Comment on “Strategic Information Management Under Leakage in a Supply Chain”

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Abstract

Anand and Goyal (2009) propose a horizontal differentiation model to study information leakage and demand signaling in a supply chain. The authors present a composite equilibrium consisting of separating and pooling outcomes in different parameter regions and claim that it satisfies the Intuitive Criterion. We show that their analysis for the pooling equilibrium has errors and also that all pooling outcomes are actually eliminated by the Intuitive Criterion. The positive note is that if the undefeated equilibrium or lexicographical maximum sequential equilibrium (LMSE) refinement is adopted, a similar composite equilibrium will be achieved, which will lead to qualitatively the same results as in the original paper.

Key words: signaling, demand uncertainty, intuitive criterion, supply chain and logistics

1 Both authors contributed equally. The authors thank the Department Editor, the Associate Editor and two reviewers for helpful comments.
1. Introduction

Anand and Goyal (2009), hereafter referred to as AG, present a horizontal differentiation model of two firms—an incumbent and an entrant—with a common upstream supplier to study information leakage and demand signaling in a supply chain where demand is uncertain. In their model, the demand intercept is uncertain—the intercept is a high value $A_H$ with probability $p$, and a low value $A_L$ with probability $1 - p$. The incumbent firm is a Stackelberg leader, which can acquire demand information to mitigate demand uncertainty before placing an order with the supplier. The supplier can decide whether to leak the incumbent’s order quantity to the entrant, which enters the market as the Stackelberg follower. The entrant can make inferences about the market demand from the supplier’s leakage decision as well as the leaked order quantity, and then place its order with the supplier.

Assuming that the incumbent has acquired the demand information, the authors examine two types of pure-strategy Perfect Bayesian Nash Equilibrium (PBNE) under information leakage in the information dissemination game—separating and pooling. In a separating equilibrium, the incumbent orders a distinct quantity in each demand state and the entrant can correctly infer the demand information. In a pooling equilibrium, the incumbent orders the same quantity under both demand states, and neither the supplier nor the entrant can tease out any demand information. Proposition 3 in AG shows that there exists a pure-strategy PBNE, which parses to the separating equilibrium when $\frac{A_H}{A_L}$ is large and the pooling equilibrium otherwise. Further, the authors claim that this composite equilibrium satisfies the Intuitive Criterion of Cho and Kreps (1987).

Equation (16) in AG’s Online Appendix specifies the entrant’s belief of the pooling equilibrium: If the incumbent’s order quantity is larger than a threshold, the entrant will believe that the demand is high ($A_H$); if the incumbent’s order quantity is lower than another threshold, the entrant will believe that the demand is low ($A_L$); if the incumbent’s order quantity falls between these thresholds, the entrant’s belief will be the same as the prior. Close examination of the proof of the pooling equilibrium (see Section 3.2.2 of AG’s Online Appendix) reveals that the authors have not considered all the IC constraints. More specifically,
the authors consider only the constraints to ensure that the incumbent (both the low type and the high type) will not deviate to a higher order quantity to be considered as the high type (i.e., equations 21 and 22). However, the analysis neglects the constraints ensuring that the incumbent cannot profitably deviate to a much lower order quantity to be considered as the low type. Next we will include these constraints to identify the pooling equilibrium in AG’s setting and then show that all pooling equilibria are actually eliminated by the Intuitive Criterion.

2. Pooling Equilibrium and Intuitive Criterion

Our analysis follows the notations in AG. First, to ensure that both types of the incumbent will pool at the same order quantity in equilibrium, the candidate pooling quantity $q_{ip}$ should not be larger than $q_{ip}^* = (A_L - \mu)^+$ (see Lemma 4 in AG’s Online Appendix). Second, to ensure that neither type of the incumbent can profitably deviate to be considered as the high type, $q_{ip} \geq q_{(ip)\min}^H \equiv A_H - \frac{\mu}{2} - \frac{1}{2} \sqrt{(A_H - \mu)(3A_H - \mu)}$ is needed (see Lemma 5 and Corollary 1 in AG’s Online Appendix). Note that the lower threshold $q_p$ in the belief system in AG is just the above lower bound of the possible pooling quantity, i.e., $q_p = q_{(ip)\min}^H$ (see Corollary 1 in AG’s Online Appendix). This belief system does not always support the pooling equilibrium identified in AG. For the pooling equilibrium to hold, we also need to include the constraints to ensure that the incumbent cannot profitably deviate to a low order quantity to be considered as the low type. First, the high-type incumbent’s profit under pooling should be (at least weakly) higher than its profit from mimicking the low type by ordering a low enough quantity:

\[
(A_H - q_{ip} - \frac{\mu - q_{ip}}{2})q_{ip} \geq \max_{q_{ul} \leq q_p} (A_L - q_{ilH} - \frac{A_L - q_{ul}}{2})q_{ilH}. \tag{1}
\]

Second, the low-type incumbent’s profit under pooling should be (at least weakly) higher than its profit from revealing its type by ordering a low enough quantity:

\[
(A_L - q_{ip} - \frac{\mu - q_{ip}}{2})q_{ip} \geq \max_{q_{il} \leq q_p} (A_L - q_{ilL} - \frac{A_L - q_{ul}}{2})q_{ilL}. \tag{2}
\]

\[\text{In a response to our comment, AG pointed out that the pooling outcome in their paper will be a pooling equilibrium if they change the belief system from equation (16) in their paper to one that “is akin to passive conjectures” (e.g., Fudenberg and Tirole 1983): if the incumbent’s order quantity is lower than a threshold, the entrant’s belief will be the same as the prior, while if the incumbent’s order quantity is larger than that threshold, the entrant’s belief will be that the demand is high ($A_H$). However, the pooling equilibrium will fail the Intuitive Criterion, which we show in this comment.}\]
In summary, for \( q \) low-type incumbent will deviate to the region of \( L \). Thus, by definition, the pooling equilibrium with \( p \) will not deviate if the entrant believes that the demand is low given \( q \). In summary, for \( q \) low-type incumbent will deviate to the region of \( L \). Thus, by definition, the pooling equilibrium with \( p \) will not deviate if the entrant believes that the demand is low given \( q \). In summary, for \( q \) low-type incumbent will deviate to the region of \( L \). Thus, by definition, the pooling equilibrium with \( p \) will not deviate if the entrant believes that the demand is low given \( q \).

Solving inequality (1), we get \( q_p \leq q_{(ip)min}^H(q_{ip}) \equiv \frac{2A_H-A_L-\sqrt{(2A_H-A_L)^2-4(2A_H-\mu-q_{ip})q_{ip}}}{2} \). Similarly from inequality (2), we have \( q_p \leq q_{(ip)min}^L(q_{ip}) \equiv \frac{A_L-\sqrt{A_L^2-4(2A_L-\mu-q_{ip})q_{ip}}}{2} \). Note that \( q_{(ip)min}^L(q_{ip}) < q_{(ip)min}^H(q_{ip}) < q_p \). In summary, for \( q_{ip} \in [q_{(ip)min}^H, q_{(ip)min}^L] \) to be a pooling-equilibrium quantity, the low threshold in the entrant’s belief system must satisfy \( q_{ip} \leq q_{(ip)min}^L(q_{ip}) \). We discuss the two cases regarding this upper bound of \( q_{ip} \) evaluated at \( q_{ip} = q_{ip}^* \), in comparison with \( q_p = q_{(ip)min}^H \), which is specified in the belief system in AG.

First, if \( q_{(ip)min}^L(q_{ip}^*) < q_{(ip)min}^H(q_{ip}) \) (which is true, for example, when \( p \to 1 \), i.e., \( \mu \to A_H \)), the outcome shown in AG’s Proposition 2 cannot constitute an equilibrium, because the low-type incumbent has an incentive to deviate from the pooling quantity \( q_{ip}^* \) to a quantity less than \( q_{(ip)min}^H \) to reveal its type. Second, if \( q_{(ip)min}^L(q_{ip}^*) \geq q_{(ip)min}^H(q_{ip}) \) (which is true, for example, when \( p \to 0 \), i.e., \( \mu \to A_L \)), the outcome in Proposition 2 is a pooling equilibrium outcome, but it does not survive the Intuitive Criterion by the following argument. Note that given AG’s pooling region \( [q_{(ip)min}^H, q_{(ip)min}^L] \), the high-type incumbent will not deviate to the region \( [q_{(ip)min}^L, q_{(ip)min}^H(q_{ip}^*)] \) even if it will be considered as the low type for sure. But the low-type incumbent will deviate to the region \( (q_{(ip)min}^L(q_{ip}^*), q_{(ip)min}^H(q_{ip}^*)) \), which is a sub-region of \( [q_{(ip)min}^H, q_{(ip)min}^L(q_{ip}^*)] \), as long as the entrant believes that such deviations cannot come from the high type. Thus, by definition, the pooling equilibrium with \( q_{ip}^* \) fails the Intuitive Criterion. Furthermore, since \( q_{(ip)min}^L(q_{ip}) < q_{(ip)min}^H(q_{ip}) \), for any pooling

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3 Please refer to the Online Appendix for details on solving constraints (1) and (2), and the formal proof of \( q_{(ip)min}^L(q_{ip}) < q_{(ip)min}^H(q_{ip}) \). In case (b) in part two of Lemma 12’s proof, AG argue that “both types are better off deviating if the entrant believes the demand is low given \( q_i \in [q_p, q_{ip}^*] \), where \( q_p = q_{(ip)min}^H \). This argument is not correct. In fact, the high-type incumbent will not deviate if the entrant believes that the demand is low given \( q_i \in [q_{(ip)min}^H, q_{(ip)min}^L(q_{ip})] \), which is a subinterval of \( [q_{(ip)min}^H, q_{ip}] \). Also, the low-type incumbent will not deviate if the entrant believes that the demand is low given \( q_i \in [q_{(ip)min}^H, q_{(ip)min}^L(q_{ip})] \).
equilibrium there exists the region \( (q_{i(p)\min}(q_{i(p)}), q_{i(p)\min}(q_{i(p)})) \), to which the high-type incumbent will not deviate even if it will be considered as the low type for sure, whereas the low-type incumbent will deviate as long as the entrant believes that such deviations cannot come from the high type. Thus, all pooling equilibria are eliminated by the Intuitive Criterion.

3. Undefeated Equilibrium and LMSE

In some signaling settings, there is a subtle, global consistency issue with the Intuitive Criterion, which has been brought up in the economics literature and is well explained in Mailath et al. (1993). In short, in some situations, the Intuitive Criterion has some logical incompleteness and rules out the pooling outcome when pooling is more intuitively plausible than separating. Mailath et al. (1993) propose the undefeated equilibrium refinement concept and show that under some regularity assumptions, the lexicographically maximum sequential equilibrium (LMSE) outcome is undefeated. In many signaling games, the LMSE and the undefeated equilibrium refinement lead to the same outcome; in essence, in AG’s setting, the LMSE outcome is the PBNE outcome that the low-type incumbent finds most profitable among all PBNE. This is intuitively reasonable: Note that the high-type incumbent has an incentive to mimic the low-type incumbent but not vice versa, i.e., the low-type incumbent is the one with an incentive to reveal its identity. So, when both separating and pooling PBNE exist, if the low-type incumbent makes a higher profit under the pooling outcome than under the separating outcome whereas the high-type incumbent makes a higher profit under the separating outcome than under the pooling outcome, then the pooling PBNE would seem more plausible whereas the separating PBE must be based on some “unreasonable” off-equilibrium belief that “forces” the low-type incumbent to separate.

The LMSE or undefeated equilibrium concepts have been used as alternative refinement concepts in many signaling situations (for recent examples, see Guo and Jiang 2016, Jiang et al. 2014, and Jiang et al. 2016). If such refinements are adopted in AG, a similar unique composite equilibrium (which is separating if \( \frac{A_H}{A_L} \) is large and pooling if \( \frac{A_H}{A_L} \) is small) will be

\[5\] The definition of LMSE applied to AG’s setting is given in part III of the Online Appendix.
achieved, which leads to the same qualitative results as in AG’s original paper. The only difference is that the lower threshold \( q_p \) in the entrant’s belief system under the pooling equilibrium should be \( q_p = \min [q^L_{(ip)\min}(q_{ip}), q^H_{(ip)\min}] \), not \( q_p = q^H_{(ip)\min} \) shown in AG.
Reference


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Online Appendix

In this appendix, we present the detailed analysis that have been omitted in the main paper. Part I provides the details of how to solve constraints (1) and (2). Part II proves $q_i \min H(q_i) < q_i \min L(q_i) < q_i$. Part III describes the undefeated equilibrium and the lexicographically maximum sequential equilibrium (LMSE).

Part I: Solving Constraints (1) and (2)

We first solve constraint (1) step by step: $(A_H - q_{ip} - \mu - \frac{q_{ip}}{2})q_{ip} \geq \max_{q_{ih} \leq q_{ip}} (A_H - q_{ih} - \frac{A_L - q_{ip}}{2})q_{ih}$, which is equivalent to solve:

\[(2A_H - \mu - q_{ip})q_{ip} \geq \max_{q_{ih} \leq q_{ip}} (2A_H - A_L - q_{ih})q_{ih}. \tag{A1}\]

We examine the two cases for $q_p$. Suppose $q_p \geq \frac{2A_H - A_L}{2}$. Then, $\max_{q_{ih} \leq q_{ip}} (2A_H - A_L - q_{ih})q_{ih} = \frac{(2A_H - A_L)^2}{4} > \frac{(2A_H - \mu)^2}{4} = (2A_H - \mu - q_{ip})q_{ip}$, which contradicts (A1). Thus, we must have $q_p < \frac{2A_H - A_L}{2}$.

Given $q_p < \frac{2A_H - A_L}{2}$, $\max_{q_{ih} \leq q_{ip}} (2A_H - A_L - q_{ih})q_{ih} = (2A_H - A_L - q_p)q_p$. That is, we need to solve $(2A_H - \mu - q_{ip})q_{ip} \geq (2A_H - A_L - q_p)q_p$, i.e., $q_p^2 - (2A_H - A_L)q_p + (2A_H - \mu - q_{ip})q_{ip} \geq 0$.

Solving this inequality, we obtain $q_p \leq \frac{2A_H - A_L - \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip})q_{ip}}}{2}$ or $q_p \geq \frac{2A_H - A_L + \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip})q_{ip}}}{2}$. Combining with the condition $q_p < \frac{2A_H - A_L}{2}$, we get $q_p \leq q_{ip}^L \min (q_{ip})$.

Next we solve constraint (2): $(A_L - q_{ip} - \mu - \frac{q_{ip}}{2})q_{ip} \geq \max_{q_{il} \leq q_{ip}} (A_L - q_{il} - \frac{A_L - q_{ip}}{2})q_{il}$, which is equivalent to solve:

\[(2A_L - \mu - q_{ip})q_{ip} \geq \max_{q_{il} \leq q_{ip}} (A_L - q_{il})q_{il}. \tag{A2}\]

We examine the two cases for $q_p$. Suppose $q_p \geq \frac{A_L}{2}$. Then, $\max_{q_{il} \leq q_{ip}} (A_L - q_{il})q_{il} = \frac{A_L^2}{4} > \frac{(2A_L - \mu)^2}{4} > (2A_L - \mu - q_{ip})q_{ip}$, which contradicts (A2). Thus, we must have $q_p < \frac{A_L}{2}$.
max \( q_{ip} \leq q_L \), \( q_{ip} = (A_i - q_p) q_p \). That is, we need to solve \( (2A_i - \mu - q_{ip}) q_{ip} \geq (A_i - q_p) q_p \), i.e.,

\[
q_{ip}^2 - A_L q_{ip} + (2A_i - \mu - q_{ip}) q_{ip} \geq 0.
\]

Solving this inequality, we obtain

\[
q_{ip} \leq \frac{A_L - \sqrt{A_L^2 - 4(2A_i - \mu - q_{ip}) q_{ip}}}{2},
\]

or \( q_{ip} \geq \frac{A_L + \sqrt{A_L^2 - 4(2A_i - \mu - q_{ip}) q_{ip}}}{2} \). Combining with the condition \( q_{ip} < A_L \), we get

\[
q_{ip} \leq \frac{A_L - \sqrt{A_L^2 - 4(2A_i - \mu - q_{ip}) q_{ip}}}{2} \left( < A_L^2 \right).
\]

**Part II:** Proof of \( q_{(ip)}^L(q_{ip}) < q_{(ip)}^H(q_{ip}) < q_{ip} \)

Here we prove \( q_{(ip)}^L(q_{ip}) < q_{(ip)}^H(q_{ip}) < q_{ip} \). First, we show \( q_{(ip)}^L(q_{ip}) < q_{(ip)}^H(q_{ip}) \).

Substituting the expressions for \( q_{(ip)}^L(q_{ip}) \) and \( q_{(ip)}^H(q_{ip}) \), we obtain

\[
q_{(ip)}^L(q_{ip}) - q_{(ip)}^H(q_{ip}) = \frac{\sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip}) q_{ip} - 2(2A_H - A_L) - \sqrt{A_L^2 - 4(2A_L - \mu - q_{ip}) q_{ip}}}}{2} = \frac{G_1}{G_2},
\]

where \( G_1 = \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip}) q_{ip}} \) and \( G_2 = 2(A_H - A_L) + \sqrt{A_L^2 - 4(2A_L - \mu - q_{ip}) q_{ip}} \).

Note that

\[
G_1^2 - G_2^2 = 4(A_H - A_L)(A_L - 2q_{ip}) - A_L^2 - 4(2A_L - \mu - q_{ip}) q_{ip} = 4(A_H - A_L)(g_1 - g_2),
\]

where \( g_1 = A_L - 2q_{ip} \) and \( g_2 = \sqrt{A_L^2 - 4(2A_L - \mu - q_{ip}) q_{ip}} \). Note that

\[
g_1^2 - g_2^2 = 4(A_L - \mu) q_{ip} < 0.
\]

Since \( g_2 > 0 \), we know \( g_1 < g_2 \), which implies \( G_1 < G_2 \). Thus we have \( q_{(ip)}^L(q_{ip}) < q_{(ip)}^H(q_{ip}) \).

Next we show \( q_{(ip)}^H(q_{ip}) < q_{ip} \). Substituting the expression for \( q_{(ip)}^H(q_{ip}) \), we obtain

\[
q_{(ip)}^H(q_{ip}) - q_{ip} = \frac{2A_H - A_L - 2q_{ip} - \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip}) q_{ip}}}{2} = \frac{F_1 - F_2}{2},
\]

where \( F_1 = 2A_H - A_L - 2q_{ip} \) and \( F_2 = \sqrt{(2A_H - A_L)^2 - 4(2A_H - \mu - q_{ip}) q_{ip}} \). Note that

\[
F_1^2 - F_2^2 = 4(A_L - \mu) q_{ip} < 0.
\]

Since \( F_2 > 0 \), we obtain \( F_1 < F_2 \), which implies \( q_{(ip)}^H(q_{ip}) < q_{ip} \). In summary, we conclude

\[
q_{(ip)}^L(q_{ip}) < q_{(ip)}^H(q_{ip}) < q_{ip}.
\]
Part III: Undefeated Equilibrium and LMSE

The undefeated equilibrium and LMSE concepts have been commonly used as an alternative equilibrium selection criterion. Mailath et al. (1993) define these outcome-based refinement concepts of undefeated equilibrium and LMSE, which are introduced to avoid the limitations of other refinement techniques such as the Intuitive Criterion (which for example may eliminate all equilibria or fail to select a unique equilibrium). Essentially, for a general class of signaling games (including AG’s setting), both undefeated equilibrium and the LMSE correspond to the same equilibrium outcome. Mailath et al. (1993) summarize and explain the advantages of these refinement concepts (Pareto-optimality, uniqueness and existence for a very general class of signaling games). Below we provide the definition of LMSE applied to AG’s setting.

**Definition (Lexicographically Maximum Sequential Equilibrium)** In a signaling game $G$, we denote the set of types by $\{H, L\}$, the $i$-type player’s payoff by $\pi_i(\cdot)$, and the set of pure-strategy perfect Bayesian equilibria by $PBE(G)$. The strategy profile $\sigma' \in PBE(G)$ lexicographically dominates (L-dominates) $\sigma \in PBE(G)$ if $\pi_L(\sigma') > \pi_L(\sigma)$, or $\pi_L(\sigma') = \pi_L(\sigma)$ and $\pi_H(\sigma') > \pi_H(\sigma)$. The strategy profile $\sigma \in PBE(G)$ is an LMSE if there does not exist $\sigma' \in PBE(G)$ that L-dominates $\sigma$.

Essentially, LMSE selects the most efficient (profitable) outcome from the perspective of the type of the informed player that has the most incentive to reveal his or her true identity. In AG’s setting, the LMSE outcome is the PBNE outcome that the low-type incumbent finds most profitable among all PBNE. The rationale for this refinement is fairly intuitive. Note that the high-type incumbent has an incentive to mimic the low-type incumbent but not vice versa, i.e., the low-type incumbent is the one with an incentive to reveal its identity. So, when both separating and pooling PBNE exist, if the low-type incumbent makes a higher profit under the pooling outcome than under the separating outcome whereas the high-type incumbent makes a higher profit under the separating outcome than under the pooling outcome, then the pooling PBNE would be selected, since the separating equilibrium can be realized only by adopting an “unreasonable” belief system.

If LMSE refinement is adopted in AG’s setting, the similar unique composite equilibrium will be achieved, which leads to the same qualitative results as in AG’s original paper. The only difference is that the low threshold ($q_p$) in the entrant’s belief system under the pooling equilibrium should be $q_p = \min\{q_{ip}^L, q_{ip}^H\}$, not $q_p = q_{ip}(ip)_{\text{min}}$ as specified in AG. This is because AG missed some incentive compatibility (IC) constraints in their analysis for the pooling equilibrium.