \frac{\hat{p}_{1,\text{sep}}^H}{q^H} - \frac{\hat{p}_{2,\text{sep}}^H}{q^H}) &\leq \frac{q^L}{2} & \text{if } \frac{q^L}{q^H} \hat{p}_{1,\text{sep}}^H \leq \hat{p}_{2,\text{sep}}^H \leq \hat{p}_{1,\text{sep}}^H \\
\hat{p}_{1,\text{sep}}^H (1 - \frac{\hat{p}_{1,\text{sep}}^H}{q^H}) + \hat{p}_{2,\text{sep}}^H (1 - \frac{\hat{p}_{2,\text{sep}}^H}{q^H}) &\leq \frac{q^L}{2} & \text{if } \hat{p}_{2,\text{sep}}^H \leq \frac{q^L}{q^H} \hat{p}_{1,\text{sep}}^H.
\end{align*}
$$

The $H$-type firm’s expected profit at its separating price scheme $(\hat{p}_{1,\text{sep}}^H, \hat{p}_{2,\text{sep}}^H)$ is given by $\hat{\pi}_{\text{sep}}^H = \hat{p}_{1,\text{sep}}^H (1 - \min\{\frac{\hat{p}_{1,\text{sep}}^H}{q^H}, 1\}) + \hat{p}_{2,\text{sep}}^H (1 - \min\{\frac{\hat{p}_{2,\text{sep}}^H}{q^H}, 1\})$.

We characterize the firm’s price schemes in the most profitable separating outcome: When price commitment is credible, the most profitable separating price scheme is

$$
(\hat{p}_{1,\text{sep}}^L, \hat{p}_{2,\text{sep}}^L) = \left(\frac{q^L}{2}, \frac{q^L}{2}\right) \text{ and } (\hat{p}_{1,\text{sep}}^H, \hat{p}_{2,\text{sep}}^H) = \begin{cases} 
\left(\frac{q^H - \sqrt{q^H(q^H - 2q^L)}}{2}, \frac{q^H}{2}\right) & \text{if } \frac{q^L}{q^H} < \frac{1}{2}, \\
\left(\frac{q^H}{2}, \frac{q^H}{2}\right) & \text{if } \frac{q^L}{q^H} = \frac{1}{2}, \\
\left(\frac{q^H}{2}, \frac{q^H + \sqrt{q^H(q^H - q^L)}}{2}\right) & \text{if } \frac{q^L}{q^H} > \frac{1}{2}.
\end{cases}
$$

Note that only when $\frac{q^L}{q^H} = \frac{1}{2}$ can the $H$-type firm costlessly separate from the $L$-type firm—achieving its first-best profit by committing to first-best prices of $\left(\frac{q^H}{2}, \frac{q^H}{2}\right)$. Otherwise, to separate, the $H$-type firm
will have to charge either a lower first-period price or a higher second-period price than its first-best price \( \frac{q^H}{2} \).

If the price scheme is separating, the \( L \)-type firm will charge its first-best price in both periods: \( \hat{p}_{t,sep}^L = \frac{q^L}{2} \). For the \( H \)-type firm, this separating price scheme must maximize its profit subject to the \( L \)-type firm’s ICs. The unconstrained optimization price scheme is \( \hat{p}_{t,sep}^H = \frac{q^H}{2} \). For this price scheme to satisfy the ICs, we need \( \frac{q^H}{2} (1 - \frac{1}{2}) + \frac{q^H}{2} (1 - \min{\frac{q^H}{2q^L}, 1}) \leq \frac{q^L}{2} \). From the ICs, \( \hat{p}_{t,sep}^H = \frac{q^H}{2} \) if and only if \( q^H = 2q^L \). Otherwise, the ICs should be binding. We then discuss the following two cases:

Case 1: \( \frac{q^L}{q^H} > \frac{1}{2} \)

We first assume that \( \hat{p}_{2,sep}^H \geq \hat{p}_{1,sep}^H \). Note that the IC is never binding if \( \hat{p}_{2,sep}^H > q^L \). If \( \hat{p}_{2,sep}^H \leq q^L \), from the binding IC, \( \hat{p}_{1,sep}^H(1 - \frac{\hat{p}_{1,sep}^H}{q^H}) + \hat{p}_{2,sep}^H(1 - \frac{\hat{p}_{2,sep}^H}{q^H}) = \frac{q^L}{2} \), the most profitable price scheme for the \( H \)-type firm is \( \frac{q^H}{2}, \frac{q^L + \sqrt{q^L(q^H - q^L)}}{2} \) and we can check that \( \hat{p}_{2,sep}^H \geq \hat{p}_{1,sep}^H \) and \( \hat{p}_{2,sep}^H \leq q^L \) immediately.

We show that this price scheme above is dominant. We define

\[
IC_1 = \{(\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H)|\hat{p}_{1,sep}^H(1 - \frac{\hat{p}_{1,sep}^H}{q^H}) + \hat{p}_{2,sep}^H(1 - \min{\frac{\hat{p}_{2,sep}^H}{q^L}, 1}) \leq \frac{q^L}{2}\}
\]

\[
IC_2 = \{(\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H)|\hat{p}_{1,sep}^H(1 - \frac{\hat{p}_{1,sep}^H}{q^H}) + \hat{p}_{2,sep}^H(1 - \min{\frac{\hat{p}_{2,sep}^H}{q^L}, 1} + \frac{\hat{p}_{1,sep}^H}{q^H} - \frac{\hat{p}_{2,sep}^H}{q^H}) \leq \frac{q^L}{2}\}
\]

\[
IC_3 = \{(\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H)|\hat{p}_{1,sep}^H(1 - \frac{\hat{p}_{1,sep}^H}{q^H}) + \hat{p}_{2,sep}^H(1 - \frac{\hat{p}_{2,sep}^H}{q^H}) \leq \frac{q^L}{2}\}
\]

It is clear that \( IC_2 \subseteq IC_1 \) if \( \hat{p}_{2,sep}^H \leq \hat{p}_{1,sep}^H \). Because \( \frac{q^H}{2}, \frac{q^L + \sqrt{q^L(q^H - q^L)}}{2} \) yields the highest \( H \)-type firm’s profit in \( IC_1 \), all the other possible outcomes in \( IC_2 \) is dominated when \( \frac{q^L}{q^H} \hat{p}_{1,sep}^H \leq \hat{p}_{2,sep}^H \leq \hat{p}_{1,sep}^H \).

From the similar argument, \( IC_3 \subseteq IC_1 \forall (\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H) \in \Re^2_+ \) so that the \( H \)-type firm’s profit is also
dominated when \( \hat{p}_{2,sep}^H \leq \frac{q_L}{q_H} \hat{p}_{1,sep}^H \). We conclude that the price scheme \( \left( \frac{q_H^H}{2}, \frac{q_L + \sqrt{q_H^H(q_H^H - q_L^H)}}{2} \right) \) is dominant.

Case 2: \( \frac{q_L}{q_H} < \frac{1}{2} \)

We first assume that \( \hat{p}_{2,sep}^H \geq \hat{p}_{1,sep}^H \) and \( \hat{p}_{2,sep}^H > q_L \). From the binding IC, \( \hat{p}_{1,sep}^H (1 - \frac{\hat{p}_{sep}}{q_H}) = \frac{q_L}{2} \), the most profitable price scheme for the \( H \)-type firm is \( \left( \frac{q_H^H}{2}, \frac{q_L + \sqrt{q_H^H(q_H^H - 2q_L^H)}}{2} \right) \), but only the price scheme \( \left( \frac{q_H^H - \sqrt{q_H^H(q_H^H - 2q_L^H)}}{2}, \frac{q_H^H}{2} \right) \) satisfies \( \hat{p}_{2,sep}^H \geq \hat{p}_{1,sep}^H \) and \( \hat{p}_{2,sep}^H > q_L \).

We then show that the price scheme above is dominant. Under \( \hat{p}_{2,sep}^H \geq \hat{p}_{1,sep}^H \) and \( \hat{p}_{2,sep}^H \leq q_L \), the binding IC is \( \hat{p}_{1,sep}^H (1 - \frac{\hat{p}_{sep}}{q_H}) + \hat{p}_{2,sep}^H (1 - \frac{\hat{p}_{sep}}{q_L}) = \frac{q_L}{2} \). Because the most profitable price scheme under this binding IC, \( \left( \frac{q_H^H}{2}, \frac{q_L + \sqrt{q_H^H(q_H^H - q_L^H)}}{2} \right) \), never satisfies \( \hat{p}_{2,sep}^H \leq q_L \), the most profitable price scheme must satisfy \( \hat{p}_{2,sep}^H = q_L \). Since \( \left( \frac{q_H^H - \sqrt{q_H^H(q_H^H - 2q_L^H)}}{2}, \frac{q_H^H}{2} \right) \) yields the highest profit in \( \hat{p}_{1,sep}^H (1 - \frac{\hat{p}_{sep}}{q_H}) = \frac{q_L}{2} \), which is the binding IC when \( \hat{p}_{2,sep}^H = q_L \), all the possible price schemes in \( \hat{p}_{2,sep}^H \leq q_L \) are dominated.

Finally, from the same argument in Case 1, \( IC_2 \subseteq IC_1 \) if \( \hat{p}_{2,sep}^H \leq \hat{p}_{1,sep}^H \) and \( IC_3 \subseteq IC_1 \cup (\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H) \in \mathbb{R}_+^2 \). Because \( \left( \frac{q_H^H - \sqrt{q_H^H(q_H^H - 2q_L^H)}}{2}, \frac{q_H^H}{2} \right) \) yields the highest profit in \( IC_1 \), the \( H \)-type firm’s profit in \( \hat{p}_{2,sep}^H \leq \hat{p}_{1,sep}^H \) is also dominated. We conclude that the price scheme \( \left( \frac{q_H^H - \sqrt{q_H^H(q_H^H - 2q_L^H)}}{2}, \frac{q_H^H}{2} \right) \) is dominant. From Cases 1 and 2, we derive the separating price scheme.

Finally we derive the LMSE by comparing the \( H \)-type firm’s profit under the pooling and separating price schemes. Under the pooling price scheme, the \( H \)-type firm’s profit is \( \frac{q_L + q_H^H}{4} \); whereas under the separating price scheme, the \( H \)-type firm’s profit is
\[
\begin{cases}
q^H(q^H + q^L) + 2(q^H - q^L)\sqrt{(q^H - q^L)q^L} \\
\quad 4q^H & \text{if } \frac{q^L}{q^H} > \frac{1}{2}
\\
\frac{q^H}{2} & \text{if } \frac{q^L}{q^H} = \frac{1}{2}
\\
\frac{q^H + 2q^L}{4} & \text{if } \frac{q^L}{q^H} < \frac{1}{2}
\end{cases}
\]

By comparing the \(H\)-type firm’s profit under the pooling and separating price schemes, we can show

that the price scheme is pooling if \(\frac{q^L}{q^H} \geq \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})}\) or \(\frac{q^L}{q^H} \leq \frac{\alpha}{\alpha + 1}\), and is separating if \(\frac{\alpha}{\alpha + 1} \leq \frac{q^L}{q^H} < \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})}\). Since the \(H\)-type firm is indifferent between the pooling and separating price scheme if \(\frac{q^L}{q^H} = \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})}\) or \(\frac{q^L}{q^H} = \frac{\alpha}{\alpha + 1}\), we need to compare the \(L\)-type firm’s profit under the pooling and separating price schemes in the two cases.

Under the separating price scheme, the \(L\)-type firm earns its first-best profit in both periods so that its profit is \(\frac{q^L}{2}\). Under the pooling price scheme, the \(L\)-type firm’s profit is

\[
\begin{cases}
\frac{\bar{q} + (2q^L - q^H)q^H}{4q^H} & \text{if } \frac{q^L}{q^H} \geq \frac{1}{2}
\\
\frac{\bar{q}}{4} & \text{if } \frac{q^L}{q^H} < \frac{1}{2}
\end{cases}
\]

By comparing the \(L\)-type firm’s profit under the pooling and separating price scheme, we show that the price scheme is pooling if \(\frac{q^L}{q^H} = \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})}\) and the \(L\)-type firm is indifferent if \(\frac{q^L}{q^H} = \frac{\alpha}{\alpha + 1}\), so the LMSE is shown.

**PROOF OF PROPOSITION 3.** In the proof to Proposition 2, we have derived the firm’s profit under the pooling and separating price schemes.

**PROOF OF PROPOSITION 4.** By Propositions 1 and 3, we can compare the profit when the price commitment is credible and not. If \(\frac{q^L}{q^H} \geq \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})}\) or \(\frac{q^L}{q^H} \leq \frac{\alpha}{\alpha + 1}\), the price scheme is pooling when the price commitment is credible. In addition, the \(H\)-type firm’s profit is the same as that when the price
commitment is not credible: \( \bar{\pi}_H^* \) = \( \bar{\pi}_L^* \). While the L-type firm’s profit is strictly lower when the price commitment is credible:

\[
\begin{align*}
\bar{\pi}_L^* &= \frac{q^L}{4} + \frac{2q^L - q^H}{q^L} < \pi_L^* = \frac{q^L}{4} \quad \text{if} \quad q^L \leq \frac{\alpha}{2(1-\sqrt{1-\alpha^2})} \\
\bar{\pi}_L^* &= \frac{q^L}{4} < \pi_L^* = \frac{q^L}{4} \quad \text{if} \quad q^L \geq \frac{\alpha}{\alpha + 1}.
\end{align*}
\]

If \( \frac{\alpha}{\alpha + 1} < \frac{q^L}{q^H} < \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})} \), the price scheme is separating when the price commitment is credible. In addition, the H-type firm’s profit is strictly higher than that when the price commitment is not credible:

\[
\begin{align*}
\bar{\pi}_L^* &= \frac{q^L}{2} + \frac{2q^L - q^H}{q^H} \sqrt{(q^H - q^L)q^L} \quad \text{if} \quad \frac{1}{2} < q^L < \frac{\alpha^2}{2(1-\sqrt{1-\alpha^2})} \\
\bar{\pi}_L^* &= \frac{q^L}{4} > \pi^*_L = \frac{q^L}{4} \quad \text{if} \quad q^L = \frac{1}{2} \\
\bar{\pi}_L^* &= \frac{q^L}{4} > \pi^*_L = \frac{q^L}{4} \quad \text{if} \quad \frac{\alpha}{\alpha + 1} < \frac{q^L}{q^H} < \frac{1}{2}.
\end{align*}
\]

While the L-type firm’s profit is also strictly lower when the price commitment is not credible: \( \bar{\pi}_L^* = \frac{q^L}{4} < \pi_L^* \).

**Proof of Proposition 5.** When the price commitment is not credible, we define \( S_t^i \) as the consumer surplus when consumers face the \( i \)-type firm in period \( t \) and \( S \) as the consumer surplus; thus, \( S = \alpha(S_t^H + S_t^L) + (1 - \alpha)(S_t^L + S_t^H) \). In the LMSE, only those consumers with \( \theta \in \left[ \frac{1}{2}, 1 \right] \) will purchase the product in both periods no matter which type of firm they face so that \( (S_t^1, S_t^2) = (\int_{\frac{1}{2}}^1 q^i \theta - \frac{q^i}{2} d\theta, \int_{\frac{1}{2}}^1 q^i \theta - \frac{q^i}{2} d\theta) \).

When the price commitment is credible, we define \( \hat{S}_t^i \) as the consumer surplus when consumers face the \( i \)-type firm in period \( t \) and \( \hat{S} \) as the consumer surplus. We then discuss the following four cases. First, if \( \frac{\alpha}{\alpha + 1} < \frac{q^L}{q^H} < \frac{1}{2} \), the price scheme is separating. For the L-type firm, it can attract those consumers
with $\theta \in \left[\frac{1}{2}, 1\right]$ in both periods so that $\hat{S}_t^L = \int_{\frac{1}{2}}^{q^L} q^L \theta - \frac{q^L}{2} d\theta$. For the $H$-type firm, it can attract those consumers with $\theta \in \left[\hat{\theta}_1^H, 1\right]$ where $\hat{\theta}_1^H < \frac{1}{2}$ in the first period because $\hat{p}_1^H < \frac{q^H}{2}$, and consumers with $\theta \in \left[\hat{\theta}_1^L, 1\right]$ will purchase its product in the second period. Thus, $(\hat{S}_1^H, \hat{S}_2^H) = (\int_{\frac{1}{2}}^{q^H} q^H \theta - \frac{q^H}{2} d\theta, \int_{\frac{1}{2}}^{q^H} q^H \theta - \frac{q^H}{2} d\theta)$. It is clear that $\hat{S} > S$ since $\hat{S}_1^H = S_1^H$ and $\alpha \hat{S}_1^H + (1 - \alpha) S_1^H > \alpha S_1^H + (1 - \alpha) S_1^L$.

Second, if $\frac{q^L}{q^H} = \frac{1}{2}$, the $H$-type firm can costlessly separate from the $L$-type firm. Since consumers with $\theta \in \left[\frac{1}{2}, 1\right]$ will purchase the product in both periods, we have $\hat{S}_t^L = \int_{\frac{1}{2}}^{q^L} q^L \theta - \frac{q^L}{2} d\theta$. It is clear that $\hat{S} = S$ since $\hat{S}_2^L = S_2^L$ and $\alpha \hat{S}_1^H + (1 - \alpha) S_1^H = \alpha S_1^H + (1 - \alpha) S_1^L$.

Third, if $\frac{1}{2} < \frac{q^L}{q^H} < \frac{\alpha^2}{2(1 - \sqrt{1 - \alpha^2})}$, the price scheme is separating so that $\hat{S}_t^L = \int_{\frac{1}{2}}^{q^L} q^L \theta - \frac{q^L}{2} d\theta$ when consumers face the $L$-type firm. For the $H$-type firm, it can attract consumers with $\theta \in \left[\frac{1}{2}, 1\right]$ in the first period, but only those consumers with $\theta \in \left[\hat{\theta}_2^H, 1\right]$ where $\hat{\theta}_2^H > \frac{1}{2}$, will purchase its product in the second period because $\hat{p}_2^H > \frac{q^H}{2}$. Hence, $(\hat{S}_1^H, \hat{S}_2^H) = (\int_{\frac{1}{2}}^{q^H} q^H \theta - \frac{q^H}{2} d\theta, \int_{\hat{\theta}_2^H}^{q^H} q^H \theta - \frac{q^H}{2} d\theta)$. We then claim that $\hat{S} < S$ since $\alpha \hat{S}_1^H + (1 - \alpha) \hat{S}_1^L = \alpha S_1^H + (1 - \alpha) S_1^L$ and $\alpha \hat{S}_1^H + (1 - \alpha) \hat{S}_1^L < \alpha S_1^H + (1 - \alpha) S_1^L$.

Fourth, if $\frac{q^L}{q^H} \geq \frac{\alpha^2}{2(1 - \sqrt{1 - \alpha^2})}$ or $\frac{q^L}{q^H} \leq \frac{\alpha}{\alpha + 1}$, the price scheme is pooling. In the first period, both types of firm can attract those consumers with $\theta \in \left[\frac{1}{2}, 1\right]$ and the consumer surplus is $\hat{S}_1^L = \int_{\frac{1}{2}}^{q^L} q^L \theta - \frac{q^L}{2} d\theta$. In the second period, the $H$-type firm attracts those consumers with $\theta \in \left[\hat{\theta}_2^L, 1\right]$ whereas the $L$-type firm can only attract those consumers with $\theta \in \left[\hat{\theta}_2^L, 1\right]$ where $\hat{\theta}_2^L > \frac{1}{2}$ since it cannot adjust its high second-
period price \( (\hat{p}^L_2 > \frac{q_L}{2}) \). Hence, the consumer surplus is \( (\hat{S}^H_2, \hat{S}^L_2) = (\int_{\frac{1}{2}}^1 q^H \theta - \frac{q^H}{2} \, d\theta, \int_{\frac{1}{2}}^1 q^L \theta - \hat{p}^L_2 \, d\theta) \).

We can show that \( \hat{S} < S \) because \( \alpha \hat{S}^H_1 + (1 - \alpha)\hat{S}^L_1 = \alpha S^H_1 + (1 - \alpha)S^L_1 \) and \( \alpha \hat{S}^H_2 + (1 - \alpha)\hat{S}^L_2 < \alpha S^H_1 + (1 - \alpha)S^L_1 \).

To sum up, from the four cases above, we showed \( \hat{S} > S \) if \( \frac{\alpha}{\alpha + 1} < \frac{q_L}{q_H} < \frac{1}{2} \); \( \hat{S} = S \) if \( \frac{q_L}{q_H} = \frac{1}{2} \); otherwise, \( \hat{S} < S \).

PROOF OF PROPOSITION 6. Social welfare is the sum of the firm’s profit and consumer surplus. It is equal to the aggregate willingness to pay among all consumers who purchase the product because the firm’s profit is from the price that consumers pay for the product. We then compare social welfare when the price commitment is credible and not.

When the price commitment is not credible, we define \( W^I_t \) as social welfare when consumers face the \( i \)-type firm in period \( t \) and social welfare as \( W \); thus, \( W = \alpha(W^H_1 + W^H_2) + (1 - \alpha)(W^L_1 + W^L_2) \).

Since only those consumers with \( \theta \in [\frac{1}{2}, 1] \) will purchase the product no matter which type of firm they face in each period, social welfare is \( (W^I_1, W^I_2) = (\int_{\frac{1}{2}}^1 q^I \theta d\theta, \int_{\frac{1}{2}}^1 q^I \theta d\theta) \).

When the price commitment is credible, we define \( \hat{W}^I_t \) as social welfare when consumers face the \( i \)-type firm in period \( t \) and \( \hat{W} \) as social welfare. We then discuss the following four cases. First, if \( \frac{\alpha}{\alpha + 1} \leq \frac{q_L}{q_H} < \frac{1}{2} \), the \( L \)-type firm attracts those consumers with \( \theta \in [\frac{1}{2}, 1] \) in both periods so that \( \hat{W}^L_t = \int_{\frac{1}{2}}^1 q^L \theta d\theta \).

The \( H \)-type firm can attract those consumers with \( \theta \in [\hat{\theta}^H_1, 1] \) where \( \hat{\theta}^H_1 < \frac{1}{2} \) in the first period and those with \( \theta \in [\frac{1}{2}, 1] \) in the second period. Hence, \( (\hat{W}^H_1, \hat{W}^H_2) = (\int_{\hat{\theta}^H_1}^1 q^H \theta d\theta, \int_{\frac{1}{2}}^1 q^H \theta d\theta) \). We can show
\[ \hat{W} > W \text{ since } \hat{W}_2^i = W_2^i \text{ and } \alpha \hat{W}_1^H + (1 - \alpha) \hat{W}_1^L > \alpha W_1^H + (1 - \alpha) W_1^L. \]

Second, if \( \frac{q_L}{q_H} = \frac{1}{2} \), consumers with \( \theta \in \left[ \frac{1}{2}, 1 \right] \) will purchase the product in both periods: \( \hat{W}_i^i = \int_{\frac{1}{2}}^{1} q^i \theta d\theta \). It is clear that \( \hat{W} = W \) because \( \hat{W}_2^i = W_2^i \) and \( \alpha \hat{W}_1^H + (1 - \alpha) \hat{W}_1^L = \alpha W_1^H + (1 - \alpha) W_1^L \).

Third, if \( \frac{1}{2} < \frac{q_L}{q_H} < \frac{2}{1 - \sqrt{1 - \alpha^2}} \), social welfare when consumers face the L-type firm is \( \hat{W}_i^L = \int_{\frac{1}{2}}^{1} q^L \theta d\theta \). For the H-type firm, it can attract consumers with \( \theta \in \left[ \frac{1}{2}, 1 \right] \) in the first period and those with \( \theta \in [\tilde{\theta}_2^H, 1] \), where \( \tilde{\theta}_2^H > \frac{1}{2} \) in the second period. Hence, \( (\hat{W}_1^H, \hat{W}_2^H) = (\int_{\frac{1}{2}}^{1} q^H \theta d\theta, \int_{\tilde{\theta}_2^H}^{1} q^H \theta d\theta) \). We claim that \( \hat{W} < W \) since \( \alpha \hat{W}_1^H + (1 - \alpha) \hat{W}_1^L = \alpha W_1^H + (1 - \alpha) W_1^L \) and \( \alpha \hat{W}_2^H + (1 - \alpha) \hat{W}_2^L < \alpha W_1^H + (1 - \alpha) W_1^L \).

Fourth, if \( \frac{q_L}{q_H} \geq \frac{2}{1 - \sqrt{1 - \alpha^2}} \) or \( \frac{q_L}{q_H} \leq \frac{\alpha}{\alpha + 1} \), both types of firm can attract those consumers with \( \theta \in \left[ \frac{1}{2}, 1 \right] \) in the first period so that \( \hat{W}_i^i = \int_{\frac{1}{2}}^{1} q^i \theta d\theta \). In the second period, the H-type firm attracts those consumers with \( \theta \in \left[ \tilde{\theta}_2^L, 1 \right] \) whereas the L-type firm can only attract those consumers with \( \theta \in [\tilde{\theta}_2^L, 1] \), where \( \tilde{\theta}_2^L > \frac{1}{2} \). Hence, \( (\hat{W}_1^H, \hat{W}_2^H) = (\int_{\frac{1}{2}}^{1} q^H \theta d\theta, \int_{\tilde{\theta}_2^L}^{1} q^L \theta d\theta) \). Since \( \alpha \hat{W}_1^H + (1 - \alpha) \hat{W}_1^L = \alpha W_1^H + (1 - \alpha) W_1^L \) and \( \alpha \hat{W}_2^H + (1 - \alpha) \hat{W}_2^L < \alpha W_1^H + (1 - \alpha) W_1^L \), we can show that \( \hat{W} < W \).

To sum up, from the four cases above, we have shown that \( \hat{W} > W \) if \( \frac{\alpha}{\alpha + 1} < \frac{q_L}{q_H} < \frac{1}{2} \); \( \hat{W} = W \) if \( \frac{q_L}{q_H} = \frac{1}{2} \); otherwise, \( \hat{W} < W \).

**Proof of Proposition 7.** If the price scheme is separating \( ((\hat{p}_1^{H, \text{sep}}, ..., \hat{p}_T^{H, \text{sep}}) \neq (\hat{p}_1^{L, \text{sep}}, ..., \hat{p}_T^{L, \text{sep}})) \), the L-type firm will charge its first-best price in all the \( T \) periods, which implies \( p_t^{L, \text{sep}} = \frac{q_L}{2} \). For the H-type firm, this separating price scheme must maximize its profit subject to the L-type firm’s ICs. The
unconstrained optimization price is \( p_{t,sep}^{H} = \frac{q^{H}}{2} \). For this price scheme to satisfy the ICs, we need \( \frac{q^{H}}{2} (1 - \frac{1}{T}) + (T - 1) \frac{q^{H}}{2} (1 - \min\{\frac{q^{H}}{2q^{L}}, 1\}) \leq T q^{L} \). From this IC, we have shown that the \( H \)-type firm can costlessly separate from the \( L \)-type firm if and only if \( T \geq \max\{\frac{q^{H}}{q^{L}}, \frac{q^{H}}{q^{H} - q^{L}}\} \).

**Online Appendix B**

This appendix provides the formal analysis for the extension to model with social learning. We assume that social learning from the early-period customers is perfect, i.e., all consumers will become informed in the second period. Within our current model framework, we find that the LMSE outcomes actually remain the same with and without social learning.

We first analyze the case that the price commitment is not credible. Since all consumers in the second period are informed via word of mouth, the equilibrium second-period price is the same as the complete-information benchmark: the \( i \)-type firm charges the first-best price \( \tilde{p}^{i}_{2} = \frac{q^{i}}{2} \) and sells to consumers with \( \theta \geq \frac{1}{2} \). Note that the first-period price is irrelevant for the second-period outcome because of perfect social learning. In this degenerate case, the first-period outcome is the same as the one-period setting: both types of firms will pool at price \( \tilde{q} \) and sells to consumer with \( \theta \geq \frac{1}{2} \) in the first period because without cost differences the \( L \)-type firm has strong incentive to mimic the \( H \)-type firm. Thus, we have shown the LMSE remain the same when the price commitment is not credible.

Second, we analyze the case that the price commitment is credible. Two types of possible equilibrium outcome are possible: pooling and separating. Given the pooling price scheme donated by \( (\tilde{p}_{1, pool}, \tilde{p}_{2, pool}) \),
all consumers evaluate the product as of average quality in the first period and those with $\theta \geq \frac{\tilde{p}_{1,\text{pool}}}{q}$ purchase the product; in the second period, all consumers become informed about true quality and those with $\theta \geq \frac{\tilde{p}_{2,\text{pool}}}{q}$ will purchase.

For the pooling outcome to hold, the $L$-type firm’s overall pooling profit must be higher than its first-best profit ($\frac{q^L}{2}$) so that it does not deviate to its first-best price scheme ($\frac{q^L}{2}, \frac{q^L}{2}$). For the nontrivial case of $\tilde{p}_{1,\text{pool}} < q$, the $L$-type firm’s incentive-compatible constraint is

$$\tilde{p}_{1,\text{pool}}(1 - \frac{\tilde{p}_{1,\text{pool}}}{q}) + \tilde{p}_{2,\text{pool}}(1 - \min\{\frac{\tilde{p}_{2,\text{pool}}}{q^L}, 1\}) \geq \frac{q^L}{2}$$

Since all consumers will know the firm’s high quality, the $H$-type firm’s total profit for both periods is computed as follows:

$$\pi^H_{\text{pool}} = \tilde{p}_{1,\text{pool}}(1 - \frac{\tilde{p}_{1,\text{pool}}}{q}) + \tilde{p}_{2,\text{pool}}(1 - \min\{\frac{\tilde{p}_{2,\text{pool}}}{q^H}, 1\})$$

Among all the pooling price schemes $(\tilde{p}_{1,\text{pool}}, \tilde{p}_{2,\text{pool}})$, $(\frac{q^L}{2}, \frac{q^H}{2})$ is the unconstrained optimization where the $H$-type firm earns the highest first-period profit when its product is evaluated as the average quality and its first-best profit in the second period. For this price scheme to satisfy the incentive-compatible constraints (IC), we need $\frac{q^H}{2} (1 - \frac{1}{2}) + \frac{q^H}{2} (1 - \min\{\frac{q^H}{2q^L}, 1\}) \geq \frac{q^L}{2}$. Hence, we showed the most profitable pooling price scheme is $(\tilde{p}_{1,\text{pool}}, \tilde{p}_{2,\text{pool}}) = (\frac{q^L}{2}, \frac{q^H}{2})$ if and only if $q^L \leq q^H \leq (1 + \alpha)q^L$ or $\frac{\alpha + 1}{\alpha}q^L \leq q^H$.

Next, we examine the separating outcome: All consumers can correctly infer the firm’s true product quality from its committed price scheme. If the $L$-type firm commits to the $H$-type firm’s price scheme
\((\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H)\), in the first period consumers will believe that its product is of high quality, but in the second period, all consumers will become informed and decide to purchase based on the true quality. For a separating outcome to hold, the \(L\)-type firm must make a lower profit than its first-best profit if it mimics the \(H\)-type firm’s price scheme. This incentive-compatible constraint (for the non-trivial case of \(\hat{p}_{1,sep}^H < q^H\)) is derived:

\[
\hat{p}_{1,sep}^H (1 - \frac{\hat{p}_{1,sep}^H}{q^H}) + \hat{p}_{2,sep}^H (1 - \min\{\frac{\hat{p}_{2,sep}^H}{q^L}, 1\}) \leq \frac{q^L}{2}.
\]

The \(H\)-type firm’s expected profit at its separating price scheme \((\hat{p}_{1,sep}^H, \hat{p}_{2,sep}^H)\) is given by

\[
\hat{\pi}_{sep}^H = \hat{p}_{1,sep}^H (1 - \frac{\hat{p}_{1,sep}^H}{q^H}) + \hat{p}_{2,sep}^H (1 - \min\{\frac{\hat{p}_{2,sep}^H}{q^H}, 1\}).
\]

If the price scheme is separating, the \(L\)-type firm will charge its first-best price in both periods:

\[
\hat{p}_{1,sep}^L = \frac{q^L}{2}. \quad \text{For the } H\text{-type firm, this separating price scheme must maximize its profit subject to the } L\text{-type firm’s IC. The unconstrained optimization price scheme is } \hat{p}_{t,sep}^H = \frac{q^H}{2}, \text{ which occurs when } \frac{q^H}{2} (1 - \frac{1}{2}) + \frac{q^H}{2} (1 - \min\{\frac{q^H}{2q^L}, 1\}) \leq \frac{q^L}{2}, \text{ i.e., } \hat{p}_{t,sep}^H = \frac{q^H}{2} \text{ if and only if } q^H = 2q^L. \text{ Otherwise, the IC should be binding. From the similar argument when there is no social learning, the } H\text{-type firm’s most profitable price scheme is } (\frac{q^H}{2}, \frac{q^L+\sqrt{q^L(q^H-q^L)}}{2}) \text{ if } \frac{q^L}{q^H} > \frac{1}{2}, \text{ whereas it is } (\frac{q^H-\sqrt{q^H(q^H-2q^L)}}{2}, \frac{q^H}{2}) \text{ if } \frac{q^L}{q^H} < \frac{1}{2}.
\]

From the argument above, we can observe that the pooling and separating outcomes are the same with and without social learning; hence, the LMSE outcomes actually remain the same in all parameter regions.