Inter-Competitor Licensing and Product Innovation

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Abstract

This paper studies how inter-competitor licensing between an incumbent and an entrant affects market competition and the entrant’s optimal product quality. In the model, the incumbent has developed some non-core technology that is used for the non-core attribute of the final product, and the entrant has a new core technology to introduce a new higher-quality product. For the non-core technology of its product, the entrant can either license it from the incumbent or incur an R&D cost to develop it in-house. The authors show that a royalty licensing contract of the non-core technology between the incumbent and the entrant has a competition-alleviation effect. More importantly, the effect of such licensing on the entrant’s optimal quality depends on whether the entrant’s core technology can significantly or only incrementally increase its quality over the incumbent’s product. The royalty contract will tend to increase the entrant’s optimal quality when the entrant’s core technology can allow for a significant quality improvement over the incumbent. By contrast, if the entrant’s technology can raise its product quality only incrementally over the incumbent’s product, the royalty licensing contract will tend to reduce the entrant’s optimal quality.

A wide range of royalty contracts are mutually acceptable; the incumbent (entrant) can benefit from a licensing contract even when the entrant pays a total royalty fee that is lower (higher) than its alternative R&D cost. These results hold even when the incumbent endogenously chooses its royalty licensing fee. The authors show that their main results are robust to several alternative modeling assumptions, e.g., alternative game sequence, endogenous quality decision by the incumbent, and alternative licensing contract.

Keywords: inter-competitor licensing; royalty; competition; pricing; product quality
INTRODUCTION

Products typically have some core attributes, which are the main determinants of the products’ value in consumers’ mind, and some non-core attributes, which are necessary for the products but are not main value drivers or product differentiators. When an innovating entrant has some innovative core technologies that allow it to introduce higher quality product than the incumbent’s, it still needs non-core technologies for its product, even though these non-core technologies or non-core product attributes do not have much differential impact on consumers’ perceived quality of the product (i.e., on the consumer’s valuation for the product). Overall, the entrant needs to decide not only what quality level of its new core technology to put in its new product, but also what to do for the non-core technology, e.g., whether to self-develop it in-house or to license it from the incumbent.

In practice, firms may often license non-core technologies to or from their direct competitors. For example, Proctor and Gamble licenses its manufacturing know-hows to competitors (Parhankangas, Holmlund, and Kuusisto 2003). Microsoft and many Android device manufacturers (e.g., Samsung and HTC) are direct competitors in many product markets such as mobile computing devices and their operating systems, and yet these manufacturers pay Microsoft a licensing royalty of $5 to $15 per Android device for some mobile operating system features.¹ Similarly, Ford licenses out its Diesel Fuel Conditioning Module (DFCM) and the Passenger-Side Air Bag Deactivation Switch IP and Technology to its competitors (Fradkin 2014). Ford also licenses its electric car patents at some undisclosed cost to other manufacturers in the industry (Arce 2015). In 2014, Tesla Motors made its electric vehicle technology patents available to other automakers. Note however, that we want to point out that typically such licensing contracts are not public information and their existence or details of the terms are usually private information between the firms. One may argue that part of the reasons for such inter-competitor licensing may be the firms’ benefits of network effects or their corporate social responsibility to promote sustainability causes. Our paper provides a potential
alternative explanation for why firms may have incentives to license their non-core technologies to direct competitors.

We focus our study on the situation in which the entrant’s product needs a non-core technology (e.g., a patent) that the incumbent firm has already developed, and that the non-core technology does not introduce much differentiation between the incumbent’s product and the entrant’s product regardless of whether the entrant licenses it or develops it in-house. So, the entrant’s product is differentiated from the incumbent’s product mainly on the core technology, which requires some marginal cost of production. In our context, the quality level of the product’s core technology represents the level of innovation by the entrant.

This paper examines two related lines of inquiry in the aforementioned licensing and product innovation context. First, should an entrant license the non-core technology from its competitor when it can self-develop it in-house? Assuming that licensing will not shorten the time-to-market for the product, then conventional wisdom suggests that firms should avoid helping competitors. For example, a firm should not strengthen its competitor’s financial wellbeing by licensing from the competitor if the firm can develop its own alternative at the same or lower cost. Neither should a firm license its technology to a competitor, definitely not at lower costs than its competitor’s alternative, if not licensing it would raise the competitor’s cost and diminish its ability to compete in the market. Our conversations with many MBA and EMBA students who have extensive work experiences in many industries reveal that many companies use an intuitive decision-making rule: Reject a contract or proposal if the alternative cost is lower. Some firms also ex post evaluate the success of contracts or proposals based on the realized costs compared with their ex ante alternative costs. We show that such intuitively sound decision-making and performance-evaluation rules can be suboptimal when competing firms’ strategic behaviors are taken into account.

Second, how does licensing of the non-core technology from the incumbent affect the equilibrium
prices and the entrant’s optimal quality for its core technology? We consider the entrant’s quality choice of its core technology as a measure of its product innovation. Though both firms should benefit if a licensing contract is mutually acceptable, it is not immediately clear how such a licensing contract will affect the entrant’s optimal quality for its core technology as compared with the case when the entrant self-develops the non-core technology for its product. We will identify the conditions under which the inter-competitor licensing contract will increase the entrant’s optimal product quality and the conditions under which it will decrease the entrant’s optimal product quality.

To address the aforementioned research questions, we analyze a model where an incumbent has already developed some non-core technology that is used for the non-core attributes of the final product while an entrant has a new core technology that enables it to introduce a new higher-quality product (e.g., having a new feature or core attribute) to compete with the incumbent in a market where consumers have heterogeneous willingness to pay for product quality. To develop and produce the new product, the entrant needs the undifferentiated non-core technology, which it can either incur a fixed R&D cost to self-develop or license from the incumbent by paying a per-unit royalty fee. To ensure analytical closed-form solutions, our main model assumes that the entrant can choose between two quality levels for its core technology, representing qualitatively the high strategy and the low strategy in real-world situations. Both firms will simultaneously set prices to compete for customers in the market. We show later that our main findings remain qualitatively the same even when the incumbent optimally decides its royalty fee after the entrant’s product quality decision, or when quality is a continuous decision variable and the incumbent can also endogenously decide its product quality in anticipation of the entrant’s entry, or when the licensing contract is a two-part tariff contract.

Our analysis shows several main findings. First, a royalty licensing contract leads to higher equilibrium prices for both firms. Price competition is softened because the royalty fee paid by the
entrant (1) directly increases its own marginal cost and induces its price increase, and (2) reduces the incumbent’s incentive to compete aggressively on price since it will collect a royalty for every sale made by the entrant. The extant literature has shown that price competition can be alleviated if a firm sources its product components from an efficient competitor in a horizontally differentiated market. Our research shows that the competition-alleviation effect of inter-competitor contracts will persist even in a quality-differentiated market, and even when product quality and royalty fee are endogenously determined.

Second, how licensing of the non-core technology affects the entrant’s optimal quality depends on whether its core technology can significantly or only incrementally improve its product quality over the incumbent’s product. If the entrant can significantly improve its quality over the incumbent’s (i.e., at least one of the entrant’s quality levels is significantly higher than the incumbent’s quality), an inter-competitor licensing contract on the undifferentiated non-core technology tends to increase the entrant’s optimal product quality. That is, the licensing contract increases the parameter region in which choosing the high strategy is optimal for the entrant. The intuition is as follows. When the entrant’s quality levels are much higher than that of the incumbent, if the entrant’s marginal cost of producing the high-quality product is in a middle range, the entrant will find it optimal to choose the low strategy in the absence of any licensing contract for the non-core technology. Under a licensing contract, even when the entrant also chooses the low strategy, the alleviated price competition will benefit the entrant by increasing its net profit margin, but the entrant will benefit even more from the licensing contract if it instead chooses the high strategy, due to the larger differentiation and its associated higher net profit margin.

In contrast, if the entrant’s core technology can only incrementally or marginally improve the entrant’s quality over the incumbent’s (i.e., with no quality levels significantly higher than the incumbent’s quality), an inter-competitor licensing contract on the undifferentiated non-core technology
tends to decrease the entrant’s optimal product quality. That is, the licensing contract increases the parameter region in which choosing the low strategy is optimal for the entrant. The intuition is as follows. If the entrant’s marginal cost of producing the high-quality product is in the middle range, in the absence of any licensing contract the entrant will optimally choose the high strategy even though producing the high-quality product is fairly inefficient in terms of cost, because if the entrant chooses the low strategy (which is more cost efficient), the differentiation of the entrant’s product from the incumbent’s product will be so incremental that intense price competition will ensue. However, when the entrant licenses the non-core technology from the incumbent, price competition will be alleviated and the entrant will not have to worry much about intense price competition when it chooses the more cost-efficient low strategy. That is, the inter-competitor licensing of the non-core technology can expand the parameter region in which choosing the low strategy is optimal for the entrant. So, the royalty licensing contract tends to make it more likely the entrant will produce a lower quality level than without licensing when the entrant’s core technology can only incrementally improve its product quality over the incumbent’s product.

Third, we find a wide range of mutually acceptable licensing contracts such that, the incumbent may offer a royalty licensing contract even when the entrant pays a total royalty fee lower than its alternative R&D cost, and the entrant may also accept a royalty licensing contract even when it pays a total royalty fee higher than its alternative R&D cost. The incumbent’s benefit from the licensing contract comes from the total licensing fee it collects from the entrant and its higher profit margin due to alleviated price competition. Hence, any contract with a positive royalty that is acceptable to the entrant will make the incumbent strictly better off than no contract. For the entrant, even when it pays a total royalty fee higher than its alternative R&D cost, it may still strictly prefer accepting the contract because of the higher net profit margin it gets due to the alleviated price competition. This result as well as the effects of licensing on the entrant’s optimal quality decision still hold when
the incumbent endogenously decides its royalty fee.

This paper is related to three streams of literature. First, our research complements the literature that studies firms’ make-or-buy decisions, including a monopoly firm’s outsourcing problem (e.g., Elmaghraby 2000), outsourcing in competitive settings with a common supplier (e.g., Cachon and Harker 2002; Shy and Stenbacka 2003; Buehler and Haucap 2006; Arya, Mittendorf, and Sappington 2008a; Arya, Mittendorf, and Sappington 2008b; Chen, Dubey, and Sen 2011; Liu and Tyagi 2011; Feng and Lu 2012; Wu and Zhang 2014), outsourcing between two competing firms (e.g., Spiegel 1993; Baake, Oechssler, and Schenk 1999; Sappington 2005; Gayle and Weisman 2007; Cai and Chen 2011), and sharing of inventory between competitors in markets with demand uncertainty (e.g., Guo and Jiang 2018). This literature of component-sourcing has focused on the firms’ make-or-buy decisions of physical components of their products, where both make-or-buy alternatives entail different levels of marginal costs of production. In contrast, we study the licensing of an incumbent’s non-core technology to a competing entrant, which can either accept the incumbent’s licensing contract or incur a fixed cost to self-develop it. More importantly, we examine how the inter-competitor licensing of the non-core technology affects the entrant’s optimal product quality in addition to market competition.

Second, our paper contributes to the broad literature on firms’ licensing decision, which includes an outside patent holder licensing its patent to other firms competing in the final market (e.g., Katz and Shapiro 1986; Kamien, Oren, and Tauman 1992; Muto 1993; Saracho 2001; Moldovanu and Sela 2003; Hernadez-Murillo and Llobet 2006), and the patent holder being one of the established firms in the industry (e.g., Gallini 1984; Gallini and Winter 1985; Katz and Shapiro 1985; Wang 1998; Wang and Yang 1999; Rockett 1990; Wang 2002; Fauli-Oller and Sandonis 2002; Kitagawa, Masuda, and Umezawa 2014). Among works in this stream of literature, the following papers are most related to our research. Gallini (1984) considers a single potential entrant and shows that
licensing may be used as a means of entry deterrence when the entrant may either buy a license or do costly research. Gallini and Winter (1985) and Katz and Shapiro (1985) examine a producer’s incentives to license a cost-reducing process innovation to a competing producer. Rockett (1990) examines licensing as a means of choosing the competitors and focuses on the equilibrium in which the weak firm is encouraged to enter as a licensee for the sole purpose of crowding the market in order to deter entry by the strong firm. Kitagawa, Masuda, and Umezawa (2014) and Kamien, Oren, and Tauman (1992) study an incumbent’s licensing of a new technology to its potential competitor, with whom it engages in Cournot competition; they show that the optimal two-part tariff licensing contract can involve a positive royalty rate. This broad stream of licensing literature has mainly focused on licensing of cost-reduction technology, that is, process innovation. In contrast, we study the licensing of an incumbent’s non-core technology, which provides an undifferentiated attribute for the final product, and the entrant can strategically choose the quality level of its core technology to differentiate with the incumbent. We expand this stream of literature to explore the effect of licensing of non-core technology on an entrant’s optimal quality in addition to the effect on price competition.

Third, we contribute to the stream of literature on firms’ quality innovation, especially in a competitive context. For example, Moorthy (1998) shows that duopoly firms should differentiate their products from each other, with the higher-quality firm choosing a higher margin. Aghion et al. (2005) show an inverted-U relationship between competition and innovation and empirically test it. Bloom, Draca, and Reenen (2015) show empirical evidence that competition from Chinese imports led to increases in R&D. Ofek and Turut (2008) study how an entrant’s innovation-imitation decision depends on the incumbent’s R&D level. This literature mainly focuses on how competition affects firms’ product innovation. There is also a stream of literature that studies how various factors (e.g., consumer regret in Jiang, Narasimhan, and Turut 2017, or consumer-to-consumer product sharing in
Jiang and Tian 2018) affect a firm’s optimal quality. We complement these streams of literature by studying how inter-competitor licensing of undifferentiated non-core technology affects an entrant’s product quality decision and the price competition in the market.

MODEL

An incumbent (firm $i$) sells product $i$ of quality $q_i$ in the market. The incumbent has already developed some non-core technology that is used for the non-core attribute of the final product. An entrant (firm $e$) has some new core technology that enables it to introduce a new higher-quality product, which may, for example, have a new feature or core attribute. Let $q_e$ denote the quality of the entrant’s product. To develop and produce its new product, the entrant needs the non-core technology for its non-core attribute. We assume that the non-core technology is not a differentiator between the two firms, so we normalize the quality of that non-core technology to zero for both firms. The entrant can either incur an R&D cost of $F$ to self-develop the non-core technology or license it from the incumbent by paying a royalty of $r$ per unit of the product sold.

The entrant can choose the quality level for its product’s core technology. Since we have normalized the quality of the non-core technology to zero, we will hereafter refer to the quality of the core technology as the quality of the product. To examine how licensing of the undifferentiated non-core technology influences the entrant’s optimal quality decision, let us consider the stylized case where the entrant can choose between two quality levels $q_e \in \{q_e^H, q_e^L\}$, both of which are higher than the incumbent’s quality, i.e., $q_e^H > q_e^L > q_i$, with the associated marginal cost such that $c_e^H > c_e^L > c_i$. These two quality choices qualitatively correspond to the high strategy and the low strategy in real-world settings. In the Robustness Check and Discussion section, we will consider the entrant’s two quality choices such that one is higher than the incumbent’s and the other is lower than the incumbent’s, and discuss the generalizability of our results. The consumer’s utility from product
$j$ is given by $u_j = \theta q_j - p_j$, where $j = \{i, e\}$, $p_j$ is the price of product $j$, and $\theta$ represents the consumer’s willingness to pay for quality. Consumers are heterogeneous in $\theta$, which is assumed to be uniformly distributed over $[0, 1]$ in the population: $\theta \sim U[0, 1]$. Without loss of generality, we normalize the total number of consumers to one. Each consumer will purchase at most one product. The consumer’s outside option has a utility of zero. Hence, the consumer will buy the product that gives her the most nonnegative utility.

In practice, typically a firm has to decide both non-core technology and core technology to develop prototype for testing before its new product can be put to market. In our context, the full product development involves decision on non-core technology (license or self-development) and decision on core technology (quality choice). Since the decision of licensing versus self-development is less flexible as compared with the quality decision for the core technology, the entrant decides its non-core technology first, followed by its decision on core technology quality. Accordingly, the game proceeds as follows. First, the incumbent decides the royalty licensing fee ($r$ per unit) for the non-core technology. Second, the entrant decides whether to accept or reject the royalty licensing contract. If it accepts, the entrant will pay a royalty fee of $r$ for each unit of the product it sells. If it rejects, it can incur a fixed cost $F$ to develop its own non-core technology. Third, the entrant chooses the quality $q_e \in \{q_e^H, q_e^L\}$ for its core technology and develops its final product. Fourth, the incumbent and the entrant simultaneously set their prices $p_i$ and $p_e$, respectively. Last, consumers make purchase decisions and firms’ profits are realized. We solve the game using backward induction.

**Analysis**

**Benchmark with No Licensing.** We first analyze the sub-game where the entrant rejects the incumbent’s licensing contract with a royalty fee $r$. This will effectively serve as the benchmark case with no licensing contract between the incumbent and the entrant. In this case, the entrant
can invest a fixed cost $F$ to self-develop the non-core technology for its product. Given $p_i$, $q_i$, $p_e$ and $q_e$, the consumer who is indifferent between buying from the entrant and buying from the incumbent is characterized by $\theta = \frac{p_e - p_i}{q_e - q_i}$ and the consumer who is indifferent between buying from the incumbent and not buying any product is characterized by $\theta = \frac{q_e}{q_i}$. Hence, the two firms’ demand functions are given by $D_i = \frac{p_i - p_e}{q_e - q_i} - \frac{p_e}{q_i}$ and $D_e = 1 - \frac{p_e - p_i}{q_e - q_i}$, and their profit functions are $\pi_i = (p_i - c_i)D_i$ and $\pi_e = (p_e - c_e)D_e - F$, respectively. Simultaneously solving the first-order conditions $\frac{d\pi_i}{dp_i} = 0$ and $\frac{d\pi_e}{dp_e} = 0$, we obtain the optimal prices $p^*_i = \frac{q_i(q_e - q_i + c_e)}{4q_i - q_e}$ and $p^*_e = \frac{q_e(2q_e - 2q_i + 2c_e + c_i)}{4q_i - q_e}$ with the corresponding profits $\pi^*_i(q_e, c_e) = \frac{q_i^2(q_e - q_i + c_e)^2}{4q_i - q_e}$ and $\pi^*_e(q_e, c_e) = \frac{(q_e(2q_e - 2q_i + c_e) - (2q_e - q_i)c_e)^2}{(4q_i - q_e)^2}$ − $F$, respectively.\(^3\)

Thus, the entrant’s profit under the high strategy (i.e., $q^H_e$) will be

$$\pi^*_e = \pi^*_e(q_e, c_e)|_{q_e=q^H_e, c_e=c^L_i} = \frac{q^H_e(2q^H_e - 2q_i + c_e) - (2q^H_e - q_i)c^L_i}{(q^H_e - q_i)(4q^H_e - q_i)^2} - F;$$

its profit under the low strategy (i.e., $q^L_e$) will be $\pi^L_e = \pi^*_e(q_e, c_e)|_{q_e=q^L_e, c_e=c^L_i} = \frac{q^L_e(2q^L_e - 2q_i + c_e) - (2q^L_e - q_i)c^L_i}{(q^L_e - q_i)(4q^L_e - q_i)^2} - F$. By comparing $\pi^*_e$ with $\pi^L_e$, one can easily show that the entrant will find the high strategy optimal when $c^H_e \leq c_F$ and the low strategy optimal otherwise, where $c_F \equiv \frac{q^H_e(2q^H_e - 2q_i + c_e) - R(2q^H_e - 2q_i + c_e) - (2q^H_e - q_i)c^L_i}{2q^H_e - q_i}$ and $R \equiv \frac{4q^H_e - q_i}{4q^L_e - q_i} \sqrt{\frac{q^H_e - q_i}{q^L_e - q_i}}$. We will later use this no-licensing benchmark to determine whether the firms will prefer entering into a licensing contract.

Next, we will analyze the firms’ optimal prices, followed by the entrant’s optimal quality of the sub-game in which the entrant accepts the incumbent’s royalty licensing contract.

**Pricing Decisions**

Suppose that the entrant accepts the royalty licensing contract with a royalty fee $r > 0$, i.e., for each unit of the product it sells, it will pay a fee of $r$ to the incumbent.\(^4\) We will add a subscript $r$ for the variables to indicate the case of royalty licensing. Given $p_{ir}$, $q_i$, $p_{er}$ and $q_{er}$, the two firms’ demand functions are $D_{ir} = \frac{p_{ir} - p_e}{q_e - q_i} - \frac{p_e}{q_i}$ and $D_{er} = 1 - \frac{p_{er} - p_e}{q_e - q_i}$, respectively, the same as in the benchmark
The firms’ profits are computed by $\pi_{ir} = (p_{ir} - c_i)D_{ir} + rD_{er}$ and $\pi_{er} = (p_{er} - c_{er})D_{er} - rD_{er}$, respectively, where $rD_{er}$ represents the total licensing fee that the entrant pays the incumbent.

With backward induction, we first analyze the firms’ pricing decisions under the royalty licensing contract. Simultaneously solving the first-order conditions $\frac{d\pi_{ir}}{dp_{ir}} = 0$ and $\frac{d\pi_{er}}{dp_{er}} = 0$ yields the firms’ optimal prices $p^*_ir = \frac{q_i(q_{er}-q_i+c_{er}+3r)+2q_{er}c_i}{4q_{er}-q_i}$ and $p^*_er = \frac{q_{er}[2(q_{er}-q_i+c_{er}+r)+c_{er}]+q_ir}{4q_{er}-q_i}$. And their corresponding profits are

$$\pi^*_ir(r, q_{er}, c_{er}) = \frac{q_{er}[q_i(q_{er}-q_i+c_{er})-c_{er}](2q_{er}-q_i)^2+q_i(q_{er}-q_i)[q_i^2+8q_{er}(q_{er}-c_{er})-c_{er}q_i]r-(8q_{er}+q_i)(q_{er}-q_i)q_ir^2}{q_i(4q_{er}-q_i)^2}$$

and

$$\pi^*_er(r, q_{er}, c_{er}) = \frac{[q_{er}(2q_{er}-2q_i+c_i)-c_{er}(2q_{er}-q_i)-r(2q_{er}-2q_i)]^2}{(q_{er}-q_i)(4q_{er}-q_i)^2}$$

for given $r$.

Comparing each firm’s price under the licensing contract with its price in the no-licensing benchmark, we obtain $p^*_ir > p^*_i$ and $p^*_er > p^*_e$ for any given $r > 0$ that is mutually acceptable. Proposition 1 shows this result. The proofs for all propositions and lemma are provided in the Appendix.

**PROPOSITION 1:** Given the entrant’s product quality, any mutually acceptable royalty licensing contract between the two firms will result in higher equilibrium prices in the market.

Compared with the no-licensing case, under a licensing contract, the royalty fee paid to the incumbent directly increases the entrant’s marginal cost, inducing it to raise its price. The entrant’s and the incumbent’s prices are strategic complement, i.e., the increase in the entrant’s price also tends to induce the incumbent to raise its price. Furthermore, with the licensing contract, the incumbent can collect a royalty $r$ for each sale the entrant makes, which in essence raises the incumbent’s opportunity cost for aggressive price competition. Thus, altogether, the inter-competitor licensing contract of the non-core technology will have a *competition-alleviation* effect. This result is consistent with the finding from the component-sourcing or make-or-buy literature (e.g., Cachon and Harker 2002; Buehler and Haucap 2006) that shows that sourcing of product components between competitors (i.e., horizontal subcontracting) can lead to higher prices in the market. In that literature, a firm can buy components for its product from a competitor or manufacture the com-
ponent itself at some different marginal costs. In our technology licensing context, an entrant can either incur a fixed R&D cost to self-develop the non-core technology for its product, or pay royalty fees (a marginal cost) to an incumbent under the licensing contract. We complement the extant literature by showing that in a quality-differentiated market, the inter-competitor royalty licensing of the non-core technology can also soften price competition, even when product quality and royalty fee are endogenously determined.

**Entrant’s Quality Decision**

To obtain the entrant’s optimal quality strategy, we need to determine which quality-cost pair yields more profit for the entrant. Let $\pi^H_{er}(r)$ denote the entrant’s profit $\pi^*_er(r,q^H_{er},c^H_{er})$ evaluated at $q^H_{er}$ and $c^H_{er}$, and $\pi^L_{er}(r)$ denote $\pi^*_er(r,q^L_{er},c^L_{er})$ evaluated at $q^L_{er}$ and $c^L_{er}$. That is, $\pi^H_{er}(r) \equiv \pi^*_er(r,q^H_{er},c^H_{er})$ and $\pi^L_{er}(r) \equiv \pi^*_er(r,q^L_{er},c^L_{er})$. Comparing $\pi^H_{er}(r)$ with $\pi^L_{er}(r)$, we find that the entrant’s optimal choice is the high strategy $q^H_{er}$ when $c^H_{er} \leq c^F$ where

$$c^F = c^H + R(2q^L-2q_i) - (2q^H-2q_i)q_i$$

and its optimal choice is the low strategy $q^L_{er}$ otherwise. Recall that we have shown earlier that without licensing the entrant will find the high strategy optimal if $c^H_{er} \leq c^F$ and the low strategy optimal otherwise. Comparison of the two thresholds $c_r$ and $c^F$, which are both functions of $q^H_{er}, q^L_{er}, q_i, c_i$, will allow us to determine how the licensing contract affects the entrant’s optimal quality.

**Lemma 1:** If $(\frac{q^H}{q_i} - 1)(\frac{q^L}{q_i} - 1) > \frac{9}{16}$, $c_r > c^F$ for any $r > 0$; if $(\frac{q^H}{q_i} - 1)(\frac{q^L}{q_i} - 1) < \frac{9}{16}$, $c_r < c^F$ for any $r > 0$.

Lemma 1 implies that when $(\frac{q^H}{q_i} - 1)(\frac{q^L}{q_i} - 1) > \frac{9}{16}$, the entrant is more likely to choose the high strategy under royalty licensing than under no licensing. And when $(\frac{q^H}{q_i} - 1)(\frac{q^L}{q_i} - 1) < \frac{9}{16}$, the entrant is more likely to choose the low strategy under royalty licensing than under no licensing.
As discussed before, the royalty licensing contract will alleviate price competition, inducing both firms to raise their prices; as a result, licensing has two potential effects on the entrant – the positive effect of increasing the entrant’s net profit margin and the negative effect of reducing its unit sales. In general, given a quality strategy, the higher the royalty fee \( r > 0 \), the lower the entrant’s profit. However, the royalty fee affects the high strategy and the low strategy differently, depending on which of the two conditions in Lemma 1 holds. In particular, when \( \left( \frac{q^H}{q_i} - 1 \right) \left( \frac{q^L}{q_i} - 1 \right) > \frac{9}{16} \), royalty fee \( r \) has a weaker negative effect on the entrant under the high strategy than under the low strategy.\(^7\) Hence, if \( \left( \frac{q^H}{q_i} - 1 \right) \left( \frac{q^L}{q_i} - 1 \right) > \frac{9}{16} \), a royalty licensing contract tends to increase the parameter region in which the entrant prefers the high strategy to the low strategy. In contrast, when \( \left( \frac{q^H}{q_i} - 1 \right) \left( \frac{q^L}{q_i} - 1 \right) < \frac{9}{16} \), royalty fee has a stronger negative effect on the entrant under the high strategy than under the low strategy.\(^8\) Hence, if \( \left( \frac{q^H}{q_i} - 1 \right) \left( \frac{q^L}{q_i} - 1 \right) < \frac{9}{16} \), a royalty licensing contract tends to reduce the parameter region in which the entrant prefers the high strategy to the low strategy.

In the ensuing analysis, we refer to the condition \( \left( \frac{q^H}{q_i} - 1 \right) \left( \frac{q^L}{q_i} - 1 \right) > \frac{9}{16} \) as “the case of highly differentiated core technology by the entrant”, which means that the entrant’s core technology can significantly improve its product quality over the incumbent’s product and at least one of the entrant’s quality levels is significantly higher than the incumbent’s product. We refer to the condition \( \left( \frac{q^H}{q_i} - 1 \right) \left( \frac{q^L}{q_i} - 1 \right) < \frac{9}{16} \) as “the case of incrementally differentiated core technology by the entrant”, which means that the entrant’s core technology can only incrementally increase its quality over the incumbent’s product. Next, we show that the inter-competitor licensing of the non-core technology can have qualitatively different effects on the entrant’s optimal product quality, depending on how significantly the entrant’s core technology can improve its quality over the incumbent’s product and how efficient the entrant can produce its different quality levels.

The Case of Highly Differentiated Core Technology by the Entrant
Clearly, anticipating its optimal quality decision, the entrant will accept the licensing contract only if it makes a (weakly) higher profit accepting than rejecting the contract. In the Appendix, we show that the entrant prefers accepting to rejecting the royalty contract only if \( r \leq \tau \), where \( \tau \) is given by (1) below and the exact definitions for the constants in \( \tau \) are provided in the proof in the Appendix.

\[
\tau = \begin{cases} 
\tau^{HH}, & \text{if } c_e^L < c_e^H \leq c_F; \\
\tau^{LH}, & \text{if } c_F < c_e^H \leq c_r; \\
\tau^{LL}, & \text{if } c_e^H > c_r.
\end{cases}
\]

(1)

To establish the range of royalty \( r \) that is acceptable to the incumbent, we need to ensure that the incumbent makes at least as much profit with the royalty contract as without it. If \( r \) is zero, the incumbent will make the same level of profit as in the no-licensing case, since zero royalty has no direct effect on the firms’ optimal pricing decisions for their respective products. If \( r \) is too high, the entrant will reject the contract, in which case the incumbent will also make the same profit as in the no-licensing case. One can easily show that, for any contract with \( r > 0 \) that is acceptable to the entrant, the incumbent will make a strictly higher profit than without the licensing contract, because it collects royalty payments and will also enjoy a higher profit margin because of alleviated price competition. Putting these all together, we can formally prove Proposition 2.

**Proposition 2:** When the entrant can significantly improve its product quality over the incumbent’s product, a mutually acceptable royalty licensing contract on the non-core technology has \( 0 < r \leq \tau \), will induce the entrant to choose higher quality if \( c_F < c_e^H \leq c_r \), and will have no effect on the entrant’s optimal quality otherwise. Moreover, \( \frac{dc}{dr} > 0 \) for \( r \in (0, \tau^{LL}) \).

Insert Figure 1 about here

Proposition 2 is further illustrated by Figure 1. In Figure 1, the vertical axis is \( c_e^H \), which starts
from the value \( c_e^L \); the horizontal axis is the royalty fee \( r \). Note that in the right-hand-side unshaded regions, the royalty fee \( r \) is too high to be acceptable to the entrant, so the market outcome is the same as the no-licensing case, in which the entrant will choose the high strategy if \( c_e^H \leq c_F \) and the low strategy if \( c_e^H > c_F \). The three shaded parameter regions ensure the licensing contract to be mutually acceptable to both firms. The first letter of each region label specifies the entrant’s optimal quality strategy in that parameter region in the absence of any licensing contract; the second letter of the region label specifies the entrant’s optimal quality strategy in the presence of the licensing contract \( r \). In the \( HH \) region (region 1 in Figure 1), the entrant’s marginal cost of producing the high-quality product is very low (i.e., \( c_e^H \leq c_F \)), the entrant will choose the high strategy \( (q_e^H) \) regardless of whether there is a licensing contract on the non-core technology between the two firms. This is intuitive. A very low \( c_e^H \) (i.e., \( c_e^H \) is not much higher than \( c_e^L \)) implies that the high-quality product is very cost-efficient to produce relative to the low-quality product; so, the entrant will find the high strategy optimal with or without the licensing contract. Similarly, in region 3 (i.e., \( LL \)), where \( c_e^H \) is very high (i.e., \( c_e^H > c_r \)), the high quality \( q_e^H \) is so cost-inefficient that the entrant will find the low strategy optimal regardless of whether it has a licensing contract with the incumbent.

However, if the entrant’s marginal cost of producing the high-quality product is in the middle range (i.e., \( c_F < c_e^H \leq c_r \) as in region 2), licensing of the non-core technology will increase the entrant’s optimal quality. The intuition is as follows. In this \( LH \) region, without licensing, the entrant will choose the low strategy because it is too cost-inefficient to produce the high-quality product. With a licensing contract, if the entrant also chooses the low strategy, then the entrant will benefit from the licensing contract due to the positive effect of licensing on increasing its net profit margin (i.e., \( p_e^{L_{er}} - c_e^L - r > p_e^{L_{er}} - c_e^L - \frac{F}{D_T} \)) even though unit sales will decrease. However, if the entrant chooses the high strategy (with much higher quality than \( q_e \)) instead, then it will benefit even more from licensing, as implied by Lemma 1. Hence, in this parameter region the entrant should
trade the relative cost efficiency of producing the low-quality product for the larger differentiation and its associated higher net profit margin afforded by choosing the high strategy.

In addition, we show that the threshold $c_r$ increases in $r$, which implies that as the royalty fee increases, the parameter region of $c_e^H$ (region 2 in Figure 1) in which licensing induces higher quality by the entrant will also increase. The intuition is as follows. Under a licensing contract, the entrant’s profit decreases with the increase of $r$. As discussed after Lemma 1, this negative effect is stronger for the low strategy than for the high strategy when the entrant can significantly increase its product quality over the incumbent’s product. A higher royalty fee will contribute to the stronger negative effect and make the low strategy even more unfavorable. In other words, a higher royalty fee will make the high strategy more favorable for the entrant.

The Case of Incrementally Differentiated Core Technology by the Entrant

We show that, with incrementally or marginally differentiated core technology, the entrant will prefer accepting to rejecting the royalty contract only if $r \leq \bar{r}$, where $\bar{r}$ is given by (2) and the exact expressions for the constants in $\bar{r}$ are provided in the proof in the Appendix.

\[
\bar{r} = \begin{cases} 
\bar{r}^{HH}, & \text{if } c_e^L < c_e^H \leq c_r; \\
\bar{r}^{HL}, & \text{if } c_r < c_e^H \leq c_F; \\
\bar{r}^{LL}, & \text{if } c_e^H > c_F.
\end{cases}
\] (2)

As explained before, any royalty contract with $r > 0$ that is acceptable to the entrant will give the incumbent strictly higher profits than under no licensing. Proposition 3 summarizes this result and shows the effect of licensing of the non-core technology on the entrant’s optimal quality.

Proposition 3: When the entrant can only incrementally improve its product quality over the inc-
cumbent’s product, a mutually acceptable royalty licensing contract on the non-core technology has 

\[ 0 < r \leq r^*, \] 

will induce the entrant to choose lower quality if \( c_r < c_r^H \leq c_F \), and will have no effect on the entrant’s optimal quality otherwise. Moreover, \( \frac{dc}{dr} < 0 \) for \( r \in (0, r^{HL}) \).

Figure 2 illustrates how royalty licensing of the non-core technology between the two firms will affect the entrant’s optimal quality decision for its core technology when the entrant can only incrementally improve its product quality over the incumbent’s. Again, the first letter of each region label specifies the entrant’s optimal quality strategy when it has no licensing contract with the incumbent, and the second letter indicates the entrant’s optimal quality strategy under a mutually acceptable royalty licensing contract. In the \( HH \) region (i.e., region 1), the entrant’s marginal cost of producing the high-quality product, \( c_e^H \), is low enough such that the entrant will find the high strategy \( (q_e^H) \) optimal regardless of whether it has any royalty licensing contract with the incumbent. In the other extreme, in the \( LL \) region (i.e., region 3), \( c_e^H \) is so high that the entrant will find the low strategy \( (q_e^L) \) optimal regardless of the existence of any royalty contract.

Our analysis reveals that for incrementally differentiated core technology by the entrant, there exists an \( HL \) parameter region where the entrant’s marginal cost of producing the high-quality product is in the middle range (i.e., \( c_r < c_r^H \leq c_F \) as in region 2 in Figure 2), licensing of the non-core technology will reduce the entrant’s optimal quality for its core technology. The intuition is as follows. In this \( HL \) region, without licensing, the entrant will optimally choose the high strategy, even though producing the high-quality product is becoming less cost-efficient. This is because for incrementally differentiated core technology, the entrant’s quality choices are not much higher than the incumbent’s quality. So, the entrant has a strong need to choose the higher of the quality choices to differentiate from the incumbent, otherwise more severe price competition will ensue. That is, even though \( c_e^H \) is a little high (i.e., somewhat inefficient to produce the high-quality product), the entrant finds it better to give up some production efficiency to try to differentiate its product from
the incumbent’s. By contrast, when there is a royalty licensing contract, because of alleviated price competition between the two firms, the entrant can now avoid the cost-inefficient high strategy and choose the cost-efficient low strategy instead, because now it does not have to worry about the low strategy’s lower level of product differentiation with the incumbent’s product. In other words, for incrementally differentiated core technology, royalty licensing can sometimes allow the entrant to choose very efficient low strategy that it otherwise would not want to choose because of the low level of product differentiation with the incumbent.

Note that the above intuition is consistent with the explanation of Proposition 2. In particular, in this $HL$ parameter region, if the entrant also chooses the high strategy as it does without licensing, its net profit margin will increase and the entrant can benefit from licensing even though its unit sales will decrease. However, if the entrant chooses the low strategy instead, then it will benefit even more from licensing, as implied by Lemma 1. Hence, in this parameter region the entrant should trade the slightly larger differentiation and its associated net profit margin of choosing the high-quality product for the efficiency of producing the low-quality product.

In addition, we show that the threshold $c_r$ decreases in $r$, which implies that as the royalty fee increases, the parameter region of $c_r^H$ (region 2 in Figure 2) in which licensing induces lower quality by the entrant will also increase. The intuition is as follows. Under a licensing contract, the entrant’s profit decreases with the increase of $r$. As discussed after Lemma 1, this negative effect is stronger for the high strategy than for the low strategy when the entrant can only incrementally increase its product quality over the incumbent’s product. A higher royalty fee will contribute to the stronger negative effect and make the high strategy even more unfavorable. In other words, a higher royalty fee will make the low strategy more favorable for the entrant.

So far we have shown a wide range of parameter values and royalty fees (the three shaded parameter regions illustrated in Figure 1 and Figure 2) for which a royalty licensing contract between
the incumbent and the entrant is mutually beneficial compared with the case of no licensing contract. In practice, the royalty fee agreed upon in licensing contracts may fall anywhere in the range of mutually acceptable fees, depending on the relative bargaining power between the two firms. One might intuit that if the total royalty fee (the per-unit royalty $r$ times the entrant’s unit sales) is much lower than the entrant’s alternative R&D cost, the incumbent might not want to license its non-core technology to the entrant, since intuitively such a contract would help the entrant save costs and perhaps become more competitive. Similarly, it may also be intuitively obvious that the entrant will prefer accepting to rejecting the licensing contract only if the total royalty fee it will pay is lower than its alternative cost. Many managers may use an intuitive decision-making rule: Reject a contract or proposal if their alternative cost is lower. Our conversation with practitioners indicates that some companies also ex post evaluate the success of contracts or proposals based on the realized costs compared with the ex ante alternative costs. Proposition 4 shows that, in the licensing context we study, such intuitively sound decision rules may not be optimal if the strategic effects of royalty licensing are taken into account.

**Proposition 4:**  (a) The incumbent can benefit from the royalty licensing contract even when the entrant pays a total royalty fee that is lower than its alternative R&D cost.

(b) The entrant can benefit from the royalty licensing contract even when the entrant pays a total royalty fee that is higher than its alternative R&D cost.

Both firms can benefit from a royalty licensing contract in wide parameter regions (shaded in Figure 1 and Figure 2). The incumbent’s benefit from the royalty contract comes from two main sources – the total licensing fee it collects from the entrant, and its higher profit margin due to alleviated price competition. Without a licensing contract, even though its competitor – the entrant – may have to incur a higher cost, that cost is a fixed R&D cost and will have no effect on the
entrant’s optimal pricing and quality decisions, nor on the entrant’s price competitiveness. Any contract with a positive royalty that is acceptable to the entrant, even when the total fee paid is lower than the entrant’s alternative cost, will make the incumbent strictly better off than no contract.

For the entrant, clearly it will benefit from a royalty licensing contract if it pays a total royalty fee lower than its alternative R&D cost. Even when it pays a total royalty fee higher than its alternative R&D cost, it may still strictly prefer accepting the licensing contract. The intuition is as follows. For instance, when the licensing contract does not affect the entrant’s optimal quality decision, that is, in regions 1 and 3 in both Figure 1 and Figure 2, the entrant’s benefit from the contract is a result of its higher net profit margin due to alleviated price competition with only a relatively slight drop in unit sales. the entrant’s benefit from royalty licensing can more than offset the total fee that it pays, even when that fee is higher than the entrant’s alternative fixed cost needed to develop the non-core technology in-house.

**Incumbent’s Royalty Fee Decision**

Using backward induction, we can now determine the incumbent’s optimal royalty fee in anticipation of the entrant’s optimal quality and the pricing decisions. The two firms’ profit functions are given by

\[
\pi_{ir}(r) = q_{er}(q_{er} - q_i + c_{er}) - c_{er}((2q_{er} - q_i)^2 + q_{er}(q_{er} - q_i)(q_{er}^2 + 8q_{er}(q_{er} - c_{er}) - c_i q_i)r - 8(q_{er} + q_i)(q_{er} - q_i)q_i r^2)
\]

and

\[
\pi_{er}(r) = \frac{q_{er}(2q_{er} - 2q_i + c_i) - c_{er}((2q_{er} - q_i) - r(2q_{er} - 2q_i))^2}{(q_{er} - q_i)(4q_{er} - q_i)^2}
\]

respectively, where \(q_{er}, c_{er}\) = \(q^H, c^H\) if \(c^H \leq c_r\) and \(q_{er}, c_{er}\) = \(q^L, c^L\) otherwise. Hence, the incumbent maximizes \(\pi_{ir}(r)\) over \(r\) subject to the constraint \(0 < r \leq r\) to obtain its optimal royalty fee. Proposition 5 and Proposition 6 show that the results in Propositions 2, 3, and 4 will still hold when the incumbent endogenously sets its royalty fee. The constants \(r^{LH*}, r^{LL*}, r^{HH*}, r^{HL*}, c_r,\) and \(c^L\) are provided in the Appendix.

**Proposition 5:** (a) When the entrant can significantly improve its quality over the incumbent’s product, if \(c_F < c^H \leq c_r\) and \(\pi_{ir}(r^{LH*}) \geq \pi_{ir}(r^{LL*})\), then the incumbent’s optimal royalty fee is
and this licensing contract will induce the entrant to choose higher quality, i.e., \( q^{*}_{er} > q^{*}_{e} \).

(b) When the entrant can only incrementally improve its quality over the incumbent’s product, if \( c_{r} < c^{H}_{e} \leq c_{F} \) and \( \pi^{L}_{ir}(r^{HL*}) \geq \pi^{H}_{ir}(r^{HH*}) \), then the incumbent’s optimal royalty fee is \( r^{HL*} \) and this licensing contract will induce the entrant to choose lower quality, i.e., \( q^{*}_{er} < q^{*}_{e} \).

According to Proposition 2, when the entrant can significantly improve its quality over the incumbent’s product, royalty licensing of the non-core technology can increase the entrant’s optimal quality for some range of \( r \). Proposition 5(a) shows that the incumbent’s optimal royalty fee, as illustrated in Figure 3(a), can fall in this range such that the quality-increasing effect of royalty licensing occurs in equilibrium of the full game. Below is an example in which the entrant can significantly improve its quality over the incumbent’s product and royalty licensing increases the entrant’s optimal quality in full equilibrium.

\textit{Illustrative Example}: \( (q_{i}, c_{i}) = (1, 0.5), (q^{L}_{e}, c^{L}_{e}) = (1.5, 0.8), (q^{H}_{e}, c^{H}_{e}) = (4, 3.04), F = 0.03 \).

Under no licensing contract, in equilibrium, \( q^{*}_{e} = q^{L}_{e}, p^{*}_{i} = 0.56, p^{*}_{e} = 0.93, \pi^{L*}_{i} = 0.0108, \pi^{L*}_{e} = 0.0038 \). Under royalty licensing, in equilibrium, \( r^{*} = r^{LH*} = 0.473, p^{*}_{ir} = 0.764, p^{*}_{er} = 3.638, q^{*}_{er} = q^{H}_{e} > q^{*}_{e}, \pi^{H*}_{ir} = 0.071, \) and \( \pi^{H*}_{er} = 0.005 \).

According to Proposition 3, when the entrant can only incrementally improve its quality over the incumbent’s product, royalty licensing of the non-core technology can decrease the entrant’s optimal quality for some range of \( r \). Proposition 5(b) shows that the incumbent’s optimal royalty fee, as illustrated in Figure 3(b), can fall in this range such that the quality-decreasing effect of royalty licensing occurs in equilibrium of the full game. The example below illustrates the quality-decreasing effect of royalty licensing under the full equilibrium for the case with the entrant having a core technology that can only incrementally improve its quality over the incumbent's product.
Illustrative Example: \((q_i, c_i) = (1, 0.7), (q_i^L, c_i^L) = (1.1, 0.78), (q_i^H, c_i^H) = (1.5, 1.18), F = 0.0025\).

Under no licensing contract, in equilibrium, \(q_e^* = q_e^H, p_i^* = 0.756, p_e^* = 1.218, \pi_i^H^* = 0.009, \pi_e^H^* = 0.00038\). Under royalty licensing, in equilibrium, \(r^* = r^{HL^*} = 0.159, p_{ir}^* = 0.852, p_{er}^* = 0.945, q_e^* = q_e^L < q_e^*, \pi_{ir}^L^* = 0.023 > \pi_{ir}^H^* = 0.0147\) and \(\pi_{er}^L^* = 0.00042 > \pi_e^H^* = 0.00038\).

**Proposition 6:** When the incumbent endogenously decides its royalty fee, (a) the incumbent can benefit from the royalty licensing contract even when the total royalty fee paid by the entrant is lower than its alternative R&D cost; and (b) the entrant can benefit from the royalty licensing contract even when the total royalty fee it pays is higher than its alternative R&D cost.

Recall that Proposition 4 shows that a royalty licensing contract can be mutually beneficial even when the entrant pays a total royalty fee lower or higher than its alternative R&D cost needed to develop the non-core technology by itself. Proposition 6 shows that this result holds true even when the incumbent endogenously and strategically chooses its royalty fee \(r\). The examples below illustrate the result of the equilibrium total royalty fee being higher than the entrant’s alternative R&D cost. Note that it is also easy to give examples to illustrate the result that the equilibrium total royalty fee being lower than the R&D cost.

**Illustrative Example for the case of highly differentiated core technology by the entrant:**

\((q_i, c_i) = (1, 0.5), (q_e^L, c_e^L) = (1.5, 0.8), (q_e^H, c_e^H) = (4, 3.3), F = 0.03\).

In equilibrium, \(r^* = 0.342, r^{L^*}D_{er}^* = 0.042 > F, \pi_{ir}^L^* = 0.0717 > \pi_i^L^* = 0.0108, \text{and } \pi_{er}^L^* = 0.0076 > \pi_e^L^* = 0.0038\).

**Illustrative Example for the case of incrementally differentiated core technology by the entrant:**

\((q_i, c_i) = (1, 0.7), (q_e^L, c_e^L) = (1.1, 0.78), (q_e^H, c_e^H) = (1.5, 1.2), F = 0.0025\).

In equilibrium, \(r^{L^*} = 0.159, r^{L^*}D_{er}^* = 0.01 > F, \pi_{ir}^L^* = 0.023 > \pi_i^L^* = 0.0015, \text{and } \pi_{er}^L^* = 0.00042 > \pi_e^L^* = 0.00002\).
ROBUSTNESS CHECK AND DISCUSSION

In this section, we will show that our main results and intuitions are robust to some alternative model settings or assumptions. First, we allow the entrant’s quality choices to be such that one is higher than the incumbent’s product quality and the other is lower than the incumbent’s product quality. Second, we allow the incumbent to decide its royalty licensing fee after the entrant decides its product quality. Third, we allow the incumbent to endogenize its quality decision in anticipation of its competitor’s entry in the market. And fourth, we consider a two-part tariff licensing contract.

**Entrant’s quality choices are such that** $q_e^H > q_i > q_e^L$

Our main model assumes that the entrant’s product quality choices are both higher than the incumbent’s, that is, $q_e^H > q_e^L > q_i$. Suppose instead that the entrant’s core technology allows for one quality level higher than the incumbent’s product quality and the other lower than the incumbent’s product quality, that is, $q_e^H > q_i > q_e^L$ and $c_e^H > c_i > c_e^L$. Other aspects of the model setting are the same as our main model analyzed before. We now analyze the game with this alternative assumption to investigate the effect of licensing of the non-core technology on the entrant’s optimal quality. The analysis of the current model follows the same procedure as in our main model, so we relegate the technical details of the analysis to the Web Appendix.

Under no licensing, the entrant will in equilibrium choose the high strategy $q_e^* = q_e^H$ if and only if

$$c_e^H \leq \frac{q_e^H(2q_e^H - 2q_i + c_i) - R[q_e^L(q_i - q_e^L + c_i) - (2q_i - q_e^L)c_e^L]}{2q_e^H - q_i} \equiv c_F$$

where $R = \sqrt{\frac{q_i(q_e^H - q_i)(4q_e^H - q_i)}{q_e^L(q_i - q_e^L)(4q_i - q_e^L)}}$. In contrast, under a licensing contract with royalty fee $r$, the entrant will find the high strategy optimal if and only if

$$c_e^H \leq c_F + 2r \frac{R(q_i - q_e^L)(q_e^H - q_i)}{2q_e^H - q_i} \equiv c_r.$$  

Let $\alpha \equiv \frac{q_e^H}{q_i}$ and $\beta \equiv \frac{q_e^L}{q_i}$, where $0 < \beta < 1 < \alpha$. Comparing $c_r$ with $c_F$, we obtain the following finding. For any $r > 0$ that is mutually acceptable to both firms, if $\beta < \frac{2}{3}[4 - \frac{32}{(27\sqrt{73} - 143)^{3/3}} + (27\sqrt{73} - 143)^{1/3}] \approx 0.82658$, then $c_r > c_F$ and licensing...
will induce the entrant to choose a weakly higher quality than under no licensing. If \( \beta > 0.82658 \), then when \( \alpha > \frac{(4-\beta)^2}{64}\left[\sqrt{\frac{\beta}{1-\beta}} + \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}\right]^2 + 1 \) or \( \alpha < \frac{(4-\beta)^2}{64}\left[\sqrt{\frac{\beta}{1-\beta}} - \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}\right]^2 + 1 \), we have \( c_r > c_F \) and licensing will induce the entrant to choose weakly higher quality; whereas when \( \frac{(4-\beta)^2}{64}\left[\sqrt{\frac{\beta}{1-\beta}} - \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}\right]^2 + 1 < \alpha < \frac{(4-\beta)^2}{64}\left[\sqrt{\frac{\beta}{1-\beta}} + \sqrt{\frac{\beta}{1-\beta} - \frac{48}{(4-\beta)^2}}\right]^2 + 1 \), we have \( c_r < c_F \) and licensing leads to the entrant’s choosing of weakly lower quality than under no licensing. It is also straightforward to show that both the incumbent and the entrant will prefer the royalty contract to no contract if \( 0 < r \leq \bar{r} \), where \( \bar{r} \) is explicitly given in the Web Appendix. In summary, we find that when \( c_F < c_e^H \leq c_r \) and \( 0 < r \leq \bar{r} \), a mutually acceptable royalty contract between the two firms will increase the entrant’s optimal quality; when \( c_r < c_e^H < c_F \) and \( 0 < r \leq \bar{r} \), a mutually acceptable royalty contract between the two firms will decrease the entrant’s optimal quality.

The insight of these findings is also qualitatively similar to that from our main model where \( q_e^H > q_e^L > q_i \). Note that in the new model, when the low strategy \( (q_e^L) \) can provide enough differentiation from \( q_i \), that is, \( q_e^L < 0.82658q_i \), (analogous to \( q_e^L > \frac{7q_i}{4} \) in the main model), a licensing contract leads to the entrant’s weakly higher quality. By contrast, when the low strategy \( (q_e^L) \) does not provide enough differentiation from \( q_i \), that is, \( q_e^L > 0.82658q_i \), (analogous to \( q_e^L < \frac{7q_i}{4} \) in the main model), a licensing contract leads to the entrant’s weakly higher quality when the differentiation between the entrant’s high strategy and the incumbent’s quality is either sufficiently high or sufficiently low (analogously, in the main model, this differentiation has to be high enough); a licensing contract leads to the entrant’s weakly lower quality when the differentiation between the entrant’s high strategy and the incumbent’s quality is in the intermediate range (analogously, in the main model, this differentiation has to be low enough).

To summarize, we find that regardless of whether the entrant’s quality choices are both higher than the incumbent’s quality or one higher and the other lower than the incumbent’s quality, our main result remains the same – the royalty licensing contract of non-core technology between the
two firms can increase or decrease the entrant’s optimal quality for its core technology. We also find that these results continue to hold when the incumbent endogenously determines the royalty fee. Furthermore, we confirm the earlier result that the incumbent (entrant) can benefit from a licensing contract even when the total royalty fee the entrant pays is lower (higher) than its alternative R&D cost. As we did in the main model, for $q_H^e > q_i > q_L^e$ we also provide two numerical examples to demonstrate the results in Propositions 5-6. Example 1: $q_H^e = 1.2, q_i = 1, q_L^e = 0.8, c_H^e = 0.7, c_i = 0.5, c_L^e = 0.4, F = 0.003$. In this example, $r^* = 0.122, q_{er}^* = q_H^e > q_i^* = q_L^e, r^* \times D_{er}^* = 0.008 > F$.

Example 2: $q_H^e = 1.1, q_i = 1, q_L^e = 0.95, c_H^e = 0.616, c_i = 0.5, c_L^e = 0.48, F = 0.0007$. In this example, $r^* = 0.111, q_{er}^* = q_L^e < q_i^* = q_H^e, r^* \times D_{er}^* = 0.0056 > F$. Note that with some care, one can potentially generalize our model to a setting where the entrant has two sets of quality options, one set with quality options higher than the incumbent’s quality and the other set with quality options lower than the incumbent’s quality.

**Royalty fee decision after entrant’s quality decision**

In our main model the incumbent sets its licensing fee for its non-core technology before the entrant chooses its quality of its core technology, which is a reasonable assumption since the entrant may would need to have the non-core technology to start developing and testing its core technology and the decision of licensing versus self-development is less flexible as compared with the quality decision for the core technology. In this extension, nevertheless, we consider an alternative game sequence where the incumbent sets its licensing fee after the entrant chooses the quality of its core technology. Other aspects of the model setting are the same as our main model. To examine the effect of licensing on the entrant’s optimal quality decision, we compare the entrant’s optimal product quality in anticipation of an acceptable licensing contract with its optimal product quality when self-developing the non-core technology. The analysis for when the entrant self-develops the non-core technology.
technology is the same as the benchmark case in the main model, in which case we have shown that if \( c_e^H \leq c_F \), then \( q_e^* = q_e^H \), whereas if \( c_e > c_F \), then \( q_e^* = q_e^L \). Note that when deciding its quality, the entrant can anticipate the incumbent’s offering of an acceptable licensing contract. We consider two cases. If the entrant chooses \( q_e^H \), then the incumbent’s and the entrant’s profit functions are

\[
\pi_{ir}^H = (p_{ir} - c_i)(\frac{p_{ir} - p_{ir}}{q_e^H - q_i} - \frac{p_{ir}}{q_i}) + r(1 - \frac{p_{ir} - p_{ir}}{q_e^H - q_i}) \quad \text{and} \quad \pi_{er}^H = (p_{er} - c_e^H - r)(1 - \frac{p_{er} - p_{ir}}{q_e^H - q_i}),
\]

respectively. Following backward induction, we first obtain the two firms’ optimal prices \( p_{ir}^{H*} = q_e(q_e^H - q_i + c_e^H + r) + q_ir \) and \( p_{er}^{H*} = \frac{q_e(r[2(q_e^H - q_i + c_e^H + r) + c_i] + q_ir)}{4q_e^H - q_i} \). Then we can obtain the incumbent’s optimal licensing fee

\[
r_{H*} = \min\{\frac{\pi_{H*}}{2}, q_e(q_e^H - q_i + 8q_e^H(q_e^H - c_e^H)) \}
\]

If the incumbent chooses \( q_e^L \), then the two firms’ profit functions are \( \pi_{ir}^L = (p_{ir} - c_i)(\frac{p_{ir} - p_{ir}}{q_e^L - q_i} - \frac{p_{ir}}{q_i}) + r(1 - \frac{p_{ir} - p_{ir}}{q_e^L - q_i}) \) and \( \pi_{er}^L = (p_{er} - c_e^L - r)(1 - \frac{p_{er} - p_{ir}}{q_e^L - q_i}) \). We follow the same procedure to solve for the two firms’ optimal prices and get \( p_{ir}^{L*} = q_e(r[2(q_e^L - q_i + c_e^L + r) + c_i] + q_ir) \) and \( p_{er}^{L*} = \frac{q_e(r[2(q_e^L - q_i + c_e^L + r) + c_i] + q_ir)}{4q_e^L - q_i} \) if \( \frac{\pi_{H*}}{2} < q_e(q_e^L - q_i + 8q_e^L(q_e^L - c_e^L)) \). And the entrant’s corresponding profit is \( \pi_{er}^{L*} = \pi_{er}^{L*} \) if \( \frac{\pi_{H*}}{2} < q_e(q_e^L - q_i + 8q_e^L(q_e^L - c_e^L)) \).

Then we compare the entrant’s profits in these two cases to determine the entrant’s optimal quality decision: \( q_{e*} = q_{e}^H \) if \( c_e^H \leq c_F \), and \( q_{e*} = q_{e}^L \) otherwise, where \( c_F \) is defined in the Web Appendix, and like \( c_F, c_e \) is an expression on parameters \( q_e^H, q_e^L, c_e^L, q_i, \) and \( c_i \). We show that licensing of non-core technology can increase or decrease the entrant’s optimal quality even when the incumbent sets the licensing fee after the entrant decides its product quality. We keep the detailed derivation in the Web Appendix and provide two numerical examples to illustrate the effect. A quality-increasing numerical example: \( q_i = 1, c_i = 0.3, q_e^L = 1.5, c_e^L = 0.8, q_e^H = 3, c_e^H = 2.2721, F = 0.00978 \). In this example, \( q_{e*} = q_{e}^H > q_{e}^L = q_{e}^L \) and \( r_{H*} = 0.363392 \). That is, licensing increases the entrant’s optimal quality. A quality-decreasing numerical example: \( q_i = 1, c_i = 0.7, q_e^L = 1.1, c_e^L =
In this example, \( q^*_e = q^L_e < q^*_e = q^H_e \) and \( r^L* = 0.1589 \). That is, licensing decreases the entrant’s optimal quality. Therefore, in summary, even if the incumbent decides its licensing fee after observing the entrant’s quality decision, our main results about the effects of licensing on the entrant’s optimal quality remain qualitatively the same.

**Incumbent’s quality is endogenous**

Our main model has assumed that the incumbent’s quality is exogenous. In an extension, we allow the incumbent to endogenously decide its product quality \( q_i \) in anticipation of the entrant’s entry in the market. We examine the effect of the licensing contract on both firms’ optimal quality decisions by comparing each firm’s optimal quality under a licensing contract to its optimal quality under no licensing. Under no licensing, the incumbent first decides \( q_i \), then the entrant self-develops the non-core technology and chooses product quality \( q_e \), and finally both firms set their prices \( p_i \) and \( p_e \), respectively. Under licensing, the incumbent first decides \( r \) and \( q_{ir} \), then the entrant decides whether to license the non-core technology from the incumbent (or self-develop it) and chooses its quality \( q_{er} \) for its core technology, and finally both firms set their prices \( p_{ir} \) and \( p_{er} \), respectively. To analytically accommodate the incumbent’s endogenous quality decision in anticipation of the entrant’s entry, we allow each firm’s quality to be a continuous variable. More specifically, let the incumbent’s and the entrant’s marginal cost be \( k_i q_i^2 \) (or \( k_{ir} q_{ir}^2 \) in the case of licensing), and \( k_e q_e^2 \) (or \( k_{er} q_{er}^2 \) in the case of licensing), respectively. In this paper, since the entrant has more recent core technology, we assume \( k_e < k_i \). And without loss of generality, we normalize \( k_i = 1 \). The complexity of our multi-stage game prevents us from obtaining closed-form analytical solutions. However, our numerical studies, with the detailed derivation procedures relegated to the Web Appendix, show that our main results from the main model remain qualitatively the same under the quadratic cost function, even when the incumbent can endogenously and strategically choose its quality, fully anticipating the entrant’s
We analyze two distinct cases. (1) The entrant’s production of its core technology has significant improvement over the incumbent’s, i.e., $k_e$ is much smaller than $k_i$. Example: $k_i = 1$, $k_e = 0.5$ and $F = 0.01$. Under no-licensing, $q_i^* \approx 0.20559$ and $q_e^* \approx 0.71007$. Under licensing, $q_{ir}^* \approx 0.10349$, $r^* \approx 0.06914$, $q_{er}^* \approx 0.73907 > q_e^*$. Therefore, we see that when the entrant’s core technology is much more efficient than the incumbent’s, licensing of the non-core technology can increase the entrant’s optimal quality. (2) The entrant’s production of its core technology has incremental improvement over the incumbent’s, i.e., $k_e$ is not much smaller than $k_i$. Example: $k_i = 1$, $k_e = 0.98$ and $F = 0.001$. Under no-licensing, $q_i^* \approx 0.26711$ and $q_e^* \approx 0.45162$. Under licensing, $q_{ir}^* \approx 0.14223$, $r^* \approx 0.06688$, $q_{er}^* \approx 0.41281 < q_e^*$. Thus, we see that when the entrant’s core technology is only a little more efficient than the incumbent’s, licensing of the non-core technology can decrease the entrant’s optimal quality.

Furthermore, our numerical studies reveal that interestingly, licensing lowers the incumbent’s optimal quality. Intuitively, with a licensing contract, the incumbent has an incentive to lower its quality to increase its differentiation from the more efficient entrant, allowing the entrant to charge higher price and to be more willing to pay the incumbent higher royalty fee. We acknowledge that these results are not analytically proven for all parameter regions, but we do find the results to hold for all numerical examples that we have analyzed.

**Two-part tariff licensing fee**

We have focused on royalty-fee licensing. In practice, licensing fees may sometimes be two-part tariff, consisting of both a fixed fee and a royalty fee (e.g., Kamien and Tauman 1986; Fauli-Oller and Sandonis 2002; Kitagawa, Masuda, and Umezawa 2014). Suppose that in our main model, the incumbent can choose a two-part tariff licensing contract instead. Let $r_t$ denote the royalty fee and $f_t$ denote the fixed fee; the total licensing fee is $r_tD_e + f_t$. Note that, as Shapiro (1985) pointed
out, if a negative fixed fee is allowed in the two-part tariff, the licensor can charge a high royalty but bribe the licensee with a side payment (i.e., a negative fixed fee), which in the extreme case will induce the licensee to exit the market and which likely violates the antitrust laws. Thus, as is typically assumed in two-part tariff models, we will also assume $f_t \geq 0$. Note that, since $f_t$ does not affect the firms’ retail pricing decisions or the entrant’s product quality decision, the incumbent’s optimal royalty $r_t^*$ in the two-part tariff remains the same as that in the main model, i.e., $r_t^* = r^*$. So, the two-part tariff contract has the same effects as the pure royalty licensing contract in terms of alleviating price competition and potentially fostering or hindering the entrant’s product innovation, depending on whether the entrant can significantly or only incrementally improve its quality over the incumbent’s. In our main model, both the incumbent and the entrant are strictly better off under royalty licensing than under no licensing (except in the boundary solution case, where the entrant is indifferent). With two-part tariffs, however, unless there is uncertainty or information asymmetry, the incumbent will always be able to extract all the benefit of the inter-competitor licensing by optimally setting the fixed component of the two-part tariff to $f_t^* = \max\{0, \pi_{er}^* - \pi_e^*\}$, in which case the entrant is just indifferent between accepting and rejecting the two-part tariff licensing contract.

**CONCLUSION**

This paper studies the economic and strategic implications of inter-competitor licensing of non-core technology in a vertically differentiated product market. The entrant has some core technology innovation that allows it to introduce an improved product with higher quality than the incumbent’s product. To develop and produce the products, the entrant also needs some non-core technology which it can either license from the incumbent on a royalty contract or self-develop by incurring some fixed R&D cost in-house. We examine the effects of the inter-competitor licensing of the non-core technology on market competition and the entrant’s optimal quality decision. Our analysis reveals
that a royalty licensing contract between the two firms has a competition-alleviation effect, inducing both firms to raise their prices. Furthermore, we show that royalty licensing can either increase or decrease the entrant’s optimal quality, depending on whether the entrant’s core technology can significantly or only incrementally improve its product quality over the incumbent’s product quality. If the entrant can significantly improve its product quality over the incumbent’s product quality, royalty licensing of the non-core technology will tend to increase the entrant’s optimal quality. If the entrant can only incrementally improve its product quality over the incumbent’s, royalty licensing will tend to reduce the entrant’s optimal quality. This provides a guideline for an entrant’s optimal quality decisions when it has a royalty licensing contract with the incumbent firm.

Conventional wisdom suggests that the incumbent should not license its non-core technology to the entrant if licensing will help the entrant save costs and become more competitive, and that the entrant should not accept the licensing contract if it will be paying a total royalty fee higher than its alternative cost of developing the non-core technology on its own. Our analysis shows that, interestingly, the royalty licensing contract can be mutually acceptable even when the entrant will pay a total licensing fee higher or lower than its alternative R&D cost. This result has important managerial implications. It shows that the intuitively sound decision-making rule of rejecting a contract when the alternative cost is lower may not be optimal. Managers should consider the strategic implications of licensing on market competition and optimal quality design.

We have demonstrated that our main findings hold qualitatively the same under some alternative assumptions, including (1) the entrant can produce quality levels either above or below the incumbent’s product, (2) the incumbent can strategically choose the royalty fee after the entrant decides the quality level for its core technology, (3) the incumbent can also endogenously decide its quality in anticipation of the entrant’s entry, and (4) the licensing contract is a two-part tariff contract.

We conclude by pointing out some caveats about our model and possible directions for future
research. First, our model has assumed that the non-core technology of the two firms’ products has no significant difference whether it is licensed from the incumbent or developed in-house by the entrant. If there is a vertical difference, e.g., the licensed one is better, there may be additional incentives for the entrant to accept the licensing contract with the incumbent and the parameter regions for the quality-increasing and quality-decreasing effects of licensing may change, but our main findings and insights should hold qualitatively the same as long as the main difference in quality between the two firms’ products comes from the entrant’s core technology rather than the baseline non-core technology. If there is a horizontal difference, i.e., consumers have different preferences over the licensed non-core technology and the in-house developed non-core technology, the firm’s licensing incentive may decrease, which may reduce the parameter regions for the mutually acceptable licensing contract and also affect the quality-increasing and quality-decreasing effects of licensing. Hence, if the non-core technology attribute is also a differentiator between the firms, we conjecture that our main findings can still qualitatively hold albeit the parameter regions can change. We leave this to future research to systematically examine the additional managerial insights of inter-competitor licensing from such multi-dimensional differentiation models. Second, we have assumed that licensing is unidirectional, i.e., the entrant licenses from the incumbent but not the other way around. In practice, licensing can go both ways – the incumbent can also license some technology from the entrant. In such cross-licensing context, the main competition-alleviation effect will be enhanced, and our main findings and insights should remain qualitatively similar but the analytical expressions will become much more cumbersome. Third, our model has assumed that, as well-established in the licensing literature, royalty fees are in terms of a fixed amount per unit of sales. This is a very reasonable assumption for licensing of non-core technology, where the licensed technology is not a key part of the product’s total value. However, in cases where the licensed technology is a crucial, core component of the product, firms may sometimes license the technology on a royalty fee that
is some percentage of the final product’s retail price. If we model the royalty fee as a percentage of retail price (i.e., revenue sharing), the mathematical complexity will dramatically increase in our multi-stage, multi-decision setting, and no closed-form analytical solutions can be obtained for the full game. We expect that the main driving forces for our findings will likely survive, but more careful future study on such revenue-sharing contracts in the licensing context is warranted. Fourth, we have assumed that licensing will not shorten the time-to-market for the product. In practice, R&D is often more time-consuming than buying a license. In a model where the entrant may also be able to shorten the time-to-market for its new product, we expect the entrant to have more incentives to license from the incumbent, which will likely increase the parameter region in which licensing will occur in equilibrium, but it will probably also induce the incumbent to raise its royalty fees. We think that studying the effect of licensing on the timing of product introduction is worth its own separate study and is beyond the scope of our paper. Finally, it may also be of practical and academic interest to examine how the firms’ licensing, pricing and quality decisions will be affected by uncertainties in market demand or R&D costs.
References


Footnotes


2Note that this paper focuses on a one-shot game. In practice, following the entrant’s introduction of new product, the incumbent may also introduce improvements to its product or add more quality attributes. One can treat that situation as the start of a new game cycle, where the “entrant” becomes the “incumbent” in the new game and vice versa. So, our current model can still be useful in shedding some light on that market interaction to the extent that the firms cannot fully anticipate all iterations of future innovations. In the Robustness Check and Discussion section, we will consider the case where the incumbent anticipates the entrant’s entry and endogenizes its product quality.

3In this paper, we focus on the parameter region in which interior solutions exist, and do not explicitly analyze the uninteresting special case in which one of the firms is competed out of the market. For instance, one set of sufficient parameter conditions for interior solutions is: \( \frac{q_i}{q_i - q_i} < \frac{q_i - c_i}{q_i - c_i}, \frac{c_i}{c_i} \) and \( F \leq \max\{\frac{q_i}{q_i - q_i}(2q_i - 2q_i + c_i), \frac{q_i}{q_i - q_i}(2q_i - 2q_i + c_i) - (2q_i - q_i)c_i^2\} \). The derivations of these conditions are provided in the Web Appendix.

4Royalty fees may sometimes be a percentage of the retail price of the licensee’s product if the licensed technology is a core part of the final product. In this paper, we study the licensing of non-core technology and focus on the per-unit royalty fee (independent of the retail price of the final product), which is most commonly used both in practice and in the literature (e.g., Rostoker 1984; Sen 2005).

5Our two-level quality model can be extended to any finite number of quality levels; the results will remain qualitatively the same albeit with more cumbersome notations. In the Robustness Check and Discussion section, we also show that our results are robust even if the entrant’s quality is a continuous variable.

6If \( c_r = c_F \), which happens when \( (2q_i^H - q_i^L)(q_i^L - 1) = \frac{q_i}{16} \), royalty licensing does not affect the entrant’s optimal quality decision.

7In our context, since \( \pi_{er}^{H*} > 0 \) and \( \pi_{er}^{L*} > 0 \), mathematically it is equivalent to compare the effect of \( r \) on \( \sqrt{\pi_{er}} \). Note that \( \frac{d\sqrt{\pi_{er}^{H*}}}{dr} = \frac{2\sqrt{q_i^H - q_i}}{4q_i^H - q_i} < \frac{d\sqrt{\pi_{er}^{L*}}}{dr} = \frac{2\sqrt{q_i^L - q_i}}{4q_i^L - q_i} \). Since the entrant’s net profit margin \( p_{er}^* - c_{er}^* = \frac{q_i^H(2q_i^H - 2q_i + c_i) - (2q_i^H - q_i)c_i^2}{4q_i^H - q_i} \) and \( \sqrt{\pi_{er}^{H*}} = \frac{2q_i^H(2q_i^H - 2q_i + c_i) - (2q_i^H - q_i)c_i^2}{4q_i^H - q_i} \), in our context, we can understand \( \sqrt{\pi_{er}} \) as the entrant’s net profit margin associated with quality differentiation. Then \( \frac{d\sqrt{\pi_{er}^{H*}}}{dr} \) refers to the effect of royalty fee on the entrant’s net profit margin associated with quality differentiation.

8Suppose that the incumbent offers the entrant a fixed-fee licensing contract (e.g., Lin 1996). Let \( f \) denote the fixed fee. It is easy to show that compared with the no-licensing case, the fixed-fee licensing contract will not affect the two firms’ retail pricing decisions; the entrant will accept the contract only if \( f \leq F \) and the incumbent will offer the licensing contract only if \( f \geq 0 \). Further, the fixed-fee licensing contract will not affect the entrant’s optimal product quality, and the incumbent sets its optimal fixed fee as \( f^* = F \). Hence, the two firms’ optimal profits under fixed-fee licensing contract are \( \pi_{ef} = \frac{q_i(q_i - q_i + c_i) - (2q_i - q_i)c_i^2}{q_i(q_i - q_i)(q_i - q_i)^2} + F \) and \( \pi_{ef}^* = \frac{q_i(q_i - q_i + c_i) - (2q_i - q_i)c_i^2}{q_i(q_i - q_i)(q_i - q_i)^2} - F \), respectively, where the entrant’s optimal quality-cost choice \((q_e, c_e)\) is \((q_e^H, c_e^H)\) when \( c_e^H \leq c_F \) and \((q_e^L, c_e^L)\) otherwise.
This indicates that competitive referral can soften the competing firms’ price competition. This is higher total licensing fee it pays. Similarly, the incumbent’s benefit from alleviated price competition. When the contract does not a

Both firms can benefit from entering into a contract in a wide range of parameter regions (shaded

Figure 1: Entrant’s optimal quality: The case of highly differentiated core technology
Figure 2: Entrant’s optimal quality: The case of incrementally differentiated core technology
benefit the entrant because, with alleviated price competition due to royalty licensing, the entrant

Recall that Proposition 2 shows that the entrant’s innovation can be fostered by the licensing conditional on the contract being accepted, firm 1 will prefer a higher unit fee (\(r_1^*\)), as Proposition 3 shows, firm 2 may accept a contract with firm 1 even if the marginal fee it pays is higher than the cost of its alternative option (\(c_{F2}^k\)).

For firm 2, even when the fee it pays is higher than the cost of its alternative option (\(c_{F2}^k\)), the entrant’s benefit can more than offset firm 2’s product quality choice (\(q_{F2}^2\)). This indicates that competitive referral can soften the competing firms’ price competition. This is because quality decrease due to competitive referral? This reason behind Proposition 6(i) is that, unlike under licensing at fixed fee, firm 2 would have a sufficient reason to accept such a contract, the profit margin with only a small loss in market share because

The industry benefits more from the licensing contract at royalty fee than from that at fixed fee;
Appendix

Proof of Proposition 1. When there is no licensing contract between the two firms, the incumbent’s demand function and profit function are \( D_i = \frac{p_e - p_i}{q_e - q_i} - \frac{p_i}{q_i} \) and \( \pi_i = (p_i - c_i)D_i \), and the entrant’s demand function and profit function are \( D_e = 1 - \frac{p_e - p_i}{q_e - q_i} \) and \( \pi_e = (p_e - c_e)D_e - F \). Since \( \frac{d^2 \pi_i}{dp_i^2} = -\frac{2q_e}{q_i(q_e - q_i)} < 0 \) and \( \frac{d^2 \pi_e}{dp_e^2} = -\frac{2}{q_e - q_i} < 0 \), \( \pi_i \) is a concave function of \( p_i \) and \( \pi_e \) is a concave function of \( p_e \). Hence, simultaneously solving the first order conditions \( \frac{d\pi_i}{dp_i} = \frac{c_i p_i + p_e q_i - 2 p_e q_e}{q_i(q_e - q_i)} = 0 \) and \( \frac{d\pi_e}{dp_e} = \frac{c_i + q_e + p_i - q_i - 2 p_e}{q_e - q_i} = 0 \) leads to two firms’ optimal prices \( p^*_i = \frac{q_i(q_e - q_i + c_e) + 2 q_e c_i}{4 q_e - q_i} \) and \( p^*_e = \frac{q_e (2 q_e - 2 q_i + 2 c_e + c_i)}{4 q_e - q_i} \).

When there is a licensing contract between the two firms, the incumbent’s and the entrant’s demand functions are \( D_{ir} = \frac{p_{er} - p_i}{q_e - q_i} - \frac{p_i}{q_i} \) and \( D_{er} = 1 - \frac{p_{er} - p_e}{q_e - q_i} \), respectively. Two firms’ profit functions are \( \pi_{ir} = (p_{ir} - c_i)D_{ir} + r D_{er} \) and \( \pi_{er} = (p_{er} - c_e - r)D_{er} \), respectively. Since \( \frac{d^2 \pi_{ir}}{dp_{ir}^2} = -\frac{2 q_{er}}{q_i(q_{er} - q_i)} < 0 \) and \( \frac{d^2 \pi_{er}}{dp_{er}^2} = -\frac{2}{q_{er} - q_i} < 0 \), \( \pi_{ir} \) is a concave function of \( p_{ir} \) and \( \pi_{er} \) is a concave function of \( p_{er} \). Hence, simultaneously solving the first order conditions \( \frac{d\pi_{ir}}{dp_{ir}} = \frac{c_i + q_{er} + p_{ir} - q_i - 2 p_{er}}{q_{er} - q_i} = 0 \) and \( \frac{d\pi_{er}}{dp_{er}} = \frac{c_{er} + r + q_{er} + p_{er} - q_i - 2 p_{er}}{q_{er} - q_i} = 0 \) leads to two firms’ optimal prices \( p^*_{ir} = \frac{q_i(q_{er} - q_i + c_e + 3 r) + 2 q_e c_i}{4 q_{er} - q_i} \) and \( p^*_{er} = \frac{q_{er} [2(q_{er} - q_i + c_e + r) + c_i] + q_i r}{4 q_{er} - q_i} \).

Hence, given \( q_{er} = q_e \), \( p^*_{ir} - p^*_e = \frac{q_i(q_e - q_i + c_e + 3 r) + 2 q_e c_i}{4 q_e - q_i} - \frac{q_i(q_e - q_i + c_e + 2 c_e + c_i)}{4 q_e - q_i} = \frac{3 q_e r}{4 q_e - q_i} > 0 \) for any \( r > 0 \), and \( p^*_{er} - p_e = \frac{q_e [2(q_e - q_i + c_e + r) + c_i] + q_i r}{4 q_e - q_i} - \frac{q_e (2 q_e - 2 q_i + 2 c_e + c_i)}{4 q_e - q_i} = \frac{r (2 q_e + q_i)}{4 q_e - q_i} > 0 \) for any \( r > 0 \).

Q.E.D.

Proof of Lemma 1. If \( \frac{(q_e^L - 1)(q_i^L - 1)}{q_i^L} > \frac{9}{16} \), then \( 16 \frac{q_e^H}{q_i^L} > \frac{q_e^L}{q_i^L} - 16(\frac{q_i^H}{q_i^L} + \frac{q_i^L}{q_i^L}) + 16 > 9 \Rightarrow 16 \frac{q_i^H}{q_i^L} = 16 \left( \frac{q_i^H}{q_i^L} + \frac{q_i^L}{q_i^L} \right) + 16 > 9 \Rightarrow 16(\frac{q_i^H}{q_i^L} + \frac{q_i^L}{q_i^L}) + 7 > 0 \Rightarrow \frac{16}{q_i^L} + \frac{q_i^L}{q_i^L} - 16 \left( \frac{q_i^H}{q_i^L} + \frac{q_i^L}{q_i^L} \right) + 16(\frac{q_i^H}{q_i^L} + \frac{q_i^L}{q_i^L}) + 7 > 0 \Rightarrow \frac{16}{q_i^L} > 8 \frac{q_i^L}{q_i^L} \Rightarrow \frac{2 q_i^L}{q_i^L} + 1 > 1 \Rightarrow \frac{2 q_i^L}{q_i^L} > 0 \Rightarrow (4 q_e^H - q_i^L)^2 > 0 \Rightarrow R(2 q_i^L - 2 q_i) - (2 q_e^H - 2 q_i) > 0 \Rightarrow c_r - c_F = \frac{R(2 q_i^L - 2 q_i) - (2 q_e^H - 2 q_i)}{2 q_e^H - q_i}, r > 0. \)
Similarly, if \((\frac{q^H}{q_i} - 1)(\frac{q^L}{q_i} - 1) < \frac{q}{16}\), then \(16\frac{q^H}{q_i} \frac{q^L}{q_i} - 16\frac{q^H}{q_i} + \frac{q^L}{q_i} + 16 < 9 \Rightarrow 16\frac{q^H}{q_i} \frac{q^L}{q_i} - 16\frac{q^H}{q_i} + \frac{q^L}{q_i} + 7 < 0 \Rightarrow 16\frac{q^H}{q_i} \frac{q^L}{q_i} - 16\frac{q^H}{q_i} + \frac{q^L}{q_i} + 7(\frac{q^H}{q_i} - \frac{q^L}{q_i}) < 0 \Rightarrow [16\frac{q^H}{q_i} - 8\frac{q^H}{q_i} + 1]\frac{q^H}{q_i} - \frac{q^L}{q_i} < 0 \Rightarrow (4\frac{q^H}{q_i} - q_i^H)^2 < (4\frac{q^H}{q_i} - q_i^L)^2 \Rightarrow q_i^H - q_i > 0 \Rightarrow R(2q_i^H - 2q_i) - (2q_i^H - 2q_i) < 0 \Rightarrow c_F - c_F = \frac{R(2q_i^H - 2q_i) - (2q_i^H - 2q_i)}{2q_i^H - q_i} r < 0.

Q.E.D.

**Proof of Proposition 2.** We prove Proposition 2 by first establishing the entrant’s optimal quality decision under no licensing contract and the entrant’s optimal quality decision under a mutually acceptable licensing contract, then comparing the entrant’s optimal quality decisions to show the effect of licensing on the entrant’s quality decision.

Under no licensing contract, the incumbent’s and the entrant’s demand functions are given by

\[
D_i = \frac{p_i - p_i}{q_i - q_i} - \frac{p_i}{q_i} \quad \text{and} \quad D_e = 1 - \frac{p_i - p_i}{q_i - q_i},
\]

and their profit functions are \(\pi_i = (p_i - c_i)D_i\) and \(\pi_e = (p_e - c_e)D_e - F\), respectively. We solve the game based on backward induction. It is straightforward that \(\pi_i\) is a concave function of \(p_i\) and \(\pi_e\) is a concave function of \(p_e\). First, simultaneously solving the first-order conditions \(\frac{d\pi_i}{dp_i} = 0\) and \(\frac{d\pi_e}{dp_e} = 0\) leads to the optimal retail prices \(p_i^* = \frac{q_i(q_i - q_i + c_i) + 2q_i c_i}{4q_i - q_i}\) and \(p_e^* = \frac{q_e(2q_e - 2q_e + 2c_e + c_i)}{4q_e - q_i}\) with the corresponding profits \(\pi_i^*(q_e, c_e) = \frac{q_i(q_i(q_i + c_i) - (2q_i - q_i)c_i)}{(4q_i - q_i)^2}\) and \(\pi_e^*(q_e, c_e) = \frac{q_i(q_i(q_i - c_i) - (2q_i - q_i)c_i)}{(4q_i - q_i)^2}\). The entrant’s profit under the high strategy \((i.e., q_i^H)\) is \(\pi_e^{H*} = \pi_e^*(q_e, c_e)|_{q_e = q_i^H, c_e = c_e^H} = \frac{q_i^H}{(4q_i^H - q_i)}\) \(-F\); its profit under the low strategy \((i.e., q_i^L)\) is \(\pi_e^{L*} = \pi_e^*(q_e, c_e)|_{q_e = q_i^L, c_e = c_e^L} = \frac{q_i^L}{(4q_i^L - q_i)^2}\) \(-F\). Next, we solve for the entrant’s optimal quality decision by comparing \(\pi_e^{H*}\) to \(\pi_e^{L*}\): \(\pi_e^{H*} \geq \pi_e^{L*}\) and \(q_e^* = q_i^H\) if \(c_e^H < c_F = \frac{q_i^H q_i^H - R q_i^L}{2q_i^H - q_i} - R(2q_i^L - q_i) - (2q_i^L - q_i)c_i^L\) \((-F\) elsewhere.

Under a royalty licensing contract, the incumbent’s and the entrant’s demand functions are
\[
\frac{p_{ir} - p_{er}}{q_{ir} - q_i} \quad \text{and} \quad D_{er} = 1 - \frac{p_{ir} - p_{er}}{q_{ir} - q_i},
\]
respectively. The two firms’ profits are
\[
\pi_{ir} = (p_{ir} - c_i)D_{ir} + rD_{er}
\]
and
\[
\pi_{er} = (p_{er} - c_e)D_{er} - rD_{er},
\]
respectively. Since \( \pi_{ir} \) is a concave function of \( p_{ir} \) and \( \pi_{er} \) is a concave function of \( p_{er} \), simultaneously solving the first-order conditions \( \frac{d\pi_{er}}{dp_{er}} = 0 \) and \( \frac{d\pi_{er}}{dp_{er}} = 0 \) leads to the firms’ optimal retail prices
\[
p_{ir}^* = \frac{q_i(q_{er} - q_i + c_{er} + 3r) + 2q_{er}c_i}{4q_{er} - 4q_i}
\]
and their corresponding profits
\[
\pi_{ir}^*(r, q_{er}, c_{er}) = \frac{q_i(q_i(q_{er} - q_i + c_{er}) - c_i(2q_{er} - q_i))^2 + q_i(q_{er} - q_i)[q_i^2 + 8q_{er}(q_{er} - c_{er}) - c_i]r - (8q_{er} + q_i)(q_{er} - q_i)q_i^2}{q_i(q_{er} - q_i)(4q_{er} - 4q_i)}
\]
and
\[
\pi_{er}^*(r, q_{er}, c_{er}) = \frac{[q_i(2q_{er} - 2q_i + c_i - c_{er}(2q_{er} - q_i) - r(2q_{er} - 2q_i))^2]{q_i(q_{er} - q_i)(4q_{er} - 4q_i)}
\]
for given \( r \). The entrant’s profit under the high strategy is
\[
\pi^H_{er}(r) = \pi^*_e(r, q_{er}, c_{er}) |_{q_{er} = q_e^H, c_{er} = c_e^H} = \frac{[q_e^H(2q_e^H - 2q_i + c_i) - c_e^H(2q_e^H - q_i) - r(2q_e^H - 2q_i))^2]{(q_e^H - q_i)(4q_e^H - 4q_i)}
\]
and its profit under the low strategy is
\[
\pi^{L^*}_{er}(r) = \pi^*_e(r, q_{er}, c_{er}) |_{q_{er} = q_e^L, c_{er} = c_e^L} = \frac{[q_e^L(2q_e^L - 2q_i + c_i) - c_e^L(2q_e^L - q_i) - r(2q_e^L - 2q_i))^2]{(q_e^L - q_i)(4q_e^L - 4q_i)}
\]
Next, for given \( r \), we solve for the entrant’s optimal quality by comparing \( \pi^H_{er}(r) \) to \( \pi^{L^*}_{er}(r) \): \( \pi^H_{er}(r) \geq \pi^{L^*}_{er}(r) \) and \( q_{er}^* = q_e^H \) if \( c_e^H \leq c_r \equiv c_F + \frac{R(2q_e^H - 2q_i) - (2q_e^H - 2q_i)q_i}{2q_e^H - 4q_i} \); \( \pi^H_{er}(r) < \pi^{L^*}_{er}(r) \) and \( q_{er}^* = q_e^L \) otherwise.

A royalty licensing contract is mutually acceptable only if both firms are (weakly) better off. For the entrant to accept the contract, \( r \) has to be such that \( \max\{\pi^H_{er}(r), \pi^{L^*}_{er}(r)\} \geq \max\{\pi^H_e, \pi^{L^*}_e\} \).

Recall from Lemma 1 that when \( \frac{q_e^H}{q_i} - 1 = \frac{q_e^L}{q_i} - 1 > \frac{9}{16} \), \( c_r > c_F \). The upper bound of \( r \) is given by solving the following problem:

\[
\begin{align*}
\pi^H_{er}(r) &\geq \pi^H_e, \quad \text{if } c_e^H < c_r \leq c_F; \\
\pi^H_{er}(r) &\geq \pi^{L^*}_e, \quad \text{if } c_F < c_e^H \leq c_r; \\
\pi^{L^*}_{er}(r) &\geq \pi^L_e, \quad \text{if } c_r > c_e^H.
\end{align*}
\]
For any mutually acceptable licensing contract, if $r > \pi$, then no licensing contract. Therefore, any licensing contract with $0 < r \leq \pi$ is mutually acceptable. Hence, the entrant will accept the royalty licensing contract only if

$$\pi = \begin{cases} 
q^{H} & (2q^{H}_e - 2q_i + c_i) - (2q^{H}_e - q_i)c^{H}_e - (4q^{H}_e - q_i)\sqrt{(q^{H}_e - q_i)\pi^{H}} = \frac{2q^{H}_e - 2q_i}{2q^{H}_e - 2q_i}, & \text{if } c^{L}_e < c^{H}_e \leq c^{H}_F; \\
q^{L} & (2q^{L}_e - 2q_i + c_i) - (2q^{L}_e - q_i)c^{L}_e - (4q^{L}_e - q_i)\sqrt{(q^{L}_e - q_i)\pi^{L}} = \frac{2q^{L}_e - 2q_i}{2q^{L}_e - 2q_i}, & \text{if } c^{L}_e < c^{H}_e \leq c^{L}_F; \\
q^{L} & (2q^{L}_e - 2q_i + c_i) - (2q^{L}_e - q_i)c^{L}_e - (4q^{L}_e - q_i)\sqrt{(q^{L}_e - q_i)\pi^{L}} = \frac{2q^{L}_e - 2q_i}{2q^{L}_e - 2q_i}, & \text{if } c^{H}_e > c^{L}_e. 
\end{cases}$$

As discussed in the paper, any royalty contract with $r > 0$ is strictly better for the incumbent than no licensing contract. Therefore, any licensing contract with $0 < r \leq \pi$ is mutually acceptable. For any mutually acceptable licensing contract, if $c^H_F < c^H_e \leq c^H_F$, then $q^{*}_{er} = q^{H}_e > q^{L}_e = q^{*}_e$; if $c^{H}_e \leq c^{F}_e$ or $c^{H}_e > c^F_F$, $q^{*}_{er} = q^{*}_e$.

From Lemma 1, when $(\frac{q^{H}_e}{q_i} - 1)(\frac{q^{L}_e}{q_i} - 1) > \frac{9}{16}$, $R(2q^{L}_e - 2q_i) - (2q^{H}_e - 2q_i) > 0$, hence $\frac{dc_F}{dr} = \frac{R(2q^{L}_e - 2q_i) - (2q^{H}_e - 2q_i)}{2q^{L}_e - 2q_i} > 0$.

Q.E.D.

**Proof of Proposition 3.** We prove Proposition 3 by following the same logic and procedure as in the proof of Proposition 2. Recall in the proof of Proposition 2, under no licensing contract, $q^{*}_e = q^{H}_e$ if $c^{H}_e \leq c^{F}_F$ and $q^{*}_e = q^{L}_e$ otherwise. Under a royalty licensing contract, $q^{*}_{er} = q^{H}_e$ if $c^{H}_e \leq c^F_F \equiv c^F_F + \frac{R(2q^{L}_e - 2q_i) - (2q^{H}_e - 2q_i)}{2q^{L}_e - 2q_i}r$ and $q^{*}_{er} = q^{L}_e$ otherwise.

A royalty licensing contract is mutually acceptable only if both firms are (weakly) better off. For the entrant to accept the contract, $r$ has to be such that $\max\{\pi^{H}_{er}(r), \pi^{L}_{er}(r)\} \geq \max\{\pi^{H}_e, \pi^{L}_e\}$. Recall from Lemma 1 that when $(\frac{q^{H}_e}{q_i} - 1)(\frac{q^{L}_e}{q_i} - 1) < \frac{9}{16}$, $c^{F}_F < c^F_F$. The upper bound of $r$ is given by
solving the following problem:

\[
\begin{align*}
\pi^{H^*}_e(r) &\geq \pi^{H^*}_e, & \text{if } c^L_e < c^H_e \leq c_r; \\
\pi^{L^*}_e(r) &\geq \pi^{H^*}_e, & \text{if } c_r < c^H_e \leq c_F; \\
\pi^{L^*}_e(r) &\geq \pi^{L^*}_e, & \text{if } c^H_e > c_F.
\end{align*}
\]

Hence, the entrant will accept the royalty licensing contract only if \( r \leq \tau \), where

\[
\tau = \begin{cases} 
\tau^{HH}, & \text{if } c^L_e < c^H_e \leq c_r; \\
\tau^{HL} = \frac{q^L_e(2q^L_e-2q_i+c_i)-(2q^L_e-q_i)c^L_e-(4q^L_e-q_i)\sqrt{(q^L_e-q_i)2\pi^H^*}}{2q^L_e-2q_i}, & \text{if } c_r < c^H_e \leq c_F; \\
\tau^{LL}, & \text{if } c^H_e > c_F.
\end{cases}
\]

As discussed in the paper, any royalty contract with \( r > 0 \) is strictly better for the incumbent than no licensing contract. Therefore, any licensing contract with \( 0 < r \leq \tau \) is mutually acceptable. For any mutually acceptable licensing contract, if \( c_r < c^H_e \leq c_F \), \( q^*_e = q^L_e < q^H_e = q^*_e \). If \( c^H_e \leq c_r \) or \( c^H_e > c_F \), \( q^*_e = q^*_e \).

From Lemma 1, when \( \left( \frac{q^H_i}{q_i} - 1 \right) \left( \frac{q^L_i}{q_i} - 1 \right) < \frac{q_i}{10} \), \( R(2q^L_e - 2q_i) - (2q^H_e - 2q_i) < 0 \), hence \( \frac{dc^*}{dr} = \frac{R(2q^L_e - 2q_i) - (2q^H_e - 2q_i)}{2q^H_e - 2q_i} < 0 \).

Q.E.D.

**Proof of Proposition 4.** Under a mutually acceptable royalty licensing contract, i.e., \( 0 < r \leq \bar{r} \), the total royalty fee the entrant pays is \( r \times D^*_e \). Recall from the proof of Proposition 2, under a licensing contract, the firms’ optimal prices are \( p^*_e = \frac{q_i(q^*_e-q_i+c^*_e+3r)+2q^*_e c_i}{4q^*_e-q_i} \) and \( p^*_e = \frac{q^*_e(2q^*_e-q_i+c^*_e+r)+q^*_e c_i}{4q^*_e-q_i} \) where \( (q^*_e, c^*_e) = (q^H_e, c^H_e) \) when \( c^H_e \leq c_r \) and \( (q^*_e, c^*_e) = (q^L_e, c^L_e) \) when \( c^H_e > c_r \).

When \( c^H_e \leq c_r \), we replace \( p^*_e \), \( p^*_e \) and \( (q^*_e, c^*_e) = (q^H_e, c^H_e) \) in \( D^*_e = 1 - \frac{p^*_e-p^*_e}{q^*_e} \) and get the entrant’s demand \( D^*_e = \frac{q^H_e(2q^H_e-2q_i+c_i)-(2q^H_e-q_i)c^H_e-2(q^H_e-q_i)r}{(q^H_e-q_i)(4q^H_e-q_i)} \) and the total royalty fee the en-
The entrant pays is \( rD^*_e = \frac{r[q^*_e(2q^*_e - 2q_i + c_i) - (2q^*_e - q_i)c^*_e - 2(q^*_e - q_i)r^2]}{(q^*_e - q_i)(4q^*_e - q_i)}. \) Note that \( rD^*_e \) is a concave function of \( r. \) If \( \frac{M - \sqrt{M^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} < r < \frac{M + \sqrt{M^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} \) where \( M = q^*_e(2q^*_e - 2q_i + c_i) - (2q^*_e - q_i)c^*_e - 2(q^*_e - q_i)r^2. \) Hence, to prove Proposition 4(b), it is sufficient to prove \( \frac{M - \sqrt{M^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} < \tilde{r} \) can hold. Note that since \( \tilde{r} = \frac{M - \sqrt{M^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{2(q^*_e - q_i)}, \) if \( c^*_e \leq \min\{c_F, c_r\}. \) Then for any \( q^*_e \) such that \( q^*_e < \frac{7q_i}{4}, \) or for any \( F \) such that \( 0 < F < \frac{3q^*_eM^2}{(2q^*_e + q_i)(4q^*_e - q_i)} \), \( \frac{M - \sqrt{M^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} < \tilde{r} \) holds. Note that since \( \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} > 0, \) there exists a royalty fee \( 0 < r < \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} \) such that \( rD^*_e < F. \) Hence, Proposition 4(a) holds.

When \( c^*_e > c_r, \) we replace \( p^*_e, p^*_e \) and \( (q^*_e, r^*_e) = (q^*_e, c^*_e) \) in \( D_{e} = 1 - \frac{p_{se} - p_{se}}{q_{se} - q_i} \) and get the entrant’s demand \( D^*_{e} = \frac{q^*_e(2q^*_e - 2q_i + c_i) - (2q^*_e - q_i)c^*_e - 2(q^*_e - q_i)r^2}{(q^*_e - q_i)(4q^*_e - q_i)} \) and the total royalty fee the entrant pays is \( rD^*_e = \frac{r[q^*_e(2q^*_e - 2q_i + c_i) - (2q^*_e - q_i)c^*_e - 2(q^*_e - q_i)r^2]}{(q^*_e - q_i)(4q^*_e - q_i)}. \) If \( \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} < r < \frac{N + \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} \) where \( N = q^*_e(2q^*_e - 2q_i + c_i) - (2q^*_e - q_i)c^*_e - 2(q^*_e - q_i)r^2. \) Then \( rD^*_e > F. \) To prove Proposition 4(b), it is sufficient to prove \( \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} < \tilde{r} \) can hold. Note that \( \tilde{r} = \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{2(q^*_e - q_i)}, \) if \( c^*_e > \max\{c_F, c_r\}. \) For any \( q^*_e \) such that \( q^*_e < \frac{7q_i}{4}, \) or for any \( F \) such that \( 0 < F < \frac{3q^*_eN^2}{(2q^*_e + q_i)^2(4q^*_e - q_i)} \), \( \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} < \tilde{r} \) holds. Note that since \( \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} > 0, \) there exists a royalty fee \( 0 < r < \frac{N - \sqrt{N^2 - 8(q^*_e - q_i)^2(4q^*_e - q_i)}F}{4(q^*_e - q_i)} \) such that \( rD^*_e < F. \) Hence, Proposition 4(a) holds.

Q.E.D.

Proof of Proposition 5. We prove Proposition 5 by showing that a licensing contract with the incumbent’s optimal royalty fee can still lead to the entrant’s higher (lower) quality decision when it can significantly (only incrementally) improve its quality over the incumbent’s.

Recall in the paper that, anticipating the entrant’s best response on quality decision, the incum-
bent’s profit is
\[
\pi_H^R(r) = \begin{cases} 
\pi^*_H(r, q^*_e, c^*_e), & \text{if } c^*_e \leq c_r; \\
\pi^*_L(r, q^*_e, c^*_e), & \text{if } c^*_e > c_r;
\end{cases} 
\]
(\text{v})
where \(\pi^*_H(r, q^*_e, c^*_e)\) and \(\pi^*_L(r, q^*_e, c^*_e)\) are concave functions of \(r\).

(a). When the entrant can significantly improve its quality over the incumbent’s, i.e., \(c_r > c_F\), then the incumbent maximizes its profit in (v) subject to the constraint \(0 < r \leq \bar{r}\), where \(\bar{r}\) is defined in (ii), and gets its optimal royalty fee \(r^*:\)

\[
r^* = \begin{cases} 
\min\left\{ \frac{8q^*_e(q^*_e-c^*_e)+q_i(q_i-c_i)}{2(q^*_e+q_i)}, \pi^{HH} \right\}, & \text{if } c^*_e \leq c_r; \\
\max\{r, \min\left\{ \frac{8q^*_e(q^*_e-c^*_e)+q_i(q_i-c_i)}{2(q^*_e+q_i)}, \pi^{LH} \right\} \}, & \text{if } c^*_e < c_r \leq c^*_e & \pi^*_H(r^{LH^*}) \geq \pi^*_H(r^{LH}); \\
\min\left\{ \frac{8q^*_e(q^*_e-c^*_e)+q_i(q_i-c_i)}{2(q^*_e+q_i)}, \pi^{LL} \right\}, & \text{if } c^*_e > c_r; 
\end{cases} 
\]
(\text{vi})
where \(r^*_e \equiv \frac{(2q^*_e-q_i)(c^*_e-c_F)}{K(2q^*_e-2q_i)-(2c^*_e-2q_i)}\) is the alternative form of writing \(c^*_e = c_r\), and \(\pi_r \equiv c_r \mid r = \pi^{LL}\). Note that, if \(c_F < c^*_e \leq c_r \& \pi^*_H(r^{LH^*}) \geq \pi^*_H(r^{LH});\) the incumbent’s optimal royalty fee is \(r^{LH^*}\) and \(q^*_e = q^*_e > q^*_e = q^*_e.\) Hence, Proposition 5(a) holds.

(b). When the entrant can only incrementally improve its quality over the incumbent’s, i.e., \(c_r > c_F\), then the incumbent maximizes its profit in (v) subject to the constraint \(0 < r \leq \bar{r}\), where
\( \bar{r} \) is defined in (iv), and gets its optimal royalty fee \( \bar{r}^* \):

\[
\bar{r}^* = \begin{cases} 
    \min \left\{ \frac{s_{q_k}^H (q_{q_k}^H - c_k^H) + q_k(q_{q_k} - c_k)}{2(8q_{q_k}^L + q_k)}, r^{HL} \right\}, & \text{if } c_k^H \leq c_r; \\
    \max \{ r_c, \min \left\{ \frac{s_{q_k}^L (q_{q_k}^L - c_k^L) + q_k(q_{q_k} - c_k)}{2(8q_{q_k}^L + q_k)}, r^{HL} \right\} \right\}, & \text{if } c_r < c_k^H \leq c_F \& \pi_{ir}(r^{HL}) \geq \pi_{ir}(r^{HH}); \\
    \min \left\{ \frac{s_{q_k}^L (q_{q_k}^L - c_k^L) + q_k(q_{q_k} - c_k)}{2(8q_{q_k}^L + q_k)}, r^{LL} \right\}, & \text{if } c_k^H > c_F; \\
\end{cases}
\]

where \( r_c \equiv \frac{(2q_{q_k}^H - q_k)(c_F - c_h^H)}{(2q_{q_k}^} - r^{HL} \right\} \) is the alternative form of writing \( c_k^H = c_r \), and \( c_r \) is the \( c_k^H \) such that \( c_r = r^{HL} \). Note that, if \( c_r < c_k^H \leq c_F \& \pi_{ir}(r^{HL}) \geq \pi_{ir}(r^{HH}), \) the incumbent’s optimal royalty fee is \( r^{HL^*} \) and \( q_{er}^* = q_{c_r}^L < q_{c_r}^H = q_{c_r}^* \). Hence, Proposition 5(b) holds.

Q.E.D.

**Proof of Proposition 6.** When the entrant can significantly improve its quality over the incumbent’s, if the incumbent’s optimal royalty fee \( r^* \) in (vi) is such that \( \frac{M^2 - \sqrt{M^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} < r^* \leq \bar{r} \) or \( \frac{N^2 - \sqrt{N^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} < r^* \leq \bar{r} \), then \( r^* D_{er}^* > F \) which means Proposition 6(b) holds; if \( r^* \) in (vi) is such that \( 0 < r^* < \frac{M^2 - \sqrt{M^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} \) or \( 0 < r^* < \frac{N^2 - \sqrt{N^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} \), then \( r^* D_{er}^* < F \) which means Proposition 6(a) holds, where \( M \) and \( N \) are given in the proof of Proposition 4. When the entrant can only incrementally improve its quality over the incumbent’s, if the incumbent’s optimal royalty fee \( r^* \) in (vii) is such that \( \frac{M^2 - \sqrt{M^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} < r^* \leq \bar{r} \) or \( \frac{N^2 - \sqrt{N^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} < r^* \leq \bar{r} \), then \( r^* D_{er}^* > F \) which means Proposition 6(b) holds; if the incumbent’s optimal royalty fee \( r^* \) in (vii) is such that \( 0 < r^* < \frac{M^2 - \sqrt{M^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} \) or \( 0 < r^* < \frac{N^2 - \sqrt{N^2 - 8(q_{q_k}^H - q_k)^2(4q_{q_k}^H - q_k)^F}}{4(q_{q_k}^H - q_k)} \), then \( r^* D_{er}^* < F \) which means Proposition 6(a) holds.

Q.E.D.
Inter-Competitor Licensing and Product Innovation

(Baojun Jiang, Hongyan Shi)

WEB APPENDIX

In this Web Appendix, we provide complementary analyses to the paper titled “Inter-Competitor Licensing and Product Innovation”. In particular, in Part A, we derive the parameter conditions for the main model such that both firms have non-negative profit in the market. In Part B, we analyze the model when assuming that one of the entrant’s quality choices is higher and the other is lower than the incumbent’s quality. In Part C, we analyze the model with a different game sequence where the incumbent decides the royalty licensing fee after the entrant’s decides its quality. In Part D, we analyze the model when the incumbent endogenously decides its quality with the anticipation of its competitor’s entry.

A. DERIVATIONS FOR SUFFICIENT PARAMETER CONDITION SUCH THAT BOTH FIRMS COEXIST IN THE MARKET FOR THE MAIN MODEL

According to the proof of Proposition 1, under no licensing contract, the incumbent’s and the entrant’s prices are

\[
p_i^* = \frac{q_e(q_e-q_i+c_e)+2q_ec_i}{4q_e-q_i} \quad \text{and} \quad p_e^* = \frac{q_e(2q_e-2q_i+2c_e+c_i)}{4q_e-q_i}.
\]

To ensure positive demand for each firm, we need \( \frac{p_i}{q_i} < \frac{p_e-p_i}{q_e-q_i} < 1 \). Plugging in the optimal prices \( p_e^* \) and \( p_i^* \) for \( \frac{p_e-p_i}{q_e-q_i} < 1 \) leads to \( \frac{q_e-c_e}{q_e-q_i} > \frac{q_e}{2q_e-q_i} \); Plugging in the optimal prices \( p_e^* \) and \( p_i^* \) for \( \frac{p_i}{q_i} < \frac{p_e-p_i}{q_e-q_i} \) leads to \( \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i} \). To summarize, to ensure positive demand for each firm, it is required \( \frac{q_e}{2q_e-q_i} < \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i} \).

To ensure positive profit margin for each firm, we need \( p_i > c_i \) and \( p_e > c_e \). Plugging in the optimal prices \( p_e^* \) and \( p_i^* \) for \( p_i > c_i \) leads to \( \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i} \); Plugging in the optimal prices \( p_e^* \) and \( p_i^* \) for \( p_e > c_e \) leads to \( \frac{q_e-c_e}{q_i-c_i} < \frac{2q_e-q_i}{q_i} \).
for \( p_e > c_e \) leads to \( \frac{q_e - c_e}{q_i - c_i} > \frac{q_e}{2q_e - q_i} \). To summarize, to ensure positive profit margin for each firm, it is required \( \frac{q_e}{2q_e - q_i} < \frac{q_e - c_e}{q_i - c_i} < \frac{2q_e - q_i}{q_i} \).

To ensure non-negative profit for the entrant, \( \pi^*_e = \frac{\left[ q_e(2q_e - 2q_i + c_i) - (2q_e - q_i)c_i \right] - (q_e - q_i)c_i^2}{(q_e - q_i)(4q_e - q_i)^2} \) and \( \pi^*_e = \frac{\left[ q_e^l(2q_e^l - 2q_i + c_i) - (2q_e^l - q_i)c_i \right] - (q_e^l - q_i)c_i^2}{(q_e^l - q_i)(4q_e^l - q_i)^2} \).

Hence, in the no licensing case, the parameters need to satisfy \( \frac{q_e}{2q_e - q_i} < \frac{q_e - c_e}{q_i - c_i} < \frac{2q_e - q_i}{q_i} \) and \( F \leq \max\left\{ \frac{\left[ q_e^H(2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_i \right] - (q_e^H - q_i)c_i^2}{(q_e^H - q_i)(4q_e^H - q_i)^2}, \frac{\left[ q_e^l(2q_e^l - 2q_i + c_i) - (2q_e^l - q_i)c_i \right] - (q_e^l - q_i)c_i^2}{(q_e^l - q_i)(4q_e^l - q_i)^2} \right\} \).

Under a licensing contract, the incumbent’s and the entrant’s prices are \( p^*_i = \frac{q_i(2q_i^e - q_i^e + c_i^e + r) + q_i r}{2(q_e - q_i)} \) and \( p^*_e = \frac{q_e[2(q_e^e - q_i^e + c_e^e) + c_e^e] + 2q_e^e c_e}{4q_e - q_i} \). To ensure positive demand for each firm, it is required that \( r < \min\left\{ \frac{q_e(c_i + 2q_e - 2q_i - c_i)}{2(q_e - q_i)}, \frac{q_e^e(c_e^e + q_e^e - q_i^e)q_i - (2q_e^e - q_i)c_{i^e}}{(q_e^e - q_i^e)q_i} \right\} \). Note that \( \frac{q_e^e(c_e^e + q_e^e - q_i^e)q_i - (2q_e^e - q_i)c_{i^e}}{(q_e^e - q_i^e)q_i} \) if and only if \( \frac{q_e - c_e}{q_i - c_i} < \frac{c_{i^e}}{c_i} \). To ensure positive profit margin for each firm, it is required that \( r > \frac{q_i(2q_{i^e} - 2q_i^e)(q_i - c_i)}{3q_i} \) (which is reduced to \( r \geq 0 \) because of the condition \( \frac{q_e - c_e}{q_i - c_i} < \frac{2q_e - q_i}{q_i} \) for the no licensing case) and \( r < \frac{q_e^e(c_e^e + 2q_e^e - 2q_i^e) - (2q_e^e - q_i)c_{i^e}}{2(q_e^e - q_i^e)} \). Note that non-negative profit for the entrant is guaranteed by \( r \). The condition of \( r \) dominates \( r < \frac{q_e^e(2q_e^e - 2q_i^e) - (2q_e^e - q_i)c_{i^e}}{2(q_e^e - q_i^e)} \).

Hence, in the case with a licensing contract, the parameters need to satisfy: \( \frac{q_e - c_e}{q_i - c_i} < \frac{c_{i^e}}{c_i} \) and \( 0 < r \leq F \); or \( \frac{q_e - c_e}{q_i - c_i} > \frac{c_{i^e}}{c_i} \) and \( 0 < r < \min\left\{ \frac{1}{2}, \frac{q_e}{q_i} \right\} \).

Therefore, combining with the condition in the no licensing case, one set of sufficient parameter conditions is: \( \frac{q_e}{2q_e - q_i} < \frac{q_e - c_e}{q_i - c_i} < \min\left\{ \frac{2q_e - q_i}{q_i}, \frac{c_{i^e}}{c_i} \right\} \) and \( F \leq \max\left\{ \frac{\left[ q_e^H(2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_i \right] - (q_e^H - q_i)c_i^2}{(q_e^H - q_i)(4q_e^H - q_i)^2}, \frac{\left[ q_e^l(2q_e^l - 2q_i + c_i) - (2q_e^l - q_i)c_i \right] - (q_e^l - q_i)c_i^2}{(q_e^l - q_i)(4q_e^l - q_i)^2} \right\} \).

Next, we show that with this set of sufficient parameter conditions, no firm will deviate from the equilibrium characterized in the paper.

First, we show that \( (p_{cm}, p_{cm}) = (p_{cm}, \infty) \) (i.e., the incumbent sets its price very high, let the entrant be the monopoly in the market and the incumbent’s profit only comes from the licensing
fee) cannot be an equilibrium, where \( p_{em} = \arg \max_{p_{em}} (p_{em} - c_{em} - r)(1 - \frac{p_{em}}{q_{em}}) = \frac{q_{em} + c_{em} + r}{2} \). When either the entrant or the incumbent is better off with deviating from \((p_{em}, \infty)\), then it is not an equilibrium.

The incumbent will deviate from \((p_{em}, \infty)\) if \( \pi_{ir}(p_{em}, p'_{ir}) > \pi_{ir}(p_{em}, \infty) = r(1 - \frac{p_{em}}{q_{em}}) = r\frac{q_{em} - c_{em} - r}{2q_{em}} \). Note that given \( p_{ir} = p_{em} \), the incumbent’s best response is given by \( p'_{ir} = \arg \max_{p_{ir}} (p_{ir} - c_{i}) (\frac{p_{em} - p_{ir}}{q_{em} - q_{i}} - \frac{p_{ir}}{q_{i}}) + r(1 - \frac{p_{em} - p_{ir}}{q_{em} - q_{i}}) = \frac{q_{i}(p_{em} + r) + q_{em} c_{i}}{2q_{em}} \), and the incumbent’s corresponding profit is \( \pi_{ir}(p_{em}, p'_{ir}) = \frac{4r^{2}q_{em}^{2} - 4c_{i} q_{em} q_{i} (c_{em} + q_{em} - r) + q_{i} c_{i}^{2}}{16 q_{em} (q_{em} - q_{i}) q_{i}} \) + \(2c_{em}(q_{em}q_{i} - 4q_{em}r + 3q_{i}r)\). Note that \( \pi_{ir}(p_{em}, p'_{ir}) = \pi_{ir}(p_{em}, \infty) \) if \( r = q_{em} + c_{em} - \frac{2c_{i} q_{em}}{q_{i}} \) and \( \pi_{ir}(p_{em}, p'_{ir}) > \pi_{ir}(p_{em}, \infty) \) otherwise. If \( \frac{q_{em} - c_{em}}{q_{i} - c_{i}} < \frac{q_{em}}{c_{i}} \) (which comes from the set of parameter condition derived above), that is, if \( \frac{q_{em}}{c_{em}} < \frac{q_{i}}{c_{i}} \), then \( q_{em} + c_{em} - \frac{2c_{i} q_{em}}{q_{i}} > q_{em} - c_{em} \), which means when the licensing fee is higher than \( q_{em} - c_{em} \) the incumbent will be indifferent. Hence, \( \pi_{ir}(p_{em}, p'_{ir}) > \pi_{ir}(p_{em}, \infty) \). So \((p_{ir}, p_{ir}) = (p_{em}, \infty)\) cannot be an equilibrium. Therefore, the incumbent setting its price extremely high and letting the entrant be the monopoly cannot be an equilibrium.

Second, we show that given \( p^{*}_{i} \), the entrant will not be better off with deviating to price-out the incumbent. If the entrant sets its price \( p'_{i} = p^{*}_{e} - \frac{q_{e}}{q_{i}} \), then the entrant can price out the incumbent and the entrant’s profit is \( \pi'_{e} = (p^{*}_{e} - \frac{q_{e}}{q_{i}} - c_{e})(1 - \frac{p^{*}_{e}}{q_{i}}) = \frac{2c_{i} q_{i}^{2} + q_{i}(q_{e} - q_{i}) - c_{e} (3q_{e} - q_{i})}{q_{i}(q_{e} - q_{i})} \). The entrant will only deviate from \((p^{*}_{i}, p^{*}_{e})\) to \((p^{*}_{i}, p'_{e})\) when making higher profit, i.e., when \( \pi'_{e} > \pi^{*}_{e} \). Note that \( \pi'_{e} = \pi^{*}_{e} \) if \( c_{i} = \frac{q_{i}(q_{e} - q_{i} + c_{e})}{2q_{e} - q_{i}} \) and \( \pi'_{e} < \pi^{*}_{e} \) otherwise. Hence, for given \( p^{*}_{i} \), the entrant will not be better off with deviating to price-out the incumbent.

Last, we show that given \( p^{*}_{e} \), the incumbent will not be better off with deviating to price-out the entrant. If the incumbent sets its price \( p'_{i} = p^{*}_{e} - q_{e} + q_{i} \), then the incumbent can price out the entrant and the incumbent’s profit is \( \pi'_{i} = (p'_{i} - c_{i})(1 - \frac{p'_{i}}{q_{i}}) = \frac{q_{i}(2q_{e} + q_{i} - 2c_{e} - c_{i}) (2q_{e} + 3q_{e}/q_{i} + q_{i} - 2q_{e}^{2} - 3c_{i} q_{i})}{q_{i}(q_{e} - q_{i})^{2}} \). The incumbent will only deviate from \((p^{*}_{i}, p^{*}_{e})\) to \((p'_{i}, p^{*}_{e})\) when making higher profit, i.e., when \( \pi'_{i} > \pi^{*}_{i} \).
Note that \( \pi_i' = \pi_i^* \) if \( c_i = q_i(2 - \frac{c_e}{q_e}) - (2q_e - 2c_e) \) and \( \pi_i' < \pi_i^* \) otherwise. Hence, for given \( p^e \), the incumbent will not be better off with deviating to price-out the entrant.

Therefore, the set of parameter conditions \( \frac{q_e}{2q_e - q_i} < \frac{q_e - c_e}{q_e - c_i} < \min\{\frac{2q_e - q_i}{q_i}, \frac{c_e}{c_i}\} \) and \( F \leq \max\{\frac{[q_e^H(2q_i^H-2q_i+c_i)-(2q_e^H-q_i)c_e^H]}{(q_i^2-q_i)(4q_i^H-q_i)^2}, \frac{[q_i^L(2q_e^L-2q_e+c_e)-(2q_i^L-q_e)c_e^L]}{(q_e^2-q_e)(4q_e^L-q_e)^2}\} \) is sufficient to ensure two firms co-exist in the market.

B. Analysis for the model with \( q_e^H > q_i > q_e^L \) and \( c_e^H > c_i > c_e^L \)

In this part of the Web Appendix, we allow the entrant’s quality choices and associated costs are such that \( q_e^H > q_i > q_e^L \) and \( c_e^H > c_i > c_e^L \). In the following, we first analyze the case when the entrant develops the non-core technology on its own, i.e., without a licensing contract; then we analyze the case when there is a licensing contract between the two competitors; last, we compare the entrant’s optimal quality in the two cases to examine the effect of licensing on the entrant’s product innovation.

Without a licensing contract, if the entrant chooses \((q_e, c_e) = (q_e^H, c_e^H)\), then \( D_e^H = 1 - \frac{p_e^H - p_i^H}{q_e^H - q_i} \) and \( D_i^H = \frac{p_e^H - p_i^H}{q_e^H - q_i} - \frac{p_i^H}{q_i} \). The entrant’s and the incumbent’s profit functions are \( \pi_e^H = (p_e^H - c_e^H)D_e^H - F \) and \( \pi_i^H = (p_i^H - c_i)D_i^H \), respectively. We solve the game based on backward induction. Since \( \pi_e^H \) is a concave function of \( p_e^H \) and \( \pi_i^H \) is a concave function of \( p_i^H \), simultaneously solving the first order conditions \( \frac{d\pi_e^H}{dp_e^H} = 0 \) and \( \frac{d\pi_i^H}{dp_i^H} = 0 \) gives: \( p_e^{H*} = \frac{q_i^H(2q_i^H-2q_i+c_i)}{4q_i^H-q_e} \) and \( p_i^{H*} = \frac{q_i^H(q_i^H-q_i+c_i^H)+2q_i^Hc_i}{4q_i^H-q_e} \). Two firms’ profits are: \( \pi_e^{H*} = \frac{[q_i^H(2q_i^H-2q_i+c_i)-(2q_i^H-q_i)c_i^H]^2}{(q_i^H-q_i)(4q_i^H-q_i)^2} - F \) and \( \pi_i^{H*} = \frac{[q_i^H(q_i^H-q_i+c_i^H)-(2q_i^H-q_i)c_i]^2}{(q_i^H-q_i)(4q_i^H-q_i)^2} \).

Alternatively, if the entrant chooses \((q_e, c_e) = (q_e^L, c_e^L)\), then \( D_e^L = \frac{p_e^L - p_i^L}{q_i - q_e^L} - \frac{p_i^L}{q_e^L} \) and \( D_i^L = 1 - \frac{p_e^L - p_i^L}{q_i - q_e^L} \). The entrant’s and the incumbent’s profit functions are \( \pi_e^L = (p_e^L - c_e^L)D_e^L - F \) and \( \pi_i^L = (p_i^L - c_i)D_i^L \), respectively. Since \( \pi_e^L \) is a concave function of \( p_e^L \) and \( \pi_i^L \) is a concave function of \( p_i^L \),
simultaneously solving the first order conditions \( \frac{dp_e}{dq_e} = 0 \) and \( \frac{dx_i^L}{dq_i^L} = 0 \) gives: \( p_e^* = \frac{q_e^c(q_e^c-q_e^c+c_i)+2q_ic_i}{4q_e^c-q_e^c} \), and \( p_i^L = \frac{q_i(2q_i-2q_i^L+2c_i+c_i^r)}{4q_i-q_i^L} \). Two firms’ profits are: \( \pi_e^L = \frac{q_e^c(q_e^c-q_e^c+c_i)(2q_e^c-q_e^c)c_i^r}{(4q_e^c-q_e^c)^2} - F \) and \( \pi_i^L = \frac{[q_i(2q_i-2q_i^L+c_i)+2q_i(2q_i^L-q_i^L)c_i^r]}{(4q_i-q_i^L)^2} \).

Hence, in the case of no licensing contract, the entrant’s optimal quality decision is \( q_e^* = q_e^H \) if \( \pi_e^H > \pi_e^L \); that is, if \( c^H_e \leq c_F = \frac{q_e^H(2q_e^H-2q_e^c+c_i)-R[2q_e^H(q_e^H-q_e^c+c_i)-(2q_e^H)(c_i^r)]}{2q_e^H-q_e^c} \) where \( R = \frac{q_e^H(q_e^H-q_e^c)}{4q_e^H-q_e^c} \). \( q_e^* = q_e^L \) otherwise.

In the case of a licensing contract with royalty fee \( r > 0 \), if the entrant chooses \( (q_{er}, c_{er}) = (q_e^H, c_e^H) \), then \( D_{er}^H = 1 - \frac{p_e^H-r^H}{q_e^H-q_e} \) and \( D_{ir}^H = \frac{p_e^H-r^H}{q_e^H-q_e} - \frac{p_e^H}{q_e} \). The entrant’s and the incumbent’s profit functions are \( \pi_{er}^H = (p_{er}^H - c_e^H)D_{er}^H - r \times D_{er}^H \) and \( \pi_{ir}^H = (p_{ir}^H - c_i)D_{ir}^H + r \times D_{ir}^H \), respectively. Simultaneously solving the first order conditions \( \frac{dp_{er}^H}{dq_{er}^L} = 0 \) and \( \frac{dx_{ir}^H}{dq_{ir}^L} = 0 \) gives: \( p_{er}^{H*} = \frac{q_e^H(q_e^H-q_e^c+c_i^H+3r)}{4q_e^H-q_e} \) and \( p_{ir}^{H*} = \frac{q_i^H(2q_i^H-2q_i^L+c_i)+2q_i^H(2q_i^H-q_i^L)r}{4q_i^H-q_i^L} \).

Two firms’ profits are: \( \pi_{er}^{H*} = \frac{[q_e^H(2q_e^H-2q_e^c+c_i)-R[2q_e^H(q_e^H-q_e^c+c_i)-(2q_e^H)(c_i^r)]]}{(4q_e^H-q_e^c)^2} \) and \( \pi_{ir}^{H*} = \frac{[q_i^H(2q_i^H-2q_i^L+c_i)+(2q_i^H)(2q_i^H-q_i^L)r]}{(4q_i^H-q_i^L)^2} \).

Alternatively, if the entrant chooses \( (q_{er}, c_{er}) = (q_e^L, c_e^L) \), then \( D_{er}^L = \frac{p_e^L-p_i^L}{q_e^L-q_i} - \frac{p_i^L}{q_i} \) and \( D_{ir}^L = 1 - \frac{p_i^L}{q_i} \). The entrant’s and the incumbent’s profit functions are \( \pi_{er}^L = (p_{er}^L - c_e^L)D_{er}^L - r \times D_{er}^L \) and \( \pi_{ir}^L = (p_{ir}^L - c_i)D_{ir}^L + r \times D_{ir}^L \), respectively. Simultaneously solving the first order conditions \( \frac{dp_{er}^L}{dq_{er}^L} = 0 \) and \( \frac{dx_{ir}^L}{dq_{ir}^L} = 0 \) gives: \( p_{er}^{L*} = \frac{q_e^L(q_e^L-q_e^L+c_i)+2q_e^L(2q_e^L+q_i^L)r}{4q_e^L-q_i^L} \) and \( p_{ir}^{L*} = \frac{(q_i-2q_i^L+2c_i+c_i^r)}{4q_i-q_i^L} \). Two firms’ profits are: \( \pi_{er}^{L*} = \frac{[q_e^L(2q_e^L-2q_e^c+c_i)-R[2q_e^L(q_e^L-q_e^c+c_i)-(2q_e^L)(c_i^r)]]}{(4q_e^L-q_e^c)^2} \) and \( \pi_{ir}^{L*} = \frac{[q_i(2q_i-2q_i^L+c_i)+(2q_i^L)(2q_i^L-q_i^L)r]}{(4q_i-q_i^L)^2} \).

Hence, given a licensing contract with royalty fee \( r > 0 \), the entrant’s optimal quality decision is \( q_{er}^* = q_e^H \) if \( \pi_{er}^{H*}(r) \geq \pi_{er}^{L*}(r) \); that is, \( c^H_e \leq c_r = c_F + 2r \frac{R(q_i-q_e^c)-(q_e^H-q_i)}{2q_e^H-q_i} \); and \( q_{er}^* = q_e^L \) otherwise.

Therefore, if \( R(q_i-q_e^L)-(q_e^H-q_i) > 0 \), then \( c_r > c_F \) and licensing can increase the entrant’s optimal quality. If \( R(q_i-q_e^L)-(q_e^H-q_i) < 0 \), then \( c_r < c_F \) and licensing can decrease the entrant’s optimal
quality. Next, we show the parameter conditions under which licensing can increase or decrease the entrant’s optimal quality.

Let \( \alpha \equiv \frac{q^H_q}{q^L_q} \) and \( \beta \equiv \frac{q^L_q}{q^H_q} \) where \( \alpha > 1 > \beta > 0 \). Then \( R(q_i - q^L_q) - (q^H_q - q_i) > 0 \) is equivalent to \( \frac{4\alpha - 1}{\beta} > \sqrt{\frac{\beta(\alpha - 1)}{1 - \beta}} \). Solving the inequality \( \frac{4\alpha - 1}{\beta} > \sqrt{\frac{\beta(\alpha - 1)}{1 - \beta}} \) leads to the following solution: When \( \beta < \frac{2}{3}(4 - \frac{32}{(27\sqrt{73} - 143)^{1/3}} + (27\sqrt{73} - 143)^{1/3}) \approx 0.82658 \) and \( \alpha > 1 \); or when \( \beta > 0.82658 \) and \( \alpha > \frac{(4 - \beta)^2}{64} \left( \sqrt{\frac{\beta}{1 - \beta}} + \sqrt{\frac{\beta}{1 - \beta} - \frac{48}{(4 - \beta)^2}} \right)^2 + 1 \) or \( \alpha < \frac{(4 - \beta)^2}{64} \left( \sqrt{\frac{\beta}{1 - \beta} - \sqrt{\frac{\beta}{1 - \beta} - \frac{48}{(4 - \beta)^2}} \right)^2 + 1 \), then \( \frac{4\alpha - q^L_q}{4q_i - q^L_q} > \sqrt{\frac{q^L_q(q^H_q - q_i)}{q_i(q^H_q - q^L_q)}} \) holds (i.e., \( c_r > c_F \) holds) and licensing can increase the entrant’s optimal quality.

Accordingly, when \( \beta > 0.82658 \) and \( \frac{(4 - \beta)^2}{64} \left( \sqrt{\frac{\beta}{1 - \beta}} + \sqrt{\frac{\beta}{1 - \beta} - \frac{48}{(4 - \beta)^2}} \right)^2 + 1 < \alpha < \frac{(4 - \beta)^2}{64} \left( \sqrt{\frac{\beta}{1 - \beta} - \sqrt{\frac{\beta}{1 - \beta} - \frac{48}{(4 - \beta)^2}} \right)^2 + 1 \) or \( \frac{(4 - \beta)^2}{64} \left( \sqrt{\frac{\beta}{1 - \beta} - \frac{48}{(4 - \beta)^2}} \right)^2 + 1 \), then \( \frac{4\alpha - 1}{\beta} < \sqrt{\frac{\beta(\alpha - 1)}{1 - \beta}} \) holds, which means that \( \frac{4\alpha - q^H_q}{4q_i - q^H_q} < \sqrt{\frac{q^L_q(q^H_q - q_i)}{q_i(q^H_q - q^L_q)}} \) holds (i.e., \( c_r < c_F \) holds) and licensing can decrease the entrant’s optimal quality.

Given the entrant’s optimal response of its quality decision, the incumbent optimally decides its royalty fee by maximizing its resulting profit function subject to the constraint that the contract is mutually acceptable. We follow the same procedure of the derivation of \( \bar{r} \) in the proof of Proposition 2 to derive the upper bound of royalty fee such that the entrant is (weakly) better off with accepting the licensing contract than developing its own non-core technology. Specifically, when \( c_r > c_F \), \( \bar{r} \) is given by solving the following problem:

\[
\begin{cases}
\pi^H_{er}(r) \geq \pi^H_{er}, & \text{if } c^L_e < c^H_e \leq c_F; \\
\pi^H_{er}(r) \geq \pi^L_{er}, & \text{if } c_F < c^H_e \leq c_r; \\
\pi^L_{er}(r) \geq \pi^L_{er}, & \text{if } c^H_e > c_r.
\end{cases}
\] (W1)
Hence, the entrant will accept the royalty licensing contract if $r \leq \bar{r}$, where

$$
\tau = \begin{cases} 
\tau^{HH} & \equiv \frac{q_e^H (2q_e^H - 2q_i + c_i) - (2q_e^H - q_i)c_e^H - (4q_e^H - q_i)\sqrt{(q_e^H - q_i)\pi_e^{H*}}}{2q_e^H - 2q_i}, & \text{if } c_e^L < c_e^H \leq c_F; \\
\tau^{LH} & \equiv \frac{q_e^L (2q_e^L - 2q_i + c_i) - (2q_e^L - q_i)c_e^L - (4q_e^L - q_i)\sqrt{(q_e^L - q_i)\pi_e^{L*}}}{2q_e^L - 2q_i}, & \text{if } c_F < c_e^H \leq c_r; \\
\tau^{LL} & \equiv \frac{q_e^L (q_i - q_e^L + c_i) - (2q_i - q_e^L)c_e^L - (4q_i - q_e^L)\sqrt{q_e^L (q_i - q_e^L)\pi_e^{L*}}}{2q_i - 2q_e^L}, & \text{if } c_e^H > c_r. 
\end{cases}
$$

When $c_r < c_F$, $\bar{r}$ is given by solving the following problem:

$$
\begin{cases} 
\pi_e^{H*}(r) \geq \pi_e^{H*}, & \text{if } c_e^L < c_e^H \leq c_r; \\
\pi_e^{L*}(r) \geq \pi_e^{H*}, & \text{if } c_r < c_e^H \leq c_F; \\
\pi_e^{L*}(r) \geq \pi_e^{L*}, & \text{if } c_e^H > c_F.
\end{cases}
$$

Hence, the entrant will accept the royalty licensing contract if $r \leq \bar{r}$, where

$$
\tau = \begin{cases} 
\tau^{HH}, & \text{if } c_e^L < c_e^H \leq c_r; \\
\tau^{HL} & \equiv \frac{q_e^L (q_i - q_e^L + c_i) - (2q_i - q_e^L)c_e^L - (4q_i - q_e^L)\sqrt{q_e^L (q_i - q_e^L)\pi_e^{H*}}}{2q_i - 2q_e^L}, & \text{if } c_r < c_e^H \leq c_F; \\
\tau^{LL}, & \text{if } c_e^H > c_F.
\end{cases}
$$

Following the same procedure as in the proof of Proposition 5, we next derive the incumbent’s optimal royalty fee $r^\ast$.

Anticipating the entrant’s best response on quality decision, the incumbent’s resulting profit function is

$$
\pi_{ir} = \begin{cases} 
\pi_{ir}^H(r) \equiv \pi_{ir}^*(q_e^H, c_e^H, r), & \text{if } c_e^H \leq c_r; \\
\pi_{ir}^L(r) \equiv \pi_{ir}^*(q_e^L, c_e^L, r), & \text{if } c_e^H > c_r;
\end{cases}
$$

where both $\pi_{ir}^H(r)$ and $\pi_{ir}^L(r)$ are concave in $r$. 
When \( c_r > c_F \), then the incumbent maximizes its profit in (W5) subject to the constraint
\[
0 < r \leq \bar{r}, \text{ where } \bar{r} \text{ is defined in (W2), and obtains its optimal royalty fee } r^*:
\]
\[
r^* = \begin{cases} 
\min \left\{ \frac{8q^H_r(i - e_r) + q_i(q_i - e_i)}{2(8q^H_r + q_i)}, \pi^{HH} \right\}, & \text{if } c^H_r \leq c_F; \\
r^{LH*} \equiv \max \left\{ r_c, \min \left\{ \frac{8q^H_r(i - e_r) + q_i(q_i - e_i)}{2(8q^H_r + q_i)}, \pi^{LH} \right\} \right\}, & \text{if } c_F < c^H_r \leq \bar{r} \text{ & } \pi^{H*}_{ir}(r^{LH*}) \geq \pi^{L*}_{ir}(r^{LH*}); \\
r^{LL*} \equiv \min \left\{ \frac{8q^H_r(i - e_r) + (q^H_r)^2(q_i - e_i)}{2q_i(8q^H_r + q_i)}, r_c \right\}, & \text{if } c_F < c^H_r \leq \bar{r} \text{ & } \pi^{H*}_{ir}(r^{LH*}) < \pi^{L*}_{ir}(r^{LH*}); \\
\min \left\{ \frac{8q^H_r(i - e_r) + (q^H_r)^2(q_i - e_i)}{2q_i(8q^H_r + q_i)}, \pi^{LL} \right\}, & \text{if } c^H_r > \bar{r};
\end{cases}
\]
\[(W6)\]
where \( r_c \equiv \frac{(2q^H_r - q_i)(c^H_r - c_F)}{R(2q_i - 2q^H_r) - (2q^H_r - 2q_i)} \) is the alternative form of writing \( c^H_r = c_r \), and \( \bar{r} \equiv c_r |_{r=\pi^{LL}}. \) Note that, if \( c_F < c^H_r \leq \bar{r} \text{ & } \pi^{H*}_{ir}(r^{LH*}) \geq \pi^{L*}_{ir}(r^{LH*}) \), the incumbent’s optimal royalty fee is \( r^{LH*} \) and \( q^*_e = q^H_e > q^L_e = q^*_c. \)

When \( c_r < c_F \), then the incumbent maximizes its profit in (W5) subject to the constraint
\[
0 < r \leq \bar{r}, \text{ where } \bar{r} \text{ is defined in (W4), and obtains its optimal royalty fee } r^*:
\]
\[
r^* = \begin{cases} 
\min \left\{ \frac{8q^H_r(i - e_r) + q_i(q_i - e_i)}{2(8q^H_r + q_i)}, \pi^{HH} \right\}, & \text{if } c^H_r \leq \bar{r}; \\
r^{HL*} \equiv \max \left\{ r_c, \min \left\{ \frac{8q^H_r(i - e_r) + (q^H_r)^2(q_i - e_i)}{2q_i(8q^H_r + q_i)}, \pi^{HL}\right\} \right\}, & \text{if } \bar{r} < c^H_r \leq c_F \text{ & } \pi^{H*}_{ir}(r^{HL*}) \geq \pi^{L*}_{ir}(r^{HL*}); \\
r^{HH*} \equiv \min \left\{ \frac{8q^H_r(i - e_r) + q_i(q_i - e_i)}{2(8q^H_r + q_i)}, r_c \right\}, & \text{if } \bar{r} < c^H_r \leq c_F \text{ & } \pi^{H*}_{ir}(r^{HL*}) < \pi^{L*}_{ir}(r^{HL*}); \\
\min \left\{ \frac{8q^H_r(i - e_r) + (q^H_r)^2(q_i - e_i)}{2q_i(8q^H_r + q_i)}, \pi^{LL}\right\}, & \text{if } c^H_r > c_F; 
\end{cases}
\]
\[(W7)\]
where \( r_c \equiv \frac{(2q^H_r - q_i)(c_F - c^H_r)}{(2q^H_r - 2q_i) - R(2q_i - 2q^H_r)} \) is the alternative form of writing \( c^H_r = c_r \), and \( \bar{r} \) is the \( c^H_r \) such that \( c_r = \pi^{HL}. \) Note that, if \( \bar{r} < c^H_r \leq c_F \text{ & } \pi^{H*}_{ir}(r^{HL*}) \geq \pi^{L*}_{ir}(r^{HL*}) \), the incumbent’s optimal royalty fee is \( r^{HL*} \) and \( q^*_c = q^L_e < q^H_e = q^*_c. \)
C. Analysis for the model with a different game sequence

In this part of the Web Appendix, the game sequence is defined as follows: first, the entrant decides its product quality; second, the incumbent sets its royalty licensing fee; third, the entrant decides to accept or not accept the contract; last, two firms set prices. Except this game sequence, the other aspects of model setup are the same as in the main model. In particular, \( q^H_e > q^L_e > q_i \), \( c^H_e > c^L_e > c_i \), and the entrant incurs a fixed cost \( F \) when developing the non-core technology on its own. In the following, we first solve the game where the entrant develops the non-core technology on its own, i.e., without a licensing contract; then we solve the game where there is a licensing contract between the two competitors; last, we compare the entrant’s optimal quality in the two cases to examine the effect of licensing on the entrant’s product innovation. The game solving is based on backward induction.

When the entrant develops the non-core technology on its own, the entrant’s optimal product quality, two firms’ optimal prices and their corresponding profits are the same as in the benchmark case in the main model. That is, \( q^*_e = q^H_e \) with the incumbent’s and entrant’s optimal profits \( \pi^H_i \) and \( \pi^H_e \) if \( c^H_e \leq c_F \) and \( q^*_e = q^L_e \) with the incumbent’s and entrant’s optimal profits \( \pi^L_i \) and \( \pi^L_e \) otherwise.

When the entrant anticipates an acceptable licensing contract from the incumbent, there are two sub-games based on the entrant’s possible quality decision: \( q_{er} = q^H_e \) or \( q_{er} = q^L_e \).

Sub-game 1: \( q_{er} = q^H_e \)

Given \( p^H_{er}, p^H_{ir}, q_{er} = q^H_e, q_i, \) and \( r^H \), the incumbent’s and the entrant’s profit functions are

\[
\pi^H_{ir} = (p^H_{ir} - c_i)(\frac{p^H_{er} - p^H_{ir}}{q^H_i - q_i}) + r^H(1 - \frac{p^H_{er} - p^H_{ir}}{q^H_i - q_i}),
\]

\[
\pi^H_{er} = (p^H_{er} - c^H_e - r^H)(1 - \frac{p^H_{er} - p^H_{ir}}{q^H_i - q_i}),
\]

Solving the first order conditions \( \frac{d\pi^H_{ir}}{dp^H_{ir}} = 0 \) and \( \frac{d\pi^H_{er}}{qp^H_{er}} = 0 \) simultaneously gives the optimal prices:
\[ p_{ir}^{H*} = \frac{q_{i}(q_{i}^{H} - q_{i} + c_{i}^{H} + 3q_{i}^{H})}{4q_{i}^{H} - q_{i}} \quad \text{and} \quad p_{er}^{H*} = \frac{q_{i}^{H}[2(q_{i}^{H} - q_{i} + c_{i}^{H} + r_{i}) + c_{i} + q_{r}r_{i}]}{4q_{i}^{H} - q_{i}}. \]

The corresponding profits are:

\[ \pi_{ir}^{H}(r_{i}) = \frac{q_{i}^{H}[q_{i}(q_{i}^{H} - q_{i} + c_{i}^{H}) - c_{i}(2q_{i}^{H} - q_{i})]^{2} + q_{i}(q_{i}^{H} - q_{i})[q_{r}^{H} + 8q_{i}^{H}(q_{i}^{H} - c_{i}) - c_{i}]r_{i} - (8q_{i}^{H} + q_{i})(q_{i}^{H} - q_{i})_{q_{i}(r_{i})^{2}}}{q_{i}(q_{i}^{H} - q_{i})(4q_{i}^{H} - q_{i})^{2}} \]

and

\[ \pi_{er}^{H}(r_{i}) = \frac{q_{i}^{H}[2q_{i}^{H} - 2q_{i} + c_{i} - c_{i}^{H}(2q_{i}^{H} - q_{i}) - r_{i}^{H}(2q_{i}^{H} - 2q_{i})]^{2}}{(q_{i}^{H} - q_{i})(4q_{i}^{H} - q_{i})^{2}}. \]

Next, we solve for the incumbent’s optimal licensing fee in this sub-game, which we denote as \( r_{i}^{H*} \).

Specifically, \( r_{i}^{H*} = \text{argmax}_{r_{i}} \pi_{ir}^{H}(r_{i}) \) s.t. \( \pi_{ir}^{H}(r_{i}) \geq \pi_{ir}^{H*}, \pi_{ir}^{H}(r_{i}) \geq \pi_{i}^{H*}. \) Constraint \( \pi_{ir}^{H}(r_{i}) \geq \pi_{ir}^{H*} \) is reduced to \( r_{i}^{H} \leq \frac{q_{i}^{H}[2q_{i}^{H} - 2q_{i} + c_{i} - c_{i}^{H}(2q_{i}^{H} - q_{i}) - r_{i}^{H}(2q_{i}^{H} - 2q_{i})]^{2}}{2q_{i}^{H} - 2q_{i}} \); and \( \pi_{ir}^{H}(r_{i}) \geq \pi_{i}^{H*} \) is reduced to \( r_{i}^{H} \geq 0. \) Therefore, given \( q_{er} = q_{i}^{H} \), the optimal licensing fee is \( r_{i}^{H*} = \min\{r_{i}^{HH}, \frac{q_{r}^{H} + 8q_{i}^{H}(q_{i}^{H} - c_{i})}{2(8q_{i}^{H} + q_{i})} \}. \)

And the entrant’s corresponding profit is

\[ \pi_{er}^{H*} = \frac{|q_{i}^{H}[2q_{i}^{H} - 2q_{i} + c_{i} - c_{i}^{H}(2q_{i}^{H} - q_{i}) - r_{i}^{H}(2q_{i}^{H} - 2q_{i})]^{2}}{(q_{i}^{H} - q_{i})(4q_{i}^{H} - q_{i})^{2}}. \]

Sub-game 2: \( q_{er} = q_{e}^{L} \)

Given \( p_{i}^{L}, p_{r}^{L}, q_{er} = q_{e}^{L}, q_{i}, \) and \( r^{L} \), the incumbent’s and the entrant’s profit functions are:

\[ \pi_{ir} = (p_{i}^{L} - c_{i})(p_{r}^{L} - p_{e}^{L}) + r^{L}(1 - \frac{p_{e}^{L} - p_{r}^{L}}{q_{e}^{L} - q_{i}}) \quad \text{and} \quad \pi_{er} = (p_{e}^{L} - r^{L} - r^{L})(1 - \frac{p_{e}^{L} - p_{r}^{L}}{q_{e}^{L} - q_{i}}), \]

respectively.

Solving the first order conditions \( \frac{d\pi_{ir}}{dp_{i}^{L}} = 0 \) and \( \frac{d\pi_{ir}}{dp_{r}^{L}} = 0 \) simultaneously gives the optimal prices:

\[ p_{i}^{L*} = \frac{q_{i}^{L} - q_{i}^{L}c_{i}^{L} + 3q_{r}^{L}r^{L}}{4q_{i}^{L} - q_{i}} \quad \text{and} \quad p_{r}^{L*} = \frac{q_{i}^{L}[2q_{i}^{L} - q_{i}^{L}c_{i}^{L} + r^{L} + c_{i} + q_{r}^{L}r^{L}]}{4q_{i}^{L} - q_{i}}. \]

The corresponding profits are:

\[ \pi_{ir}^{L}(r^{L}) = \frac{q_{i}^{L}[q_{i}^{L} - q_{i}^{L}c_{i}^{L} + c_{i}(2q_{i}^{L} - q_{i})]^{2} + q_{i}(q_{i}^{L} - q_{i})[q_{r}^{L} + 8q_{i}^{L}(q_{i}^{L} - c_{i}) - c_{i}]r^{L} - (8q_{i}^{L} + q_{i})(q_{i}^{L} - q_{i})r^{L}_{q_{i}}(r^{L})^{2}}{q_{i}(q_{i}^{L} - q_{i})(4q_{i}^{L} - q_{i})^{2}} \]

and

\[ \pi_{er}^{L}(r^{L}) = \frac{|q_{i}^{L}[2q_{i}^{L} - 2q_{i} + c_{i} - c_{i}^{L}(2q_{i}^{L} - q_{i}) - r^{L}(2q_{i}^{L} - 2q_{i})]^{2}}{(q_{i}^{L} - q_{i})(4q_{i}^{L} - q_{i})^{2}}. \]

Next, we solve for the incumbent’s optimal licensing fee in this sub-game, which we denote as \( r^{L*} \).

Specifically, \( r^{L*} = \text{argmax}_{r^{L}} \pi_{ir}^{L}(r^{L}) \) s.t. \( \pi_{ir}^{L}(r^{L}) \geq \pi_{i}^{L*}, \pi_{ir}^{L}(r^{L}) \geq \pi_{i}^{L*}. \) Constraint \( \pi_{ir}^{L}(r^{L}) \geq \pi_{ir}^{L*} \) is reduced to \( r^{L} \leq \frac{q_{i}^{L}[2q_{i}^{L} - 2q_{i} + c_{i} - c_{i}^{L}(2q_{i}^{L} - q_{i}) - 4q_{i}^{L} - q_{i}](q_{i}^{L} - q_{i})^{2}}{2q_{i}^{L} - 2q_{i}} \); and \( \pi_{ir}^{L}(r^{L}) \geq \pi_{i}^{L*} \) is reduced to \( r^{L} \geq 0. \) Therefore, given \( q_{er} = q_{i}^{L} \), the optimal licensing fee is \( r^{L*} = \min\{r^{LL}, \frac{q_{r}^{L} + 8q_{i}^{L}(q_{i}^{L} - c_{i})}{2(8q_{i}^{L} + q_{i})} \}. \)

And the entrant’s corresponding profit is \( \pi_{er}^{L*} = \frac{|q_{i}^{L}[2q_{i}^{L} - 2q_{i} + c_{i} - c_{i}^{L}(2q_{i}^{L} - q_{i}) - r^{L*}(2q_{i}^{L} - 2q_{i})]^{2}}{(q_{i}^{L} - q_{i})(4q_{i}^{L} - q_{i})^{2}}. \)

Last, we solve for the entrant’s optimal quality decision by comparing the entrant’s profit

\[ \pi_{er}^{H*} = \frac{|q_{i}^{H}[2q_{i}^{H} - 2q_{i} + c_{i} - c_{i}^{H}(2q_{i}^{H} - q_{i}) - r^{H*}(2q_{i}^{H} - 2q_{i})]^{2}}{(q_{i}^{H} - q_{i})(4q_{i}^{H} - q_{i})^{2}} \]

to its profit \( \pi_{er}^{L*} = \frac{|q_{i}^{L}[2q_{i}^{L} - 2q_{i} + c_{i} - c_{i}^{L}(2q_{i}^{L} - q_{i}) - r^{L*}(2q_{i}^{L} - 2q_{i})]^{2}}{(q_{i}^{L} - q_{i})(4q_{i}^{L} - q_{i})^{2}}, \)


where \( r^{H_*} = \min\{\bar{r}^{HH}, \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)}\} \) and \( r^{L_*} = \min\{\bar{r}^{LL}, \frac{q^2_i + 8q_i^L (q^L - c^L_i) - c_q q_i}{2(q^L_i + q_i)}\} \). Hence, under a licensing contract, the entrant’s optimal quality decision is: \( q^*_e = q^H_e \) if \( \pi^{H_*}_{er} \geq \pi^{L_*}_{er} \), that is, if \( c^H_e \leq \frac{q^H_i (2q_i^H - 2q_i + c_i) - R(q^H_i (2q_i^H - 2q_i + c_i) - c_q (2q_i^H - q_i))}{2q_i^H - q_i} + \frac{R(2q_i^H - 2q_i) r^{L_*} - (2q_i^H - 2q_i) r^{H_*}}{2q_i^H - q_i} = c_F + \frac{R(2q_i^H - 2q_i) r^{L_*} - (2q_i^H - 2q_i) r^{H_*}}{2q_i^H - q_i} \) and \( q^*_e = q^L_e \) otherwise. Next, we show that with this different game sequence, the quality-increasing effect and quality-decreasing effect of licensing can still occur under certain conditions by considering two cases: (i) at \( c^H_e = c_F \), \( \bar{r}^{HH} < \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \), (ii) at \( c^H_e = c_F \), \( \bar{r}^{HH} > \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \).

(i) If at \( c^H_e = c_F \), \( \bar{r}^{HH} < \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \), then \( r^{H_*} = \bar{r}^{HH} \) for \( c^H_e \leq c_F \) since \( \bar{r}^{HH} \) is increasing in \( c^H_e \) and \( \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \) is decreasing in \( c^H_e \). Note that then \( \pi^{H_*}_{e,c_F} = \pi^{HH}_{c_F,c_F} \). That is, \( \pi^{HH}_{c_F,c_F} = \pi^{LL}_{c_F,c_F} = \pi^{L_*}_{c_F,c_F} = 0 \). By definition, \( r^{L_*} \leq \bar{r}^{LL} \), hence, \( \pi^{HH}_{c_F,c_F} = \pi^{LL}_{c_F,c_F} = \pi^{L_*}_{c_F,c_F} = 0 \). Let \( c_r \) be the value of \( c^H_e \) such that \( c^H_e = c_F + \frac{R(2q_i^H - 2q_i) r^{L_*} - (2q_i^H - 2q_i) r^{H_*}}{2q_i^H - q_i} \). Then \( c_r < c_F \); \( q^*_e = q^H_e \) when \( c^H_e \leq c_r \), and \( q^*_e = q^L_e \) when \( c_r < c^H_e \leq c_F \). Hence, when \( c^H_e \leq c_r \), licensing does not affect the entrant’s optimal quality; when \( c_r < c^H_e \leq c_F \), licensing decreases the entrant’s optimal quality. One numerical example for the quality-decreasing effect of licensing is: \( q_i = 1, c_i = 0.7, q^L_e = 1.1, q^L_e = 0.78, q^H_e = 1.5, c^H_e = 1.18, F = 0.0025 \). In this example, \( q^*_e = q^L_e \leq q^*_e = q^L_e \) and \( r^{L_*} \leq 0.1589 \).

Since \( \bar{r}^{HH} \) is increasing in \( c^H_e \) and \( \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \) is decreasing in \( c^H_e \), there exists a unique value of \( c^H_e \), which we denote as \( c_1 \), such that \( \bar{r}^{HH} = \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \) at \( c_1 \). Since \( c^H_e = c_F \), \( \bar{r}^{HH} < \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \), then \( c_1 > c_F \). Hence, when \( c_F < c^H_e \leq c_1 \), \( q^*_e = q^L_e \) and licensing does not change the entrant’s optimal quality. When \( c^H_e > c_1 \), then \( \bar{r}^{HH} > \frac{q^2_i + 8q_i^H (q^H - c^H_i) - c_q q_i}{2(q^H_i + q_i)} \), \( \pi^{H_*}_{er} (r^{H_*}) = \frac{(q^H_i - c^H_i)^2 (2q^H_i + q_i)^2}{(q^H_i - c^H_i)^2 (2q^H_i + q_i)^2} \), which is decreasing in \( c^H_e \). Hence, for any \( c^H_e > c_1 \), \( \pi^{H_*}_{er} (r^{H_*}, c^H_e > c_1) < \pi^{H_*}_{er} (r^{H_*}, c^H_e = c_1) = \pi^{H_*}_{er} (\bar{r}^{HH}, c^H_e = c_1) = \pi^{H_*}_{Er} (c^H_e = c_1) < \pi^{L_*}_{er} \leq \pi^{L_*}_{er} \). Hence, when \( c^H_e > c_1 \), \( q^*_e = q^L_e \) and licensing does not affect the entrant’s optimal quality decision.
To summarize, we analytically proved that if \( c_e^H = c_F \), \( \bar{r}^{HH} < \frac{q_i^2 + 8q_i^H(q_i^H - c_e^H) - c_i}{2(q_i^H + q_i)} \), then licensing leads to the entrant’s same or lower optimal quality.

(ii) If at \( c_e^H = c_F \), \( \bar{r}^{HH} > \frac{q_i^2 + 8q_i^H(q_i^H - c_e^H) - c_i}{2(q_i^H + q_i)} \), then \( r^{H*} = \frac{\bar{r}^{HH} - c_i}{q_i^H + q_i} \) for \( c_e^H \geq c_F \) since \( \bar{r}^{HH} \) is increasing in \( c_e^H \) and \( \frac{q_i^2 + 8q_i^H(q_i^H - c_e^H) - c_i}{2(q_i^H + q_i)} \) is decreasing in \( c_e^H \). At \( c_e^H = c_F \), \( r^{H*}|_{c_e^H=c_F} = \frac{q_i^2 + 8q_i^H(q_i^H - c_F) - c_i}{2(q_i^H + q_i)} \). We first show that under certain conditions, the quality-increasing effect and the quality-decreasing effect of licensing can occur, then we discuss the effect of licensing under other conditions. Note that \( \bar{r}^{HH}|_{c_e^H=c_F} > \frac{q_i^2 + 8q_i^H(q_i^H - c_e^H) - c_i}{2(q_i^H + q_i)} \) requires \( \frac{q_i^2 + 8q_i^H(q_i^H - c_e^H) - c_i}{2(q_i^H + q_i)} > F \geq \max\{0, F_1\} \) where

\[
F_1 = \frac{q_i(q_i-c_i) + 8q_i^H(q_i^H-c_F)}{2(q_i^H+q_i)} \frac{2q_i^H(2q_i^H-2q_i+c_i) - 2c_F(2q_i^H-q_i) - (q_i^H-q_i)q_i(q_i-c_i) + 8q_i^H(q_i^H-c_F)}{8q_i^H+q_i}.
\]

We denote this condition as condition (a). We next consider two cases: (1) \( \bar{r}^{LL} \leq \frac{q_i^2 + 8q_i^L(q_i^L-c_L)-c_i}{2(q_i^L+q_i)} \), (2) \( \bar{r}^{LL} > \frac{q_i^2 + 8q_i^L(q_i^L-c_L)-c_i}{2(q_i^L+q_i)} \).

(1) If \( \bar{r}^{LL} \leq \frac{q_i^2 + 8q_i^L(q_i^L-c_L)-c_i}{2(q_i^L+q_i)} \) holds, then it requires \( \frac{q_i^L(2q_i^L-2q_i+c_i) - c_i}{2q_i^L+q_i} \leq \frac{q_i^2 + 8q_i^L(q_i^L-c_L)-c_i}{2(q_i^L+q_i)} \), or \( \frac{q_i^L(2q_i^L-2q_i+c_i) - c_i}{2q_i^L+q_i} > \frac{q_i^2 + 8q_i^L(q_i^L-c_L)-c_i}{2(q_i^L+q_i)} \) and

\[
F \leq F_2 = \frac{q_i(q_i-c_i) + 8q_i^L(q_i^L-c_L)}{2(q_i^L+q_i)} \frac{2q_i^L(2q_i^L-2q_i+c_i) - 2c_L(2q_i^L-q_i) - (q_i^L-q_i)q_i(q_i-c_i) + 8q_i^L(q_i^L-c_L)}{8q_i^L+q_i}.
\]

We denote this condition as condition (b). Note that only if both conditions (a) and (b) hold, this case of at \( c_e^H = c_F \), \( r^{L*} = \frac{q_i^2 + 8q_i^L(q_i^L-c_L)-c_i}{2(q_i^L+q_i)} \) and \( r^{L*} = \bar{r}^{LL} \) occurs. Then \( [R(q_e^L-q_i)r^{L*}-(q_e^H-q_i)r^{H*})|_{c_e^H=c_F} = R(q_e^L-q_i)\bar{r}^{LL} - (q_e^H-q_i)\frac{q_i^2 + 8q_i^H(q_i^H-c_F)-c_i}{2(q_i^H+q_i)} \) \( \) and \( r^{L*} = \frac{q_i^2 + 8q_i^H(q_i^H-c_F)-c_i}{2(q_i^H+q_i)} \) > 0 as condition (c). Let \( c_r \) be the value of \( c_e^H \) such that \( c_e^H = c_F + \frac{R(2q_i^L-2q_i)r^{L*}-(2q_i^H-2q_i)}{2(q_i^H+q_i)} \).

Then when condition (c) holds, \( c_r > c_F \); \( q_{er}^* = q_e^H \) when \( c_F < c_e^H \leq c_r \), and \( q_{er}^* = q_e^L \) when \( c_e^H > c_r \).

Hence, when the parameters are such that conditions (a), (b) and (c) hold and \( c_F < c_e^H \leq c_r \), then licensing increases the entrant’s optimal quality. One numerical example for this quality-increasing effect of licensing is: \( q_i = 1, c_i = 0.3, q_e^L = 1.5, c_e^L = 0.8, q_e^H = 3, c_e^H = 2.2721, F = 0.00978 \). In this example, \( q_{er}^* = q_e^H > q_{er}^* = q_e^L \) and \( r^{H*} = 0.363392 \). When the parameters are such that conditions
(a), (b) and (c) hold and \( c_e^H > c_r \), licensing does not change the entrant’s optimal quality. When the parameters are such that conditions (a), (b) and (c) hold and \( c_e^H \leq c_F \), licensing leads to the entrant’s same or lower optimal quality. When the parameters are such that conditions (a) and (b) hold but condition (c) does not hold, then licensing may decrease but cannot increase the entrant’s optimal quality.

\[(2) \text{ If } \bar{r}^{LL} > \frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} \text{ holds, then it requires } \frac{q_e^L(2q_e^L - 2q_i + c_i) - c_i q_i}{2q_e^L - 2q_i} > \frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} \text{ and } F > \max\{0, F_2\}. \]

This means that the case at \( c_e^H, r^{H*} = \frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} \) and \( r^{L*} = \frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} \text{ occurs only if } \frac{q_e^L(2q_e^L - 2q_i + c_i) - c_i q_i}{2q_e^L - 2q_i} > \frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2q_e^L - 2q_i} \text{ and } \frac{q_e^L(2q_e^L - 2q_i + c_i) - c_i q_i}{(q_e^L - q_i)(4q_e^L - q_i)^2} > F > \max\{0, F_1, F_2\} \text{ hold. We denote this condition as condition (d). Note that } \frac{R(q_e^L - q_i)r^{L*} - (q_e^H - q_i)r^{H*}}{c_e^H = c_F} = \frac{R(q_e^L - q_i)}{c_e^H - c_F} = \frac{\frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} - (q_e^H - q_i)\frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} - (q_e^H - q_i)\frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} > 0 \text{ as condition (e). Let } c_r \text{ be the value of } c_e^H \text{ such that } c_e^H = c_F + \frac{\frac{R(2q_e^L - 2q_i)r^{L*} - (2q_e^H - 2q_i)\frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} > 0}{2q_e^L - 2q_i}}. \]

Then when condition (e) holds, \( c_r > c_F; q_{er}^* = q_e^H \) when \( c_F < c_e^H \leq c_r \), and \( q_{er}^* = q_e^L \) when \( c_e^H > c_r \).

Hence, when the parameters are such that conditions (d) and (e) both hold, and \( c_F < c_e^H \leq c_r \), then licensing increases the entrant’s optimal quality.

One numerical example for this quality-increasing effect of licensing is: \( q_i = 1, c_i = 0.31, q_e^L = 1.5, c_e^L = 0.8, q_e^H = 3, c_e^H = 2.2649, F = 0.01064. \) In this example, \( q_{er}^* = q_e^H > q_e^* = q_e^L \) and \( r^{H*} = 0.366648. \) When the parameters are such that conditions (d) and (e) both hold, and \( c_e^H < c_F \), licensing leads to the entrant’s same or lower optimal quality. When the parameters are such that condition (d) holds, but condition (e) does not hold, then licensing may decrease but cannot increase the entrant’s optimal quality.

To summarize, we showed that if at \( c_e^H = c_F, \bar{r}^{HH} > \frac{q_e^2 + 8q_e^L(q_e^L - c_i^e) - c_i q_i}{2(8q_e^L + q_i)} \), then licensing leads to
the entrant’s same, higher, or lower optimal quality.

D. Analysis for the model with the incumbent’s endogenous quality

In this part of the Web Appendix, the incumbent endogenously decides its product quality in anticipation of its competitor’s entry in the market. Let the incumbent’s and the entrant’s marginal cost be $k_iq_i^2$ (or $k_eq_e^2$ in the case of licensing), and $k_eq_e^2$ (or $k_eq_{er}^2$ in the case of licensing), respectively. Because of tractability issues, next, we introduce two numerical examples to demonstrate the effect of licensing on the entrant’s optimal quality when the incumbent endogenously decides its product quality.

Example 1: When the entrant’s production of its core technology has significant improvement over the incumbent’s, let $k_i = 1$, $k_e = 0.5$, and $F = 0.01$. In the case of no licensing, for given $q_i$, $q_e$, $p_i$ and $p_e$, the incumbent’s and the entrant’s demand functions are: $D_i = \frac{p_i-p_e}{q_i-q_e}$ and $D_e = 1 - \frac{p_e-p_i}{q_e-q_i}$, respectively, if $q_e > q_i$ and $p_e > p_i$; $D_i = 1 - \frac{p_i-p_e}{q_i-q_e}$ and $D_e = \frac{p_e-p_i}{q_i-q_e} - \frac{p_i}{q_i}$ if $q_e < q_i$ and $p_e < p_i$; $D_i = 1 - \frac{p_i}{q_i}$ and $D_e = 0$ if $q_e < q_i$ and $p_e > p_i$; and $D_i = 0$ and $D_e = 1 - \frac{p_e}{q_e}$ if $q_e > q_i$ and $p_e < p_i$. Their profit functions are $\pi_i = (p_i - q_i^2)D_i$ and $\pi_e = (p_e - 0.5q_e^2)D_e - F$. One can easily show that in this example, the entrant will optimally respond with $q_e > q_i$ and the incumbent will optimally set $q_i < q_e$ in anticipation of a more efficient competitor’s entry. In the licensing case, we will only list the demand function where both firms compete in the market. We solve the game based on backward induction. First, since $\pi_i$ is a concave function of $p_i$ and $\pi_e$ is a concave function of $p_e$, simultaneously solving the first order conditions $\frac{d\pi_i}{d p_i} = 0$ and $\frac{d\pi_e}{d p_e} = 0$ leads to the optimal prices $p_i^* = \frac{0.125q_i^2+q_i(0.25+0.5q_i)-0.25q_i}{q_i-0.25q_i}$ and $p_e^* = \frac{q_e[0.5q_e+0.25q_i^2-(0.25q_i-0.5)q_i]}{q_e-0.25q_i}$. Then the two firms’ profits are: $\pi_i(q_e, q_i) = \frac{0.015625q_i^2q_e^2-q_e(q_i-2)q_i(2-2q_i)^2}{(q_i-q_e)(q_i-0.25q_i)^2}$ and $\pi_e(q_e, q_i) = \frac{0.0625q_i^2(q_e^2-q_e(2+0.5q_i)-q_i(q_i-2)^2)}{(q_e-q_i)(q_e-0.25q_i)^2}$. 
Next, we maximize the entrant’s profit $\pi_e(q_e, q_i)$ over $q_e$ to obtain its optimal quality $q^*_e(q_i)$ for given $q_i$. Let $q^*_e(q_i) = \arg\max_{q_e} \pi_e(q_e, q_i)$ (i.e., $q^*_e(q_i)$ is the solution to the first order condition $\frac{d\pi_e(q_e, q_i)}{dq_e} = 0$ that maximizes the entrant’s profit). Last, we maximize the incumbent’s profit $\pi_i(q^*_i(q_i), q_i)$ over $q_i$ to obtain its optimal product quality $q^*_i$ in anticipation of $q^*_e(q_i)$, $p^*_i$ and $p^*_e$. Let $q^*_i = \arg\max_{q_i} \pi_i(q^*_i(q_i), q_i)$. Our numerical analysis procedure is: among the seven solutions to the first order condition $\frac{d\pi_e(q_e, q_i)}{dq_e} = 0$, we can analytically rule out three solutions that give $\pi_e(q_e, q_i) = 0$ and there are four solutions left. We denote these four solutions as $q^*_{e1}(q_i)$, $q^*_{e2}(q_i)$, $q^*_{e3}(q_i)$, and $q^*_{e4}(q_i)$. For each of these four solutions, we numerically maximize the incumbent’s profit $\pi_i(q^*_{ij}(q_i), q_i)$ where $j = 1, 2, 3, 4$ to compare the maximum of each $\pi_i(q^*_{ij}(q_i), q_i)$ and obtain the optimal $q^*_i$ and the corresponding $q^*_e(q^*_i)$ as follows: $q^*_i \approx 0.20559$, $q^*_e(q^*_i) \approx 0.71007$, $\pi^*_i \approx 0.01071$, and $\pi^*_e \approx 0.04535$.

In the case of a licensing contract with royalty fee $r$, for given $q_{ir}$, $q_{er}$, $p_{ir}$ and $p_{er}$, the incumbent’s and the entrant’s demand functions are $D_{ir} = \frac{p_{ir} - p_{er}}{q_{ir}} - \frac{p_{ir}}{q_{ir}}$ and $D_{er} = 1 - \frac{p_{er} - p_{ir}}{q_{er}}$, respectively. Their profit functions are $\pi_{ir} = (p_{ir} - q_{ir}^2)D_{ir} + r \times D_{er}$ and $\pi_{er} = (p_{er} - 0.5q_{er}^2 - r)D_{er}$. First, simultaneously solving the first order conditions $\frac{d\pi_{ir}}{dp_{ir}} = 0$ and $\frac{d\pi_{er}}{dp_{er}} = 0$ leads to the optimal prices

\[
p^*_{ir} = \frac{0.125q^2_{ir} + q_{ir}(0.25 + 0.5q_{ir}) - 0.25q_{ir} + 0.75r}{q_{ir} - 0.25q_{ir}} \quad \text{and} \quad p^*_{er} = \frac{q_{er}[0.5q_{er} + 0.25q^2_{er} + (0.25q_{er} - 0.5)q_{ir} + 0.5r] + 0.25r}{q_{er} - 0.25q_{ir}}.
\]

Let $\pi_{ir}(q_{er}, q_{ir}, r)$ and $\pi_{er}(q_{er}, q_{ir}, r)$ be the incumbent and the entrant’s corresponding profit. Next, we maximize the entrant’s profit $\pi_{er}(q_{er}, q_{ir}, r)$ over $q_{er}$ to obtain its optimal quality $q^*_{er}(q_{ir}, r)$. Let $q^*_{er}(q_{ir}, r) = \arg\max_{q_{er}} \pi_{er}(q_{er}, q_{ir}, r)$ (i.e., one of the solutions to the first order condition $\frac{d\pi_{er}(q_{er}, q_{ir}, r)}{dq_{er}} = 0$ that maximizes the entrant’s profit). Last, we numerically maximize the incumbent’s profit $\pi_{ir}(q^*_{ir}(q_{ir}, r), q_{ir}, r)$ jointly over $q_{ir}$ and $r$ to obtain its optimal product quality $q^*_{ir}$ and its optimal royalty fee $r^*$ subject to the constraint that neither firm is worse off with the licensing contract, that is, $\pi_{ir}(q^*_{ir}(q_{ir}, r), q_{ir}, r) \geq \pi^*_i$ and $\pi_{er}(q^*_{er}(q_{ir}, r), q_{ir}, r) \geq \pi^*_e$. Following this procedure, $\pi^*_i$ is achieved at $(q^*_{ir}, r^*) = (0.10349, 0.06914)$. So, $q^*_{er}(q^*_{ir}, r^*) \approx 0.73907 > q^*_e$. Therefore, with
consideration of the incumbent’s response in quality decision, licensing leads to the entrant’s higher optimal quality.

Example 2: When the entrant’s production of its core technology has incremental improvement over the incumbent’s, let $k_i = 1$, $k_e = 0.98$, and $F = 0.001$. One can easily show that in this example, the entrant will optimally respond with $q_e > q_i$ and the incumbent will optimally set $q_i < q_e$ in anticipation of a more efficient competitor’s entry. Hence, in the case of no licensing, for given $q_i$, $q_e$, $p_i$ and $p_e$, the incumbent’s and the entrant’s demand functions are $D_i = \frac{p_e - p_i}{q_e - q_i} - \frac{p_i}{q_i}$ and $D_e = 1 - \frac{p_e - p_i}{q_e - q_i}$, respectively. Their profit functions are $\pi_i = (p_i - q_i^2)D_i$ and $\pi_e = (p_e - 0.98q_e^2)D_e - F$.

We solve the game based on backward induction. First, simultaneously solving the first order conditions $\frac{d\pi_i}{dp_i} = 0$ and $\frac{d\pi_e}{dp_e} = 0$ leads to the optimal prices $p_i^* = \frac{0.245q_i^2 + q_e(0.25 + 0.5q_i) - 0.25q_i^2}{q_e - 0.25q_i}$ and $p_e^* = \frac{q_e[0.5q_i + 0.49q_i^2 + (0.25q_i - 0.5)q_i]}{q_e - 0.25q_i}$. Then the two firms’ profits are:

$$\pi_i(q_e, q_i) = \frac{0.066025q_i^3[2q_e^2 - q_e(2.04082q_i - 1.02041) - q_i(1.02041 - 1.02041q_i)]^2}{(q_e - q_i)(q_e - 0.25q_i)^2}$$

and

$$\pi_e(q_e, q_i) = \frac{0.2401q_e^2(q_e - 0.25q_i)^2(q_e - 0.510204q_i - 1.02041)^2}{(q_e - q_i)(q_e - 0.25q_i)^2}$$.

Next, we maximize the entrant’s profit over $q_e$ to obtain its optimal quality $q_e^*(q_i)$. Let $q_e^*(q_i) = argmax_{q_e} \pi_e(q_e, q_i)$ (i.e., $q_e^*(q_i)$ is the solution to the first order condition $\frac{d\pi_e(q_e, q_i)}{dq_e} = 0$ that maximizes the entrant’s profit). Last, we maximize the incumbent’s profit $\pi_i(q_e^*(q_i), q_i)$ over $q_i$ to obtain its optimal product quality $q_i^*$ in anticipation of $q_e^*(q_i)$, $p_i^*$ and $p_e^*$. Then $q_i^* = argmax_{q_i} \pi_i(q_e^*(q_i), q_i)$. Following the same procedure as in Example 1, we numerically obtain the optimal solutions: $q_i^* \approx 0.26711$, $q_e^*(q_i) \approx 0.45162$, $\pi_i^* \approx 0.01269$, and $\pi_e^* \approx 0.01076$.

In the case of a licensing contract with royalty fee $r$, for given $q_{ir}$, $q_{er}$, $p_{ir}$ and $p_{er}$, the incumbent’s and the entrant’s demand functions are $D_{ir} = \frac{p_{er} - p_{ir}}{q_{er} - q_{ir}} - \frac{p_{ir}}{q_{ir}}$ and $D_{er} = 1 - \frac{p_{er} - p_{ir}}{q_{er} - q_{ir}}$, respectively. Their profit functions are $\pi_{ir} = (p_{ir} - q_{ir}^2)D_{ir} + r \times D_{er}$ and $\pi_{er} = (p_{er} - 0.5q_{er}^2 - r)D_{er}$. First,
simultaneously solving the first order conditions $\frac{d\pi_{ir}}{dp_{ir}} = 0$ and $\frac{d\pi_{er}}{dp_{er}} = 0$ leads to the optimal prices $p^*_{ir} = \frac{0.245q_{ir}^2 + q_{er}(0.25 + 0.5q_{ir}) - 0.25q_{ir} + 0.75r}{q_{er} - 0.25q_{ir}}$ and $p^*_{er} = \frac{q_{er}[0.5q_{er} + 0.49q_{ir}^2 + (0.25q_{ir} - 0.5)q_{ir} + 0.5r] + 0.25r q_{ir}}{q_{er} - 0.25q_{ir}}$. Let $\pi_{ir}(q_{er}, q_{ir}, r)$ and $\pi_{er}(q_{er}, q_{ir}, r)$ be the incumbent and the entrant’s corresponding profit. Next, we maximize the entrant’s profit over $q_{er}$ to obtain its optimal quality $q^*_{er}(q_{ir}, r)$. Let $q^*_{er}(q_{ir}, r) = \arg\max_{q_{er}} \pi_{er}(q_{er}, q_{ir}, r)$ (i.e., one of the solutions to the first order condition $\frac{d\pi_{er}(q_{er}, q_{ir}, r)}{dq_{er}} = 0$ that maximizes the entrant’s profit). Last, we numerically maximize the incumbent’s profit $\pi_{ir}(q^*_{er}(q_{ir}, r), q_{ir}, r)$ jointly over $q_{ir}$ and $r$ to obtain its optimal product quality $q^*_{ir}$ and its optimal royalty fee $r^*$ subject to the constraint that neither firm is worse off with the licensing contract, that is, $\pi_{ir}(q^*_{ir}(q_{ir}, r), q_{ir}, r) \geq \pi_i^*$ and $\pi_{er}(q^*_{er}(q_{ir}, r), q_{ir}, r) \geq \pi_e^*$. Following this procedure, $\pi_{ir}^*$ is achieved at $(q^*_{ir}, r^*) = (0.14223, 0.06688)$. So, $q^*_{er}(q^*_{ir}, r^*) \approx 0.41282 < q^*_{ir}$. Therefore, with consideration of the incumbent’s response in quality decision, licensing leads to the entrant’s lower optimal quality.